ERGODICITY OF FLUID SERVER QUEUEING SYSTEM IN RANDOM ENVIRONMENT

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ABSTRACT

There are sufficient conditions of the ergodicity for queuing systems in a random environment. But as theoretically so practically it is very important to obtain a criterion of the ergodicity which defines an ability to handle customers of these systems and a possibility to analyze them in a regime of heavy traffic. Among queuing systems in the random environment there are systems with the hysteresis control which are very important in modern applications. In this paper the criterion of the ergodicity is obtained for one server queuing system in the random environment. This criterion is based on a reduction of this queuing system to classical Lindley chain. Some asymptotic formulas in the heavy traffic regime are obtained for this queuing system also.

INTRODUCTION

Mathematical models of queuing systems and networks in the random environment attract an attention of specialists in the queuing theory (see for an example [1] and its bibliography) because of manifold applications to transport models [2, p. 430-432, 438] and systems with the hysteresis control [3], [4].

Deterministic models of technical systems with the hysteresis control (periodic systems close to discontinuous) are considered in the theory of ordinary differential equations with a small parameter under high-order derivatives [5], [6], [7]. But a presence of the small parameter in these models does not allow to obtain visible formulas for solutions of these equations. It is connected with sufficiently complicated behavior of their solutions - an availability of few adjacent boundary layers in vicinities of a discontinuous point.

At a same moment stochastic models of queuing systems in the random environment as a rule obtain solutions only in a form of sufficient not necessary and sufficient conditions [1, theorem 1, Formula (2)]. An importance of the ergodicity criteria is in their capability to define an ability to handle customers of queuing system [8]. So a work in this direction is actual in spite of an abundance of results in which there are formulas and algorithms of limit distributions calculations of queuing systems in the random environment.

In this paper the ergodicity criteria are obtained not by an reinforcement of known results of limit distributions calculations for queuing systems in the random environment but by a construction of sufficiently general stochastic models for queuing systems with a type of the Lindley chain [9, P. 20-36]. In frames of this approach a fluid model of one server queuing system [10], [11, p. 8-12] is considered and for this model an amount of a fluid in the system is defined in moments of their regime changes. Besides of the ergodicity criteria for the considered model asymptotic formulas for limit distributions in the heavy traffic regime are obtain also.

1. ERGODICITY CRITERIA

Consider the following fluid model of one server queuing system. Divide nonnegative half-axis $t \geq 0$ onto half-intervals $[T_0, T_1)$, $T_0 = 0$, $T_1 = T_0 + t_0$, $[T_1, T_2)$, $T_2 = T_1 + t_1, \ldots$ Here independent and identically distributed random variables (i.i.d.r.v.`s) $t_0, t_1, \ldots,$ have the distribution $G(t) = P(t_n < t)$, $t \geq 0$, $n \geq 0$, concentrated on the half-axis $t \geq 0$ and $Mt_n < \infty$. Assume that on the half-interval $[T_{n-1}, T_n)$, $n > 0$, some reservoir is replenished by a fluid with the intensity $a_n > 0$ and the fluid is pumped out with the intensity $b_n > 0$ if the fluid volume is positive. If the fluid volume is zero then for $a_n < b_n$ the outflow intensity becomes equal the inflow intensity a_n and the initial volume of the fluid in the reservoir equals $w_0 \geq 0$. Further suppose that the differences $(a_n - b_n)$, $n \geq 0$, characterizing random behavior of the environment in which the one server queuing system is situated is the sequence of i.i.d.r.v`s with the mean $M|a_n - b_n| < \infty$ and random sequences $(a_n - b_n)$, $n \geq 0$, and $t_n, n \geq 0,$ are independent.

Denote $W(t)$, $t \geq 0$, the fluid volume in the reservoir at the moment t. The function $W(t)$ is the polygonal line with the inflection points T_n, $n \geq 0$. This function is analogous to the virtual waiting time in the one server queuing system but it is not identical to it. Suppose that $w_n = W(T_n)$, $n \geq 0$, then from previous assumptions the fluid volume $w_{n+1} = W(T_{n+1})$ in the reservoir at the moment T_{n+1} satisfies the equality

$$w_{n+1} = (w_n + \xi_n)^+, \; n \geq 0, \text{ where } d^+ = \max(0, d). \tag{1}$$

From the ergodicity theorem for the Lindley chain $w_n, n \geq 0,$ [9, §3, theorem 7] the necessary and sufficient condition of its ergodicity is the inequality

$$M\xi_n = Mt_n M(a_n - b_n) < 0. \tag{2}$$

Remark 1. This ergodicity criterion is true for more general assumptions for a stationarity of the random sequence ξ_n, $n \geq 0$, in the narrow sense.

2 ASSIMPTOTIC ANALYSIS IN REGIME OF HEAVY TRAFFIC

Obtained results allow to transfer well known asymptotic formulas for the Lindley chain onto fluid one server queuing system in random environment which may be represented as the queuing system with the hysteresis control. If

$$c = |M\xi_n| \to 0, \; d = D\xi_n = const,$$

then in the condition $M|\xi_n|^3 < \infty$ (9,[chapter 1, formulas (57), (58)],[13]) we have well known asymptotic formula for the limit distribution of the Markov chain w_n, $n \geq 0$: for any $x > 0$

$$\lim_{n\to\infty} P(w_n > x/|c|) \sim \exp(-2x/d), \; |c| \to 0.$$

Refinements of these results may be found in [9, p. 65-67],[14,chapter. III]. These refinements are based on the diffusion approximation of the random sequence (1).

In the conclusion consider the case when $c \to 0$, $d = d(c)$. Assume that the random variables ξ_n satisfy the following conditions. There is the sequence of i.i.d.r.v`s Δ_n, $n \geq 0$,

$$M\Delta_n = 0\,,\ D\Delta_n = f\,,\ M|\Delta_n|^3 < \infty\,,$$

so that $\xi_n = -\varepsilon + \varepsilon^{\gamma}\Delta_n$, $n \ge 0$, and consequently $c = -\varepsilon$, $d = f\varepsilon^{2\gamma} = f|c|^{2\gamma}$. Define the random variable $R_\gamma = R_\gamma(\varepsilon)$ by the equality

$$\lim_{n\to\infty} P(w_n > x) = P(R_\gamma > x)\,,\ x > 0\,.$$

Then from the theorem [15, theorem 1] for $\varepsilon \to 0$, $x > 0$, the following relations are true

$$R_\gamma \to +\infty\,,\ 0 \le \gamma < 1/2\,;\ R_\gamma \to 0\,, \gamma > 1/2\,; P(R_\gamma > x) \to \exp(-2x/f)\,, \gamma = 1/2\,.$$

Remark 2. A reduction of the constructed model of the one server fluid queuing system in the random environment to the Lindley chain allows to transfer onto this model known results on the stability of limit and prelimit distributions (see for an example, [9,§20], [16],[17, chapters V. VI]).

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