MODELING SAFETY OF MULTISTATE SYSTEMS WITH APPLICATION TO MARITIME FERRY TECHNICAL SYSTEM

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ABSTRACT

Basic notions of the ageing multistate systems safety analysis are introduced. The system components and the system safety functions are defined. The mean values and variances of the multistate system lifetimes in the safety state subsets and the mean values of its lifetimes in the particular safety states are defined. The notions of the multi-state system risk function and the moment of exceeding by the system the critical safety state are introduced. A series and a parallel-series safety structures of the multistate systems with ageing components are defined and their safety function are determined. As a particular case, the safety functions of the considered multi-state systems composed of components having exponential safety functions are determined. An applications of the proposed multistate system safety models to the prediction of safety characteristics of a maritime ferry operating at winter conditions technical system is presented as well.

1 INTRODUCTION

Taking into account the importance of the safety and operating process effectiveness of real technical systems it seems reasonable to expand the two-state approach to multi-state approach (Kolowrocki, 2004; Kolowrocki, Soszynska-Budny, 2011; Kolowrocki, Soszynska-Budny, 2012) in system safety analysis. The assumption that the systems are composed of multi-state components with safety states degrading in time (Kolowrocki, 2004; Kolowrocki, Soszynska-Budny, 2011; Kolowrocki, Soszynska-Budny, 2012) gives the possibility for more precise analysis of their safety and operational processes' effectiveness. This assumption allows us to distinguish a system safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operation process effectiveness. Then, an important system safety characteristic is the time to the moment of exceeding the system safety critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state safety function that is a basic characteristics of the multi-state system. The safety models of the considered here typical multistate system structures can be applied in the safety analysis of real complex technical systems. They may be successfully applied, for instance, to safety analysis, identification, prediction and optimization of the maritime transportation systems.

2 MULTISTATE APPROACH TO SAFETY ANALYSIS

In the multistate safety analysis to define a system composed of n, $n \in N$, ageing components we assume that:

- E_i, $i = 1,2,\ldots,n$, are components of a system,
- all components and a system under consideration have the set of safety states $\{0,1,...,z\}$, $z \geq 1$,
- the safety states are ordered, the state 0 is the worst and the state z is the best,
- the component and the system safety states degrade with time t,

- $T_i(u)$, $i = 1,2,\dots,n$, $n \in N$, are independent random variables representing the lifetimes of components E_i in the safety state subset $\{u, u+1, ..., z\}$, while they were in the safety state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the safety state subset $\{u, u+1, ..., z\}$, while it was in the safety state z at the moment $t = 0$,
- $s_i(t)$ is a component E_i safety state at the moment t, $t \in <0,\infty)$, given that it was in the safety state z at the moment $t = 0$,
- $s(t)$ is the system safety state at the moment t, $t \in <0,\infty)$, given that it was in the safety state z at the moment $t = 0$.

The above assumptions mean that the safety states of the ageing system and components may be changed in time only from better to worse.

Definition 1. A vector

$$S_i(t,\cdot) = [S_i(t,0), S_i(t,1), \dots, S_i(t,z)] \tag{1}$$

for $t \in <0,\infty)$, $i = 1,2,\dots,n,$ where

$$S_i(t,u) = P(s_i(t) \geq u \mid s_i(0) = z) = P(T_i(u) > t) \tag{2}$$

for $t \in <0,\infty)$, $u = 0,1,\dots,z,$ is the probability that the component E_i is in the safety state subset $\{u, u+1, ..., z\}$ at the moment t, $t \in <0,\infty)$, while it was in the safety state z at the moment $t = 0$, is called the multistate safety function of a component E_i.

Definition 2. A vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,0), \boldsymbol{S}(t,1), \dots, \boldsymbol{S}(t,z)],\ t \in <0,\infty), \tag{3}$$

where

$$\boldsymbol{S}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t) \tag{4}$$

for $t \in <0,\infty)$, $u = 0,1,...,z$, is the probability that the system is in the safety state subset $\{u, u+1, ..., z\}$ at the moment t, $t \in <0,\infty)$, while it was in the safety state z at the moment $t = 0$, is called the multi-state safety function of a system.

The safety functions $S_i(t,u)$ and $\boldsymbol{S}(t,u)$, $t \in <0,\infty)$, $u = 0,1,...,z$, defined by (2) and (4) are called the coordinates of the components and the system multistate safety functions $S_i(t,\cdot)$ and $\boldsymbol{S}(t,\cdot)$ given by respectively (1) and (3). It is clear that from Definition 1 and Definition 2, for $u = 0,$ we have

$S_i(t,0) = 1$ and $\boldsymbol{S}(t,0) = 1$.

The mean lifetime of the system in the safety state subset $\{u, u+1, ..., z\}$ is defined by

$$\mu(u) = \int_0^\infty \boldsymbol{S}(t,u)dt,\ u = 1,2,...,z, \tag{5}$$

and the standard deviation of the system lifetime in the safety state subset $\{u, u+1, ..., z\}$ is given by

$$\sigma(u) = \sqrt{n(u) - [\mu(u)]^2}\ , \ u = 1,2,...,z, \tag{6}$$

where

$$n(u) = 2\int_0^\infty t\, \boldsymbol{S}(t,u)dt,\ u = 1,2,...,z. \tag{7}$$

Moreover, the mean lifetimes of the system in the safety state u, $u = 1,2,...,z$,

$$\bar{\mu}(u) = \int_0^\infty p(t,u)dt,\ u = 1,2,...,z, \tag{8}$$

where

$$p(t,u) = P(\mathrm{s}(t) = u \mid s(0) = z) = \boldsymbol{S}(t,u) - \boldsymbol{S}(t, u+1),$$

for $u = 0,1,...,z-1$, $t \in < 0,\infty)$, can be found from the following relationships (Kolowrocki, Soszynska-Budny, 2011)

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ u = 0,1,...,z-1,\ \bar{\mu}(z) = \mu(z). \tag{9}$$

Definition 3. A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \le t),\ t \in < 0,\infty), \tag{10}$$

that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the safety state z at the moment t = 0 is called a risk function of the multi-state system (Kolowrocki, Soszynska-Budny, 2011).
Under this definition, from (4), we have

$$\boldsymbol{r}(t) = 1 - P(s(t) \ge r \mid s(0) = z) = 1 - \boldsymbol{S}(t,r),\ t \in < 0,\infty), \tag{11}$$

and if τ is the moment when the system risk exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{12}$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the system risk function $\boldsymbol{r}(t)$.

3 SAFETY OF SERIES AND PARALLEL-SERIES SYSTEMS

Now, after introducing the notion of the multistate safety analysis, we may define basic multi-state safety structures.
Definition 4. A multistate system is called series if its lifetime $T(u)$ in the safety state subset $\{u, u+1,..., z\}$ is given by

$$T(u) = \min_{1 \le i \le n}\{T_i(u)\},\ u = 1,2,...,z.$$

The number n is called the system structure shape parameter.
The above definition means that a multi-state series system is in the safety state subset $\{u, u+1, ..., z\}$ if and only if all its n components are in this subset of safety states. That meaning is very close to the definition of a two-state series system considered in a classical reliability analysis that is not failed if all its components are not failed. This fact can justify the safety structure scheme for a multistate series system presented in Figure. 1.

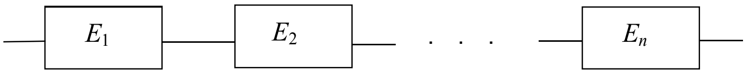

Figure 1. The scheme of a series system safety structure

It is easy to work out that the safety function of the multi-state series system is given by the vector (Kolowrocki, Soszynska-Budny, 2011)

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)] \tag{13}$$

with the coordinates

$$\boldsymbol{S}(t,u) = \prod_{i=1}^{n} S_i(t,u), \ t \in <0,\infty), \ u = 1,2,...,z. \tag{14}$$

Hence, if the system components have exponential safety functions, i.e.

$$S_i(t,\cdot) = [1, S_i(t,1), \ldots, S_i(t,z)], \ t \in <0,\infty), \ i = 1,2,\ldots,n, \tag{15}$$

where

$$S_i(t,u) = \exp[-\lambda_i(u)t], \ t \in <0,\infty), \ u = 1,2,\ldots,z, \ i = 1,2,\ldots,n, \tag{16}$$

the formula (14) takes the following form

$$\boldsymbol{S}(t,u) = \prod_{i=1}^{n} \exp[-\lambda_i(u)t], \ t \in <0,\infty), \ u = 1,2,...,z. \tag{17}$$

Definition 5. A multistate system is called parallel-series if its lifetime $T(u)$ in the safety state subset $\{u, u+1, ..., z\}$ is given by

$$T(u) = \min_{1 \le i \le k}\{\max_{1 \le j \le l_i}\{T_{ij}(u)\}\}, \ u = 1,2,...,z.$$

The above definition means that the multistate parallel-series system is composed of k multistate parallel subsystems and it is in the safety state subset $\{u, u+1, ..., z\}$ if and only if all its k parallel subsystems are in this safety state subset. In this definition l_i, $i = 1,2,...,k,$ denote the numbers of components in the parallel subsystems. The numbers k and $l_1, l_2, ..., l_k$ are called the system structure shape parameters. The scheme of a multistate parallel-series system given in Figure 2.

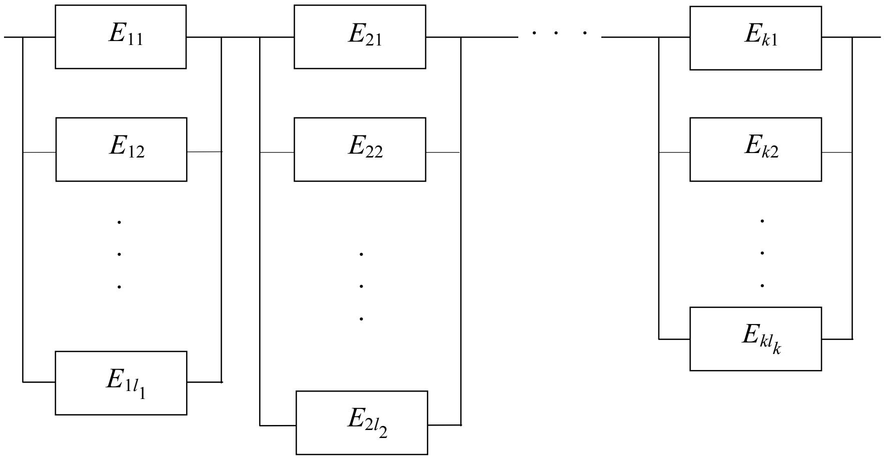

Figure 2. The scheme of a parallel-series system

The safety function of the multi-state parallel-series system is given by the vector (Kolowrocki, Soszynska-Budny, 2011)

$$\boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,\cdot) = [1, \boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,1), ..., \boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,z)], \tag{18}$$

with the coordinates

$$\boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,u) = \prod_{i=1}^{k}[1 - \prod_{j=1}^{l_i}[1 - S_{ij}(t,u)]], \ t \in < 0, \infty), \ u = 1,2,...,z, \tag{19}$$

where k is the number of its parallel subsystems linked in series and l_i, $i = 1,2,...,k$, are the numbers of components in the parallel subsystems.
Hence, if the system components have exponential safety functions, i.e.

$$S_{ij}(t,\cdot) = [1, S_{ij}(t,1), \ldots, S_{ij}(t,z)], \ \ t \in < 0, \infty), \ i = 1,2,...,k, j = 1,2,...,l_i, \tag{20}$$

where

$$S_i(t,u) = \exp[-\lambda_{ij}(u)t], \ \ t \in < 0, \infty), \ u = 1,2,\ldots,z, \ \ i = 1,2,...,k, j = 1,2,...,l_i, \tag{21}$$

the formula (19) takes the following form

$$\boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,u) = \prod_{i=1}^{k}[1 - \prod_{j=1}^{l_i}[1 - \exp[-\lambda_{ij}(u)t]], \ t \in < 0, \infty), \ u = 1,2,...,z. \tag{22}$$

4 SAFETY OF MARITME FERRY TECHNICAL SYSTEM

The considered maritime ferry is a passenger Ro-Ro ship operating at the Baltic Sea between Gdynia and Karlskrona ports on regular everyday line. We assume that the ferry is composed of a number of main subsystems having an essential influence on its safety. These subsystems are illustrated in Figure 3 and Figure 4.

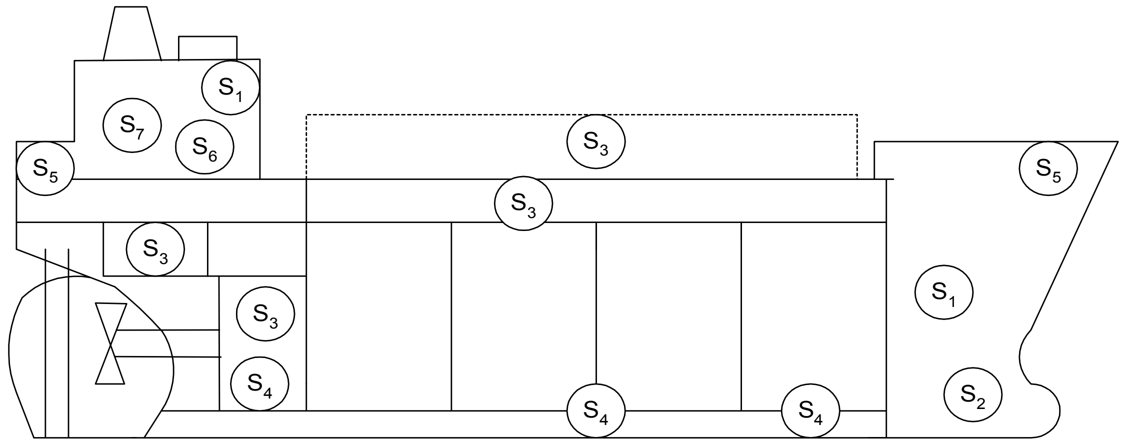

Figure 3. Subsystems having an essential influence on the ferry safety

On the scheme of the ferry presented in Figure 3, there are distinguished its following subsystems:
S_1 - a navigational subsystem,
S_2 - a propulsion and controlling subsystem,
S_3 - a loading and unloading subsystem,
S_4 - a stability control subsystem,
S_5 - an anchoring and mooring subsystem,
S_6 - a protection and rescue subsystem,
S_7 - a social subsystem.
In the safety analysis of the ferry, we omit the protection and rescue subsystem s_6 and the social subsystem s_7 and we consider its strictly technical subsystems S_1, S_2, S_3, S_4 and S_5 only, further called the ferry technical system.

The navigational subsystem S_1 is composed of one general component $E_{11}^{(1)}$, that is equipped with GPS, AIS, speed log, gyrocompass, magnetic compass, echo sounding system, paper and electronic charts, radar, ARPA, communication system and other subsystems.
The propulsion and controlling subsystem S_2 is composed of :
- the subsystem S_{21} which consist of 4 main engines $E_{11}^{(2)}$, $E_{12}^{(2)}$, $E_{13}^{(2)}$, $E_{14}^{(2)}$;
- the subsystem S_{22} which consist of 3 thrusters $E_{21}^{(2)}$, $E_{22}^{(2)}$, $E_{31}^{(2)}$;
- the subsystem S_{23} which consist of twin pitch propellers $E_{41}^{(2)}$, $E_{51}^{(2)}$;
- the subsystem S_{24} which consist of twin directional rudders $E_{61}^{(2)}$, $E_{71}^{(2)}$.

The loading and unloading subsystem S_3 is composed of :
- the subsystem S_{31} which consist of 2 remote upper trailer decks to main deck $E_{11}^{(3)}$, $E_{21}^{(3)}$;
- the subsystem S_{32} which consist of 1 remote fore car deck to main deck $E_{31}^{(3)}$;
- the subsystem S_{33} which consist of passenger gangway to Gdynia Terminal $E_{41}^{(3)}$;
- the subsystem S_{34} which consist of passenger gangway to Karlskrona Terminal $E_{51.}^{(3)}$.

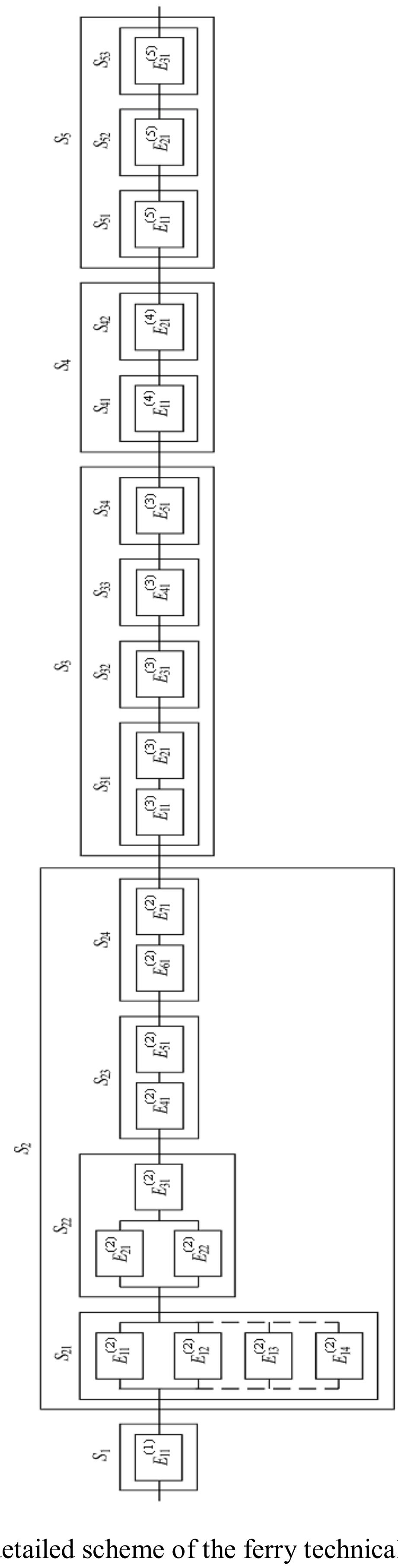

Figure 4. The detailed scheme of the ferry technical system structure

The stability control subsystem S_4 is composed of :

- the subsystem S_{41} which consist of an anti-heeling system $E_{11}^{(4)}$, which is used in port during loading operations;
- the subsystem S_{42} which consist of an anti-heeling system $E_{21}^{(4)}$, which is used at sea to stabilizing ships rolling.

The anchoring and mooring subsystem S_5 is composed of :

- the subsystem S_{51} which consist of aft mooring winches $E_{11}^{(5)}$;
- the subsystem S_{52} which consist of fore mooring and anchor winches $E_{21}^{(5)}$;
- the subsystem S_{53} which consist of fore mooring winches $E_{31}^{(5)}$.

The detailed scheme of these subsystems and components is illustrated in Figure 4.
The subsystems S_1, S_2, S_3, S_4, S_5, indicated in Figure 4 are forming a general series safety structure of the ferry technical system presented in Figure 5.

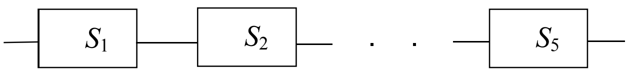

Figure 5. The general scheme of the ferry technical system safety structure

After discussion with experts, taking into account the safety of the operation of the ferry, we distinguish the following five safety states ($z = 4$) of the ferry technical system and its components:

- a safety state 4 – the ferry operation is fully safe,
- a safety state 3 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution,
- a safety state 2 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution and causing small accidents,
- a safety state 1 - the ferry operation is much less safe and much more dangerous because of the possibility of serious environment pollution and causing extensive accidents,
- a safety state 0 – the ferry technical system is is destroyed.

Moreover, by the expert opinions, we assume that there are possible the transitions between the components' safety states only from better to worse ones and we assume that the system and its components critical safety state is $r = 2$.
From the above, the subsystems S_υ, $\upsilon = 1,2,3,4,5,$ are composed of five-state, i.e. z = 4, components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,3,4,5,$ having the safety functions

$$S_{ij}^{(\upsilon)}(t,\cdot) = [1,\ S_{ij}^{(\upsilon)}(t,1), S_{ij}^{(\upsilon)}(t,2), S_{ij}^{(\upsilon)}(t,3), S_{ij}^{(\upsilon)}(t,4)],$$

with the coordinates that by the assumption are exponential of the forms

$$S_{ij}^{(\upsilon)}(t,1) = \exp[-\lambda_{ij}^{(\upsilon)}(1)t],\ S_{ij}^{(\upsilon)}(t,2) = \exp[-\lambda_{ij}^{(\upsilon)}(2)t],$$

$$S_{ij}^{(\upsilon)}(t,3) = \exp[-\lambda_{ij}^{(\upsilon)}(3)t],\ S_{ij}^{(\upsilon)}(t,4) = \exp[-\lambda_{ij}^{(\upsilon)}(4)t].$$

The subsystem S_1 consists of one component $E_{ij}^{(1)}$, $i=1$, $j=1$, i.e. we may consider it either as a series system composed of $n=1$ components or for instance as a parallel-series system with parameters $k=1$, $l_1=1$, with the exponential safety functions on the basis of data coming from experts and given below.
The coordinates of the subsystem S_1 component five-state safety function are:

$$S_{11}^{(1)}(t,1) = \exp[-0.033t],\ S_{11}^{(1)}(t,2) = \exp[-0.04t],$$

$$S_{11}^{(1)}(t,3) = \exp[-0.045t],\ S_{11}^{(1)}(t,4) = \exp[-0.05t].$$

Thus, the subsystem S_1 safety function is identical with the safety function of its component, i.e.

$$\boldsymbol{S}^{(1)}(t,\cdot) = [1,\ \boldsymbol{S}^{(1)}(t,1),\ \boldsymbol{S}^{(1)}(t,2),\ \boldsymbol{S}^{(1)}(t,3),\ \boldsymbol{S}^{(1)}(t,4)],\quad t \in <0,\infty), \tag{23}$$

where, according to the formulae (18)-(19), we have

$$\boldsymbol{S}^{(1)}(t,u) = \boldsymbol{S}_{1;1}(t,u) = \prod_{i=1}^{1}[1-\prod_{j=1}^{1}[1-S_{ij}^{(1)}(t,u)]] = S_{11}^{(1)}(t,u),\ t \in <0,\infty),\ u = 1,2,3,4, \tag{24}$$

and particularly

$$\boldsymbol{S}^{(1)}(t,1) = \boldsymbol{S}_{1;1}(t,1) = \exp[-0.033t], \tag{25}$$

$$\boldsymbol{S}^{(1)}(t,2) = \boldsymbol{S}_{1;1}(t,2) = \exp[-0.04t], \tag{26}$$

$$\boldsymbol{S}^{(1)}(t,3) = \boldsymbol{S}_{1;1}(t,3) = \exp[-0.045t], \tag{27}$$

$$\boldsymbol{S}^{(1)}(t,4) = \boldsymbol{S}_{1;1}(t,4) = \exp[-0.05t]. \tag{28}$$

The subsystem S_2 is a five-state parallel-series system composed of components $E_{ij}^{(2)}$, $i=1,2,...,k$, $j=1,2,...,l_i$, $k=7$, $l_1=4$, $l_2=2$, $l_3=1$, $l_4=1$, $l_5=1$, $l_6=1$, $l_7=1$, with the exponential safety functions identified on the basis of data coming from experts given below. The coordinates of the subsystem S_2 components' five-state safety functions are:

$$S_{1j}^{(2)}(t,1) = \exp[-0.033t],\ S_{1j}^{(2)}(t,2) = \exp[-0.04t],$$
$$S_{1j}^{(2)}(t,3) = \exp[-0.05t],\ S_{1j}^{(2)}(t,4) = \exp[-0.055t],\ j=1,2,3,4,$$

$$S_{2j}^{(2)}(t,1) = \exp[-0.066t],\ S_{2j}^{(2)}(t,2) = \exp[-0.07t],$$
$$S_{2j}^{(2)}(t,3) = \exp[-0.075t],\ S_{2j}^{(2)}(t,4) = \exp[-0.08t],\ j=1,2,$$

$$S_{31}^{(2)}(t,1) = \exp[-0.066t],\ S_{31}^{(2)}(t,2) = \exp[-0.07t],$$
$$S_{31}^{(2)}(t,3) = \exp[-0.075t],\ S_{31}^{(2)}(t,4) = \exp[-0.08t],$$

$$S_{i1}^{(2)}(t,1) = \exp[-0.033t],\ S_{i1}^{(2)}(t,2) = \exp[-0.04t],$$
$$S_{i1}^{(2)}(t,3) = \exp[-0.045t],\ S_{i1}^{(2)}(t,4) = \exp[-0.05t],\ i = 4,5,6,7.$$

Hence, according to the formulae (18)-(19), the subsystem S_2 safety function is given by

$$\boldsymbol{S}^{(2)}(t,\cdot) = [1,\ \boldsymbol{S}^{(2)}(t,1),\ \boldsymbol{S}^{(2)}(t,2),\ \boldsymbol{S}^{(2)}(t,3),\ \boldsymbol{S}^{(2)}(t,4)],\quad t \in <0,\infty), \tag{29}$$

where

$$\boldsymbol{S}^{(2)}(t,u) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,u) = \prod_{i=1}^{7}[1-\prod_{j=1}^{l_i}[1-s_{ij}^{(2)}(t,u)]],\ t \in <0,\infty),\ u = 1,2,3,4, \tag{30}$$

and particularly

$$\boldsymbol{S}^{(2)}(t,1) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,1) = 6[\exp[-0.033t]]^2[1-\exp[-0.033t]]^2$$

$$+ 4[\exp[-0.033t]]^3[1-\exp[-0.033t]] + [\exp[-0.033t]]^4\ ]\ [1 - [1 - \exp[-0.066t]]^2]\exp[-0.066t]$$

$$\exp[-0.033t]\exp[-0.033t]\ \exp[-0.033t]\ \exp[-0.033t]$$

$$= 12\exp[-0.33t] + 8\exp[-0.429t] - 16\exp[-0.363t] - 3\exp[-0.462t] \tag{31}$$

$$\boldsymbol{S}^{(2)}(t,2) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,2) = [\ 6[\exp[-0.04t]]^2[1-\exp[-0.04t]]^2 + 4[\exp[-0.04t]]^3[1-\exp[-0.04t]]$$

$$+ [\exp[-0.04t]]^4\ ]\ [1 - [1 - \exp[-0.07t]]^2]\exp[-0.07t]\ \exp[-0.04t]\exp[-0.04t]\ \exp[-0.04t]\ \exp[-0.04t]$$

$$= 12\exp[-0.38t] + 8\exp[-0.49t] + 6\exp[-0.46t] - 16\exp[-0.42t] - 6\exp[-0.45t] - 3\exp[-0.53t] \tag{32}$$

$$\boldsymbol{S}^{(2)}(t,3) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,3) = 6[\exp[-0.05t]]^2[1-\exp[-0.05t]]^2$$

$$+ 4[\exp[-0.05t]]^3[1-\exp[-0.05t]] + [\exp[-0.05t]]^4\ ]\ [1 - [1 - \exp[-0.075t]]^2]\exp[-0.075t]$$

$$\exp[-0.045t]\exp[-0.045t]\ \exp[-0.045t]\ \exp[-0.045t]$$

$$= 12\exp[-0.43t] + 8\exp[-0.555t] + 6\exp[-0.53t] - 16\exp[-0.48t]$$

$$- 6\exp[-0.505t] - 3\exp[-0.605t] \tag{33}$$

$$\boldsymbol{S}^{(2)}(t,4) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,4) = 6[\exp[-0.055t]]^2[1-\exp[-0.055t]]^2$$

$$+ 4[\exp[-0.055t]]^3[1-\exp[-0.055t]] + [\exp[-0.055t]]^4\ ]$$

$$[1 - [1 - \exp[-0.08t]]^2]\exp[-0.08t]\ \exp[-0.05t]\exp[-0.05t]\ \exp[-0.05t]\ \exp[-0.05t]$$

$$= 12\exp[-0.47t] + 8\exp[-0.605t] + 6\exp[-0.58t] - 16\exp[-0.525t]$$

$$- 6 \exp[-0.55t] - 3 \exp[-0.66t]. \tag{34}$$

The subsystem S_3 is a five-state series system composed of $n = 5$ components that can also be considered as a parallel-series system composed of components $E_{ij}^{(3)}$, $i = 1,2,...,k$, $j = l_i$, $k = 5$, $l_1 = 1$, $l_2 = 1$, $l_3 = 1$, $l_4 = 1$, $l_5 = 1$, with the exponential safety functions identified on the basis of data coming from experts given below. The coordinates of the subsystem S_3 components' five-state safety functions are:

$$S_{11}^{(3)}(t,1) = \exp[-0.02t],\ S_{11}^{(3)}(t,2) = \exp[-0.03t],$$
$$S_{11}^{(3)}(t,3) = \exp[-0.035t],\ S_{11}^{(3)}(t,4) = \exp[-0.04t],$$

$$S_{21}^{(3)}(t,1) = \exp[-0.02t],\ S_{21}^{(3)}(t,2) = \exp[-0.025t],$$
$$S_{21}^{(3)}(t,3) = \exp[-0.03t],\ S_{21}^{(3)}(t,4) = \exp[-0.04t],$$

$$S_{31}^{(3)}(t,1) = \exp[-0.033t],\ S_{31}^{(3)}(t,2) = \exp[-0.04t],$$
$$S_{31}^{(3)}(t,3) = \exp[-0.045t],\ S_{31}^{(3)}(t,4) = \exp[-0.05t],$$

$$S_{41}^{(3)}(t,1) = \exp[-0.033t],\ S_{41}^{(3)}(t,2) = \exp[-0.04t],$$
$$S_{41}^{(3)}(t,3) = \exp[-0.045t],\ S_{41}^{(3)}(t,4) = \exp[-0.05t],$$

$$S_{51}^{(3)}(t,1) = \exp[-0.033t],\ S_{51}^{(3)}(t,2) = \exp[-0.04t],$$
$$S_{51}^{(3)}(t,3) = \exp[-0.045t],\ S_{51}^{(3)}(t,4) = \exp[-0.05t],$$

Hence, according to the formulae (18)-(19), the subsystem S_3 five-state safety function is given by

$$\boldsymbol{S}^{(3)}(t,\cdot) = [1,\ \boldsymbol{S}^{(3)}(t,1),\ \boldsymbol{S}^{(3)}(t,\ 2),\ \boldsymbol{S}^{(3)}(t,\ 3),\ \boldsymbol{S}^{(3)}(t,\ 4)],\ t \in <0,\infty), \tag{35}$$

where

$$\boldsymbol{S}^{(3)}(t,\ u) = \boldsymbol{S}_{5;1,1,1,1,1}(t,u) = \prod_{i=1}^{5}[1-\prod_{j=1}^{1}[1-S_{ij}(t,u)]] = \prod_{i=1}^{5} S_{i1}(t,u)\ ,\ t \in <0,\infty),\ u = 1,2,3,4, \tag{36}$$

and particularly

$$\boldsymbol{S}^{(3)}(t,1) = \boldsymbol{S}_{5;1,1,1,1,1}(t,1) = \exp[-0.02t] \exp[-0.02t] \exp[-0.033t] \exp[-0.033] \exp[-0.033]$$

$$= \exp[-0.139t], \tag{37}$$

$$\boldsymbol{S}^{(3)}(t,\ 2) = \boldsymbol{S}_{5;1,1,1,1,1}(t,2) = \exp[-0.03t] \exp[-0.025t] \exp[-0.04t] \exp[-0.04t] \exp[-0.04t]$$

$$= \exp[-0.175t], \tag{38}$$

$$\boldsymbol{S}^{(3)}(t,\ 3) = \boldsymbol{S}_{5;1,1,1,1,1}(t,3) = \exp[-0.035t] \exp[-0.03t] \exp[-0.045t] \exp[-0.045t] \exp[-0.045t]$$

$$= \exp[-0.200t], \quad (39)$$

$$\boldsymbol{S}^{(3)}(t,\ 4) = \boldsymbol{S}_{5;1,1,1,1,1}(t,\ 4) = \exp[-0.04t]\ \exp[-0.04t]\ \exp[-0.05t]\ \exp[-0.05t]\ \exp[-0.05t]$$

$$= \exp[-0.230t], \quad (40)$$

The subsystem S_4 is a five-state series system composed of $n = 2$ components that can also be considered as a parallel-series system composed of components $E_{ij}^{(4)}$, $i = 1,...,k$, $j = l_i$, $k = 2$, $l_1 = 1$, $l_2 = 1$, with the exponential safety functions identified on the basis of data coming from experts and given below. The coordinates of the subsystem S_4 components' multi-state safety functions are:

$$S_{11}^{(4)}(t,1) = \exp[-0.05t],\ S_{11}^{(4)}(t,2) = \exp[-0.06t],$$
$$S_{11}^{(4)}(t,3) = \exp[-0.065t],\ S_{11}^{(4)}(t,4) = \exp[-0.07t],$$

$$S_{21}^{(4)}(t,1) = \exp[-0.033t],\ S_{21}^{(4)}(t,2) = \exp[-0.04t],$$
$$S_{21}^{(4)}(t,3) = \exp[-0.045t],\ S_{21}^{(4)}(t,4) = \exp[-0.05t].$$

Hence, according to the formulae (18)-(19), the subsystem S_4 five-state safety function is given by

$$\boldsymbol{S}^{(4)}(t,\cdot) = [1,\ \boldsymbol{S}^{(4)}(t,1),\ \boldsymbol{S}^{(4)}(t,2),\ \boldsymbol{S}^{(4)}(t,\ 3),\ \boldsymbol{S}^{(4)}(t,4)],\ t \in <0,\infty), \quad (41)$$

where

$$\boldsymbol{S}^{(4)}(t,u) = \boldsymbol{S}_{2;1,1}(t,u) = \prod_{i=1}^{2}[1 - \prod_{j=1}^{1}[1 - S_{ij}(t,u)]] = \prod_{i=1}^{2} S_{ij}(t,u),\quad t \in <0,\infty),\quad u = 1,2,3,4, \quad (42)$$

and particularly

$$\boldsymbol{S}^{(4)}(t,1) = \boldsymbol{S}_{2;1,1}(t,1) = \exp[-0.05t]\ \exp[-0.033t] = \exp[-0.083t], \quad (43)$$

$$\boldsymbol{S}^{(4)}(t,2) = \boldsymbol{S}_{2;1,1}(t,2) = \exp[-0.06t]\ \exp[-0.04t] = \exp[-0.100t], \quad (44)$$

$$\boldsymbol{S}^{(4)}(t,\ 3) = \boldsymbol{S}_{2;1,1}(t,\ 3) = \exp[-0.065t]\ \exp[-0.045t] = \exp[-0.110t] \quad (45)$$

$$\boldsymbol{S}^{(4)}(t,4) = \boldsymbol{S}_{2;1,1}(t,4) = \exp[-0.07t]\ \exp[-0.05t] = \exp[-0.120t]. \quad (46)$$

The subsystem S_5 is a five-state series system composed of $n = 3$ components that can also be considered as a parallel-series system composed of components $E_{ij}^{(5)}$, $i = 1,2,..,k$, $j = l_i$, $k = 3$, $l_1 = 1$, $l_2 = 1$, $l_3 = 1$, with the exponential safety functions identified on the basis of data coming from experts given below. The coordinates of the subsystem S_5 components' five-state safety functions are:

$$S_{11}^{(5)}(t,1) = \exp[-0.033t],\ S_{11}^{(5)}(t,2) = \exp[-0.04t],$$
$$S_{11}^{(5)}(t,3) = \exp[-0.045t],\ S_{11}^{(5)}(t,4) = \exp[-0.05t],$$

$$S_{21}^{(5)}(t,1) = \exp[-0.033t],\ S_{21}^{(5)}(t,2) = \exp[-0.04t],$$
$$S_{21}^{(5)}(t,3) = \exp[-0.05t],\ S_{21}^{(5)}(t,4) = \exp[-0.055t],$$

$$S_{31}^{(5)}(t,1) = \exp[-0.033t],\ S_{31}^{(5)}(t,2) = \exp[-0.04t],$$
$$S_{31}^{(5)}(t,3) = \exp[-0.05t],\ S_{31}^{(5)}(t,4) = \exp[-0.06t].$$

Hence, according to the formulae (18)-(19), the subsystem S_5 five-state safety function is given by

$$\boldsymbol{S}^{(5)}(t,\cdot) = [1,\ \boldsymbol{S}^{(5)}(t,1),\ \boldsymbol{S}^{(5)}(t,2),\ \boldsymbol{S}^{(5)}(t,3),\ \boldsymbol{S}^{(5)}(t,4)],\quad t \in <0,\infty), \tag{47}$$

where

$$\boldsymbol{S}^{(5)}(t,u) = \boldsymbol{S}_{3;1,1,1}(t,u) = \prod_{i=1}^{3}[1-\prod_{j=1}^{1}[1-S_{ij}(t,u)]] = \prod_{i=1}^{3} S_{i1}(t,u),\ t \in <0,\infty),\ u = 1,2,3,4, \tag{48}$$

and particularly

$$\boldsymbol{S}^{(5)}(t,1) = \boldsymbol{S}_{3;1,1,1}(t,1) = \exp[-0.033t]\ \exp[-0.033t]\ \exp[-0.033t] = \exp[-0.099t], \tag{49}$$

$$\boldsymbol{S}^{(5)}(t,2) = \boldsymbol{S}_{3;1,1,1}(t,2) = \exp[-0.04t]\ \exp[-0.04t]\ \exp[-0.04t] = \exp[-0.12t], \tag{50}$$

$$\boldsymbol{S}^{(5)}(t,3) = \boldsymbol{S}_{3;1,1,1}(t,3) = \exp[-0.045t]\ \exp[-0.05t]\ \exp[-0.05t] = \exp[-0.145t], \tag{51}$$

$$\boldsymbol{S}^{(5)}(t,4) = \boldsymbol{S}_{3;1,1,1}(t,4) = \exp[-0.05t]\ \exp[-0.055t]\ \exp[-0.06t] = \exp[-0.165t]. \tag{52}$$

Considering that the ferry technical system is a five-state series system, after applying (13)–(14), its safety function is given by

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \geq 0, \tag{53}$$

where by (25)-(28), (31)-(34), (37)-(40), (43)-(46) and (49)-(52), we have

$$\boldsymbol{S}(t,u) = \boldsymbol{S}_5(t,u) = \boldsymbol{S}^{(1)}(t,u)\ \boldsymbol{S}^{(2)}(t,u)\ \boldsymbol{S}^{(3)}(t,u)\ \boldsymbol{S}^{(4)}(t,u)\ \boldsymbol{S}^{(5)}(t,u) \text{ for } u = 1,2,3,4,$$

and particularly

$$\boldsymbol{S}(t,1) = \exp[-0.033t]\ [12\exp[-0.33t] + 8\exp[-0.429t] - 16\exp[-0.363t]$$
$$- 3\exp[-0.462t]]\ \exp[-0.139t]\ \exp[-0.083t]\ \exp[-0.099t]$$
$$= 12\exp[-0.684t] + 8\exp[-0.783t] - 16\exp[-0.717t] - 3\exp[-0.816t], \tag{54}$$

$$\boldsymbol{S}(t,2) = \exp[-0.040t]\ [12\exp[-0.38t] + 8\exp[-0.49t] + 6\exp[-0.46t]$$

$$- 16\exp[-0.42t] - 6\exp[-0.45t] - 3\exp[-0.53t]]\ \exp[-0.175t]\ \exp[-0.100t]\ \exp[-0.12t]$$

$$= 12\exp[-0.815t] + 8\exp[-0.925t] + 6\exp[-0.895t]$$

$$- 16\exp[-0.855t] - 6\exp[-0.885t] - 3\exp[-0.965t], \quad (55)$$

$$\boldsymbol{S}(t,3) = \exp[-0.045t]\ [12\exp[-0.43t] + 8\exp[-0.555t] + 6\exp[-0.53t]$$

$$- 16\exp[-0.48t] - 6\exp[-0.505t] - 3\exp[-0.605t]]\ \exp[-0.200t]\exp[-0.110t]\ \exp[-0.145t]$$

$$= 12\exp[-0.930t] + 8\exp[-1.055t] + 6\exp[-1.030t]$$

$$- 16\exp[-0.980t] - 6\exp[-1.005t] - 3\exp[-1.105t], \quad (56)$$

$$\boldsymbol{S}(t,4) = \exp[-0.05t][12\exp[-0.47t] + 8\exp[-0.605t] + 6\exp[-0.58t]$$

$$- 16\exp[-0.525t] - 6\exp[-0.55t] - 3\exp[-0.66t]]\ \exp[-0.230t]\ \exp[-0.120t]\ \exp[-0.165t]$$

$$= 12\exp[-1.035t] + 8\exp[-1.170t] + 6\exp[-1.145t]$$

$$- 16\exp[-1.090t] - 6\exp[-1.115t] - 3\exp[-1.225]. \quad (57)$$

The safety function of the ferry five-state technical system is presented in Figure 6.

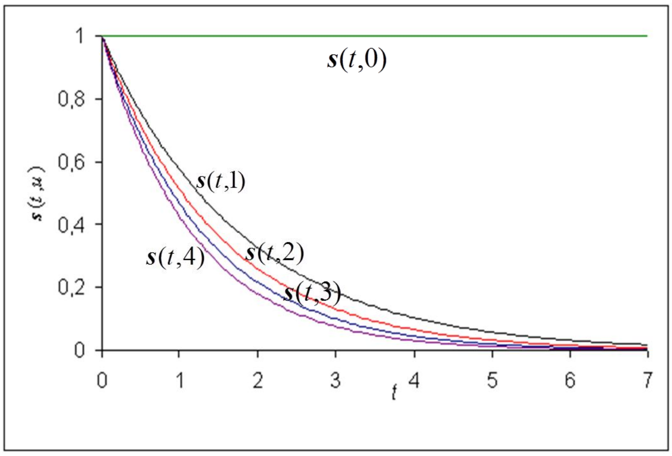

Figure 6. The graph of the ferry technical system safety function $\boldsymbol{s}(t,\cdot)$ coordinates

The expected values and standard deviations of the ferry technical system lifetimes in the safety state subsets calculated from the results given by (54)-(57), according to the formulae (5)-(7), are:

$$\mu(1) \cong 1.770,\ \ \mu(2) \cong 1.476,\ \ \mu(3) \cong 1.300,\ \ \mu(4) \cong 1.164 \text{ year}, \quad (58)$$

$$\sigma(1) \cong 1.733,\ \sigma(2) \cong 1.447,\ \sigma(3) \cong 1.277,\ \sigma(4) \cong 1.144 \text{ year,} \quad (59)$$

and further, using (58), from (9), the mean values of the ferry technical system conditional lifetimes in the particular safety states are:

$$\bar{\mu}(1) \cong 0.294,\ \bar{\mu}(2) \cong 0.176,\ \bar{\mu}(3) \cong 0.136,\ \bar{\mu}(4) \cong 1.164 \text{ year.} \quad (60)$$

As the critical safety state is r =2, then the system risk function, according to (10), is given by

$$\boldsymbol{r}(\mathrm{t}) = 1 - \boldsymbol{S}(t,2) = 1\text{-}[12 \exp[-0.815t] + 8 \exp[-0.925t] + 6 \exp[-0.895t]$$

$$- 16 \exp[-0.855t] - 6 \exp[-0.885t] - 3 \exp[-0.965t]], \text{ for } t \geq 0. \quad (61)$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, by (12), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.077. \quad (62)$$

The graph of the risk function $\boldsymbol{r}(t)$ of the ferry five-state technical

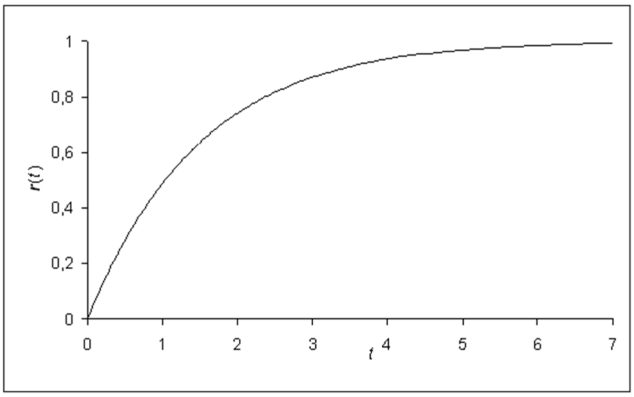

Figure 7. The graph of the risk function $\boldsymbol{r}(t)$ of the ferry technical system

5 CONCLUSION

The proposed in this paper model for safety evaluation and prediction of the considered here typical multistate system structures are applied for safety analysis of the maritime ferry technical system operating at Baltic Sea. The safety function, the risk function and other safety characteristics of the considered system are find. The system safety structures are fixed generally with not high accuracy in details concerned with the subsystems structures because of their complexity and concerned with the components safety characteristics because of the luck of statistical data necessary for their estimation. However, the results presented in the paper suggest that it seems reasonable to continue the investigations focusing on the methods of safety analysis for other more complex multi-state systems.

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