BLOCK INSPECTION POLICY MODEL WITH IMPERFECT INSPECTIONS FOR MULTI-UNIT SYSTEMS

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ABSTRACT

In this paper, the authors' research work is focused on imperfect inspection policy investigation, when not all defects are identified during inspection action performance and probability of defect identification is not a constant variable. They are interested in Block Inspection Policy performance for multi-unit systems, the maintenance policy which is one of the most commonly used in practice. As a result, at the beginning, few words about delay time modelling approach and a brief literature overview is given. Later, the model of Block-Inspection Policy is provided. The numerical example with the use of QNU Octave program is given. In the next Section, the sensitivity analysis of the developed model is characterized. The article ends up with summary and directions for further research.

1 INTRODUCTION

The main dependability characteristics of any technical system are mainly a function of its inspection and maintenance strategy. To model the inspection interaction, a concept called delay-time may be used.

The mentioned approach was developed by Christer et al. (see e.g. (Christer 1987, 1982, Christer & Waller 1984a, b, Christer & Whitelaw 1983)). The basic idea rests on an observation that a failure does not usually occur suddenly, but is preceded by a detectable fault for some time prior to actual failure, the delay time (Christer & Redmond, 1992). So, the delay time h is defined as the time lapses from the moment when a fault could first be noticed till the moment when a subsequent failure occurs, if left unattended (Christer 1999, Christer & Redmond 1992). During period h there is an opportunity to identify and prevent failure (Nowakowski & Werbińska-Wojciechowska 2012). For more information see e.g. (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a, b, c, Nowakowski & Werbińska-Wojciechowska 2012).

A literature review, in which delay-time models are investigated along with other PM models are given in (Dekker & Scarf 1998, Guo et al. 2001, 2000, Mazzuchi et al. 2007, Ozekici 1996, Thomas 1986, Valdez-Flores & Feldman 1989). The state of art works, dedicated strictly to DT modelling are given in (Alzubaidi 1993, Baker & Christer 1994, Christer 2002, 1999, Christer & Redmond 1992, 1990, Lee 1999, Nowakowski & Werbińska-Wojciechowska 2012, Redmond 1997, Wang 2012, 1992).

In real-world situation, inspections may not reveal all defects present in a system, especially for large complex systems. Moreover, the quality of performed inspections depends on inspection techniques used, inspection training, inspection practices imposed or the nature of any supervision.

As a result, the focus of this paper is to investigate the Block Inspection Policy model with imperfect inspections and to define their influence on maintenance policy results.

Moreover, most of the imperfect inspection maintenance models assume, that the probability p of a defect not detection during inspection action performance is constant, what may not be valid for real-life technical objects like means of transport (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a). Thus, in the presented paper the probability p is not a constant (like e.g. in (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a)) but linearly changes according to the defect symptoms visibility increase.

As a result, the paper is organized as follows: In the introduction Section, few words about delay time modelling approach and a brief literature overview is given. Later, the model of Block-Inspection Policy is provided. The numerical example with the use of QNU Octave program is given. In the next Section, the sensitivity analysis of the developed model is characterised. The article ends up with summary and directions for further research.

In conclusion, this article is a continuation of the delay time modelling problems being investigated in (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a, b, c, Nowakowski & Werbińska-Wojciechowska 2012, Werbińska-Wojciechowska 2012).

2 DELAY-TIME MODELS WITH IMPERFECT MAINTENANCE – LITERATURE OVERVIEW

The basic delay-time models for single- or multi-component systems assume that a visible defect is always found in a system if it is there. However, performance process of real technical systems like transportation systems indicates, that this assumption is insufficient. Thus, the imperfect inspection case should be investigated. The problem of imperfect inspection is analysed and overviewed e.g. in (Alzubaidi 1993, Choi 1997, Christer 1999, Das & Sarmah 2010, Kobbacy & Murthy 2008, Lee 1999, Redmond 1997, Sarkar et al. 2011, Wang 2012).

Ones of the first works which investigate the delay-time model with imperfect inspection are (Christer & Waller 1984a, Christer & Redmond 1990). In these works the basic inspection model for industrial plant maintenance is provided. In the next work (Christer & Waller 1984b), authors present the variation of this imperfect-inspection model for a vehicle fleet maintenance. Later, Baker & Wang (1993), present an extended delay-time model, in which the age of an object influence both the period u and delay time h.

Inspection models for single-component system are presented in (Christer 1992, Okumura 1997, Okumura et al. 1996, Pellegrin 1992, Zhao et al. 2007). In (Christer 1992), author develops the model of condition-monitoring for a production plant, when the initial point of a defect is measured as the time from the as-new condition. Pellegrin (1992) considers on-condition maintenance based on periodic inspection of productive equipment. The condition of equipment is described by the wear and its deterioration. Author also builds a graphical procedure to choose the best inspection interval for different criteria. Following this, in (Baker & Wang 1992), there is analysed the repairable machine that may fail or suffer breakdown many times during the course of its service lifetime, and is inspected for visible faults at intervals. The authors mostly focus on the problem of model parameter estimation with the use of maximum likelihood method and Akaike information criterion, providing also a model for imperfect inspections performance. Later, in (Okumura et al. 1996), authors develop a method for determining the discrete time points of inspection for a deteriorating single-unit system under condition-based maintenance. The delay-time model is here utilized to describe the transition of the system's states. This problem is later investigated by Okumura (1997). In the next work, (Zhao et al. 2007), authors investigate the model to evaluate the reliability and optimise the inspection schedule for a multi-defect component. There is also considered the situation of non-constant inspection intervals. In (Wang 2011), author presents an extended delay –time model for production plant and investigates its three-stage failure process.

An inspection-replacement model for a multi-component system is proposed e.g. in (Christer & Wang 1995, Wang 2008). In (Christer & Wang 1995), authors assume also that system is inspected not only on a planned basis, but also when a component fails. The model let the total expected cost per unit time minimize with respect to the inspection intervals and the system replacement time. Wang in his work (2008), focuses on the basic inspection model for complex or multi-component system, as a continuation of research work presented in e.g. (Christer & Wang 1995). Later, he extends the model in work (Lv & Wang 2011). In work (Baker et al. 1997), authors focus on the estimation of the model parameters and their errors from records of failure times and number of defects found at inspections of a machine which has been operated for some time under some inspection (also imperfect) regime. Later, in work (Christer & Lee 2000), authors present a delay-time-based PM model with an assumption of non-negligible downtime. The model is provided for perfect and non-perfect homogeneous processes and for a perfect non-homogeneous process. Moreover, Wang (2010) author develops a delay-time-based PM model when three types of maintenance (inspections, repairs, other PM actions) activities are performed.

Delay-time based models are also investigated for various types of technical objects, e.g. production plant maintenance processes development. In (Christer et al. 1995), authors present a Preventive Maintenance (PM) model applying the delay-time modelling technique to optimize the PM of the key machine in cooper products manufacturing company. An inspection model is developed to describe the relationship between the total downtime and the PM interval. Later, in (Christer et al. 1998), authors develop a delay-time model for PM of production plant, when assuming that defects identified at PM may not all be removed. They use objective estimation of model parameters, and the results are provided for NHPP arrival rate of failures and perfect-/imperfect-inspection cases. The problem of parameters estimation process for production plants maintenance modelling is later also investigated by Christer et al. (2000) or Wang (2009b). In (Wen-Yuan & Wang 2006) authors focus on optimising the preventive maintenance interval of a production plant. As in previous models, authors use likelihood formulation to model the problem. This problem is continued for the case of complex plant in (Ben-Daya et al. 2009). The model is analysed for a case example of exploitation process of extrusion press working in Cooper Company. One step further goes author in (Wang 2009a). In this work, author model the production process which may be subjected to two types of deteriorations. The first type of deterioration is a shift in product quality caused by minor process defects that may be identified and rectified by routine inspections and repair. Minor inspections are assumed to be perfect. The second deterioration type is a major defect caused by a major mechanical or electrical problem that can be observed only when the defect has led to a breakdown of the process or the defect is revealed by a major inspection followed by an appropriate major repair action at the time of the inspection. In the investigated model major inspections are assumed to be imperfect. The inspection model is focused on optimising the inspection intervals for both types of inspections. Later, the availability model based on delay-time modelling with imperfect maintenance is investigated in (Wang et al. 2011). Moreover, in (Dagg & Newby 1998, Wang & Sheu 2003) complex production systems inspection maintenance is optimized with the use of Markov theory.

Other applications of delay-time models with imperfect maintenance regard to aircraft structure (see e.g. (Cai & Zhu 2011)), railway tracks inspection and maintenance procedures (see e.g. (Podofillini et al. 2006)), inter-city express bus fleets maintenance (see e.g. (Desa & Christer 2001)), wind turbine maintenance (see e.g. (Andrawus et al. 2007)), or offshore oil platform plant reliability (see e.g. (Wang & Majid 2000)). The methodology for the application of delay time analysis via Monte Carlo Simulation is given in (Cunningham et al. 2011).

Following this, in the next Sections the Block Inspection (BI) Policy is investigated and its sensitivity for type of inspection action performance.

3 BLOCK INSPECTION POLICY MODEL WITH IMPERFECT MAINTENANCE

The investigated system is comprised of k identical elements, in a *k-out-of-n* (e.g. 2-out-of-3 in the Fig. 1) reliability structure, working independently under the same conditions. Moreover, components may be in one of three states: operating properly, operating with defects or down. They prone to become defective independently of each other when the system is in operating. The performed maintenance policy bases on Block Inspection policy which assumes, that inspections take place at regular time intervals of T, and each requires constant time. The inspections are assumed to be imperfect. Thus, any component's defect, which occurred in the system till the moment of inspection, will be unnoticed with probability p or correctly identified with probability $1-p$. All elements with identified defects will be replaced within the inspection period.

The performance of the investigated system, being illustrated in Figure 1, is also defined by the additional assumptions:

- the system is a two state system where, over its service life, it can be either operating or down for necessary repair or planned maintenance,
- maintenance actions restores maintained components to as good as new condition,
- the system can remain functioning in an acceptable manner until breakdown (despite having elements' defects),
- defects which may have arisen in the system, deteriorate over an operating time,
- the breakdown will be assumed to have been caused by $n - k + 1$ defects which has deteriorated sufficiently to affect the operating performance of the system as a whole,
- failures of the system are identified immediately and repairs or replacements are made as soon as possible,
- system incurs costs of: new elements, when they are replaced, inspection costs, and some additional, consequence costs, when system fails,
- elements' lifetime, repair time, replacement time and the length of the delay time before element's failure are random and their probability distributions are known.

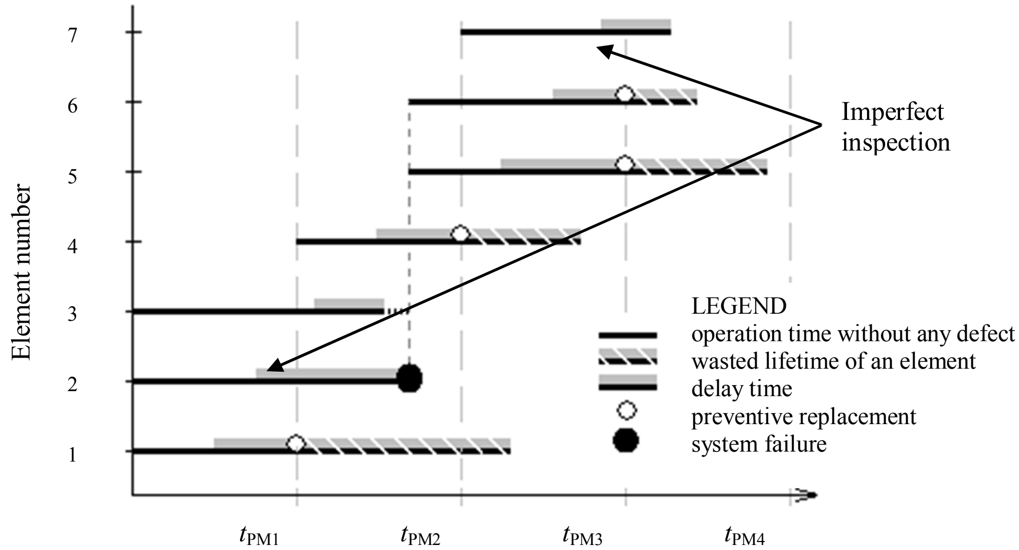

Figure 1. Idea of the Block Inspection Policy with imperfect inspection (2-out-of-3 reliability structure)

The system in Figure 1 is inspected at t_{PM} moments. Diagnosis of defect symptoms is imperfect thus elements 2 and 7 are allowed to further work although their defects might be noticed. Because of the fact, during one of following periods between inspections (t_{PM1} - t_{PM2}) two consecutive elements fail what causes a system failure. On the other hand, some elements' defects are properly diagnosed at the first possible inspection (elements: 1,4,5,6) and the components are preventively replaced but their potential lifetime is wasted.

The system presented above was modelled in GNU Octave software. The list of tested system parameters, which were used in the simulation model of the system exploitation process, is given in the Table 1.

Table 1. Modelled system parameters

Notation	Description	Basic value
c_e	the cost of a new element	1
c_i	the cost of an inspection	1
c_c	the cost of a system failure	1000
T_i	the time required for inspection	0
$F(t)$	C.d.f. of single element's lifetime	$F(t) = 1 - e^{-(100)^{3,5}}$
$G_r(t)$	C.d.f. of single element's replacement time when corrective action is taken	$G_r(t) = 1 - e^{-(100)^{2,3}}$
$G_p(t)$	C.d.f of single element's replacement time when preventive action is taken	$G_p(t) = 1 - e^{-(10)^{2,3}}$
$F(h)$	C.d.f. of delay time	$F_h(h) = 1 - e^{-(35)^{3,5}}$

4 INFLUENCE OF IMPERFECT INSPECTION ON BI POLICY RESULTS

Practical usage of the *BI* policy requires results of theoretical studies to be transformed into practical guidelines for maintenance staff. For this reason, Authors decided to study the problem, which may arise in practice, of imperfect inspection and its influence on results of the *BI* policy. We especially focus on the relation of inspection imprecision and the best length of inspection period in systems with different characteristics, maintained according to the *BI* policy.

Inspection precision is understood as the ability of system to detect (and correctly interpret) its elements' defects during inspection if their symptoms may be observable. This ability is given by p factor describing the probability that an element defect will not be noticed by maintenance service. The greater value of p means more difficult diagnosis of a real state of system during inspection.

The two basic cases of imperfect inspection, analysed in the paper, assume:

- the probability of defect omitting (p) in the course of inspection is constant and may result from, e.g. too little knowledge and experience of maintenance staff who inspect a system,
- the probability p is the effect of difficulties with observing of defect signal during inspection and depends on the strength of the signal. Thus, the probability p decreases with the time when an element is closer to its failure, because the signal strength usually increases in time and a defect may be easily noticed and correctly interpreted when a time to failure is short.

For the purpose of comparison of the costs and the availability results of the *BI* policy for the cases of perfect and imperfect inspection the analysis has been done and its results are presented in Figures 2 – 5. The chosen effects depicted in the figures assume various length of the inspection period (T) and system reliability structure given by the number of elements (k) which have to be in up-state in order a system to be up. However, the research was also conducted for various levels of inspection imprecision determined by probability p (p = 0,1÷1 or p growing linearly) and for various delay time lengths (the mean delay time of elements ranges from 0 to the half of the mean element's lifetime).

The Figure 2 shows the total costs (*C*) of new elements, when they are replaced, inspection and consequence costs for *k-out-of-n* systems which are perfectly inspected. The same features characterize the system whose cost results are presented in the Figures 3 and 6, but the systems are inspected imperfectly. Figures 6-7 present the cost and the availability ratio achieved by a system inspected with various levels of inspection precision 1-*p*, constant during the whole delay time of an element (the case of little experienced maintenance staff), while Figures 3 and 5 demonstrate the results assuming that the chance of defect detection increases linearly in time (inspection closer to failure moment returns better information).

The analysis has proved the expected fact that all tested costs and the availability ratio depend on inspection precision. The strength of this impact is much greater in series structured (*n-out-of-n*) and similar systems, more liable to system failures. Lower precision of inspection increases the system failure cost, which is dominant cost component, but at the same time it decreases the summary cost of new elements that are used in the system. The total cost, directly proportional to the number of system failures, is much higher in the cases when inspection is not perfect. The corresponding effect is observable in availability ratio analysis – low reliability of inspection reduces meaningfully the availability ratio of maintained system if it is not substituted by oftener inspections. On the other hand, some expected effect may be observable – even relatively low probability of defect omission does not cause any severe cost or availability consequences if the inspection period is shortened (e.g. $T \leq 5$ in Fig. 3 or 5). The effect is unequivocal in series systems ($k = n$), but when a reliability structure of a system changes and becomes more failure-resistant ($k << n$), the analysis of the availability ratio demonstrates also the other result. Parallel system should be inspected in long intervals if one wants to maximize its availability. Moreover the time between inspections (*T*) should be carefully determined in systems with "intermediate" reliability structures because there is a relatively short range of *T*, which give "near optimum" results for such systems. The fact of existing of the length of an inspection period that yields "the nearly best" results for simulated system was the reason for authors to carry out the research whose results are presented in the next section of the paper.

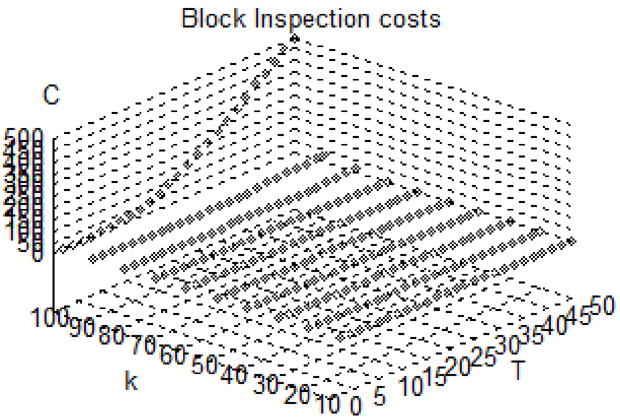

Figure 2. Block Inspection costs for k-out-of-100 system for the case of perfect inspection

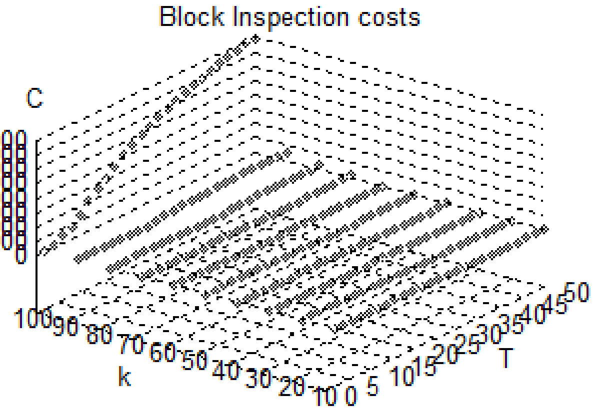

Figure 3. Block Inspection costs for *k-out-of-100* system imperfectly inspected (*p* decreases linearly)

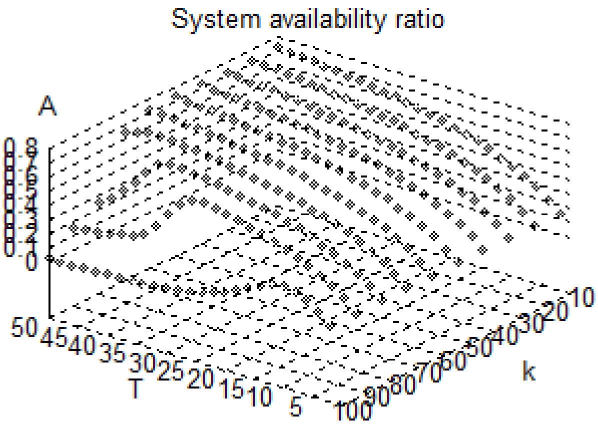

Figure 4. Availability ratio of *k-out-of-100* system for the case of perfect inspection

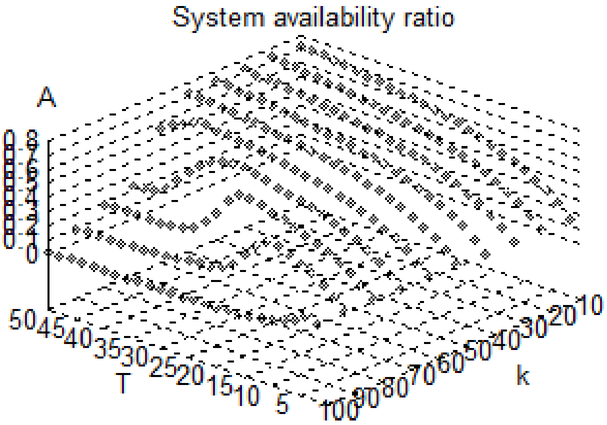

Figure 5. Availability ratio of *k-out-of-100* system imperfectly inspected (p decreases linearly)

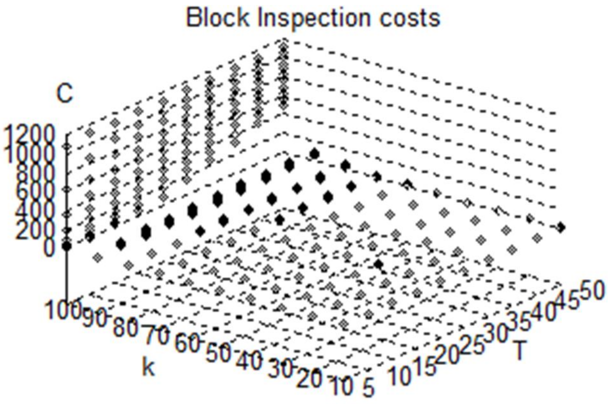

Figure 6. Block Inspection costs for *k-out-of-100* system imperfectly inspected ($p = 0 \div 1$)

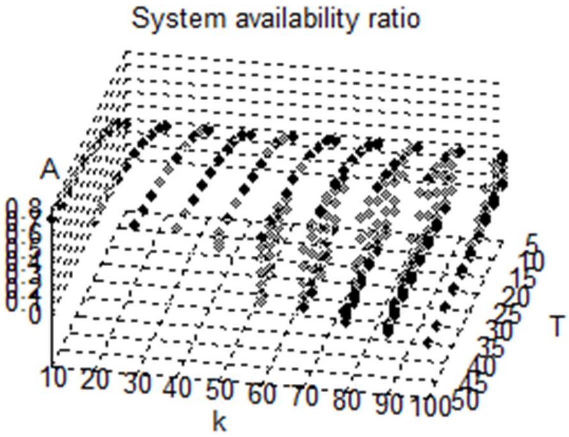

Figure 7. Availability ratio of *k-out-of-100* system imperfectly inspected ($p = 0 \div 1$)

2.1 The optimum inspection period

The main advantage of the *BI* policy, as well as all the policies basing on the state of a maintained system, comes from inspection findings which should determine if any preventive replacement is required. When an inspection does not give proper results, it should be executed more often than if a diagnosis is perfect, in order to increase the chance to notice an element defect. The inspection uncertainty may be neutralized by shortening the period between inspections as it was shown in the previous part of the paper. That was the reason to carry out the further research, how the optimal inspection period changes when inspections become less reliable (growing value of p).

The "near optimal" time between preventive inspections T, according to literature findings (see e.g. (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012b)), depends mainly on a system lifetime determined by lifetime of system components and its reliability structure as well as elements' delay time. The period T that yields "low" costs and "high" availability ratio of maintained system should fulfil following conditions in a series structured system:

$$\frac{h}{T} \approx 2 \tag{1}$$

where: T = the period between inspections, h = the mean delay time value.

System with parallel reliability structure may be inspected even at longer time intervals because of its higher resistance to single element failures.

The expressions 1 has been found to be true for perfectly inspected systems. The goal of the following analysis is to show how the above expressions should be changed when inspection becomes less precise. The research results are presented in Figures 8 – 11.

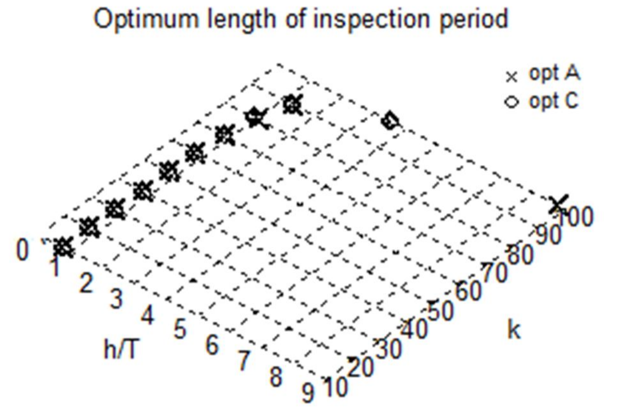

Figure 8. The length of inspection period in the relation to element's delay time, which yields the best availability (opt *A*) and cost (opt *C*) results in a *k-out-of-100* system which is perfectly inspected

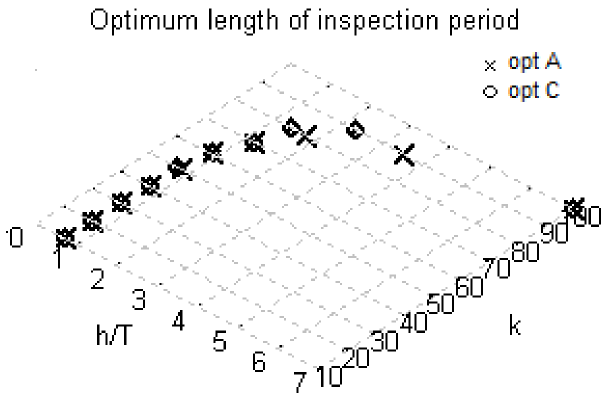

Figure 9. The length of inspection period in the relation to element's delay time, which yields the best availability (opt *A*) and cost (opt *C*) results in a *k-out-of-100* system which is imperfectly inspected (*p* is growing linearly)

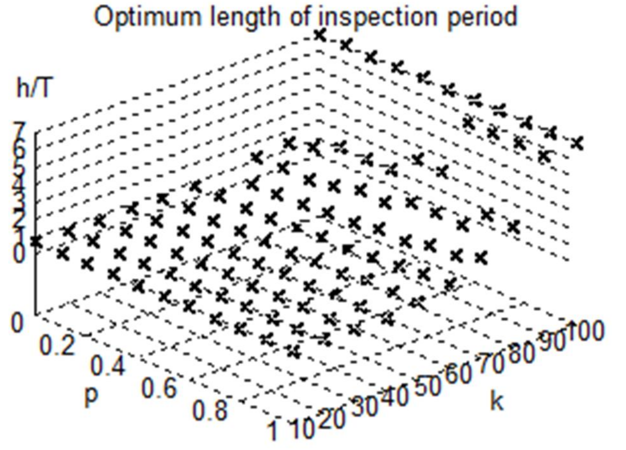

Figure 10. The length of inspection period in the relation to element's delay time, which yields the best cost results in a *k-out-of-100* system which is imperfectly inspected

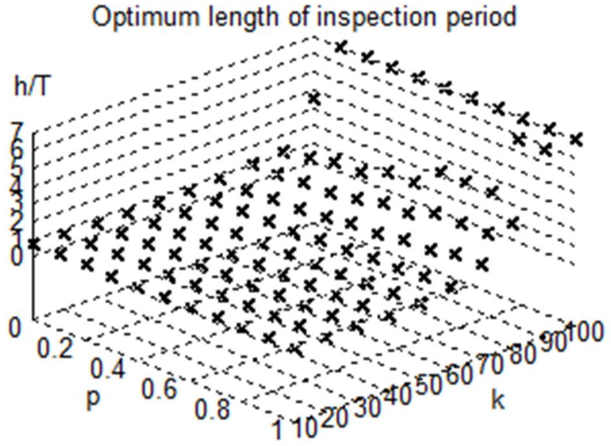

Figure 11. The length of inspection period in the relation to element's delay time, which yields the best availability results in a *k-out-of-100* system which is imperfectly inspected

The Figure 8 depicts the optimum relation of the mean delay time of a system element (h) and inspection period length (T), found among the tested range, from the point of view of the cost and availability of a *k-out-of-n* system, which is perfectly inspected. When $k = 1$ in a system (a parallel structure), the cheapest solution have been found for the cases when inspection period length is close to the mean value of element's delay time ($h/T<1$). If the number of elements required for system operation rises ($1 < k < n$), inspection period should be reduced ($1 \leq h/T \leq 2$) in order a system to obtain the best maintenance results. When a systems becomes a series system ($k = n$) the cheapest solutions exist for the condition $3 \leq h/T \leq 4$, while the highest availability ratio of a system is observable for the shortest inspection period, which was tested in the study ($h/T \rightarrow max$). The next figures (Fig. 9-11) present the optimum results obtained for the same values of variables *h/T* and *k* but the assumption about perfect inspection is released. The direction of the curve (Fig. 9) marking the best found cost and availability relations of *h*/*T* is the same independently on the inspection precision, but the slope of the curve is not. When an inspection is imperfect (Fig. 9), the same systems (for whose $k \geq 0{,}7n$) should be inspected approximately two times more often than if a system is perfectly inspected ($2 > h/T > 4$). In order to confirm the fact is true not only for the chosen case, the analysis of *h/T* for various values of the mean delay time *h* was conducted and for various system sizes. The exemplary results are shown in Figures 12-15.

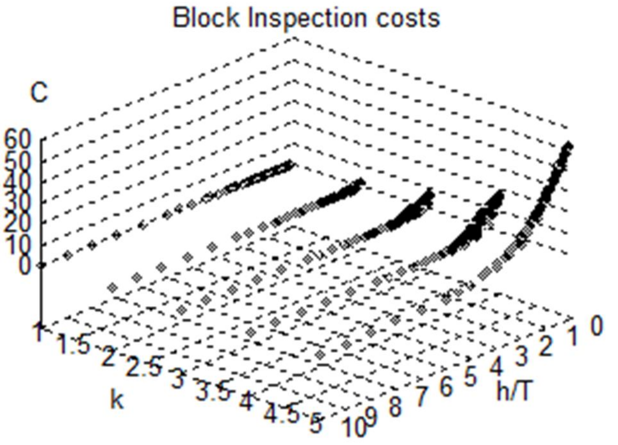

Figure 12. Block Inspection costs for *k-out-of-5* system imperfectly inspected (*p* decreases linearly)

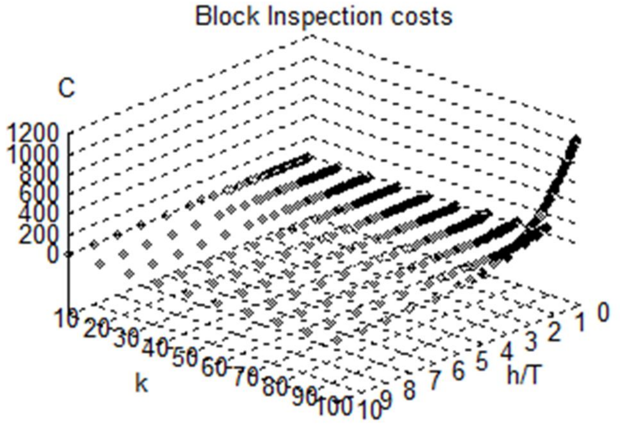

Figure 13. Block Inspection costs for *k-out-of-100* system imperfectly inspected (*p* decreases linearly)

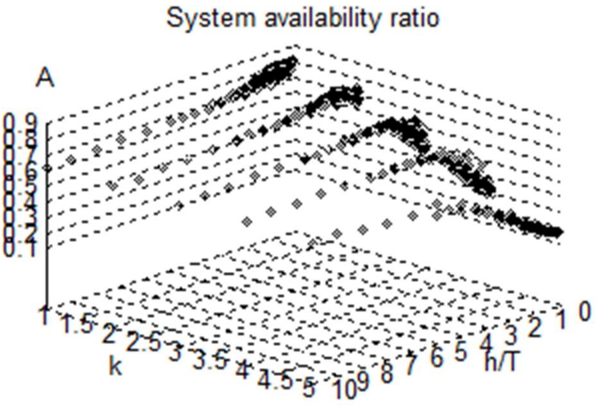

Figure 14. Availability ratio of *k-out-of-5* system imperfectly inspected (*p* decreases linearly)

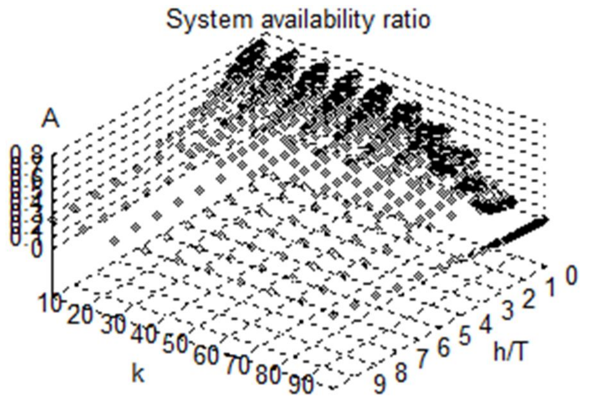

Figure 15. Availability ratio of *k-out-of-5* system imperfectly inspected (*p* decreases linearly)

Independently on the value of mean delay time of elements constituting a system, the best inspection period, with maximum level of system availability or minimum costs, is some part of the mean delay time. The absolute values of *h* and *T* do not have any greater meaning, thus it seems to be reasonable to generalize the conclusions resulting from the research. The fact is especially useful from the practical point of view. When one realizes that inspection in a system maintained according to the *BI* policy is not perfect and is able to estimate the mean delay time of elements, he has some reference range of inspection period lengths which might be applied in practice in order to get “good” availability and cost results.

5 CONCLUSIONS

The presented sensitivity analysis of investigated BI policy model gives the possibility to obtain some rules for definition of the principal relations between the system performance under given PM policy with imperfect maintenance and chosen PM policy parameters.

The research issues analysed in the paper are the continuation of research analyses made in (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a). In the presented paper authors develop extended DT model with imperfect maintenance, when the probability p is not a constant parameter. Such a model parameter definition is more adjusted to real-life technical objects performance, like means of transport. Thus, in the next step, authors will make an effort to define some rules how to choose a PM policy from an engineering point of view.

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