APPLICATION OF GEOMETRIC PROCESS IN ACCELERATED LIFE TESTING ANALYSIS WITH TYPE-I CENSORED WEIBULL FAILURE DATA

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ABSTRACT

In Accelerated life testing (ALT), generally, the estimates of original parameters of the life distribution are obtained by using the log linear function between life and stress which is just a simple re-parameterization of the original parameter but from the statistical point of view, it is preferable to work with the original parameters instead of developing inferences for the parameters of the log-linear link function. By the use of geometric process one can easily deal with the original parameters of the life distribution in accelerated life testing. In this paper the geometric process is used in accelerated life testing to estimate the parameters of Weibull distribution with type-I censored data. The maximum likelihood estimates of the parameters are obtained by assuming that the lifetimes under increasing stress levels form a geometric process. In addition, asymptotic confidence interval estimates of the parameters using Fisher information matrix are also obtained. A Simulation study is also performed to check the statistical properties of estimates of the parameters and the confidence intervals.

Keywords: Maximum Likelihood Estimation; Survival Function; Fisher Information Matrix; Asymptotic Confidence Interval; Simulation Study.

1 INTRODUCTION

Nowadays, there is a big competition among manufacturing industries to provide quality products to their customers and hence the customer expectations are also very high which makes the products in recent era very reliable and dependable. As in life testing experiments the failure time data is used to obtain the product life characteristics under normal operating conditions, therefore, such life data has become very difficult to obtain as a result of the great reliability of today's products and hence under normal operating conditions, as products usually last long, the corresponding life-tests become very time consuming and expensive. In these cases, an accelerated life test (ALT) which is a quick way to obtain information about the life distribution of a material, component or product can be applied to reduce the experimental time and the cost incurred in the experiment. In ALT items are subjected to conditions that are more severe than the normal ones, which yields shorter life but, hopefully, do not change the failure mechanisms. Failure information collected under this severe test stresses can be extrapolated to obtain an estimate of lifetime under normal operating condition based on some life-stress relationship.

ALTs, generally deal with three types of stress loadings i.e. constant stress, step stress and linearly increasing stress. The constant stress loading is a time-independent test setting and others are the time-dependent test setting. The constant stress loading has several advantages over time-dependent test settings, for example, most of the products in real life are operated at a constant stress. Therefore, a constant stress test describes the actual use of the product. Also, it is comparatively easy to run and to quantify a constant stress test. Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some of failure data are missing). For more details about ALTs one can consult Bagdonavicius and Nikulin [1], Meeker and Escobar [2], Nelson [3, 4], Mann and Singpurwalla [5].

Constant stress ALT with different types of data and test planning has been studied by many authors. For example, Yang [6] proposed an optimal design of 4-level constant-stress ALT plans considering different censoring times. Pan et al. [7] proposed a bivariate constant stress accelerated degradation test model by assuming that the copula parameter is a function of the stress level that can be described by a logistic function. Chen et al. [8] discuss the optimal design of multiple stress constant accelerated life test plan on non-rectangle test region. Watkins and John [9] considers constant stress accelerated life tests based on Weibull distributions with constant shape and a log-linear link between scale and the stress factor which is terminated by a Type-II censoring regime at one of the stress levels. Fan and Yu [10] discuss the reliability analysis of the constant stress accelerated life tests when a parameter in the generalized gamma lifetime distribution is linear in the stress level. Ding et al. [11] dealt with Weibull distribution to obtain accelerated life test sampling plans under type I progressive interval censoring with random removals. Ahmad et al. [12], Islam and Ahmad [13], Ahmad and Islam [14], Ahmad, et al. [15] and Ahmad [16] discuss the optimal constant stress accelerated life test designs under periodic inspection and Type-I censoring.

Geometric process (GP) is first used by Lam [17] in the study of repair replacement problem. Since then a large amount of studies in maintenance problems and system reliability have been shown that a GP model is a good and simple model for analysis of data with a single trend or multiple trends, for example, Lam and Zhang [18], Lam [19] and Zhang [20]. So far, there are only four studies in the analysis of accelerated life test that utilize the GP. Huang [21] introduced the GP model for the analysis of constant stress ALT with complete and censored exponential samples. Kamal et al. [22] extended the GP model for the analysis of complete Weibull failure data in constant stress ALT. Zhou et al. [23] implement the GP in ALT based on the progressive Type-I hybrid censored Rayleigh failure data. More recently Kamal et al. [24] used the geometric process for the analysis of constant stress accelerated life testing for Pareto Distribution with complete data.

In this paper, the constant stress ALT with geometric process and type-I censoring for Weibull distribution is considered. Estimates of Parameters are obtained by maximum likelihood estimation technique and Confidence intervals for parameters are obtained by using the asymptotic properties. Lastly statistical properties of estimates and confidence intervals are examined through a simulation study.

2 THE MODEL AND TEST PROCEDURE

2.1 The Geometric Process (GP)

A GP is a stochastic process $\{X_n, n = 1,2,...\}$ such that $\{\lambda^{n-1} X_n, n = 1,2,...\}$ forms a renewal process where $\lambda > 0$ is real valued and called the ratio of the GP. It is easy to show that if $\{X_n, n = 1,2,...\}$ is a GP and the probability density function of X_1 is $f(x)$ with mean μ and variance σ^2 then the probability density function of X_n will be $\lambda^{n-1} f(\lambda^{n-1} x)$ with mean μ / λ^{n-1} and variance $\sigma^2 / \lambda^{2(n-1)}$.

It is clear to see that a GP is stochastically increasing if $0 < \lambda < 1$ and stochastically decreasing if $\lambda > 1$. Therefore, GP is a natural approach to analyse the data from a series of events with trend. For more details about GP and its properties see Braun et al. [25].

2.2 The Weibull Distribution

The probability density function, the cumulative distribution function, the survival function and the failure rate (or hazard rate) of a two parameter Weibull distribution with scale parameter $\alpha > 0$ and shape parameter $\beta > 0$, are given respectively by

$$f(x|\alpha,\beta) = \frac{\beta}{\alpha^{\beta}} x^{\beta-1} \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad x \geq 0$$

(1)

$$F(x|\alpha,\beta) = 1 - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad x \geq 0$$

$$S(x) = \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad x \geq 0$$

$$h(x|\alpha,\beta) = \frac{k}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}$$

It is easy to verify that failure rate (or hazard rate) decreases over time if $\beta < 1$ (or increases with time if $\beta > 1$) and $\beta = 1$ indicates that the failure rate is constant over time.

2.3 Assumptions and test procedure

i. Suppose that an accelerated life test with s increasing stress levels in which a random sample of n identical items is placed under each stress level and start to operate at the same time. Let $x_{ki}, i = 1,2,...,n,\ k = 1,2,...,s$ denote observed failure time of i^{th} test item under k^{th} stress level. Whenever an item fails, it will be removed from the test and the test is terminated at a prespecified censoring time t at each stress level and the exact failure times $x_{ki} \leq t$ of items are observed.
ii. The product life follows Weibull distribution given by (1) at any stress.
iii. The scale parameter is a log-linear function of stress. That is, $\log(\alpha_k) = a + bS_k$, where a and b are unknown parameters depending on the nature of the product and the test method.
iv. Let random variables $X_0, X_1, X_2, ..., X_s$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily and sequence $\{X_k, k = 1,2,...,s\}$ forms a geometric process with ratio $\lambda > 0$.

Assumptions (i-iii) are very usually discussed in literature of ALTs but assumption (iv) which will be used in this study may be better than the usual one without increasing the complexity of calculations. The next theorem discusses how the assumption of geometric process (assumption iv) is satisfied when there is a log linear relationship between a life and stress (assumption iii).

Theorem 2.1: *If the stress level in a constant stress ALT is increasing with a constant difference then the lifetimes under each stress level forms a* GP *that is, If* $S_{k+1} - S_k$ *is constant for* $k = 1, 2, ..., s-1$*, then* $\{X_k, k = 0, 1, 2, ..., s\}$ *forms a* GP. *Or log linear relationship and GP model are equivalent when the stress increases arithmetically in constant stress ALT.*

Proof: From assumption (iii), it can easily be shown that

$$\log\left(\frac{\alpha_{k+1}}{\alpha_k}\right) = b(S_{k+1} - S_k) = b\Delta S \tag{2}$$

Now eq. (2) can be rewritten as

$$\frac{\alpha_{k+1}}{\alpha_k} = e^{b\Delta S} = \frac{1}{\lambda} \qquad \text{(Assumed)} \tag{3}$$

This shows that stress levels increases arithmetically with a constant difference ΔS. Therefore, It is clear from (3) that

$$\alpha_k = \frac{1}{\lambda}\alpha_{k-1} = \frac{1}{\lambda^2}\alpha_{k-2} = ... = \frac{1}{\lambda^k}\alpha$$

The PDF of the product lifetime under the k^{th} stress level is

$$f_{X_k}(x) = \frac{\beta}{\alpha_k{}^{\beta}} x^{\beta-1} \exp\left\{-\left(\frac{x}{\alpha_k}\right)^{\beta}\right\}$$

$$= \frac{\beta}{\left(\frac{1}{\lambda^k}\alpha\right)^{\beta}} x^{\beta-1} \exp\left\{-\left(\frac{x}{\frac{1}{\lambda^k}\alpha}\right)^{\beta}\right\} = \left(\frac{\lambda^k}{\alpha}\right)^{\beta} \beta\, x^{\beta-1} \exp\left\{-\left(\frac{\lambda^k}{\alpha}x\right)^{\beta}\right\}$$

This implies that

$$f_{X_k}(x) = \lambda^k f_{X_0}(\lambda^k x) \tag{4}$$

Now, the definition of GP and (4) have the evidence that, if density function of X_0 is $f_{X_0}(x)$, then the probability density function of X_k will be given by $\lambda^k\ f(\lambda^k x)$, $k = 0,1,2,\cdots,s$. Therefore, it is clear that lifetimes under a sequence of arithmetically increasing stress levels form a geometric process with ratio λ.

3 MAXIMUM LIKELIHOOD ESTIMATION AND FISHER INFORMATION MATRIX

Here the maximum likelihood method of estimation is used because ML method is very robust and gives the estimates of parameter with good statistical properties. In this method, the estimates of parameters are those values which maximize the sampling distribution of data. However, ML estimation method is very simple for one parameter distributions but its implementation in ALT is mathematically more intense and, generally, estimates of parameters do not exist in closed form, therefore, numerical techniques such as Newton Method, Some computer programs are used to compute them.

Let the test at each stress level is terminated at time t and only $x_{ki} \le t$ failure times are observed. Assume that $r_k (\le n)$ failures at the k^{th} stress level are observed before the test is suspended and $(n - r_k)$ units are still survived the entire test without failing.

Now the likelihood function for constant stress ALT with Type I censored Weibull failure data using GP at one of the stress level is given by

$$L_k(\alpha,\theta,\lambda)=\frac{n!}{(n-r_k)!}\left[\left(\frac{\lambda^k}{\alpha}\right)^{r_k\beta}\beta^{r_k}\prod_{i=1}^{r_k}x_{k(i)}^{\beta-1}\exp\left\{-\left(\frac{\lambda^k x_{k(i)}}{\alpha}\right)^{\beta}\right\}\right]\left[\exp\left\{-\left(\frac{\lambda^k t}{\alpha}\right)^{\beta}\right\}\right]^{n-r_k}$$

Therefore, now the likelihood function of observed data for total s stress levels is

$$L_k(\alpha,\theta,\lambda)=L_1\times L_2\ldots\times L_s$$

$$=\prod_{k=1}^{s}\left[\frac{n!}{(n-r_k)!}\left(\frac{\lambda^k}{\alpha}\right)^{r_k\beta}\beta^{r_k}\left\{\prod_{i=1}^{r_k}x_{k(i)}^{\beta-1}\exp\left\{-\left(\frac{\lambda^k x_{k(i)}}{\alpha}\right)^{\beta}\right\}\right\}\left[\exp\left\{-\left(\frac{\lambda^k t}{\alpha}\right)^{\beta}\right\}\right]^{n-r_k}\right]$$

(5)

The log-likelihood function corresponding (5) takes the form

$$l=\log L_k(\alpha,\theta,\lambda)=\sum_{k=1}^{s}\left[\begin{array}{l}\log\left(\frac{n!}{(n-r_k)!}\right)+kr_k\beta\log\lambda-r_k\beta\log\alpha+r_k\log\beta\\ \qquad+(\beta-1)\sum_{i=1}^{r_k}\log x_{k(i)}-\left(\frac{\lambda^k}{\alpha}\right)^{\beta}\left(\sum_{i=1}^{r_k}x_{k(i)}^{\beta}+(n-r_k)t^{\beta}\right)\end{array}\right]$$

MLEs of α,β and λ are obtained by solving the following normal equations

$$\frac{\partial l}{\partial\alpha}=\sum_{k=1}^{s}\left[-\frac{r_k\beta}{\alpha}+\beta\lambda^{k\beta}\left(\frac{1}{\alpha}\right)^{\beta+1}\left(\sum_{i=1}^{r_k}x_{k(i)}^{\beta}+(n-r_k)t^{\beta}\right)\right]=0$$

$$\frac{\partial l}{\partial\lambda}=\sum_{k=1}^{s}\left[\frac{kr_k\beta}{\lambda}-\frac{k\beta}{\lambda}\left(\frac{\lambda^k}{\alpha}\right)^{\beta}\left(\sum_{i=1}^{r_k}x_{k(i)}^{\beta}+(n-r_k)t^{\beta}\right)\right]=0$$

$$\frac{\partial l}{\partial\beta}=\sum_{k=1}^{s}\left[\begin{array}{l}kr_k\log\lambda-r_k\log\alpha+\frac{r_k}{\beta}+\sum_{i=1}^{r_k}\log(x_{k(i)})-\left(\frac{\lambda^k}{\alpha}\right)^{\beta}\sum_{i=1}^{r_k}(x_{k(i)})^{\beta}\left\{\log(x_{k(i)})+\log\left(\frac{\lambda^k}{\alpha}\right)\right\}\\ -(n-r_k)\left(\frac{\lambda^k}{\alpha}\right)^{\beta}t^{\beta}\left\{\log t+\log\left(\frac{\lambda^k}{\alpha}\right)\right\}\end{array}\right]=0$$

Above equations are nonlinear; therefore, it is very difficult to obtain a closed form solution. So, Newton-Raphson method is used to solve these equations simultaneously to obtain $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\lambda}$.

The Fisher's information matrix composed of the negative second partial derivatives of log likelihood function can be written as

$$F=\begin{bmatrix}-\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda} & -\frac{\partial^2 l}{\partial\alpha\partial\beta}\\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda^2} & -\frac{\partial^2 l}{\partial\lambda\partial\beta}\\ -\frac{\partial^2 l}{\partial\beta\partial\alpha} & -\frac{\partial^2 l}{\partial\beta\partial\lambda} & -\frac{\partial^2 l}{\partial\beta^2}\end{bmatrix}$$

Where the elements of the Fisher Information matrix are obtained as

$$\frac{\partial^2 l}{\partial \alpha^2} = \sum_{k=1}^{s} \left[\frac{r_k \beta}{\alpha^2} - \beta(\beta+1)\lambda^{k\beta}\left(\frac{1}{\alpha}\right)^{\beta+2} \left\{ \sum_{i=1}^{r_k} x_{k(i)}^{\beta} + (n-r_k)t^{\beta} \right\} \right]$$

$$\frac{\partial^2 l}{\partial \lambda^2} = \sum_{k=1}^{s} \left[-\frac{k r_k \beta}{\lambda^2} - \frac{k\beta}{\lambda^2}(k\beta-1)\left(\frac{\lambda^k}{\alpha}\right)^{\beta} \left\{ \sum_{i=1}^{r_k} x_{k(i)}^{\beta} + (n-r_k)t^{\beta} \right\} \right]$$

$$\frac{\partial^2 l}{\partial \beta^2} = \sum_{k=1}^{s} \left[\begin{array}{l} -\dfrac{r_k}{\beta^2} - \left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \left\{ \log(x_{k(i)}) + \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\}^2 \\ -(n-r_k)\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} t^{\beta} \left\{ \log t + \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\}^2 \end{array} \right]$$

$$\frac{\partial^2 l}{\partial \alpha \partial \lambda} = \frac{\partial^2 l}{\partial \lambda \partial \alpha} = \sum_{k=1}^{s} \left[\frac{k\beta^2}{\alpha\lambda}\left(\frac{\lambda^k}{\alpha}\right)^{\beta} \left\{ \sum_{i=1}^{r_k} x_{k(i)}^{\beta} + (n-r_k)t^{\beta} \right\} \right]$$

$$\frac{\partial^2 l}{\partial \beta \partial \lambda} = \frac{\partial^2 l}{\partial \lambda \partial \beta} = \sum_{k=1}^{s} \left[\begin{array}{l} \dfrac{k r_k}{\lambda} - \dfrac{k\beta}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \log(x_{k(i)}) \\ -\displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \left\{ \dfrac{k}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} + \dfrac{k\beta}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\} - (n-r_k)\dfrac{k\beta}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} t^{\beta} \log t \\ -(n-r_k)t^{\beta} \left\{ \dfrac{k}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} + \dfrac{k\beta}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\} \end{array} \right]$$

$$\frac{\partial^2 l}{\partial \alpha \partial \beta} == \frac{\partial^2 l}{\partial \beta \partial \alpha} = \sum_{k=1}^{s} \left[\begin{array}{l} -\dfrac{r_k}{\alpha} + \dfrac{\beta}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \log(x_{k(i)}) \\ +\displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \left\{ \dfrac{1}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} + \dfrac{\beta}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\} + (n-r_k)\dfrac{\beta}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} t^{\beta} \log t \\ +(n-r_k)t^{\beta} \left\{ \dfrac{1}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} + \dfrac{\beta}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\} \end{array} \right]$$

4 ASYMPTOTIC CONFIDENCE INTERVAL ESTIMATES

According to large sample theory, the maximum likelihood estimators, under some appropriate regularity conditions, are consistent and normally distributed. Since ML estimates of parameters are not in closed form, therefore, it is impossible to obtain the exact confidence intervals, so asymptotic confidence intervals based on the asymptotic normal distribution of ML estimators instead of exact confidence intervals are obtained here.

Now, the variance covariance matrix can be written as

$$\Sigma = \begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} & -\frac{\partial^2 l}{\partial \alpha \partial \beta} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda^2} & -\frac{\partial^2 l}{\partial \lambda \partial \beta} \\ -\frac{\partial^2 l}{\partial \beta \partial \alpha} & -\frac{\partial^2 l}{\partial \beta \partial \lambda} & -\frac{\partial^2 l}{\partial \beta^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\lambda}) & ACov(\hat{\alpha}\hat{\beta}) \\ ACov(\hat{\lambda}\hat{\alpha}) & AVar(\hat{\lambda}) & ACov(\hat{\lambda}\hat{\beta}) \\ ACov(\hat{\beta}\hat{\alpha}) & ACov(\hat{\beta}\hat{\lambda}) & AVar(\hat{\beta}) \end{bmatrix}$$

The $100(1-\gamma)\%$ asymptotic confidence interval for α, β and λ are then given respectively by

$$\left[\hat{\alpha} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\alpha})}\right], \left[\hat{\beta} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\beta})}\right] \text{and} \left[\hat{\lambda} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\lambda})}\right]$$

5 SIMULATION STUDY

The performance of the estimates can be evaluated through some measures of accuracy which are the mean squared error (MSE), relative absolute bias (RAB) and the 95% asymptotic confidence intervals for different sample sizes and stress levels. Now for this purpose following simulation study is conducted.

To perform the simulation study, first a random sample x_{ki}, $k=1,2,...,s$, $i=1,2,...,r$ is generated from Weibull distribution which is censored at $t = 4, 6$. The values of the parameters and number of stress levels are chosen to be $\alpha = 0.80$, $\beta = 2.50$, $\lambda = 1.50$ and $s = 4$. For different sample sizes $n = 50,100,...,250$ the MLEs, MSEs, RABs and lower and upper CI limits (LCL and UCL) of the 95% confidence interval of parameters based on 600 simulations are obtained by the model discussed in this paper and summarized in Table 1 and 2.

Table 1: Simulation Study Results with $\alpha = 0.80$, $\beta = 2.50$, $\lambda = 1.50$, $s = 4$ and $t = 4$

n	$\hat{\alpha}$ $\hat{\lambda}$ $\hat{\beta}$	MSE $(\hat{\alpha})$ MSE $(\hat{\lambda})$ MSE $(\hat{\beta})$	RAB $(\hat{\alpha})$ RAB $(\hat{\lambda})$ RAB $(\hat{\beta})$	95 % Confidence Interval	
				LCL	UCL
50	0.846 1.439 2.534	0.0203 0.0076 0.0295	0.0575 0.0407 0.0136	0.5816 1.3166 2.2043	1.1104 1.5614 2.8637
100	0.839 1.446 2.529	0.0194 0.0085 0.0210	0.0488 0.0360 0.0116	0.5768 1.2993 2.2504	1.1012 1.5927 2.8076
150	0.810 1.512 2.511	0.0133 0.0079 0.0170	0.0125 0.0080 0.0044	0.5848 1.3389 2.2562	1.0352 1.6851 2.7658
200	0.792 1.520 2.502	0.0115 0.0096 0.0157	0.0100 0.0133 0.0008	0.5827 1.3320 2.2564	1.0013 1.7080 2.7476
250	0.784 1.534 2.498	0.0105 0.0175 0.0121	0.0200 0.0227 0.0008	0.5860 1.2838 2.2824	0.9820 1.7842 2.7136

Table 2: Simulation Study Results with $\alpha = 0.80$, $\beta = 2.50$, $\lambda = 1.50$, $s = 4$ and $t = 6$

n	$\hat{\alpha}$ $\hat{\lambda}$ $\hat{\beta}$	MSE$(\hat{\alpha})$ MSE$(\hat{\lambda})$ MSE$(\hat{\beta})$	RAB$(\hat{\alpha})$ RAB$(\hat{\lambda})$ RAB$(\hat{\beta})$	95 % Confidence Interval	
				LCL	UCL
50	0.893	0.0225	0.1163	0.6619	1.1241
	1.594	0.0130	0.0627	1.4669	1.7210
	2.587	0.0237	0.0348	2.3383	2.8357
100	0.889	0.0204	0.1113	0.6699	1.1081
	1.568	0.0113	0.0453	1.4076	1.7284
	2.542	0.0201	0.0168	2.2769	2.8071
150	0.874	0.0313	0.0925	0.5592	1.1888
	1.502	0.0083	0.0013	1.3234	1.6806
	2.499	0.0204	0.0004	2.2191	2.7789
200	0.832	0.0178	0.0400	0.5779	1.0860
	1.483	0.0054	0.0113	1.3430	1.6229
	2.474	0.0199	0.0104	2.2024	2.7456
250	0.804	0.0087	0.0050	0.6212	0.9868
	1.492	0.0122	0.0053	1.4799	1.7076
	2.482	0.0231	0.0072	2.1860	2.7780

6 DISCUSSION AND CONCLUSIONS

In this paper the problem of constant stress ALT with type-I censored Weibull failure data using GP has been considered. The MLEs, MSEs, RABs the 95% asymptotic confidence intervals estimates of the model parameters were also obtained.

From the results in Table 1 and 2, it is easy to find that estimates of the parameter perform well. For fixed θ, α and λ, the MSEs and the RABs of θ, α and λ decreases as n increases. This indicates that the ML estimates provide asymptotically normally distributed and consistent estimator for the parameters. For the fixed sample sizes, as the termination time t gets larger the MSEs and RABs of the estimators decrease. This is very usual because more failures are obtained due to large values of censored time and thus increase the efficiency of the estimators.

From above discussion and results it may be concluded that the present model work well under type-I censored data for Weibull distribution and would be a good choice to be considered in ALTs in future. For the perspective of further research in this direction one can choose some other lifetime distribution with different types of censoring schemes.

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