MULTI PHASE RELIABILITY ANALYSIS WITH VARIABLE PHASE DURATION AND VARIABLE PHASE SEQUENCE

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ABSTRACT

This paper proposes a new simulation model for evaluating reliability of phased mission systems with variable phase durations and variable sequencing. The model simulates the system failures within target operating period using a modularized phased mission system model. Monte Carlo method has been used to simulate the reliability of phased mission and mission phases. A case study on electronic lighting control system is given to demonstrate the proposed simulation model. The effects of assuming constant duration and constant sequencing in the mission have been discussed during the case study.

1. Introduction

Many systems work in mission which can be divided into several phases. In different phases, the system configurations and the requirements of the system may be different. For example, the voyage of an aircraft can be divided into several tasks, such as take-off, cruise, and landing, each with completely different reliability requirements and behaviour. Usually each task can be treated as a phase of the system. These kinds of systems are called as multi phase systems. If the system successfully operates throughout all of the phases then the mission said to be a success. Phased mission systems are encountered in many industrial fields, such as nuclear, aerospace, chemical, electronic, navigation, and military fields, etc.

Compared with single-phase systems, reliability analysis of multi phase system is much more complex, because of the dependence across the phases. For instance, the state of a component at the beginning of a new phase is identical to its state at the end of the previous phase [1]. The dynamic structure and configuration of a multi phase system usually requires a distinct model for each phase, which also increases the complexity of modelling & analysis. Although reliability analysis of multi phase systems has been studied for more than 20 years, the size of problems that can be solved is still very small due to the high computational complexity of the known methods [2].

There are two classes of models applied in reliability evaluation for phased mission system: combinatorial and state-based.

The most widely used combinatorial models are Fault tree (FT) and Reliability block diagram (RBD). The combinational models are able to describe the mapping relation of model elements and system faults. FT and RBD can be conveniently constructed with classical qualitative analysis method, such as FMEA. However, the combinational models exhibit a lot of drawbacks and limitations considering the dependency relation of components, the maintenance of broken-down components. The assumption of combinatorial models for multi phase analysis is that all the states of all components in the system are s-independent [2]. This assumption can simplify the analysis, however it limits the applicability of the models because s-dependence within as well as across the phases does occur in some cases. When there is no s-dependence within a phase, the s-dependence across the phases still needs to be accounted. Esary Introduced a method which can deal with the s-dependence across the phases using a set of s-independent mini-components to replace the component in each phase [1]. However, this method causes the size of the problem to become very large as the number of phases increases. As the number of components and phases becomes large, the number of disjoint products also increases rapidly, which consequently increases the amount of storage and computation-time. The computational complexity of combinatorial models is much less than that of state based models. However, these models usually need to find mincuts of systems, and to calculate the sum of disjoint products, which is still computationally intensive.

State-based model are more feasible to formalize phased mission. The state-based models such as Markov Chains and Petri Net can explain the complex dependency exist in the components. The main idea of Markov-chain based models is to, directly or indirectly construct a Markov chain to represent the system behaviour. These models at once account for the dependence among the components within a phase and the dependence across the phases. However, Markov-chain based models suffer from the state explosion problem when the number of components becomes large.

In recent times, Amari [3] presented a methodology for analytical analysis of phased missions which is based on the solution of cumulative exposure model for k-out-of-n systems. Chew [4] described the use of a Petri net to model the reliability of the maintenance-free operating period (MFOP) and phased mission scenario.

Multi phase systems sometimes consist phases which may not be characterized as sequential, fixed duration or both. We call these systems as multi phase system with variable duration and variable sequencing. A typical example of a phased mission with variable duration and variable sequencing is job shop where the sequence of machines used and duration of machines used is not fixed. Usually these systems are analysed with an assumption of constant sequencing and constant duration. This may lead to over estimate or under estimate the reliability characteristics of the system.

Most of the analytical approaches discussed earlier will become almost impossible to apply when the system is following variable sequence of phases with variable phase durations. Because of this reason Simulation based approach has been selected for analysis of multi phase systems with variable sequence of phases and variable phase durations. It allows the modelling of any reliability distribution without particular restrictions. This paper presents a simulation model for evaluating phased mission system reliability with variable duration variable sequence of phases. The simulation based model focuses on the logic rather than mathematical relationship, which makes it perfectly suitable for analysis of variable sequence, variable duration phased mission systems.

Section 2 describes the problem statement and presents a methodology for user profile data collection and analysis. Section 3 describes the simulation algorithm. Section 4 explains the proposed algorithm with the help of a case study.

2. Problem statement

The phased mission systems studied in this paper are non repairable phased mission system, and at the beginning of mission all the components in the system are as good as new. All the components are having only two states either success or failure. System is also having only two states either success or failure. All the component failures are independent of each other.

The basic method proposed in this paper assumes that durations of all phases are variable and the sequence in which the phase's occur is also not fixed. So from mission to mission the sequence of operation of phases and duration of phases may change. In any particular mission if the system is successfully able to complete the required phase sequence for required phase durations, the mission can be treated as success. Missions will be continuously performed one after one till the completion of target time period. The objective of the model is to calculate the probability of completion of all the missions within the target time period with out a single failure.

The Phase durations are assumed to be following a weibull distribution. The patterns of the phase's durations and phase sequences have to be studied based on the system usage data. Mathematically modelled phase durations and sequences will be used to simulate the system behaviour.

2.2. User profile data collection and analysis

User profile captures the way product will be used by the end user. If a product is having "n" different functions or phases of operation, the user may not be using all the "n" phases at a time or in a particular sequence. There are two parameters which decide the usage profile of the product. First, the sequence of phases in which the product is used. Second, the duration of each phase. For example if a system has three phases A,B,C then the some of possible sequences in a mission will be ABC, BCA,CAB,AB,BC,CA,A,B,C etc. By assuming that these sequences are mutually exclusive the probability of occurrence can be calculated. For this the data has to be collected over certain number of missions regarding the occurrence of various phase sequences. The duration of each individual phase may also follow a particular pattern which has to be modelled using the duration data of various phases.

In the subsequent case, the data is collected as shown in Table 1 for calculating the probability of occurrences of various sequences. For example sequence ABC has been occurred S1 times out of S number of observed missions. So the probability of occurrence of ABC sequence is the ratio of S1 to S. similarly other sequence probabilities also calculated. CP is the cumulative probability of sequences. For example CP2 is the sum of CP1 and P2. Similarly remaining cumulative probabilities are calculated. Here in the Table 1 only certain number of possible sequences is shown. There can be many possible sequences in a mission. Sequences with negligible possibility of occurrence should be avoided to reduce the unnecessary complexity in analysis.

Table 1: The probability of occurrences of various sequences

S.No (i)	Sequence	Si	Pi ($S = \sum Si$)	$CP_i = \sum_1^i P_x$
1	ABC	S1	S1 / S	CP1
2	BCA	S2	S2 / S	CP2
3	CAB	S3	S3 / S	CP3
4	AB	S4	S4 / S	CP4
5	BC	S5	S5 / S	CP5
6	CA	S6	S6 / S	CP6
7	A	S7	S7 / S	CP7
8	B	S8	S8 / S	CP8
9	C	S9	S9 / S	1

Si - No of times i^{th} sequence occurred during observation period
Pi - Probability of occurrence of sequence
CPi - Cumulative probability of occurrence

The system should be monitored for certain number of missions to collect the data regarding the phase duration. For modelling the duration of various phases should be collected as shown in the Table 2.

Table 2: Data for modeling duration of phases

Mission No. (i)	A phase duration	B phase duration	C phase duration
1	t_{A1}	t_{B1}	t_{C1}
2	t_{A2}	t_{B2}	t_{C2}
3	t_{A3}	t_{B3}	t_{C3}
4	t_{A4}	t_{B4}	-
5	-	t_{B5}	t_{C5}
6	t_{A6}	-	t_{C6}
7	t_{A7}	-	-
8	-	t_{B8}	-
9	-	-	t_{C9}

Using the collected data the respective duration distributions of various phases should be determined. The parameters as shown in Table 3 should be calculated by assuming Weibull distribution as suitable distribution. For example using the data points t_{A1}, t_{A2}, t_{A3}, t_{A4}, t_{A5}, t_{A6}, t_{A7} the parameters of phase A duration distribution $\eta 1$, $\beta 1$ should be calculated. Similarly other phase's distribution parameters will be calculated.

Table 3: Parameters of modeling duration of phases

Phase Name	Eta	Beta
Phase A	η1	β1
Phase B	η2	β2
Phase C	η3	β3

3. Simulation algorithm for variable phase sequence and variable phase duration

Five modules constitute the Simulation-Based Evaluation Model of Phased-Mission System with variable phase sequence and variable phase duration. They are reliability analysis module, mission simulation module, Phase sequence simulation module, Phase duration simulation module, phase simulation module. If all the required phases are success in a mission then mission will be treated as success. Missions will be simulated one after one until total accumulated age of system is equal to the target operating life. If required operating life is successfully completed then the corresponding simulation run will be treated as success. The ratio of number of successful simulations to total number of simulation runs gives the Reliability of the product at the end of target operating hours.

Reliability analysis module calculates the probability of the system for completing target operating hours by running the preset number of simulations. It calls for the *mission simulation module* in each simulation run. It stores the out come of *mission simulation module*. The procedure will be repeated until all the simulation runs are completed. Probability of completing the target time period is the ratio of number of success outcomes of *mission simulation module* to the total number of simulation runs. Figure 1 shows the flow chart of reliability analysis module.

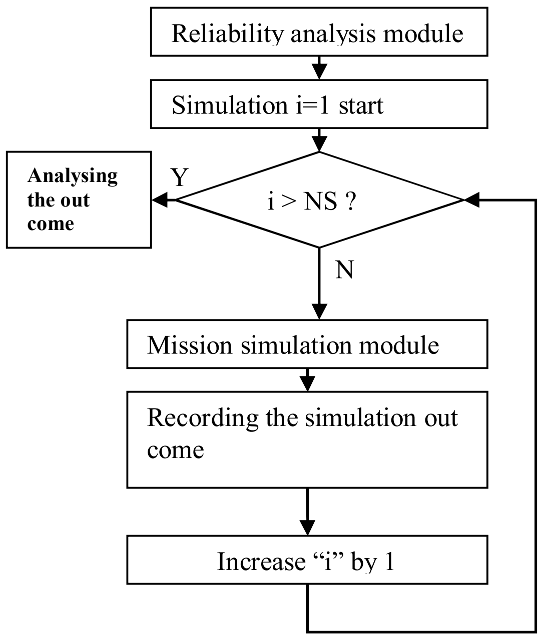

Figure 1: Flow chart of the Reliability analysis module

NS : Number of simulations

Mission simulation module returns out come of individual simulation run. This module simulates missions in loop until target operating hours are completed. For this it calls *phase sequence simulation module,* which generates a random phase sequence. Then for all the phases of the generated sequence, random duration will be generated in loop using *phase duration simulation module.* The generated two parameters phase sequence and phase duration will be passed to *phase simulation module.* This module will calculate the status of the system at the end of each phase of the sequence in loop. If outcome is success in all the phases of generated sequence then mission will be treated as success. Then it Stores the outcome of each mission and total accumulated age of the system. This process is repeated till accumulated age of the system is equal to the target operating hours. If outcome is success in all the simulated missions then success will be returned from *mission simulation module.* Flow chart of mission simulation module is as shown in Figure 2.

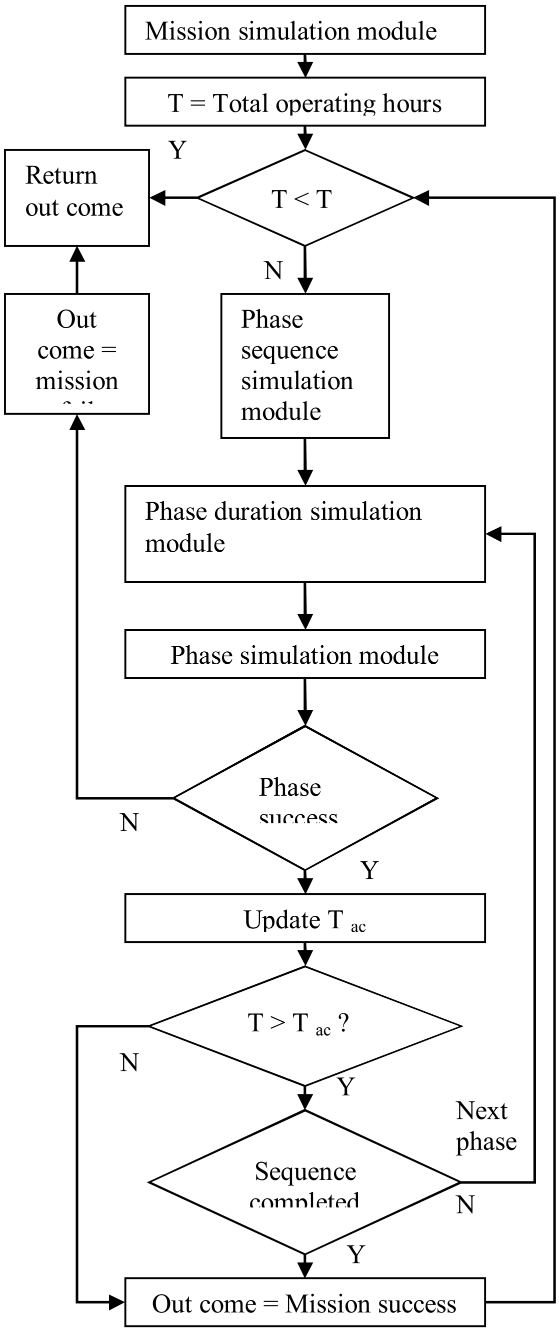

Figure 2: Flow chart of the mission simulation module

T_{ac}: Total accumulated age of system

Phase sequence simulation module function is to generate a random sequence of phases based on probabilities calculated in the usage profile analysis. For this a random number between “0” to “1” will be generated. Then it is compared with the cumulative probabilities of the sequences to select any particular sequence. For example, as per Table 1 if generated random number lies between 0 to CP1 then ABC sequence will be selected. If it lies between CP1 to CP2 then BCA will be selected. Then this randomly selected sequence is returned by *Phase sequence simulation module.*

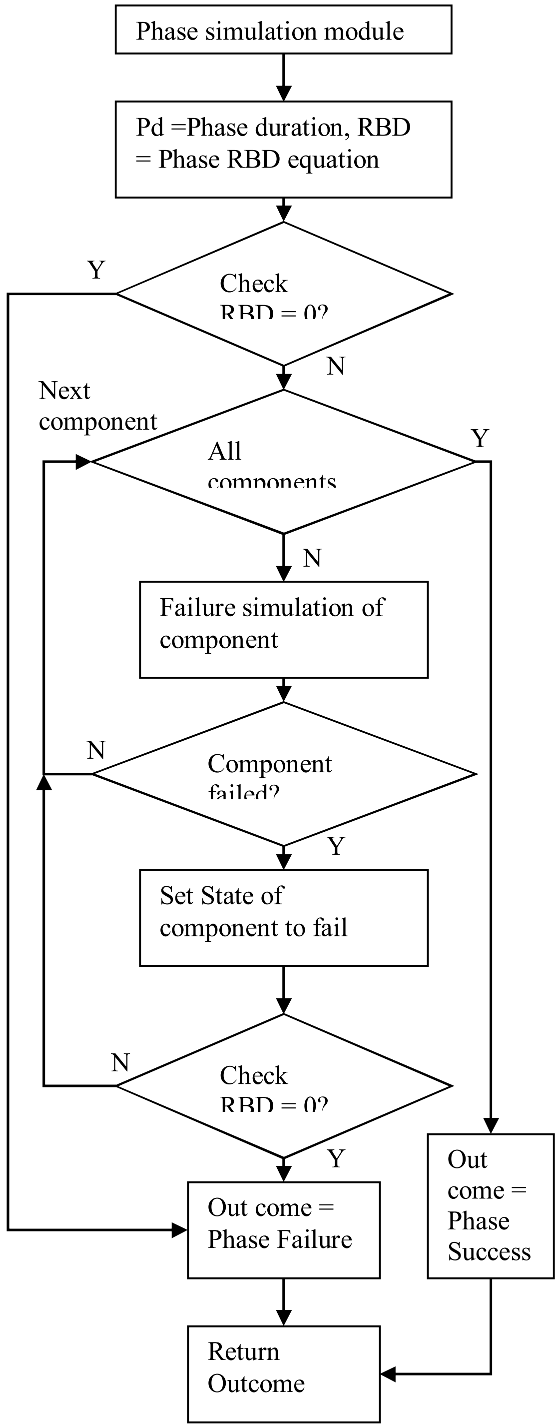

Figure 3: Flow chart phase simulation module

Phase duration simulation module generates duration of phases based on phase duration distribution parameters calculated in usage profile analysis. For this a random number 'R' should be generated between "0" to "1". By assuming Weibull distribution parameters as discussed in Table 3, phase1 duration will be calculated as follows

$$\text{Phase 1 duration} = \eta 1 * ((\ln (1/R)) \wedge (1/ \beta 1))$$

In this way required phase duration is randomly generated and returned by *Phase duration simulation module.* Then this generated phase duration will be passed to *phase simulation module*. This process is repeated till all the phases in the generated phase sequence are completed.

Phase simulation module returns status of the system at the end of called phase. It takes phase duration generated by *Phase duration simulation module* and phase sequence generated by *Phase sequence simulation module* as input. Phase simulation module simulates a phase by using classic motecarlo simulation technique. But while deciding the status of the components state dependencies across the phases have to be considered. For example, a redundant component failure may not cause the system failure in a phase but same may lead to system failure as soon as it enters into next phase because of changes in the system configuration. Ref [5], [6] can be referred for simulation of general RBD, but as discussed they has to be carefully integrated with phase dependencies. Flow chart for *phase simulation module* is as shown in Figure 3.

4. Case study

A consumer electronic "lighting controller system" having three phases A, B, C has been considered for analysis. "Lighting control system" is an electronic lighting management system which is used for controlling various lighting effects in interiors of buildings like conference rooms, office areas etc. The system composed of five circuit cards C1, C2, C3, C4 and C5. Different configuration of circuit cards will provide various lighting effect inside the interior. For study purpose three effects class 1(welcome lighting effect), class 2 (meeting lighting effect), class 3 (lecture lighting effect) are considered. For realistic calculation of reliability multi phase modelling and analysis has to be carried out by considering each class as one phase. During phase A (class 1) system will be working in k-out-of-n configuration. C1, C5 and any 2 out 3 among C2, C3, C4 has to work. Under phase B (class 2), C1, C2, C3, C4 has to work for the system functioning. Phase C (class 3) requires all cards functioning. In each phase the configuration of the system RBD is as shown in Figure 4 to Figure 6.

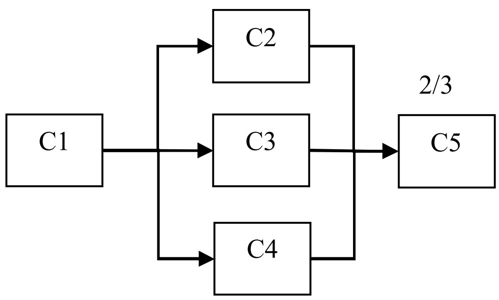

Figure 4: RBD of Phase A

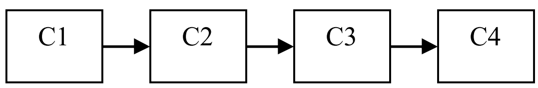

Figure 5: RBD of Phase B

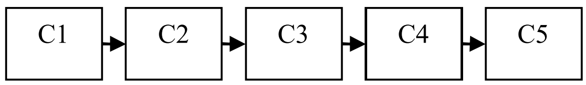

Figure 6: RBD of Phase C

In this kind of systems the user can use the system in any sequence of phases for any duration to fulfil his requirements. If the mission is defined as operation of the product in one ON to OFF, then the mission may consists any sequence of phases and for any phase duration. So the system is assumed to be following varying sequence and varying duration. If the mission is defined as operation of the product in one ON to OFF, then the mission may consists any sequence of phases and for any phase duration. So one particular customer base usage patterns are observed for certain number of missions and corresponding data has been analyzed to calculate the probability of occurrence of various sequences. Data for durations of phases A, B, C in various missions also have been collected which helps to determine the distribution parameters of the phase durations. A, B, C Phase durations distributions are calculated as shown in Table 4. The failure distributions of C1, C2, C3, C4 and C5 are as discussed in Table 5. Phase sequence patterns are having the probabilities as shown in Table 6.

Table 4: Phase Duration distribution

Duration distribution	**Eta**	**Beta**
Phase A	1.08	2.6
Phase B	2.66	3.5
Phase C	3.05	4.8

Table 5: Circuit failure distribution

S. No	**Circuit card**	**Constant Failure rate (FPMH)**
1	C1	0.83
2	C2	5.7634
3	C3	8.942
4	C4	6.97
5	C5	9.7024

Table 6: Phase sequence probabilities

S.No (i)	**Sequence**	**Si**	**Pi (S = ∑Si)**	**$CP_i = \sum_1^i P_x$**
1	A	5	5/ 60 = 0.08333	0.083333
2	B	10	10/ 60 = 0.16667	0.25
3	C	5	5/60 = 0.08333	0.333333
4	AB	5	5/60 = 0.08333	0.416667
5	BC	5	5/60 = 0.08333	0.5
6	ABC	30	30/60 = 0.5	1

The algorithm discussed in section 3 has been utilized to analyze the lighting management system. After running the simulation for 10000 runs the reliability of the system for life of 1000 hours has been calculated as 0.9739. Unsuccessful rates of each phase can also be estimated with the same algorithm by further analyzing the simulation out come.

The same system has been analyzed using Blocksim8 software with an assumption of constant phase duration and constant phase sequencing. The system has been modeled as shown in the Figure 7 the duration of each phase has been taken as the mean value of the phase duration distributions as mentions in Table 4.

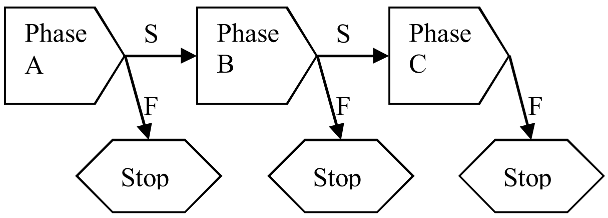

Figure 7: Phase diagram by assuming constant sequence

The BlockSim8 analysis estimates a reliability figure of 0.972 which is less comparing with the reliability calculated using the proposed algorithm. So In this particular example, assuming constant duration and constant sequencing may lead to wrong estimates of reliability. This may call unnecessary design improvements. The suggested algorithm can also be used for constant sequencing and constant duration of phases by eliminating the phase duration simulation module, phase sequence simulation module. Then it simulates the reliability similar to Blocksim8.

5. Conclusion

In this paper, a new simulation model for evaluating reliability of phased mission systems with variable phase duration and variable sequence is proposed. This model is modular in nature constitutive of reliability analysis module, mission simulation module, Phase sequence simulation module, Phase duration simulation module, phase simulation module. A methodology to model the usage profile data is also discussed in detail. The case study in section 4 demonstrates the usage of the model.

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7. References

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