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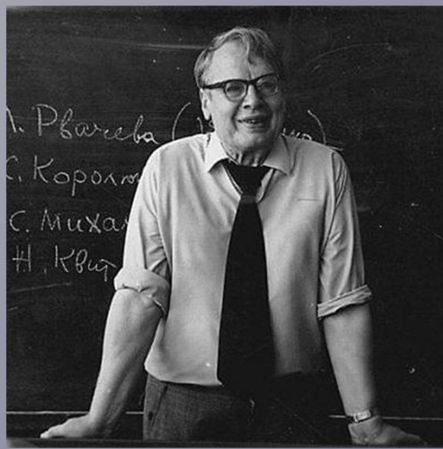

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RELIABILITY:
THEORY&APPLICATIONS

RELIABILITY
RISK ANALYSIS
MAINTENANCE
SAFETY

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RELIABILITY:

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Table of Contents

Depending on the goals of studies, the object of studies may be either an energy complex as a whole, or individual energy systems it includes, or separate elements of the systems.

There are sufficient conditions of the ergodicity for queuing systems in a random environment. But as theoretically so practically it is very important to obtain a criterion of the ergodicity which defines an ability to handle customers of these systems and a possibility to analyze them in a regime of heavy traffic. Among queuing systems in the random environment there are systems with the hysteresis control which are very important in modern applications. In this paper the criterion of the ergodicity is obtained for one server queuing system in the random environment. This criterion is based on a reduction of this queuing system to classical Lindley chain. Some asymptotic formulas in the heavy traffic regime are obtained for this queuing system also.

Winter sports can be associated with risk of sustaining injuries. The risk reduction is possible as a result of an analysis, portraying the most dangerous incidents and undesired events. Decreasing the frequency of such events or reducing their consequences can limit the overall risk associated with snowboarding. First, a preliminary selection of undesired events was performed using the MIL-STD-882 matrix method. Then, a graph showing the most likely categories of body injuries that may occur during one day of snowboarding was developed. The graph allowed for determining events associated with the highest risk of injury.

Basic notions of the ageing multistate systems safety analysis are introduced. The system components and the system safety functions are defined. The mean values and variances of the multistate system lifetimes in the safety state subsets and the mean values of its lifetimes in the particular safety states are defined. The notions of the multi-state system risk function and the moment of exceeding by the system the critical safety state are introduced. A series and a parallel-series safety structures of the multistate systems with ageing components are defined and their safety function are determined. As a particular case, the safety functions of the considered multi-state systems composed of components having exponential safety functions are determined. An applications of the proposed multistate system safety models to the prediction of safety characteristics of a maritime ferry operating at winter conditions technical system is presented as well.

Nowadays renewable energy sources attract attention of humanity because the depletion of conventional nonrenewable energy (coal, gas, oil, etc.) is getting increasingly obvious. Wind energy is characterized by a considerable potential among the renewable resources. Human civilizations have harnessed wind since long ago. In the ancient times wind was used to propel boats. It is known that even 3000 years BC the citizens of Alexadria had used "wind wheels". In the 16th century the Netherlands had more than ten thousand wind-driven plants that were used to dry lakes for cultivation area. In 1888 the USA constructed a large wind power plant for electricity production. The multi-blade wind motors invented by the engineer Davydov appeared at the Russian Exhibition in Nizhny Novgorod in 1896. Wind mills found wide application. In the USSR the first 100 kV wind power plant was built in the Crimea in 1931 and was in operation until World War II. Currently wind energy is widely used in more than 60 countries of the world. Today 10 leading countries account for about 86% of all wind power capacities installed in the world, of which more than 38% are situated in China and the USA. In Europe wind energy is mostly used in Germany, Denmark, Spain, Portugal, and France. The total installed capacity in the world reached 194 GW in 2011 and continues to soar.

In different applications (for an example in the mining engineering) a problem of a definition of a set in two or three dimension spaces by a finite set of points origins. This problem consists of a determination in the finite set of some subset of points sufficiently close to each other. A solution of this problem consists of two parts. Primarily initial finite set of points is approximated by point Poisson flow in some area which is widely used in the stochastic geometry [1, sections 5, 6]. But a concept of a proximity is analyzed using methods of the random graph theory like a concept of maximal connectivity component [2] - [4]. This concept origins in a junction of the combinatory probability theory and of the graph theory. An analysis of these concepts and mathematical constructions leads to a generalization of the random graph theory theorems onto graphs generated by point Poisson flow in some area.

The result of comparison of criteria, which statistics characterize differing properties of random variables of sample, depends on the importance of these properties. In turn, the importance of properties can essentially change for modeled analogues of sample.

In this paper, the authors' research work is focused on imperfect inspection policy investigation, when not all defects are identified during inspection action performance and probability of defect identification is not a constant variable. They are interested in Block Inspection Policy performance for multi-unit systems, the maintenance policy which is one of the most commonly used in practice. As a result, at the beginning, few words about delay time modelling approach and a brief literature overview is given. Later, the model of Block-Inspection Policy is provided. The numerical example with the use of QNU Octave program is given. In the next Section, the sensitivity analysis of the developed model is characterized. The article ends up with summary and directions for further research.

In Accelerated life testing (ALT), generally, the estimates of original parameters of the life distribution are obtained by using the log linear function between life and stress which is just a simple re-parameterization of the original parameter but from the statistical point of view, it is preferable to work with the original parameters instead of developing inferences for the parameters of the log-linear link function. By the use of geometric process one can easily deal with the original parameters of the life distribution in accelerated life testing. In this paper the geometric process is used in accelerated life testing to estimate the parameters of Weibull distribution with type-I censored data. The maximum likelihood estimates of the parameters are obtained by assuming that the lifetimes under increasing stress levels form a geometric process. In addition, asymptotic confidence interval estimates of the parameters using Fisher information matrix are also obtained. A Simulation study is also performed to check the statistical properties of estimates of the parameters and the confidence intervals.

We extend the Probabilistic ideas from stochastic processes (queuing theory and reliability) on creation of some realistic models for studying several governing political formations, and find their survival characteristics. These models were presented at the Sixth and Seventh International Conferences on Mathematical Models in Reliability (Moscow 2009, and Beijing 2011). Our focus is on a “democracy” model, where the times of survival (existence at the political scene, duration of stay in leading coalition, governing survivability, life time distribution, longevity, etc.) can be derived from the model. Markovian models of spending time in certain sets of states are explored, and some discussion on statistical properties and evaluations are presented. We are confident that other political schemes also can be modeled using appropriate probabilistic tools.

RELIABILITY OF ENERGY SYSTEMS

Dubitsky M.A.

Introduction. Depending on the goals of studies, the object of studies may be either an energy complex as a whole, or individual energy systems it includes, or separate elements of the systems. The studied object may perform different functions.

The main specified functions include:

a) *purpose* of the object;
b) the fact of its *creation* [1].

Ability of the object to perform the function associated with its *purpose* is referred to as power supply reliability (Fig. 1). *Power supply reliability as applied to energy systems is ability of the facility to supply the target product of the required quality to consumers following the given schedule of consumption.* An energy complex unites power systems, systems of gas, oil, coal and heat supply, and nuclear energy systems. Purpose of the electric power system (EPS) in particular is power supply for consumers. *Power supply reliability of EPS is its ability to supply power of the required quality following the given power consumption schedule.* Power supply reliability is reliability in "narrow sense" as consideration is given to the specified function only (a function related to the object purpose) [2].

Power supply reliability is a complex property that may include several unit properties: failure-free operation, maintainability, survivability, stability, controllability, durability and conservability. Durability and conservability are characteristic of the system's elements rather than of the system as a whole.

Another specified function of the object conditioned by the fact of its *creation* is avoidance of situations hazardous for people and environment that are caused by failures during the object operation [1]. *Ability of the object to avoid situations that are hazardous for* people and environment is referred to as safety (Fig. 1).

Reliability is ability of the object to perform all the specified functions in the required scope under certain operating conditions (Fig. 1). Reliability is a complex property. Difference between reliability and power supply reliability lies in the fact that reliability includes *safety* as additional unit property.

The notion of reliability is being shaped. Should one compare the first (1980) and second (2007) editions of glossaries on the energy systems reliability he would notice that the definition of reliability and of its unit properties has changed [3, 4]. At the moment there is a need to change (clarify) the content of such unit properties as failure-free operation, controllability and survivability.

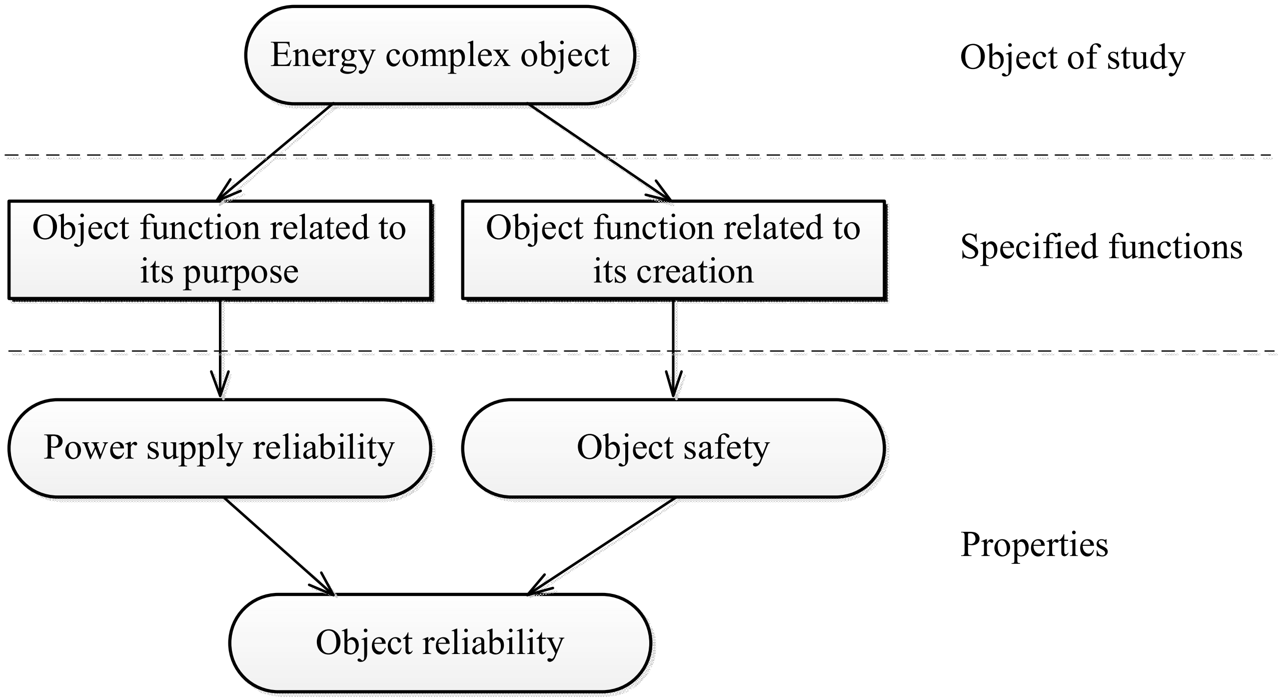

Fig. 1. The ratio of notions: an object, its specified functions and properties

(Object of energy complex; Object function related to its purpose; Object function related to its creation; Power supply reliability; Object safety; Object reliability. Object of studies. Specified functions. Properties)

Failure-free operation *is ability of the object to continuously maintain operability or operable state during a certain period of time [1, 3].* The facility is subjected to different disturbances. All the disturbances effecting the facility can be divided into two main groups: whether they are *external* with respect to the object or they are of *internal* origin. It was recommended that assessment of failure-free operation should take into account all the disturbances irrespective of the fact whether they are major or minor, internal or external with respect to the object [1].

A different approach is also possible that is more constructive and, therefore, more preferable as it allows one to pay special attention to extreme disturbances, to principles (criteria) of decision-making and to reliability support activities in the extreme conditions. The essence of the method lies in that the disturbances are identified and considered separately, within the survivability property rather than within the failure-free operation property. Such approach to accounting different disturbances is preferred by well-known specialists in the field of reliability of technical systems [5-7].

Therefore, in the study of the failure-free operation it is sufficient to take into account all the disturbances of the *internal* origin, i.e. equipment failures (drawbacks of operation, maintenance defects, manufacturing defects, and end of service life); and errors of operating personnel [1]. As to external disturbances, one should take into account only the disturbances the object is designed for (thunderstorms, earthquakes (but within the design seismicity of the area) etc.).

The effect of factors reducing failure-free operation ability and, hence, reliability could be fully or partially balanced by [8]: selection of the appropriate design of the system; higher reliability and enhanced equipment performances (including equipment and devices of control means and systems); redundancy in all the elements of the system; perfection of the system operation management.

Controllability is ability of the object to maintain normal operating conditions [1,3,4]. Such a definition does not give a comprehensive idea of the term. It does not take into account that an object can be controlled in different operating conditions, including control in emergency conditions. It does not take into account requirements to control in different operating conditions. For example, the object control in emergency conditions at high controllability shall not result in cascade emergency with large-scale interruption of power supply for consumers.

The main requirements to the definition of controllability could be:

- the object control at high controllability in emergency shall not result in cascade emergency with large-scale interruption of power supply for consumers;
- high controllability shall ensure parameters control and their input into the feasible region [6];
- high controllability of the object shall allow maintenance of normal operating conditions using by control [1].

Those requirements being taken into account, the following definition of controllability could be proposed: *Controllability is ability of the object: not to allow cascade development of emergencies with large-scale interruption of power supply for consumers; bring the operating conditions back into the allowable region and maintain the specified parameters by control.*

Controllability is ensured by: sufficient control range and mobility of the main equipment; mobility of standby capacity; redundancy of transfer capability of the network; selection of the architecture and parameters of the automatic control devices, and of automatic emergency and on-line control.

Survivability is ability of the object to resist to disturbances avoiding their cascade propagation resulting in large-scale interruption of power supply for consumers. This definition of 'survivability' term is given in the first and second editions of Reliability of Energy Systems glossary [3,4]. Nevertheless, 'cascade propagation' does not mean survivability failure. As it has already been noted, it is a feature of insufficiently high controllability. Large-scale interruption of power supply for consumers is not an indicator of survivability failures either. It can be caused, for example, by technological process breakdown in the system that (as evidences the analysis of major emergencies) can be promptly mitigated and the object continue operation in normal conditions.

The main indicator of survivability failure is a failure at extreme external impact on the system. Extreme *external* disturbances include [5]:

- External impacts on the object that were not considered during its design (hurricanes, earthquakes, tsunami, etc.);
- Deliberate actions (sabotage attacks, acts of terror, military actions, etc.).

Such disturbances result in partial or even complete destruction of the object. Let 'survivability' denote the reliability in extreme conditions [1]. *'Survivability' is ability of the object to resist to external disturbances it is not designed for in normal operating conditions.* 'To resist' means that the level of the system operation at extreme external impact shall not be lower than the minimum feasible one.

Minimum feasible level of operation at extreme external impact could be ensured by: emergency power backup; selection of the system structure and operating regimes (preventive reduction of the system operating level); automatic and on-line control; reserve stock of the main equipment and consumables and their optimum location in the system; anti-terrorism activities.

Safety is a unit property of reliability. As a matter of fact, assessment of the relevancy of safety problems for energy systems is noted to be interpreted in different ways by the same authors. On the one hand, "research in this field has become more intense" [9]; on the other hand, "the main reason why safety was excluded from the list of reliability properties of energy facilities is the fact that this property has not been in demand for 27 years" [4]. *Firstly, such wording does not contain a substantial justification for exclusion of this property, and, secondly, with account of major* *emergencies that occurred at energy systems in the past 30 years, it is evident that the problem of safety assurance needs to be more thoroughly considered [2].*

As opposed to power supply reliability, safety is a multi-aspect property. For assessment of power supply reliability, it is sufficient to control supply of the target product for the customers, while it is not sufficient for safety analysis. Situations causing hazards to people and environment may not be related to supply of the target product to the customers. It is thus advisable to review a ratio between safety, failure-free operation, controllability and survivability (particularly in view of the fact that the content of the notions for unit properties of power supply reliability has been revised).

Ratio between failure-free operation, survivability and controllability Figure 2 shows a set of disturbances (denote it as A) which may upset failure-free operation. Some of the disturbances are associated with cascade propagation of emergencies and massive interruption of power supply for consumers due to insufficiently high controllability of the object. Let *B* be a set of disturbances that upset failure-free operation and were associated with cascade propagation of emergencies and massive limitation of power supply to consumers due to insufficiently high controllability (Fig. 2).

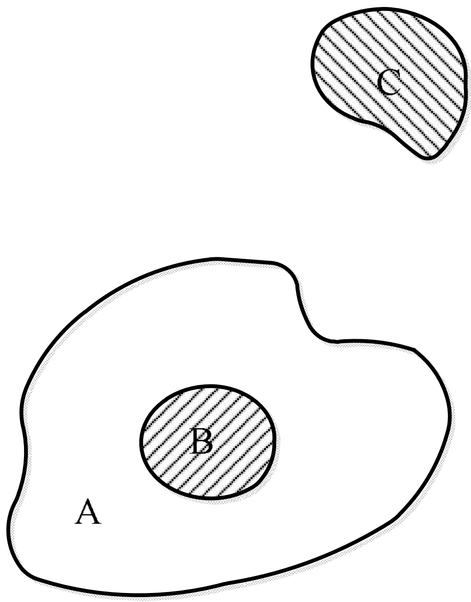

Fig. 2. Ratio between sets of disturbances that upset failure-free operation and survivability: *A* is a set of disturbances that upset failure-free operation; *B* – is a set of disturbances that upset failure-free operation and are associated with cascade propagation of emergencies due to insufficiently high controllability; *C* is a set of disturbances that upset survivability.

$$B \subset A. \qquad (1)$$

Cascade emergencies, for instance at EPS, may occur due to the following causes [1]:

- Short circuit in elements of the system (about 10 per cent of all the failures of elements in the main 400-750 kV networks are accompanied by failures of other elements);
- Overload or power jump in power transmission due to capacity disbalance in the interconnected parts of the system.
- False tripping of power line, buses or transformers by relay protection or automatic emergency devices.
- Non full-phase regime due to failure of breakers at operative switching-overs.
- Erroneous shut down of power supply by the operating personnel.

Disturbances that result in survivability failure are not considered in studies of failure-free operation. As it has already been said, extreme disturbances may be the consequence of floods, tsunami, typhoons, earthquakes, snowfalls of deliberate external acts. Therefore, a set of disturbances resulting in survivability failure has no elements common with a set of disturbances that upset failure-free operation. Let C be a set of extreme disturbances that may result in survivability failure then

$$A \cap C = \emptyset \,. \tag{2}$$

Survivability failures are, as a rule, associated with the cascade propagation of emergencies. For this reason the set C in Fig. 2 is shaded.

Ratio between survivability, controllability, and safety . All the disturbances that may cause survivability failures will also be the cause of safety failures (Fig. 3) as survivability failures are associated with the target product shortage, destruction of the object, and situations hazardous for people and environment. Studies on the object safety shall be preceded by analysis of their survivability. Higher survivability of the facility enhances its safety.

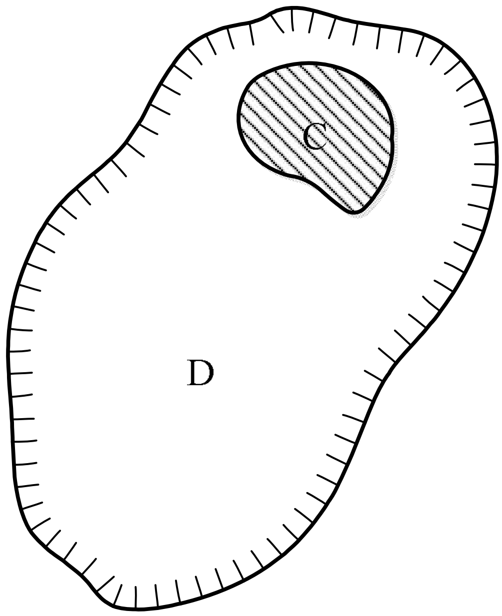

Fig. 3. Ratio between sets of disturbances that upset survivability and safety. *C* is a set of disturbances that cause survivability failure; D is a set of disturbances that cause safety failures.

But, on the other hand, not all the disturbances that cause safety failure would cause the survivability failure. For example, the emergency at Sayano-Shushenskaya Power Plant that caused considerable destruction and death of people was an upset of failure-free operation, but it was not a survivability failure. Failure of electric precipitators at thermal power plants that may cause higher emission of pollutants with flue gases and, hence, safety failure, is not a survivability failure.

Not all the activities used to raise safety enhance survivability. Location of some or other EPS facilities (of power plants, in particular) impacts EPS safety, but it has no impact on the EPS survivability and failure-free operation. Therefore, higher safety does not always result in higher survivability. Let D be a set of disturbances that may result in safety failures. then

$$C \subset D \,. \tag{3}$$

Ratio between failure-free operation, controllability, and safety. Disturbance of failure-free operation may be at the same time the cause of safety failures. First, safety failures may be caused by failure-free operation disturbances associated with cascade propagation of emergencies and notable shortage of the target product supply to consumers.

Then

$$B \subset D \quad (4)$$

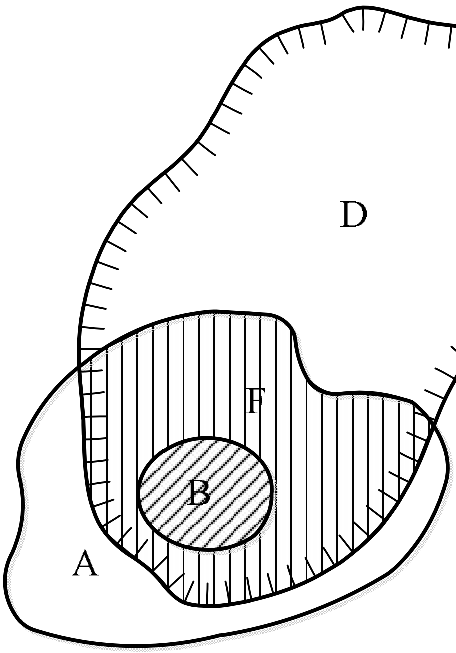

Fig. 4. Ratio between sets of disturbances that upset failure-free operation and safety: A is a set of disturbances upsetting failure-free operation; D is a set of disturbances causing the safety failures; B is a set of disturbances upsetting the failure-free operation associated with the cascade propagation of emergencies due to insufficient controllability; F is a set of disturbances that cause the safety failures and upset the failure-free operation, but do not cause the cascade propagation of emergencies.

Such emergencies are frequent at EPS of different countries. They occur on the average once in two years (there were 20 major emergencies in the 40-years period between 1965 and 2005). Some emergencies were major ones. Hundreds of thousands and even millions of people had no power supply. Thousands of people were 'prisoned' in lifts and metro trains, there were fires, robbery, etc. Examples of such emergencies are: emergency in the USA on November 9-10, 1965; emergency at EPS of the USA and Canada in 2003, Moscow systems emergency in 2005, systems emergency in St.- Petersburg on August 20, 2010; emergency in Canada on January 23, 2005, and emergency in Brazil in 2009. Such emergencies are in the focus of attention of the countries' authorities.

Secondly, safety failures without cascade propagation of emergencies are also possible, for example, due to emergency disconnection of certain consumers of Grade 1 or due to emergency shut down of electric pumps in the sewage system. It would result in emergency discharge of wastes, pollution of the area and water reservoirs. Let F be a set of disturbances that upset safety and failure-free operation without cascade propagation (Fig. 4). Then

$$A \cap D - B = F \quad (5)$$

Ratio between failure-free operation, controllability, survivability and safety of EPS. Not all the disturbances that cause safety failure would cause the survivability failure and upset failure-free operation. The non-shaded area in Fig. 5 corresponds to the set of disturbances D.

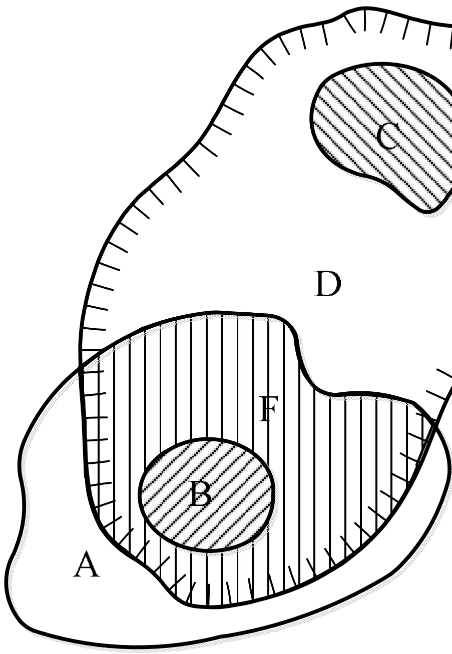

Fig. 5. Ratio between sets of disturbances that upset failure-free operation, survivability, and safety: A is a set of disturbances upsetting failure-free operation; D is a set of disturbances causing the safety failures; B is a set of disturbances upsetting the failure-free operation associated with the cascade propagation of emergencies due to insufficient controllability; F is a set of disturbances causing the safety failures and upsetting the failure-free operation, but they do not cause the cascade propagation of emergencies; C is a set of disturbances causing the survivability failures.

They include, for example, disturbances caused by destruction of ash dumps of Thermal Power Plants that is followed by destruction of other facilities, pollution of the area, water reservoirs, etc. Let L be a set of such disturbances. Then

$$D - A \cap D - C = L. \quad (6)$$

Safety failures can be replaced by disturbances of failure-free operation. For example, at a stage of EPS design it is not always possible to avoid the excess of maximum permissible concentrations of hazardous substances. Sometimes for a short period of time the concentration of hazardous substances in the surface air may rapidly increase, for example, under availability of raised inversions located immediately over the stacks of thermal power plants, and in windless conditions. For avoiding the rapid growth of hazardous substances concentrations in the surface air the regimes of power plants are varied, up to complete shut down of boilers fired by fuel with high content of sulphur and ash in highly dangerous periods.

Therefore, higher safety does not always enhance failure-free operation. In some instances higher safety may cause some reduction in failure-free operation. For instance, failures of safety systems of nuclear power plants may cause emergency tripping of power generating equipment.

Therefore, causes, affecting factors and activities to ensure failure-free operation and survivability, failure-free operation and safety, safety and survivability, respectively, may differ from one another.

CONCLUSIONS

1. Consideration is given to the ratio between notions: an object, its specified functions and properties

2. Content of such notions as power supply reliability, failure-free operation, controllability, survivability and safety is discussed.

3. Definitions of such terms as power supply reliability, controllability and survivability are given.

4. The ratio between such unit properties of reliability as failure-free operation, controllability, survivability and safety is discussed.

5. Causes, affecting factors and activities to ensure failure-free operation, survivability and safety may differ.

6. New edition of glossary on "Power System Reliability. Terminology" should, firstly, take into account changes in the essence of such terms as failure-free operation, controllability, and, secondly, bring back the 'safety' term and include it into the unit properties of power systems reliability.

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ERGODICITY OF FLUID SERVER QUEUEING SYSTEM IN RANDOM ENVIRONMENT

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●

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ABSTRACT

There are sufficient conditions of the ergodicity for queuing systems in a random environment. But as theoretically so practically it is very important to obtain a criterion of the ergodicity which defines an ability to handle customers of these systems and a possibility to analyze them in a regime of heavy traffic. Among queuing systems in the random environment there are systems with the hysteresis control which are very important in modern applications. In this paper the criterion of the ergodicity is obtained for one server queuing system in the random environment. This criterion is based on a reduction of this queuing system to classical Lindley chain. Some asymptotic formulas in the heavy traffic regime are obtained for this queuing system also.

INTRODUCTION

Mathematical models of queuing systems and networks in the random environment attract an attention of specialists in the queuing theory (see for an example [1] and its bibliography) because of manifold applications to transport models [2, p. 430-432, 438] and systems with the hysteresis control [3], [4].

Deterministic models of technical systems with the hysteresis control (periodic systems close to discontinuous) are considered in the theory of ordinary differential equations with a small parameter under high-order derivatives [5], [6], [7]. But a presence of the small parameter in these models does not allow to obtain visible formulas for solutions of these equations. It is connected with sufficiently complicated behavior of their solutions - an availability of few adjacent boundary layers in vicinities of a discontinuous point.

At a same moment stochastic models of queuing systems in the random environment as a rule obtain solutions only in a form of sufficient not necessary and sufficient conditions [1, theorem 1, Formula (2)]. An importance of the ergodicity criteria is in their capability to define an ability to handle customers of queuing system [8]. So a work in this direction is actual in spite of an abundance of results in which there are formulas and algorithms of limit distributions calculations of queuing systems in the random environment.

In this paper the ergodicity criteria are obtained not by an reinforcement of known results of limit distributions calculations for queuing systems in the random environment but by a construction of sufficiently general stochastic models for queuing systems with a type of the Lindley chain [9, P. 20-36]. In frames of this approach a fluid model of one server queuing system [10], [11, p. 8-12] is considered and for this model an amount of a fluid in the system is defined in moments of their regime changes. Besides of the ergodicity criteria for the considered model asymptotic formulas for limit distributions in the heavy traffic regime are obtain also.

1. ERGODICITY CRITERIA

Consider the following fluid model of one server queuing system. Divide nonnegative half-axis $t \ge 0$ onto half-intervals $[T_0, T_1)$, $T_0 = 0$, $T_1 = T_0 + t_0$, $[T_1, T_2)$, $T_2 = T_1 + t_1$,... Here independent and identically distributed random variables (i.i.d.r.v.`s) $t_0, t_1, \ldots,$ have the distribution $G(t) = P(t_n < t)$, $t \ge 0$, $n \ge 0$, concentrated on the half-axis $t \ge 0$ and $Mt_n < \infty$. Assume that on the half-interval $[T_{n-1}, T_n)$, $n > 0$, some reservoir is replenished by a fluid with the intensity $a_n > 0$ and the fluid is pumped out with the intensity $b_n > 0$ if the fluid volume is positive. If the fluid volume is zero then for $a_n < b_n$ the outflow intensity becomes equal the inflow intensity a_n and the initial volume of the fluid in the reservoir equals $w_0 \ge 0$. Further suppose that the differences $(a_n - b_n)$, $n \ge 0$, characterizing random behavior of the environment in which the one server queuing system is situated is the sequence of i.i.d.r.v`s with the mean $M|a_n - b_n| < \infty$ and random sequences $(a_n - b_n)$, $n \ge 0$, and $t_n, n \ge 0,$ are independent.

Denote $W(t)$, $t \ge 0$, the fluid volume in the reservoir at the moment t. The function $W(t)$ is the polygonal line with the inflection points T_n, $n \ge 0$. This function is analogous to the virtual waiting time in the one server queuing system but it is not identical to it. Suppose that $w_n = W(T_n)$, $n \ge 0$, then from previous assumptions the fluid volume $w_{n+1} = W(T_{n+1})$ in the reservoir at the moment T_{n+1} satisfies the equality

$$w_{n+1} = (w_n + \xi_n)^+, \ n \ge 0, \text{ where } d^+ = \max(0, d). \tag{1}$$

From the ergodicity theorem for the Lindley chain $w_n, n \ge 0,$ [9, §3, theorem 7] the necessary and sufficient condition of its ergodicity is the inequality

$$M\xi_n = Mt_n M(a_n - b_n) < 0. \tag{2}$$

Remark 1. This ergodicity criterion is true for more general assumptions for a stationarity of the random sequence ξ_n, $n \ge 0$, in the narrow sense.

2 ASSIMPTOTIC ANALYSIS IN REGIME OF HEAVY TRAFFIC

Obtained results allow to transfer well known asymptotic formulas for the Lindley chain onto fluid one server queuing system in random environment which may be represented as the queuing system with the hysteresis control. If

$$c = |M\xi_n| \to 0, \ d = D\xi_n = const,$$

then in the condition $M|\xi_n|^3 < \infty$ (9,[chapter 1, formulas (57), (58)],[13]) we have well known asymptotic formula for the limit distribution of the Markov chain w_n, $n \ge 0$: for any $x > 0$

$$\lim_{n\to\infty} P(w_n > x/|c|) \sim \exp(-2x/d), \ |c| \to 0.$$

Refinements of these results may be found in [9, p. 65-67],[14,chapter. III]. These refinements are based on the diffusion approximation of the random sequence (1).

In the conclusion consider the case when $c \to 0$, $d = d(c)$. Assume that the random variables ξ_n satisfy the following conditions. There is the sequence of i.i.d.r.v`s Δ_n, $n \ge 0$,

$$M\Delta_n = 0\,,\ D\Delta_n = f\,,\ M|\Delta_n|^3 < \infty\,,$$

so that $\xi_n = -\varepsilon + \varepsilon^{\gamma}\Delta_n$, $n \ge 0$, and consequently $c = -\varepsilon$, $d = f\varepsilon^{2\gamma} = f|c|^{2\gamma}$. Define the random variable $R_\gamma = R_\gamma(\varepsilon)$ by the equality

$$\lim_{n\to\infty} P(w_n > x) = P(R_\gamma > x)\,,\ x > 0\,.$$

Then from the theorem [15, theorem 1] for $\varepsilon \to 0$, $x > 0$, the following relations are true

$$R_\gamma \to +\infty\,,\ 0 \le \gamma < 1/2\,;\ R_\gamma \to 0\,, \gamma > 1/2\,; P(R_\gamma > x) \to \exp(-2x/f)\,, \gamma = 1/2\,.$$

Remark 2. A reduction of the constructed model of the one server fluid queuing system in the random environment to the Lindley chain allows to transfer onto this model known results on the stability of limit and prelimit distributions (see for an example, [9,§20], [16],[17, chapters V. VI]).

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RISK ANALISYS OF A SNOWBOARDER

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ABSTRACT

Winter sports can be associated with risk of sustaining injuries. The risk reduction is possible as a result of an analysis, portraying the most dangerous incidents and undesired events. Decreasing the frequency of such events or reducing their consequences can limit the overall risk associated with snowboarding.

First, a preliminary selection of undesired events was performed using the MIL-STD-882 matrix method. Then, a graph showing the most likely categories of body injuries that may occur during one day of snowboarding was developed. The graph allowed for determining events associated with the highest risk of injury.

1 INTRODUCTION

Research on safety and therefore conduct of various health risk analyses has made a relatively recent entry into the fields of science. Safety research started with the recognition that safety problems in many branches of technology and human life are common in character and therefore can be described in the same way.

Snowboarding probably was discovered at the beginning of the 20th century. A major increase in the popularity of this discipline occurred in the sixties and the growth trend continues into the present day. The rising number of snowboarders on slopes leads in turn to more accidents. To counteract this effect it is necessary to perform a risk analysis. Results and conclusions from the analysis can be helpful in increasing snowboarders safety. That is the purpose of this paper.

2 RISK ASSESSMENT METHOD

Risk analysis was made in accordance with a method presented in reference Szopa 2009. First, threats facing a snowboarder were identified. This hazard identification process allowed pinpointing Undesired Events (UE) that can possibly lead to an accident. Then UEs with relatively high risk score were selected for further analysis. The procedure was carried out using the matrix method presented in fig. 1. The method had been adjusted to the needs of this analysis, i.e. the severity of accidents was expressed as injuries sustained by a snowboarder. The categories of losses are defined in fig. 1.

negligible – no actual injuries
minor – insignificant injuries or bruises that do not impair snowboarding skills
average – significant injuries requiring at least one day's rest before returning to the slope
critical – severe injury disabling the snowboarder for the rest of the trip, requiring medical attention

Frequency of occurrence ↑	Negligible	Minor	Average	Critical
Common	13	7	3	1
Possible	16	9	5	2
Rare	18	11	6	4
Improbable	19	14	10	8
Unbelievable	20	17	15	12

Severity of accident →

Figure 1. Risk level classification according to MIL-STD-882 2000.

The most dangerous undesired events, i.e. events with the lowest rating in the table (fig. 1) were signed $A^{(k)}$, where k is the number of event. Then the events were analysed more thoroughly. To estimate the risks to a snowboarder, firstly we have to determine the probabilities of the chosen undesired events and secondly evaluate the most likely measure of the losses if a particular event takes place. The risk level in this paper is described as the most probable level of injury $c_0(1)$ (Szopa 2009) during one day of snowboarding. The risk can be calculated as follows:

$$c_0^{(k)}(1) = Q^{(k)}(1) \cdot Z_0^{(k)} \tag{1}$$

where: k – number of UE chosen during the initial risk analysis conducted with the matrix method, $c_0^{(k)}(1)$ – partial risk of UE marked with the index k, $Q^{(k)}(1)$ – probability of event $A^{(k)}$ occurring during one day, $Z_0^{(k)}$ – the most probable level of injury under the condition that the event k has occurred.

The next step during risk analysis is determining the probability of $Q^{(k)}(1)$ happening. Because of a lack of statistical data, these probability levels were determined using a ranking method (Swain & Guttmann 1983). For this purpose 11 experts were asked to fill in a special questionnaire. The experts were mostly people with vast experience in snowboarding; however the group included a few members who were relatively new to the discipline.

Their task was to rank the chosen events $A^{(k)}$ by probability from the rarest to the most common. To aggregate the expert opinion For a specific UE the sum of positions of $A^{(k)}$ divided by the number of experts is the average position of the occurrence according to the entire group of experts. Mean positions of UEs create an expert scale $S^{(k)}$. These mean positions can be converted into the probability $Q^{(k)}(1)$ of an UE occurrence with the following formula taken from the SLIM method (Kirwan 1994):

$$\log Q^{(k)}(1) = a \cdot S^{(k)} + b \tag{2}$$

where: a and b are independent parameters of a linear equation, calculated as part of the scale calibration process. The parameter values are usually appointed based on two or more known probabilities of event occurrence. However it must be noted that results obtained in this fashion are not very precise, because data used for calibration of the expert scale is usually overestimated. To draw conclusions pointing to UEs with the highest risk does not require specific values of $Q^{(k)}(1)$ – only the relative frequency of the events. Therefore the formula (2) was used here without separately calculating values a and b, which instead have been estimated based on a large number of similar individual risk analyses performed at the Faculty of Power and Aeronautical Engineering of Warsaw University of Technology.

The hazard level $Z_0^{(k)}$ was estimated using a direct judgment expert method (Ayyub 2001, Matyjewski 2009). Five categories of harm were considered (Szopa 2009): c_1 – no loss, c_2 – small loss, c_3 – moderate loss, c_4 – severe loss, c_5 – fatal loss.

3 DESCRIPTION OF SNOWBOARDER-SKI LIFT-SLOPE SYSTEM

This paper considers snowboard riding only in the winter. It is assumed that the slope is properly prepared, and snow levels meet basic snowboarding needs.

The most common types of ski lifts in Poland were taken into account. Research included both T-bar lifts and chairlifts. One- and two-person T-bar lifts were taken under consideration.

The snowboarder is assumed to be using a wooden board laminated with fiberglass. The base of the board is covered with polythene p-tex which keeps wax on the board. The metal edges of the board are inclined at an angle of 87°-90°.

This paper does not include activities that are not connected with snowboard riding directly, like going to the slope, lunch breaks or going back toward lodgings. Consequently, basic snowboarding activities include approaching the slope, warm-up exercises, putting on an equipment, approaching lifts and transport to the top of the slope, checking and fastening the board, riding down straight and in slalom fashion, performing snowboarding tricks, resting on the slope while sitting up, riding to the lift and unfastening the board.

4 THE RISK ANALYSIS AND RESULTS

After recognition of the system elements, primary UEs with a high risk level were selected using the matrix method (MIL-STD-882 2000). A list of UEs was prepared with corresponding numbers appointed depending on an event's frequency of occurrence and the severity of its consequences (tab.1). Afterwards 10 events associated with the highest risk level were chosen. The names and symbols of the events are presented in the table 1.

The expert scale was calibrated using arbitrarily chosen values $a = 0.3$ and $b = -4$, the selection was based on similar analyses, e.g. (Matyjewski & Sztuka 2010). The resulting probability values are presented in table 2.

Table 1. Preliminary risk analysis results

Activity	Name of undesired event	Rating
Riding downhill	Colliding with a moving skier/snowboarder	3
	Colliding with a standing skier/snowboarder	13
	Being hit by a skier/snowboarder while taking a break	5
	Hitting a stationary object e.g. tree, fence	13
	Falling down due to loss of stability	13
	Falling out of the designated route	16
Jumping	Falling down during landing phase of a jump	7
	Approaching the jump incorrectly	16
	Landing too short/too far	5
Approaching and leaving the slope	Falling down due to loss of stability	18
	Sores because of carrying equipment	16
Chairlift ride	Getting hit on the calves by the lift	18
	Falling down due to loss of stability while leaving the chairlift	13
Riding a single person rope tow	Developing sores on the thighs due to the T-bar	13
	Falling down as a result of a too rapid start	18
	Hitting a slope-user crossing the lift's trail	14
	Falling down as a result of the slope being uneven	18

Activity	Name of undesired event	Rating
	Getting hit by the T-bar Falling down while getting off the T-bar lift A skier/snowboarder falling on the lift before us	18 16 11
Riding a two-person rope tow	Falling down due to the other person losing stability Trampling the second person Being trampled by a person	18 19 19

Table 2. Final analysis results

Symbol	Name of undesired event	$S^{(k)}$	$Q^{(k)}(1)$	$Z_0^{(k)}$
$A^{(1)}$	Colliding with a moving skier/snowboarder	5,27	0,0038	c_3
$A^{(2)}$	Riding into a standing skier/snowboarder	3,82	0,0014	c_1
$A^{(3)}$	Being hit by a skier/snowboarder while taking a break	5,45	0,0043	c_3
$A^{(4)}$	Hitting a stationary object e.g. tree, fence	3,09	0,0008	c_1
$A^{(5)}$	Falling down due to loss of stability	7	0,0126	c_1
$A^{(6)}$	Falling down during landing phase of a jump	8,36	0,0323	c_2
$A^{(7)}$	Landing too short/too far	6,82	0,0111	c_3
$A^{(8)}$	Falling down due to loss of stability while leaving the chairlift	4	0,0016	c_1
$A^{(9)}$	Developing sores on the thighs due to a T-bar	4,45	0,0022	c_1
$A^{(10)}$	A skier/snowboarder falling off the T-bar lift before us	5,45	0,0043	c_2

The level of risk in the form of the most probable loss caused by a specific UE occurrence was judged based on the authors' snowboarding experinece. The chosen categories of loss are included in the last column of table 2. In accordance with (1), the probability of an UE occurrence $Q^{(k)}(1)$ multiplied by the hazard level $Z_0^{(k)}$ is equal to the level of risk for each of the considered UEs. This measure of risk is represented in graphic form in fig. 2.

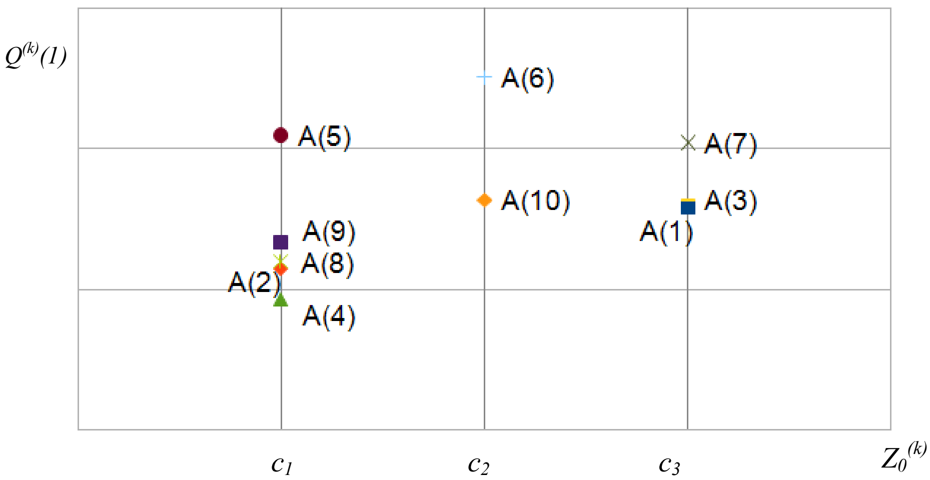

Fig. 2. The risk analysis results

As the risk level increases towards the right and the top of the graph (fig. 2.) the following undesired events represent the highest risk of sustaining injuries while snowboarding: $A^{(6)}$, $A^{(7)}$, $A^{(3)}$ and $A^{(1)}$, that is: falling down during landing phase of a jump, landing too short/too far, being hit by a skier/snowboarder while taking a break and colliding with a moving skier/snowboarder.

5 CONCLUSIONS

Even though it might seem that snowboarding, as an extreme sport is highly dangerous health loss associated with snowboarding is not severe. Usually the most probable consequences can be classified as either negligible or minor, at most bruises, cuts and sicknesses disallowing riding for up to one day.

Quite big differences can be found in the questionnaires filled out by the experts. Dependence between the expert's experience and the probability of an undesired event occurrence can be observed. Snowboarders with little experience ranked falling due to loss of stability as the most important. Among the more experienced snowboarders there is a higher probability of falling while landing after a jump and collision caused by a snowboarder/skier while resting on the slope. These easy to predict results are the consequence of experience gained on slopes during snowboarding.

In order to increase the safety of snowboarders, attention must be paid to UEs with the highest associated risks. Therefore properly profiled ski jumps should be built. A snowboarder can lower the risk of injury by choosing suitable to their skills slopes and ski routs. The level of risk of getting hit by a skier or snowboarder while resting does not seem to depend on experience. Dressing in flashily collared clothing could be a solution, but snowboarders have their own dress-code, so this idea would be hard to implement. The ignorance and reluctance to obey the Ski Code of Conduct should be addressed properly by the authorities and media. The consequences of collisions can be limited by wearing proper protectors and helmets.

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MODELING SAFETY OF MULTISTATE SYSTEMS WITH APPLICATION TO MARITIME FERRY TECHNICAL SYSTEM

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ABSTRACT

Basic notions of the ageing multistate systems safety analysis are introduced. The system components and the system safety functions are defined. The mean values and variances of the multistate system lifetimes in the safety state subsets and the mean values of its lifetimes in the particular safety states are defined. The notions of the multi-state system risk function and the moment of exceeding by the system the critical safety state are introduced. A series and a parallel-series safety structures of the multistate systems with ageing components are defined and their safety function are determined. As a particular case, the safety functions of the considered multi-state systems composed of components having exponential safety functions are determined. An applications of the proposed multistate system safety models to the prediction of safety characteristics of a maritime ferry operating at winter conditions technical system is presented as well.

1 INTRODUCTION

Taking into account the importance of the safety and operating process effectiveness of real technical systems it seems reasonable to expand the two-state approach to multi-state approach (Kolowrocki, 2004; Kolowrocki, Soszynska-Budny, 2011; Kolowrocki, Soszynska-Budny, 2012) in system safety analysis. The assumption that the systems are composed of multi-state components with safety states degrading in time (Kolowrocki, 2004; Kolowrocki, Soszynska-Budny, 2011; Kolowrocki, Soszynska-Budny, 2012) gives the possibility for more precise analysis of their safety and operational processes' effectiveness. This assumption allows us to distinguish a system safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operation process effectiveness. Then, an important system safety characteristic is the time to the moment of exceeding the system safety critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state safety function that is a basic characteristics of the multi-state system. The safety models of the considered here typical multistate system structures can be applied in the safety analysis of real complex technical systems. They may be successfully applied, for instance, to safety analysis, identification, prediction and optimization of the maritime transportation systems.

2 MULTISTATE APPROACH TO SAFETY ANALYSIS

In the multistate safety analysis to define a system composed of n, $n \in N$, ageing components we assume that:

- E_i, $i = 1,2,\ldots,n$, are components of a system,
- all components and a system under consideration have the set of safety states $\{0,1,...,z\}$, $z \geq 1$,
- the safety states are ordered, the state 0 is the worst and the state z is the best,
- the component and the system safety states degrade with time t,

- $T_i(u)$, $i = 1,2,\dots,n$, $n \in N$, are independent random variables representing the lifetimes of components E_i in the safety state subset $\{u, u+1, ..., z\}$, while they were in the safety state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the safety state subset $\{u, u+1, ..., z\}$, while it was in the safety state z at the moment $t = 0$,
- $s_i(t)$ is a component E_i safety state at the moment t, $t \in < 0, \infty)$, given that it was in the safety state z at the moment $t = 0$,
- $s(t)$ is the system safety state at the moment t, $t \in < 0, \infty)$, given that it was in the safety state z at the moment $t = 0$.

The above assumptions mean that the safety states of the ageing system and components may be changed in time only from better to worse.

Definition 1. A vector

$$S_i(t,\cdot) = [S_i(t,0), S_i(t,1), \dots, S_i(t,z)] \tag{1}$$

for $t \in < 0, \infty)$, $i = 1,2,\dots,n,$ where

$$S_i(t,u) = P(s_i(t) \geq u \mid s_i(0) = z) = P(T_i(u) > t) \tag{2}$$

for $t \in < 0, \infty)$, $u = 0,1,\dots,z,$ is the probability that the component E_i is in the safety state subset $\{u, u+1, ..., z\}$ at the moment t, $t \in < 0, \infty)$, while it was in the safety state z at the moment $t = 0$, is called the multistate safety function of a component E_i.

Definition 2. A vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,0), \boldsymbol{S}(t,1), \dots, \boldsymbol{S}(t,z)],\ t \in < 0, \infty),$$

(3)

where

$$\boldsymbol{S}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t) \tag{4}$$

for $t \in < 0, \infty)$, $u = 0,1,...,z$, is the probability that the system is in the safety state subset $\{u, u+1, ..., z\}$ at the moment t, $t \in < 0, \infty)$, while it was in the safety state z at the moment $t = 0$, is called the multi-state safety function of a system.

The safety functions $S_i(t,u)$ and $\boldsymbol{S}(t,u)$, $t \in < 0, \infty)$, $u = 0,1,...,z$, defined by (2) and (4) are called the coordinates of the components and the system multistate safety functions $S_i(t,\cdot)$ and $\boldsymbol{S}(t,\cdot)$ given by respectively (1) and (3). It is clear that from Definition 1 and Definition 2, for $u = 0$, we have

$S_i(t,0) = 1$ and $\boldsymbol{S}(t,0) = 1$.

The mean lifetime of the system in the safety state subset $\{u, u+1, ..., z\}$ is defined by

$$\mu(u) = \int_0^\infty \boldsymbol{S}(t,u)dt,\ u = 1,2,...,z, \tag{5}$$

and the standard deviation of the system lifetime in the safety state subset $\{u, u+1, ..., z\}$ is given by

$$\sigma(u) = \sqrt{n(u) - [\mu(u)]^2}\ , u = 1,2,...,z, \quad (6)$$

where

$$n(u) = 2\int_0^\infty t\, \boldsymbol{S}(t,u)dt,\ u = 1,2,...,z. \quad (7)$$

Moreover, the mean lifetimes of the system in the safety state u, $u = 1,2,...,z$,

$$\bar{\mu}(u) = \int_0^\infty p(t,u)dt,\ u = 1,2,...,z, \quad (8)$$

where

$$p(t,u) = P(\mathrm{s}(t) = u \mid s(0) = z) = \boldsymbol{S}(t,u) - \boldsymbol{S}(t, u+1),$$

for $u = 0,1,...,z-1,\ t \in < 0,\infty)$, can be found from the following relationships (Kolowrocki, Soszynska-Budny, 2011)

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ u = 0,1,...,z-1,\ \bar{\mu}(z) = \mu(z). \quad (9)$$

Definition 3. A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \le t),\ t \in < 0,\infty), \quad (10)$$

that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the safety state z at the moment t = 0 is called a risk function of the multi-state system (Kolowrocki, Soszynska-Budny, 2011).
Under this definition, from (4), we have

$$\boldsymbol{r}(t) = 1 - P(s(t) \ge r \mid s(0) = z) = 1 - \boldsymbol{S}(t,r),\ t \in < 0,\infty), \quad (11)$$

and if τ is the moment when the system risk exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \quad (12)$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the system risk function $\boldsymbol{r}(t)$.

3 SAFETY OF SERIES AND PARALLEL-SERIES SYSTEMS

Now, after introducing the notion of the multistate safety analysis, we may define basic multi-state safety structures.
Definition 4. A multistate system is called series if its lifetime $T(u)$ in the safety state subset $\{u, u+1,..., z\}$ is given by

$$T(u) = \min_{1 \le i \le n}\{T_i(u)\},\ u = 1,2,...,z.$$

The number n is called the system structure shape parameter.
The above definition means that a multi-state series system is in the safety state subset $\{u, u+1, ..., z\}$ if and only if all its n components are in this subset of safety states. That meaning is very close to the definition of a two-state series system considered in a classical reliability analysis that is not failed if all its components are not failed. This fact can justify the safety structure scheme for a multistate series system presented in Figure. 1.

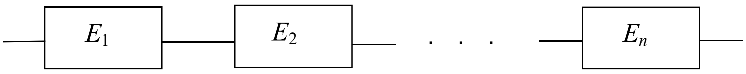

Figure 1. The scheme of a series system safety structure

It is easy to work out that the safety function of the multi-state series system is given by the vector (Kolowrocki, Soszynska-Budny, 2011)

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)] \quad (13)$$

with the coordinates

$$\boldsymbol{S}(t,u) = \prod_{i=1}^{n} S_i(t,u), \; t \in <0,\infty), \; u = 1,2,...,z. \quad (14)$$

Hence, if the system components have exponential safety functions, i.e.

$$S_i(t,\cdot) = [1, S_i(t,1), \dots, S_i(t,z)], \; t \in <0,\infty), \; i = 1,2,\dots,n, \quad (15)$$

where

$$S_i(t,u) = \exp[-\lambda_i(u)t], \; t \in <0,\infty), \; u = 1,2,\dots,z, \; i = 1,2,\dots,n, \quad (16)$$

the formula (14) takes the following form

$$\boldsymbol{S}(t,u) = \prod_{i=1}^{n} \exp[-\lambda_i(u)t], \; t \in <0,\infty), \; u = 1,2,...,z. \quad (17)$$

Definition 5. A multistate system is called parallel-series if its lifetime $T(u)$ in the safety state subset $\{u, u+1, ..., z\}$ is given by

$$T(u) = \min_{1 \le i \le k} \{\max_{1 \le j \le l_i} \{T_{ij}(u)\}\}, \; u = 1,2,...,z.$$

The above definition means that the multistate parallel-series system is composed of k multistate parallel subsystems and it is in the safety state subset $\{u, u+1, ..., z\}$ if and only if all its k parallel subsystems are in this safety state subset. In this definition l_i, $i = 1,2,...,k$, denote the numbers of components in the parallel subsystems. The numbers k and $l_1, l_2, ..., l_k$ are called the system structure shape parameters. The scheme of a multistate parallel-series system given in Figure 2.

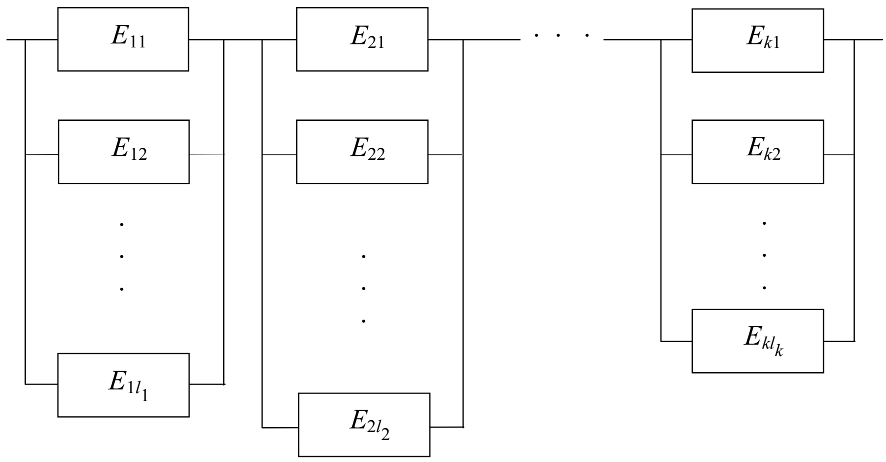

Figure 2. The scheme of a parallel-series system

The safety function of the multi-state parallel-series system is given by the vector (Kolowrocki, Soszynska-Budny, 2011)

$$\boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,\cdot) = [1, \boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,1), \ldots, \boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,z)], \quad (18)$$

with the coordinates

$$\boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,u) = \prod_{i=1}^{k}[1-\prod_{j=1}^{l_i}[1-S_{ij}(t,u)]], \ t \in <0,\infty), \ u = 1,2,\ldots,z,$$

(19)

where k is the number of its parallel subsystems linked in series and l_i, $i = 1,2,...,k$, are the numbers of components in the parallel subsystems.
Hence, if the system components have exponential safety functions, i.e.

$$S_{ij}(t,\cdot) = [1, S_{ij}(t,1), \ldots, S_{ij}(t,z)], \ t \in <0,\infty), \ i = 1,2,\ldots,k, j = 1,2,\ldots,l_i, \quad (20)$$

where

$$S_i(t,u) = \exp[-\lambda_{ij}(u)t], \ t \in <0,\infty), u = 1,2,\ldots,z, \ i = 1,2,\ldots,k, j = 1,2,\ldots,l_i, \quad (21)$$

the formula (19) takes the following form

$$\boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,u) = \prod_{i=1}^{k}[1-\prod_{j=1}^{l_i}[1-\exp[-\lambda_{ij}(u)t]], \ t \in <0,\infty), \ u = 1,2,\ldots,z. \quad (22)$$

4 SAFETY OF MARITME FERRY TECHNICAL SYSTEM

The considered maritime ferry is a passenger Ro-Ro ship operating at the Baltic Sea between Gdynia and Karlskrona ports on regular everyday line. We assume that the ferry is composed of a number of main subsystems having an essential influence on its safety. These subsystems are illustrated in Figure 3 and Figure 4.

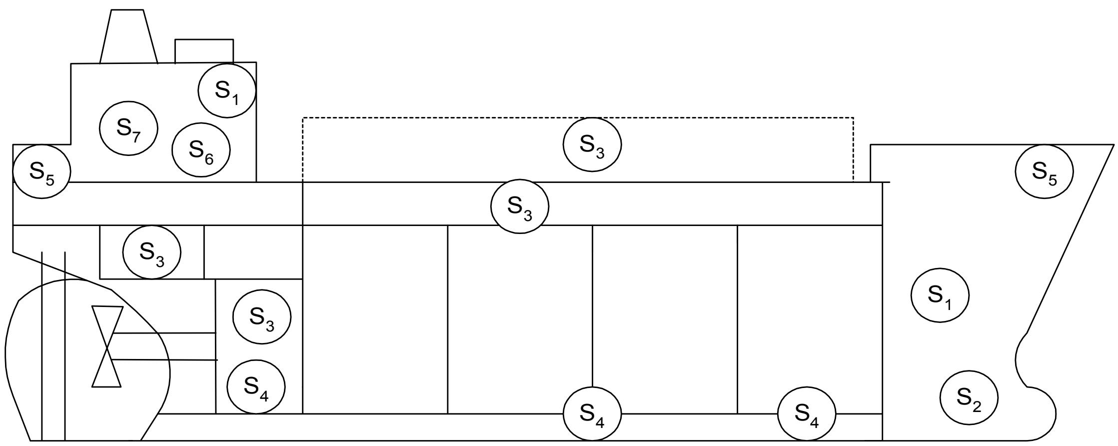

Figure 3. Subsystems having an essential influence on the ferry safety

On the scheme of the ferry presented in Figure 3, there are distinguished its following subsystems:
S_1 - a navigational subsystem,
S_2 - a propulsion and controlling subsystem,
S_3 - a loading and unloading subsystem,
S_4 - a stability control subsystem,
S_5 - an anchoring and mooring subsystem,
S_6 - a protection and rescue subsystem,
S_7 - a social subsystem.
In the safety analysis of the ferry, we omit the protection and rescue subsystem S_6 and the social subsystem S_7 and we consider its strictly technical subsystems S_1, S_2, S_3, S_4 and S_5 only, further called the ferry technical system.

The navigational subsystem S_1 is composed of one general component $E_{11}^{(1)}$, that is equipped with GPS, AIS, speed log, gyrocompass, magnetic compass, echo sounding system, paper and electronic charts, radar, ARPA, communication system and other subsystems.
The propulsion and controlling subsystem S_2 is composed of :
- the subsystem S_{21} which consist of 4 main engines $E_{11}^{(2)}$, $E_{12}^{(2)}$, $E_{13}^{(2)}$, $E_{14}^{(2)}$;
- the subsystem S_{22} which consist of 3 thrusters $E_{21}^{(2)}$, $E_{22}^{(2)}$, $E_{31}^{(2)}$;
- the subsystem S_{23} which consist of twin pitch propellers $E_{41}^{(2)}$, $E_{51}^{(2)}$;
- the subsystem S_{24} which consist of twin directional rudders $E_{61}^{(2)}$, $E_{71}^{(2)}$.

The loading and unloading subsystem S_3 is composed of :
- the subsystem S_{31} which consist of 2 remote upper trailer decks to main deck $E_{11}^{(3)}$, $E_{21}^{(3)}$;
- the subsystem S_{32} which consist of 1 remote fore car deck to main deck $E_{31}^{(3)}$;
- the subsystem S_{33} which consist of passenger gangway to Gdynia Terminal $E_{41}^{(3)}$;
- the subsystem S_{34} which consist of passenger gangway to Karlskrona Terminal $E_{51.}^{(3)}$.

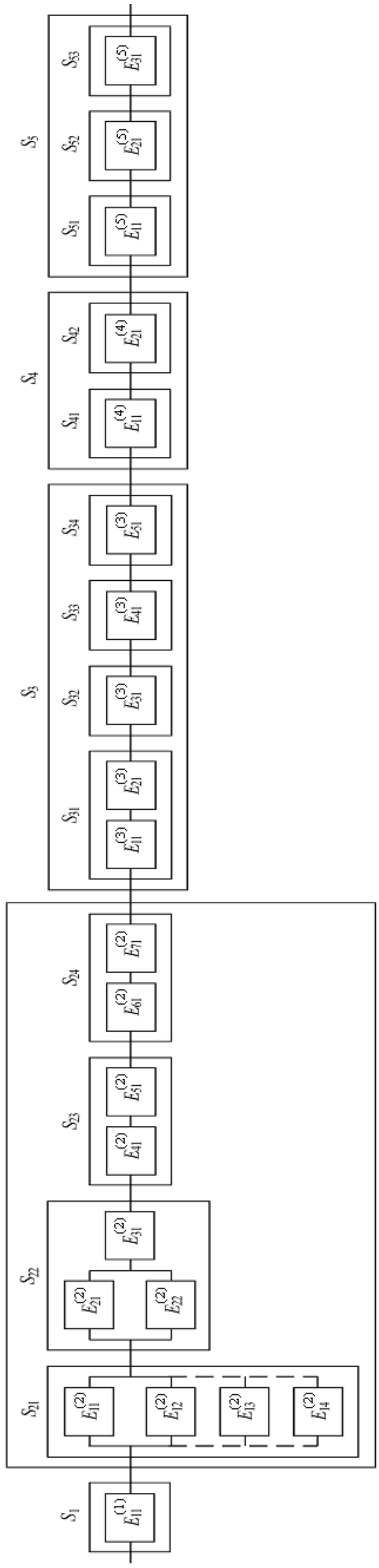

Figure 4. The detailed scheme of the ferry technical system structure

The stability control subsystem S_4 is composed of :

- the subsystem S_{41} which consist of an anti-heeling system $E_{11}^{(4)}$, which is used in port during loading operations;
- the subsystem S_{42} which consist of an anti-heeling system $E_{21}^{(4)}$, which is used at sea to stabilizing ships rolling.

The anchoring and mooring subsystem S_5 is composed of :

- the subsystem S_{51} which consist of aft mooring winches $E_{11}^{(5)}$;
- the subsystem S_{52} which consist of fore mooring and anchor winches $E_{21}^{(5)}$;
- the subsystem S_{53} which consist of fore mooring winches $E_{31}^{(5)}$.

The detailed scheme of these subsystems and components is illustrated in Figure 4.
The subsystems S_1, S_2, S_3, S_4, S_5, indicated in Figure 4 are forming a general series safety structure of the ferry technical system presented in Figure 5.

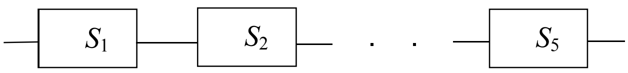

Figure 5. The general scheme of the ferry technical system safety structure

After discussion with experts, taking into account the safety of the operation of the ferry, we distinguish the following five safety states ($z = 4$) of the ferry technical system and its components:

- a safety state 4 – the ferry operation is fully safe,
- a safety state 3 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution,
- a safety state 2 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution and causing small accidents,
- a safety state 1 - the ferry operation is much less safe and much more dangerous because of the possibility of serious environment pollution and causing extensive accidents,
- a safety state 0 – the ferry technical system is is destroyed.

Moreover, by the expert opinions, we assume that there are possible the transitions between the components' safety states only from better to worse ones and we assume that the system and its components critical safety state is $r = 2$.
From the above, the subsystems S_υ, $\upsilon = 1,2,3,4,5,$ are composed of five-state, i.e. z = 4, components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,3,4,5,$ having the safety functions

$$S_{ij}^{(\upsilon)}(t,\cdot) = [1,\ S_{ij}^{(\upsilon)}(t,1),\ S_{ij}^{(\upsilon)}(t,2),\ S_{ij}^{(\upsilon)}(t,3),\ S_{ij}^{(\upsilon)}(t,4)\ ],$$

with the coordinates that by the assumption are exponential of the forms

$$S_{ij}^{(\upsilon)}(t,1) = \exp[-\lambda_{ij}^{(\upsilon)}(1)t],\ S_{ij}^{(\upsilon)}(t,2) = \exp[-\lambda_{ij}^{(\upsilon)}(2)t],$$

$$S_{ij}^{(\upsilon)}(t,3) = \exp[-\lambda_{ij}^{(\upsilon)}(3)t],\ S_{ij}^{(\upsilon)}(t,4) = \exp[-\lambda_{ij}^{(\upsilon)}(4)t].$$

The subsystem S_1 consists of one component $E_{ij}^{(1)}$, $i=1$, $j=1$, i.e. we may consider it either as a series system composed of $n=1$ components or for instance as a parallel-series system with parameters $k=1$, $l_1=1$, with the exponential safety functions on the basis of data coming from experts and given below.
The coordinates of the subsystem S_1 component five-state safety function are:

$$S_{11}^{(1)}(t,1) = \exp[-0.033t],\ S_{11}^{(1)}(t,2) = \exp[-0.04t],$$

$$S_{11}^{(1)}(t,3) = \exp[-0.045t],\ S_{11}^{(1)}(t,4) = \exp[-0.05t].$$

Thus, the subsystem S_1 safety function is identical with the safety function of its component, i.e.

$$\boldsymbol{S}^{(1)}(t,\cdot) = [1,\ \boldsymbol{S}^{(1)}(t,1),\ \boldsymbol{S}^{(1)}(t,2),\ \boldsymbol{S}^{(1)}(t,3),\ \boldsymbol{S}^{(1)}(t,4)],\quad t\in<0,\infty), \tag{23}$$

where, according to the formulae (18)-(19), we have

$$\boldsymbol{S}^{(1)}(t,u) = \boldsymbol{S}_{1;1}(t,u) = \prod_{i=1}^{1}[1-\prod_{j=1}^{1}[1-S_{ij}^{(1)}(t,u)]] = S_{11}^{(1)}(t,u),\ t\in<0,\infty),\ u=1,2,3,4, \tag{24}$$

and particularly

$$\boldsymbol{S}^{(1)}(t,1) = \boldsymbol{S}_{1;1}(t,1) = \exp[-0.033t], \tag{25}$$

$$\boldsymbol{S}^{(1)}(t,2) = \boldsymbol{S}_{1;1}(t,2) = \exp[-0.04t], \tag{26}$$

$$\boldsymbol{S}^{(1)}(t,3) = \boldsymbol{S}_{1;1}(t,3) = \exp[-0.045t], \tag{27}$$

$$\boldsymbol{S}^{(1)}(t,4) = \boldsymbol{S}_{1;1}(t,4) = \exp[-0.05t]. \tag{28}$$

The subsystem S_2 is a five-state parallel-series system composed of components $E_{ij}^{(2)}$, $i=1,2,...,k$, $j=1,2,...,l_i$, $k=7$, $l_1=4$, $l_2=2$, $l_3=1$, $l_4=1$, $l_5=1$, $l_6=1$, $l_7=1$, with the exponential safety functions identified on the basis of data coming from experts given below. The coordinates of the subsystem S_2 components' five-state safety functions are:

$$S_{1j}^{(2)}(t,1) = \exp[-0.033t],\ S_{1j}^{(2)}(t,2) = \exp[-0.04t],$$
$$S_{1j}^{(2)}(t,3) = \exp[-0.05t],\ S_{1j}^{(2)}(t,4) = \exp[-0.055t],\ j=1,2,3,4,$$

$$S_{2j}^{(2)}(t,1) = \exp[-0.066t],\ S_{2j}^{(2)}(t,2) = \exp[-0.07t],$$
$$S_{2j}^{(2)}(t,3) = \exp[-0.075t],\ S_{2j}^{(2)}(t,4) = \exp[-0.08t],\ j=1,2,$$

$$S_{31}^{(2)}(t,1) = \exp[-0.066t],\ S_{31}^{(2)}(t,2) = \exp[-0.07t],$$
$$S_{31}^{(2)}(t,3) = \exp[-0.075t],\ S_{31}^{(2)}(t,4) = \exp[-0.08t],$$

$$S_{i1}^{(2)}(t,1) = \exp[-0.033t],\ S_{i1}^{(2)}(t,2) = \exp[-0.04t],$$
$$S_{i1}^{(2)}(t,3) = \exp[-0.045t],\ S_{i1}^{(2)}(t,4) = \exp[-0.05t],\ i = 4,5,6,7.$$

Hence, according to the formulae (18)-(19), the subsystem S_2 safety function is given by

$$\boldsymbol{S}^{(2)}(t,\cdot) = [1,\ \boldsymbol{S}^{(2)}(t,1),\ \boldsymbol{S}^{(2)}(t,2),\ \boldsymbol{S}^{(2)}(t,\ 3),\ \boldsymbol{S}^{(2)}(t,4)\ ],\quad t \in <0,\infty), \tag{29}$$

where

$$\boldsymbol{S}^{(2)}(t,u) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,u) = \prod_{i=1}^{7}[1-\prod_{j=1}^{l_i}[1-s_{ij}^{(2)}(t,u)]],\ t \in <0,\infty),\ u = 1,2,3,4, \tag{30}$$

and particularly

$$\boldsymbol{S}^{(2)}(t,1) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,1) = 6[\exp[-0.033t]]^2[1-\exp[-0.033t]]^2$$

$$+4[\exp[-0.033t]]^3[1-\exp[-0.033t]] + [\exp[-0.033t]]^4\ ]\ [1 - [1 - \exp[-0.066t]]^2]\exp[-0.066t]$$

$$\exp[-0.033t]\exp[-0.033t]\ \exp[-0.033t]\ \exp[-0.033t]$$

$$= 12\ \exp[-0.33t] + 8\ \exp[-0.429t]\ -16\exp[-0.363t]\ -\ 3\exp[-0.462t] \tag{31}$$

$$\boldsymbol{S}^{(2)}(t,2) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,2) = [\ 6[\exp[-0.04t]]^2[1-\exp[-0.04t]]^2 + 4[\exp[-0.04t]]^3[1-\exp[-0.04t]]$$

$$+[\exp[-0.04t]]^4\ ]\ [1 - [1 - \exp[-0.07t]]^2]\exp[-0.07t]\ \exp[-0.04t]\exp[-0.04t]\ \exp[-0.04t]\ \exp[-0.04t]$$

$$= 12\ \exp[-0.38t] + 8\ \exp[-0.49t] + 6\ \exp[-0.46t]\ -\ 16\ \exp[-0.42t]\text{-}\ 6\ \exp[-0.45t]\ -\ 3\ \exp[-0.53t] \tag{32}$$

$$\boldsymbol{S}^{(2)}(t,\ 3) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,3) = 6[\exp[-0.05t]]^2[1-\exp[-0.05t]]^2$$

$$+4[\exp[-0.05t]]^3[1-\exp[-0.05t]] + [\exp[-0.05t]]^4\ ]\ [1 - [1 - \exp[-0.075t]]^2]\exp[-0.075t]$$

$$\exp[-0.045t]\exp[-0.045t]\ \exp[-0.045t]\ \exp[-0.045t]$$

$$= 12\ \exp[-0.43t] + 8\ \exp[-0.555t] + 6\ \exp[-0.53t]\ -\ 16\ \exp[-0.48t]$$

$$-\ 6\ \exp[-0.505t]\ -\ 3\ \exp[-0.605t] \tag{33}$$

$$\boldsymbol{S}^{(2)}(t,4) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,4) = 6[\exp[-0.055t]]^2[1-\exp[-0.055t]]^2$$

$$+4[\exp[-0.055t]]^3[1-\exp[-0.055t]] + [\exp[-0.055t]]^4\ ]$$

$$[1 - [1 - \exp[-0.08t]]^2]\exp[-0.08t]\ \exp[-0.05t]\exp[-0.05t]\ \exp[-0.05t]\ \exp[-0.05t]$$

$$= 12\ \exp[-0.47t] + 8\ \exp[-0.605t] + 6\ \exp[-0.58t]\ -\ 16\ \exp[-0.525t]$$

$$- 6 \exp[-0.55t] - 3 \exp[-0.66t]. \quad (34)$$

The subsystem S_3 is a five-state series system composed of $n = 5$ components that can also be considered as a parallel-series system composed of components $E_{ij}^{(3)}$, $i = 1,2,...,k$, $j = l_i$, $k = 5$, $l_1 = 1$, $l_2 = 1$, $l_3 = 1$, $l_4 = 1$, $l_5 = 1$, with the exponential safety functions identified on the basis of data coming from experts given below. The coordinates of the subsystem S_3 components' five-state safety functions are:

$$S_{11}^{(3)}(t,1) = \exp[-0.02t],\ S_{11}^{(3)}(t,2) = \exp[-0.03t],$$
$$S_{11}^{(3)}(t,3) = \exp[-0.035t],\ S_{11}^{(3)}(t,4) = \exp[-0.04t],$$

$$S_{21}^{(3)}(t,1) = \exp[-0.02t],\ S_{21}^{(3)}(t,2) = \exp[-0.025t],$$
$$S_{21}^{(3)}(t,3) = \exp[-0.03t],\ S_{21}^{(3)}(t,4) = \exp[-0.04t],$$

$$S_{31}^{(3)}(t,1) = \exp[-0.033t],\ S_{31}^{(3)}(t,2) = \exp[-0.04t],$$
$$S_{31}^{(3)}(t,3) = \exp[-0.045t],\ S_{31}^{(3)}(t,4) = \exp[-0.05t],$$

$$S_{41}^{(3)}(t,1) = \exp[-0.033t],\ S_{41}^{(3)}(t,2) = \exp[-0.04t],$$
$$S_{41}^{(3)}(t,3) = \exp[-0.045t],\ S_{41}^{(3)}(t,4) = \exp[-0.05t],$$

$$S_{51}^{(3)}(t,1) = \exp[-0.033t],\ S_{51}^{(3)}(t,2) = \exp[-0.04t],$$
$$S_{51}^{(3)}(t,3) = \exp[-0.045t],\ S_{51}^{(3)}(t,4) = \exp[-0.05t],$$

Hence, according to the formulae (18)-(19), the subsystem S_3 five-state safety function is given by

$$\boldsymbol{S}^{(3)}(t,\cdot) = [1,\ \boldsymbol{S}^{(3)}(t,1),\ \boldsymbol{S}^{(3)}(t,\ 2),\ \boldsymbol{S}^{(3)}(t,\ 3),\ \boldsymbol{S}^{(3)}(t,\ 4)],\ t \in < 0,\infty), \quad (35)$$

where

$$\boldsymbol{S}^{(3)}(t,\ u) = \boldsymbol{S}_{5;1,1,1,1,1}(t,u) = \prod_{i=1}^{5}[1-\prod_{j=1}^{1}[1-S_{ij}(t,u)]] = \prod_{i=1}^{5} S_{i1}(t,u),\ t \in < 0,\infty),\ u = 1,2,3,4, \quad (36)$$

and particularly

$$\boldsymbol{S}^{(3)}(t,1) = \boldsymbol{S}_{5;1,1,1,1,1}(t,1) = \exp[-0.02t]\ \exp[-0.02t]\ \exp[-0.033t]\ \exp[-0.033]\ \exp[-0.033]$$
$$= \exp[-0.139t], \quad (37)$$

$$\boldsymbol{S}^{(3)}(t,\ 2) = \boldsymbol{S}_{5;1,1,1,1,1}(t,\ 2) = \exp[-0.03t]\ \exp[-0.025t]\ \exp[-0.04t]\ \exp[-0.04t]\ \exp[-0.04t]$$
$$= \exp[-0.175t], \quad (38)$$

$$\boldsymbol{S}^{(3)}(t,\ 3) = \boldsymbol{S}_{5;1,1,1,1,1}(t,\ 3) = \exp[-0.035t]\ \exp[-0.03t]\ \exp[-0.045t]\ \exp[-0.045t]\ \exp[-0.045t]$$

$$= \exp[-0.200t], \tag{39}$$

$$\boldsymbol{S}^{(3)}(t,\ 4) = \boldsymbol{S}_{5;1,1,1,1,1}(t,\ 4) = \exp[-0.04t]\ \exp[-0.04t]\ \exp[-0.05t]\ \exp[-0.05t]\ \exp[-0.05t]$$

$$= \exp[-0.230t], \tag{40}$$

The subsystem S_4 is a five-state series system composed of $n = 2$ components that can also be considered as a parallel-series system composed of components $E_{ij}^{(4)}$, $i = 1,...,k$, $j = l_i$, $k = 2$, $l_1 = 1$, $l_2 = 1$, with the exponential safety functions identified on the basis of data coming from experts and given below. The coordinates of the subsystem S_4 components' multi-state safety functions are:

$$S_{11}^{(4)}(t,1) = \exp[-0.05t],\ S_{11}^{(4)}(t,2) = \exp[-0.06t],$$
$$S_{11}^{(4)}(t,3) = \exp[-0.065t],\ S_{11}^{(4)}(t,4) = \exp[-0.07t],$$

$$S_{21}^{(4)}(t,1) = \exp[-0.033t],\ S_{21}^{(4)}(t,2) = \exp[-0.04t],$$
$$S_{21}^{(4)}(t,3) = \exp[-0.045t],\ S_{21}^{(4)}(t,4) = \exp[-0.05t].$$

Hence, according to the formulae (18)-(19), the subsystem S_4 five-state safety function is given by

$$\boldsymbol{S}^{(4)}(t,\cdot) = [1,\ \boldsymbol{S}^{(4)}(t,1),\ \boldsymbol{S}^{(4)}(t,2),\ \boldsymbol{S}^{(4)}(t,\ 3),\ \boldsymbol{S}^{(4)}(t,4)\ ],\ t \in < 0,\infty), \tag{41}$$

where

$$\boldsymbol{S}^{(4)}(t,u) = \boldsymbol{S}_{2;1,1}(t,u) = \prod_{i=1}^{2}[1 - \prod_{j=1}^{1}[1 - S_{ij}(t,u)]] = \prod_{i=1}^{2} S_{ij}(t,u),\quad t \in < 0,\infty),\quad u = 1,2,3,4,$$

(42)

and particularly

$$\boldsymbol{S}^{(4)}(t,1) = \boldsymbol{S}_{2;1,1}(t,1) = \exp[-0.05t]\ \exp[-0.033t] = \exp[-0.083t], \tag{43}$$

$$\boldsymbol{S}^{(4)}(t,2) = \boldsymbol{S}_{2;1,1}(t,2) = \exp[-0.06t]\ \exp[-0.04t] = \exp[-0.100t], \tag{44}$$

$$\boldsymbol{S}^{(4)}(t,\ 3) = \boldsymbol{S}_{2;1,1}(t,\ 3) = \exp[-0.065t]\ \exp[-0.045t] = \exp[-0.110t] \tag{45}$$

$$\boldsymbol{S}^{(4)}(t,4) = \boldsymbol{S}_{2;1,1}(t,4) = \exp[-0.07t]\ \exp[-0.05t] = \exp[-0.120t]. \tag{46}$$

The subsystem S_5 is a five-state series system composed of $n = 3$ components that can also be considered as a parallel-series system composed of components $E_{ij}^{(5)}$, $i = 1,2,..,k$, $j = l_i$, $k = 3$, $l_1 = 1$, $l_2 = 1$, $l_3 = 1$, with the exponential safety functions identified on the basis of data coming from experts given below. The coordinates of the subsystem S_5 components' five-state safety functions are:

$$S_{11}^{(5)}(t,1) = \exp[-0.033t],\ S_{11}^{(5)}(t,2) = \exp[-0.04t],$$
$$S_{11}^{(5)}(t,3) = \exp[-0.045t],\ S_{11}^{(5)}(t,4) = \exp[-0.05t],$$

$$S_{21}^{(5)}(t,1) = \exp[-0.033t],\ S_{21}^{(5)}(t,2) = \exp[-0.04t],$$
$$S_{21}^{(5)}(t,3) = \exp[-0.05t],\ S_{21}^{(5)}(t,4) = \exp[-0.055t],$$

$$S_{31}^{(5)}(t,1) = \exp[-0.033t],\ S_{31}^{(5)}(t,2) = \exp[-0.04t],$$
$$S_{31}^{(5)}(t,3) = \exp[-0.05t],\ S_{31}^{(5)}(t,4) = \exp[-0.06t].$$

Hence, according to the formulae (18)-(19), the subsystem S_5 five-state safety function is given by

$$\boldsymbol{S}^{(5)}(t,\cdot) = [1,\ \boldsymbol{S}^{(5)}(t,1),\ \boldsymbol{S}^{(5)}(t,2),\ \boldsymbol{S}^{(5)}(t,3),\ \boldsymbol{S}^{(5)}(t,4)],\quad t \in <0,\infty), \tag{47}$$

where

$$\boldsymbol{S}^{(5)}(t,u) = \boldsymbol{S}_{3;1,1,1}(t,u) = \prod_{i=1}^{3}[1-\prod_{j=1}^{1}[1-S_{ij}(t,u)]] = \prod_{i=1}^{3} S_{i1}(t,u),\ t \in <0,\infty),\ u = 1,2,3,4, \tag{48}$$

and particularly

$$\boldsymbol{S}^{(5)}(t,1) = \boldsymbol{S}_{3;1,1,1}(t,1) = \exp[-0.033t]\ \exp[-0.033t]\ \exp[-0.033t] = \exp[-0.099t], \tag{49}$$

$$\boldsymbol{S}^{(5)}(t,2) = \boldsymbol{S}_{3;1,1,1}(t,2) = \exp[-0.04t]\ \exp[-0.04t]\ \exp[-0.04t] = \exp[-0.12t], \tag{50}$$

$$\boldsymbol{S}^{(5)}(t,3) = \boldsymbol{S}_{3;1,1,1}(t,3) = \exp[-0.045t]\ \exp[-0.05t]\ \exp[-0.05t] = \exp[-0.145t], \tag{51}$$

$$\boldsymbol{S}^{(5)}(t,4) = \boldsymbol{S}_{3;1,1,1}(t,4) = \exp[-0.05t]\ \exp[-0.055t]\ \exp[-0.06t] = \exp[-0.165t]. \tag{52}$$

Considering that the ferry technical system is a five-state series system, after applying (13)–(14), its safety function is given by

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \geq 0, \tag{53}$$

where by (25)-(28), (31)-(34), (37)-(40), (43)-(46) and (49)-(52), we have

$$\boldsymbol{S}(t,u) = \boldsymbol{S}_5(t,u) = \boldsymbol{S}^{(1)}(t,u)\ \boldsymbol{S}^{(2)}(t,u)\ \boldsymbol{S}^{(3)}(t,u)\ \boldsymbol{S}^{(4)}(t,u)\ \boldsymbol{S}^{(5)}(t,u)\ \text{for}\ u = 1,2,3,4,$$

and particularly

$$\boldsymbol{S}(t,1) = \exp[-0.033t]\ [12\ \exp[-0.33t] + 8\ \exp[-0.429t] - 16\exp[-0.363t]$$
$$- 3\exp[-0.462t]]\ \exp[-0.139t]\ \exp[-0.083t]\ \exp[-0.099t]$$
$$= 12\ \exp[-0.684t] + 8\ \exp[-0.783t] - 16\exp[-0.717t] - 3\exp[-0.816t], \tag{54}$$

$$\boldsymbol{S}(t,2) = \exp[-0.040t]\,[12\exp[-0.38t] + 8\exp[-0.49t] + 6\exp[-0.46t]$$

$$- 16\exp[-0.42t] - 6\exp[-0.45t] - 3\exp[-0.53t]]\exp[-0.175t]\exp[-0.100t]\exp[-0.12t]$$

$$= 12\exp[-0.815t] + 8\exp[-0.925t] + 6\exp[-0.895t]$$

$$- 16\exp[-0.855t] - 6\exp[-0.885t] - 3\exp[-0.965t], \quad (55)$$

$$\boldsymbol{S}(t,3) = \exp[-0.045t]\,[12\exp[-0.43t] + 8\exp[-0.555t] + 6\exp[-0.53t]$$

$$- 16\exp[-0.48t] - 6\exp[-0.505t] - 3\exp[-0.605t]]\exp[-0.200t]\exp[-0.110t]\exp[-0.145t]$$

$$= 12\exp[-0.930t] + 8\exp[-1.055t] + 6\exp[-1.030t]$$

$$- 16\exp[-0.980t] - 6\exp[-1.005t] - 3\exp[-1.105t], \quad (56)$$

$$\boldsymbol{S}(t,4) = \exp[-0.05t][12\exp[-0.47t] + 8\exp[-0.605t] + 6\exp[-0.58t]$$

$$- 16\exp[-0.525t] - 6\exp[-0.55t] - 3\exp[-0.66t]]\exp[-0.230t]\exp[-0.120t]\exp[-0.165t]$$

$$= 12\exp[-1.035t] + 8\exp[-1.170t] + 6\exp[-1.145t]$$

$$- 16\exp[-1.090t] - 6\exp[-1.115t] - 3\exp[-1.225] . \quad (57)$$

The safety function of the ferry five-state technical system is presented in Figure 6.

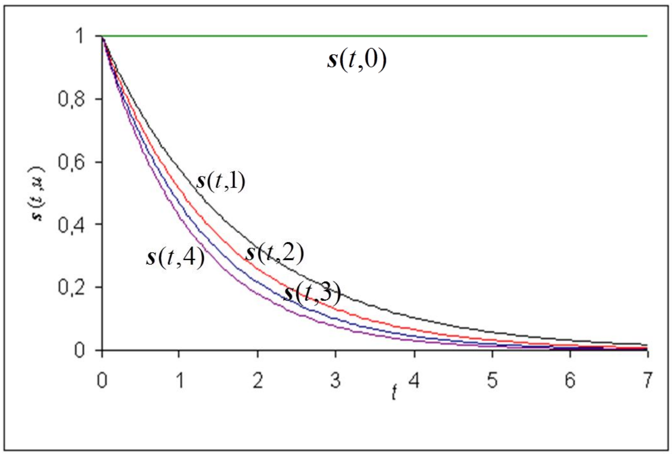

Figure 6. The graph of the ferry technical system safety function $\boldsymbol{s}(t,\cdot)$ coordinates

The expected values and standard deviations of the ferry technical system lifetimes in the safety state subsets calculated from the results given by (54)-(57), according to the formulae (5)-(7), are:

$$\mu(1) \cong 1.770, \ \mu(2) \cong 1.476, \ \mu(3) \cong 1.300, \ \mu(4) \cong 1.164 \text{ year}, \quad (58)$$

$$\sigma(1) \cong 1.733,\ \sigma(2) \cong 1.447,\ \sigma(3) \cong 1.277,\ \sigma(4) \cong 1.144 \text{ year}, \quad (59)$$

and further, using (58), from (9), the mean values of the ferry technical system conditional lifetimes in the particular safety states are:

$$\bar{\mu}(1) \cong 0.294,\ \bar{\mu}(2) \cong 0.176,\ \bar{\mu}(3) \cong 0.136,\ \bar{\mu}(4) \cong 1.164 \text{ year}. \quad (60)$$

As the critical safety state is r =2, then the system risk function, according to (10), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) = 1\text{-}[12 \exp[-0.815t] + 8 \exp[-0.925t] + 6 \exp[-0.895t]$$

$$- 16 \exp[-0.855t] - 6 \exp[-0.885t] - 3 \exp[-0.965t]], \text{ for } t \geq 0. \quad (61)$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, by (12), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.077. \quad (62)$$

The graph of the risk function $\boldsymbol{r}(t)$ of the ferry five-state technical

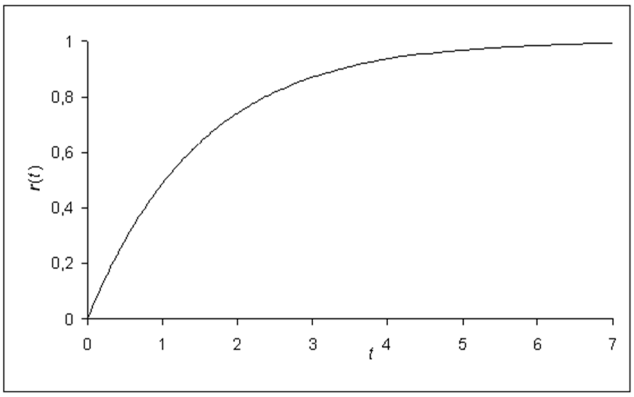

Figure 7. The graph of the risk function $\boldsymbol{r}(t)$ of the ferry technical system

5 CONCLUSION

The proposed in this paper model for safety evaluation and prediction of the considered here typical multistate system structures are applied for safety analysis of the maritime ferry technical system operating at Baltic Sea. The safety function, the risk function and other safety characteristics of the considered system are find. The system safety structures are fixed generally with not high accuracy in details concerned with the subsystems structures because of their complexity and concerned with the components safety characteristics because of the luck of statistical data necessary for their estimation. However, the results presented in the paper suggest that it seems reasonable to continue the investigations focusing on the methods of safety analysis for other more complex multi-state systems.

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WIND HYDROPOWER SYSTEM AS A VARIANT ON DIVERSIFICATION OF DISTRIBUTED GENERATION

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M.A. Rychkov

INTRODUCTION

Nowadays renewable energy sources attract attention of humanity because the depletion of conventional nonrenewable energy (coal, gas, oil, etc.) is getting increasingly obvious. Wind energy is characterized by a considerable potential among the renewable resources.

Human civilizations have harnessed wind since long ago. In the ancient times wind was used to propel boats. It is known that even 3000 years BC the citizens of Alexadria had used "wind wheels". In the 16th century the Netherlands had more than ten thousand wind-driven plants that were used to dry lakes for cultivation area. In 1888 the USA constructed a large wind power plant for electricity production. The multi-blade wind motors invented by the engineer Davydov appeared at the Russian Exhibition in Nizhny Novgorod in 1896 [1]. Wind mills found wide application. In the USSR the first 100 kV wind power plant was built in the Crimea in 1931 and was in operation until World War II.

Currently wind energy is widely used in more than 60 countries of the world. Today 10 leading countries account for about 86% of all wind power capacities installed in the world, of which more than 38% are situated in China and the USA. In Europe wind energy is mostly used in Germany, Denmark, Spain, Portugal, and France. The total installed capacity in the world reached 194 GW [2] in 2011 and continues to soar.

When used as distributed generation, modern wind power plants along with advantages (free primary energy) have some drawbacks:

- Lack of regularity and constancy in electricity generation due to variability of wind parameters;
- Relatively high cost and low reliability;
- Complexity of automated control of wind power plants both in case of their autonomous operation and in case of their operation within a grid;
- Environmental problems (noise and allocation of large territories).

Elimination of these drawbacks is associated with additional costs of creating storage devices to replace generation capacities, sophisticated distributed automation of control system of parallel operation of a large number of wind generators "virtual power plant"; and removal of wind power plants from populated settlements to the uninhabited areas.

In this paper the authors take account of all the above circumstances and consider the advantages and disadvantages of a wind hydropower system (WHPS) that consists of wind - driven pumps, a storage capacity (water reservoir) and a hydropower plant.

An advantage of the system is its principal simplicity versus other designs of wind power plants and consists mainly in a simple scheme of converting power generated by the wind power plant.

The research aims to find out the conditions to make this system more efficient as compared to the other types of wind power plants. The authors suggest a technique for feasibility study on the efficiency of the wind hydropower system. Moreover, special attention is paid to reliability of power supply to consumers connected to such systems.

The main stages of the technique for calculation of parameters and estimation of the WHPS efficiency include:

1. Study on electricity consumption, load curves and requirements for electricity supply to the existing consumers;
2. Analysis of database on wind conditions (wind speeds and duration) in the studied area.
3. In the case of sufficient wind conditions – study and choice of water sources the most appropriate for the considered local conditions to be used to fill the hydropower plant reservoir with the aid of wind-driven pumps (available nearby water source: sea, lake, river, underground sources, etc.).
4. Determination of a required installed capacity of hydropower plant, characteristics of the main equipment and construction of the hydropower plant, taking account of electricity demand, load curves and reliability.
5. Collection of information on nomenclature and parameters of commercially manufactured hydropower units. Choice of an effective number of units and their rated capacity for concrete conditions.
6. Determination of a required capacity of reservoir and its main characteristics on the basis of local topographic and weather climatic conditions. The reservoir capacity can be increased depending on other economic needs of the region. Calculations of structures and hydro constructions of the reservoir.
7. Determination of the required installed capacity of wind-driven pumps and their characteristics, on the basis of requirements for reservoir filling within a calculation period determined by the wind speeds in this area and reliability requirements.
8. Acquisition of information about nomenclature and parameters of commercially manufactured wind-driven pumps. Choice of an effective number and delivery of the pumps to meet specific conditions, reliable and sufficient to fill the reservoir to the required level. A special order can be placed to manufacture exclusive pumps.
9. Preparation of technical and economic data for comparative estimation of the suggested and alternative variants, including the case of receiving electricity from power grid, and traditional wind power plants with capacities to backup them, etc.
10. Choice of the final variant of electricity supply in the region on the basis of feasibility study of the variants.
11. Solving the other problems related to the construction of WHPS. For example, consideration of possibility of WHPS operation using the reservoir in the low head of hydropower plant (HPP) to pump water from it to the upper reservoir with the aid of wind-driven pumps (a closed cycle scheme), etc.

BRIEF CHARACTERISTIC OF THE COMPLEX

The flowchart of the WHPS is presented in Fig.1. Its main modules are:

1. Reservoir filling module – wind-driven pumps.
2. Energy storage module – reservoir (water reservoir).
3. Generation module – HPP.

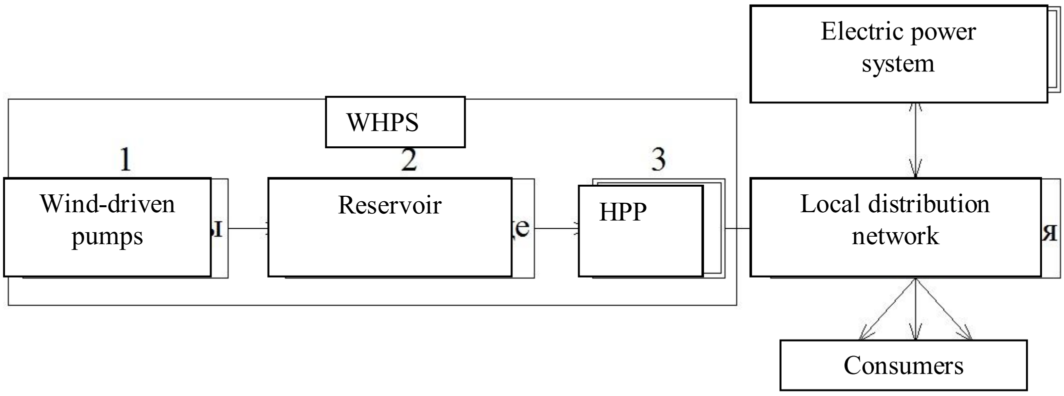

Fig.1. Flowchart of WHPS and its interfaces with the system of electricity supply to the area.

As Figure 1 shows the idea of WHPS is not original. However, the reason why we address this issue is related to the circumstances that are growing urgent nowadays.

The essence of the suggested plant lies in the fact that wind being primary energy resource for WHPS operation is harnessed to fill reservoir which represents the energy storage stage in the cycle of electricity production. The major advantage of the WHPS is the combination of wind power plant advantages and the idea of pumped water storage. This excludes the main flaw of wind power generation, i.e. a mismatch between unpredictable variations in wind speed and electricity consumption schedule. This distinguishes WPHS from similar plants in which wind energy is used directly to generate electricity to meet the demand and to charge storage battery. When there is no wind and energy storage systems such plants are backed up by diesel power plants (DPP) or gas-turbine power plants.

The wind hydropower system is in many parameters similar to the pumped storage power plants [3]. However, the main difference is the use of free wind power to fill the reservoir. The hydropower plant included in the system covers not only peak loads but the entire local load. Moreover, for WHPS the connection (at least a weak one) with power grid is desirable but not mandatory. With this connection WHPS can perform the functions imposed on distributed generation.

Compared to other wind power plants this system has the following advantages:

1. The WHPS designed according to such a flowchart makes it possible to separate and consider individually two random non-correlated processes:
   – the use of wind power for electricity supply under any wind conditions;
   – reliable (continuous) high quality electricity supply to consumers, irrespective of wind conditions at any time moment.
2. The possibility of applying relatively simple (and hence cheap and reliable wind-driven plants including wind-driven pumps with mechanical transmission of wind power to hydro pump (piston or centrifugal).
3. Application of reservoir as an energy storage which is environmentally friendlier and simpler than storage batteries of the same capacity, pressed air, hydrogen etc. as well as backup diesel units with diesel fuel stocks, or gas turbine plants.
4. Use of reservoir for the purposes other than storage, i.e. as a reservoir for tap water supply to the nearest populated settlements and productions, as a drinking place, for fish breeding, poultry farming, irrigation of agricultural lands, recreation needs, etc.

Application of such a system is efficient first of all for remote populated settlements because their full and reliable electricity supply from centralized power grid can be difficult or expensive. Although for the reasons of reliability and cost effectiveness WHPS can be constructed when consumers are supplied with electricity from grid. In this case the connection to the power grid can:

- increase the electricity supply reliability;
- decrease the reservoir capacity, particularly, if there are consumers of all categories. Then it is sufficient to use hydropower under emergency conditions to meet only the demand of consumers of the first category and partially of the second category;
- considerably improve power quality if transmission lines connected to the grid are very long and have a low rated voltage (110 kV and lower, down to 10-6 kV at lengths of 50-150 km and longer). Owing to HPP there will be local surplus active and reactive power for continuous electricity supply and voltage control;
- enable transmission of surplus power to the grid.

An additional advantage of the suggested system is the fact that to fill the reservoir it is not necessary to use high-speed wind machines, on the contrary it is more expedient to apply slow-speed wind-driven pumps. This increases the period of wind use and does not require such high aerodynamic characteristics as those necessary to use wind turbines to directly supply an electric load. The possibility of using natural relief of an area in order to construct a reservoir should also be considered as a benefit of the WHPS. The authors consider the possibility of applying such a system for electricity and water supply particularly in the arid regions.

However, along with the advantages the system has some flaws. First of all this is the impact of climatic conditions on water storage in the reservoir. With allocation of the plant in severe climatic conditions there appears a danger of reservoir and water supply system freezing and as a consequence the impossibility of their further use in the winter period. Elimination of this drawback requires additional investment. The relief of the territory may not always be suitable for the reservoir construction. Then it can be necessary to create an artificial water reservoir because of flat ground or insufficient ground strength which can lead to additional investment.

Taking into account the known electricity consumption variability over time electricity generation from WHPS should provide a reliable power supply to meet the demand.

Bearing in mind the advantages and disadvantages of the suggested system to clearly understand its efficiency as applied to specific conditions as well as to make a specific design it is necessary to develop an efficiency estimation technique as a calculation tool for solving the problem of electricity supply in specific conditions on the basis of renewable energy sources.

Thus, the technique for determining the WHPS parameters in general should include the following steps:

1. Calculation of HPP parameters on the basis of electricity consumption forecast and the need to provide reliable electricity supply to consumers;
2. Calculation of reservoir characteristics on the basis of calculated HPP parameters.
3. Determination of parameters for wind power plants on the basis of calculation results for p.2 and, bearing in mind wind characteristics of the area in which the plant is situated and reliability of wind-driven pumps.
4. Solving the other problems related to the construction of WHPS.

The HPP parameters, reservoir characteristics and parameters of wind power units are calculated using the known techniques but taking into account specific operating features of these facilities within the WHPS and estimation of the power supply reliability. Below the authors present the specific features of selecting the parameters and characteristics for the indicated components of WHPS.

DETERMINATION OF HPP PARAMETERS

Determination of HPP parameters in fact implies selection of a rated capacity and type of hydropower units with respect to selected head, and their number bearing in mind the required level of reliability of electricity supply.

The algorithm for calculation is as follows. After determining a regular annual load peak from the forecast of social and economic development of the region $N^L_{reg.peak}$ for the respective time period we find an irregular annual peak $N^L_{ir.peak}$ by the expression:

$$N^L_{ir.peak} = (1+3\sigma)N^L_{reg.peak}. \quad (1)$$

In (1) σ is standard deviation of load from a regular value of capacity, per unit. It is known [4] that these deviations follow the normal distribution.

The optimal reliability of HPP is calculated according to the Bernoulli formula [5].

The method based on the Bernoulli formula makes it possible to estimate the required reserve and probability of shortage-free operation of the facilities consisting of *n* components according to their rated parameters. In order to assess reliability of HPP these parameters will be represented by rated capacity N_r, number of hydropower units *n* and probability of failure-free operation *p*. With the assumed N_r we determine the required number of units which will provide supply of the required load with a specified (rated) probability of shortage-free operation under the minimum capacity reserve.

The calculations are made according to the formula of binomial distribution

$$\begin{aligned}(p[N_r]+q[0])^n &= p^n[nN_r]+C_n^1p^{n-1}q[(n-1)N_r]+C_n^2p^{n-2}q^2[(n-2)N_r]+\ldots+ \\ &+C_n^ip^{n-i}q^i[(n-i)N_r]+\ldots+q^n[0]=1\ ,\end{aligned} \quad (2)$$

where *p* – probability of operable state of hydropower unit (taken from the data of manufacturer or according to the emergency rate statistics for HPP equipment); $q = 1 - p$ – probability of emergency downtime of the hydropower unit; *n* – the number of units to be installed at HPP; $i = \overline{1,n}$ - the number of units that can be in an inoperable state; C_n^i – number of combinations from *n* units with respect to *i*; expressions in square brackets characterize the values of the HPP available capacity in respective calculated states.

The presented binomial expansion is a full group of events with different possible states of the HPP components. In this case this is a combination of operable (*n* - *i*) and inoperable *i* components from their total number *n*.

From (2) we find the probability of shortage-free load supply:

$$\boldsymbol{P} = \sum_{i=0}^{I} C_n^i p^{n-i} q^i [(n-i)N_r]$$

for all *i*, for which

$$(n-i)N_r \geq N^н_{ir.peak}. \quad (3)$$

In this case we choose *n* for which

$$\mathbf{P} \geq \mathbf{P}_{stnd} \quad (4)$$

In (4) $\mathbf{P}_{stnd}$ is a standard value of probability of shortage-free electricity supply to consumers. In Russia $\mathbf{P}_{stnd}$ is assumed at the level of 0.996 [4], in Western Europe – 0.9996.

If (4) is not provided then N_r and/or n, at which (4) is met, varies.

Normally N_r is taken equal to the capacity given in the catalogues of plant manufacturers of hydropower equipment and the number of units n of the required rated capacity is specified.

Thus, from the above calculation we determine the main electric parameters of HPP: rated capacity of units N_r^0, number of units n^0 and installed capacity of HPP

$$N_{HPP}^{inst} = n^0 N_r^0 \text{ kW}$$

CALCULATION OF RESERVOIR CHARACTERISTICS

At the second stage of calculation of the WHPS parameters we determine the required reservoir capacity.

Since wind conditions vary water supply to water reservoir varies too. The main objective here is to provide such volume and conditions for reservoir filling as to have sufficient water to meet the demand for electricity in a required amount and at a required time throughout the entire calculation period T, that depends on the wind conditions.

The wind conditions are estimated on the basis of data from climatologic reference books on wind for the area where the WHPS is going to be constructed. These data are used to determine the duration of periods with wind speed insufficient for operation of wind turbines and duration of energy inefficient wind speed. The two parameter Weibull distribution [1] is used in calculations to determine the repetition of wind speeds. The obtained information is then used to determine the duration of period with a wind speed that ensures useful work of wind-driven pumps.

In this case the calculation period T should be considered as

$$T=T_w+T_{i/w}, \quad (5)$$

where T_w – time of sufficient wind conditions, day; $T_{i/w}$ – time of insufficient wind conditions, day.

Energy storage is used to solve the following problems:

- Reciprocal matching of energy production and consumption schedules in order to provide uninterrupted electricity supply to consumers;
- Increase in the efficiency of wind energy utilization through complete use of the total output of wind turbines.

When resolving the issues related to storage of energy produced by wind power plants we should take into account the following characteristics:

- relative sizes;
- duration of energy storage;
- admissible amount of energy to be stored;
- complexity of energy transformations (rectification, inversion, frequency transformation, etc.)
- simplicity and safety of maintenance, etc.

The main criterion for determining the reservoir capacity is the need to provide the required water flow rate Q by operating hydropower units. Flow rate decreases with an increase in the water head H. This condition is taken into account to design the reservoir in terms of the area relief.

The calculated water head is assumed according to the possibilities of reservoir construction in a specified area. With the assumed calculated head on the basis of manufacturer's data for the chosen type of hydropower unit we determine a specific water flow rate Q_0 (m^3/kWh). The required available reservoir capacity is determined by the formula

$$V_{avlb} = W_{req} \cdot Q_0 \text{ m}^3, \quad (6)$$

where W_{req} –required HPP output determined by the load curve for a respective calculation period T:

$$W_{req} = \int_0^T N(t)dt \text{ kWh},$$

where $N(t)$ – required power of electricity consumption at hour t of the load curve.

Knowing the reservoir surface area $F_з$ (m^2), we estimate the depth of water layer of the available reservoir capacity:

$$\Delta H = V_{avlb} / F_з \text{ m}$$

Based on the known H_{min} (from manufacturer's data) we determine the maximum head water level:

$$H_{max.} = H_{min} + \Delta H \text{ m.}$$

To estimate the reservoir surface area and depth of the available reservoir capacity we should seek to reduce the surface area (which decreases the alienation of land surface for reservoir, evaporation surface etc.), and the depth of periodic reservoir drawdown since large variations in the water level have a negative impact on the flora and fauna of the reservoir itself and its seashore. Generally, the reservoir surface area can be regulated by diking the reservoir of small sizes. The regulation of the reservoir surface area makes it possible to choose the depth of drawdown and vice versa, depending on the specific circumstances.

The selection of the hydropower units for WHPS implements the key principle of determining the reservoir capacity, i.e. makes it possible to provide the minimum possible flow rate Q in order to minimize the reservoir capacity and as a result decrease investment in its construction and operation.

Since an HPP supplies electricity to consumers, covering the whole of the load curve, the available reservoir capacity should satisfy the water flow rate by hydropower units for the assumed calculation period T of power supply. The calculation period can be taken equal to the time interval from a day (daily storage) to a year (yearly storage). The calculation period is chosen based on specific wind parameters – the more frequent is the wind, the shorter is the calculation period and hence, the smaller is the water reservoir and the lower is the investment in its construction.

In addition to the available reservoir capacity V_{av} to meet the water flow rate by hydropower units, account should be taken of losses caused by water evaporation V_{evap} from reservoir surface, by filtration V_f through ground and by ice formation V_i for the areas of cold climate [6, 7].

The methods for determination of water flow rate to compensate for these losses are empirical and applied depending on every specific case.

The total storage capacity will be

$$V_{total} = V_{av} + V_{evap} + V_f + V_i + V_{dead} = V_{av} + V_{loss} + V_{dead} \text{ m}^3,$$

where $V_{loss} = V_{evap} + V_f + V_i$; V_{dead} – dead storage capacity.

Since construction of a purely man-made lake is an expensive measure, it is more expedient to arrange it on the basis of natural relief roughness with minimum involvement of materials and labor inputs in construction of a storage reservoir. The possibilities for provision of V_{total} at the site are evaluated by calculating the storage capacity of a prospective reservoir through the sequential summation of capacities ΔV_i of individual layers between two adjacent contour lines on the topographic maps. This is done for determination of reservoir surface F by way of their planimetering based on the knowledge of topographic characteristics of the site.

In general the storage capacity V_{total} can be increased in case of the need to solve other economic problems in the area of WHPS construction that were mentioned above. This is, however, a separate problem.

The method of reservoir arrangement and its type by its design features are determined based on technical and economic indices of one or another variant for specific conditions. However, preference should be given to reservoir arrangement, taking advantage of natural relief as much as possible [6, 7].

At the initial stage of reservoir filling the wind-driven pumps will have to fill the total storage capacity V_{total}, which will require some time. Then after filling of the dead and available capacities, the wind-driven pumps will have to fill only the capacity $V_{av} + V_{loss}$ to be emptied during the calculation period T. And the capacity filled in advance is emptied at the current period and simultaneously the new volume $V_{av} + V_{loss}$ is stored for HPP operation at the next period.

Reservoir dislocation can be chosen based on a great number of variants: gorge, ravine, notch, depression on the upland. Apart from natural conditions, it is possible to consider creation of a man-made diked lake, a lake with consolidation of its bed with impermeable materials, etc.

CALCULATION OF WIND-DRIVEN PUMP PARAMETERS

In operation of any plant using wind energy, including WHPS, the wind parameters are of prime importance as a source of energy production.

Wind depends on many complex geophysical and climatic factors. Its variability, therefore, can be predicted only with some probability that is determined as a result of statistical processing of the results of wind speed observations in the considered area for a long-term period.

Wind speed is the most important energy characteristic that estimates its kinetic energy. Under the impact of some meteorological factors (atmosphere perturbations, changes in solar activity and amount of heat energy arriving from the space to the Earth, etc.), and also the relief of the site, the wind speed changes in rate and direction. The powerful winds favorable for operation of wind power plants alternate thereby with calms.

The wind-driven pump capacity depends on the wind speed and the surface area swept by the wind wheel and is calculated by the formula:

$$N_w = \frac{\upsilon^3 D^2}{7000} \text{ kW,} \tag{7}$$

where υ – wind speed, m/s; D – wind wheel diameter, m.

The wind-driven pump converts part of this capacity into effective capacity that is estimated by the wind energy utilization factor ζ:

$$N_{w\,av} = \zeta \cdot N_w. \tag{8}$$

In general it is not expedient to use wind power plants for direct covering of electric loads without additional expensive smoothing and replacing facilities and also automatic controllers by virtue of essential distinctions between the consumer load curve and the curve of wind speed variation as random functions of time.

As was noted, the wind-driven pump (module for reservoir filling) capacity can be determined on the basis of the necessary storage capacity $V_{av} + V_{loss}$ and the reservoir filling time T_r to be known, during the calculation period T. It is apparent that the dead storage capacity is filled once before the beginning of WHPS operation, and installation of additional wind-driven pumps for this purpose will be inexpedient.

The total pumping capacity of wind-driven pumps $Q_{w\Sigma}$ is calculated by the formula:

$$Q_{w\Sigma} = \frac{V_{av} + V_{loss}}{T_w} \text{ m}^3/\text{s},$$

where T_w is in seconds.

The total delivery of wind-driven pumps is determined by the expression:

$$N_{w\Sigma} = \frac{9.81 Q_{w\Sigma} \text{H}}{\eta} \text{ kW}, \quad (9)$$

where η – pump efficiency; H – height of water, m.

The time of wind-driven pump operation T_w is determined from the formula in [1]:

$$T_w = \frac{f(\upsilon \geq \upsilon_0) \cdot T}{100}, \quad (10)$$

where $f(\upsilon \geq \upsilon_0)$ – probability that the initial speed of the wind-driven pump will be exceeded, %; υ_0 – initial speed of the wind wheel, m/s. In calculations υ_0 is taken equal to 3 m/s. The multi-blade wind-driven pumps that are targeted for use in WHPS start to operate at this speed.

The values of $f(\upsilon \geq \upsilon_0)$ as a function of the wind parameters $\upsilon_0 / \overline{\upsilon}$ and c_υ are determined as tabular data in accordance with the Weibull distribution [1].

The down time of the wind-driven pumps T_{calm} is determined as:

$$T_{calm} = T - T_w \text{ , h.} \quad (11)$$

The relation between T_w and T_{calm} may be arbitrary, and the time of sufficient wind speed T_w may be both longer and shorter than T_{calm}. The wind-driven pump capacity depends on the relation between T_w and T_{calm}. The longer is T_w, the lower is the capacity $N_{w\Sigma}$.

The rated capacity of the wind-driven pump $N_{w\ rat}$ is chosen based on the machine industry capabilities. It is obvious that for these purposes the choice should be made of maximum possible capacity in terms of specific design conditions.

Then the minimum needed number of wind-driven pumps is calculated as

$$N_w = N_{w\Sigma} / N_{w\,r}. \quad (12)$$

The number of wind-driven pumps and their rated capacity considering reliability of wind-driven pumps can be evaluated more accurately on the basis of their emergency rate q_w to be determined and formula (2). The standard reliability of all the wind-driven pumps is taken as a function of wind conditions in the considered region, however, not lower than the probability of shortage-free power supply (for RF – 0.996).

The described stages of WHPS calculation are basic for designing the considered plant. Technical and economic problems dealing with power supply from WHPS are solved at the next stage.

From the technical standpoint they include:

- generation of an electric circuit of hydropower plant, choice of voltages of generator, auxiliaries, master switchgear;
- generation of an electric circuit of local distribution network;
- assurance of reliability of power supply to consumers and required power quality in accordance with the standards of electric installation code, maintenance rules, and other documents;
- consideration of specific features of natural-climatic and geographical conditions for WHPS construction when being designed;
- implementation of capabilities for additional utilization of the man-made reservoir in socio-economic development of the region;
- assessment of WHPS security.

From the economic standpoint it is necessary to carry out a feasibility study on the effectiveness of the proposed system in comparison with other alternative options of power supply:

1) wind power plants in combination with replacing power sources (diesel power plants, geothermal power plants, etc.).
2) wind power plants in combination with storage facilities of other types (thermal, chemical, mechanical, etc.).

In addition one should bear in mind advantages of the suggested complex over the mentioned ones:
- simplicity of design;
- simplicity of meeting the main requirements to reliability and quality of power supply to consumers;
- wider range of economic usage (not only power generation).

Thus, the optimal decision for all these options can be chosen only based on the specific feasibility analysis that considers secondary advantages such as environmental.

ASSESSMENT OF TECHNICAL AND ECONOMIC EFFECTIVENESS

Technical and economic characteristics of the suggested WHPS should be determined to compare them with similar characteristics of other alternative power generation sources in the considered region and to estimate their dependence on local conditions.

The technical and economic characteristics of power supply options should be determined on the basis of their functional comparability: full satisfaction of demand, power supply reliability and power quality. Besides, one should bear in mind such advantages of WHPSs over other options as absence of fuel costs, simplicity, low cost and high reliability in comparison with wind power plants, substantially simplified control of WHPS, possibility for solving other (in addition to power supply) socio-economic problems in the considered region, higher environmental compatibility of WHPS, etc.

Effectiveness can be assessed based on the information about expenditures for both WHPS and alternative power supply options such as renewable and non-renewable energy sources, i.e. expenditures for construction of replacing diesel or gas-fired power plants, fuel cost, cost of diverse storage facilities and their practical limits on capacity.

For the most general case the expression for the simplified technical and economic evaluation of WHPS can be written in the following form:

$$aN_{WP} + bV_{total} + cN_{HPP} < dN_{WPP} + eN_{SF} + fN_{DPP} + gB_f W \frac{Tcalm}{T} n. \qquad (13)$$

In this expression:

a – unit cost of wind-driven pump (WP), RUR/kW;
N_{WP} – installed capacity of WP, kW;
b – unit cost of reservoir construction, RUR/m^3;
V_{total} – reservoir storage capacity, m^3;
c – unit cost of hydropower plant (HPP) construction (without reservoir cost), RUR/kW;
N_{HPP} – installed capacity of HPP, kW;
d – unit cost of wind power plant (WPP), RUR/kW;
N_{WPU} – installed capacity of WPP, kW;
e – unit cost of storage facility (SF), RUR/kW;
N_{SU} – installed capacity of SF, kW;
f – unit cost of additional power plant (DPP) construction, RUR/kW;
N_{DPP} – installed capacity of DPP, kW;
g – fuel cost for DPP, RUR/kg;
B_f – specific fuel consumption by DPP, kg/kWh;
W – required power generation for the calculation period T, kWh;

T – calculation period corresponding to cyclic recurrence of wind activity at the considered site, hours;

T_{calm}– duration of calm period at the calculation period, hours;

n – recurrence number of periods T during the WHPS service life.

Effectiveness is assessed on the basis of the following simplifications and assumptions.

The unit cost includes the cost of land allotted for facilities to be constructed.

Only the costs that differ in the options compared are calculated. Therefore, the proceeds from electricity trade that are assumed to be equal and the fixed costs for operation of compared power facilities are not taken into account.

Discounting costs during the service life of power facilities, which can hardly influence the basic compared option are not considered. Their service life is taken equal, i.e. 30 years.

If the cost of plants compared proves to be almost equal, preference should be given to WHPS owing to the indicated additional effects of its use.

The hydropower plant that is considered in this statement differs from the traditional run-of-river plant in the essential decrease of the probabilistic nature of water inflow to the reservoir. Here the necessary water volume is provided to a sufficiently high degree by installation of additional wind-driven pumps that deliver the needed volume of water at the periods of sufficient wind speed.

Expression (13) is of universal character to compare WHPS to any types of alternative options. Therefore, in the right-hand side there are zero values for the plants that are not used in the corresponding option.

The financial efficiency can be assessed by calculating the payback period:

$$T_{payback} = S/P = S/((C - Z)W_{year}),$$

where S – expenditures for the corresponding project (the right- or left-hand side of expression (13));

P – annual profit from produced electricity sales;

C – electricity price in the energy market;

Z – electricity production cost;

W_{year} – volume of annual electricity sales.

Preliminary analysis of the field of WHPS use has shown that these systems prove to be attractive in the range of power consumption from 30–50 kW to 10–15 MW. In this case at low loads it is possible to have small ponds filled by two or three wind-driven pumps that deliver water at a height of 20-30 m rather than conventional reservoirs. WHPSs of higher capacity will require water heads of 100–150 m and higher (considering employment of diversion schemes). WHPSs of larger capacity become irrational because of vast areas required for allocation of wind-driven pumps and a reservoir.

An example of the comparative technical and economic evaluation of WHPS and an alternative power supply option (an additional diesel power plant as the cheapest alternative option) is presented below.

EXAMPLE OF COMPARATIVE ASSESSMENT OF TECHNICAL AND ECONOMIC EFFECTIVENESS OF WHPS AND AN ALTERNATIVE OPTION

The authors consider power supply to a coastal area of Lake Baikal [8, 9].The annual regular maximum load is 650 kW. In accordance with (1) the irregular maximum load is taken equal to 1.09*650 = 710 kW with σ =0.03. The required power consumption for the calculation period T considering losses and auxiliary power supply will make up 2147040 kWh. The required HPP capacity considering uninterrupted power supply (P = 0.9996) will amount to 142*7 = 994 kW. Lake Baikal is the water source. The necessary reservoir capacity is 0.13 km^3. Wind speed in the area allows the calculation period T to be taken equal to 6 months and the total time of energy-effective wind strength during the calculation period T_w to be taken equal to 3 months.

Two options are studied:

1. Wind hydropower system.
2. Wind power plant with an additional diesel power plant.

According to calculations in the first option the wind-driven pump **capacity** should be 2300 kW.

In the second option the wind power plant capacity equals 1100 kW, the diesel power plant capacity –100 kW (considering power supply reliability).

The technical and economic analysis was carried out based on the following averaged economic indices (see (13)):

a = 8000 RUR/kW;
b= 10 RUR/ m^3;
c = 5000 RUR/ kW;
d = 25000 RUR/ kW;
f = 10000 RUR/ kW;
g = 30 RUR/ kg;
B_f = 0.4 kg / kWh;
n = 60 – number of occurrences of periods T during the power supply system life (30 years).

The costs of construction and operation of the considered power supply options are calculated based on (13):

1. $aN_{WP} + bV_{total} + cN_{HPP} = 8000 \cdot 2300 + 10 \cdot 0,13 \cdot 10^9 + 5000 \cdot 994 = 18400000 + 1300000000 + 4970000 = 1\,323\,370\,000$ RUR $\cong$ 1,323 billion RUR.
2. $dN_{WPP} + eN_{SF} + fN_{DPP} + gB_fW\frac{T_{calm}}{T}n = 25000 \cdot 1100 + 0 + 10000 \cdot 1100 +$

   $+ 30 \cdot 0.4 \cdot 2147040 \cdot \frac{3}{6} \cdot 60 = 27500000 + 0 + 11000000 + 772934400 = 811434400$

   RUR $\cong$ 0.811 billion RUR.

The calculations show that the reservoir is the most expensive structure of WHPS (98.3 %). Hence, special attention should be paid to decrease of its construction costs. To make the calculations more accurate it is necessary first of all to estimate the real value of b because of its impact on the cost of the first option. The costs of both options are comparable even with b = 6 RUR/m^3, and with b = 1 RUR/m^3 the WHPS costs become equal to 0.152570 billion RUR, i.e. 5.3 times cheaper than the second option. The WHPS competitiveness will increase with the fuel price rise (the costs of fuel delivery to remote areas are not taken into account and they are comparable to fuel cost and even exceed it).

The main conclusion from the indicated calculations is: effectiveness of the WHPS option depends on reservoir parameters. The smaller is the reservoir capacity, and correspondingly the unit costs, the more profitable will be the WHPS option. The indicated reservoir parameters can be decreased by increasing the head H, shortening the calculation period T and maximum possible utilization of natural relief elements for reservoir construction at the specific site.

CONCLUSION

1. In the context of increasing shortage of fossil fuel resources and topicality of environmental problems the necessity of using renewable energy resources rises.
2. Investigations in the field of the most economical and technologically expedient renewable energy sources for specific areas result in designs of different systems, WHPS as an example.
3. The design works and commissioning of such a system can be realized on the basis of the technique for determination of its technical and economic effectiveness. The technique is to solve a great number of problems: from choice of the primary WHPS link – wind-driven

pump to the final result – generation of power of the required quality for its reliable supply to consumers.

4. The technique should be applied as a tool for assessment of the efficiency of using the suggested system. All sorts of difficulties cannot be overcome successfully without flexible consideration of wind energy utilization forms.
5. Parameters of the required reservoir depend on the electric load, on the one hand, and the wind speed in the area of WHPS construction, on the other hand. The calculation period of reservoir drawdown is chosen based on the wind speed in the considered area. Therewith T_w is always shorter than *T,* and the shorter is *T* and the longer is T_w, the smaller will be the reservoir capacity and hence, the cheaper will be the WHPS construction.
6. The paper suggests a sequence of the WHPS calculation, choice of its basic parameters including power supply reliability and technical and economic effectiveness. The sequence of calculation is universal, i.e. it is applicable to any conditions of WHPS operation.
7. In general, WHPS plays a part of “distributed” generation that is defined as power generation at the point of its consumption. In this case the power losses and the costs of its transmission by regional power grids are excluded. Power supply reliability improves.
8. Availability of even a weak tie line with the power grid enhances flexibility, reliability and effectiveness of the local power supply system. Power quality in the considered area improves considerably. Besides, in this case excess power can be supplied to the common system network.
9. WHPS as distributed generation is of diversification character, allowing the variety of plants on renewable energy resources that utilize wind energy to be increased.

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CONNECTIVITY PROBABILITY OF RANDOM GRAPH GENERATED BY POINT POISSON FLOW

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ABSTRACT

In different applications (for an example in the mining engineering) a problem of a definition of a set in two or three dimension spaces by a finite set of points origins. This problem consists of a determination in the finite set of some subset of points sufficiently close to each other. A solution of this problem consists of two parts. Primarily initial finite set of points is approximated by point Poisson flow in some area which is widely used in the stochastic geometry [1, sections 5, 6]. But a concept of a proximity is analyzed using methods of the random graph theory like a concept of maximal connectivity component [2] - [4]. This concept origins in a junction of the combinatory probability theory and of the graph theory. An analysis of these concepts and mathematical constructions leads to a generalization of the random graph theory theorems onto graphs generated by point Poisson flow in some area.

1. MAIN RESULTS AND THEIR PROOFS

Following [1] consider the Poisson flow $\Lambda(n)$ of points $x(1),...,x(\tau(n))$ with the intensity $\lambda_n = n$ in three dimension cube A with unit length edges. Contrast each pair of points $x(i), x(j)$ the Euclidean distance $\rho(x(i), x(j))$ and introduce Boolean variable $z(x(i), x(j)) = 1$, if $\rho(x(i), x(j)) < r(n)$ and $z(x(i), x(j)) = 0$ in opposite case. Here $r(n)$ is some positive number dependent on n. Construct the random graph $\Gamma(n)$ with $\tau(n)$ vertices and the adjacency matrix $\|z(x(i), x(j))\|_{i,j=1}^{\tau(n)}$. Denote $P(n)$ the connectivity probability of this graph, $\overline{P}(n) = 1 - P(n)$.

Theorem 1. Suppose that $r(n) = (n\varphi(n))^{-1/3}$, $\varphi(n) \to \infty$, $n \to \infty$. Then the disconnection probability $\overline{P}(n) \to 1$.

Proof. Denote the center of the cube A by O and describe around O the ball $U_1(n)$ with the radius $r_1(n) = (n/\varphi_1(n))^{-1/3}$, $\varphi_1(n) \to \infty$, $\varphi_1(n)/n \to 0$, $\varphi_1^2(n)/\varphi(n) \to 0$, $n \to \infty$.

Then the probability that a positive number of the flow $\Lambda(n)$ points turns out in the ball $U_1(n)$ is

$$P_1(n) = 1 - \exp\left(-\frac{4\pi n \cdot \left((n/\varphi_1(n))^{-1/3}\right)^3}{3}\right) = 1 - \exp\left(-\frac{4\pi\varphi_1(n)}{3}\right) \to 1,\ n \to \infty.$$

Construct further the ball $U_2(n)$ with the center O and with the radius

$$r_2(n) = r_1(n) + r(n) = (n/\varphi_1(n))^{-1/3} + (n\varphi(n))^{-1/3}.$$

Consider the ball layer $U_2(n) \setminus U_1(n)$ and using the condition $\varphi_1^2(n)/\varphi(n) \to 0$, $n \to \infty$, calculate its volume

$$V_2(n) = \frac{4\pi\left(r_2^3(n) - r_1^3(n)\right)}{3} = 4\pi n^{-1}\left(\varphi_1^2(n)/\varphi(n)\right)^{1/3}\left(1 + \mathrm{O}\left(\left(\varphi_1^2(n)/\varphi(n)\right)^{1/3}\right)\right), \ n \to \infty$$

Then for $n \to \infty$ the probability $P_2(n)$ of an absence in the set $U_2(n) \setminus U_1(n)$ points of the flow $\Lambda(n)$ satisfies the formula:

$$P_2(n) = \exp(-nV_2(n)) = \exp\left(-4\pi n n^{-1}\left(\varphi_1^2(n)/\varphi(n)\right)^{1/3}\left(1 + \mathrm{O}\left(\left(\varphi_1^2(n)/\varphi(n)\right)^{1/3}\right)\right)\right) \to 1 .$$

Build now the set $A \setminus U_2(n)$ and calculate the probability $P_3(n)$ that a positive number of the flow $\Lambda(n)$ points belong to this set using the condition $\varphi_1^2(n)/\varphi(n) \to 0$, $n \to \infty$,

$$P_3(n) = 1 - \exp(-n(1 - V_2(n))) = 1 - \exp\left(-n\left(1 - 4\pi n^{-1}\left(\varphi_1^2(n)/\varphi(n)\right)^{1/3}\left(1 + \mathrm{O}\left(\left(\varphi_1^2(n)/\varphi(n)\right)^{1/3}\right)\right)\right)\right) \to 1,$$
$$n \to \infty .$$

So the probability of an intersection of the following events:

1) in the ball $U_1(n)$ there is positive number of the flow $\Lambda(n)$ points;
2) in the ball layer $U_2(n) \setminus U_1$ there are not points of the flow $\Lambda(n)$;
3) in the set $A \setminus U_2(n)$ there is a positive number of the flow $\Lambda(n)$ points

tends to the unit for $n \to \infty$. As a distance from any point of the ball $U_1(n)$ to each point of the set $A \setminus U_2(n)$ is not smaller than $r(n)$, so these events intersection contains in the event: the random graph $\Gamma(n)$ is not connected. Consequently the following limit relation is true: $\overline{P}(n) \to 1$, $n \to \infty$.

Theorem 2. Suppose that $r(n) = (n/\psi(n))^{-1/3}$ where $\psi(n)/n \to 0$, $\psi(n) - \ln n \to \infty$, $n \to \infty$. Then the connectivity probability $P(n) \to 1$, $n \to \infty$.

Proof. From the condition $\psi(n) - \ln n \to \infty$, $n \to \infty$, we obtain that $r(n) \to 0$, $n \to \infty$. Assume that the cube A and the cube A_n with the edge length $r(n)/\sqrt{6}$ have common centre and are homothetic. By a parallel hyphenation of the cube A_n on distances fold to $r(n)/\sqrt{6}$ by each coordinate construct a family of all cubes intersected with the cube A. An bundling of these cubes contains the cube A. A complete number $N(n)$ of all cubes from this family for some finite and positive number C is smaller than $Cn/\psi(n)$. The probability $P'(n)$ that a positive number of the flow $\Lambda(n)$ points occurs in some cube of this family equals

$$P'(n) = 1 - \exp\left(-n \cdot (n/\psi(n))^{-1}/6\sqrt{6}\right) = 1 - \exp\left(-\psi(n)/6\sqrt{6}\right).$$

Consequently the probability $P''(n)$ of these $N(n)$ events intersection $L(n)$ equals

$$P''(n) = (P'(n))^{N(n)} = \left(1 - \exp\left(-\psi(n)/6\sqrt{6}\right)\right)^{N(n)} \geq \left(1 - \exp\left(-\psi(n)/6\sqrt{6}\right)\right)^{Cn/\psi(n)},$$

so

$$0 \geq (\ln P''(n))^{N(n)} = Cn \ln\left(1 - \exp\left(-\psi(n)/6\sqrt{6}\right)\right)/\psi(n) \geq$$
$$\geq -\frac{Cn\exp(-\psi(n))}{6\sqrt{6}\psi(n)\left(1 - \exp\left(-(n)/6\sqrt{6}\right)\right)} = R(n).$$

From the condition $\psi(n) - \ln n \to \infty, n \to \infty$, we have that $R(n) \to 0$, $n \to \infty$. As a result the probability of the event $L(n)$ satisfies the formula $P''(n) \to 1$, $n \to \infty$. A distance between each two points belonging to incident cubes (with common face) is not larger than $r(n)$. So the event that the graph $\Gamma(n)$ is connected contains the event $L(n)$ and the inequality $P(n) \geq P''(n)$ is true. As a result we have that

$$1 \geq \liminf_{n\to\infty} P(n) \geq \lim_{n\to\infty} P''(n) \geq 1$$

The theorem 2 is proved completely.

2. CONCLUSION

Remark 1. In the conditions of Theorem 2 for any γ, $1/2 < \gamma < 1$, with a probability tending to the unit for $n \to \infty$, the graph $\Gamma(n)$ has single connectivity component with a number of vertices larger than $\gamma\tau(n)$. Indeed the event that there is single connectivity component with the number of vertices larger than $\gamma\tau(n)$, includes the event that the graph $\Gamma(n)$ is connected. From Theorem 2 the probability of the last event tends to the unit for $n \to \infty$. Analogous concept of a giant connectivity component but for a complete graph with independently working edges was introduced and was analysed in [2].

Remark 2. Theorems 1, 2 may be reformulated in terms of the threshold function of the connectivity as a graph theory property [2], [3, Section 10]. If $r(n) << n^{-1/3}$ then with the probability $\overline{P}(n) \to 1$, $n \to \infty$, the graph $\Gamma = \Gamma(n)$ in unconnected. If $r(n) << n^{-1/3}$ then with the probability $P(n) \to 1$, $n \to \infty$, the graph $\Gamma(n)$ is connected. In the last statement it is necessary to make a single refinement of the condition $r(n) << n^{-1/3}$ in the form $r(n) = (n/\psi(n))^{-1/3}$, $\psi(n) - \ln n \to \infty$, $n \to \infty$.

Remark 3. More modern treatment of introduced concepts but for complete graphs with independent and randomly working edges was made in [4].

Remark 4. In Theorems 1, 2 the cube A may be replaced by another geometrical bodies for an example by a ball.

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PRINCIPLES OF CLASSIFICATION RELIABILITY STATISTICAL DATA OF THE ELECTRIC EQUIPMENT OF POWER SUPPLY SYSTEMS

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ABSTRACT

The result of comparison of criteria, which statistics characterize differing properties of random variables of sample, depends on the importance of these properties. In turn, the importance of properties can essentially change for modeled analogues of sample

I. INSTRUCTION

It is known, that the basic requirements shown to the decision of numerous operational problems in electro power systems (EPS), is maintenance of reliability of work and decrease in operational expenses [1]. The bright example are problems of the organization of maintenance service and repair (MS&R) electric equipments. In turn, maintenance of reliability provides an opportunity of comparison estimations parameters of reliability (PR) concrete electric equipment, i.e. transition from average PR to parameters of individual reliability. Average PR electric equipments are important and traditionally used, for example, at comparison PR of schemes of projected switching centre, at an estimation of size of a reserve of capacity in EPS. At calculation PR of the concrete equipment on known, simple enough, to formulas and algorithms, experts meet essential difficulties. PR calculated on population (i.e. the average estimations) unsuitable for the decision of operational problems. In addition, data on refusals and defects of the concrete equipment are so poor, that or do not allow to calculate PR, or accuracy of estimations appears unacceptable. Therefore, maintenance of reliability of work in practice carried out, unfortunately, mainly at an intuitive level. The certain contribution to the decision of this problem is brought by the automated information systems providing information support of dispatching personnel EPS. But the objective estimation of parameters is still actually on individual reliability.

II. FEATURES OF STATISTICAL DATA

Necessary to note, that at the analysis of reliability of equipment EPS classification of statistical data of operation on one, and sometimes and to the two attributes, set by nameplate data and data of conditions of operation, it is spent. For example, in [2] are resulted PR electric equipments of a various class of a voltage. Are occasionally resulted PR electric equipment grouped as or to purpose, a design, service life, a manufacturer and other attributes. Classification of statistical data more than to two attributes does not practice. The reason for that is the variety of versions of attributes (VA) and decrease in accuracy of estimations PR (increase in width of a confidential interval). Decrease in accuracy occurs within the limits of the assumption of conformity of statistical data to casual sample of some general population.

Actually:

1. The statistical data describing reliability of equipment EPS (data on non-working conditions), depend on the big number of passport and operational data (installation sites, a class of a voltage, a design, service life, etc.) and consequently <u>cannot be considered</u> neither as analogue of general population, nor <u>as final sample of homogeneous data</u>. In the mathematician such data it is accepted

to name multivariate. Unfortunately, analytical methods of the analysis of multivariate data developed only for the assumption of conformity of distribution of random variables to some to one, to mainly normal law of distribution. It at all does not correspond to the real histograms of distribution constructed on statistical data of operation of electric equipment. As an example on fig.1 histograms of duration of emergency switching-off are resulted (τ_a) power units 300MVt [3]. The first histogram characterizes distribution according to operation of eight power units for the period 1992-2006 years. The second histogram characterizes distribution $P^*(\tau_a)$ all power units for 2005 year. The number of cases of emergency switching-off for this sample has decreased with 634 up to 48. On the third histogram distribution P* is shown $P^*(\tau_a)$ for the first power unit in 2005year.
2.

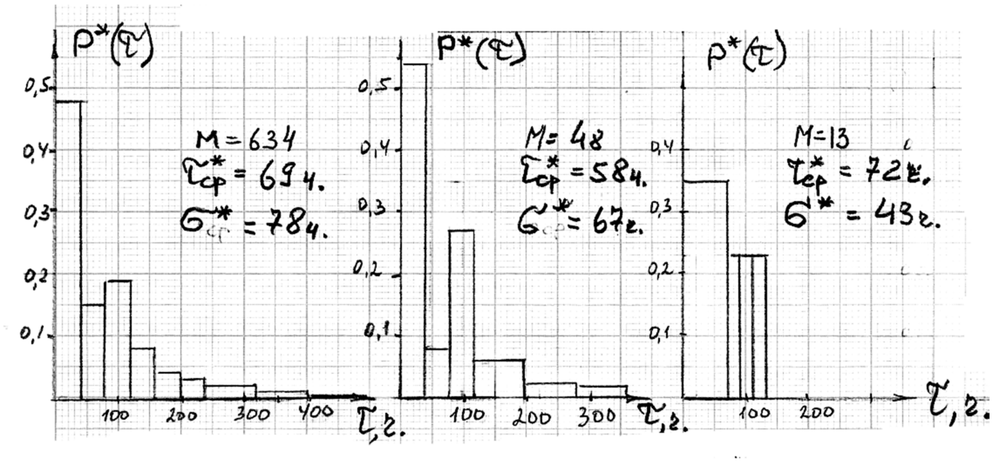


Fig.1 Histograms of duration of emergency switching-off of power units 300MVt

Comparison of character of change of these histograms and laws of change of normal distribution confirms small probability of conformity $P^*(\tau_a)$ to one concrete and, in particular, to the normal law of distribution.

3. At classification of multivariate statistical data on set VA, selective data taken from final population of multivariate data not casually. For example, all switches with rated voltage 110Kv not casually get out. We shall specify this feature. Not casual sample:

- consists of random variables;
- number of random variables of sample n_v is casual, changes in time, for example, increases;
- features of distribution in an interval of change random variables final population of multivariate data depends from VA;

3. The type of the law of distribution of final population of multivariate statistical data not only is not known. It regularly casually varies in process of accumulation of statistical data
4. The interval of change random variable in sample of final population of multivariate statistical data on set VA is no more, than an interval of change random variable in the most final population. We shall remind, that for general population of a random variable the average quadratic deviation always is less, than the average quadratic deviation for any on number of representative sample and with reduction of number of random variables in sample, n_v an estimation of an average quadratic deviation increases. These features allow concluding, that application of classical methods of the analysis samples from general population for the analysis samples from final population of multivariate data it is necessary to be careful.

III. ABOUT SET STATISTIC, DESCRIBING RANDOM VARIABLES OF SAMPLE

The most objective approach to the decision of the statistical problems arising at classification of multivariate data is application of computer modeling possible samples and checks of assumptions (hypotheses) about expedient classification of data on everyone VA. Difficulties arise at an estimation of expediency of classification of data. As matter of fact - this know problem about a finding significant VA. We spent the decision of this problem within the limits of methodology of the theory of check of statistical hypotheses. In mathematical statistics, it considered two types of the problems connected with comparison of functions of distribution:

1. Check of the assumption that sample of random variables X casually taken from general population of random variables with the set type of distribution $F_\Sigma(X)$.
2. Check of the assumption of uniformity two or several sample the random variables casually taken from same general populations with the known continuous law of distribution $F_\Sigma(X)$.

The estimation of expediency of classification of multivariate statistical data offered to be carried out by comparison of statistical functions of distribution (s.f.d.). Final population of multivariate data $F^*_\Sigma(X)$ with s.f.d. samples $F^*_V(Y)$, of this population. Comparison $F^*_\Sigma(X)$ also $F^*_V(Y)$ theoretically carried out on number of numerical characteristics of a random variable a vertical divergence of distributions $F^*_\Sigma(X)$ and $F^*_V(Y)$, which we shall designate as Δ(Y) also we shall calculate under the formula:

$$\Delta(Y_i) = F^*_\Sigma(Y_i) - F^*_V(Y_i) \tag{1}$$

where: $1 < \Delta(Yi_) < 1$ c $i=1,n_v$

According to the established practice, these numerical characteristics we shall name statistics and we shall designate S (Δ). To S(Δ) concern:

1. The greatest vertical divergence between $F^*_\Sigma(X)$ and $F^*_V(Y)$. It is calculated on following algorithm:
   1.1. n_v realizations Δ(Y) are placed in ascending order;
   1.2. Absolute values of the first and n_v -th values ranking of some Δ(Y) are compared and the greatest value is defined Δ_m;
   1.3. It also is the greatest vertical deviation $F^*_\Sigma(X)$ and $F^*_V(Y)$ with the sign;
2. Average value of a vertical divergence. It is calculated under the formula:

$$M^*[\Delta(Y)] = n_v^{-1}\sum_{i=1}^{n_v}|\Delta(Y_i)| = \Delta^*_{av} \tag{2}$$

It is necessary to note, that

$$M^*[\Delta(Y)] \neq n_v^{-1}\sum_{i=1}^{n_v}\Delta(Y_i) \tag{3}$$

As under this formula, average value of a random variable Δ(Y), instead of an average deviation is calculated. Distinction between formulas (2) and (3) shown, when among realizations Δ(Y) there are both positive, and negative sizes.

3. Average quadratic deviation Δ(Y). Are calculated under the formula:

$$\sigma^*[\Delta(Y)] = \sqrt{\frac{\sum_{i=1}^{n_v}\left[\Delta^*_{av} - |\Delta(Y_i)|\right]^2}{(n_v - 1)}} = \Delta^*_{ск} \tag{4}$$

By analogy with p.2

$$\sigma^*[\Delta(Y)] \neq \sqrt{\frac{\sum_{i=1}^{n_v}\left[\Delta^*_{av} - \Delta(Y_i)\right]^2}{(n_v - 1)}} \quad (5)$$

4. Scope of dispersion of a random variable $L^*_V(\Delta)$. It is calculated under the formula:

$$L^*_V(\Delta) = \Delta_{max} - \Delta_{min} \quad (6)$$

This list could be continued. But also it is enough resulted numerical characteristics for an illustration of the mechanism of comparison efficiency of criteria check assumptions of character a divergence s.f.d. $F^*_\Sigma(X)$ and $F^*_V(Y)$

IV. A QUESTION ON EFFICIENCY OF CRITERIA

According to the established practice efficiency of criteria is characterized by function of capacity of criterion W[S(Δ)]. In turn W[S(Δ)] =1-β[S(Δ)], where-β[S(Δ)] - an error II type for statistics S(Δ). The essence of considered criteria is same and is reduced to comparison of empirical value $S_э(\Delta)$ with boundary value of distribution $R[S_{\alpha}(\Delta)]=\alpha[S(\Delta)]$, where α[S(Δ)] - an error I type.

It is accepted, for the fixed value α[S(Δ)] to consider criterion as more effective if its function of capacity has the greatest value. So that to compare with efficiency of criteria it is enough to construct dependences W[S(Δ)] from α[S(Δ)] and to compare W[S(Δ)] for 0<α[S(Δ)] <1

The algorithm of construction of this dependence reduced to following calculations:

1. Construction s.f.d. realizations of statistics $S_1(\Delta)$ for initial assumption H_1 according to which distributions $F^*_\Sigma(X)$ also $F^*_V(Y)$ differ casually. We shall designate this distribution as $F^*[S_1(\Delta)]$. The sequence of calculations, features of imitating modeling of realizations representative sample, results of calculations for of some n_v is resulted in [4] on an example of statistics of the greatest vertical deviation Δ_m;
2. Here the sequence of construction s.f.d. is resulted. Realizations of statistics $S_2(\Delta)$ for assumption H_2 according to which distributions $F^*_\Sigma(X)$ also $F^*_V(Y)$ differ not casually.
3. It is systematized realizations $F^*[S_1(\Delta)]$ and $F^*[S_2(\Delta)]$ at $S_1(\Delta)=S_2(\Delta)$. As quantile distributions $F^*[S_1(\Delta)]$ are not equal quantile distributions $F^*[S_2(\Delta)]$, performance p.3. It appears impossible. The analysis of realizations quantile these distributions after ranking shows, that distinction of some realizations $S_1(\Delta)$ and $S_2(\Delta)$ takes place not less than in the fourth category of their quantitative estimations. If to neglect this difference, the number of equal realizations $S_1(\Delta)$ and $S_2(\Delta)$ reaches 10%. Unfortunately, this quantity is often not enough for the full characteristic of dependence W[S(Δ)]=φ{α[S(Δ)]}. The decision of this problem found the basis of the assumption of linear character of change s.f.d. Intervals between quantile distributions.

Considering, that the number quantile estimated in hundreds, the size of an entered error calculations corresponding quantile probabilities appears less accuracy of calculation quantile.

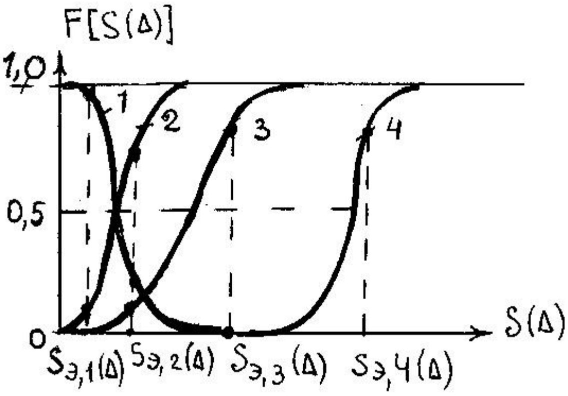


Fig.2. Curves

1. $R_1[S(\Delta)] = 1\text{-}F_1[S(\Delta)]; M_1[S(\Delta)]$
2. $F_2[S(\Delta)]; M_2[S(\Delta)] = M_1[S(\Delta)]$
3. $F_3[S(\Delta)]; M_3[S(\Delta)] = 2.0M_1[S(\Delta)]$
4. $F_4[S(\Delta)]; M_4[S(\Delta)]>>M_1[S(\Delta)]$

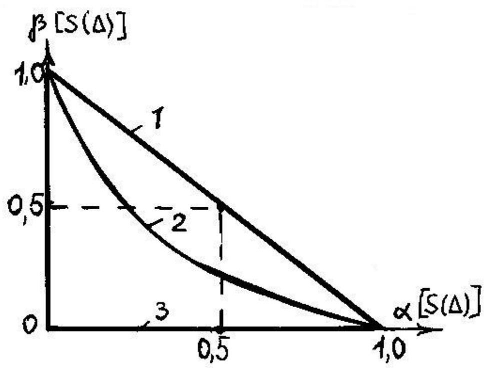


Fig.3. Typical dependences

$\beta[S(\Delta)]=\varphi\{\alpha[S(\Delta)]\}$
Are constructed according to $R_1[S(\Delta)]$ and
1. $F_2[S(\Delta)]$; 2. $F_3[S(\Delta)]$; 3. $F_4[S(\Delta)]$

Let's consider features of application of this approach to the sample analysis from final population of multivariate data on set VA. On fig.2 typical functions of statistic distribution, describing a divergence $F^*_\Sigma(X)$ are resulted and $F^*_V(Y)$. As simplification, s.f.d. F * [S (Δ)] are represented by continuous functions of distribution. Three variants of sample distributions are shown. Curves 2 and 4 characterize limiting parities s.f.d. Final population of multivariate data $F^*_\Sigma(X)$ and s.f.d. the second and the fourth sample $F^*_{V,2}(X)$ and $F^*_{V,4}(Y)$.

The parity $F^*_\Sigma(X)$ also $F^*_{V,2}(Y)$ characterizes a case, when functions of distribution $[1\text{-}R_1[S(\Delta)]]$ and F_2 $[S(\Delta)]$ are practically identical, and a parity $F^*_\Sigma(X)$ and $F^*_{V,4}(Y)$ - a case, when a divergence $[1\text{-}R_1[S(\Delta)]]$ and $F_4[S(\Delta)]$ it is not casual. A parity of functions of distribution $R_1[S(\Delta)]$ and $F_3[S(\Delta)]$ borrows intermediate position.

As follows from fig.2:

$$\text{As}\quad \left.\begin{aligned} &R_1[S_{Э,1}(\Delta)]>>F_2[S_{Э,1}(\Delta)],\ H\Rightarrow H_1\\ &R_1[S_{Э,2}(\Delta)]<<F_2[S_{Э,2}(\Delta)],\ H\Rightarrow H_2\\ &R_1[S_{Э,3}(\Delta)]<<F_3[S_{Э,3}(\Delta)],\ H\Rightarrow H_2\\ &R_1[S_{Э,2}(\Delta)]>F_3[S_{Э,3}(\Delta)],\ H\Rightarrow H_1 \end{aligned}\right\}\qquad (7)$$

For these parities are constructed and represented on fig.3 dependences $\beta[S(\Delta)]=\varphi\{\alpha[S(\Delta)]\}$. In particular, a curve 1 according to $R_1[S(\Delta)]$ and $F_2[S(\Delta)]$, a curve 2 according to $R_1[S(\Delta)]$ and F_3 $[S(\Delta)]$ a curve 3-on data $R_1[S(\Delta)]$ and $F_4[S(\Delta)]$.

V. EXPERIMENTAL RESEARCHES

In practice for statistical check of the assumption that sample of random variables X is casually taken from general population of random variables with the set law of distribution $F_\Sigma(X)$, the greatest distribution was received Kolmogorov's based on statistics Dn [4] with the criterion. This criterion concerns to group nonparametric. In other words, this criterion with success can be used as for comparison $F_\Sigma(X)$ and $F^*_v(X)$, $F^*_\Sigma(X)$ and $F^*_v(X)$. Formulas and tables for application of this criterion are resulted in many monographers and manuals. And practically in all these sources the inaccuracy of a finding of size of the greatest vertical divergence of distributions $F_\Sigma(X)$

and $F_v^*(X)$ is marked as maximal value absolute sizes of observable values Δ. However, in one of many seen managements on mathematical statistics the reason of this mistake is not underlined.

The analysis of statistics Δ_m

Realizations of statistics Δ_m, were calculated on following algorithm [5].

$$\left.\begin{array}{ll} \text{- Pay off} & \Delta_i = (\xi_i - i/n_v);\ i=1,n_v \\ \text{- Are defined} & \Delta_{m,1} = \min\{\Delta_1, \Delta_2,\ldots, \Delta_i, \ldots, \Delta_{nv}\} \\ & \Delta_{m,1} = \max\{\Delta_1, \Delta_2.., \Delta_i, \ldots, \Delta_{nv}\} \\ \text{- If} & |\Delta_{m,1}| > |\Delta_{m,2}|,\ \text{that } \Delta_m = \Delta_{m,1} \\ \text{- Differently} & \Delta_m = \Delta_{m,2} \end{array}\right\} \quad (8)$$

Here ξ- random numbers with uniform expansion in an interval [0,1], simulating true values quantile uniform distribution. S.f.d. $F^*(\Delta_m)$ constructed on 25000 realizations Δ_m and for of some n_v are resulted on fig.4. Importance of these researches consist first of all that with sufficient accuracy for practice borders of change could be established Δ_m, describing the greatest vertical divergence s.f.d. $F_\Sigma(\xi)$ and $F_v^*(\xi)$ with the set significance value. And by that to have an opportunity to estimate character of a divergence any s.f.d. $F_\Sigma(X)$ and $F_v^*(X)$.

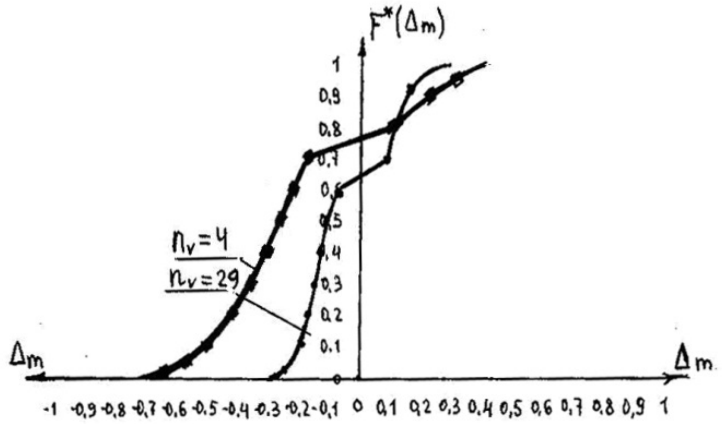


Fig.4. Statistical functions of distribution $F^*(\Delta_m)$ for n_v=4 both 29 and number of iterations N=25000

It is established, that quantile distributions $F^*(\Delta_m) = \alpha \le 0{,}1$ for $n \ge 2$ are equal on size and opposite on a sign quantile distributions $F(D_n) = 2\alpha$. In other words D_n though characterizes the greatest divergence of expansions $F_\Sigma(X)$ and $F_v^*(X)$, but there is no the greatest vertical divergence of these distributions. Distributions Δ_m it is original. Here Δ_m it considered simply as some numerical value, ранжированное in ascending order. If Δ_m to consider as the greatest vertical divergences s.f.d. $F_\Sigma(\xi)$ and $F_v^*(\xi)$, function $F^*(\Delta_m)$ - not is s.f.d. The reason for that presence of positive and negative values Δ_m. And than more negative value Δ_m, i.e. than divergence $F_\Sigma(\xi)$ and $F_v^*(\xi)$ there is more, that probability of acceptance of hypothesis H_1 ($F_\Sigma(\xi)$ and $F_v^*(\xi)$ miss casually) there will be more, and a probability of event Δ_m=0 essentially more zero. Laws of change of distribution of positive and negative values Δ_m for n_v=4 are resulted on fig. 5, and a parity of their number for of some n_v - in table 1.

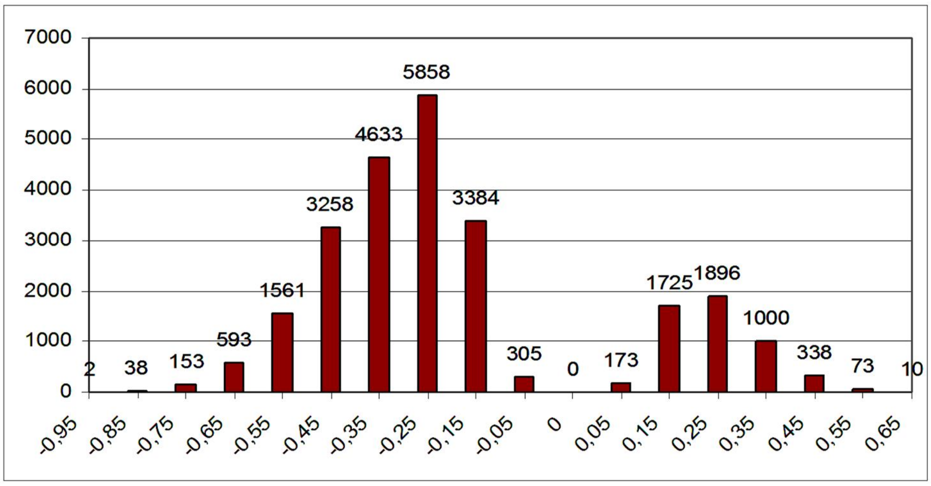


Fig.5. The histogram of distribution of the greatest vertical distribution $F_\Sigma(\xi)$ and $F_v^*(\xi)$

Table 1.

Data on a parity of positive and negative values Δ_m

Number of random variables in sample	2	4	7	11	16	22	29	150
Relative number of negative values Δ_m	0,87	0,79	0,73	0,68	0,65	0,63	0,61	0,55
Parity of negative and positive values Δ_m	6,7	3,8	2,7	2,1	1,9	1,7	1,6	1,2

As follows from table 1, with increase n_v a parity of negative and positive values Δ_m decreases, but at n_v=150 it still is not equal to unit. In table 2 experimental and settlement values quantile distributions $F_v^*(\Delta_m)$ for of some n_v and probabilities $R_v^*(\Delta_m)=[1-F_v^*(\Delta_m)]=\alpha$ are resulted. We shall remind that experimental values received by imitating modeling on the computer [5], and settlement value-under the formula:

$$\Delta_{m,(1-0.5\alpha)}^{settl} = -\left[\Delta_{m,0.5\alpha}^{\exp er} + n_v^{-1}\right] \quad (9)$$

Table 2

Experimental and settlement values quantiles distributions $F_v^*(X)$ for of some n_v and probabilities $R_v^*(\Delta_m)=[1-F_v^*(\Delta_m)]$.

$R_v^*(\Delta_m)$	Δm	Number of random variables in sample (Π_1)						
		2	4	6	11	40	90	150
0.025	experiment	0,343	0,377	0,358	0,302	0,185	0,131	0,104
	settlement	0,342	0,373	0,356	0,298	0,183	0,130	0,103
0.05	experiment	0,285	0,319	0,303	0,260	0,164	0,116	0,092
	settlement	0,285	0,317	0,302	0,262	0,162	0,116	0,092
0.1	experiment	0,184	0,240	0,244	0,216	0,140	0,100	0,079
	settlement	0,184	0,244	0,244	0,218	0,139	0,100	0,079
0.2	experiment	0,060	0,160	0,171	0,160	0,112	0,091	0,065
	settlement	0,061	0,165	0,171	0,164	0,116	0,081	0,064
0.3	experiment	-0,239	-0,173	-0,127	-0,097	0,089	0,067	0,053
	settlement	-0,027	0,105	0,125	0,128	0,094	0,068	0,055

Given tables 2 show, that the formula (9) precisely enough displays interrelation of boundary values of an interval of change of statistics Δ_m provided that 0,25. We shall enter into consideration three statistics based on random variables of an absolute vertical divergence of distributions $F_\Sigma(X)$ and $F_v^*(X)$:

- The greatest value of an absolute divergence. The algorithm of calculation looks like:

$$\left.\begin{array}{ll} \text{- calculated} & \Delta_i = (\xi_i - i/n_v);\ i=1,n_v \\ \text{- defined} & B_v = \max\{\Delta_1, \Delta_2.., \Delta_i, \dots \Delta_{nv}\} \end{array}\right\} \quad (10)$$

- Average value of an absolute divergence $M^*_{v,j}(\Delta)$ with j=1,N, where N – number of iterations. It is calculated under the formula (3)
- Average quadratic value of an absolute divergence $\sigma^*_{v,j}(\Delta)$. It is calculated under the formula (5)

The analysis of statistics B_v

Distribution $F^*(B_v)$ has essential advantage in comparison $F^*(\Delta_m)$. It characterizes distribution of size of the greatest deviation of functions of distribution $F_\Sigma(X)$ and $F^*_v(X)$ without taking into account a sign on a deviation, i.e. it is considered equivalent both positive, and negative value of a deviation Δ_m. In table 3 are resulted quantile distributions B_v for of some values n_v and probabilities $F^*(B_v)$. If with them to compare to data table.2 it is easy to notice essential distinction of their critical values. So at $F^*(D_n)=F^*(B_v)=F^*(\Delta_m)=0{,}05$ and $n_v=4$ corresponding quantile will be equal $D_n=0.624$, $B_v=0.570$ and $\Delta_m=0{,}319$. Thus, the essence of a mistake at practical applications of criterion of Kolmogorov consists more often that statistics B_v is compared not to critical value of distribution $R^*(B_v)$, and with critical value of statistics of Kolmogorov D_n. If to sum up the aforesaid it is necessary to note, each of entered in consideration statistics, for example, D_n, B_v or Δ_m, at check of the assumption it should be compared to the critical values, calculated on distributions, accordingly, $F^*(D_n)$, $F^*(B_v)$ and $F^*(\Delta_m)$. In the illustrative purposes according to table 3 on fig.6 statistical distributions $R^*(B_v)=1-F^*(B_v)$ for $n_v=4$, 22 and 150 are resulted. As one would expect with increase n_v critical values B_v decrease. Character of distribution $R^*(B_v)$ changes also

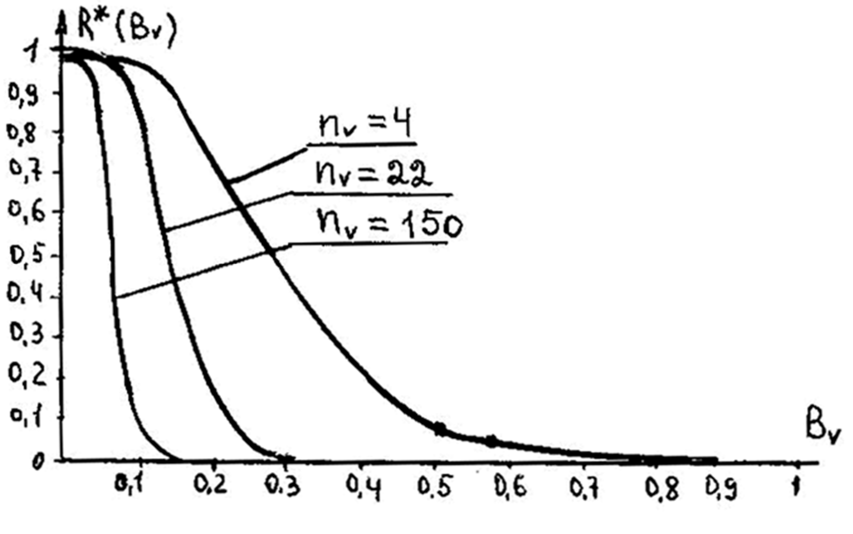


Fig.6 Statistical distributions $R^*(B_v)=1-F^*(B_v)$ for of some n_v

In table 4 the factors of the equation $B_v = A \cdot n_v^{-b}$ calculated according to table 3 and factor determination R^2 are resulted.

Table 4

Factors of the equation of regress

$F^*(B_v)$	a	b	R^2
0.9	1.079	0.459	0.9998
0.9	0.942	0.453	0.9997
0.8	0.774	0.439	0.9986
0.7	0.668	0.430	0.9982
0.6	0.590	0.422	0.9985
0.5	0.518	0.412	0.9975
0.4	0.447	0.396	0.9956
0.3	0.384	0.382	0.9922
0.2	0.317	0.360	0.9862
0.1	0.236	0.321	0.9829

As an example on fig.7 laws of change of a curve $B_v = A \cdot n_v^{-b}$ for $F^*(B_v)$=0,95 (for a significance value are shown α=0,05 and 0,50)

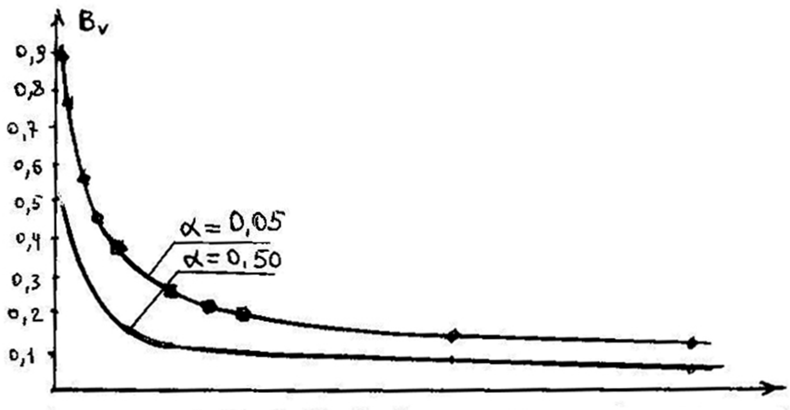


Fig.7 Laws of change of absolute size of the greatest vertical deviation from number of sample units n_v at α=0,05 and 0,05

Table 3

Quantile distributions of statistics for of some values n_v and probabilities $F^*(B_v)$.

N	$F^*(B_v)$	Number of sample units (n_v)								
		2	4	7	11	22	29	40	90	150
1	0,05	0,112	0,127	0,116	0,104	0,083	0,075	0,067	0,048	0,038
2	0,1	0,157	0,154	0,136	0,120	0,094	0,084	0,075	0,053	0,042
3	0,15	0,193	0,175	0,151	0,131	0,103	0,092	0,081	0,057	0,045
4	0,2	0,223	0,191	0,164	0,142	0,110	0,098	0,087	0,061	0,048
5	0,25	0,249	0,208	0,177	0,152	0,117	0,104	0,092	0,064	0,051
6	0,3	0,274	0,222	0,189	0,160	0,124	0,110	0,097	0,067	0,053
7	0,35	0,300	0,236	0,201	0,170	0,130	0,115	0,101	0,071	0,056
8	0,4	0,324	0,250	0,213	0,179	0,136	0,121	0,106	0,074	0,058
9	0,45	0,348	0,268	0,225	0,189	0,143	0,127	0,111	0,077	0,061
10	0,5	0,376	0,286	0,236	0,198	0,150	0,132	0,116	0,080	0,063
11	0,55	0,401	0,306	0,249	0,209	0,157	0,139	0,121	0,083	0,066
12	0,6	0,426	0,326	0,262	0,219	0,164	0,145	0,127	0,087	0,069
13	0,65	0,449	0,348	0,277	0,231	0,172	0,152	0,133	0,091	0,072
14	0,7	0,473	0,370	0,294	0,244	0,181	0,160	0,139	0,095	0,075
15	0,75	0,499	0,393	0,313	0,258	0,191	0,169	0,147	0,100	0,079
16	0,8	0,548	0,421	0,334	0,276	0,203	0,179	0,155	0,106	0,083
17	0,85	0,620	0,454	0,358	0,295	0,217	0,191	0,166	0,112	0,088
18	0,9	0,683	0,497	0,391	0,322	0,235	0,206	0,179	0,122	0,096
19	0,95	0,778	0,568	0,442	0,362	0,263	0,232	0,201	0,136	0,107
20	0,99	0,902	0,689	0,538	0,440	0,320	0,283	0,240	0,164	0,129

As is known, average arithmetic value of random variables is the basic numerical characteristic of their center of grouping. Distinguish also a geometrical average, a harmonious average, a fashion and a median. In spite of the fact that all these numerical characteristics is united with concept of the center of grouping of random variables, each of them, so to say, «has the center» and only it and characterizes. Hence, «the center of grouping of random variables» considered as an attribute, and its versions will be the numerical characteristics noted above.

Each of VA will characterize features of distinction of distributions $F_\Sigma(X)$ and $F^*_v(X)$ peculiar only to it. Below formulas for an estimation of these numerical characteristics are resulted.

Calculation of average arithmetic value $n^*_v(\Delta)$ spent under the formula (2), calculation of average geometrical value - under the formula:

$$\sigma^*_v(\Delta) = \sqrt[n_v]{\prod_{i=1}^{n_v} |\Delta_i|}, \qquad (11)$$

and calculation of an average harmonious – under the formula:

$$H^*_v(\Delta) = \left[\frac{1}{n_v}\sum_{i=1}^{n_v}\frac{1}{|\Delta_i|}\right]^{-1} \qquad (12)$$

The estimation of a fashion spent under the histogram as average value of an interval, probability of hit in which random variables of sample the greatest.

The estimation of a median is spent by a finding $0{,}5n_v$ values ranging random variables of sample, if n_v even, and

$$\Delta_{med} = 0.5\left[\left|\Delta_{\frac{n_v+1}{2}}\right| + \left|\Delta_{\frac{n_v-1}{2}}\right|\right] \qquad (13)$$

Values – if n_v uneven.

According to algorithm of classification of data with a view of decrease in duration of calculations, the expediency of classification of data supervised for the sample having the greatest value of statistics. Thus considered, that if for this sample divergence $F_\Sigma(X)$ and $F^*_v(X)$ with the minimal risk of the erroneous decision it can be accepted casual the divergence of all others sample on set VA casual also.

In table 5 some results of calculation quantile distributions of statistics $M^*_v(\Delta)$ for of some n_v and probabilities $F^*[M^*_v(\Delta)]$ are resulted at N=25000

In the illustrative purposes according to table 5 on fig.8 distributions $R^*[M^*_v(\Delta)] = \{1 - F^*[M^*_v(\Delta)]\}$ for n_v=4 are resulted; 10 and 50, and on fig.9 a curve of dependence of size of statistics $M^*_v(\Delta)$ from n_v for $R^*[M^*_v(\Delta)] = \alpha = 0.05$ и 0.5.

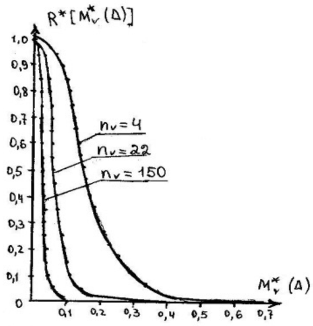


Fig.8. Laws of change $R^*[M^*_v(\Delta)]$

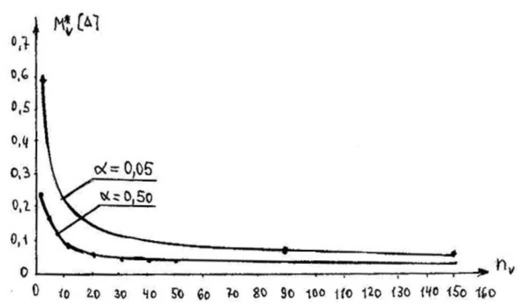


Fig.9. Dependence of critical values $M^*_v(\Delta)$ from number of sample units n_v

Table 5

Quantile distributions of statistics $M_v^*(\Delta)$ for of some n_v and probabilities $F^*[M_v^*(\Delta)]$

N	$F^*[M_v^*(\Delta)]$	Number of sample units (n_v)								
		2	4	7	11	22	29	40	90	150
1	0,05	0,077	0,0714	0,056	0,045	0,032	0,028	0,024	0,016	0,013
2	0,1	0,111	0,085	0,065	0,052	0,037	0,032	0,027	0,018	0,014
3	0,15	0,136	0,097	0,072	0,057	0,040	0,035	0,030	0,020	0,015
4	0,2	0,158	0,106	0,078	0,062	0,044	0,038	0,032	0,021	0,016
5	0,25	0,177	0,115	0,084	0,067	0,046	0,040	0,034	0,023	0,017
6	0,3	0,194	0,124	0,090	0,071	0,049	0,043	0,036	0,024	0,019
7	0,35	0,209	0,133	0,096	0,075	0,052	0,045	0,038	0,025	0,020
8	0,4	0,224	0,142	0,102	0,080	0,055	0,048	0,041	0027	0,021
9	0,45	0,238	0,152	0,109	0,085	0,058	0,050	0,043	0,028	0,022
10	0,5	0,250	0,163	0,116	0,090	0,062	0,053	0,045	0,030	0,023
11	0,55	0,276	0,173	0,123	0,095	0,065	0,056	0,048	0,032	0,024
12	0,6	0,303	0,185	0,131	0,101	0,069	0,059	0,051	0,033	0,026
13	0,65	0,331	0,198	0,140	0,107	0,073	0,063	0,054	0,035	0,027
14	0,7	0,361	0,213	0,150	0,115	0,078	0,068	0,058	0,038	0,029
15	0,75	0,395	0,232	0,162	0,123	0,084	0,073	0,062	0,040	0,031
16	0,8	0,433	0,254	0,176	0,134	0,091	0,078	0,067	0,043	0,034
17	0,85	0,475	0,281	0,194	0,147	0,099	0,085	0,073	0,048	0,037
18	0,9	0,526	0,314	0,217	0,164	0,112	0,095	0,081	0,053	0041
19	0,95	0,591	0,363	0,254	0,192	0,130	0,111	0,095	0,062	0,048
20	0,99	0,68	0,448	0,318	0,250	0,168	0,145	0,121	0,080	0,062

As follows from fig.8, s.f.d. the sums of random variables with uniform distribution in an interval [0,1] even for n_v=150 it is dissymmetric. And consequently, critical values quantile these distributions cannot be calculated according to average arithmetic value $M^*_{v;0.5}(\Delta)$ and average quadratic value $\sigma^*[M^*_v(\Delta)]$. The analysis shows, that the equation of interrelation $M^*_v(\Delta)$ and n_v for the fixed value $R^*[M^*_v(\Delta)] = \alpha$ (fig.9) can be presented by sedate function $M^*_v(\Delta) = An_v^{-b}$ with factor of determination R^2> 0.99

In table 6 the constant factors of this equation calculated under the standard program of sedate transformation for of some values $R^*[M^*_v(\Delta)] = \alpha$ are resulted.

Table 6

Estimations of constant factors of the equations of regress and factor of determination

N	$R^*[M^*_v(\Delta)] = \alpha$.	Factors of regress		R2
		AND	IN	
1	0.05	0.81	0.58	0.9961
2	0.1	0.71	0.58	0.9949
3	0.2	0.57	0.58	0.9941
4	0.3	0.48	0.57	0.995
5	0.4	0.41	0.56	0.996
6	0.5	0.35	0.55	0.998
7	0.6	0.30	0.54	0.998
8	0.7	0.26	0.53	0.998
9	0.8	0.22	0.52	0.999
10	0.9	0.16	0.49	0.999

So that to find critical value of statistics $M^*_v(\Delta)$, for example, α=0,5 and n_v=5 it is necessary to calculate $M^*_{v,0.5}(\Delta) = 0.81/5^{0.58}$ only and if we shall compare with empirical value of statistics $M^*_{v;Э}(\Delta)$ with $M^*_{v;0.5}(\Delta)$ at $M^*_{v;Э}(\Delta) < M^*_{v;0.5}(\Delta)$ it is possible to approve, that sample with a high probability is homogeneous with final population of multivariate data. In other words, classification of data on set VA is inexpedient.

Analysis of statistics $\sigma^*_v(\Delta)$.

At the analysis of statistical data of operation, EPS a degree of dispersion n_v realizations of a random variable Δ concerning the center of grouping $M^*_v(\Delta)$ it is characterized by an average statistical deviation $\sigma^*_v(\Delta)$ more often. The factor of a variation, and size of scope of dispersion calculated under formula $L^*(\Delta)=(\Delta_{max}-\Delta_{min})$ is less often used it is not applied. Practice of classification of multivariate data shows, that sample of random variables X on significant VA is concentrated to some interval $[X_j; X_{j+n_v}]$, which according to recommended algorithm is located in the top part of an interval of change final population of multivariate data since, $M^*_\Sigma(\Delta) < M^*_v(\Delta)$ and it is essential less it.

In table 7 some results of calculation quantile distributions of statistics $\sigma^*_v(\Delta)$ for of some n_v and probabilities $F^*[\sigma^*_v(\Delta)]$ with step 0,05 for number of iterations N=25000 are resulted. In the illustrative purposes on fig.10 are resulted in the form of continuous curves s.f.d. $F^*[\sigma^*_v(\Delta)]$ for of some n_v. On fig.11 the curve changes $\sigma_v^*(\Delta) = \varphi(n_v)$ received under table 7 and the standard program of sedate transformation of statistical data are resulted

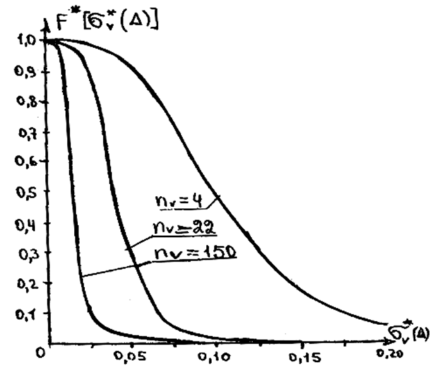


Fig.10 Character of change s.f.d. $F^*[\sigma_v^*(\Delta)]$ depending on n_v

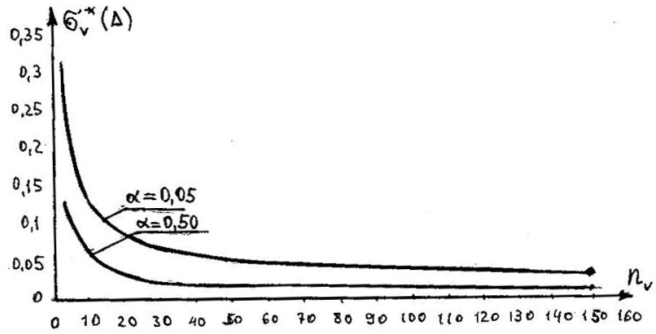


Fig.11. Laws of change critical values of statistics $\sigma_v^*(\Delta)$ depending on n_v

In table 8 factors of the equation the regresses calculated under the standard program of sedate transformation for of some values $R^*[\sigma_v^*(\Delta)]$ are resulted

Table 8

Estimations of constant factors of the equations of regress and factor of determination

N	$R^*[\sigma_v^*(\Delta)] = \alpha$.	Factors of regress		R^2
		AND	IN	
1	0.05	0,428	0,54	0,997
2	0.1	0,385	0,55	0,996
3	0.2	0,322	0,545	0,996
4	0.3	0,276	0,54	0,998
5	0.4	0,237	0,53	0,999
6	0.5	0,197	0,50	0,999
7	0.6	0,163	0,48	0,995
8	0.7	0,129	0,46	0,991
9	0.8	0,094	0,43	0,996
10	0.9	0,061	0,40	0,992

VI. SOME RESULTS COMPARISON OF CRITERIA

Results of the analysis of laws of change s.f.d. статистик B_v, $M_v^*(\Delta)$ also $\sigma_v^*(\Delta)$ have allowed estimating probabilities $F^*[B_{v,э}]$, $F^*[M_v^*(\Delta_э)]$ and $F^*[\sigma_v^*(\Delta_э)]$, where the index «e» designates "experimental" value of probability of display of each of statistics. And as these of statistics characterize those or other properties of casual values of a vertical divergence of distributions $F_\Sigma(X)$ and $F_v^*(X)$, the probability of display of statistics will characterize, as a matter of fact, the importance of this property. In other words, an attribute of divergence $F_\Sigma(X)$ and $F_v^*(X)$ is the vertical distance between these distributions, and versions of an attribute – statistics.

By comparison, of this statistic the question on that, first, is of interest, probabilities of display of each of statistics, calculated on the same sample of general population how much essentially differs. Some results of calculations are resulted in table 9.

Table 7

Quantile distributions of statistics $\sigma^*_v(\Delta)$ for of some n_v and probabilities $F^*[\sigma^*_v(\Delta)]$ on N=25000

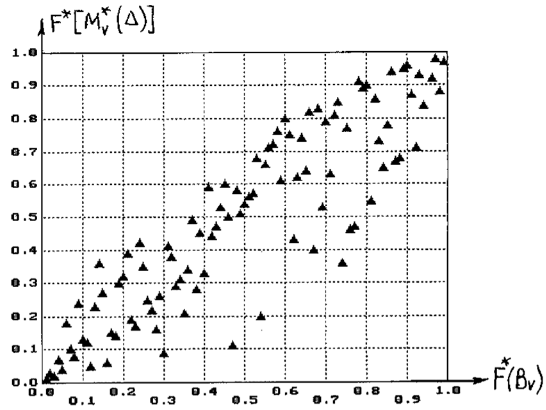
N	$F^*[\sigma^*_v(\Delta)]$	Number of sample units (n_v)								
		2	4	7	11	22	29	40	90	150
1	0,05	0,012	0,038	0,037	0,031	0,023	0,020	0,017	0,012	0,009
2	0,1	0,025	0,050	0,044	0,036	0,026	0,023	0,019	0,013	0,010
3	0,15	0,037	0,059	0,048	0,039	0,028	0,025	0,021	0,014	0,011
4	0,2	0,049	0,066	0,053	0,042	0,030	0,026	0,023	0,015	0,012
5	0,25	0,062	0,073	0,056	0,045	0,032	0,028	0,024	0,016	0,012
6	0,3	0,075	0,079	0,060	0,048	0,034	0,030	0,025	0,017	0,013
7	0,35	0,089	0,085	0,064	0,051	0,036	0,031	0,027	0,018	0,014
8	0,4	0,103	0,091	0,067	0,053	0,038	0,033	0,028	0,019	0,014
9	0,45	0,118	0,096	0,071	0,056	0,040	0,034	0,029	0,019	0,015
10	0,5	0,130	0,102	0,075	0,059	0,041	0,036	0,031	0,020	0,016
11	0,55	0,150	0,107	0,079	0,062	0,044	0,038	0,032	0,021	0,017
12	0,6	0,168	0,114	0,084	0,065	0,046	0,040	0,034	0,022	0,017
13	0,65	0,186	0,120	0,088	0,069	0,048	0,042	0,036	0,024	0018
14	0,7	0,204	0,128	0,094	0,073	0,051	0,044	0,038	0,025	0,019
15	0,75	0,224	0,137	0,100	0078	0,054	0,047	0,040	0,026	0,021
16	0,8	0,244	0,147	0,107	0,083	0,058	0,050	0,043	0,028	0,022
17	0,85	0,267	0,159	0,116	0,090	0,062	0,054	0,046	0,030	0,024
18	0,9	0,292	0,175	0,127	0,098	0,068	0,059	0,051	0,033	0,026
19	0,95	0,320	0,199	0,145	0,112	0,078	0,068	0,058	0,038	0,030
20	0,99	0,347	0,244	0,178	0,140	0,097	0,084	0,072	0,048	0,037

Table 9

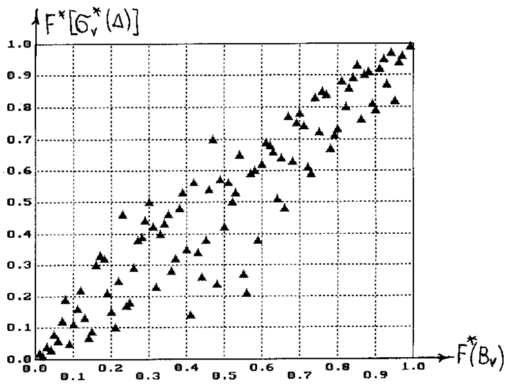
Parities $R^*[M^*_v(\Delta)]$ and $R^*[\sigma^*_v(\Delta)]$, calculated for the same samples with n_v=4

N	ξ_1	ξ_2	ξ_3	ξ_4	$R^*[M^*_v(\Delta)]$	$R^*[\sigma^*_v(\Delta)]$
1	0,399	0,363	0,688	0,524	0,32	0,64
2	0,945	0,781	0,225	0,848	0,81	0,43
3	0,429	0,488	0,724	0,682	0,68	0,36
4	0,921	0,812	0,913	0,432	0,29	0,69
5	0,778	0,459	0,402	0,1	0,25	0,77

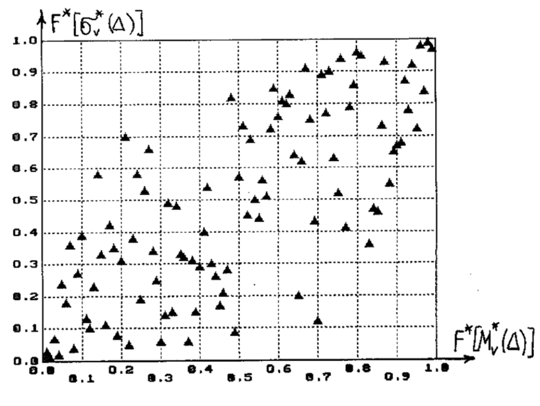
The examples resulted in table 10 testify that probabilities $R^*[M^*_v(\Delta)]$ and $R^*[\sigma^*_v(\Delta)]$ can essentially differ. The reasons of such distinction are known. Average value of realizations of a vertical deviation s.f.d. $F_\Sigma(X)$ and $F^*_v(X)$ there can be small enough, and their average quadratic deviation – greater and on the contrary. In other words, examples tables 10 testify that comparison of efficiency of criteria of check of hypotheses differing статистик not always is justified. First, because the result of comparison depends on distribution $F^*_v(X)$, i.e. the result of comparison not is a rule, and secondly because statistics of criteria can have various physical sense, for example $M^*_v(\Delta)$ and $\sigma^*_v(\Delta)$.



a) Illustration of interrelation F*[B_v] and $F^*[M^*_v(\Delta)]$ at n_v=3



b) Illustration of interrelation F*[B_v] and $F^*[\sigma^*_v(\Delta)]$ at n_v=3



c) Illustration of interrelation $F^*[M^*_v(\Delta)]$ and $F^*[\sigma^*_v(\Delta)]$ at n_v=3

Fig.12. Correlation field of interrelation of probabilities of display of realizations statistic

Moreover, as they can be independent as well as $M^*_v(\Delta)$ and $\sigma^*_v(\Delta)$, casual character of divergence $F_\Sigma(X)$ from $F^*_v(X)$ by criterion with statistics $M^*_v(\Delta)$ yet does not mean, that divergence $F_\Sigma(X)$ from $F^*_v(X)$ by criterion with statistics $\sigma^*_v(\Delta)$ it will appear also casual. In the illustrative purposes on рис.10 the correlation field of interrelation of probabilities of display of realizations statistics к B_v, $M^*_v(\Delta)$ and $\sigma^*_v(\Delta)$, calculated for the same samples from n_v=3 random variables is resulted.

Calculations spent in following sequence:

- For each sample from n_v random variables ξ, In regular intervals distributed in an interval [0,1], realizations B_v are calculated, $M^*_v(\Delta)$ and $\sigma^*_v(\Delta)$. Calculations are spent N time, where N- number of iterations. Results of calculations brought in the table and which form is shown on fig.13.

N	B_v,	$M^*_v(\Delta)$	$\sigma^*_v(\Delta)$	$F^*(B_v)$	$F^*[M^*_v(\Delta)]$	$F^*[\sigma^*_v(\Delta)]$

Fig.13. Table of initial data

- Ranking of realizations B_v of the table and by way of increase in numerical values B_v is spent. Together with $B_{v,i}$ move and corresponding $B_{v,i}$ values $M^*_{v,i}(\Delta)$ and $\sigma^*_{v,i}(\Delta)$;
- Pays off $F^*_i[B_v] = i/N$ with i=1,N and are brought in a column $F^*(B_v)$ of the table A
- In the table B, the similar table A, ranking of realizations $M^*_v(\Delta)$ is spent and further under the formula $F^*_i[M^*_v(\Delta)] = i/N$ corresponding $M^*_{v,i}(\Delta)$ probabilities $F^*_i[M^*_v(\Delta)]$ are calculated
- For each value of statistics $M^*_v(\Delta)$ from the table A there is a value equal to it in the table B and corresponding value of probability $F^*[M^*_v(\Delta)]$ which is brought in a column $F^*[M^*_v(\Delta)]$ of the table A
- In the table B ranking of realizations $\sigma^*_v(\Delta)$ is spent and further under the formula $F^*_i[\sigma^*_v(\Delta)] = i/N$ corresponding $\sigma^*_{v,i}(\Delta)$ probabilities $F^*_i[\sigma^*_v(\Delta)]$ are calculated
- For each value of statistics $\sigma^*_v(\Delta)$ from the table A there is a value equal to it in the table B and corresponding value of probability $F^*[\sigma^*_v(\Delta)]$ which is brought in a column $F^*[\sigma^*_v(\Delta)]$ of the table A

As follows from the resulted figures the essential interrelation between probabilities $F^*[B_v]$ and $F^*[M^*_v(\Delta)]$ or $F^*[\sigma^*_v(\Delta)]$ observed. This interrelation has evident physical interpretation: with growth B_v grow, on the average, $M^*_v(\Delta)$ and $\sigma^*_v(\Delta)$.

Figure 12 full enough characterizes weak interrelation between $M^*_v(\Delta)$ and $\sigma^*_v(\Delta)$. Therefore and the answer to a question on, whether is enough to check up character of divergence $F_\Sigma(X)$ from $F^*_v(X)$ only on one statistics B_n it appears ambiguous, and the priority is given expediency of attraction to the decision of all statistic.

VII. RECOGNITION EXPEDIENCY OF CLASSIFICATION OF MULTIVARIATE DATA

Above-stated testifies to necessity of check of the assumption expediency of classification of multivariate data by the criteria reflecting the basic properties of random variables of a vertical divergence of distributions $F^*_\Sigma(X)$ and $F^*_V(Y)$. Conditions of check of possible assumptions recommended by authors look like:

$$\left.\begin{array}{lll} \text{If} & S_e(\Delta)<S_{0.5}(\Delta), & \text{that } H_1 \\ \text{If} & S_e(\Delta)>S_{0.05}(\Delta), & \text{that } H_2 \\ \text{If} & S_{0,5}(\Delta)<S(\Delta)>S_{0,05}(\Delta), & \text{that } H_3 \end{array}\right\} \quad (14)$$

where $S(\Delta)$ – one of possible statistics a random variable Δ; $S_{0.05}(\Delta)$ and $S_{0,5}(\Delta)$ – critical values of statistics with a significance value, accordingly 0,05 and 0,5; H_1, H_2 and H_3 – assumptions, accordingly, about casual character of a divergence $F^*_\Sigma(X)$ and $F^*_V(Y)$ and inexpediency of classification of data; about not casual divergence $F^*_\Sigma(X)$ and $F^*_V(Y)$ and expediency of classification of data; expediency of an estimation and comparison of risk of the erroneous decision for H_1 and H_2.

Algorithm of an estimation of expediency of classification of multivariate data we shall consider on a following example. Let sample with n_v=4 is set: {0,151, 0,341, 0,259, 0,120}. Random numbers are received by program way, are called pseudo-casual, and have uniform distribution in an interval [0;1]. The basic assumption: random numbers of sample have uniform distribution in an interval [0;1]. For check of this assumption that is identical to the assumption of inexpediency of classification of final population of statistical data, we shall calculate realizations of values of a vertical divergence of distributions $F_\Sigma(X)$ from $F^*_v(X)$. It is easy to be convinced, that they are equal: Δ_1= -0,349; Δ_2= -0,660; Δ_3= -0,498; Δ_4= -0,130. Results of calculations $\Delta_{m,E}$; $B_{v,E}$; $M^*_{v,E}(\Delta)$ and $\sigma^*_{v,E}(\Delta)$ under formulas 8,10; 2 and 4 are resulted in table 10. Here probabilities $R^*(\Delta_m)$; $R^*(B_{v,E})$; $R^*[M^*_{v,E}(\Delta)]$ and $R^*[\sigma^*_{v,E}(\Delta)]$ are resulted

Table 10

Results of calculations at an estimation of expediency of classification of statistical data

N	Statistics	Estimations статистик	$R^*[S_v(\Delta)]$	The decision
1	$\Delta_{m,E}$	-0,660	0,02	H_2
2	$B_{v,E}$	0,660	0,02	H_2
3	$M^*_{v,E}(\Delta)$	0,407	0,03	H_2
4	$\sigma^*_{v,E}(\Delta)$	0,224	0,27	H_3

As follows from table 10, three from four criteria testify that the set sample is unpresentable, and the lead classification is expedient (H_2). And only size of an average quadratic deviation of realizations of vertical divergence $F_\Sigma(X)$ from $F^*_v(X)$ has probability 0,27 that value of a error II type testifies about necessities of the account of this property and attraction to check of hypothesis H_1.

Thus, one criterion testifies to expediency of classification of data, others – on the contrary, testify to uniformity of compared data. Thus, it is necessary to answer a question: whether there are properties of compared random variables on which they cannot be acceptance homogeneous? In our example is. This question and character of the answer are natural.

CONCLUSION

1. The expediency of classification of final population of multivariate data, in other words, presence significant VA established based on the theory of check of statistical hypotheses.
2. Criterion of check of a hypothesis is the condition non-ascendance empirical value of statistics of its critical value. As statistics, Kolmogorov's D_n statistics is most often used. However random variables of vertical divergence $F_\Sigma(X)$ from $F_v^*(X)$ are characterized also:
   - the greatest on absolute size and constant on a sign a vertical divergence Δ_m;
   - the greatest on absolute value of divergences B_v;
   - average arithmetic value of an absolute divergence $M_v^*(|\Delta|)$
   - average quadratic value of an absolute divergence $\sigma_v^*(|\Delta|)$;

   This list could be continued. But the main thing here is that each of considered above statistics characterizes, distinct from other properties of random variables Δ property and has the distribution $R^*[S_i(\Delta)]$. The importance of properties is defined by a parity $B_v = A \cdot n_v^{-b}$, which calculated on concrete sample. The $M_n^*(\Delta)$ is less the importance VA above and on the contrary.
3. To compare with these curves on capacity, certainly, it is possible. As a result, of comparison we shall be convinced that the greatest capacity has criterion which statistics characterizes properties of random variables the samples having among all other properties the greatest importance. For following sample with considerable probability, the importance of properties of random variables can essentially change.

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BLOCK INSPECTION POLICY MODEL WITH IMPERFECT INSPECTIONS FOR MULTI-UNIT SYSTEMS

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ABSTRACT

In this paper, the authors' research work is focused on imperfect inspection policy investigation, when not all defects are identified during inspection action performance and probability of defect identification is not a constant variable. They are interested in Block Inspection Policy performance for multi-unit systems, the maintenance policy which is one of the most commonly used in practice. As a result, at the beginning, few words about delay time modelling approach and a brief literature overview is given. Later, the model of Block-Inspection Policy is provided. The numerical example with the use of QNU Octave program is given. In the next Section, the sensitivity analysis of the developed model is characterized. The article ends up with summary and directions for further research.

1 INTRODUCTION

The main dependability characteristics of any technical system are mainly a function of its inspection and maintenance strategy. To model the inspection interaction, a concept called delay-time may be used.

The mentioned approach was developed by Christer et al. (see e.g. (Christer 1987, 1982, Christer & Waller 1984a, b, Christer & Whitelaw 1983)). The basic idea rests on an observation that a failure does not usually occur suddenly, but is preceded by a detectable fault for some time prior to actual failure, the delay time (Christer & Redmond, 1992). So, the delay time h is defined as the time lapses from the moment when a fault could first be noticed till the moment when a subsequent failure occurs, if left unattended (Christer 1999, Christer & Redmond 1992). During period h there is an opportunity to identify and prevent failure (Nowakowski & Werbińska-Wojciechowska 2012). For more information see e.g. (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a, b, c, Nowakowski & Werbińska-Wojciechowska 2012).

A literature review, in which delay-time models are investigated along with other PM models are given in (Dekker & Scarf 1998, Guo et al. 2001, 2000, Mazzuchi et al. 2007, Ozekici 1996, Thomas 1986, Valdez-Flores & Feldman 1989). The state of art works, dedicated strictly to DT modelling are given in (Alzubaidi 1993, Baker & Christer 1994, Christer 2002, 1999, Christer & Redmond 1992, 1990, Lee 1999, Nowakowski & Werbińska-Wojciechowska 2012, Redmond 1997, Wang 2012, 1992).

In real-world situation, inspections may not reveal all defects present in a system, especially for large complex systems. Moreover, the quality of performed inspections depends on inspection techniques used, inspection training, inspection practices imposed or the nature of any supervision.

As a result, the focus of this paper is to investigate the Block Inspection Policy model with imperfect inspections and to define their influence on maintenance policy results.

Moreover, most of the imperfect inspection maintenance models assume, that the probability p of a defect not detection during inspection action performance is constant, what may not be valid for real-life technical objects like means of transport (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a). Thus, in the presented paper the probability p is not a constant (like e.g. in (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a)) but linearly changes according to the defect symptoms visibility increase.

As a result, the paper is organized as follows: In the introduction Section, few words about delay time modelling approach and a brief literature overview is given. Later, the model of Block-Inspection Policy is provided. The numerical example with the use of QNU Octave program is given. In the next Section, the sensitivity analysis of the developed model is characterised. The article ends up with summary and directions for further research.

In conclusion, this article is a continuation of the delay time modelling problems being investigated in (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a, b, c, Nowakowski & Werbińska-Wojciechowska 2012, Werbińska-Wojciechowska 2012).

2 DELAY-TIME MODELS WITH IMPERFECT MAINTENANCE – LITERATURE OVERVIEW

The basic delay-time models for single- or multi-component systems assume that a visible defect is always found in a system if it is there. However, performance process of real technical systems like transportation systems indicates, that this assumption is insufficient. Thus, the imperfect inspection case should be investigated. The problem of imperfect inspection is analysed and overviewed e.g. in (Alzubaidi 1993, Choi 1997, Christer 1999, Das & Sarmah 2010, Kobbacy & Murthy 2008, Lee 1999, Redmond 1997, Sarkar et al. 2011, Wang 2012).

Ones of the first works which investigate the delay-time model with imperfect inspection are (Christer & Waller 1984a, Christer & Redmond 1990). In these works the basic inspection model for industrial plant maintenance is provided. In the next work (Christer & Waller 1984b), authors present the variation of this imperfect-inspection model for a vehicle fleet maintenance. Later, Baker & Wang (1993), present an extended delay-time model, in which the age of an object influence both the period u and delay time h.

Inspection models for single-component system are presented in (Christer 1992, Okumura 1997, Okumura et al. 1996, Pellegrin 1992, Zhao et al. 2007). In (Christer 1992), author develops the model of condition-monitoring for a production plant, when the initial point of a defect is measured as the time from the as-new condition. Pellegrin (1992) considers on-condition maintenance based on periodic inspection of productive equipment. The condition of equipment is described by the wear and its deterioration. Author also builds a graphical procedure to choose the best inspection interval for different criteria. Following this, in (Baker & Wang 1992), there is analysed the repairable machine that may fail or suffer breakdown many times during the course of its service lifetime, and is inspected for visible faults at intervals. The authors mostly focus on the problem of model parameter estimation with the use of maximum likelihood method and Akaike information criterion, providing also a model for imperfect inspections performance. Later, in (Okumura et al. 1996), authors develop a method for determining the discrete time points of inspection for a deteriorating single-unit system under condition-based maintenance. The delay-time model is here utilized to describe the transition of the system's states. This problem is later investigated by Okumura (1997). In the next work, (Zhao et al. 2007), authors investigate the model to evaluate the reliability and optimise the inspection schedule for a multi-defect component. There is also considered the situation of non-constant inspection intervals. In (Wang 2011), author presents an extended delay –time model for production plant and investigates its three-stage failure process.

An inspection-replacement model for a multi-component system is proposed e.g. in (Christer & Wang 1995, Wang 2008). In (Christer & Wang 1995), authors assume also that system is inspected not only on a planned basis, but also when a component fails. The model let the total expected cost per unit time minimize with respect to the inspection intervals and the system replacement time. Wang in his work (2008), focuses on the basic inspection model for complex or multi-component system, as a continuation of research work presented in e.g. (Christer & Wang 1995). Later, he extends the model in work (Lv & Wang 2011). In work (Baker et al. 1997), authors focus on the estimation of the model parameters and their errors from records of failure times and number of defects found at inspections of a machine which has been operated for some time under some inspection (also imperfect) regime. Later, in work (Christer & Lee 2000), authors present a delay-time-based PM model with an assumption of non-negligible downtime. The model is provided for perfect and non-perfect homogeneous processes and for a perfect non-homogeneous process. Moreover, Wang (2010) author develops a delay-time-based PM model when three types of maintenance (inspections, repairs, other PM actions) activities are performed.

Delay-time based models are also investigated for various types of technical objects, e.g. production plant maintenance processes development. In (Christer et al. 1995), authors present a Preventive Maintenance (PM) model applying the delay-time modelling technique to optimize the PM of the key machine in cooper products manufacturing company. An inspection model is developed to describe the relationship between the total downtime and the PM interval. Later, in (Christer et al. 1998), authors develop a delay-time model for PM of production plant, when assuming that defects identified at PM may not all be removed. They use objective estimation of model parameters, and the results are provided for NHPP arrival rate of failures and perfect-/imperfect-inspection cases. The problem of parameters estimation process for production plants maintenance modelling is later also investigated by Christer et al. (2000) or Wang (2009b). In (Wen-Yuan & Wang 2006) authors focus on optimising the preventive maintenance interval of a production plant. As in previous models, authors use likelihood formulation to model the problem. This problem is continued for the case of complex plant in (Ben-Daya et al. 2009). The model is analysed for a case example of exploitation process of extrusion press working in Cooper Company. One step further goes author in (Wang 2009a). In this work, author model the production process which may be subjected to two types of deteriorations. The first type of deterioration is a shift in product quality caused by minor process defects that may be identified and rectified by routine inspections and repair. Minor inspections are assumed to be perfect. The second deterioration type is a major defect caused by a major mechanical or electrical problem that can be observed only when the defect has led to a breakdown of the process or the defect is revealed by a major inspection followed by an appropriate major repair action at the time of the inspection. In the investigated model major inspections are assumed to be imperfect. The inspection model is focused on optimising the inspection intervals for both types of inspections. Later, the availability model based on delay-time modelling with imperfect maintenance is investigated in (Wang et al. 2011). Moreover, in (Dagg & Newby 1998, Wang & Sheu 2003) complex production systems inspection maintenance is optimized with the use of Markov theory.

Other applications of delay-time models with imperfect maintenance regard to aircraft structure (see e.g. (Cai & Zhu 2011)), railway tracks inspection and maintenance procedures (see e.g. (Podofillini et al. 2006)), inter-city express bus fleets maintenance (see e.g. (Desa & Christer 2001)), wind turbine maintenance (see e.g. (Andrawus et al. 2007)), or offshore oil platform plant reliability (see e.g. (Wang & Majid 2000)). The methodology for the application of delay time analysis via Monte Carlo Simulation is given in (Cunningham et al. 2011).

Following this, in the next Sections the Block Inspection (BI) Policy is investigated and its sensitivity for type of inspection action performance.

3 BLOCK INSPECTION POLICY MODEL WITH IMPERFECT MAINTENANCE

The investigated system is comprised of k identical elements, in a *k-out-of-n* (e.g. 2-out-of-3 in the Fig. 1) reliability structure, working independently under the same conditions. Moreover, components may be in one of three states: operating properly, operating with defects or down. They prone to become defective independently of each other when the system is in operating. The performed maintenance policy bases on Block Inspection policy which assumes, that inspections take place at regular time intervals of T, and each requires constant time. The inspections are assumed to be imperfect. Thus, any component's defect, which occurred in the system till the moment of inspection, will be unnoticed with probability p or correctly identified with probability $1-p$. All elements with identified defects will be replaced within the inspection period.

The performance of the investigated system, being illustrated in Figure 1, is also defined by the additional assumptions:

- the system is a two state system where, over its service life, it can be either operating or down for necessary repair or planned maintenance,
- maintenance actions restores maintained components to as good as new condition,
- the system can remain functioning in an acceptable manner until breakdown (despite having elements' defects),
- defects which may have arisen in the system, deteriorate over an operating time,
- the breakdown will be assumed to have been caused by $n - k + 1$ defects which has deteriorated sufficiently to affect the operating performance of the system as a whole,
- failures of the system are identified immediately and repairs or replacements are made as soon as possible,
- system incurs costs of: new elements, when they are replaced, inspection costs, and some additional, consequence costs, when system fails,
- elements' lifetime, repair time, replacement time and the length of the delay time before element's failure are random and their probability distributions are known.

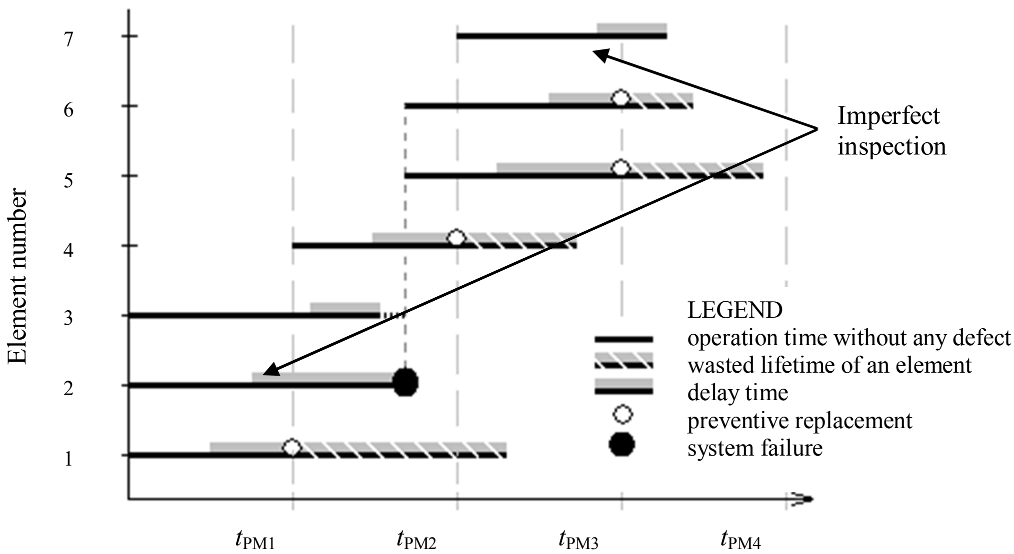

Figure 1. Idea of the Block Inspection Policy with imperfect inspection (2-out-of-3 reliability structure)

The system in Figure 1 is inspected at t_{PM} moments. Diagnosis of defect symptoms is imperfect thus elements 2 and 7 are allowed to further work although their defects might be noticed. Because of the fact, during one of following periods between inspections (t_{PM1} - t_{PM2}) two consecutive elements fail what causes a system failure. On the other hand, some elements' defects are properly diagnosed at the first possible inspection (elements: 1,4,5,6) and the components are preventively replaced but their potential lifetime is wasted.

The system presented above was modelled in GNU Octave software. The list of tested system parameters, which were used in the simulation model of the system exploitation process, is given in the Table 1.

Table 1. Modelled system parameters

Notation	Description	Basic value
c_e	the cost of a new element	1
c_i	the cost of an inspection	1
c_c	the cost of a system failure	1000
T_i	the time required for inspection	0
$F(t)$	C.d.f. of single element's lifetime	$F(t)=1-e^{-(100)^{3,5}}$
$G_r(t)$	C.d.f. of single element's replacement time when corrective action is taken	$G_r(t)=1-e^{-(100)^{2,3}}$
$G_p(t)$	C.d.f of single element's replacement time when preventive action is taken	$G_p(t)=1-e^{-(10)^{2,3}}$
$F(h)$	C.d.f. of delay time	$F_h(h)=1-e^{-(35)^{3,5}}$

4 INFLUENCE OF IMPERFECT INSPECTION ON BI POLICY RESULTS

Practical usage of the *BI* policy requires results of theoretical studies to be transformed into practical guidelines for maintenance staff. For this reason, Authors decided to study the problem, which may arise in practice, of imperfect inspection and its influence on results of the *BI* policy. We especially focus on the relation of inspection imprecision and the best length of inspection period in systems with different characteristics, maintained according to the *BI* policy.

Inspection precision is understood as the ability of system to detect (and correctly interpret) its elements' defects during inspection if their symptoms may be observable. This ability is given by p factor describing the probability that an element defect will not be noticed by maintenance service. The greater value of p means more difficult diagnosis of a real state of system during inspection.

The two basic cases of imperfect inspection, analysed in the paper, assume:

- the probability of defect omitting (p) in the course of inspection is constant and may result from, e.g. too little knowledge and experience of maintenance staff who inspect a system,
- the probability p is the effect of difficulties with observing of defect signal during inspection and depends on the strength of the signal. Thus, the probability p decreases with the time when an element is closer to its failure, because the signal strength usually increases in time and a defect may be easily noticed and correctly interpreted when a time to failure is short.

For the purpose of comparison of the costs and the availability results of the *BI* policy for the cases of perfect and imperfect inspection the analysis has been done and its results are presented in Figures 2 – 5. The chosen effects depicted in the figures assume various length of the inspection period (T) and system reliability structure given by the number of elements (k) which have to be in up-state in order a system to be up. However, the research was also conducted for various levels of inspection imprecision determined by probability p (p = 0,1÷1 or p growing linearly) and for various delay time lengths (the mean delay time of elements ranges from 0 to the half of the mean element's lifetime).

The Figure 2 shows the total costs (C) of new elements, when they are replaced, inspection and consequence costs for *k-out-of-n* systems which are perfectly inspected. The same features characterize the system whose cost results are presented in the Figures 3 and 6, but the systems are inspected imperfectly. Figures 6-7 present the cost and the availability ratio achieved by a system inspected with various levels of inspection precision 1-p, constant during the whole delay time of an element (the case of little experienced maintenance staff), while Figures 3 and 5 demonstrate the results assuming that the chance of defect detection increases linearly in time (inspection closer to failure moment returns better information).

The analysis has proved the expected fact that all tested costs and the availability ratio depend on inspection precision. The strength of this impact is much greater in series structured (*n-out-of-n*) and similar systems, more liable to system failures. Lower precision of inspection increases the system failure cost, which is dominant cost component, but at the same time it decreases the summary cost of new elements that are used in the system. The total cost, directly proportional to the number of system failures, is much higher in the cases when inspection is not perfect. The corresponding effect is observable in availability ratio analysis – low reliability of inspection reduces meaningfully the availability ratio of maintained system if it is not substituted by oftener inspections. On the other hand, some expected effect may be observable – even relatively low probability of defect omission does not cause any severe cost or availability consequences if the inspection period is shortened (e.g. $T \leq 5$ in Fig. 3 or 5). The effect is unequivocal in series systems ($k = n$), but when a reliability structure of a system changes and becomes more failure-resistant ($k << n$), the analysis of the availability ratio demonstrates also the other result. Parallel system should be inspected in long intervals if one wants to maximize its availability. Moreover the time between inspections (T) should be carefully determined in systems with "intermediate" reliability structures because there is a relatively short range of T, which give "near optimum" results for such systems. The fact of existing of the length of an inspection period that yields "the nearly best" results for simulated system was the reason for authors to carry out the research whose results are presented in the next section of the paper.

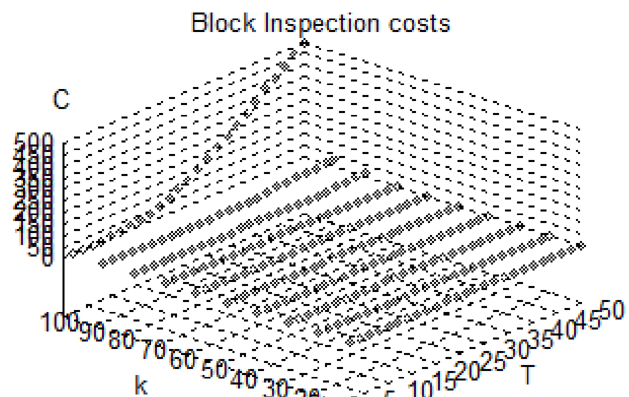

Figure 2. Block Inspection costs for k-out-of-100 system for the case of perfect inspection

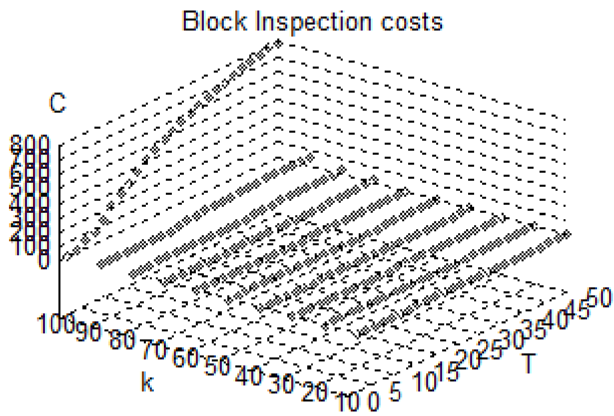

Figure 3. Block Inspection costs for *k-out-of-100* system imperfectly inspected (p decreases linearly)

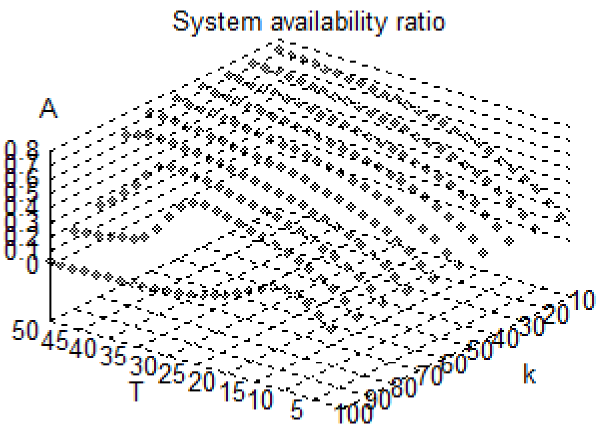

Figure 4. Availability ratio of *k-out-of-100* system for the case of perfect inspection

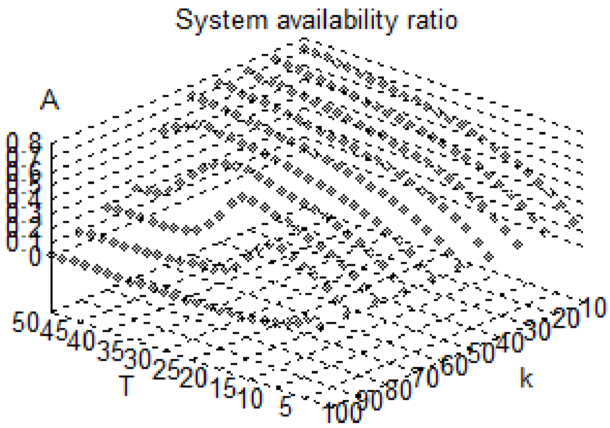

Figure 5. Availability ratio of *k-out-of-100* system imperfectly inspected (*p* decreases linearly)

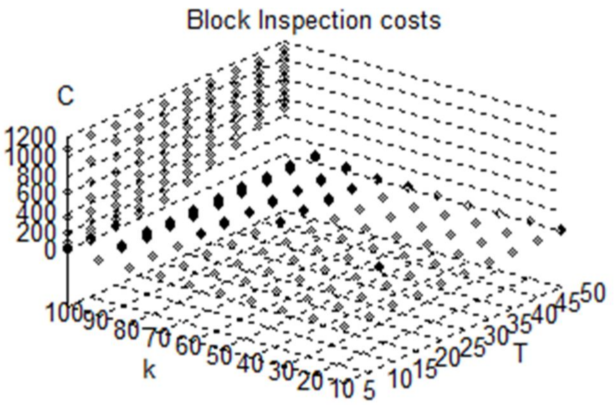

Figure 6. Block Inspection costs for *k-out-of-100* system imperfectly inspected ($p = 0 \div 1$)

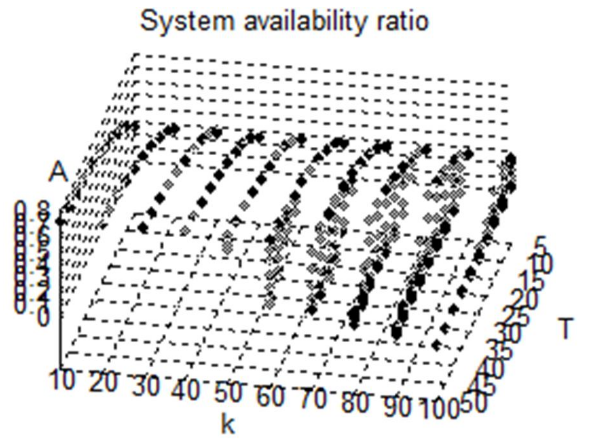

Figure 7. Availability ratio of *k-out-of-100* system imperfectly inspected ($p = 0 \div 1$)

2.1 The optimum inspection period

The main advantage of the *BI* policy, as well as all the policies basing on the state of a maintained system, comes from inspection findings which should determine if any preventive replacement is required. When an inspection does not give proper results, it should be executed more often than if a diagnosis is perfect, in order to increase the chance to notice an element defect. The inspection uncertainty may be neutralized by shortening the period between inspections as it was shown in the previous part of the paper. That was the reason to carry out the further research, how the optimal inspection period changes when inspections become less reliable (growing value of *p*).

The "near optimal" time between preventive inspections *T*, according to literature findings (see e.g. (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012b)), depends mainly on a system lifetime determined by lifetime of system components and its reliability structure as well as elements' delay time. The period *T* that yields "low" costs and "high" availability ratio of maintained system should fulfil following conditions in a series structured system:

$$\frac{h}{T} \approx 2 \tag{1}$$

where: T = the period between inspections, h = the mean delay time value.

System with parallel reliability structure may be inspected even at longer time intervals because of its higher resistance to single element failures.

The expressions 1 has been found to be true for perfectly inspected systems. The goal of the following analysis is to show how the above expressions should be changed when inspection becomes less precise. The research results are presented in Figures 8 – 11.

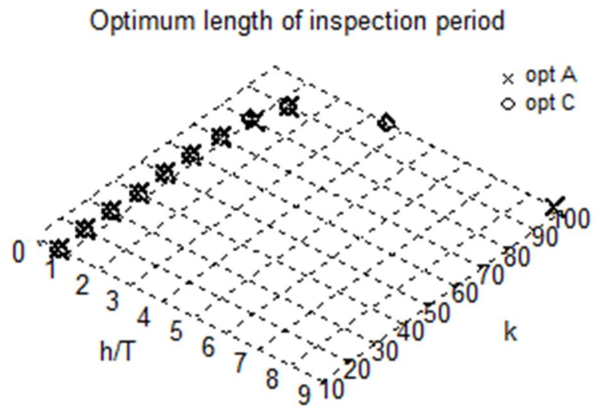

Figure 8. The length of inspection period in the relation to element's delay time, which yields the best availability (opt *A*) and cost (opt *C*) results in a *k-out-of-100* system which is perfectly inspected

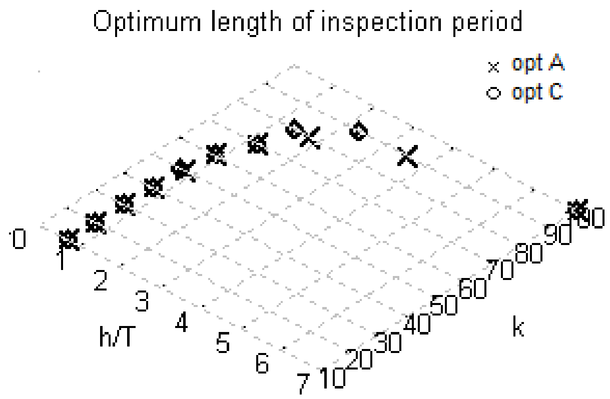

Figure 9. The length of inspection period in the relation to element's delay time, which yields the best availability (opt *A*) and cost (opt *C*) results in a *k-out-of-100* system which is imperfectly inspected (*p* is growing linearly)

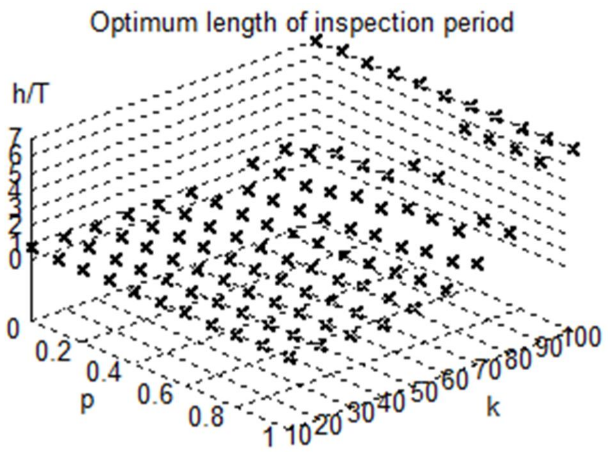

Figure 10. The length of inspection period in the relation to element's delay time, which yields the best cost results in a *k-out-of-100* system which is imperfectly inspected

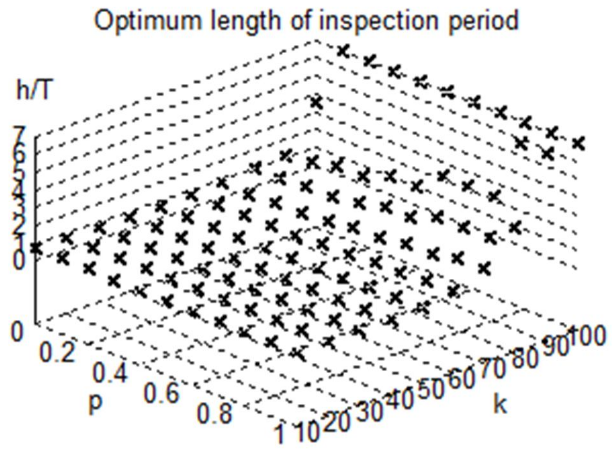

Figure 11. The length of inspection period in the relation to element's delay time, which yields the best availability results in a *k-out-of-100* system which is imperfectly inspected

The Figure 8 depicts the optimum relation of the mean delay time of a system element (h) and inspection period length (T), found among the tested range, from the point of view of the cost and availability of a *k-out-of-n* system, which is perfectly inspected. When $k = 1$ in a system (a parallel structure), the cheapest solution have been found for the cases when inspection period length is close to the mean value of element's delay time ($h/T<1$). If the number of elements required for system operation rises ($1 < k < n$), inspection period should be reduced ($1 \leq h/T \leq 2$) in order a system to obtain the best maintenance results. When a systems becomes a series system ($k = n$) the cheapest solutions exist for the condition $3 \leq h/T \leq 4$, while the highest availability ratio of a system is observable for the shortest inspection period, which was tested in the study ($h/T \rightarrow max$). The next figures (Fig. 9-11) present the optimum results obtained for the same values of variables *h/T* and *k* but the assumption about perfect inspection is released. The direction of the curve (Fig. 9) marking the best found cost and availability relations of *h*/*T* is the same independently on the inspection precision, but the slope of the curve is not. When an inspection is imperfect (Fig. 9), the same systems (for whose $k \geq 0{,}7n$) should be inspected approximately two times more often than if a system is perfectly inspected (2 > *h/T* > 4). In order to confirm the fact is true not only for the chosen case, the analysis of *h/T* for various values of the mean delay time *h* was conducted and for various system sizes. The exemplary results are shown in Figures 12-15.

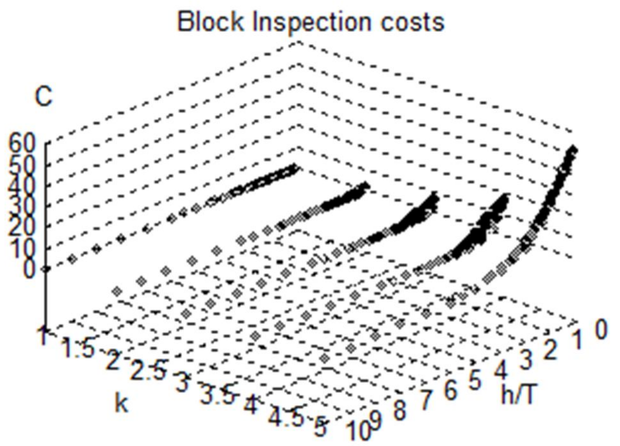

Figure 12. Block Inspection costs for *k-out-of-5* system imperfectly inspected (*p* decreases linearly)

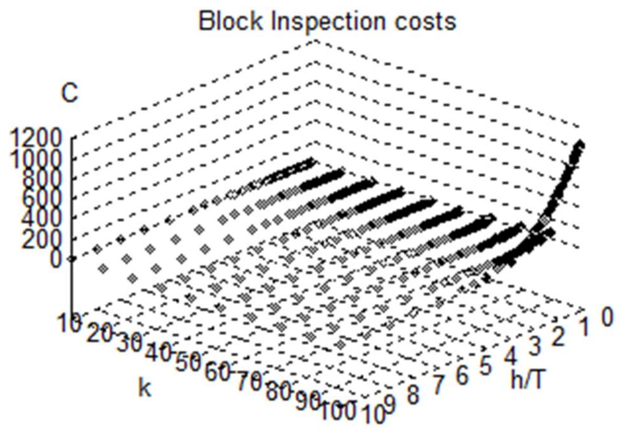

Figure 13. Block Inspection costs for *k-out-of-100* system imperfectly inspected (*p* decreases linearly)

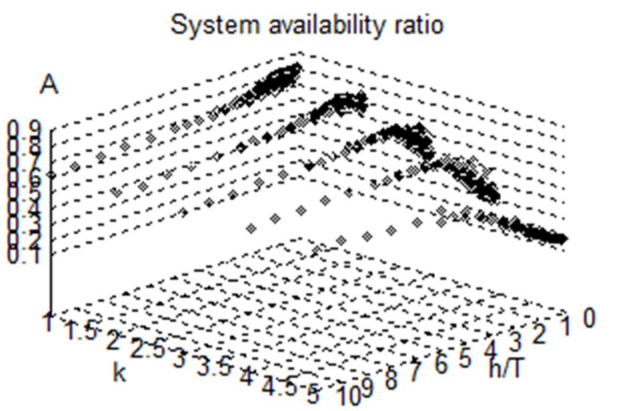

Figure 14. Availability ratio of *k-out-of-5* system imperfectly inspected (*p* decreases linearly)

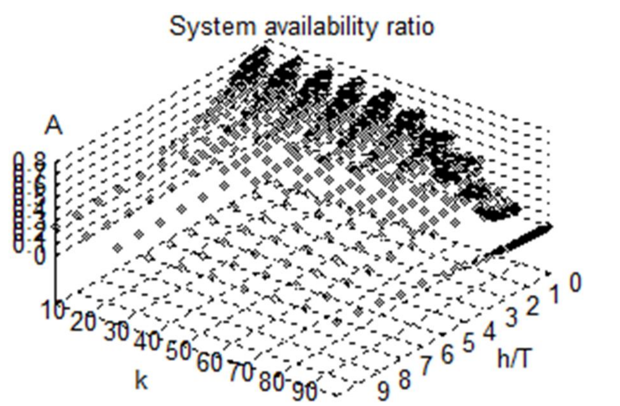

Figure 15. Availability ratio of *k-out-of-5* system imperfectly inspected (*p* decreases linearly)

Independently on the value of mean delay time of elements constituting a system, the best inspection period, with maximum level of system availability or minimum costs, is some part of the mean delay time. The absolute values of *h* and *T* do not have any greater meaning, thus it seems to be reasonable to generalize the conclusions resulting from the research. The fact is especially useful from the practical point of view. When one realizes that inspection in a system maintained according to the *BI* policy is not perfect and is able to estimate the mean delay time of elements, he has some reference range of inspection period lengths which might be applied in practice in order to get “good” availability and cost results.

5 CONCLUSIONS

The presented sensitivity analysis of investigated BI policy model gives the possibility to obtain some rules for definition of the principal relations between the system performance under given PM policy with imperfect maintenance and chosen PM policy parameters.

The research issues analysed in the paper are the continuation of research analyses made in (Jodejko-Pietruczuk & Werbińska-Wojciechowska 2012a). In the presented paper authors develop extended DT model with imperfect maintenance, when the probability p is not a constant parameter. Such a model parameter definition is more adjusted to real-life technical objects performance, like means of transport. Thus, in the next step, authors will make an effort to define some rules how to choose a PM policy from an engineering point of view.

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APPLICATION OF GEOMETRIC PROCESS IN ACCELERATED LIFE TESTING ANALYSIS WITH TYPE-I CENSORED WEIBULL FAILURE DATA

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ABSTRACT

In Accelerated life testing (ALT), generally, the estimates of original parameters of the life distribution are obtained by using the log linear function between life and stress which is just a simple re-parameterization of the original parameter but from the statistical point of view, it is preferable to work with the original parameters instead of developing inferences for the parameters of the log-linear link function. By the use of geometric process one can easily deal with the original parameters of the life distribution in accelerated life testing. In this paper the geometric process is used in accelerated life testing to estimate the parameters of Weibull distribution with type-I censored data. The maximum likelihood estimates of the parameters are obtained by assuming that the lifetimes under increasing stress levels form a geometric process. In addition, asymptotic confidence interval estimates of the parameters using Fisher information matrix are also obtained. A Simulation study is also performed to check the statistical properties of estimates of the parameters and the confidence intervals.

Keywords: Maximum Likelihood Estimation; Survival Function; Fisher Information Matrix; Asymptotic Confidence Interval; Simulation Study.

1 INTRODUCTION

Nowadays, there is a big competition among manufacturing industries to provide quality products to their customers and hence the customer expectations are also very high which makes the products in recent era very reliable and dependable. As in life testing experiments the failure time data is used to obtain the product life characteristics under normal operating conditions, therefore, such life data has become very difficult to obtain as a result of the great reliability of today's products and hence under normal operating conditions, as products usually last long, the corresponding life-tests become very time consuming and expensive. In these cases, an accelerated life test (ALT) which is a quick way to obtain information about the life distribution of a material, component or product can be applied to reduce the experimental time and the cost incurred in the experiment. In ALT items are subjected to conditions that are more severe than the normal ones, which yields shorter life but, hopefully, do not change the failure mechanisms. Failure information collected under this severe test stresses can be extrapolated to obtain an estimate of lifetime under normal operating condition based on some life-stress relationship.

ALTs, generally deal with three types of stress loadings i.e. constant stress, step stress and linearly increasing stress. The constant stress loading is a time-independent test setting and others are the time-dependent test setting. The constant stress loading has several advantages over time-dependent test settings, for example, most of the products in real life are operated at a constant stress. Therefore, a constant stress test describes the actual use of the product. Also, it is comparatively easy to run and to quantify a constant stress test. Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some of failure data are missing). For more details about ALTs one can consult Bagdonavicius and Nikulin [1], Meeker and Escobar [2], Nelson [3, 4], Mann and Singpurwalla [5].

Constant stress ALT with different types of data and test planning has been studied by many authors. For example, Yang [6] proposed an optimal design of 4-level constant-stress ALT plans considering different censoring times. Pan et al. [7] proposed a bivariate constant stress accelerated degradation test model by assuming that the copula parameter is a function of the stress level that can be described by a logistic function. Chen et al. [8] discuss the optimal design of multiple stress constant accelerated life test plan on non-rectangle test region. Watkins and John [9] considers constant stress accelerated life tests based on Weibull distributions with constant shape and a log-linear link between scale and the stress factor which is terminated by a Type-II censoring regime at one of the stress levels. Fan and Yu [10] discuss the reliability analysis of the constant stress accelerated life tests when a parameter in the generalized gamma lifetime distribution is linear in the stress level. Ding et al. [11] dealt with Weibull distribution to obtain accelerated life test sampling plans under type I progressive interval censoring with random removals. Ahmad et al. [12], Islam and Ahmad [13], Ahmad and Islam [14], Ahmad, et al. [15] and Ahmad [16] discuss the optimal constant stress accelerated life test designs under periodic inspection and Type-I censoring.

Geometric process (GP) is first used by Lam [17] in the study of repair replacement problem. Since then a large amount of studies in maintenance problems and system reliability have been shown that a GP model is a good and simple model for analysis of data with a single trend or multiple trends, for example, Lam and Zhang [18], Lam [19] and Zhang [20]. So far, there are only four studies in the analysis of accelerated life test that utilize the GP. Huang [21] introduced the GP model for the analysis of constant stress ALT with complete and censored exponential samples. Kamal et al. [22] extended the GP model for the analysis of complete Weibull failure data in constant stress ALT. Zhou et al. [23] implement the GP in ALT based on the progressive Type-I hybrid censored Rayleigh failure data. More recently Kamal et al. [24] used the geometric process for the analysis of constant stress accelerated life testing for Pareto Distribution with complete data.

In this paper, the constant stress ALT with geometric process and type-I censoring for Weibull distribution is considered. Estimates of Parameters are obtained by maximum likelihood estimation technique and Confidence intervals for parameters are obtained by using the asymptotic properties. Lastly statistical properties of estimates and confidence intervals are examined through a simulation study.

2 THE MODEL AND TEST PROCEDURE

2.1 The Geometric Process (GP)

A GP is a stochastic process $\{X_n, n = 1,2,...\}$ such that $\{\lambda^{n-1} X_n, n = 1,2,...\}$ forms a renewal process where $\lambda > 0$ is real valued and called the ratio of the GP. It is easy to show that if $\{X_n, n = 1,2,...\}$ is a GP and the probability density function of X_1 is $f(x)$ with mean μ and variance σ^2 then the probability density function of X_n will be $\lambda^{n-1} f(\lambda^{n-1} x)$ with mean μ / λ^{n-1} and variance $\sigma^2 / \lambda^{2(n-1)}$.

It is clear to see that a GP is stochastically increasing if $0 < \lambda < 1$ and stochastically decreasing if $\lambda > 1$. Therefore, GP is a natural approach to analyse the data from a series of events with trend. For more details about GP and its properties see Braun et al. [25].

2.2 The Weibull Distribution

The probability density function, the cumulative distribution function, the survival function and the failure rate (or hazard rate) of a two parameter Weibull distribution with scale parameter $\alpha > 0$ and shape parameter $\beta > 0$, are given respectively by

$$f(x|\alpha,\beta) = \frac{\beta}{\alpha^{\beta}} x^{\beta-1} \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad x \ge 0 \tag{1}$$

$$F(x|\alpha,\beta) = 1 - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad x \ge 0$$

$$S(x) = \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad x \ge 0$$

$$h(x|\alpha,\beta) = \frac{k}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}$$

It is easy to verify that failure rate (or hazard rate) decreases over time if $\beta < 1$ (or increases with time if $\beta > 1$) and $\beta = 1$ indicates that the failure rate is constant over time.

2.3 Assumptions and test procedure

i. Suppose that an accelerated life test with s increasing stress levels in which a random sample of n identical items is placed under each stress level and start to operate at the same time. Let $x_{ki}, i = 1,2,...,n,\ k = 1,2,...,s$ denote observed failure time of i^{th} test item under k^{th} stress level. Whenever an item fails, it will be removed from the test and the test is terminated at a prespecified censoring time t at each stress level and the exact failure times $x_{ki} \le t$ of items are observed.

ii. The product life follows Weibull distribution given by (1) at any stress.

iii. The scale parameter is a log-linear function of stress. That is, $\log(\alpha_k) = a + bS_k$, where a and b are unknown parameters depending on the nature of the product and the test method.

iv. Let random variables $X_0, X_1, X_2, ..., X_s$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily and sequence $\{X_k, k = 1,2,...,s\}$ forms a geometric process with ratio $\lambda > 0$.

Assumptions (i-iii) are very usually discussed in literature of ALTs but assumption (iv) which will be used in this study may be better than the usual one without increasing the complexity of calculations. The next theorem discusses how the assumption of geometric process (assumption iv) is satisfied when there is a log linear relationship between a life and stress (assumption iii).

Theorem 2.1: *If the stress level in a constant stress ALT is increasing with a constant difference then the lifetimes under each stress level forms a* GP *that is, If* $S_{k+1} - S_k$ *is constant for* $k = 1, 2, ..., s-1$*, then* $\{X_k, k = 0, 1, 2, ..., s\}$ *forms a* GP. *Or log linear relationship and GP model are equivalent when the stress increases arithmetically in constant stress ALT.*

Proof: From assumption (iii), it can easily be shown that

$$\log\left(\frac{\alpha_{k+1}}{\alpha_k}\right) = b(S_{k+1} - S_k) = b\Delta S \tag{2}$$

Now eq. (2) can be rewritten as

$$\frac{\alpha_{k+1}}{\alpha_k} = e^{b\Delta S} = \frac{1}{\lambda} \quad \text{(Assumed)} \tag{3}$$

This shows that stress levels increases arithmetically with a constant difference ΔS. Therefore, It is clear from (3) that

$$\alpha_k = \frac{1}{\lambda}\alpha_{k-1} = \frac{1}{\lambda^2}\alpha_{k-2} = \ldots = \frac{1}{\lambda^k}\alpha$$

The PDF of the product lifetime under the k^{th} stress level is

$$f_{X_k}(x) = \frac{\beta}{\alpha_k{}^{\beta}} x^{\beta-1} \exp\left\{-\left(\frac{x}{\alpha_k}\right)^{\beta}\right\}$$

$$= \frac{\beta}{\left(\frac{1}{\lambda^k}\alpha\right)^{\beta}} x^{\beta-1} \exp\left\{-\left(\frac{x}{\frac{1}{\lambda^k}\alpha}\right)^{\beta}\right\} = \left(\frac{\lambda^k}{\alpha}\right)^{\beta} \beta x^{\beta-1} \exp\left\{-\left(\frac{\lambda^k}{\alpha}x\right)^{\beta}\right\}$$

This implies that

$$f_{X_k}(x) = \lambda^k f_{X_0}(\lambda^k x) \tag{4}$$

Now, the definition of GP and (4) have the evidence that, if density function of X_0 is $f_{X_0}(x)$, then the probability density function of X_k will be given by $\lambda^k f(\lambda^k x)$, $k = 0,1,2,\cdots,s$. Therefore, it is clear that lifetimes under a sequence of arithmetically increasing stress levels form a geometric process with ratio λ.

3 MAXIMUM LIKELIHOOD ESTIMATION AND FISHER INFORMATION MATRIX

Here the maximum likelihood method of estimation is used because ML method is very robust and gives the estimates of parameter with good statistical properties. In this method, the estimates of parameters are those values which maximize the sampling distribution of data. However, ML estimation method is very simple for one parameter distributions but its implementation in ALT is mathematically more intense and, generally, estimates of parameters do not exist in closed form, therefore, numerical techniques such as Newton Method, Some computer programs are used to compute them.

Let the test at each stress level is terminated at time t and only $x_{ki} \leq t$ failure times are observed. Assume that $r_k (\leq n)$ failures at the k^{th} stress level are observed before the test is suspended and $(n - r_k)$ units are still survived the entire test without failing.

Now the likelihood function for constant stress ALT with Type I censored Weibull failure data using GP at one of the stress level is given by

$$L_k(\alpha,\theta,\lambda)=\frac{n!}{(n-r_k)!}\left[\left(\frac{\lambda^k}{\alpha}\right)^{r_k\beta}\beta^{r_k}\prod_{i=1}^{r_k}x_{k(i)}^{\beta-1}\exp\left\{-\left(\frac{\lambda^k x_{k(i)}}{\alpha}\right)^{\beta}\right\}\right]\left[\exp\left\{-\left(\frac{\lambda^k t}{\alpha}\right)^{\beta}\right\}\right]^{n-r_k}$$

Therefore, now the likelihood function of observed data for total s stress levels is

$$L_k(\alpha,\theta,\lambda)=L_1\times L_2\ldots\times L_s$$

$$=\prod_{k=1}^{s}\left[\frac{n!}{(n-r_k)!}\left(\frac{\lambda^k}{\alpha}\right)^{r_k\beta}\beta^{r_k}\left\{\prod_{i=1}^{r_k}x_{k(i)}^{\beta-1}\exp\left\{-\left(\frac{\lambda^k x_{k(i)}}{\alpha}\right)^{\beta}\right\}\right\}\left[\exp\left\{-\left(\frac{\lambda^k t}{\alpha}\right)^{\beta}\right\}\right]^{n-r_k}\right]$$

(5)

The log-likelihood function corresponding (5) takes the form

$$l=\log L_k(\alpha,\theta,\lambda)=\sum_{k=1}^{s}\left[\begin{array}{l}\log\left(\frac{n!}{(n-r_k)!}\right)+kr_k\beta\log\lambda-r_k\beta\log\alpha+r_k\log\beta\\ \qquad+(\beta-1)\sum_{i=1}^{r_k}\log x_{k(i)}-\left(\frac{\lambda^k}{\alpha}\right)^{\beta}\left(\sum_{i=1}^{r_k}x_{k(i)}^{\beta}+(n-r_k)t^{\beta}\right)\end{array}\right]$$

MLEs of α,β and λ are obtained by solving the following normal equations

$$\frac{\partial l}{\partial\alpha}=\sum_{k=1}^{s}\left[-\frac{r_k\beta}{\alpha}+\beta\lambda^{k\beta}\left(\frac{1}{\alpha}\right)^{\beta+1}\left(\sum_{i=1}^{r_k}x_{k(i)}^{\beta}+(n-r_k)t^{\beta}\right)\right]=0$$

$$\frac{\partial l}{\partial\lambda}=\sum_{k=1}^{s}\left[\frac{kr_k\beta}{\lambda}-\frac{k\beta}{\lambda}\left(\frac{\lambda^k}{\alpha}\right)^{\beta}\left(\sum_{i=1}^{r_k}x_{k(i)}^{\beta}+(n-r_k)t^{\beta}\right)\right]=0$$

$$\frac{\partial l}{\partial\beta}=\sum_{k=1}^{s}\left[\begin{array}{l}kr_k\log\lambda-r_k\log\alpha+\frac{r_k}{\beta}+\sum_{i=1}^{r_k}\log(x_{k(i)})-\left(\frac{\lambda^k}{\alpha}\right)^{\beta}\sum_{i=1}^{r_k}(x_{k(i)})^{\beta}\left\{\log(x_{k(i)})+\log\left(\frac{\lambda^k}{\alpha}\right)\right\}\\ -(n-r_k)\left(\frac{\lambda^k}{\alpha}\right)^{\beta}t^{\beta}\left\{\log t+\log\left(\frac{\lambda^k}{\alpha}\right)\right\}\end{array}\right]=0$$

Above equations are nonlinear; therefore, it is very difficult to obtain a closed form solution. So, Newton-Raphson method is used to solve these equations simultaneously to obtain $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\lambda}$.

The Fisher's information matrix composed of the negative second partial derivatives of log likelihood function can be written as

$$F=\begin{bmatrix}-\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda} & -\frac{\partial^2 l}{\partial\alpha\partial\beta}\\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda^2} & -\frac{\partial^2 l}{\partial\lambda\partial\beta}\\ -\frac{\partial^2 l}{\partial\beta\partial\alpha} & -\frac{\partial^2 l}{\partial\beta\partial\lambda} & -\frac{\partial^2 l}{\partial\beta^2}\end{bmatrix}$$

Where the elements of the Fisher Information matrix are obtained as

$$\frac{\partial^2 l}{\partial \alpha^2} = \sum_{k=1}^{s} \left[\frac{r_k \beta}{\alpha^2} - \beta(\beta+1)\lambda^{k\beta}\left(\frac{1}{\alpha}\right)^{\beta+2} \left\{ \sum_{i=1}^{r_k} x_{k(i)}^{\beta} + (n-r_k)t^{\beta} \right\} \right]$$

$$\frac{\partial^2 l}{\partial \lambda^2} = \sum_{k=1}^{s} \left[-\frac{k r_k \beta}{\lambda^2} - \frac{k\beta}{\lambda^2}(k\beta-1)\left(\frac{\lambda^k}{\alpha}\right)^{\beta} \left\{ \sum_{i=1}^{r_k} x_{k(i)}^{\beta} + (n-r_k)t^{\beta} \right\} \right]$$

$$\frac{\partial^2 l}{\partial \beta^2} = \sum_{k=1}^{s} \left[\begin{array}{l} -\dfrac{r_k}{\beta^2} - \left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \left\{ \log(x_{k(i)}) + \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\}^2 \\ -(n-r_k)\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} t^{\beta} \left\{ \log t + \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\}^2 \end{array} \right]$$

$$\frac{\partial^2 l}{\partial \alpha \partial \lambda} = \frac{\partial^2 l}{\partial \lambda \partial \alpha} = \sum_{k=1}^{s} \left[\frac{k\beta^2}{\alpha\lambda}\left(\frac{\lambda^k}{\alpha}\right)^{\beta} \left\{ \sum_{i=1}^{r_k} x_{k(i)}^{\beta} + (n-r_k)t^{\beta} \right\} \right]$$

$$\frac{\partial^2 l}{\partial \beta \partial \lambda} = \frac{\partial^2 l}{\partial \lambda \partial \beta} = \sum_{k=1}^{s} \left[\begin{array}{l} \dfrac{k r_k}{\lambda} - \dfrac{k\beta}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \log(x_{k(i)}) \\ -\displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \left\{ \dfrac{k}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} + \dfrac{k\beta}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\} - (n-r_k)\dfrac{k\beta}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} t^{\beta} \log t \\ -(n-r_k)t^{\beta} \left\{ \dfrac{k}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} + \dfrac{k\beta}{\lambda}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\} \end{array} \right]$$

$$\frac{\partial^2 l}{\partial \alpha \partial \beta} == \frac{\partial^2 l}{\partial \beta \partial \alpha} = \sum_{k=1}^{s} \left[\begin{array}{l} -\dfrac{r_k}{\alpha} + \dfrac{\beta}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \log(x_{k(i)}) \\ +\displaystyle\sum_{i=1}^{r_k} (x_{k(i)})^{\beta} \left\{ \dfrac{1}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} + \dfrac{\beta}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\} + (n-r_k)\dfrac{\beta}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} t^{\beta} \log t \\ +(n-r_k)t^{\beta} \left\{ \dfrac{1}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} + \dfrac{\beta}{\alpha}\left(\dfrac{\lambda^k}{\alpha}\right)^{\beta} \log\left(\dfrac{\lambda^k}{\alpha}\right) \right\} \end{array} \right]$$

4 ASYMPTOTIC CONFIDENCE INTERVAL ESTIMATES

According to large sample theory, the maximum likelihood estimators, under some appropriate regularity conditions, are consistent and normally distributed. Since ML estimates of parameters are not in closed form, therefore, it is impossible to obtain the exact confidence intervals, so asymptotic confidence intervals based on the asymptotic normal distribution of ML estimators instead of exact confidence intervals are obtained here.

Now, the variance covariance matrix can be written as

$$\Sigma = \begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} & -\frac{\partial^2 l}{\partial \alpha \partial \beta} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda^2} & -\frac{\partial^2 l}{\partial \lambda \partial \beta} \\ -\frac{\partial^2 l}{\partial \beta \partial \alpha} & -\frac{\partial^2 l}{\partial \beta \partial \lambda} & -\frac{\partial^2 l}{\partial \beta^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\lambda}) & ACov(\hat{\alpha}\hat{\beta}) \\ ACov(\hat{\lambda}\hat{\alpha}) & AVar(\hat{\lambda}) & ACov(\hat{\lambda}\hat{\beta}) \\ ACov(\hat{\beta}\hat{\alpha}) & ACov(\hat{\beta}\hat{\lambda}) & AVar(\hat{\beta}) \end{bmatrix}$$

The $100(1-\gamma)\%$ asymptotic confidence interval for α, β and λ are then given respectively by

$$\left[\hat{\alpha} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\alpha})}\right], \left[\hat{\beta} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\beta})}\right] \text{and} \left[\hat{\lambda} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\lambda})}\right]$$

5 SIMULATION STUDY

The performance of the estimates can be evaluated through some measures of accuracy which are the mean squared error (MSE), relative absolute bias (RAB) and the 95% asymptotic confidence intervals for different sample sizes and stress levels. Now for this purpose following simulation study is conducted.

To perform the simulation study, first a random sample x_{ki}, $k = 1,2,...,s$, $i = 1,2,...,r$ is generated from Weibull distribution which is censored at $t = 4, 6$. The values of the parameters and number of stress levels are chosen to be $\alpha = 0.80$, $\beta = 2.50$, $\lambda = 1.50$ and $s = 4$. For different sample sizes $n = 50,100,...,250$ the MLEs, MSEs, RABs and lower and upper CI limits (LCL and UCL) of the 95% confidence interval of parameters based on 600 simulations are obtained by the model discussed in this paper and summarized in Table 1 and 2.

Table 1: Simulation Study Results with $\alpha = 0.80$, $\beta = 2.50$, $\lambda = 1.50$, $s = 4$ and $t = 4$

n	$\hat{\alpha}$ $\hat{\lambda}$ $\hat{\beta}$	MSE $(\hat{\alpha})$ MSE $(\hat{\lambda})$ MSE $(\hat{\beta})$	RAB $(\hat{\alpha})$ RAB $(\hat{\lambda})$ RAB $(\hat{\beta})$	95 % Confidence Interval	
				LCL	UCL
50	0.846 1.439 2.534	0.0203 0.0076 0.0295	0.0575 0.0407 0.0136	0.5816 1.3166 2.2043	1.1104 1.5614 2.8637
100	0.839 1.446 2.529	0.0194 0.0085 0.0210	0.0488 0.0360 0.0116	0.5768 1.2993 2.2504	1.1012 1.5927 2.8076
150	0.810 1.512 2.511	0.0133 0.0079 0.0170	0.0125 0.0080 0.0044	0.5848 1.3389 2.2562	1.0352 1.6851 2.7658
200	0.792 1.520 2.502	0.0115 0.0096 0.0157	0.0100 0.0133 0.0008	0.5827 1.3320 2.2564	1.0013 1.7080 2.7476
250	0.784 1.534 2.498	0.0105 0.0175 0.0121	0.0200 0.0227 0.0008	0.5860 1.2838 2.2824	0.9820 1.7842 2.7136

Table 2: Simulation Study Results with $\alpha = 0.80,\ \beta = 2.50,\ \lambda = 1.50,\ s = 4$ and $t = 6$

n	$\hat{\alpha}$ $\hat{\lambda}$ $\hat{\beta}$	MSE $(\hat{\alpha})$ MSE $(\hat{\lambda})$ MSE $(\hat{\beta})$	RAB $(\hat{\alpha})$ RAB $(\hat{\lambda})$ RAB $(\hat{\beta})$	95 % Confidence Interval	
				LCL	UCL
50	0.893 1.594 2.587	0.0225 0.0130 0.0237	0.1163 0.0627 0.0348	0.6619 1.4669 2.3383	1.1241 1.7210 2.8357
100	0.889 1.568 2.542	0.0204 0.0113 0.0201	0.1113 0.0453 0.0168	0.6699 1.4076 2.2769	1.1081 1.7284 2.8071
150	0.874 1.502 2.499	0.0313 0.0083 0.0204	0.0925 0.0013 0.0004	0.5592 1.3234 2.2191	1.1888 1.6806 2.7789
200	0.832 1.483 2.474	0.0178 0.0054 0.0199	0.0400 0.0113 0.0104	0.5779 1.3430 2.2024	1.0860 1.6229 2.7456
250	0.804 1.492 2.482	0.0087 0.0122 0.0231	0.0050 0.0053 0.0072	0.6212 1.4799 2.1860	0.9868 1.7076 2.7780

6 DISCUSSION AND CONCLUSIONS

In this paper the problem of constant stress ALT with type-I censored Weibull failure data using GP has been considered. The MLEs, MSEs, RABs the 95% asymptotic confidence intervals estimates of the model parameters were also obtained.

From the results in Table 1 and 2, it is easy to find that estimates of the parameter perform well. For fixed θ, α and λ, the MSEs and the RABs of θ, α and λ decreases as n increases. This indicates that the ML estimates provide asymptotically normally distributed and consistent estimator for the parameters. For the fixed sample sizes, as the termination time t gets larger the MSEs and RABs of the estimators decrease. This is very usual because more failures are obtained due to large values of censored time and thus increase the efficiency of the estimators.

From above discussion and results it may be concluded that the present model work well under type-I censored data for Weibull distribution and would be a good choice to be considered in ALTs in future. For the perspective of further research in this direction one can choose some other lifetime distribution with different types of censoring schemes.

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SURVIVAL MODELS OF SOME POLITICAL PROCESSES

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ABSTRACT

We extend the Probabilistic ideas from stochastic processes (queuing theory and reliability) on creation of some realistic models for studying several governing political formations, and find their survival characteristics. These models were presented at the Sixth and Seventh International Conferences on Mathematical Models in Reliability (Moscow 2009, and Beijing 2011). Our focus is on a "democracy" model, where the times of survival (existence at the political scene, duration of stay in leading coalition, governing survivability, life time distribution, longevity, etc.) can be derived from the model. Markovian models of spending time in certain sets of states are explored, and some discussion on statistical properties and evaluations are presented. We are confident that other political schemes also can be modeled using appropriate probabilistic tools.

1 INSTRUCTIONS

Modeling politics based on certain scientific concepts and its inclusion into quantitative models is a challenging task. The authors of this work did an extensive review of some successful attempts in political modeling by mathematical means such as: Cioffi-Revilla (2009) recent model of a political system; the recent textbook of Clark et al. (2008) on modeling of preferences in political science; the Taylor's book (1995) with ideas for discrete and deductive mathematical approaches in international conflict resolution; Doyle nonlinear methods (2000) to describe and solve existing political questions; Monroe's (1997) evaluations of the current state of empirical political theory and guidelines to future developments in political science; the Ordeshook (1986), Morrow (1994), Hafer (2007) approaches based on the use of games theory in modeling political processes, and some non mathematical ideas as these, presented in J. March (1994), Then we started working on an approach based on construction of specific probabilistic analysis of components that form political processes inn conditions of an open democracy. In our opinion, formal political theory seeks to develop formal, mathematical models of political, demographic, and economic processes. The above mentioned authors in their books and articles, make an attempt to integrate the modern developments of the applied mathematical theories (games, decision making, multiple, interactive decisions) with models of demand and supply of public goods, and social-choice theory, part of what the political structures are considered.

The articles of Esa and Dimitrov (2009, 2011) are an illustration of how probability tools can be used to model basic components in the big political games. Reliability and service system theories provide a good assembly of approaches in analysis of various sides of the products (risks, and costs inclusive) and characteristics in the political activities. Encouraged by the welcome reception of these presentations on behalf of the experts in applied statistical modeling at the MMR' (2009 and 2011) Forums, we continue working in the same direction. We found more sites our approach may reveal. In the present work we use the results of these simple models to illustrate various important additional characteristics of the political subjects. In our opinion, appropriate models can be made to study totalitarian schemes, monarchy, parliamentary kingdoms, even some non existing, but virtually possible political structures. And this is our reason to consider probability models in politics a challenging area of applied probability and statistics.

2 CHARACTERISTICS OF THE DEMOCRATIC POLITICAL PARTY MODEL

First we use the results from the model of a political party, considered as a formation within a finite population of active individuals N. The existence (formation) of a party requires certain minimum of members, say $M+1$. Each free individual may decide to join a party at any time, as well as a member can quit the party at any time. Simultaneously, there exist a numerous pool of other parties which operate in a similar way, and their members are not allowed to join (or switch to) another party.

After discussing this mechanism and dynamics, Esa and Dimitrov (2009) introduced the following mathematical model of the political life in a country:

There is a population of N individuals (citizens of a country). These are r parties in the society. Each party is considered as a service system of N available seats (servers) in each. The rate of inputs from each free individual towards party (service system) j is λ_j . $j=1,\ldots,r$. At system j the individual spends some random time S_j, and goes free of politics (back to its source). A simple Markovian model in the case of exponentially distributed service times S_j of parameters μ_j shows the stationary probabilities P_j that an individual is free (then we use $j=0$), or is member of the party $j=1,2,\ldots,r$ are given by the expressions

$$P_0 = \left(1+\frac{\lambda_1}{\mu_1}+\cdots+\frac{\lambda_r}{\mu_r}\right)^{-1}; \quad P_j = \frac{\lambda_j}{\mu_j}\left(1+\frac{\lambda_1}{\mu_1}+\cdots+\frac{\lambda_r}{\mu_r}\right)^{-1}, \quad j=1,2,\ldots,r. \tag{1}$$

A multinomial model describes the entire spectrum of the party's life in the country, with N independent active *free i*ndividuals. The coordinates of the random vector $\vec{X}=(X_0, X_1, \ldots, X_r)$ represent the number of individuals members of each party $X_0+X_1+\ldots+X_r=N$. They are distributed according to the multinomial law

$$P(X_0=k_0, X_1=k_1, \ldots, X_r=k_r) = \frac{N!}{k_0!k_1!\ldots k_r!}P_0^{k_0}P_1^{k_1}\ldots P_r^{k_r}, \quad k_0+k_1+\ldots+k_r=N. \tag{2}$$

Hence, the chance of the j^{th} party to exist (at a minimum M_j+1 *members* required for this purpose) is having Binomial probability $P(X_j \geq M_j+1)=1-B(M_j; N, P_j)$ with the P_j, given by (1), and $B(k; N, p)$ is notation for c.d.f. of Binomial distribution witn parameters N and p, and k its argument. The probability generating function of this distribution (2) is given by the expression

$$P(z_0,z_1,\ldots,z_r) = E(z_0^{X_0}z_1^{X_1}\ldots z_r^{X_r}) = \left(P_0z_0+P_1z_1+\ldots+P_rz_r\right)^N, \quad |z_j|\leq 1. \tag{3}$$

It allows particular calculation of various average characteristics, correlations etc.

The average number of members of a party j is $E(X_j)=NP_j$, and its variation equals $V(X_j)=NP_j(1-P_j)$.

The correlation coefficients between the counts X_i and X_j of the parties labeled as i and j are given by the expressions

$$\rho_{ij} = Corr(X_i, X_j) = -\sqrt{\frac{P_i}{1-P_i}}\sqrt{\frac{P_j}{1-P_j}}, \quad i\neq j, \quad i, j=0,1, \ldots, r.$$

The correlation is always negative, since parties compete for one and the same pool of potential members. Interesting quantities are the correlation coefficients of the parties $i=1,\ldots,r$ with the "party" of the free individuals, for which subscript j is zero. Their absolute values may be used as a kind of rating indices for the parties in the country. The more the value, the higher the rating is. Interesting observation here is, that the correlation coefficients between parties (as well as the rating) do no depend on the total population size N.

Probability for party j to be dissolved must be considered under condition that party is active, i.e. given that $X_j \geq M_j+1$. This is the probability that someone of its members will decide to quit the platform when its members are at the critical number M_j+1. Thus this probability equals

$$d_j = P\{X_j(t+\Delta t) = M_j \mid X_j(t) \geq M_j + 1\} = \binom{N}{M_j+1} P_j^{M_j+1}(1-P_j)^{N-M_j+1} \frac{(M_j+1)\mu_j \Delta t}{1-B(M_j; N, P_j)}.$$

Probability for party (platform) j not to exist (to be in a "sleep" state) equals to the measure of the chance for the model to spend in one of the states X_j=0, 1, …, M_j, and therefore, is given by the expression

$$P(X_j \leq M_j) = B(M_j; N, P_j)$$

with the P_j, determined by (1).

Further dynamic analysis will allow to determine the **duration of existence of a party**. Let us note that when we look at the Markovian process of the changes in the platform (party's) states, we may imaginary consider another, absorbing Markov chain whose absorbing states are these when X_j=0, 1, …, M_j, and the states $X_j=M_j+1$, M_j+2, …, N are transient. Then, the duration of the existence of party (platform) j on the political scene will be equal to the time the above described Markovian process spends in the sets of its transient states.

Denote by $B_{k_0,k_1,\cdots,k_r}$ the average time the process spends at the transition set of states if it starts at a state given by the left hand side of equation 2. Taking into account the infinitesimal intensities of the process interstate transitions, and the respective average spending times immediately before a transition and the respective sojourn times in the same state and in the transient sets after the transition, we arrive to the following system of equations for the sojourn times $B_{k_0,k_1,\cdots,k_r}$, valid for $k_j > M_j+1$, for all j = 1, …, r:

$$B_{k_0,k_1,\cdots,k_r} = \sum_{i=1}^{r} \sum_{k_i=M_i+1}^{N} k_i \mu_i \frac{B_{k_0,k_1,\cdots,k_i-1,\cdots,k_r}}{k_0 \sum_{l=1}^{r} \lambda_l + \sum_{l=1}^{r} k_l \mu_l} + \frac{1}{k_0 \sum_{l=1}^{r} \lambda_l + \sum_{l=1}^{r} k_l \mu_l} +$$

$$+ \sum_{i=1}^{r} \sum_{k_i=M_i+1}^{N} (k_0+1)\lambda_i \frac{B_{k_0+1,k_1,\cdots,k_i,\cdots,k_r}}{k_0 \sum_{l=1}^{r} \lambda_l + \sum_{l=1}^{r} k_l \mu_l}. \quad (4)$$

On the boundary layers where some, or several $k_i = M_i + 1$ the equations (4) are still valid with $B_{k_0,k_1,\cdots,k_i-1,\cdots,k_r} = 0$. Also is true that $B_{k_0,k_1,\cdots,k_r} = 0$ if $k_j \leq M_j$, for any j = 1, …, r. System (4) always has a solution since the chain is absorbing. This solution can be found by the method of inverse matrices.

The expected life time of party j is then given by the expression

$$B_j = \sum_{k_j=M_j+1}^{N} \binom{N}{k_j} P_j^{k_j}(1-P_j)^{N-k_j} \sum_{\substack{i=0 \\ i\neq j}}^{r} \sum_{\substack{\mathrm{k_j}=M_j+1 \\ i\neq j}}^{N} \mathrm{P}(\mathrm{X}_0 = \mathrm{k}_0, \mathrm{X}_1 = \mathrm{k}_1, \ldots, \mathrm{X}_\mathrm{r} = \mathrm{k}_\mathrm{r}) B_{k_0,k_1,\cdots,k_r} \quad (5)$$

The use of the introduced parameters in respective statistical data may allow practical estimation of these parameters and give the answer to various interesting statistical questions.

3 CHARACTERISTICS OF THE ELECTION MODEL

The party model of the previous section was used in [5] to create respective election model. It is assumed the following configurations before the elections. There are C coalitions registered for the peoples vote. A coalition may consist of one ore several parties. The “Zero” party is made by those who are not members of any party. They vote with probability p, or not vote with probability q=1-p. A vote goes to coalition C_j with probability Q_j proportional to the intensities to join the parties with indices $i_1,\ldots,i_{C_j}$ from this coalition, i.e.

$$Q_j = \frac{\lambda_{i_1} + \cdots + \lambda_{i_{C_j}}}{\lambda_1 + \lambda_2 + \cdots + \lambda_r}, \quad j=1,\ \ldots,\ C. \tag{6}$$

Party members vote for the coalition where their party belongs. Under some additional assumptions it is found that the random vector $(T_0,T_1,\ldots T_C)$ of votes given to the coalitions in the elections has multinomial distribution with probability generating function of the voting results $(T_0,T_1,\ldots T_C)$ given by the expression

$$T(z_0,z_1,\ldots,z_C) = \left((1-p)P_0 z_0 + \sum\nolimits_{j=1}^{C} (pP_0 Q_j + P_{i_1} + \cdots P_{i_{C_j}}) z_j\right)^N, \tag{7}$$

Here P_i, Q_j are given by expressions in (1) and (6), and T_0 is the number of those who do not vote. Hence, the random number of voters for coalition C_j has Binomial distribution with parameters N and $\alpha_j = pP_0 Q_j + P_{i_1} + \cdots + Pi_{C_j}$. Roughly speaking, the number of votes for a coalition equals to the sum of its party members and the votes of non-party people who may vote for this coalition.

Knowledge of the distribution of the random variables Tj allows calculation of various interesting explicit and/or average characteristics related to the specific electoral laws. For instance:

The average number of votes for coalition T_j equals $N\alpha_j$.

The probability that coalition T_j does not survive the requirements to pass the minimum γ% percentage barrier is given by the expression

$$P\{T_j < \gamma\ V/100) = B(\frac{\gamma\ V}{100};\ N,\ \alpha_j). \tag{8}$$

Here $V=pE(X_0)$ is the expected number of voters in the elections, X_0 is the number of members of “the zero party” with the marginal distribution as given by equation (2), and $B(.)$ is notation for the Binomial c.d.f. The complement to 1 of the probability in (8) is the probability for this coalition to survive the elections. Since here the work is mostly with binomially distributed r.v.’s, all the conventional approximations to the Binomial distribution (Poisson, normal) are legitimate tools to simplify the calculations. We omit these details.

The provability that coalition C_j is the winner in the elections will be determined from the requirement

$$T_j = \max\{T_1,\ldots,T_C). \tag{9}$$

When take into account (7) we easily find that the probability (9) to be fulfilled equals

$$\beta_j = \sum_{\substack{k_j > k_1,\cdots,k_C \\ k_0+k_1+\cdots+k_C=N}}^{N} \frac{N!}{k_0!k_1!\cdots k_C!}[(1-p)P_p]^{k_0}\alpha_1^{k_1}\ldots\alpha_C^{k_C}. \tag{10}$$

It is intuitively clear that the coalition with the highest value of the probability α_j is the expected winner. However, (10) allows to evaluate probabilities for any other coalition to win.

One last remark here is that the results in votes are negatively correlated random variables. The correlation matrix among the coordinates of the random vector $(T_0,T_1,\ldots T_C)$ is given by the entries as shown in the case for the counts X_i and X_j of the parties in the previous section, where instead the probabilities P_j the quantities α_j must be used.

In many cases, the votes from the losing coalitions are distributed proportionally between the winning coalitions, according to the numbers of actual votes approved for the winners. These potential extra votes may increase the elected members from a coalition. Respective conditional distributions are also available based on the described here model. This will be subject of another study.

4 CHARACTERISTICS OF THE GOVERNING MODEL

The process of formation of governing coalition between the winners in the election has a complex structure. Here we enter in the complexity of issues well described in chapters 3 and 4 by J. March (1994). It is a challenging task to make mathematical models based on these descriptions of decision making. Usually, the coalition with major sits in the National Assembly (NA) takes responsibility to form a governing coalition. It negotiates with groups with smaller sits in the NA, until gets more than K supporters among the sits in the NA. Then the government is ready to be formed. This government survives as long as it is supported by at least K members in the NA. Assuming that the government is supported by G coalitions, with $M_1,\dots,M_G$ representatives in the NA, we modeled the government as a system with subsystems connected in series, with variable number of functioning components in each subsystem (the coalitions in the government). With the notation $F_j(t)$ for the distribution function of the time of random duration that a member of the NA from coalition j keeps his/her loyalty to the governmental formation (and decisions assumed independent between the members of the NA), we derive in [5] that the government survives time of duration t is given by the rule

$$P\{L>t\}=\prod_{j=1}^{G}\sum_{i=K_j}^{M_j}\binom{M_j}{i}\left(1-F_j(t)\right)^i F^{M_j-i}(t). \tag{11}$$

Here K is a number showing the minimum number of members required for a coalition to exist as an entity in the NA, and L is the life time of this government.

If the next elections are scheduled after expiration of time of duration T, the average life time of this government will be presented by the quantity

$$\mu_T = E(L\mid T)=\int_0^T P\{L>t\}\ dt \tag{12}$$

However, this is politics, and the life is dynamically changed. There are times when the parties in opposition call for non-confidentiality vote against this particular government. The results from the vote are modeled by making use of the models from sections 2 and 3. Such votes are binary (Pro or Con where the votes “abstain” are actually in favor of no confidence). Each coalition may have its own probability p_j for a member to vote “Pro”. The random number N_j of “pro”-votes in each coalition C_j is binomial of parameters (Y_j, p_j), where Y_j is the random number of surviving supporters in the j^{th} governing coalition. Therefore, the chance to survive a non-confidentiality vote at time t has probability

$$P(N_1+\cdots N_C\ge K,\ L>t)=\sum_{j=1}^{C}\ \sum_{k_1+\cdots k_C\ge K}\ \sum_{i=K_j}^{M_j}\binom{M_j}{i}\left(1-F_j(t)\right)^i F^{M_j-i}(t)\binom{i}{k_j}p_j^{k_j}(1-p_j)^{i-k_j} \tag{13}$$

If the calls from opposition for non-confidentiality vote for the government form a point process of certain kind, the survival probability of the government will decrease proportionally to the product of probabilities to survive each non- confidentiality vote. If just one no confidence vote is supposed to be induced with a uniform distribution within the assumed interval between elections, the probability, say *a,* for surviving it equals to the integral of the expression on the right hand side of (3). Then the expected life of this government will be evaluated by the expression

$$\mu(a) = a\mu_T + \frac{1}{2}(1-a)\mu_T = \frac{\mu_T}{2}(1+a).$$

Complications under other assumptions are evident, but not worthless to be discussed.

5 CONCLUSIONS

Political processes offer interesting area of applications of various mathematical modeling approaches and theories. We discuss probability models by keeping close to the processes of formation of political units and activities to their natural components. The uncertain elements are naturally included into specific probabilistic relationships. The obtained analytical results produce promising particular characteristics, and offer a lot of field for discussions, statistical considerations and interesting applications.

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