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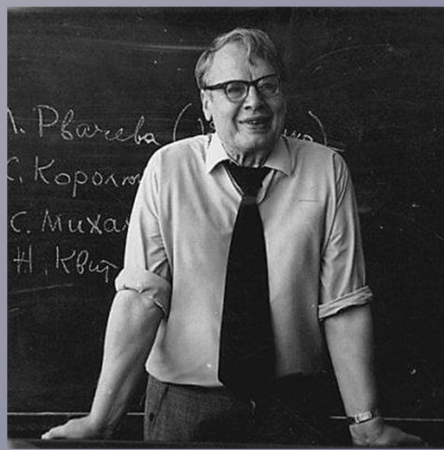

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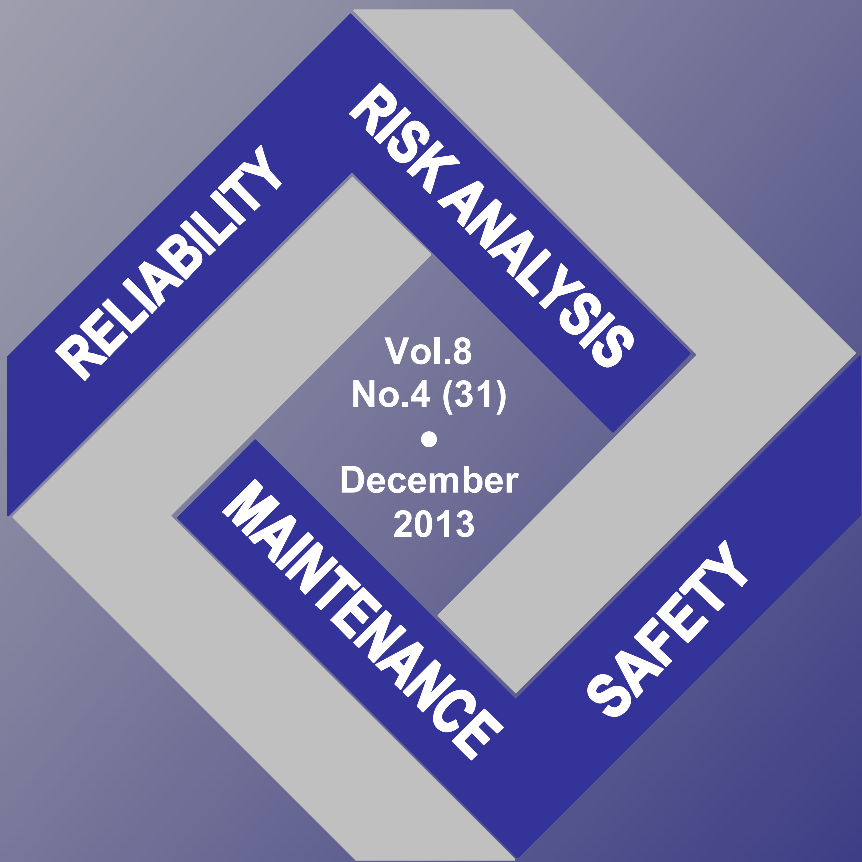
VOL.8 NO.4 (31)
DECEMBER, 2013

RELIABILITY:
THEORY&APPLICATIONS



San Diego

ISSN 1932-2321

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RELIABILITY:

THEORY & APPLICATIONS

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Table of Contents

The paper deals with possibilities of stochastic approaches in risk modelling from a point of costs and data relating reliability aiming to define needed parameters of distribution of stochastic variables. It presents outputs from developed and applied simulation models, to assess reliability, maintainability, availability, costs and risks from operational data.

The question of synthesis of base of fuzzy knowledge ANFIS - networks for control streams of reactive capacity and voltage of knots considered in distributive electric networks. Synthesis base of fuzzy knowledge and term on basis of the received regime reports executed for schemes 14 knots electric networks with method application subtractive clustering. Root mean square error parameter adjustment of a method shows to adequacy of control of a mode in DEN.

This article is about how the confusion of terms can give rise to confusion in the approaches and methods, and bring to a deadlock the whole research area. It doesn't mean that we would like to teach someone or to present "the ultimate truth". It is rather another attempt to declare that "software reliability» does not exist in the form it is treated in most of "traditional" sources.

This paper is addressed to a problem of development of the resources used for solving challenges in co-design and testing of the computer systems and their components. Both target and natural are considered. Target resources include models, methods and means. Natural resources like particularities of the target resources are examined. Natural development of target resources with structuring under particularities of the Universe which is considered like parallel and approximate is analysed. Natural resources in remove of contradicts between target resources are shown.

Most of the engineers finds it very tedious to do accelerated reliability test of any motor product if it has to be mechanically loaded as it involves lot of difficulties in motor's loading capabilities and its size, which also directly impacts the development time and cost. This is why all the sequences in accelerated reliability test are done on no-load condition which gives results but are not accurate since in field motors are always under loading condition. To overcome all these difficulties, the paper proposes a new approach to perform accelerated reliability testing of low tension 3 phase motors under full load without actually coupling mechanical load to the motor shaft. The approach is simple and easy to implement. In this approach, instead of mechanical loading motor is equivalently loaded just by giving a supply voltage approximately 20-25% greater than the rated voltage to get same load current similar to fully loaded motor across all the three phases. Therefore, the application of proposed approach to perform accelerated reliability testing is experimented on low tension motor and results

SOME SUBJECTS OF COMPENSATION OF THE INDUCTION GENERATORS REACTIVE POWER IN WIND-POWER AND SMALL HYDROELECTRIC POWER PLANTS

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ABSTRACT

Reactive power compensation of asynchronous generators wind power and small hydroelectric power stations increases the reliability of connecting them to the so-called "weak" power grids of power systems. The methods of reactive power compensation for asynchronous generators of various designs.

Asynchronous generators are widely used as an electromechanical converter in wind-power and small hydro-power plants. The main advantages of these generators are low cost, simple design, reliability in operation, resistance to the external accidents and etc. But along with this, they have some disadvantages such as reactive power consumption, voltage control inability (unable to control voltage), a significant voltage reduction in start-up of the power plant, that is particularly affected by the plants unit rating and a "weak" power grid/electrical network at the place of their installation.

Having used as above-mentioned electromechanical converters - AC machines have various designs and layouts. The simplest of them is squirrel-cage rotor induction generator, which is mainly used in small hydropower plants at the early stages of their power range. The next ones are with induction generators cage rotor with frequency inverters (VFD-Variable Frequency Drive) with fully controlled thyristors in the generators stator circuit. These converters have been applied in medium and high power wind-power plants, and practically are not used in small hydro-power engineering.

Double-fed induction machine (DFIM) is the most widely used generator in wind-power engineering. They equipped with frequency inverters connected to the rotor winding of the machine. At the same time considering undeniable advantages, these machines can be also recommended to be used in medium and relatively high power small hydro power plant.

This paperwork studies several subjects of reactive power compensation of various types and configurations of asynchronous generators rotated by renewable energy sources. It should be noted that, in considerable amount of papers works reactive power compensation of these electromechanical converters is covered [1,2,3]. But in a greater degree, they study steady state and quasi steady state operation modes. The reactive power compensation in the start-up conditions remains poorly studied [3]. Once more it should be noted that considerable amount of reactive power compensation is required namely during the start-up mode, its availability may hinder the voltage drop when the generator rotated by renewable energy sources and connected to the "weak" electric grids, thereby substantially improve their reliability.

Induction generator with cage rotor

Let us consider reactive power compensation of the asynchronous generator with cage rotor, which is widely used as an electromechanical transducer in small hydropower plants. As a result of a full-scale experiment on the generator with rated power of $P_{nom.}$=132 kW, it was revealed that reactive power consumption of asynchronous generators slightly depends on its active/resistive load, therefore for this class of generators using non-adjustable capacitor battery is recommended. However it should be taken into consideration that, the experiment was carried out on under-loaded AC machine.

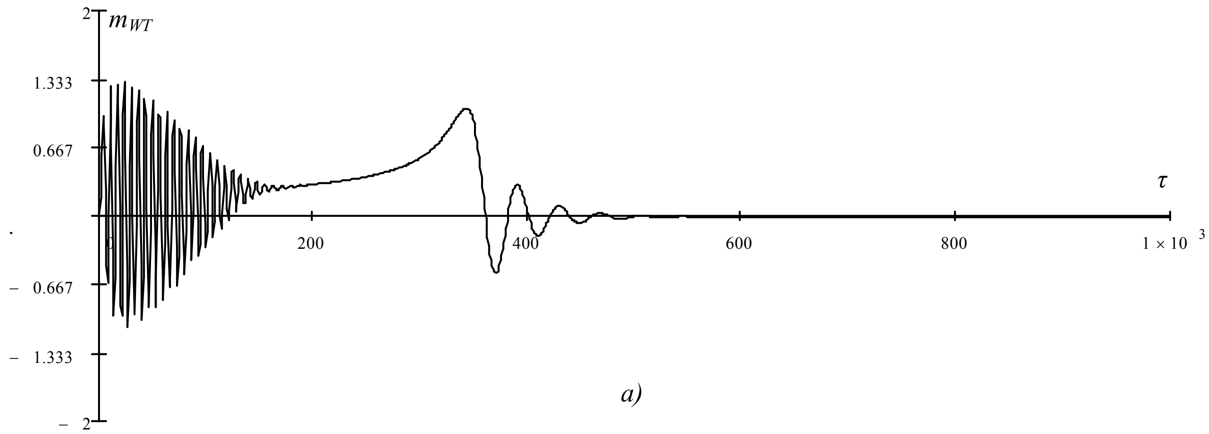
Computer simulation of asynchronous generator, made on the base of induction motor 4AH250M4 with rated power P=110 kW was performed. Operating mode of small hydropower plant generator is simulated–torque on the generator shaft is changed by changing of opening angle of mechanically driven hydro turbine wicket gate. Change of generator operation condition data is shown in Table 1.

Table 1.

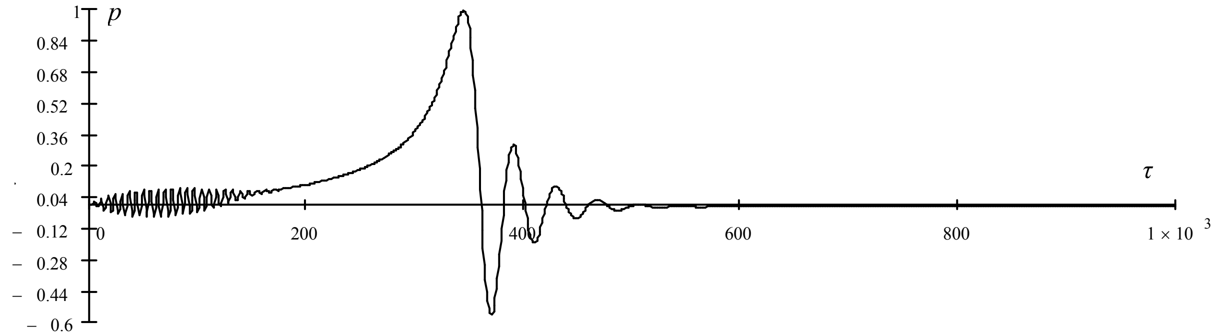
№	m_{wt}, *r.u.*	*p*, *r.u.*	*ω*, *r.u.*	*q*, *r.u.*	*s*, *r.u.*
1	–0,01	–0,01	1	0,227	0,228
2	–0,1	–0,1	1,002	0,231	0,252
3	–0,2	–0,201	1,003	0,239	0,312
4	–0,3	–0,303	1,005	0,25	0,392
5	–0,4	–0,40	1,007	0,265	0,482
6	–0,5	–0,50	1,009	0,283	0,578
7	–0,6	–0,60	1,01	0,304	0,678
8	–0,7	–0,709	1,012	0,328	0,781
9	–0,8	–0,811	1,014	0,357	0,887
10	–0,85	–0,863	1,015	0,374	0,94

Table data analysis shows that in practice, reactive power changing from non-load running (in m_{WT}=–0,01) to rated load (in m_{WT}=–0,85) is approximately 64%.

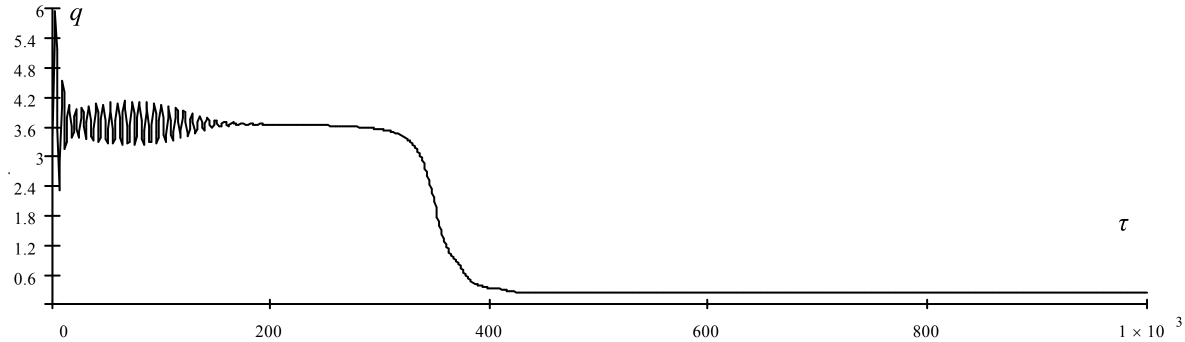
But if it was knowingly selected oversized in terms generator power and its maximum loading conformed to the half of the rated power (row 5 on Table 1), then reactive power (q_5= 0,265) exceeds the non-load running reactive power only by 16%, which conforms with full-scale experiment results, shown in [1]. In this case it is reasonable to use non-adjustable static capacitor batteries, and their value at q_5=0,265 is approximately (S_{base}=129,2 kVA) Q_5=34,2 kVAR. In the case of rated loading *m*=–0,85 and q_{10}=0,374 the power of capacitor batteries should be q_{10}= 48,3 kVAR. Generator start-up condition is also interesting enough to consider. Figure 1 (*a, b, c, d*) shows corresponding changes of generator operation conditions – electromagnetic torque m_{EM}, reactive power q, rotor speed ω_r, and total power S at the minimal torque of the wind(water) turbine on the generator shaft m_{WT} =–0,01 (minus denotes generator mode, and plus denotes motor mode). Fluctograms shows that the average value of the reactive power at start-up (Fig.1,*b*) is *q*=3,6 (rel.unit) and it remains constant within *τ*=350 rad. (*t*=1,1 sec.).Although relative reactive power at the start-up has a considerable value, but start-up duration for this type of generator is small and the absolute value of start-up reactive power in “weak” electrical grid causes small voltage drop at generator connection point for a short while.) Therefore it is not reasonable to make special efforts for reactive power compensation at generators start-up.


m_{WT}
2
1.333
0.667
– 0.667
– 1.333
– 2
0
200
400
600
800
1 × 10 3
τ

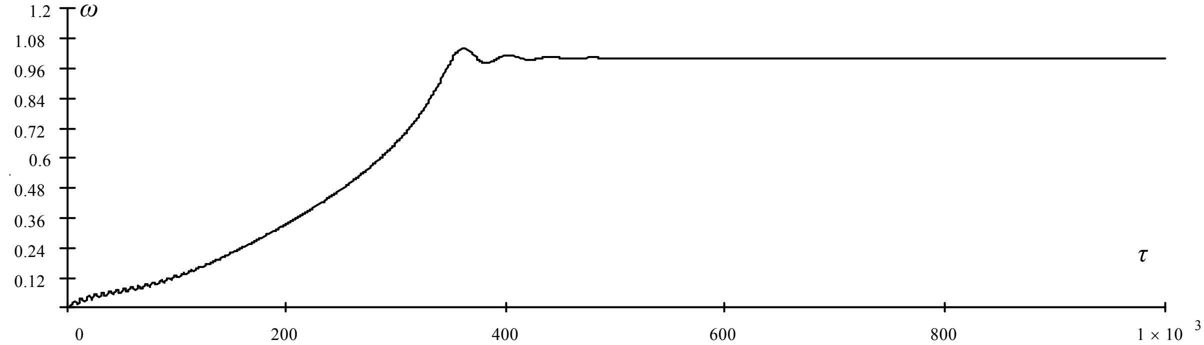
a)


p
1
0.84
0.68
0.52
0.36
0.2
0.04
– 0.12
– 0.28
– 0.44
– 0.6
0
200
400
600
800
1 × 10 3
τ

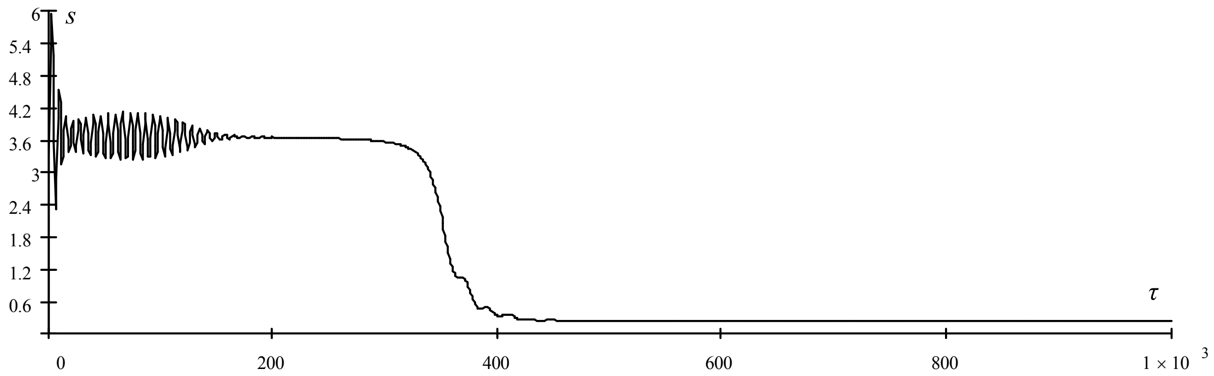
b)


q
6
5.4
4.8
4.2
3.6
3
2.4
1.8
1.2
0.6
0
200
400
600
800
1 × 10 3
τ

c)


ω
1.2
1.08
0.96
0.84
0.72
0.6
0.48
0.36
0.24
0.12
0
200
400
600
800
1 × 10 3
τ

d)



e)

Figure 1.

When generator power is 2 MW, start-up time is t_n=4÷5 sec and reactive power start-up value is q_n=4,7 rel. unit, that may have substantial influence on electrical grid at the connection point. In addition to that, reactive power reaches to considerable value at the steady state mode. For example, for the same type of generator at rated load pg=0,832, reactive power will reach to value qg=0,493 relative units, which is in absolute value Qg=1130 kVAR. All above mentioned points indicates that while attempting using "pure" asynchronous generators with cage rotor for the wind-power and small hydro-power plants at the relatively high power range it creates virtually undecidable problem in reactive power compensation at start-up and regular operation, which adversely affects electrical grid functioning, especially at the "weak" points.

Parameters and equations of mathematical models of generators with rated power of $P_{nom.}$=110 kW and $P_{nom.}$=2000 kW are listed in Appendix 1.

Asynchronous generator with squirrel-cage rotor and frequency inverters in generator stator circuit

Above mentioned electromechanical converters are used in relatively high power wind-power plants – with rated power of 1200–3600 kW (e.g. Siemens Wind Power GmbH). In this case frequency inverters are made on the basis of fully controlled IGBTs- transistors with PWM control.

Availability of the frequency inverters on the stator circuit of asynchronous generator rotated by renewable energy sources, allows substantially expand functional capabilities of the electromechanical converters, including reactive power compensation at the start-up mode. Let's consider this on the wind-power plant (WPP) example. As known [4], WPP is equipped with frequency inverters for the efficiency improvement (i.e. energy generation improvement) at the specific range of the wind speed variation. This range is determined by the maximum value of wind power utilization factor. As a rule of thumb, for the big WPP the power transmission into the electrical grid occurs at initial values of the wind speed – 3,5÷4 m/sec. In this case the asynchronous generator speed is minimal (i.e. frequency inverters functions at the lower borderline frequency).If wind speed range is from 6÷7 m/sec to 10÷12 m/sec, the generator rotating frequency is regulated from the minimum to the maximum value. For the wind speed range from 3,5÷4 m/sec to 6÷7 m/sec and from10÷12 m/sec to wind speed calculated value, at which the rated power $V_{calc.}$ (14÷15 m/sec) is generated; the generator operates at the constant speed, with lowest in the first case and with the highest in the second case.

Let's build the mathematical model of above mentioned operation algorithm of asynchronous generator with frequency inverter on stator circuit and choose more preferred frequency inverter control with the reactive power consumption minimization and its compensation point of view.

For this purpose it is preferable to use fixed space axes – d_s, q_s, for equations of wind power unit asynchronous machine [5]:

$$\left.\begin{aligned}
p\Psi_{ds} &= U_s \cdot \cos(\tau) - r_s \cdot i_{ds} \\
p\Psi_{qs} &= -U_s \cdot \sin(\tau) - r_s \cdot i_{qs} \\
p\psi_{dr} &= \psi_{qr} \cdot \omega_r - r_r \cdot i_{dr} \\
p\psi_{qr} &= -\psi_{dr} \cdot \omega_r - r_r \cdot i_{qr} \\
p\omega_r &= \frac{1}{T_j} \cdot m_{EM} - \frac{1}{T_j} \cdot m_{WT} \\
m_{EM} &= \psi_{dr} \cdot i_{qr} - \psi_{qr} \cdot i_{dr} \\
i_{ds} &= k_s \cdot \psi_{ds} - k_m \cdot \psi_{dr} \\
i_{qs} &= k_s \cdot \psi_{qs} - k_m \cdot \psi_{qr} \\
i_{dr} &= k_r \cdot \psi_{dr} - k_m \cdot \psi_{ds} \\
i_{qr} &= k_r \cdot \psi_{qr} - k_m \cdot \psi_{qs}
\end{aligned}\right\} \qquad (1)$$

In these equations: ψ_{ds}, ψ_{qs}, ψ_{dr}, ψ_{qr} are stator and rotor magnetic linkages at the fixed axes d_s and q_s, and i_{ds}, i_{qs}, i_{dr}, i_{qr} are currents accordingly.

$U_{ds} = U_s \cdot \cos(\tau)$; $U_{qs} = -U_s \cdot \sin(\tau)$ are stator voltage components, where U_s is a module of; r_s, r_r– active resistance of asynchronous generator stator and rotor circuits; Tj–inertia constant of asynchronous generator rotor and wind (water) turbine in [rad]; t – time in radian; p –differentiation symbol for time τ; m_{EM} and m_{WT} – electromagnetic moments of wind (water) turbine. Coefficients are determined by the following expressions:

$$k_s = \frac{x_r}{x_s \cdot x_r \cdot x_m^2}; \quad k_r = \frac{x_s}{x_s \cdot x_r \cdot x_m^2}; \quad k_m = \frac{x_m}{x_s \cdot x_r \cdot x_m^2}.$$

This notation allows considering occurrence and control of frequency inverter output parameters, installed in stator circuit of asynchronous generator with cage rotor. In this case voltage components in equations (1) will be equated as:

$$\left.\begin{aligned}
U_{ds} &= k_{us} \cdot \cos(k_f \cdot \tau) \\
U_{qs} &= -k_{us} \cdot \sin(k_f \cdot \tau)
\end{aligned}\right\} \qquad (2),$$

where $k_{us} = \frac{U_s}{U_{s.base}}$ – amplitude component controlled value of stator voltage;

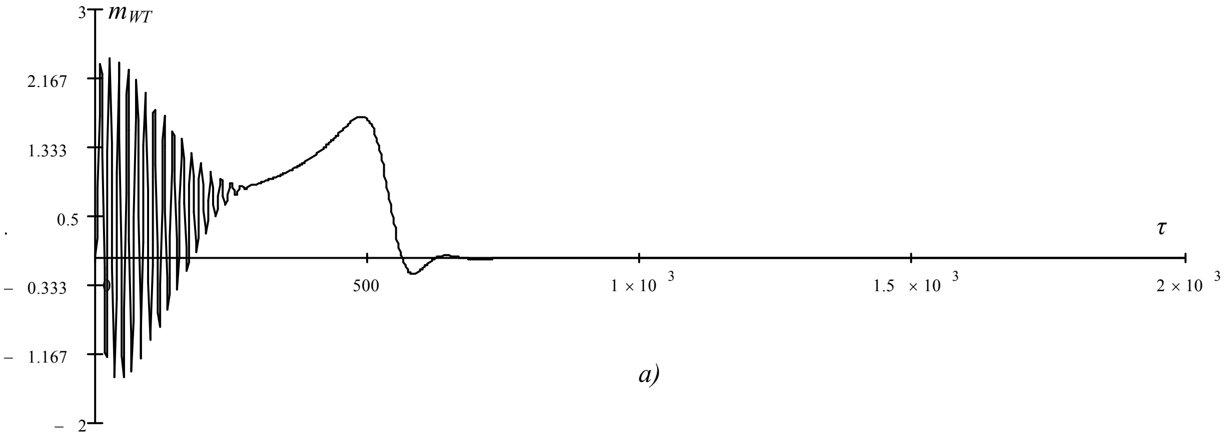
$U_{s.base}$ – reference value of stator voltage; $U_{s.base} = \sqrt{2} \cdot U_{f.n.}$;

$k_f = \frac{f}{f_{base}}$ – frequency component controlled value of stator voltage;

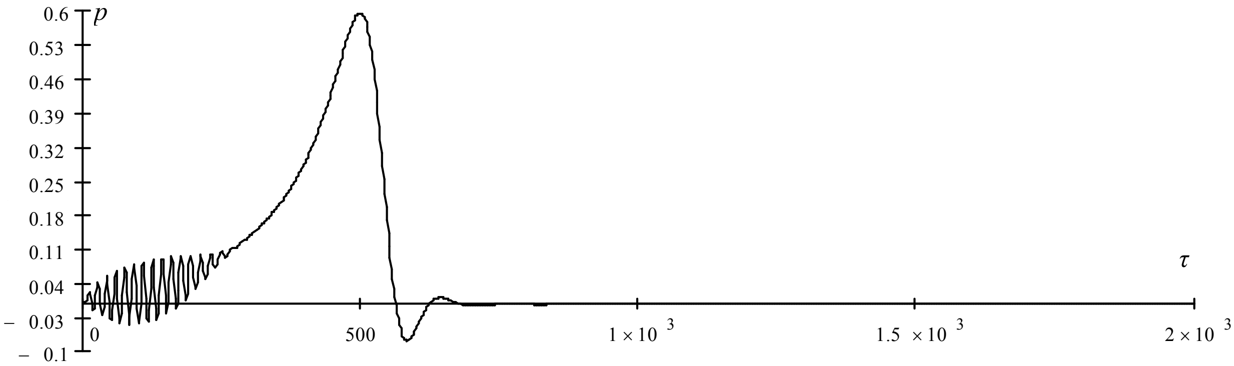
f_{base}=$f_{н}$=50 Hz – reference value of current frequency.

As noticed, frequency start-up is available in the wind-power plants equipped with the frequency inverters. It would appear reasonable that this start-up will be implemented prior to minimum value of wind power plant rotor speed. Let us consider the asynchronous generator with rated power of $P_{nom.}$=2000 kW, which plays electromechanical converter roleat WPP (wind power plant).For example, in Siemens 2,3 MW/93-45 WPP lower rotary speed is ~550 rev. per min., and upper is 1450 rev. per min., in other words lower rotary speed $\omega_r \approx 0{,}4$. Considering that WPP start-up directly at wind speed 3÷4 m/sec and k_{us}=k_{fs}=1, then reactive power value would be q_{max}=4,7 rel. units with duration of $\tau \approx 2180$ rad. (~7 sec), which is surely unallowable.

Fluctogramm of operation conditions change at the start-up in the constant frequency $f = 0{,}4 \cdot f_{ftd}$ is shown in Fig. 2 (*a, b, c, d, e*). Fig. 2.*a* shows the fluctogramm generator electromagnetic torque change in driving moment of the wind-turbine engine at the generator shaft; m_{WT}=–0,01 (that corresponds to the wind speed $V_{nom.} \approx 3{,}5$ m/sec). After ending the transient process, the torque value switches from the positive, that corresponds to the motor mode, to negative value of $m_{EM}=m_{WT}=-0{,}01$, which is generator mode. The WPP rotor speed reaches the value $\omega_r=0{,}40013$, which is indicating that machine is operating in generator mode (slip $S_{0,4}=-0{,}000325$) (Fig. 2,*b*). In Fig. 2 (*c,d*) fluctogramms of change of reactive power q and total power S of the generator is shown, and both have positive sign. The positive sign shows that these powers are consumed from the electrical grid; moreover they slightly differ from each other. It also corresponds to the physics of the process, as steady-state value of active power is scanty $p = m \cdot \omega_r = -0{,}004$. Steady-state value of power factor's $\cos\varphi_{0,4} = \dfrac{p}{S} = 0{,}03$.



a)



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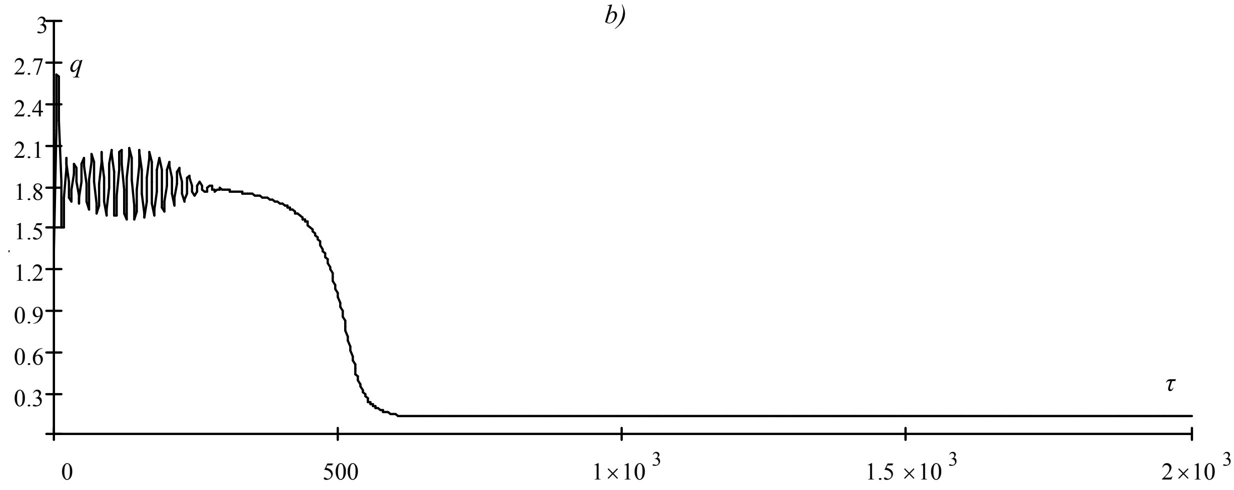
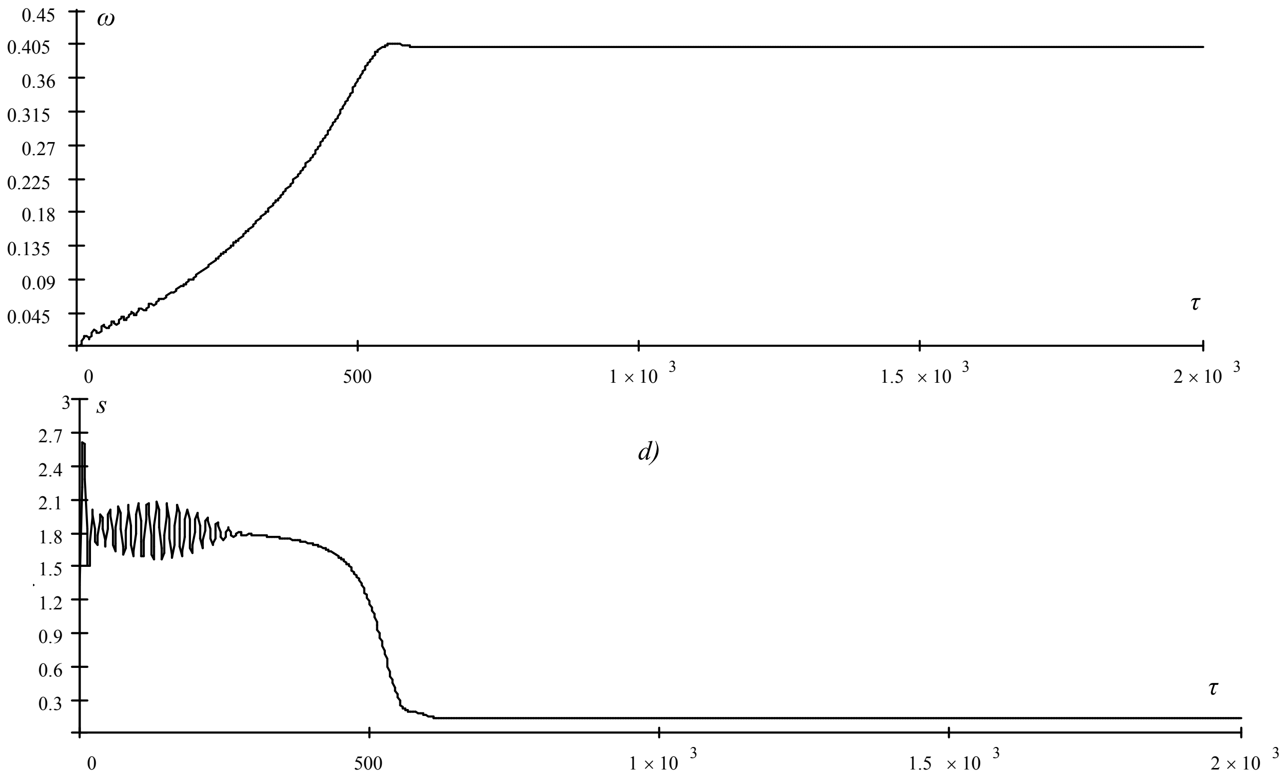


c)



d)

e)

Figure 2.

Analyzing fluctogramm at Fig. 2 following conclusions can be made: Although during the low frequency start-up, the starting reactive power value considerably decreases and its lifetime is (q_{max}=1,9; $t_{cont.}$=520 rad.), the reactive power value remains substantially high. The availability of frequency inverter on the rotor circuit of asynchronous generator gives an opportunity of frequency starting of WPP. For sake of simplicity in implementation, let's assume that, linear frequency start-up is carried out, which means voltage amplitude and frequency applied to generator stator windings are changing as per the following correlation:

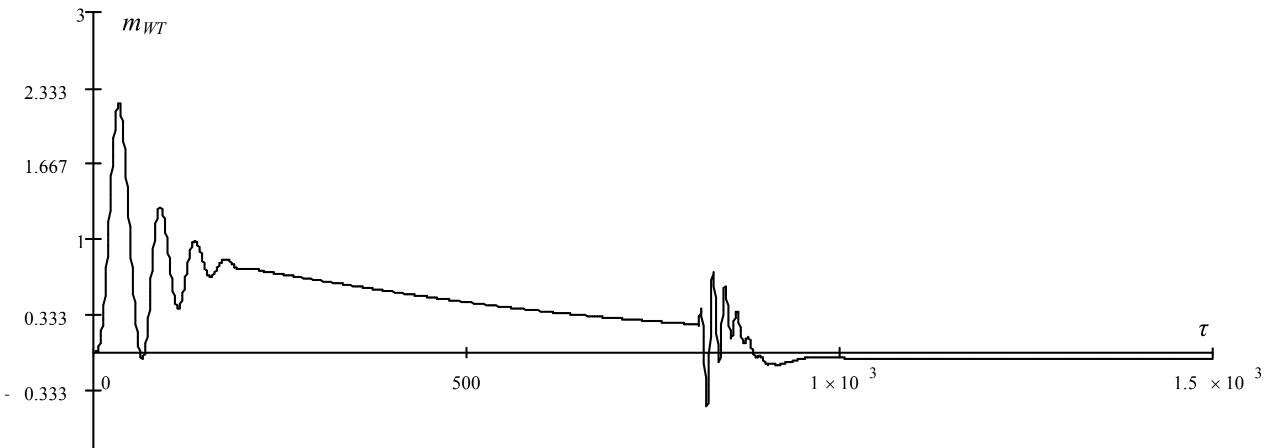
$$k_{us} = k_{fs} = k_0 + k_1 \cdot \tau \qquad (3)$$

Upon that, speed of frequency rise kf must be synchronized with the WPP shaft speed growth rate.

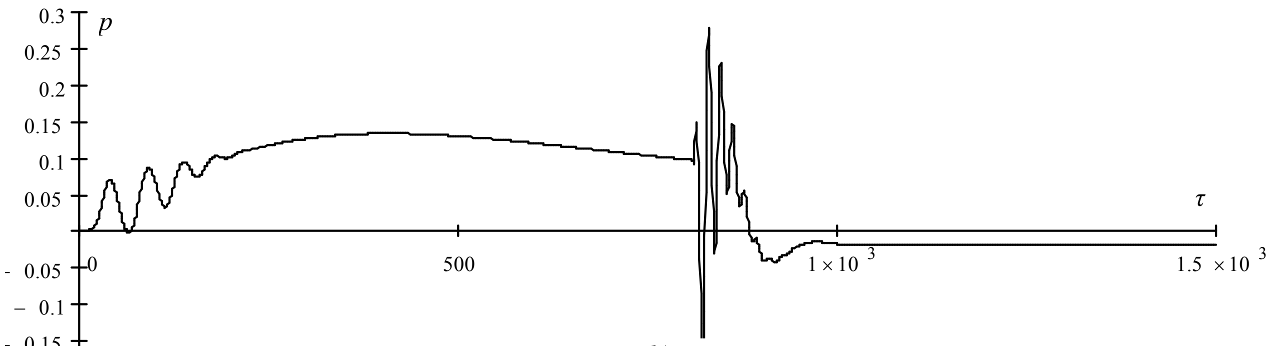
At the Fig. 3 fluctogramm of operating conditions change at the frequency start-up is shown. Amplitude and frequency are changing linearly as per correlation:

$$k_{us} = k_{fs} = 0{,}1 + 0{,}00037 \cdot \tau \qquad (4)$$

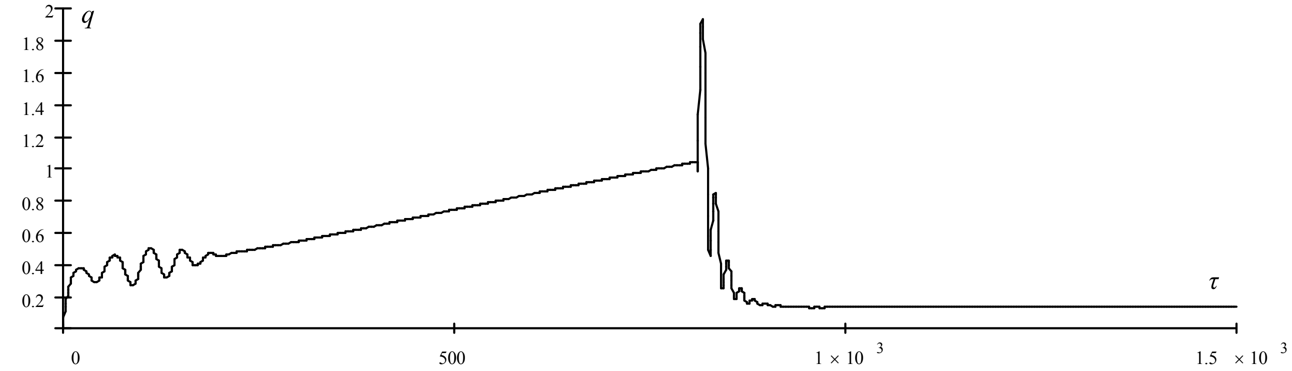
In Fig. 3,*a* electromagnetic torque curve at the m_{WM}=–0,05 is shown. From the curve it can be seen evidently that the average value m_{EM} at the frequency start-up at ω_r=0 does not exceed m_{EM}=1, then with relatively light oscillationstorque curve changes from the motor mode to the generator mode and sets in value m_{EM}=m_{WM}=–0,05. In Fig. 3,*b* fluctogramm of rotor rotation frequency change of WWP at the frequency speedup is shown k_f=k_u=0,4. Rotation frequency is set at n=0,40065. And finally, in Fig. 3,*c* reactive power curve at the frequency start-up is shown. The power is changing in the range of $Q_o \approx 0{,}3$ to $q_{0,4} \approx 0{,}95$, notably average value of reactive power is tangible, and its duration is $\tau_{cont.}$~ 800 rad.


m_{WT}
3
2.333
1.667
1
0.333
- 0.333
– 1
0
500
1 × 10 3
1.5 × 10 3
τ

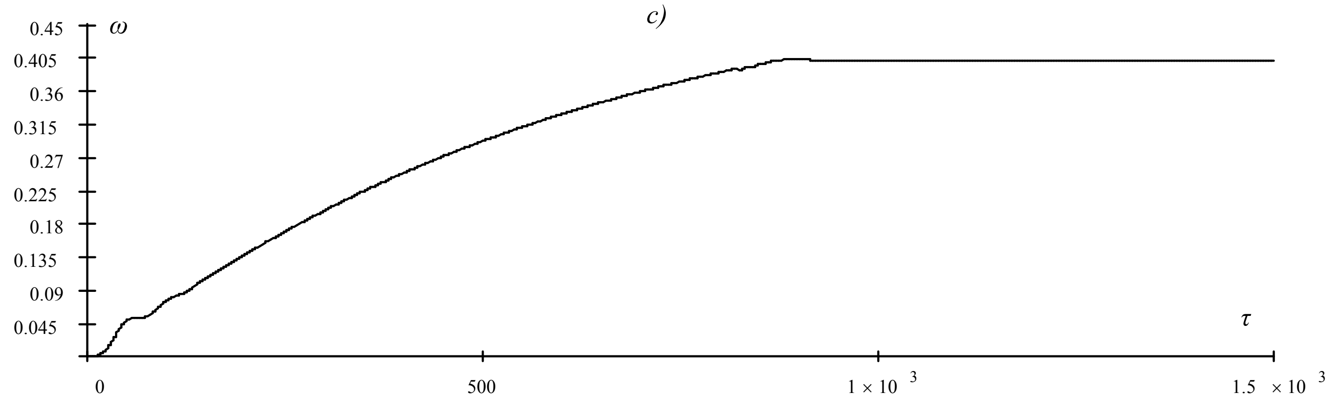
a)


p
0.3
0.25
0.2
0.15
0.1
0.05
- 0.05
– 0.1
- 0.15
– 0.2
0
500
1 × 10 3
1.5 × 10 3
τ

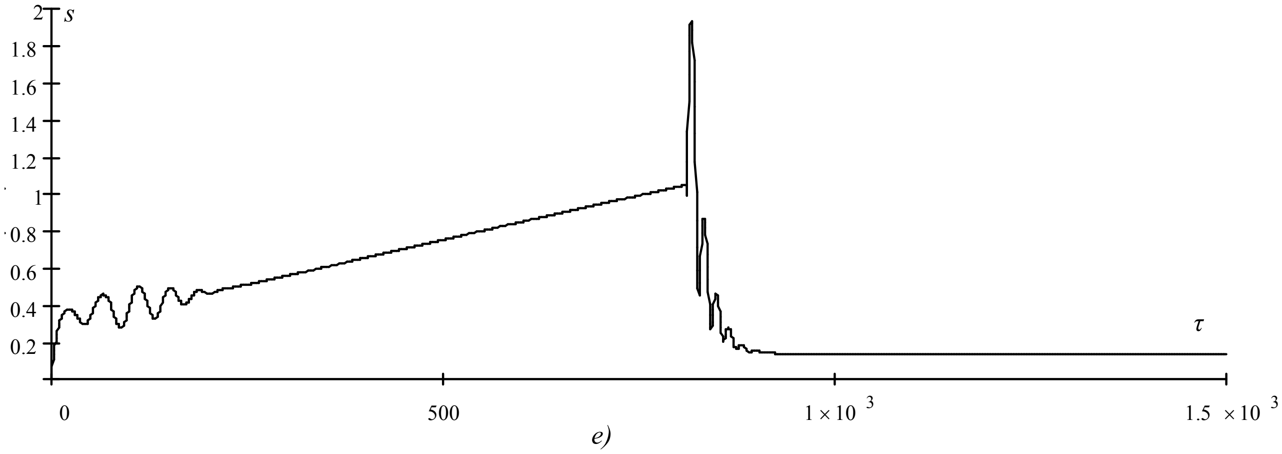
b)


q
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
500
1 × 10 3
1.5 × 10 3
τ

c)


ω
0.45
0.405
0.36
0.315
0.27
0.225
0.18
0.135
0.09
0.045
0
500
1 × 10 3
1.5 × 10 3
τ

d)



e)

Figure 3.

After frequency start-up, let us consider reactive power at the steady state mode at low frequency in rotation frequency control zone in proportion to wind speed and at the frequency maximum value. All results are tabulated in Table 2.

Analyzing results shown in Table 2, the following conclusions can be drawn. At the start-up mode (frequency start-up) average capacity compensating reactive power starting value is $Q_{com.\,strt} = 0{,}62$ in relative units, at the maximum loads this value equals to $Q_{com.\,\max} = 0{,}45$ and at the medium loads this value is $Q_{com\,med} \approx 0{,}3$, and at least at the minimum loads $Q_{com\,\min} \approx 0{,}15$.) Thereby, for the reactive power compensation of WPP with rated power Prtd =2000 kW ideally 4 sets of static capacitor batteries [static compensator], with Qbat~325 kVAR power each, are required. Based on operation mode, controllable set of SCB (statistic capacitor battery) will provide capacitive power from 1300 kVAR at the start-up; 975 kVAR at the maximum load; 650kVAR at the medium loads (duration of which is absolute majority), and 325 kVAR - at the minimum loads. As only 4 stages of SCB are required, then it is possible to limit by the step control through semiconductor (thyristor) switches.

Table 2.

№	$k_u=k_f$	n	m_{EM}	p_{EM}	S	q
1	Frequency start up	from 0 to 0,401	–0,05	–0,02	0,132	0,13
2	0,4	0,402	–0,15	–0,06	0,145	0,132
3	0,4	0,402	–0,16	–0,064	0,147	0,132
4	0,5	0,502	–0,174	–0,087	0,188	0,166
5	0,6	0,603	–0,251	–0,151	0,253	0,203
6	0,7	0,704	–0,342	–0,241	0,343	0,244
7	0,8	0,806	–0,448	–0,361	0,464	0,291
8	0,9	0,907	–0,568	–0,516	0,622	0,348
9	1	1,009	–0,703	–0,709	0,824	0,419
10	1	1,01	–0,75	–0,758	0,872	0,432
11	1	1,011	–0,8	–0,809	0,923	0,446
12	1	1,0116	0,83	–0,839	0,955	0,455
13	1	1,012	–0,85	0,86	0,976	0,461

As a matter of practice in compensating device power and amount selections following factors are considered: harmonics, compensability of the grid and etc.

Induction generator with phase-wound rotor and frequency converter at the rotor circuit (double-feed induction machine)

Above mentioned generator is used as an electromechanical converter especially in wind-power plant (70-80% of all WPP field), but over the last years it tends to be used as a hydroelectric generator in small hydroelectric power plants.

Double-feed induction machine with frequency inverter at the rotor circuit allows to regulate rotor rotation frequency of the WPP in the range of ±25÷30% by the insertion of e.m.f. (electromotive force) into rotor winding with slip frequency [6].

It is easy to write and solve generator differential equations on the axes rotating at the generator rotor speed. This allows to simulate rotor voltage amplitude and frequency [6] control mode relatively simply:

$$\left.\begin{aligned}
p\Psi_{ds} &= -U_s \cdot \sin(\theta) + \psi_{qs} - \psi_{qs} \cdot s - r_s \cdot i_{ds} \\
p\Psi_{qs} &= U_s \cdot \cos(\theta) - \psi_{ds} + \psi_{ds} \cdot s - r_s \cdot i_{qs} \\
p\psi_{dr} &= -U_r \cdot k_{ur} \cdot \sin(k_{fr} \cdot \tau) - r_r \cdot i_{dr} \\
p\psi_{qr} &= U_r \cdot k_{ur} \cdot \cos(k_{fr} \cdot \tau) - r_r \cdot i_{qr} \\
ps &= \frac{1}{T_j} \cdot m_{\text{WP}} - \frac{1}{T_j} \cdot m_{EM} \\
p\theta &= s \\
m_{EM} &= \psi_{\text{ds}} \cdot i_{qs} - \psi_{qs} \cdot i_{ds} \\
i_{ds} &= k_s \cdot \psi_{ds} - k_m \cdot \psi_{dr} \\
i_{qs} &= k_s \cdot \psi_{qs} - k_m \cdot \psi_{qr} \\
i_{dr} &= k_r \cdot \psi_{dr} - k_m \cdot \psi_{ds} \\
i_{qr} &= k_r \cdot \psi_{qr} - k_m \cdot \psi_{qs}
\end{aligned}\right\} \quad (5)$$

Here, k_{ur} and k_{fr} are factors considering rotor voltage amplitude and frequency control.

As mentioned before, in these generators rotor speed is controlled based on wind speed up and down from synchronous speed not more than 25÷30% (from 0,75÷0,7 to 1,25÷1,3 in relative units). The rated power of frequency inverter is not more than 30% of the generator rated power, and this gives comparative advantage to the WPP equipped with double-feed asynchronous generator over the WPP equipped with squirrel-cage asynchronous generator with frequency converter at the stator circuit. In case of asynchronous generator with cage rotor the rated power of the frequency inverter must not be less than the rated power of the generator.

If these WPP perform direct starting, then in the equation (5) rotor voltage component U_r must be equated to zero.

$$\left.\begin{aligned}
p\psi_{dr} &= 0 - r_r \cdot i_{dr} \\
p\psi_{qr} &= 0 - r_r \cdot i_{qr}
\end{aligned}\right\} \quad (6)$$

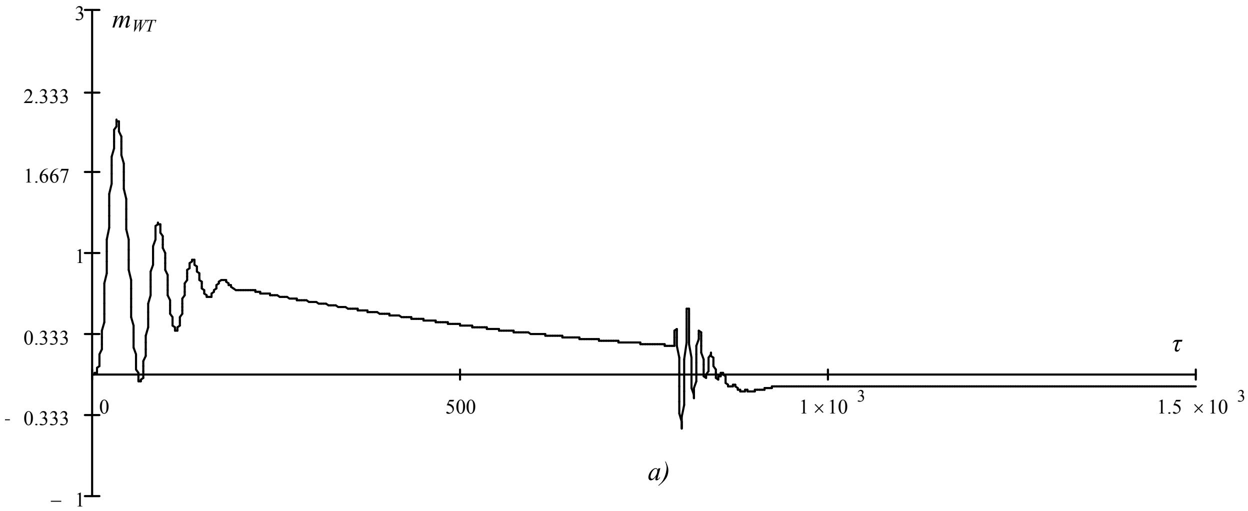
But in this case start-up value of reactive power reaches up to 5-6 multiple values, and its duration depends on plant flyweight.

Therefore, direct connection of WPP with double-fed induction machine to “weak” electric grids is undesirable.

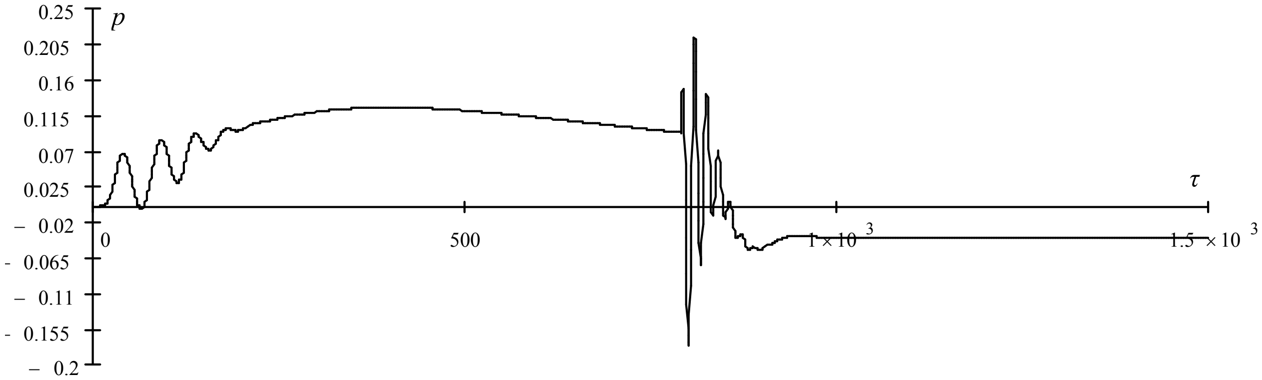
Frequency inverter on the rotor circuit enables frequency start-up, if inverter power range makes it possible.) However, it is important to note that, frequency start-up is efficient at the frequency variation from (0÷0,1) to (0,3÷0,4) units of rated value. As the lower control limit of these generators is in the range of 0,75÷0,7 relative units, and the upper control limit is in range of

1,25÷1,3 relative units, so after frequency start-up it is important to get frequency of 0,75÷0,7 rel. unit. At the Fig. 4 fluctogram of generator operating conditions change in frequency start-up at the rotor voltage amplitude and frequency changing as per following expression is shown:

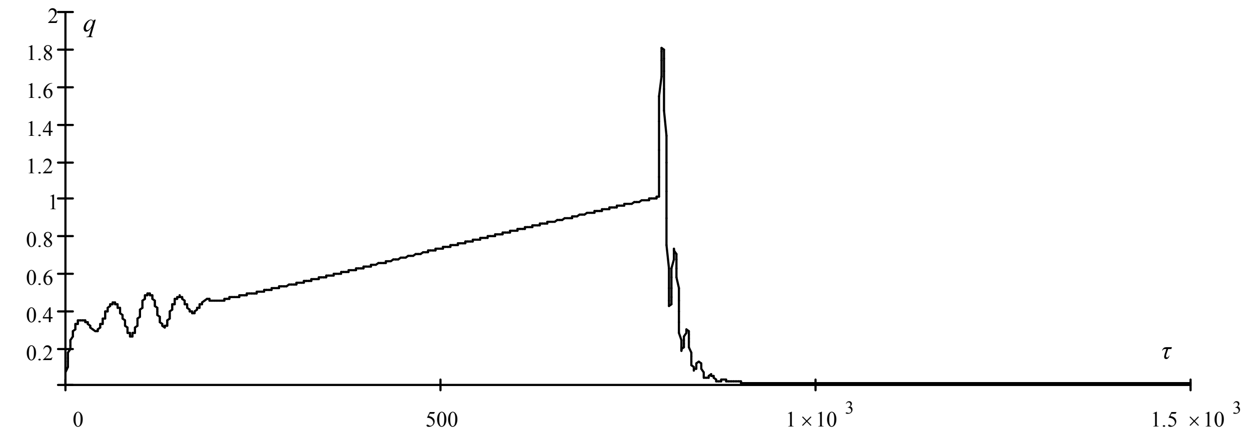
$$k_{ur} = k_{fr} = 0{,}1 + 0{,}00038 \cdot \tau \qquad (7)$$



a)

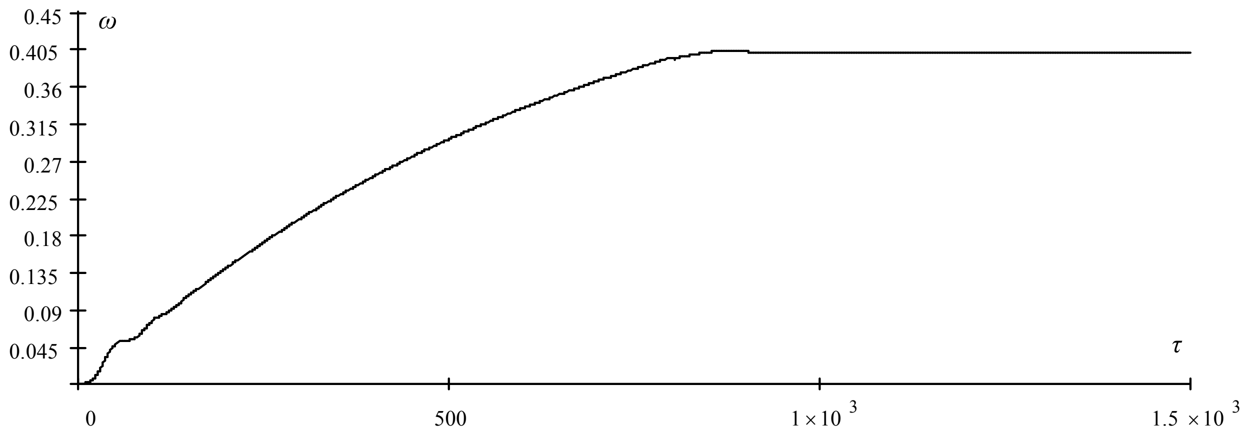


b)



c)

τ



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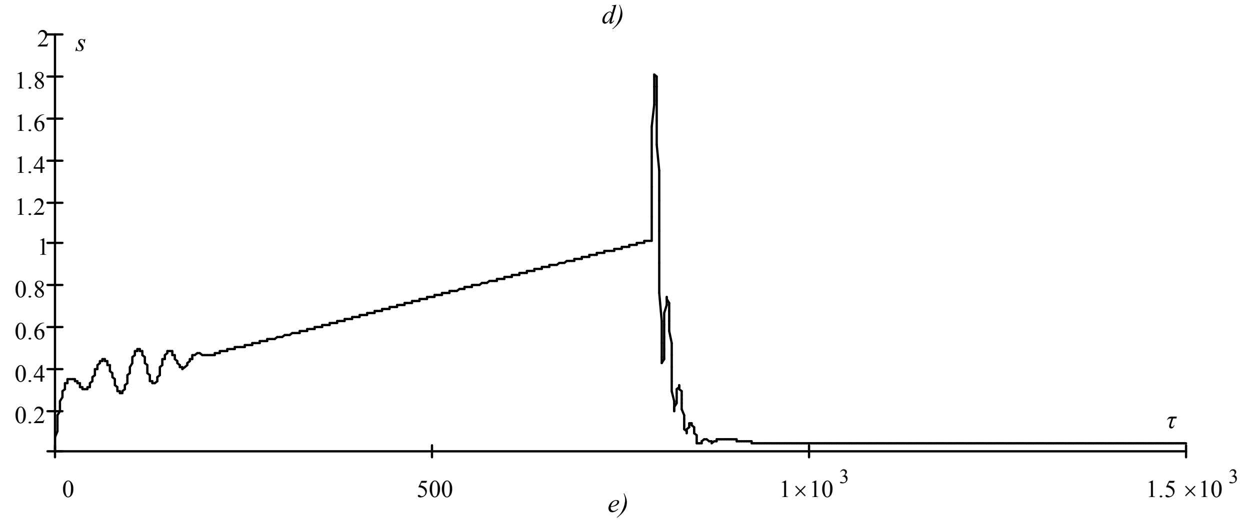

e)

Figure .4

Electromagnetic torque m_{EM}, reactive power q and WPP rotation frequency shown in Fig.4 (*a*, *b*, *c*) practically same with appropriate curves at the current frequency changing from the part of asynchronous generator stator.

It is well known that WPP with double-fed asynchronous generator allows rotation frequency control in proportion to wind speed at the range of 25-30%. Upon this power factor changes from 0,9 (inductive) to 0,9 (capacitive). When reactive power is consumed from electrical grid, inductive power factor cosφ=0,9 accompanied by light load, corresponding to low shaft rotation frequencies of WPP. Average value of consumed reactive power in this operating mode is qconsm≈0,5. Based upon frequency start-up, if assumed that the compensating capacitor battery power is Q_{cap}=0,5 (approx. Q_{cap}=1150 kVAR in absolute units in base power P=2300 kW), in this case from reactive power compensation viewpoint at the start-up and before compensation operating mode in inductive section/range, it is possible in some way to minimize negative influence of WPP to the electrical grids.)

CONCLUSIONS

1. As a solution of reactive power compensation in electrical grids, to which WPP with induction generators is connected, at start-up and steady-state operating conditions it is reasonable to use complete mathematical model of induction machines.

2. Using induction generators with cage rotor as an electromechanical converter in WPP (wind power plant) and small HPP (hydroelectric power plant), reactive power at the start-up mode is within the range of 3,5—4,5 relative units (larger values for high power). At the steady-state modes, the reactive power from no-load to rated load operation is changing in range of 50÷60%, that states on substantive dependence of consumed reactive power from load.

3. Having frequency converter in stator or rotor circuit of cage and phase-wound rotor induction generator it is important to carry out frequency start-up. At the amplitude and frequency changing as per linear law, it is possible considerably reduce average value of reactive power from 3,5-4,5 relative units to 0,5-0,6 relative units. In this case the calculated reactive power value of static capacitor batteries used for the compensation will be in reasonable limits.

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APPENDIX

a) Characteristics of small induction power generator on the base of AC motor model of 4AH 250M4:

P=110 kW	cosφ=0,908	J=0,968 kg·m2
$U_{н.ф}$=220 V	R_1=0,03 Ohm	$U_{base} = \sqrt{2} \cdot U_{н.ф} = 310$ V
$I_{н.ф}$=195,5 A	$R_2^{'} = 0{,}0172$ Ohm	$I_{base} = \sqrt{2} \cdot I_{н.ф} = 277{,}85$A
m=3	x_1=0,19 Ohm	$Z_{base} = \frac{U_{base}}{I_{base}} = 1{,}125$ Ohm
2p=4	x_0=4,83 Ohm	$P_{base} = \frac{3}{2} U_{base} \cdot I_{base} = 129{,}2$ kW
$S_н$=0,015		

b) Characteristics of asynchronous generator on base of double fed induction motor model of AKN 2-53-12. MUKH L4:

P=2000 kW	$в_н$=2,5	R_1=0,17Ohm	$x_0 = 44{,}59$ Ohm
$U_{н.ф}$=3468V	cosφ=0,86	$R_2^{'} = 0{,}18$ Ohm	$Z_{base} = \frac{U_{base}}{I_{base}}$=14,758 Ohm
$I_{н.ф}$=235A	$\eta_н$=0,951	$x_1 = 1{,}55$ Ohm	$U_{base} = \sqrt{2} \cdot U_{н.ф}$
$M_н$=38,3 kNm	$U_{2н}$=1045	$x_2^{'} = 1{,}64$ Ohm	$I_{base} = \sqrt{2} \cdot I_{н.ф}$
n_{rtd} =500 rpm			

MULTI PHASE RELIABILITY ANALYSIS WITH VARIABLE PHASE DURATION AND VARIABLE PHASE SEQUENCE

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ABSTRACT

This paper proposes a new simulation model for evaluating reliability of phased mission systems with variable phase durations and variable sequencing. The model simulates the system failures within target operating period using a modularized phased mission system model. Monte Carlo method has been used to simulate the reliability of phased mission and mission phases. A case study on electronic lighting control system is given to demonstrate the proposed simulation model. The effects of assuming constant duration and constant sequencing in the mission have been discussed during the case study.

1. Introduction

Many systems work in mission which can be divided into several phases. In different phases, the system configurations and the requirements of the system may be different. For example, the voyage of an aircraft can be divided into several tasks, such as take-off, cruise, and landing, each with completely different reliability requirements and behaviour. Usually each task can be treated as a phase of the system. These kinds of systems are called as multi phase systems. If the system successfully operates throughout all of the phases then the mission said to be a success. Phased mission systems are encountered in many industrial fields, such as nuclear, aerospace, chemical, electronic, navigation, and military fields, etc.

Compared with single-phase systems, reliability analysis of multi phase system is much more complex, because of the dependence across the phases. For instance, the state of a component at the beginning of a new phase is identical to its state at the end of the previous phase [1]. The dynamic structure and configuration of a multi phase system usually requires a distinct model for each phase, which also increases the complexity of modelling & analysis. Although reliability analysis of multi phase systems has been studied for more than 20 years, the size of problems that can be solved is still very small due to the high computational complexity of the known methods [2].

There are two classes of models applied in reliability evaluation for phased mission system: combinatorial and state-based.

The most widely used combinatorial models are Fault tree (FT) and Reliability block diagram (RBD). The combinational models are able to describe the mapping relation of model elements and system faults. FT and RBD can be conveniently constructed with classical qualitative analysis method, such as FMEA. However, the combinational models exhibit a lot of drawbacks and limitations considering the dependency relation of components, the maintenance of broken-down components. The assumption of combinatorial models for multi phase analysis is that all the states of all components in the system are s-independent [2]. This assumption can simplify the analysis, however it limits the applicability of the models because s-dependence within as well as across the phases does occur in some cases. When there is no s-dependence within a phase, the s-dependence across the phases still needs to be accounted. Esary Introduced a method which can deal with the s-dependence across the phases using a set of s-independent mini-components to replace the component in each phase [1]. However, this method causes the size of the problem to become very large as the number of phases increases. As the number of components and phases becomes large, the number of disjoint products also increases rapidly, which consequently increases the amount of storage and computation-time. The computational complexity of combinatorial models is much less than that of state based models. However, these models usually need to find mincuts of systems, and to calculate the sum of disjoint products, which is still computationally intensive.

State-based model are more feasible to formalize phased mission. The state-based models such as Markov Chains and Petri Net can explain the complex dependency exist in the components. The main idea of Markov-chain based models is to, directly or indirectly construct a Markov chain to represent the system behaviour. These models at once account for the dependence among the components within a phase and the dependence across the phases. However, Markov-chain based models suffer from the state explosion problem when the number of components becomes large.

In recent times, Amari [3] presented a methodology for analytical analysis of phased missions which is based on the solution of cumulative exposure model for k-out-of-n systems. Chew [4] described the use of a Petri net to model the reliability of the maintenance-free operating period (MFOP) and phased mission scenario.

Multi phase systems sometimes consist phases which may not be characterized as sequential, fixed duration or both. We call these systems as multi phase system with variable duration and variable sequencing. A typical example of a phased mission with variable duration and variable sequencing is job shop where the sequence of machines used and duration of machines used is not fixed. Usually these systems are analysed with an assumption of constant sequencing and constant duration. This may lead to over estimate or under estimate the reliability characteristics of the system.

Most of the analytical approaches discussed earlier will become almost impossible to apply when the system is following variable sequence of phases with variable phase durations. Because of this reason Simulation based approach has been selected for analysis of multi phase systems with variable sequence of phases and variable phase durations. It allows the modelling of any reliability distribution without particular restrictions. This paper presents a simulation model for evaluating phased mission system reliability with variable duration variable sequence of phases. The simulation based model focuses on the logic rather than mathematical relationship, which makes it perfectly suitable for analysis of variable sequence, variable duration phased mission systems.

Section 2 describes the problem statement and presents a methodology for user profile data collection and analysis. Section 3 describes the simulation algorithm. Section 4 explains the proposed algorithm with the help of a case study.

2. Problem statement

The phased mission systems studied in this paper are non repairable phased mission system, and at the beginning of mission all the components in the system are as good as new. All the components are having only two states either success or failure. System is also having only two states either success or failure. All the component failures are independent of each other.

The basic method proposed in this paper assumes that durations of all phases are variable and the sequence in which the phase's occur is also not fixed. So from mission to mission the sequence of operation of phases and duration of phases may change. In any particular mission if the system is successfully able to complete the required phase sequence for required phase durations, the mission can be treated as success. Missions will be continuously performed one after one till the completion of target time period. The objective of the model is to calculate the probability of completion of all the missions within the target time period with out a single failure.

The Phase durations are assumed to be following a weibull distribution. The patterns of the phase's durations and phase sequences have to be studied based on the system usage data. Mathematically modelled phase durations and sequences will be used to simulate the system behaviour.

2.2. User profile data collection and analysis

User profile captures the way product will be used by the end user. If a product is having "n" different functions or phases of operation, the user may not be using all the "n" phases at a time or in a particular sequence. There are two parameters which decide the usage profile of the product. First, the sequence of phases in which the product is used. Second, the duration of each phase. For example if a system has three phases A,B,C then the some of possible sequences in a mission will be ABC, BCA,CAB,AB,BC,CA,A,B,C etc. By assuming that these sequences are mutually exclusive the probability of occurrence can be calculated. For this the data has to be collected over certain number of missions regarding the occurrence of various phase sequences. The duration of each individual phase may also follow a particular pattern which has to be modelled using the duration data of various phases.

In the subsequent case, the data is collected as shown in Table 1 for calculating the probability of occurrences of various sequences. For example sequence ABC has been occurred S1 times out of S number of observed missions. So the probability of occurrence of ABC sequence is the ratio of S1 to S. similarly other sequence probabilities also calculated. CP is the cumulative probability of sequences. For example CP2 is the sum of CP1 and P2. Similarly remaining cumulative probabilities are calculated. Here in the Table 1 only certain number of possible sequences is shown. There can be many possible sequences in a mission. Sequences with negligible possibility of occurrence should be avoided to reduce the unnecessary complexity in analysis.

Table 1: The probability of occurrences of various sequences

S.No (i)	Sequence	Si	Pi $(S = \sum Si)$	$CP_i = \sum_1^i P_x$
1	ABC	S1	S1 / S	CP1
2	BCA	S2	S2 / S	CP2
3	CAB	S3	S3 / S	CP3
4	AB	S4	S4 / S	CP4
5	BC	S5	S5 / S	CP5
6	CA	S6	S6 / S	CP6
7	A	S7	S7 / S	CP7
8	B	S8	S8 / S	CP8
9	C	S9	S9 / S	1

Si - No of times i^{th} sequence occurred during observation period
Pi - Probability of occurrence of sequence
CPi - Cumulative probability of occurrence

The system should be monitored for certain number of missions to collect the data regarding the phase duration. For modelling the duration of various phases should be collected as shown in the Table 2.

Table 2: Data for modeling duration of phases

Mission No. (i)	A phase duration	B phase duration	C phase duration
1	t_{A1}	t_{B1}	t_{C1}
2	t_{A2}	t_{B2}	t_{C2}
3	t_{A3}	t_{B3}	t_{C3}
4	t_{A4}	t_{B4}	-
5	-	t_{B5}	t_{C5}
6	t_{A6}	-	t_{C6}
7	t_{A7}	-	-
8	-	t_{B8}	-
9	-	-	t_{C9}

Using the collected data the respective duration distributions of various phases should be determined. The parameters as shown in Table 3 should be calculated by assuming Weibull distribution as suitable distribution. For example using the data points t_{A1}, t_{A2}, t_{A3}, t_{A4}, t_{A5}, t_{A6}, t_{A7} the parameters of phase A duration distribution $\eta 1$, $\beta 1$ should be calculated. Similarly other phase's distribution parameters will be calculated.

Table 3: Parameters of modeling duration of phases

Phase Name	Eta	Beta
Phase A	η1	β1
Phase B	η2	β2
Phase C	η3	β3

3. Simulation algorithm for variable phase sequence and variable phase duration

Five modules constitute the Simulation-Based Evaluation Model of Phased-Mission System with variable phase sequence and variable phase duration. They are reliability analysis module, mission simulation module, Phase sequence simulation module, Phase duration simulation module, phase simulation module. If all the required phases are success in a mission then mission will be treated as success. Missions will be simulated one after one until total accumulated age of system is equal to the target operating life. If required operating life is successfully completed then the corresponding simulation run will be treated as success. The ratio of number of successful simulations to total number of simulation runs gives the Reliability of the product at the end of target operating hours.

Reliability analysis module calculates the probability of the system for completing target operating hours by running the preset number of simulations. It calls for the *mission simulation module* in each simulation run. It stores the out come of *mission simulation module*. The procedure will be repeated until all the simulation runs are completed. Probability of completing the target time period is the ratio of number of success outcomes of *mission simulation module* to the total number of simulation runs. Figure 1 shows the flow chart of reliability analysis module.

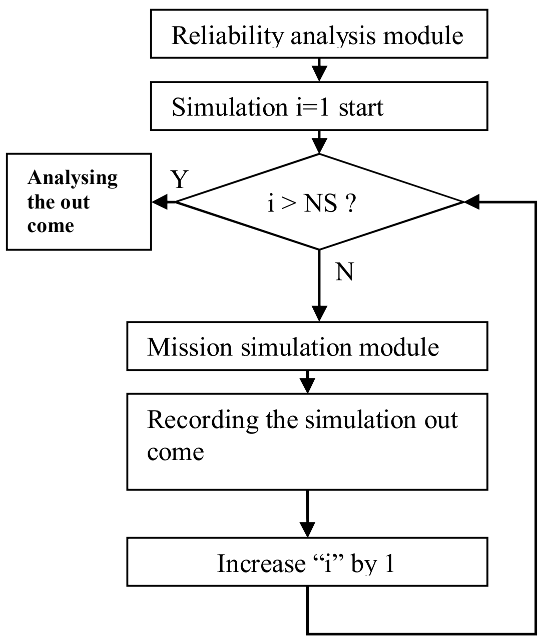

Figure 1: Flow chart of the Reliability analysis module

NS : Number of simulations

Mission simulation module returns out come of individual simulation run. This module simulates missions in loop until target operating hours are completed. For this it calls *phase sequence simulation module,* which generates a random phase sequence. Then for all the phases of the generated sequence, random duration will be generated in loop using *phase duration simulation module.* The generated two parameters phase sequence and phase duration will be passed to *phase simulation module.* This module will calculate the status of the system at the end of each phase of the sequence in loop. If outcome is success in all the phases of generated sequence then mission will be treated as success. Then it Stores the outcome of each mission and total accumulated age of the system. This process is repeated till accumulated age of the system is equal to the target operating hours. If outcome is success in all the simulated missions then success will be returned from *mission simulation module.* Flow chart of mission simulation module is as shown in Figure 2.

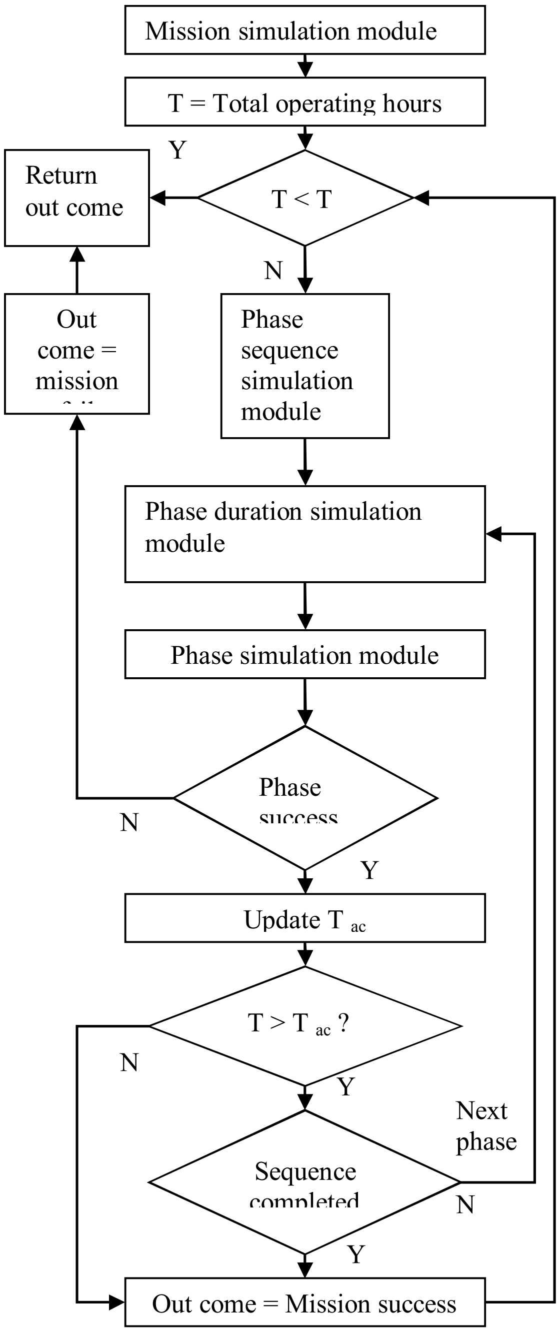

Figure 2: Flow chart of the mission simulation module

T_{ac}: Total accumulated age of system

Phase sequence simulation module function is to generate a random sequence of phases based on probabilities calculated in the usage profile analysis. For this a random number between "0" to "1" will be generated. Then it is compared with the cumulative probabilities of the sequences to select any particular sequence. For example, as per Table 1 if generated random number lies between 0 to CP1 then ABC sequence will be selected. If it lies between CP1 to CP2 then BCA will be selected. Then this randomly selected sequence is returned by *Phase sequence simulation module.*

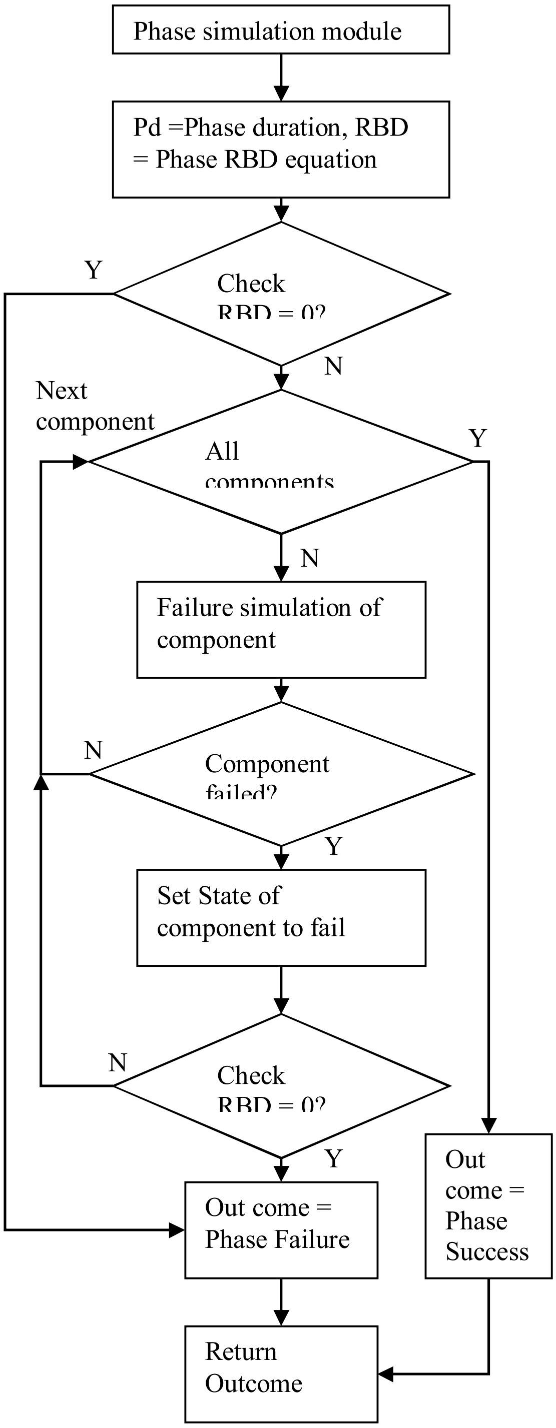

Figure 3: Flow chart phase simulation module

Phase duration simulation module generates duration of phases based on phase duration distribution parameters calculated in usage profile analysis. For this a random number ‘R’ should be generated between “0” to “1”. By assuming Weibull distribution parameters as discussed in Table 3, phase1 duration will be calculated as follows

$$\text{Phase 1 duration} = \eta 1 * ((\ln (1/R)) \wedge (1/ \beta 1))$$

In this way required phase duration is randomly generated and returned by *Phase duration simulation module.* Then this generated phase duration will be passed to *phase simulation module*. This process is repeated till all the phases in the generated phase sequence are completed.

Phase simulation module returns status of the system at the end of called phase. It takes phase duration generated by *Phase duration simulation module* and phase sequence generated by *Phase sequence simulation module* as input. Phase simulation module simulates a phase by using classic motecarlo simulation technique. But while deciding the status of the components state dependencies across the phases have to be considered. For example, a redundant component failure may not cause the system failure in a phase but same may lead to system failure as soon as it enters into next phase because of changes in the system configuration. Ref [5], [6] can be referred for simulation of general RBD, but as discussed they has to be carefully integrated with phase dependencies. Flow chart for *phase simulation module* is as shown in Figure 3.

4. Case study

A consumer electronic "lighting controller system" having three phases A, B, C has been considered for analysis. "Lighting control system" is an electronic lighting management system which is used for controlling various lighting effects in interiors of buildings like conference rooms, office areas etc. The system composed of five circuit cards C1, C2, C3, C4 and C5. Different configuration of circuit cards will provide various lighting effect inside the interior. For study purpose three effects class 1(welcome lighting effect), class 2 (meeting lighting effect), class 3 (lecture lighting effect) are considered. For realistic calculation of reliability multi phase modelling and analysis has to be carried out by considering each class as one phase. During phase A (class 1) system will be working in k-out-of-n configuration. C1, C5 and any 2 out 3 among C2, C3, C4 has to work. Under phase B (class 2), C1, C2, C3, C4 has to work for the system functioning. Phase C (class 3) requires all cards functioning. In each phase the configuration of the system RBD is as shown in Figure 4 to Figure 6.

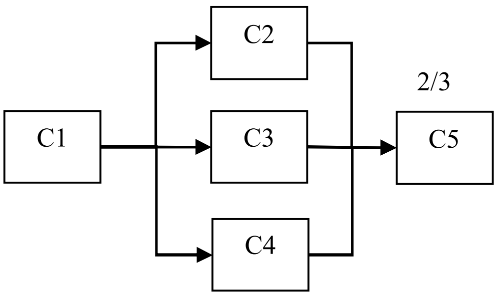

Figure 4: RBD of Phase A

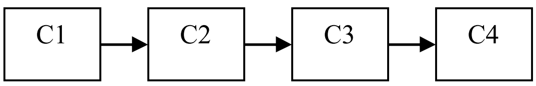

Figure 5: RBD of Phase B

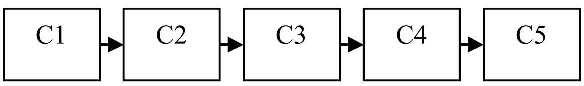

Figure 6: RBD of Phase C

In this kind of systems the user can use the system in any sequence of phases for any duration to fulfil his requirements. If the mission is defined as operation of the product in one ON to OFF, then the mission may consists any sequence of phases and for any phase duration. So the system is assumed to be following varying sequence and varying duration. If the mission is defined as operation of the product in one ON to OFF, then the mission may consists any sequence of phases and for any phase duration. So one particular customer base usage patterns are observed for certain number of missions and corresponding data has been analyzed to calculate the probability of occurrence of various sequences. Data for durations of phases A, B, C in various missions also have been collected which helps to determine the distribution parameters of the phase durations. A, B, C Phase durations distributions are calculated as shown in Table 4. The failure distributions of C1, C2, C3, C4 and C5 are as discussed in Table 5. Phase sequence patterns are having the probabilities as shown in Table 6.

Table 4: Phase Duration distribution

Duration distribution	**Eta**	**Beta**
Phase A	1.08	2.6
Phase B	2.66	3.5
Phase C	3.05	4.8

Table 5: Circuit failure distribution

S. No	**Circuit card**	**Constant Failure rate (FPMH)**
1	C1	0.83
2	C2	5.7634
3	C3	8.942
4	C4	6.97
5	C5	9.7024

Table 6: Phase sequence probabilities

S.No (i)	**Sequence**	**Si**	**Pi (S = ∑Si)**	**$CP_i = \sum_1^i P_x$**
1	A	5	5/ 60 = 0.08333	0.083333
2	B	10	10/ 60 = 0.16667	0.25
3	C	5	5/60 = 0.08333	0.333333
4	AB	5	5/60 = 0.08333	0.416667
5	BC	5	5/60 = 0.08333	0.5
6	ABC	30	30/60 = 0.5	1

The algorithm discussed in section 3 has been utilized to analyze the lighting management system. After running the simulation for 10000 runs the reliability of the system for life of 1000 hours has been calculated as 0.9739. Unsuccessful rates of each phase can also be estimated with the same algorithm by further analyzing the simulation out come.

The same system has been analyzed using Blocksim8 software with an assumption of constant phase duration and constant phase sequencing. The system has been modeled as shown in the Figure 7 the duration of each phase has been taken as the mean value of the phase duration distributions as mentions in Table 4.

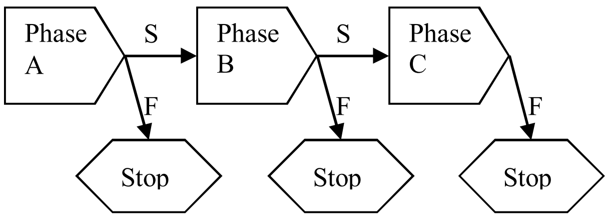

Figure 7: Phase diagram by assuming constant sequence

The BlockSim8 analysis estimates a reliability figure of 0.972 which is less comparing with the reliability calculated using the proposed algorithm. So In this particular example, assuming constant duration and constant sequencing may lead to wrong estimates of reliability. This may call unnecessary design improvements. The suggested algorithm can also be used for constant sequencing and constant duration of phases by eliminating the phase duration simulation module, phase sequence simulation module. Then it simulates the reliability similar to Blocksim8.

5. Conclusion

In this paper, a new simulation model for evaluating reliability of phased mission systems with variable phase duration and variable sequence is proposed. This model is modular in nature constitutive of reliability analysis module, mission simulation module, Phase sequence simulation module, Phase duration simulation module, phase simulation module. A methodology to model the usage profile data is also discussed in detail. The case study in section 4 demonstrates the usage of the model.

6. Acknowledgement

The authors would like to express their gratitude to the management of Crompton Greaves Ltd., for providing the necessary authorization and opportunity to present this paper.

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SEQUENTIAL ALGORITHMS OF GRAPH NODES FACTORIZATION

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In this paper algorithms of a factorization of graph nodes which are used in a processing of data connected with different extreme situations are constructed. We assume that information about a new node and incident edges arrives at each step sequentially. In non oriented graph we suppose that two nodes are equivalent if there is a way between them. In oriented graph we assume that two nodes are equivalent if there is a cycle which includes these nodes. In this paper algorithms of such factorization are constructed. These algorithms need $O(n^2)$ arithmetic operations where n is a number of graph nodes.

1. Factorization of nodes in non oriented graph

Two nodes of a non oriented graph belong to a same connectivity component (are equivalent [1,§3]), if in the graph there is a way which connects these nodes.

1. On the step 1 we put that there is the single node 1 and the number of connectivity components equals $q=1$ and the connectivity component $K_1=\{1\}$.
2. Assume that on the step n there are nodes $1,...,n;$ and $q \le n$ connectivity components $K_1,...,K_q$

$$K_i \cap K_j = \varnothing,\ i \ne j,\ \bigcup_{i=1}^{q} K_i = \{1,...,n\}.$$

3. On the step $n+1$ we receive an information about $a_j, 1 \le j \le n$: $a_j = 1$, if the nodes $n+1, j$ are connected by (non oriented) edge, in opposite case $a_j = 0$. Calculate $c_j = \vee_{j\in K_i} a_j$, $1 \le i \le q$, using n arithmetic operations.
4. Define the indexes set $I=\{i : c_i = 1\} = \{i_0, i_1,...,i_r\}$, $1 < i_0 < i_1 < ... < i_r \le q$, using no more than n arithmetic operations.
5. From the sequence $\{i_0+1,...,i_1-1,i_1+1,...,i_2-1,i_2+1,...,i_r-1,i_r+1,...,q\}$ with $q-i_0$ integers remove the numbers $i_1,...,i_r$ and transform it into the sequence with $q-i_0-r$ integers $\{i_0+1,...,i_1-1,i_1+1,...,i_2-1,i_2+1,...,i_r-1,i_r+1,...,q\} = \{l(i_0+1),...,l(q-r)\}$ using no more than n arithmetic operations.
6. Assume that $I \notin \varnothing$ then put $K_i := K_i$, $1 \le i \le i_0$, $K_0 = \{n+1\} \cup \left[\bigcup_{i\in I} K_i\right]$, $K_i := K_{l(i)}$, $i_0+1 \le i \le q-r$, $q := q-r$, using no more than n arithmetic operations.

If $I \in \varnothing$ then put $K_{q+1} := \{n+1\}$, $q := q+1$ using 2 arithmetic operations. Consequently the transformation of the connectivity components set on the step $n+1$ needs no more then $4n$ aarithmetic operations for the non oriented graph. So the calculation complexity of suggested algorithm on n steps is no more then $2n^2$ aarithmetic operations.

2. Factorization of nodes in oriented graph

Say that two nodes of an oriented graph are equivalent [1,§3] if there is a cycle which contains them. It is obvious that this binary relation is reflexive, symmetric and transitive and so is a relation of equivalence on the set of nodes of the oriented graph. On the set of equivalence classes introduce the following binary relation: $p \succeq q$ if in the initial graph there is a way from any node of the class p to any node of the class q. It is clear that this binary relation is reflexive, transitive and anti symmetric and so is a relation of a partial order. [1,§4]. Then we contrast to this relation a zero-one matrix in which in the cell (p,q) there is 1 if $p \succeq q$ and 0 in other cases.

We suggest the following sequential algorithm of the oriented graph nodes factorization. On the first step we have a single node which is in the single cluster. The matrix a of the partial order here consists of the single unit. Assume that on the n-the step there is the set I of clusters and the one-zero matrix a which characterizes the relation of the partial order $\succeq$ between them.

On the step $n+1$ new node $n+1$ and two edges appear. One of these edges runs into the cluster p and another runs from the cluster q into the node $n+1$. Then new clusters and the partial order matrix a are constructed as follows. Denote

$$K_p = \{k \in I_n : p \succeq k\}, \quad R_q = \{k \in I_n : k \succeq q\}, \tag{1}$$

$$A = K_p \cap R_q,\ A_1 = K_p \setminus A,\ A_2 = R_q \setminus A,\ B = I \setminus \left(A \cup A_1 \cup A_2\right). \tag{2}$$

The new node $n+1$ and the clusters from the set A form new cluster $(n+1)$. The matrix a is divided into 16 rectangular boxes which are created by the cluster $n+1$ and the sets of clusters A_1, A_2, B. To describe these boxes introduce auxiliary designations $\mathcal{A}$ of a sub matrix which coincides with analogous sub matrix in previous version of a, $\mathcal{E}$ is a sub matrix consisting of units and $\mathcal{O}$ is a sub matrix consisting of zeros. In a cell "$\ldots \to \ldots$" of a left set "..." is a vector-column and right set "..." is
a vector-string consisting of clusters. The matrix a contains the following cells:

1. $(n+1) \to (n+1)$ which has the form $\mathcal{E}$ because $(n+1)$ is the cluster;
2. $(n+1) \to A_1$ has the form $\mathcal{E}$ by a definition of the clusters set A_1;
3. $A_2 \to (n+1)$ has the form $\mathcal{E}$ by a definition of the clusters set A_2;
4. $A_2 \to A_1$ has the form $\mathcal{E}$ because in the graph on the step n+1 there is a way from a cluster $j \in A_2$ to a cluster $i \in A_1$ which passes through the node $n+1$;
5. $A_1 \to (n+1)$ has the form $\mathcal{O}$ because in opposite case a part of clusters from A_1 joins the set A;
6. $(n+1) \to A_2$ has the form $\mathcal{O}$ because in opposite case a part of clusters from the set A_2 joins the set A;
7. $A_1 \to A_2$ has the form $\mathcal{O}$ because in opposite case a part of clusters from the sets A_1, A_2 joins the set A_0;
8. $(n+1) \to B$ has the form $\mathcal{O}$ because in opposite case a part of clusters from the set B joins the set A_1;
9. $B \to (n+1)$ has the form $\mathcal{O}$ because in opposite case a part of clusters from the set B joins the set A_2;
10. $A_1 \to B$ has the form $\mathcal{O}$ because in opposite case a part of clusters from the set B joins the set A_1;
11. $B \to A_2$ has the form $\mathcal{O}$ because in opposite case a part of clusters from the set B joins the set A_2;
12. $B \to B$ has the form $\mathcal{A}$ because in opposite case a way $i \to k_1 \to k_2 \to j \to (n+1) \to i$, $i \in A_1$, $j \in A_2$, $k_1 \in B, k_2 \in B$ appears and so the clusters $k_1, k_2 \in B$;

13. $A_1 \to A_1$ has the form $\mathcal{A}$ because in opposite case a way $i \to k \to (n+1), i \in A_1, k \in B$, $j \in A_2$ appears and so $k \in A_1 \bigcap A_2$;
14. $A_2 \to A_2$ has the form $\mathcal{A}$ because in opposite case a way $i \to k \to (n+1)$, $i \in A_1$, $k \in B$, $j \in A_2$ appears and so $k \in A_1 \bigcap A_2$;
15. $B \to A_1$ has the form $\mathcal{A}$ because in opposite case a way $k \to j \to (n+1) \to i$, $i \in A_1$, $k \in B$, $j \in A_2$ appears and so $k \in A_2$;
16. $A_2 \to B$ has the form $\mathcal{A}$ because in opposite case a way $k \to i \to (n+1) \to j$, $i \in A_1$, $k \in B$, $j \in A_2$ appears and so $k \in A_1$.

The transformation of the matrix a on the transition from the step n to the step $n+1$ needs the sets A, A_1, A_2, B definition by the formulas (1), (2) and demands $O(n)$ arithmetic operations and $O(n^2)$ operations of an assignment. Consequently a calculation of the connectivity components and of the partial order matrix up to the step n demands $O(n^2)$ arithmetic operations and $O(n^3)$ assignment operations.

Remark 1. This solution remains correct if the single edge from the node $n+1$ is replaced by a few edges and the single edge to the node $n+1$ is replaced by a few edges also. Numbers of these additional edges is no more than some finite m which does not depend on n. Denote by P the set of clusters in which new edges come into and by Q the set of clusters from which new edges come into the node $n+1$. Then the sets A, A_1, A_2, B are defined as follows. Assume that

$$K_p = \{k \in I_n : p \succeq k\},\ p \in P,\ R_q = \{k \in I_n : k \succeq q\},\ q \in Q,$$
$$K' = \bigcup_{p \in P} K_p,\ R = \bigcup_{q \in Q} R_q,\ A = K' \bigcap R'.$$

New node $n+1$ and clusters from the set A create new cluster which we denote by $(n+1)$ and put $A_1 := K' \setminus A,\ A_2 := R' \setminus A,\ B := I \ \setminus (A \bigcup A_1 \bigcup A_2), I := I \bigcup n+1.$

Describe now an algorithm of a construction of the clusters set and the matrix of partial order on this set in general case. On the step 1 there is the single node 1, the set of clusters $K = \{1\}$ and the matrix of the partial order characterized by the formula $a(1,1) = 1$. Assume that on the step n there is the set of clusters K. These clusters create a partitioning of the set $I = \{1,...,n\}$ into non intersected subsets,

Each cluster $k \in K$ is indexed by a maximal number of its nodes.

On the set K we have unit-zero matrix $a = \|a(p,q)\|_{p,q \in K}$ which characterizes the relation of partial order "$\succeq$": $a(p,q)=1$ if $p \succeq k$ and $a(p,q)=0$ in opposite case.
From the new node $n+1$ no more than m edges exit into the set $\mathcal{P} \subseteq I$ of nodes and no more than come into the node $n+1$ from the set $\mathcal{Q} \subseteq I$ of n nodes. Denote by P, Q the sets of clusters generated by nodes of the sets $\mathcal{P}, \mathcal{Q}$ and define

$$K_p = \{k \in K : a(p,k) = 1\},\ p \in P;\ R_q = \{k \in K : a(k,q) = 1\},\ q \in Q;$$

$$K' = \bigcup_{p\in P} K_p, \ R = \bigcup_{q\in Q} R_q, \ A = K' \bigcap R'.$$

The new node $n+1$ and the clusters from the set A create new cluster $(n+1)$, put

$A_1 := K' \setminus A, \ A_2 := R' \setminus A, \ B := K \setminus \left(A \bigcup A_1 \bigcup A_2 \right), \quad I := I \bigcup (n+1)$.

Then a calculation of the matrix a on the step $n+1$ satisfies the formulas

$a(n+1, n+1) = 1, \ a(n+1, A_1) := \mathcal{E}, \ a(A_2, n+1) := \mathcal{E}, \ a(A_2, A_1) := \mathcal{E},$

$a(A_1, n+1) := O, \ a(n+1, A_2) = O, \ a(A_1, A_2) = O, \ a(B, B) = O,$

$a(B, n+1) = O, \ \ a(A_1, B) = O, \ a(B, A_2) = O.$

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DETECT FAILURES IN COMPLEX TECHNICAL SYSTEMS

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ABSTRACT

The paper analyses possibilities of failures detection in specific technical systems. On the basis of preliminary analysis a suitable type of combined analysis is determined which detects failures with an emphasis on non-destructive approach. This method can be used for diagnostics and failure detection in systems of central power supplies used in transportation, especially in researching phase of the product life cycle. The paper presents the use of modern thermo visual technology which is documented by direct material and marking the measured temperatures on the locations of particular failures.

1 GENERAL INSTRUCTIONS

Recently, a strong emphasis has been put on ensuring safety of machines and systems that might, under certain conditions, endanger public safety. However, the issue of safety and dependability relates to the issue of failure occurrence and timely identification. Such failures may cause the system fail or stop functioning. In order to stop such failures to occur, diagnostic methods and tools are both appropriate and reliable. Technical diagnostics is instrumental in detecting primary failures, whereas even the most critical failures can be detected through non-destructive approach. Presently, detection of failures in technical systems does not only include meeting legal requirements, but also the overall safety and dependability of the system in question. It is required not only to identify and analyse failures, but also minimize their possible causes in the phase of researching, developing and designing new products. At this stage of the life cycle of the product or equipment is therefore necessary to establish a number of logical steps that can draw attention to failures which may endanger public safety in a systematic way**.**

The aim of the paper is to describe the methodology of failure detection process in complex technical systems. This method is very sensitive to preparation of individual steps of solution and requires the knowledge of functionality of a monitored object. The monitored object is a central power unit, which is presently used in railway transportation. Central power units are used as a substitution of rotary electrical generators (dynamos) for supplying electric equipment in passenger railway wagons. Central unit is multisystem equipment which supplies a wagon at all required AC&DC voltages which can occur in a main line of railway systems across the Europe.

2 PRELIMINARY ANALYSIS AND DEFINING BASIS FOR DETECTING POSSIBLE FAILURES

The first step in an initial phase of project is the analyzing and then specification which subsystem is the most critical from failure occurrence point of view and its impact on the whole system functionality, i.e. potential failure of which subsystem could be the main reason of a total shut down of the system. At present, the requirements for production growth and its quality are closely connected to requirements for dependability of technical system. An early diagnostics and checking of technical condition in many cases allow detecting potential failure, which could cause serious damage of the system in operation. The most effective way of failure detection in machinery is the process which does not require machine disassembly. After preliminary analysis of a suitable method selection we chose the method of thermovisual diagnostics followed by heat stability tests. Thermovisual diagnostics has several advantages, such as a non-invasive character, and it can detect an area with abnormal heat generation during operation of central power unit. Heat stability tests can be split into two parts: dry heat tests and cold tests.

2.1 Thermovisual measuring analysis

Thermovisual measuring enables in full operation detection of overheating in a non-desctructive way and provides a possibility to measure contactlessly surface temperature of the objects using an infrared thermo sensor. The results of temperature measuring are digitally displayed with the recognition of temperature levels which significantly helps during failure detection and analyzing. A thermocamera works on a basis of an infrared radiation, which is emitted by examined objects. This radiation is concentrated by opto-mechanical system of an infracamera to a semiconductor detector, which converts the radiation into electrical signal. Made images are possible to work into a final image called a thermogram, which displays a heating curve (temperature changes) of the examined surface area. It is suitable to use this technique in the initial phases of research because it can detect abnormally hot electrical contacts or components; thus proceed in risk analysis of the central energy source as a whole.

At the beginning of measuring by a thermovision camera we were focused on a whole central power unit. Gradually, the parts with no heat dependence were selected; thus it could be concluded that these parts of the central unit would not be responsible for system failure followed by equipment failure. Real temperatures were simulated at which the research institute provides the operation of equipment. These temperatures meet technical conditions and specified ranges of the equipment. Temperature range is from -25°C to + 55 °C.

The main reason of these failures are electric current connections which were preliminary recognized as the root cause of thermal hazard and subsequently cause the malfunction of central power unit or some of its subsystems. Heat failure criterion is a fact, that in steady thermal conditions an electrical current connection and its input have the same temperature. At the beginning of measurement it was necessary to set threshold temperatures to avoid incorrect colour interpretation. To measure accurately it is important to set a minimal range; thus to ensure the finest step between temperature levels. Through mutual comparison of equipment surface temperatures (between phases, between fields, etc.) it is possible to localize the places with higher temperatures which indicates a possible failure. An initial idea is that during scanning by a thermovision camera an infrared radiation, emitted by the area of a defective connection of two conductors leading an electric current, is detected.

The main criterion to decide whether it is an inconvenient connection is not only an absolute temperature of connection, but especially temperature difference to other connections, possibly its temperature gradient. By measuring, observing and comparing to archived data we set possible decisions on quality of an examined connection. Found heat failure is judged according to its importance in regard to a measured rise of temperature, relevant current load and expected current load. In this way we can get a track of a found failure and its importance in a place. The next phase of an analysis is to elaborate a complex report including general information on colour thermogram supplemented with colour photography and marking a site of a failure on the graph. In the next figure there are outputs of a thermovision camera on an anticipated failure place which were made in an area of a high voltage convertor. Repeated measuring showed a heat failure, so it confirmed a hypothesis on a site of failure.

This fact significantly conduced to a decision which direction to take when deducing hazard formation and its solution. It is evident, that this kind of analysis can reveal only possible hazards, which are caused by a physical quantity heat. All discordant locations phases were found in the sites of the high voltage converter, but in different areas. The high voltage converter was scanned in operation in specified time intervals. The aim is to observe temperature and heat focuses distribution and especially changes of these temperatures which can indicate possible sources of hazard.

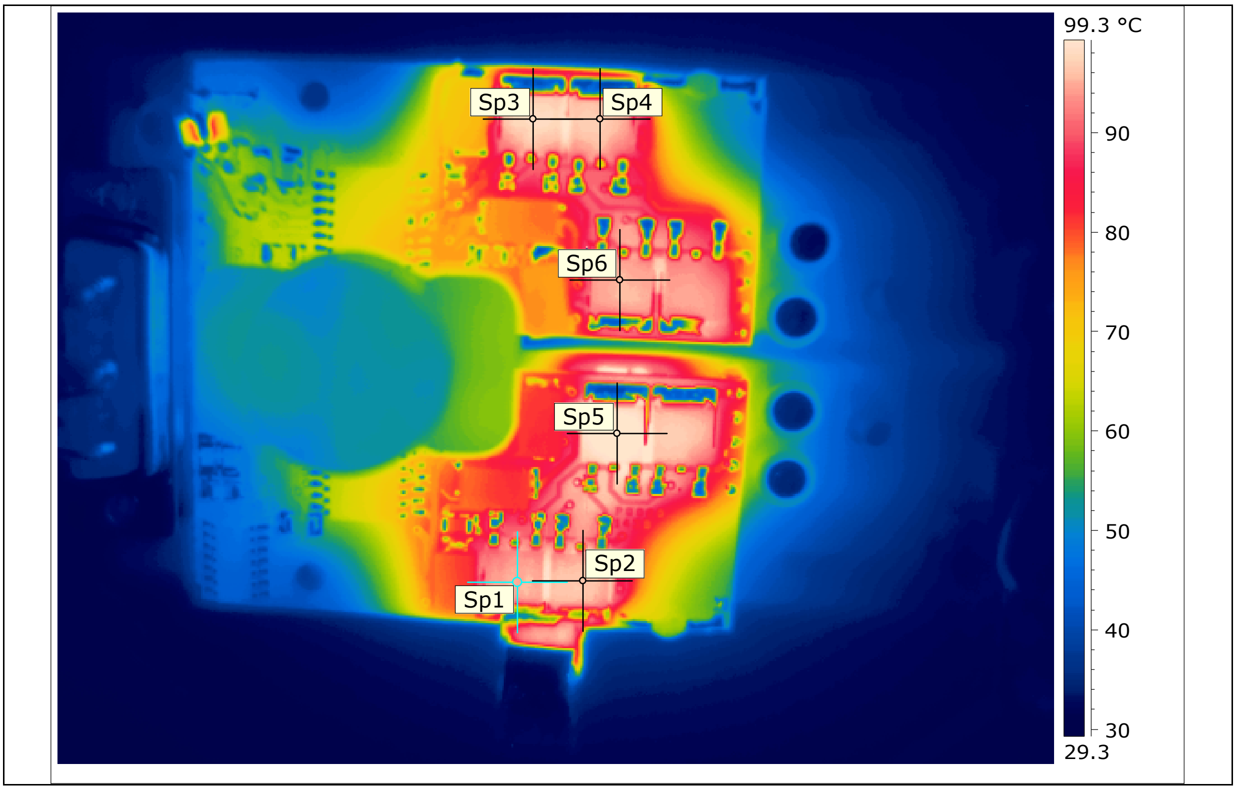

Figure 1 Thermovisual image, convertor

As can be seen from a thermograph in **Figure 1**, in some measurements the temperatures go over up to 45°C, which is unaccepted. The findings suggest that the central power unit does not work in a defined working temperature range, because thermal junctions between the components appear.

Emittance	0.96	Description, Image Title : IR_0886.jpg Measurement Time : 13,18 hod.
Judged	36.0°C	

Temperature	
Surrounding Temperature	22.0°C
Object Distance	0.2 m
Sp1 Temperature	95.0°C
Sp2 Temperature	96.0°C
Sp3 Temperature	98.2°C
Sp4 Temperature	96.7°C
Sp5 Temperature	100.3°C
Sp6 Temperature	95.3 °C

f[kHz]	Ugh1[V]	Ugl1[V]	Ugh2[V]	Ugl2[V]	Uusm.[V]	C[nF]
50	10,83	-5,4	11,39	-4,82	20,25	180

Figure 2 Analysis thermovisual image, convertor

2.2 Hear tests analysis

In the following text the list of individual steps of thermal stability tests is presented and they will be the issue of a further research. To solve this task a special software has been used which is used for diagnostics in an environmental chamber Heraeus- VÖTSCH VSKZ 06/210/S. From the previous outputs of thermovision tests it was necessary to concentrate on a high voltage converter, because it looks that this subsystem causes power unit failure. Both thermal stability tests are performed by software, which provides complex information on operating conditions and failure state of a power unit, including detection and failure data processing. The software is also able to localize a failure on a level of replaceable logical unit with the output of diagnostic results for maintenance and service, and a long-term storage of diagnostic results. The third level of diagnostics enables connecting of service equipment (PC) through a suitable interface (RS232 line, CAN interface…), and optionally it can contain an interface for connection to a central diagnostic computer of a railway carriage. This test is carried out according to a norm "STN 50155/C1 Runway application/ Electronic devices of rail vehicles" and consists of two tests: a cold test and a dry heat test. Parameters which are kept in these tests (temperature in a chamber, device load, supply voltage) are determined by an operator of a device in technical conditions.

2.2.2 Specifield cold test outputs

A device which is not under voltage is placed into an environmental chamber. After cooling down the chamber to a minimum temperature, the device stays at this temperature for 2 hours. After this (keeping at -25°C for 2 hours – in terms of a data sheet related to a particular device) the device is powered and its proper function is checked. After the test and warming the device to an ambient temperature the test for its function is performed again. Because the sources of potential failure are determined in the part of a high voltage converter; attention is concentrated on this area. By performing of tests we can get a view of possible failures.

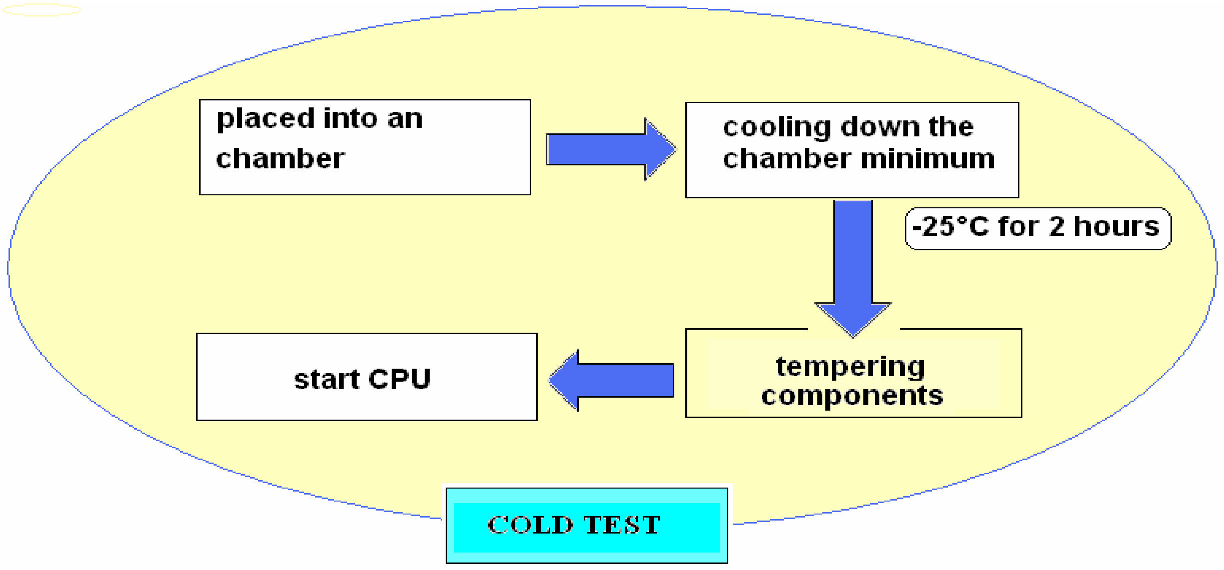

Figure 3 Cold test

A primary failure is identified in a part of converter block, which provides regulation of energizing of driving motors. The root cause of the failure was an error in control circuits of a driver converter.

2.2.2 Specifield dry heat test outputs

During a dry heat test the attention was focused especially on a converter block. The test for function of the device was done through a dry heat test at a temperature + 55 °C. The chamber was heated up to the temperature 70 °C. After reaching this temperature, the test for function of the device was performed.

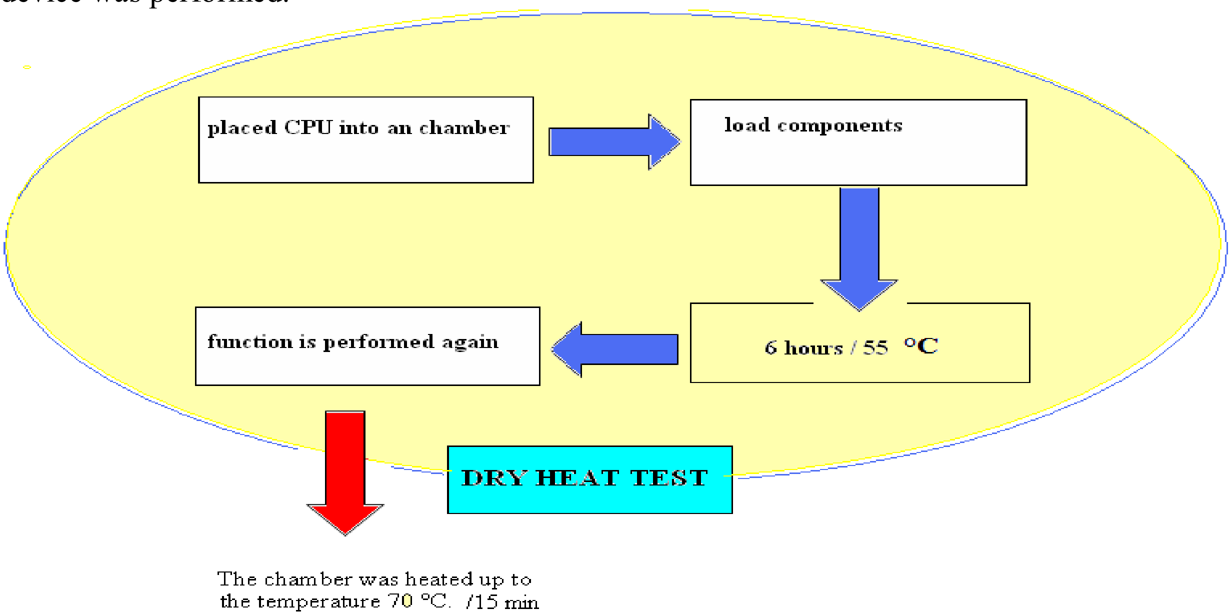

Figure 4 Dry heat test

After connecting a notebook to a diagnostic interface of environmental chamber diagnostic software evaluated that the failure is on the converter. This failure was in the place of diodes assembly. An error was made during a design of this protection block. After component data sheet checking, it was recognized, that improper type of a diode was selected. This type cannot meet the requirements specified for this device. At low current it is not necessary to install a cooler on a diode body. However; rising temperature of a chip causes a higher cut-off current of the diode and this caused a device collapse. It means that at a temperature 70°C and particular thermal resistance, the diode body should not exceed a standard temperature 70°C on a chip. The output of test results was a corrective action and recently an appropriate cooler has been designed.

3 CONCLUSION

One of the practical tasks that the paper solves is to define a suitable method which will be used in failure analysis. We determined a combination of methods of analysis because of complexity, which was necessary to follow. The essence of combined (mixed) methods is coincident use of some procedures of failure detection. Thermovisual diagnostics helped to detect provisional places of hazard and it was deduced to a subsystem of a central power unit, in a place where a failure occurred, the system becomes malfunctioned. This part of a central power unit is in a place of a driver converter and a failure was caused by a wrong design and selection of components on a board. In this case the faulty components were diodes; they are already replaced by a more powerful one with higher thermal load level. Another very serious failure occurred in a driver converter block, it was in control circuits of a sub-driver.

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MODELING THE DEPENDABILITY AND COST OF MAINTENANCE

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ABSTRACT

The paper deals with possibilities of stochastic approaches in risk modelling from a point of costs and data relating reliability aiming to define needed parameters of distribution of stochastic variables. It presents outputs from developed and applied simulation models, to assess reliability, maintainability, availability, costs and risks from operational data.

1 INTRODUCTION

Dependability issue can not been dealt in an insulated way as how it has unfortunately happened till now, but, only in a thorough full system approach in respecting technical, technological, economic and other interactions to assess a risk. From a general understanding of reliability it is not essential to express partial features of reliability, but to assess an optimum of a whole system taking all effecting components into consideration. A rate of optimum can be expressed using an interrelationship between relating models, that evaluate more partial features and they express e.g. an operational effectiveness of the system in a form:

$$PU(t) = (R(t), U(t), A(t), N(t), RM(t)) \quad (1)$$

where:
PU (t) ... operational effectiveness of the system,
R (t) ... system reliability,
U (t) ... system maintainability,
A (t) ... system availability.
N (t) ... maintenance costs,
RM (t) ... risk of a mission

In an above mentioned simplified approach we can understand an operational effectiveness as an ability of a given technical system to meet given function in certain time period and in given conditions. An operational effectiveness understood in such a way is a certain quality and utility criterion of the system and it is expressed by its technical and economic parameters, in conditions in which the parameter has risen, in which it is operated and is maintained. It features an operational effectiveness as a relation of a level of a fulfilment of reliability and maintainability, costs and a risk.

It expresses an operational effectiveness through a probability, that a level of fulfilment of a system function will meet requirements, that a system will be working in a given time period with no breakdowns and that in a specified time period it will be ready for operation at allocated costs. It defines significant elements and evaluated cost kinds and amounts; it reviews an object – a personal land-rover and costs aiming to provide a high level of readiness.

Quantitative analysis through a simulation modelling provides a computation (an estimate) of some selected quantitative numeric values of chosen indicators of reliability.

A numeric value of an indicator can be obtained through experimenting with a model with a computer taking into consideration simple events, which the model structurally relates with behaviour and analysed states of the system. Model and all input parameters are of a stochastic nature, and a result of the analysis is stochastic, burdened with a certain rate of uncertainty, which can be reduced, but not completely eliminated.

It is very difficult, sometimes impossible to express an intersection of probability of several events with different kinds of distribution. Aim of a submitted paper is to explain simulation approaches to analyses on a particular example of an analysis of operational events with a sufficient sample of a reviewed type of personal land-rovers. Analyzing of data is to be used to define parameters of distribution of variables as reliability, maintainability, readiness, costs and assessment of risk rate. To carry out simulation experiments and their assessment.

2. APPROACH TO A PROBLEM BEING SOLVED, USED METHODS, EXPERIMENTS

2.1 Operational data a analysis

The above mentioned analysis represents an evaluation of 1451 failures of all kinds of particular group and systems. It comes from reports on failures in operation and from user service centres. The failures are statistically processed in a needed form for a next use in form of variables distribution parameters and used in simulation experiments. The subject-matter of the review was a set composed of 56 personal land-rovers with a different number of failures and kilometres driven by operational units. **Figure 1** and **Figure 2.**

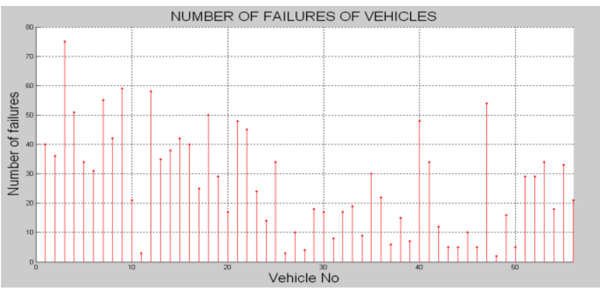

Figure 1 Number of vehicle failures

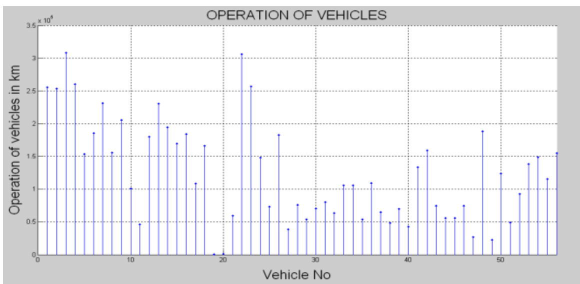

Figure 2 Number of km driven

The reviewed data were evaluated by groups as an engine with systems, gear system, steering, brake system, electrical installation, bodywork and a framework. Percentage portion of failures in particular groups are shown in **Figure 3**. Number of failures in particular groups in relation with number of km driven by cars is in the **Figure 4**.

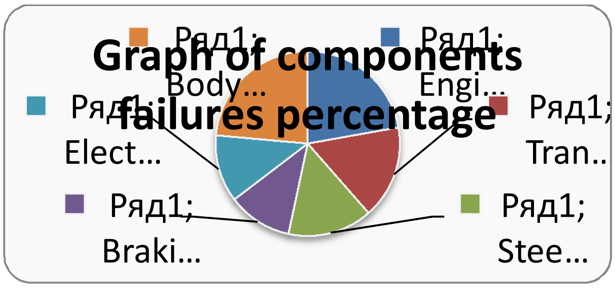

Figure 3 A chart of a portion of failure groups in percentage

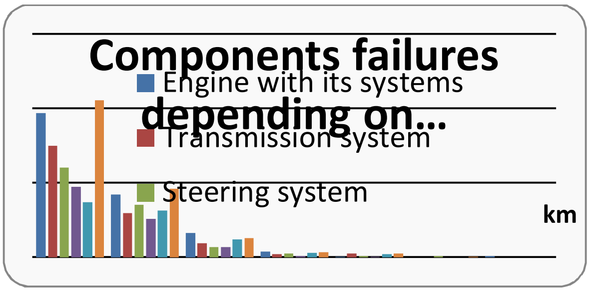

Figure 4 Failures in groups in relation with kms driven

From above mentioned data we assessed a drive turn failure– it is a distance driven by a vehicle between two failures - DTF and payload for a failure removal. We computed a distance between failures into time between failures –TBF in hours. We deduced a time to repair from payload for a failure removal (time to repair) – TTR in hours.

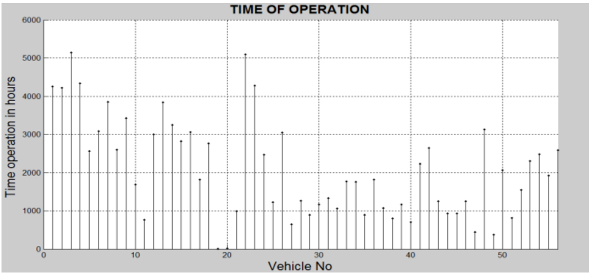

Figure 5 Time for operation of vehicles

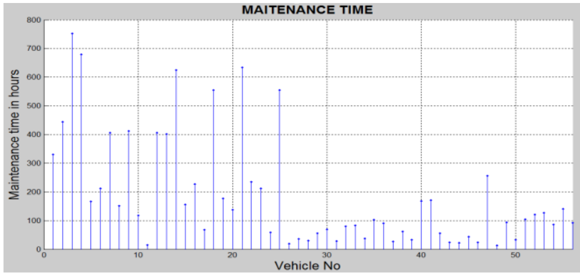

Figure 6 Maintenance time of vehicles

The costs have been reviewed by number of vehicle failures, number of kms driven and by costs on preventive and follow-up maintenance of particular groups.

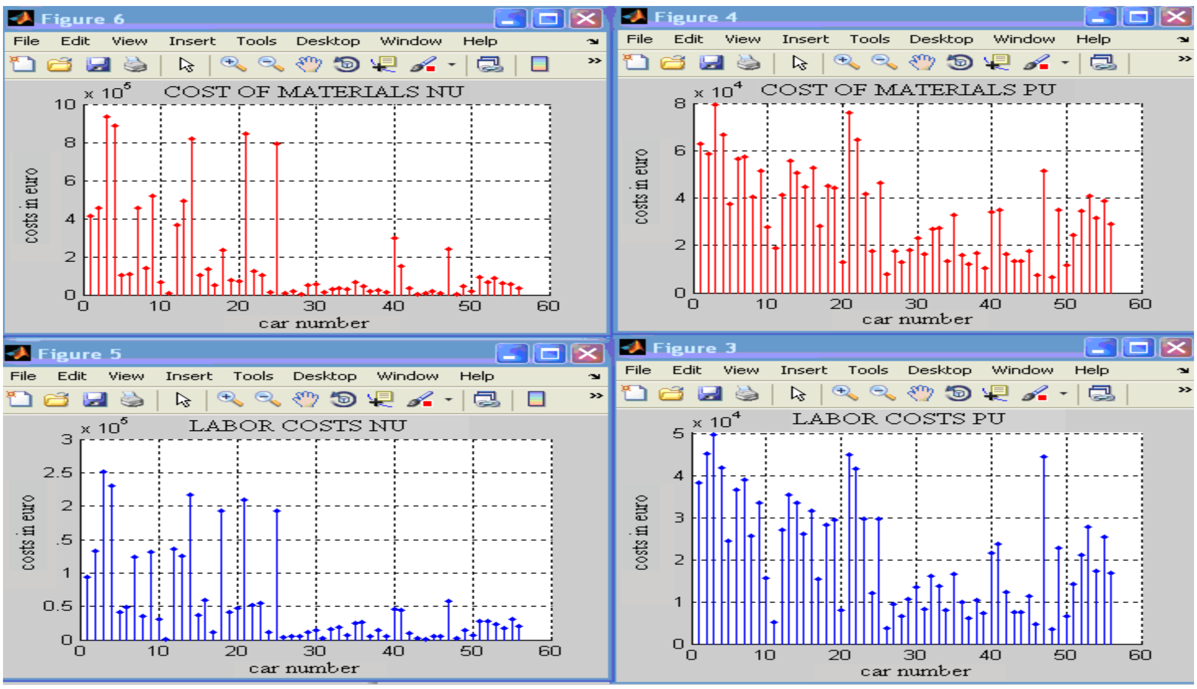

Figure 7 Input data on costs on individual vehicles

Other data have been deduced by simulation modelling.

We assessed the gained data; we expressed hypotheses on their probability distribution. We dealt with thesis Exponential probability and Weibull probability hypotheses.

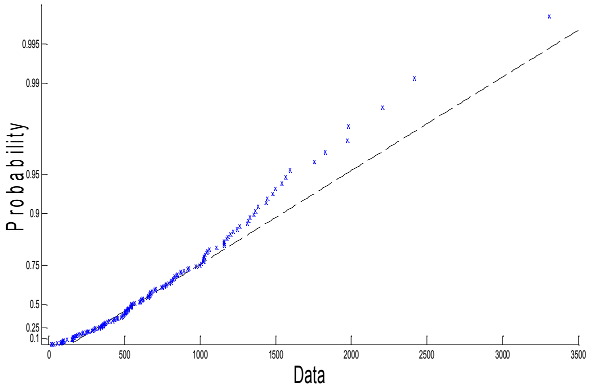

Figure 8 Verification of the probability distribution of the time between failure by means of exponential distribution

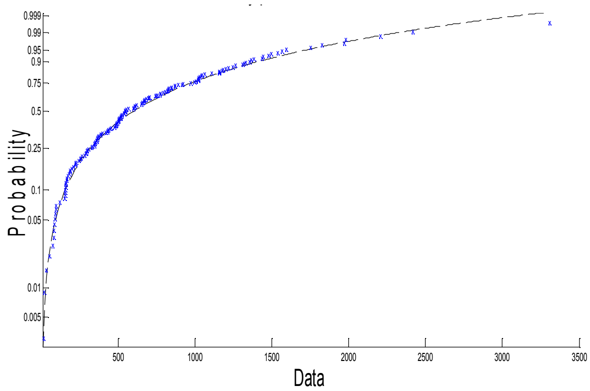

Figure 9 Verification of the probability distribution of the time between failure by means of Weibull distribution

Figure 8 and **Figure 9** show that TTF can be simulated by Weibull distribution.

We chose a more suitable hypothesis, that we have verified and defined the parameters of distribution with a confidence level 100(1 - alpha) = 99.9. We acted in the same way in defining all distribution parameters.

2.2 A Stochastic approach to the analysis of reliability, maintainability and availability

It stems from a term model of a Reliability Block Diagram. It makes provision for a probability of rise of events in complex systems, representing various arranged structures being formed by elements. A structure of the system can be expressed by a record:

$$M_k = \{ E_1, E_2, \ldots\ldots E_k \} \tag{2}$$

Particular symbols mean:
M_k – a system with „k“ number of elements,
E_i – i-th element of the system.

A structure of elements can be serial one, parallel or combined. The systems with a serial connection, which are typical for mechanical systems, are arranged one behind the other and they are independent. A failure of one element causes a loss of an operation capability of a whole system. The system is operational, if all elements are in a serviceable state. If we designate a reliability of the i-th element E_i as R_i, so the reliability of the system is a product of reliabilities of all elements:

$$R_S = R_1 \times R_2 \times R_3 \cdots R_n = \prod_{i=1}^{n} R_i \tag{3}$$

The reliability of a serial system is lower that the most reliable element of the system. Confidence of a reliable operation of a serial system with an increasing number of elements decreases and a probability of a failure occurrence increases. A vehicle in a model can be presented as a system with a serial arrangement of elements, where the elements represent the main design groups of the vehicle. A serial arrangement is chosen because if one system fails, so the whole vehicle becomes operationally incapable. If we can express a probability of a reliable operation of elements through the parameters of distribution of random variable intervals between failures, a total probability of a reliable operation can be expressed through aggregate statistical parameters defined from values of a total number of generated data.

A simulation model of an analysis of reliability with a selection of a decisive event is based on the following principles:

- The system will be divided in serial subsystems.
- We generate values of periods between failures of particular elements.
- From a time of a failure rise of subsystems we are selecting the highest values of the time rise of failure and we use them in incorporating the system in a serial structure.
- From the times of a rise of a failure of serial arranged subsystems we select the least value of time of a failure rise.
- Process of generating a selection of events is repeated until number of executions is completed.
- We collect statistical data about periods of operation, a total number of failures and other needed data.

As a rule we assess only non-reparable elements and systems. The elements of repairable system can be analyzed separately aiming to define statistic parameters of reliability or to define a final probability of a reliable operation of the system. Generally we have to suppose a final level of reliability characterized by an availability of a system in a draft of repairable mechanical systems composed of several subsystems or elements. One of possible ways is an assessment, resulting from a known or an estimated level of availability of particular subsystems. The aim is to define an availability of a system from knowing indicators of partial features of reliability maintainability and provision of maintenance of individual components.

A starting point for a draft of a model of system availability is a so called state analysis, in which the state may occur. The system may be in many and different states, whereby each of them is defined by a combination of individual elements. Likewise each element of the system may occur in different states that randomly take turns. Such process, when the states of objects being monitored change randomly in time, are called the Markov random process.

The states in mechanical system are expressed the most often through a bi-state model. The system may occur either in a functional or a non-functional state depending of a state of particular elements. If the transition between these states take turn randomly and they can appear any time, that random process is called a simple process of a restoration. Reliability of objects being repaired is characterized first of all by indicators of availability, that describe in a complex way their reliability and maintainability. Indicator of availability is a function or a numerical value being used for a description of a distribution of probability of a particular (random) parameter characterizing an availability of the object. Generally such parameter is a state of object that changes randomly in time.

The probability, in which state the object (an element, a system) occurs in a given time period is describe by a function of an immediate availability $A(t)$ for a serviceable state or a function of an immediate non-availability $U(t)$ for a non-functional (unable for operation) state. The $A(t)$ and $U(t)$ function are complementary each other, an aggregate of their values in a given moment is equal to 1.

The function of an immediate availability $A(t)$ expresses a probability, that an object is able in a given time period to fulfil a requested function in given conditions, provided that the necessary external means are assured.

This indicator is not used in practice very often, as an immediate availability is not generally a point of interest, but a level of its availability related to a certain time period.

A factor of an asymptotic availability A is used very often in a technical practice and for a stabilized process of renovation provided that:

- logistic, administrative and technical delays are negligible,

- distributions of a random variable for a reliability and maintainability are exponential ones.

The relation for a deterministic computation:

$$A = \frac{MTBF}{MTTR + MTBF}$$

(4)

where

MTBF – mean time between failures,

MTTR – mean time to repair.

A factor of an asymptotic availability characterizes a certain stable level of availability that the object gradually approaches with an increasing time of operation.

All other statistic methods established based on stochastic principles lead always to a non-constant function of availability *A*, i.e. a function of availability dependent on an operational time *t*.

Building of deterministic models of availability stems from a notion, that a time function between failures and a time needed to remove a failure in failing of the *Ei* element have the same distribution of parameter probability, as the elements, they consist thereof.

They lead the most often to an exponential or Weibull distribution of probability. The time flowcharts of failure rate and repairs, or other stochastic effects during provision of reliability of complex systems in real operation are not considered in such models.

In a real case, the operational reliability, or its partial features are related with processes, needed to provide a removal of a failure (a control process, a supply system, repair process, etc.). Therefore a model can have as well several states and distributions of randomly variables.

These facts can be expressed by a simulation modelling.

We use the fact, that probabilities of the time of a rise of part failure and time of a failure removal are parameters with stochastic nature that may occur in wide range of parameters.

A proposed solution can be expressed in the following way:

A. We divide the *Mk* system in subsystems, or elements

Partial systems are analyzed separately and the conclusions are final ones for a system assessment.

B. Statistic regularities of subsystems (elements) of a model are described by:

- distribution of probabilities of intervals of the rise of failures,
- distribution of probabilities of an active period of maintenance,
- or distribution of probabilities of next shut-downs.

C. We define, which states are important for analysis of a system and which we want to express through a simulation. We can merge some states.

D. We define output parameters; we want to process statistically and to express them graphically.

E. We establish a computer-aided simulation model and we perform experiment that we evaluate.

2.3 Simulation modelling of costs and their risk rate

Specification of requirements for reliability of a transport means is first of all an issue of looking for an acceptable compromise between a requested level of reliability and a level of costs, which will be needed for its achievement. Provision of reliability in a stage of application is however dependent on allocated sources for a provision of maintenance.

If we start from a definition of a reliability, which is understood as an ability of the object in meeting a needed function in given conditions and in a given time interval, we can note, that a main reviewed feature of a reliability is a functionality of the object. From this view knowledge of function is a starting point in defining requirements for reliability, whose fulfilment is expected from an object. Amount of costs for provision of maintenance is conditioned on maintainability in addition to reliability. We achieve knowledge of assessment functions by above mentioned approaches. Costs on maintenance of particular groups and a vehicle as a whole and their division may serve as a background for assessment of risks related to the failures of equipment. It represents an effective tool and an important source of information for decision-making in area of management.

There is a certain rate of uncertainty connected with each function of transport means, that it will be carried out in a different way than requested and that possible deviations from an expected function will have an unwanted consequence on a result of the function of the object as a whole. Therefore there is a certain risk, understood as a combination of probability, that a certain event occurs (a failure) and consequences (costs), which would occur, if an event would happen.

The existence and knowledge of consequence itself or a probability of an event rise does not need to lead to a rise of an unacceptable risk. If an unwanted event has no serious consequence, then even at a high probability of its rise, the risk related with it can be acceptable as well. It is valid vice-versa, of course.

Risk is a quantitative and a qualitative expression of a menace; it is a rate of a menace or a level of a menace. It defines a combination of a probability of a rise of an unwanted event (a failure) and a range of relevance of a possible amount of costs, resulted from a rise of a transport means failure. Such definition being in line with valid legislation enables monitoring of a rate of occurrence of negative events (failures, accidents, disasters) but their consequences as well, that means a great advantage for a strategy of risk management.

The risk expression is made the most often through a function of a probability of a rise of a negative case or an event and as a consequence of an unwanted event (Note: not always it relates a product of these events):

$$R = P \ x \ D \, , \qquad (5)$$

where: R – is a value of a risk,
P – probability of a rise of a negative case, event,
D – consequence of an unwanted event.

The input parameters of risk expression are random parameters; we can express and model their relations, which will be mentioned in the next point of the paper.

3 DESCRIPTION OF ACHIEVED RESULTS

3.1 Analysis of reliability, maintainability and availability

We performed simulation experiments with parameters of distribution of accepted hypotheses, through programmes developed in MATLAB. Simulation experiments were done by a discontinuous simulation with a variable time step. We registered necessary data of simulation experiments at advance of simulation time.

Periods between failures of particular groups of vehicle and a maintenance period are illustrated by probability density in the **Figure 10** and **Figure 11**.

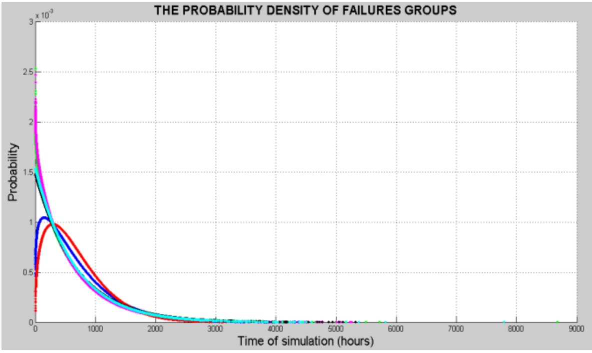

Figure 10 Density of a probability of time between failures of groups

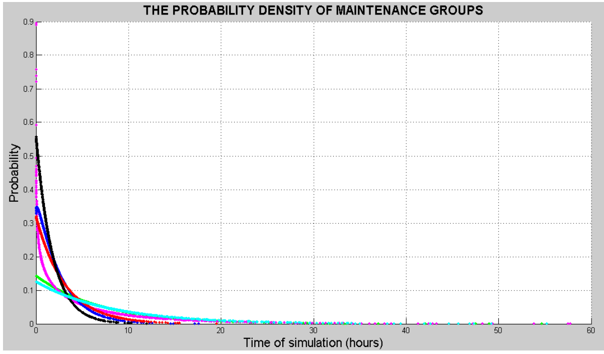

Figure 11 Density of probability of time of maintenance of groups

The reliability is assessed with a model mentioned in Chapter 3.1. Generating failures and their visualization creates a notion about a rise of failures and a ratio of particular groups in failures and failures of vehicle as a whole **Figure 12** and about time between failures **Figure 13**.

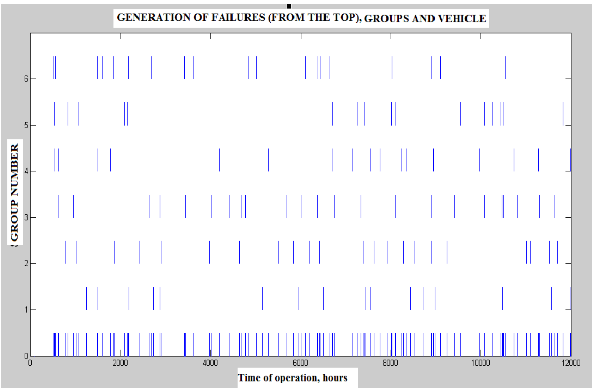

Figure 12 Ratio of particular groups in failures and failures of vehicle

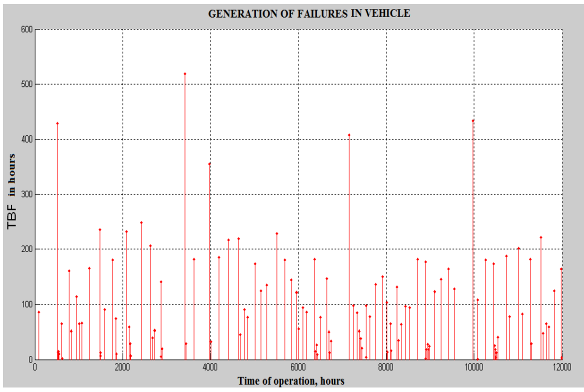

Figure 13 Generated periods between failures of vehicle

The same procedure was used in analyzing maintainability. Availability of particular groups was assessed through simulating 10000 hours of operation, representing 600000 km driven, with parameters of periods between failures and time of maintenance distributions for accepted hypotheses. The availability of a land-rover was simulated with a precondition, that in a case of a failure of any of groups the vehicle is to be repaired, it means, and that a serial repairable system is dealt. Principle of a shift by a time step accepted a choice of a decisive event. They have been generated from a time of rise of group failures. A minimum event of a group with the least time to failure has been chosen. A time till repair has been chosen for a group failure and a next period between failures was included into events calendar. The simulation time has been shifted.

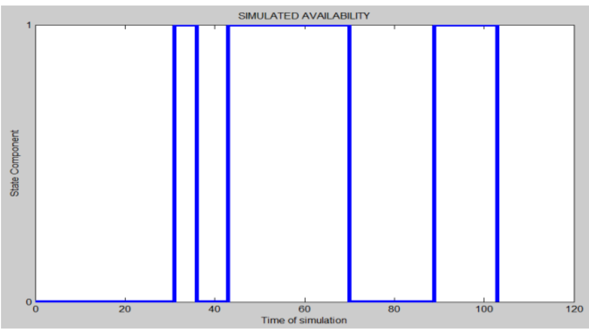

Figure 14 Simple process of restoration

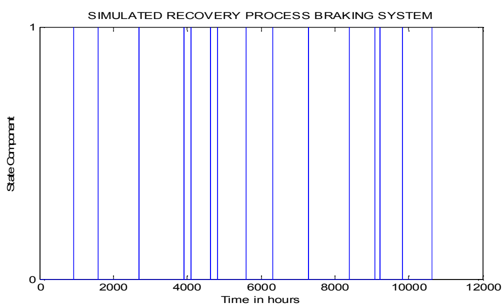

Figure 15 Process of renovation for a braking system

Particular groups show high factors of an asymptotic availability. The values of the factors do not differ significantly the groups are balanced from a view of reliability.

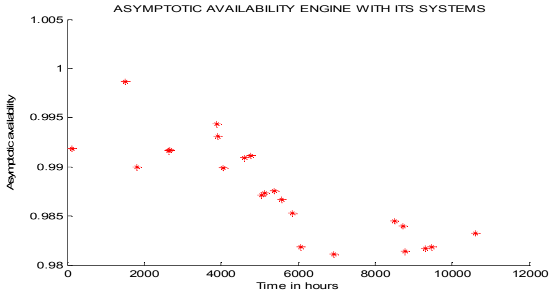

Figure 16 Asymptotic availability for an engine and its systems

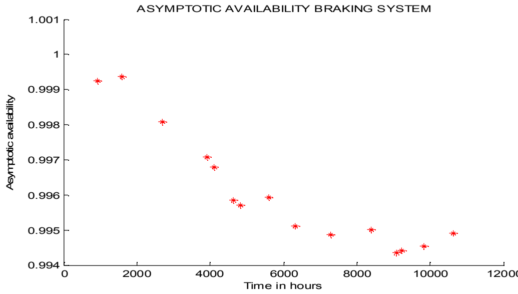

Figure 17 Asymptotic availability for a braking system

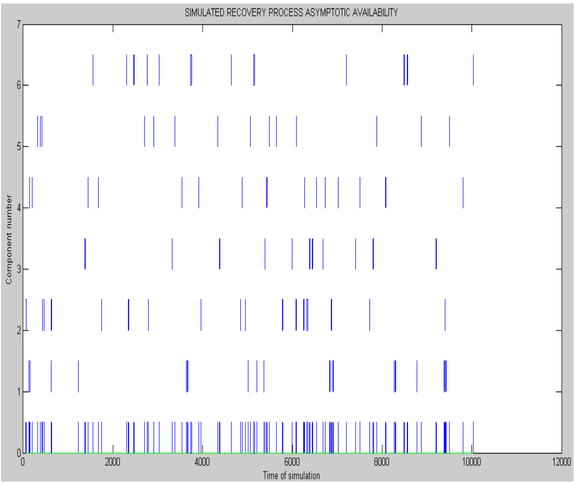

Figure 18 Renovation process of the vehicle and its groups

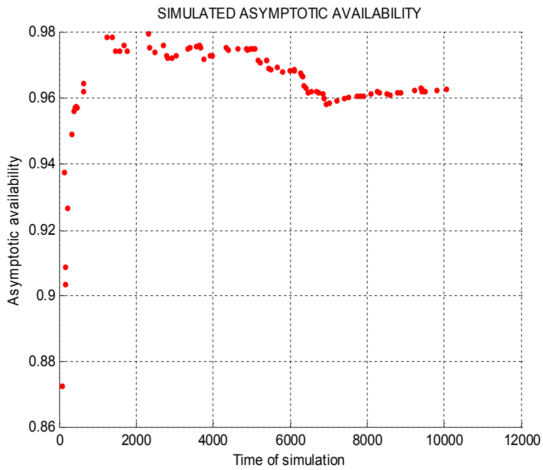

Figure 19 Asymptotic availability of the vehicle

3.2.Analysis of costs and a rate of their risk

Maintenance costs for particular groups of vehicle statistically processed and they show the distribution parameters illustrated in the **Figure 20** and **Figure 21**.
and courses of function for a probability of density and distribution function in the **Figure 22** and **Figure 23.**

Components	Probability distribution Parameters of distribution NP
Engine with its systems	Exponential distribution muhat = 1.076844859813086e+002
Transmission system	Exponential distribution muhat = 85.338893617021384
Steering system	Exponential distribution muhat = 39.993674418604691
Braking system	Exponential distribution muhat = 1.756717073170731e+002
Electrical installation	Exponential distribution muhat = 26.907053140096636
Bodywork and frame	Exponential distribution muhat = 4.124709411764700e+002

Figure 20 Distribution parameters of labor cost groups

Components	Probability distribution Parameters of distribution NM
Engine with its systems	Exponential distribution muhat =3.124684112149525e+002
Transmission system	Exponential distribution muhat = 2.960842553191487e+002
Steering system	Exponential distribution muhat =1.046163720930233e+002
Braking system	Weibull distribution parmhat = 1.0e+002 * 1.831692349885320 0.011053278631922
Electrical installation	Weibull distribution parmhat = 64.364482095181003 0.993711850690347
Bodywork and frame	Weibull distribution parmhat = 1.0e+002 * 1.139124640766775 0.009847012574545

Figure 21 Distribution parameters cost of materials groups

From a course of costs distribution functions we can conclude a range in which the costs would occur. The costs for material, assessing a mean of probability 0.5, define an increasing order for costs in cost groups as electric installation, steering, body, and a frame, braking system, gear system, engine with systems **Figure 22**.

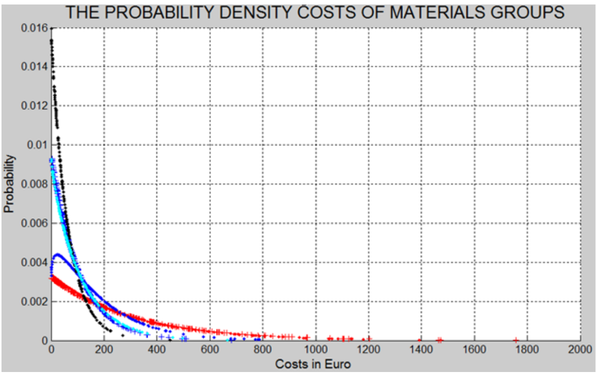

Figure 22 The probability of density costs of material groups

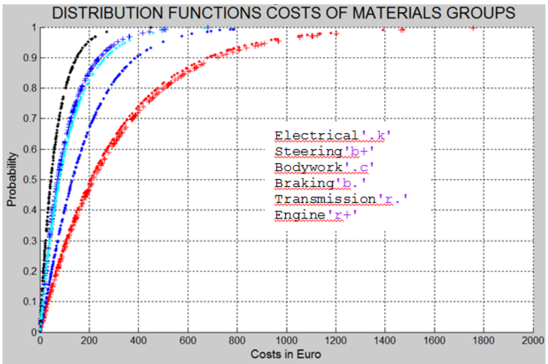

Figure 23 Distribution functions of costs of material groups.

At labor costs the order is electric installation, steering, gear system, an engine with systems, a braking system, body and a frame **Figure 25.**

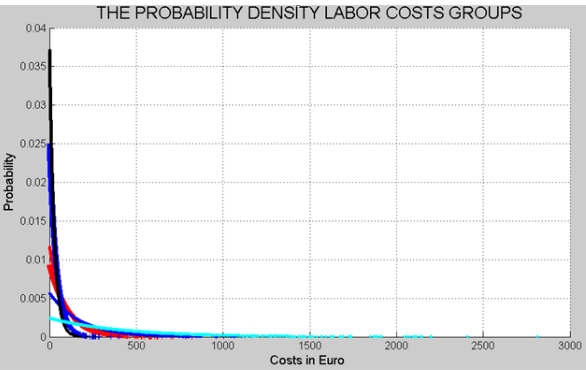

Figure 24 Probability of density of labor costs groups

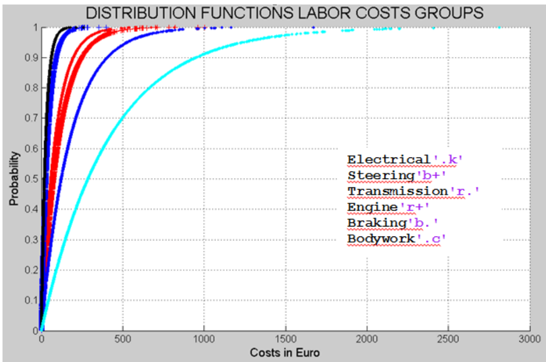

Figure 25 Distribution functions of labor costs groups

We use a function of density of a failure probability as a rise of a negative event – a failure and an amount of total costs as a result of an unfavourable event.

Visual expression of an intersection of these events gives us a notion about a rate of rise of critical situation. We can quantify this fact and to express it by probability of risk matrix.

We will use a distribution of a failure probability to generate a rise of a negative event – a failure and a distribution of a probability of costs to generate the amount of costs resulted from an unwanted event.

Graphic expression of an intersection of these events in a point of costs matrix and operation in hours provides us with a perception relating with quantification if a risk situation rises. Aggregations of their occurrence and their quantification on the legs enable comparing the risks from costs for maintenance of objects being assessed.

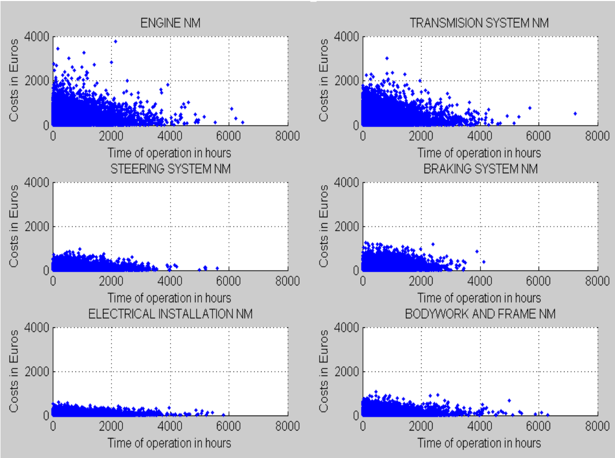

Figure 26 Probable risk matrix of costs of materials

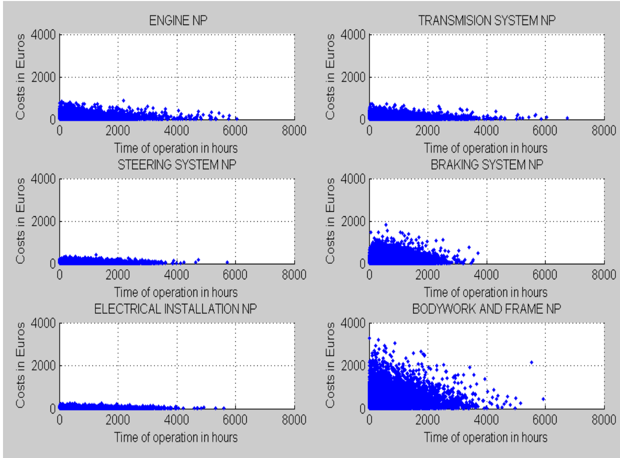

Figure 27 Probable risk matrix of labor costs groups

We can quantify this probability and to define it with a probability of elements, lines or columns of the risk matrix.

With an increased number of simulated events, representing a longer distance of kilometres driven, the ranges of affected risks increase as well. **Figure 28** and **Figure 29**.

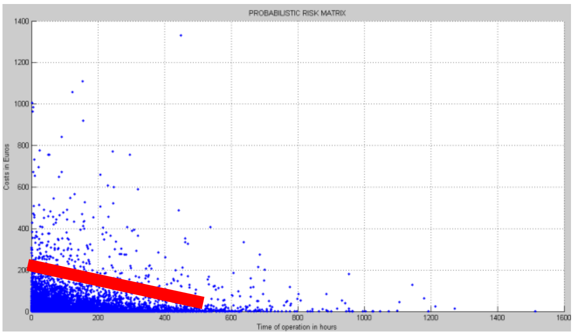

Figure 28 Probable risk matrix 1000 simulations

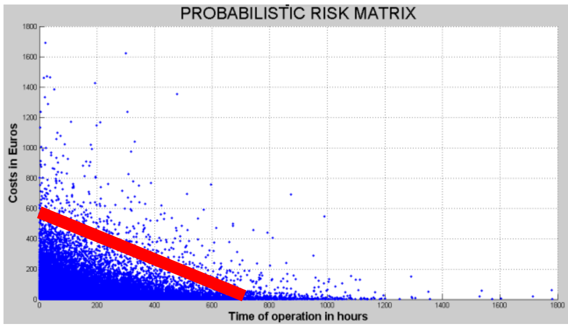

Figure 29 Probable risk matrix 10000 simulations

If we use a leg to define the most often occurrence of events and we compare the risk matrices of particular groups, we can obtain an opinion relating a risk in expanding costs for maintenance of particular groups of expenses **Figure 30**.

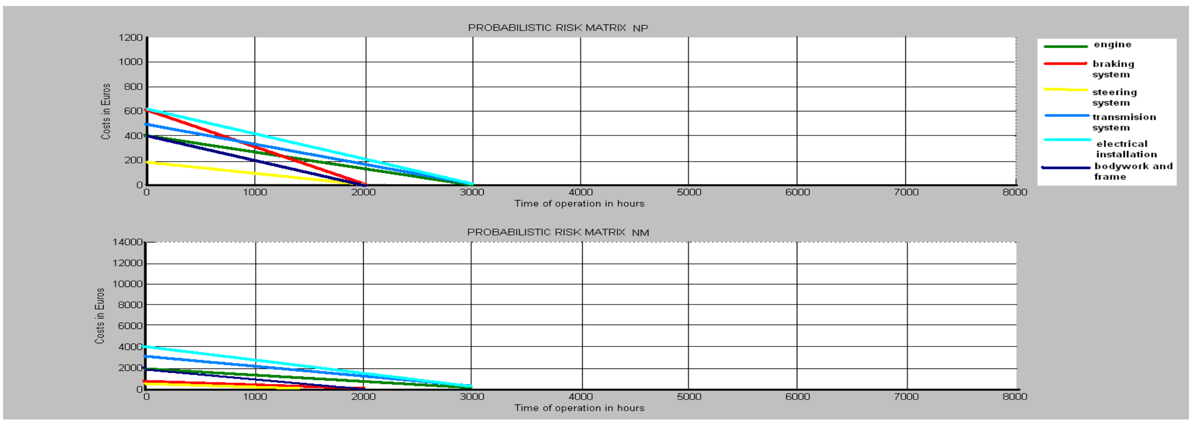

Figure 30 Matrices of risks of Figure 26 and Figure 27

If we use probability distributions of failure and costs probability for a discrete simulation with a variable time step, we can obtain a perception on amount of costs for maintenance of particular groups depending on hours of operation **Figure 31**.

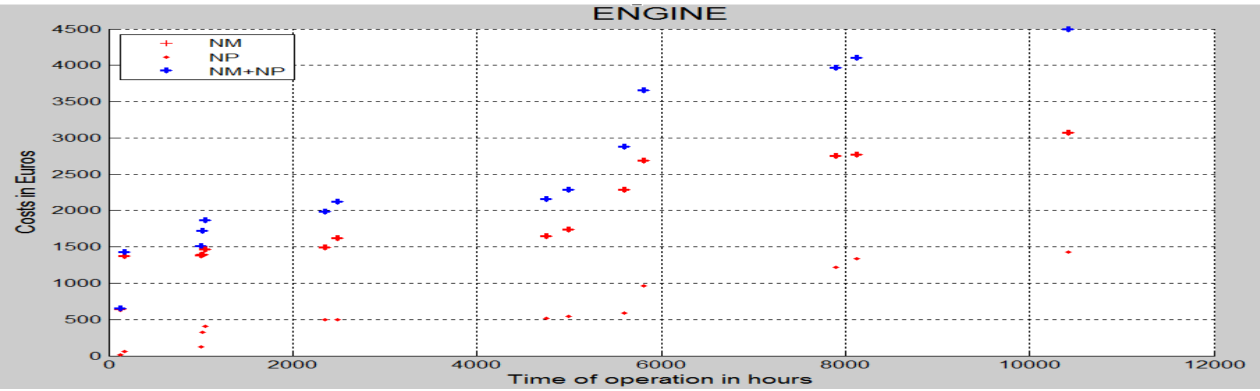

Figure 31 Simulation of failures and costs for engine maintenance.

4. CONCLUSIONS

From above mentioned results of data processing and from executed experiments results the following.

The results have shown that a level of reliability of particular groups is on a comparable high level. None of the groups is significantly different in a viewpoint of reliability except of a braking system. The same is for maintainability. The number of simulated failures of groups in operation up to 300000km ranges in particular simulation experiments from 6 to 15, a total number of a vehicle failures ranges from 40 to 48.

A sense of a mathematic expression of an availability factor has been supported, that relationship between reliability and maintainability expresses possibilities of an increase of availability of designed and operated devices that interfere with technological limits of periods when the activities are performed. Availability can be increased practically only through shortening of intervals of components of the device maintainability that interferes with technological limits of the action being performed. Asymptotic availability of a terrain vehicle is lower than the availability of groups, it becomes stabilized on 0.958- 0.966 level.

The statistic characteristics of a failure-free operation of vehicles and particular groups and statistic characteristics of costs are used for application of theory of risks and solution of tasks related to issues of maintenance and logistics issues.

They provide details for assessment of different risks and their quantification. They are more suitable than qualitative assessment and thy give a better visualization than balance methods resulting from mean values, or semi-quantitative methods of a risk assessment.

Mathematic and simulation modelling is for an analysis, modelling and prediction of stochastic phenomena in the operation, maintenance, logistics, risk assessment very favourable, first of all for a possibility of monitoring through graphic outputs, which give more visual perception about stochastic processes.

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SYNTHESIS BASE OF FAZZY KNOWLEDGE ANFIS-CONTROL OF REACTIVE CAPACITY AND VOLTAGE IN DISTRIBUTIVE ELECTRIC NETWORKS

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ABSTRACT

The question of synthesis of base of fuzzy knowledge ANFIS - networks for control streams of reactive capacity and voltage of knots considered in distributive electric networks. Synthesis base of fuzzy knowledge and term on basis of the received regime reports executed for schemes 14 knots electric networks with method application subtractive clustering. Root mean square error parameter adjustment of a method shows to adequacy of control of a mode in DEN.
Keywords. Distributive electric networks, reactive capacity, voltage, clustering, fuzzy neural network, control.

I. INSTRUCTION

Scientific and technical progress at the present stage of development has created preconditions for creation of intellectual control systems of the continuous technological processes having certain complexities and uncertainty. Besides, in the majority of cases of a problem of modern control because of complexity of the mathematical models describing them cannot effectively solved by classical methods. In these conditions application of modern technologies of processing of the information, including paradigms Soft Computing and their hybrids (neural networks, likelihood methods, fuzzy logic, genetic algorithms, and theory of chaos) has special values [1].

Based on the hybrid neuro-fuzzy network including paradigms of fuzzy logic and neural networks it is possible to synthesize the effective control systems, allowing overcome lacks peculiar to these paradigms separately. On the basis the device of fuzzy logic, the logic decision makes, and based on algorithm of training of a neural network parameters corresponding function of an accessory [2] adjusted.

From the point of view of uncertainty and quality of the multivariate initial information actual is application of neuro -fuzzy systems (ANFIS-network) for control of modes of complex distributive electric networks (DEN). Uncertainty at the decision of questions control of streams of reactive capacity and knots voltages caused stochastic by changes of active and reactive loadings, discrepancy or absence of the information on them, insufficient maintenance with measuring devices, and circuit changes at operation of electric networks. Also full maintenance of objects with measuring complexes does not give an opportunity of the account stochastic change of electric loadings in all time intervals.

In view of abovementioned, in the given work questions of synthesis of the fuzzy knowledge base for control streams of reactive capacity and knots voltages in DEN with use of an ANFIS-network are considered.

II. ANFIS-MODEL CONTROL OF REACTIVE CAPACITY AND KNOTS VOLTAGES IN DEN

At control of reactive capacity and a voltages in DEN power supply systems with numerous central point effectively to present system of fuzzy logic conclusions in the form of hybrid neuro - fuzzy networks (ANFIS-model [2,3]). It caused by that in fuzzy logic expert knowledge of structure of object in the form of linguistic statements used, and neural networks create an opportunity of effective training. On fig.1 shown structure of ANFIS-network with *n* in entrance parameters. The architecture and a rule of work of each layer of an ANFIS-network are given in [1,4]. The system ANFIS-control based on use of fuzzy knowledge base Sugeno.

In Sugeno model, the fuzzy knowledge base, describing dependences between entrance parameters $X = (x_1, x_2, ..., x_4)$ and target parameter*,* described by a polynomial of the first order in the following form [4]:

$$d_j = b_{j,o} + \sum_{i=1}^{m} b_{j,i} x_i \tag{1}$$

where, $b_{j,i}$ - some real numbers $i = \overline{0,n}$.

The degree of membership of an entrance vector $X^* = (x_1^*, x_2^*, ..., x_n^*)$ to values d_j defined by following system of the fuzzy logic equations:

$$\mu_{d_j}(X^*) = \underset{p=\overline{1,k_j}}{\vee} \underset{i=\overline{1,n}}{\wedge} \left[\mu_{jp}(x_i^*)\right], \qquad j = \overline{1,m} \tag{2}$$

where $\vee(\wedge)$- operation from *S-norm (t-norms)*, i.e. from set of realizations of logic operations *or (and)*.

As membership function analytical models of membership functions of a variable x to any fuzzy term in the form of calumniated functions [4] are used:

$$\mu(x) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}} \tag{3}$$

where *a*, *b*, *c* – some numerical parameters accepting any valid values and ordered by a parity $a \langle b \langle c$, where parameter $b \rangle 0$; *a* – factor of concentration of membership function; *b* – factor of a steepness of membership function; *c* – coordinate of a maximum of membership function.

The algorithm of control in the beginning demands formalization of the knowledge base depending on entrance signals (a deviation of reactive capacity and a voltage on each unit) and their quantities. With this purpose, the quantity a term-subsets of linguistic variables of reactive capacity and a voltage defined. For the decision of the considered question the method subtractive clustering (mountain algorithm clustering) [4] is used.

III. ALGORITHM SUBTRACTIVE CLUSTERING

On a first step of algorithm it is necessary to generate the potential centers *(Q)* cluster having final numbers q_r , where $r = \overline{1,n}$. The quantity possible cluster equal $Q = q_1 q_2 ... q_n$.

On the second step of algorithm, the potential of the centers cluster under the following formula pays off:

$$P(Z_h) = \sum_{k=\overline{1,M}} \exp(-\eta D(Z_h, X_k)), \; h = \overline{1,Q} \tag{4}$$

Where $Z_h = z_{1,h}, z_{2,h}, ..., z_{n,h}$ – the potential center of h-th cluster;
η – Positive constant; $D(Z_h, X_k)$ – distance between the potential center cluster (Z_h) and object clustering (X_k). In Euclidean space, this distance pays off under the formula:

$$D(Z_h, X_k) = \sqrt{\left\| Z_h - X_k \right\|^2} \tag{5}$$

On the third step of algorithm as the centers cluster, choose coordinates of «mountain tops». The center of the first cluster appoints a point with the greatest potential. Before a choice, following cluster the center it is necessary to exclude influence just found cluster. For this purpose from current values of potential, subtract the contribution of the center just found cluster.

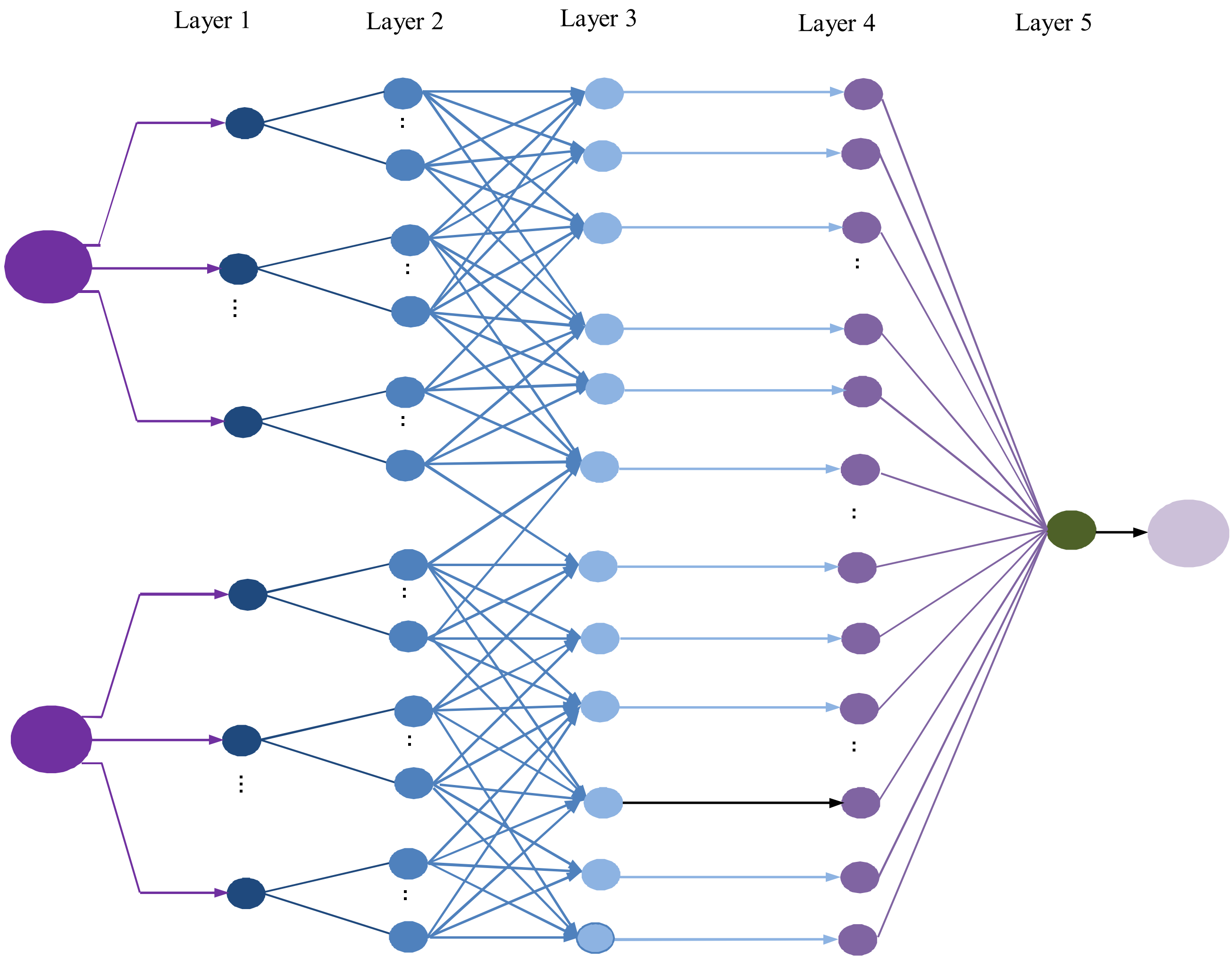


Fig. 1. Structure ANFIS – networks control of reactive capacity

Recalculation of potential occurs under the formula:

$$P_2(Z_h) = P_1(Z_h) - P_1(V_1)\exp(-\beta D(Z_h, V_1)),\ Z_h \neq V_1,\ h = \overline{1,Q} \tag{6}$$

where P(·) - potential on 1-st iteration; P_2(.) – potential on 2-th iteration; β- A positive constant; V_1 – the center of the first found cluster.

$$V_1 = \arg_{Z_1, Z_1, \ldots, Z_k} \max(P_1(Z_1), P_1(Z_2), \ldots, P_1(Z_Q)) \tag{7}$$

The center of the second cluster defined on the maximal value of the updated potential:

$$V_2 = \arg_{Z_h: Z_h \neq V_1, h=\overline{1,Q}} \max(P_2(Z_1), P_2(Z_2), \ldots, P_2(Z_Q)) \tag{8}$$

Then value of potentials again recalculated:

$$P_3(Z_h) = P_2(Z_h) - P_2(V_2)\exp(-\beta D(Z_h, V_2)),\ Z_h \neq V_1,\ h = \overline{1,Q} \tag{9}$$

Iterative procedure of allocation of the centers clusters proceeds until the maximal value of potential exceeds some threshold. The algorithm subtractive clustering not fuzzy, however it often use at synthesis of fuzzy rules from data.

IV. RESULTS OF PRACTICAL REALIZATION

The first stage of procedure of construction of fuzzy Sugeno model is definition of quantity a term-subsets and fuzzy rules on experimental data or by expert estimations. The volume of the knowledge base is defined as F=TG, where T – quantity of terms of one entrance variable; G – quantity of entrance variables, for example, at T=4 and G=5 we receive quantity of rules 45=1024, and already at T=7 and G=14 the quantity of rules considerably increases up to 714.

In conformity with it for complex DEN, the quantity of fuzzy rules turns out significant and use of expert knowledge connected with the certain difficulties. Therefore with the purpose of control reactive capacity and a voltage in DEN uses algorithm subtractive clustering for synthesis of fuzzy rules. The fuzzy knowledge bases reflecting every possible variant of rules as a result received.

Data subtractive clustering used as fast algorithm for synthesis of fuzzy rules. Besides for ANFIS-algorithm this method is as though an index point for training synthesized fuzzy model. The basic advantage of application clustering for synthesis of fuzzy model consists that rules of the knowledge base turn out object-oriented. It reduces an opportunity so-called «combinatory explosion», i.e. catastrophic increase in volume of the knowledge base at a plenty of entrance parameters. Below questions of synthesis of system of a fuzzy logic conclusion of type Sugeno (fig.2) with application of a method subtractive clustering are considered.

By means of subroutine Clustering [5] packages Fuzzy Logic Toolbox, are executed allocation cluster pair initial data (value of knots voltages and reactive capacities). On fig.3, results clustering on initial data shown and in table 1 value of their fashion given.

For synthesis of fuzzy rules of developed neuro- fuzzy network ANFIS at control of reactive capacity in DEN, results of regime calculations 14 central electric networks IEEE14BUS are used. Calculations executed by means of program complex ETAP [6].

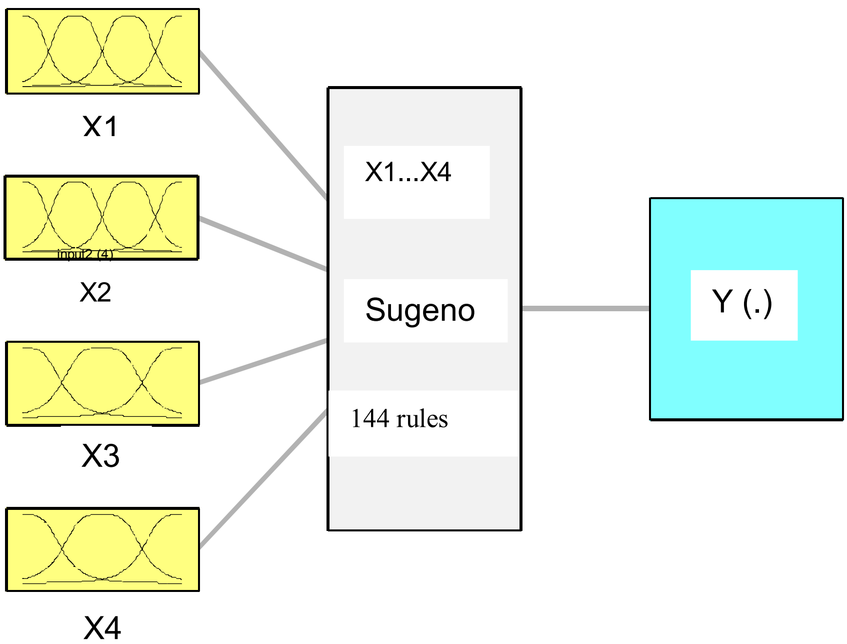
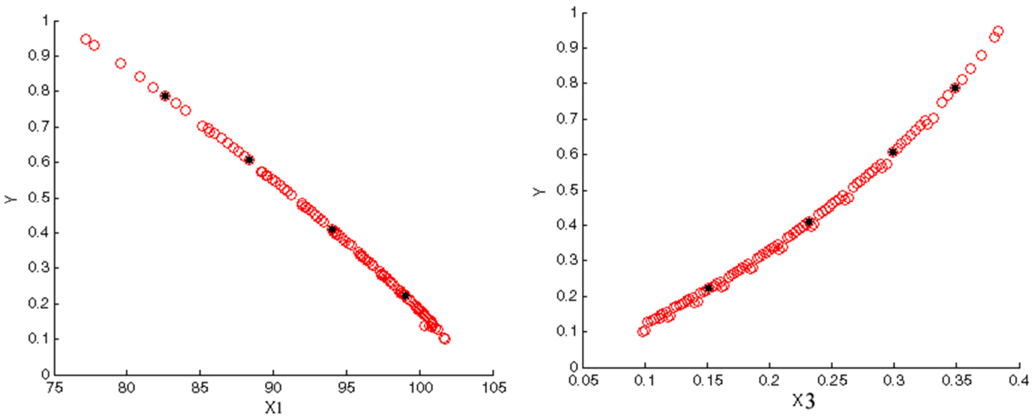


Fig.2. Block diagram of fuzzy Sugeno model



a) b)

Fig. 3. Results subtractive clustering initial selective data

Tab. 1

Value of a fashion found cluster

X1	X2	X3	X4	Y
99,030	99,390	0,152	0,151	0,222
94,060	94,330	0,232	0,231	0,409
88,340	88,320	0,299	0,299	0,606
82,630	82,180	0,348	0,350	0,787

Apparently, from fig.3, for linguistic variables it is enough to accept 4 terms.

At a following stage Sugeno model of the first order with fuzzy rules in quantity 144 synthesized.

Computer realization of fuzzy rules ANFIS-network shown on fig.4.

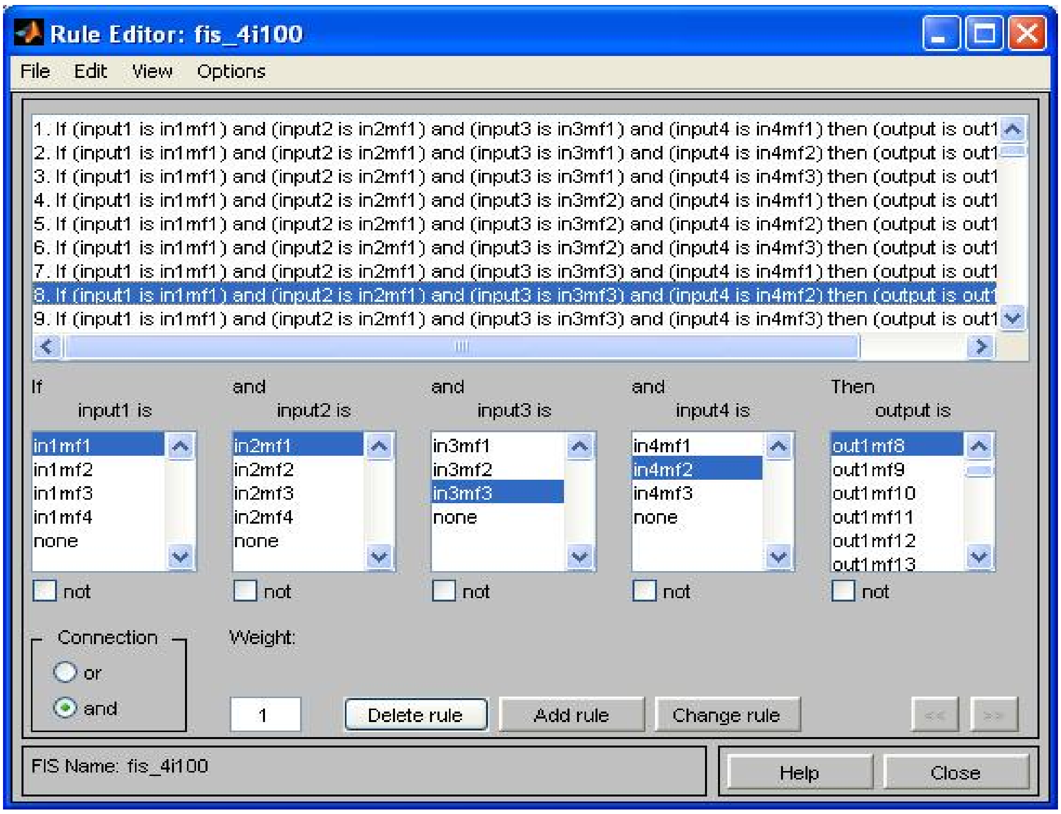


Fig. 4. Computer realization of fuzzy rules of an ANFIS-network

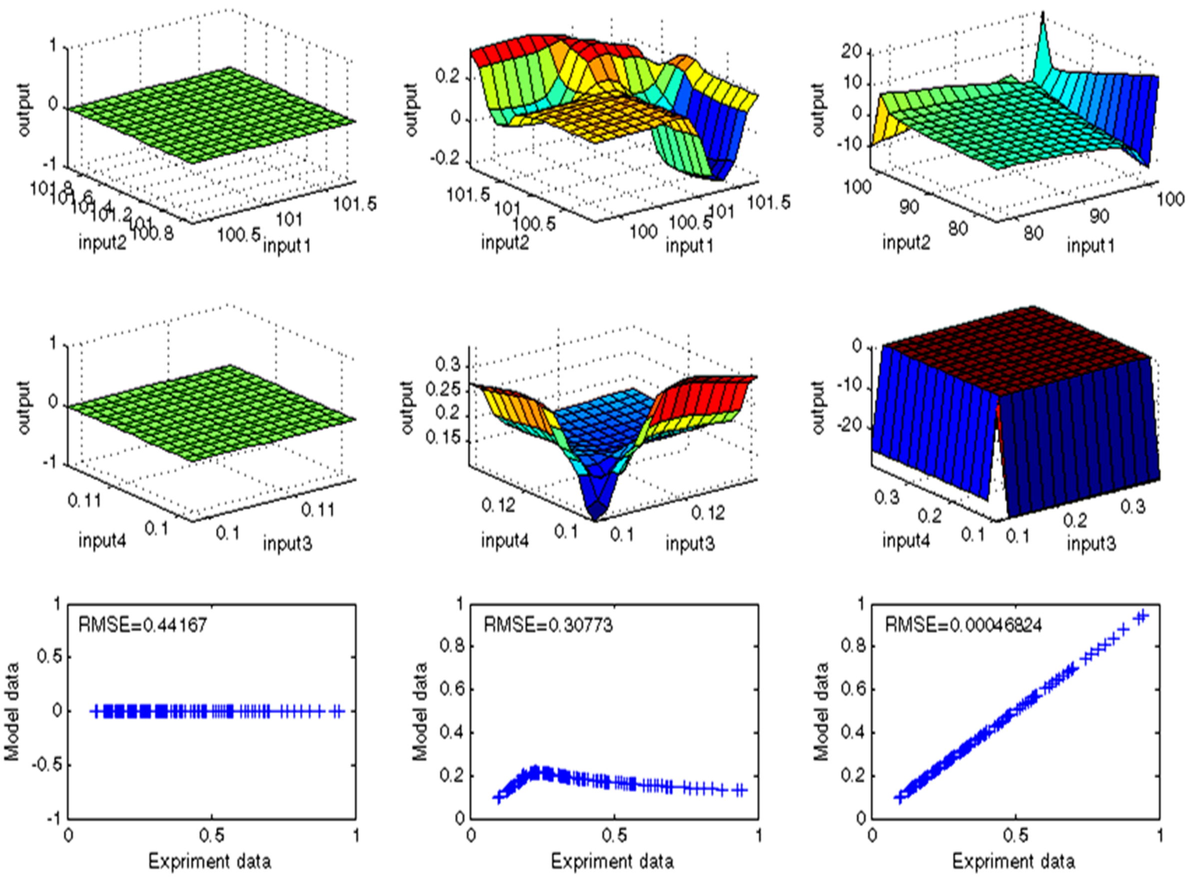
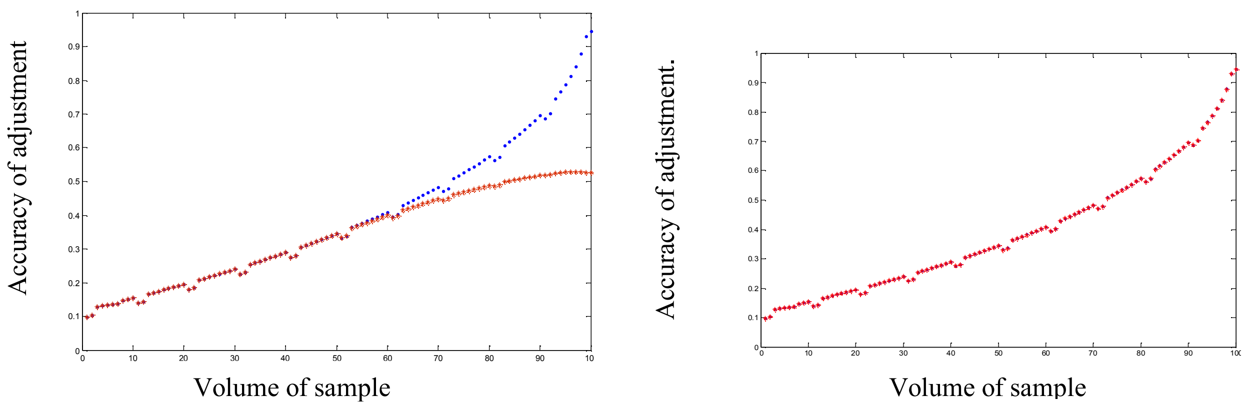


Fig. 5. Adjustment of synthesized fuzzy Sugeno model
a- adjustment; b-after adjustment on 20 samples; c-after adjustment on 100 samples

The surface "inputs-outputs" of initial fuzzy model shown on fig.5a. Apparently, from figure, before adjustment the fuzzy model badly reflects the basic features of identified dependence. Testing of model shows, that an average quadratic deviation (a.q.d.) between settlement data and results of fuzzy modeling makes 0,442. On fig.5b surface of training and distribution a.q.d. between settlement data and results of fuzzy modeling on 20 samples shown. As a result of adjustment, a.q.d. On training sample has decreased with 0,442 up to 0,308. Apparently from figure, after adjustment the fuzzy model well reflects behavior of identified dependence and quality of the synthesized base of fuzzy knowledge improves. On fig.5c the surface of training and distribution с.к.о given accordingly. Between settlement data and results of fuzzy modeling on 100 samples. Apparently, a.q.d. between settlement data and results of fuzzy modeling has decreased up to 0,00047 and the surface reflects identified dependence is better.

On fig.6 dependences of accuracy adjustment on volume of sample shown.

Apparently from fig.6a, the volume of sample equal 60 is not sufficient for full training system that is shown in a divergence between settlements and modeled in operating parameters. From fig. 6б accuracy of adjustment at volume of sample 100 is equal 0,94, that it is possible to consider sufficient for training system.



a) b)

Fig.6. A curve of dependence of errors of adjustment from volume of sample
a-after adjustment on 60 samples; b-after adjustment on 100 samples

CONCLUSIONS

1. Questions of synthesis of terms-subsets and bases of fuzzy knowledge Sugeno of ANFIS-model on the basis of algorithm subtractive clustering for control of reactive capacity and knots voltages in DEN power supply systems are considered.
2. Values a.q.d. between settlement data and results of fuzzy modeling of the synthesized neuro - fuzzy network, received on the basis of results of the limited regime calculations for test scheme IEEE14BUS have shown its adequacy to control of reactive capacity and knots voltages in DEN.

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SOFTWARE RELIABILITY. WHAT IS IT?

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ABSTRACT

This article is about how the confusion of terms can give rise to confusion in the approaches and methods, and bring to a deadlock the whole research area. It doesn't mean that we would like to teach someone or to present "the ultimate truth". It is rather another attempt to declare that "software reliability» does not exist in the form it is treated in most of "traditional" sources.

1 INTRODUCTION

More than 40 years have passes since the term "software reliability" firmly entrenched in the everyday life of modern science. In the early seventies of the last century, programming started to convert "from an art to a science." The growth of software (SW) complexity and its extensive penetration into all spheres of human life have shown that its errors can lead to serious financial losses, human casualties, technological and environmental disasters. In this regard, there arose a need of prediction of software failure and modeling of their consequences that led to the rapid development of a new field of science. In [1, 2, 3] the models of software reliability designed to analyze and forecast the process of defect detection in software products were first proposed.

Over the last years a large number of reliability models have been created. Valuation models allow assessing the approximate number of defects in them on the basis of special metrics before testing or operation of the program. Predictive models based on the study of the process of identifying defects allow to foresee of the development of this process over time, and to obtain estimated reliability of the program. The most comprehensive and systematic presentation of the modern theory of software reliability with a description of the existing models can be found in [4]. However, as the experience shows, none of these models can claim to be a universal one, each model "serves" its class of software systems, boundaries between the classes remain very shaky.

At present, the reliability of software is considered as a special case of the general reliability theory. Practically all reputable textbooks on reliability certainly contain the relevant section with regularly repeated attempts to use the same mathematical tools as for the analogous process in technical systems for the description of defect detection in software. Nevertheless, in the authors' opinion triumph on this way has not been reached and is not expected to occur. If you pay attention to the essence of scientific publications on the theory of software reliability, we can see that in the last twenty years, no fundamentally new ideas in this area have been proposed. In [5] the attempts of mechanical introduction into the existing variable models, designed to take into account the secondary defects, were made. However, according to the authors [5], it is practically impossible to introduce these variables into some models and for some of them such complicated mathematical expressions are obtained, that their practical application becomes difficult. It seems to us that all of the above-stated just give evidence to the above mentioned dead-end, the theory of software reliability has come to.

To find a way out of the dead end, it is necessary to go back to the roots - basic principles, which laid the foundation of a new theory forty years ago. We should find out whether the principles themselves turned out to be false, or incorrect premises were made out of the right principles. It's impossible to break the deadlock without such seditious attacks on fundamentals and recognized experts, only by hanging another patch on an improperly working model.

At the dawn of the development of software reliability theory the phenomenon of obtaining "wrong result" from the program, by analogy with the theory of complex systems reliability, became known as "failure". The first step towards the truth has been made. It was followed by the second one - the ability of a software system to operate without failures was called "reliability." A theory of reliability of complex systems by that time was, first - sufficiently well developed, and second - tested through practice. Therefore, the implementation of methods and mathematical tools of the existing reliability theory to a new physical phenomenon was absolutely natural.

Talking of criticism of modern software reliability, we are not the first here. The inherent differences between software and technical systems were once and again referred to by many authors [6, 7, 8, 9]. Getting back to the roots we should find out whether the use of the term "reliability" to the software was competent. Now, let's talk about terms.

2 WHAT IS "RELIABILITY"?

In theory of technical systems reliability the term "reliability" is defined as a property of the system to perform reliably the functions it is designed to. Consequently, the inherent attributes of reliability are:

1. The time in which the system performs the specified function.
2. Randomly occurring system element failures, leading to a definite inefficiency of functioning or even to a complete cessation of functioning of the system.

Technical systems are subdivided into recoverable and unrecoverable. The first include, for example, vehicles, air traffic control systems, etc., and the second - a system designed to perform a single mission (for example, from household light bulbs to satellites).

The random nature of failures in the system leads to such natural probabilistic characteristics (parameters) as the average run time, the probability of failure-free operation for a specified time, and the availability factor. With the appearance of the theory of software reliability the direct transfer of the concept of complex systems reliability on the software systems was, of course, the easiest way. For example, the standard [10] defines software reliability as the probability of failure-free operation for a specified period of time in an appropriate environment. Similar definition is in monograph «Handbook of Software Reliability Engineering» [4]. Herewith, software failure itself is treated as an event where the given result is different from that which would be expected from an ideal program. Most often the fact of software failure is actually determined in a forensic way, with the help of a man - an operator. If there were no operator control, the development of operated that way software process could become unpredictable.

Further, the failure in the existing theory of software reliability is thought of as a random event. However, its randomness of this event is placed in question by many authors [6, 7, 8, 9]. In particular, in [6] it is noted: «… Errors caused by software have no stochastic nature: they will repeat as soon as some conditions will be repeated. Errors of software, in a sense, are not "objective", they depend on type of operations, type of inputs and, at last, on type of users. … Errors caused by software do not depend on time in a usually understandable way: if you don't use software it cannot fail! At a pinch, in this sense software can be compared with a spare unit which can be used but nobody knows the timing of this usage. At last independence of errors. There is no such concept as a "sample" for software: there is a phenomenon of cloning. "Replacement" of "failed" software has no sense! You will change one Mollie for another Mollie with the same genes, with the same illness, with the same properties».

Six years later, in [7] the author draws the attention again to the impossibility of mechanical transfer of the reliability theory fundamental principles of technical systems to the software. The chapter on the software reliability ends like this: «… attempts to put "hardware reliability shoes" on "software legs" are absolutely wrong and, moreover, will lead only to a logical dead end". Thus we can come to a conclusion that for the six years between the publications of the said articles, the theory of software reliability has not undergone major changes, and the term “reliability” as it is understood in the technical systems cannot be used for software.

Its reason lies in the fact that stochastic apparatus of reliability theory is practically fully used for the description of processes in software. But having questioned the random nature of software failures, we automatically put into question all modern probability theory of reliability as well! Is it good or bad? We consider it to be very good. In the modern theory of software reliability there are more questions than answers. Therefore, the rejection of willful false assumptions may allow the creation of a new, more reasonable and “workable” theory of software reliability..

3 WHAT IS “SOWTWARE”?

Let’s study the notion of “software” or “program”.

The international standard ISO/IEC 2382-1 in the section “01.01.08 . Software” indicates that the software is “Any part of Programs, Procedures, rules and associated documentation of an Information processing system”. Thus, the reliability of the software – means “the reliability … of programs, procedures, rules and documentation”. Already here, in this only line there is so much confusion that such document should be treated either as the Scriptures (taking all without hesitation and doubt), or we should try all the same to critically evaluate the resulting. After all, it turns out that when talking about the reliability of software, we talk about the “failure-free” procedures, rules, and even the documentation! And what, if I may ask, is “failure-free documentation” or “MTBF”? So where is, I beg your pardon, a coincidence here? And where is the “time of use”? Let's see, if the term “reliability” can be used only for the part of this definition for the program? The same standard in the section “01.05.01. Program; Computer program” defines program as “Syntactic unit that conforms to the rules of a particular Programming language and that is composed of Declarations and Statements or Instructions needed to solve a certain function, task, or problem”. A close examination of this definition raises a number of questions again (Fig. 1).

Suppose that some programmer wrote his program in “C” in strict accordance with its rules. This correspondence is confirmed by the fact that during the compilation of the program no warning messages were even issued. After the compiler operation, this program turned out to be converted into an object code, but by the rules of a completely different language, which corresponds to the assembly language of the used processor. Formally, according to the definition, we are dealing with another program that conforms to a completely different language. However, on the other hand, from the point of view of the main functions of this software – controlling the computer hardware - the program has not changed. Just the form of its presentation has changed, nothing more, and the information content of the program has remained the same. Obviously, we assume herewith that the compiler translates high-level language operators into object code correctly. But then it turns out that the program is not a “syntactic unit that conforms to a particular programming language rules”. From the point of view of the main control functions of the program, it is nothing more than information that controls the computer's hardware. And then the syntactic rules of the language are nothing but the rules of coding this information. The encoding rules are subject to change - information in such a case should remain unchanged. This information defines a sequence of computer operations to transform input (initial) data to output data- that is the result of its operation. Therefore, we can state that the program is encoded programming information in the form of instructions of a specific programming language about the way of converting the input data set into the output.

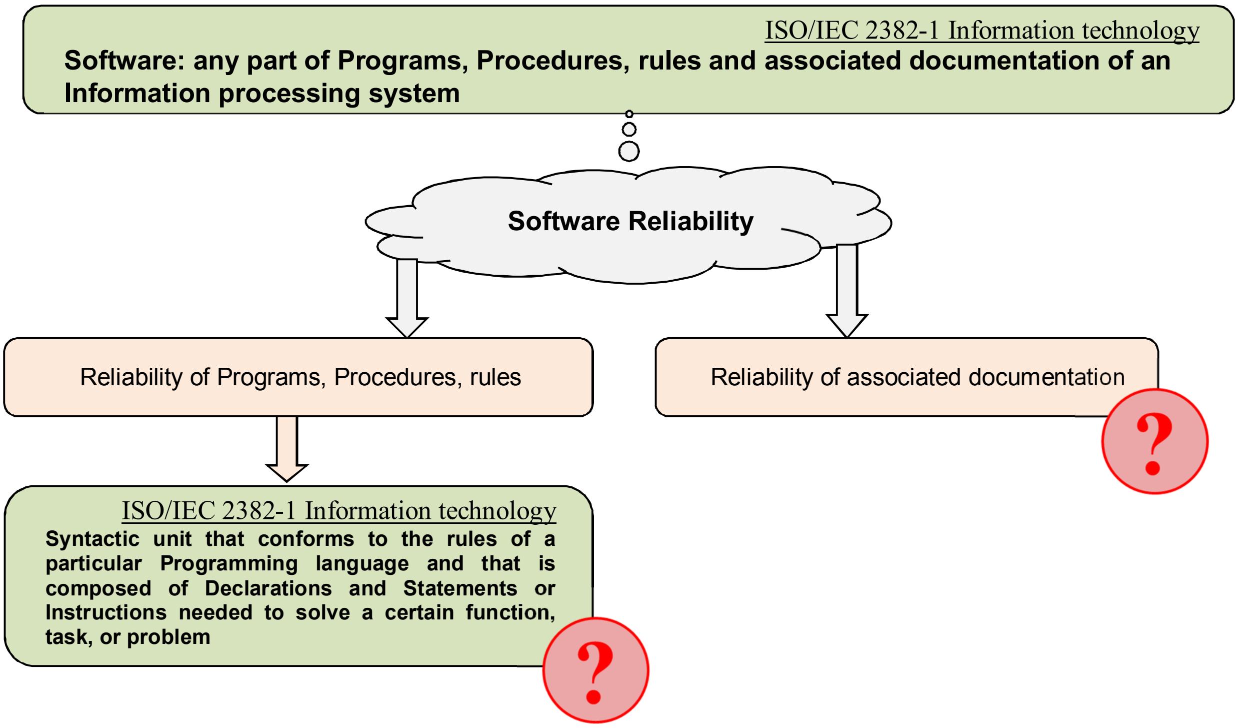

Fig. 1 Structure of the definition “Software”

Thus, the program is information. This information should be presented in a certain way. Such presentation comprehends a particular programming language with its own rules. Anything can act as a storage medium - from a sheet of paper with hand-written source code to the hard drive with binary code. The essence of the program, as information, herewith remains absolutely unchanged. Thus, speaking of the reliability of the software, we have to comprehend the reliability of information. However, the term “reliability” is unacceptable for information.

4 FAILURES OF SOFTWARE – DOES IT EXIST?

In 1995 Nozer D. Singpurvalla (George Washington University) published an article [8], the name of which posed the question: “The failure rate of software: does it exist?” In the article it was stated that the notion of failure rate with respect to the software has no mathematical sense. Thus the author writes: “… it does not make mathematical sense to talk about the failure rate of a piece of software. Furthermore, even when it does make sense, the failure rate is personal, and does not exist outside of the software engineer's mind”. This is due to the fact that the occurrence or non-occurrence of software failures depends not only on the software itself, but also on the person using it. Indeed, let’s imagine a case where the used software performs merely two functions of any kind. (Fig. 2).

The program code of the first function is perfect, and the second code is written very negligently. Then the first user who only uses the first function of the program, will state with full right that it always works correctly. However, the second user, who needs only the second function for work will constantly deal with incorrect results and assume that the program is unworkable. And most interestingly, both of them will be right. It turns out that the failure rate is characterized not so much by the software itself as by its members. The result obtained is remarkable, especially when you consider that all models of software reliability in some way try to calculate specifically the failure rate and use it to obtain the other reliability factors [4].

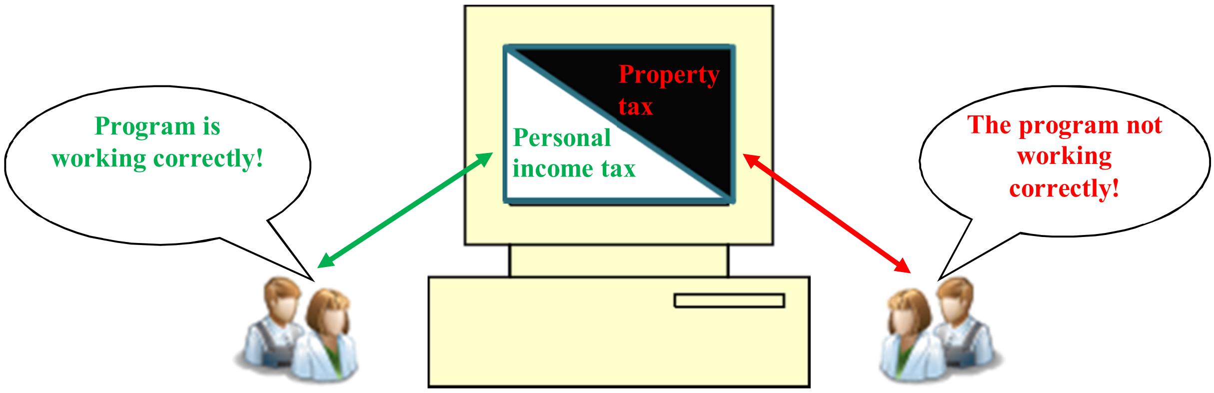

Fig. 2. So how does the program work?

All the above mentioned allows us to go further and ask the question – do any software failures exist at all? The traditional reliability theory defines failure as the event of violation of operational condition of an object. With the failure of the software it's not all that simple. In fact, the program may fail, performing some task that lies "outside" its tested area. Or the software developer can even make a mistake in the very logic of the program. After identifying the program defect (in coding or solution logic), the program gets finalized and becomes able to handle the problem unsolvable before (assuming that during updating the new "secondary" defects were not introduced).

5 CONCLUSION

The authors are well aware that, after fourty years of use in many thousands of scientific publications and hundreds of textbooks, the use of a new term may lead to embarrassment, as if it is something parallel, and the "software reliability" in the old sense, continues to live and thrive. However, it seemed to us that it is necessary to dot once again all the "I"s .

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MODELS, METHODS AND MEANS FOR SOLVING THE CHALLENGES IN CO-DESIGN AND TESTING OF COMPUTER SYSTEMS AND THEIR COMPONENTS

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ABSTRACT

This paper is addressed to a problem of development of the resources used for solving challenges in co-design and testing of the computer systems and their components. Both target and natural are considered. Target resources include models, methods and means. Natural resources like particularities of the target resources are examined. Natural development of target resources with structuring under particularities of the Universe which is considered like parallel and approximate is analysed. Natural resources in remove of contradicts between target resources are shown.

1 INTRODUCTION

The artificial Computer World created by the human takes a special place in development of the Universe and its components. The Computer World is most dynamically developing area of knowledge and human creativity. For short historical term the huge way of development which analysis allows coming nearer to understanding of laws of this development and development of the World as a whole is gone.

Whether creation of the Computer World casual or natural is? What role is allocated to the computer World in modern development? What role is played in this development by the human, and in what degree he is free in a choice of solved problems and decisions? These questions are considered in section 2 operating with concepts of models, methods, means and target resources as a whole for the decision of problems of synthesis and the analysis, including challenges of co-design and testing of computer systems and their components.

What basic directions of development of target resources in co-design and testing of computer systems? What are the natural resources? How they can be shown and used? How the development of resources is stimulated? Answers to these questions are offered in section 3 using examples of development of the models, methods and means in information and computer technologies..

2 THE WORLD CREATED BY THE HUMAN

2.1 THE WORLD CREATED BY THE HUMAN

Development of the Computer World can be analysed as process of decision of the challenges, including problems of co-design and testing in computer systems and their components.

The problem can be solved at performance of two conditions: execution of a set of works for limited time achieving the certain productivity, and reception of reliable results. The decision of the problem has also the third condition – investments in this decision of the certain resources which further refer to target.

2.2 TARGET RESOURCES

Target resources contain all necessary for solving a problem: models, methods and means.

Models are our ideas of the Universe and its components. We think using models. Methods describe the transformations which are carried out with resources. We operate using methods. Means allow realizing these transformations. Models and methods can be related to an information part of target resources, and means – to technological one, including materials and tools, made in one's turn with use of models, methods and tools.

2.3 STAGES OF ACCUMULATION OF THE INFORMATION

The human is the tool in the decision of a challenge connected with development of models, methods and means. Models are our knowledge, and methods are skills. These information resources make two parties of human experience. In this respect creation of the Computer World is not casual. It is one of stages in accumulation of the information in the form of human experience. The previous stages: both the writing and the publishing were marked in speech keynote of academic A.P. Ershov (Ershov 1981). They were preceded with a stage of accumulation of human experience using its transfer "by word of mouth". Development of writing has raised accuracy of both kept knowledge and skills. Publishing also has considerably increased a level of replication, which promotes distribution of information and increase of reliability in process of its accumulation.

With development of both information and computer technologies the stage of human experience formalization began. On the one hand, this is creation of databases and the knowledge connected by networks. Search systems provide a high degree of accessibility to the data that becomes a necessary condition for using these data in view of prompt growth of their amount. On the other hand, formalization of human experience was representing in development of the software with the prepared decisions of problems. Once the written program of function evaluation *sin x* for ever has attributed this problem to set of closed problems as well as the program of joining of spacecrafts the Soyuz - Apollo.

It is necessary to note, that process of information accumulation passed and occurs also outside of the human cerebration, for example at a genic level. Training which begins with fastening conditioned reflexes at worms and proceeds up to complex behaviour at mammal and birds (Akimushkin 1991), also are stages of accumulation of the information. These stages have begun before accumulation of human experience. Therefore it is possible to assume, that development of resources by the human concerns to intermediate stages of accumulation of the information which has more common nature.

3 DEVELOPMENT OF RESOURCES

3.1 A RESOURCE AS AN ELEMENT OF THE UNIVERSE

Development of resources can be considered from two positions: organizational and functional. In the first case the resource is represented as system of elements. The second position shows a resource as an element of system in interrelation with other elements.

All resources are elements of such system as our Universe, which itself cannot be investigated as an element of system. Studying of features of the Universe probably only by research of its elements as well as these elements inherit features of the Universe, being structured under its realities.

Development of the Computer World most obviously shows particularities of the Universe, characterizing it as parallel and approximate. The growing level of parallelism and fuzziness testifies to it in the decisions of challenges in co-design and testing of computer systems and their components. As an example development of personal computers can serve. Parallelism of structural decisions has passed a way of development at both a circuit and system level from realization of consecutive and series-parallel operations in structure of arithmetic-logic units up to single-cycle iterative array circuits, projected for performance of each operation in structure of pipeline.

Processing of the approximate data has received development from co-processors of non-obligatory delivery up to several floating point pipelines in structure of the CPU and up to many thousand floating point pipelines in the graphic processor with its use for performance of parallel calculations on technology CUDA (Guk 97, NVIDIA CUDA 2007).

The Universe is the generator of the approximate data. All in this Universe exists in admissions (tolerance). Results of measurements are the approximate data. Therefore the importance of computer processing of the approximate data permanently grows.

Both models and methods also develop from exact to approximate, changing representations about their adequacy in relation to features of the Universe.

For example, the number has passed a way of development from the codeword up to approximate representation in floating-point formats with two components: a significant and an exponent. Codeword size determines strong dependence between accuracy and size of range for exact data. The sizes of significant and exponent independent determine accuracy and range size in a floating-point number. This independence or parallelism follows from the features of parallel and fuzzy Universe which produces different requirements to accuracy and range of data.

The model of arithmetic operation has passed a way from complete exact operation to approximated truncated one executed in simultaneous units (parallel adders and shifters, iterative array multipliers and dividers) using floating point formats with single accuracy (Kahan 1996).

A model of the calculated result is transformed from exact representation of a number up to approximate one, which has high exact most significant bits (MSB) and low non-exact least significant bits (LSB). Such transformation develops correct result into reliable one which can contain the errors caused by the circuit faults in LSB. These errors are inessential for reliability of result (Drozd 2003).

Increase of a level of parallelism in solved problems makes exact methods inefficient, replacing them on approximated. For example, testing of the software products containing thousand of modules becomes approximated (Pomorova 2009).

Thus, development of target resources occurs by the natural way being structured under features of the Universe including first of all its parallelism and fuzziness.

3.2 NATURAL RESOURCES

Target resources are a cost-based part of the task decision. First of all it is obvious to technological resources. Payment of information resources can enter cost of the tools developed with use of valuable models and methods. Information resources can be also freely distributed as are paid by work of the previous generations of researchers.

Decision of a challenge can use not only paid target resources but also the natural resources which are free as well as they are given to target resources like their particularities. Free character of natural resources makes their attractive defining a problem of their study.

Two kinds of natural resources are most known. There are natural information redundancy and natural time redundancy used in on-line testing of the digital components (Savchenko 1977, Romankevich 1979).

Definition of natural resources as particularities of the target resources essentially expands set of their kinds. As a rule, the challenge is solved stage by stage. Particularities of the target resources involved at the previous stages can be used as natural resources at the following stages.

Use of both kinds of natural redundancy is completely described by such model of natural resources activation as these kinds are pawned on a design stage of digital systems and their components, and are used at the decision of the following problem of on-line testing.

It is necessary to note, that resources grow together being structured under the same features of the Universe. It can be observed regarding their organization. For example, expansion of a set of solved problems is carried out by increase of productivity, reliability and also due to resource-saving. The basic approach to increase of productivity will consist in replicating operational elements and perfection of functions for a choice of results from parallel branches of calculations (Guk 2003). Reliability of results increases using fault-tolerant structures which contain replicated operational elements and functions for a choice of reliable results (Ushakov 2003). One of approaches to power-saving consists in downturn of frequency of the operational elements work that is compensated by duplicating of operational elements and use of functions of a choice (Chandracasan 1992).

It is important to notice that replication of operational elements is the first step of parallelism development. At the following steps parallelism transforms operational elements into versions with various kinds of diversity (Kharchenko 2008) reducing of both amount and significance of the common parts of versions, rising of their independent degree and development of their particularities.

Following development of the structures from replication up to diversity are supported by a method of the results preparation which under various names grasps space of decisions behind a clear advantage (Drozd 2004).

Thus, resources in the development get the similar structure working on all components of the decision of a problem: productivity, reliability, and resource-saving. Natural narrowing of target resources to each other on uniform base developed under common features of the Universe eliminates contradictions between them demonstrating natural resources.

3.3 MOTIVATION OF RESOURCES DEVELOPMENT

Natural development of resources is stimulated by the method of carrot and stick. A stick is natural selection, and carrot is gifts as realization of natural resources. Increase of both efforts and spent resources leads to decline of result in case of move against the stream of Universe development.

This issue can be shown considering reliability of the on-line testing methods which is estimated for digital circuits with use of both probability of an essential error and error detection probability. The probability of an essential error is the basis characteristic of the computing circuit as object of on-line testing. The main characteristic of the on-line testing methods is the error detection probability. Reliability of on-line testing methods can be visually considered using both of probabilities in unit-side square shown in Figure 1 (Drozd 2006).

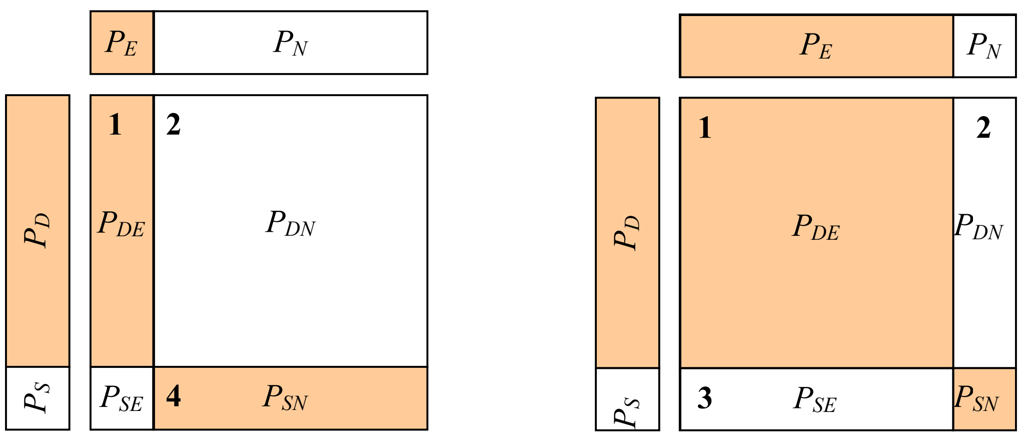

Figure 1. Reliability of on-line testing methods
a – for traditional methods; b – for residue checking of truncated operation

Horizontal side of this square contains a sum of the probabilities P_E and $P_N = 1 - P_E$ that the occurred error is essential and inessential. Vertical side of the square contains a sum of the probabilities P_D and $P_S = 1 - P_D$ of error detection and error skipping.

The square is splitting into four parts that define the probabilities connected by the following formula:

$$P_{DE} + P_{DN} + P_{SE} + P_{SN} = 1,$$

where $P_{DE} = P_D P_E$ and $P_{DN} = P_D P_N$ are detection probabilities of an essential and inessential error accordingly; $P_{SE} = P_S P_E$ and $P_{SN} = P_S P_N$ are skipping probabilities of an essential and inessential error accordingly.

The on-line testing method has reliability in checking the result detecting essential errors and skipping inessential ones. This reliability contains the probabilities of the first and last parts of the square:

$$R = P_{DE} + P_{SN} = P_D P_E + (1 - P_D)(1 - P_E). \quad (1)$$

The features of approximate calculations significantly reduce the probability P_E. Operation of multiplication used in representation of approximate data in floating point formats twice reduces probability P_E for complete arithmetic operations. Additionally this probability is reduced in both operations of denormalization and normalization of the mantissas in results of all previous and following operations accordingly. That's why the traditional on-line testing methods, such as both parity prediction and residue checking (based on self-checking circuit theory (Anderson, 1973)), which check the complete arithmetic operations with high error detection probability $P_D >> P_S$ have very low reliability. This fact is shown in Figure 1, a, where high error detection probability is basically used for detection of inessential errors. It leads to reject erroneous but reliable results and reduces reliability of the traditional on-line testing methods. Increase of error detection probability reduces reliability of the traditional on-line testing methods.

According to the formula (1), the high probability of error detection $P_D > 0.5$ can provide high reliability of on-line testing methods $R > 0.5$ only in case of high probability of an essential error $P_E > 0.5$. It is achievable only at performance of the truncated operations. These operations become effective at performance of two conditions. First, computing circuits receive development up to a parallelism level of the simultaneous. The second condition will consist in development of operation up to a level of calculation execution with single accuracy when size of mantissa in result is inherited from mantissa of operand that is typical for floating point formats (Goldberg 1991).

In these conditions the truncated operation almost twice simplifies the digital circuit and reduces time of calculations (Savelyev 1987, Rabinovich 1980). Except increase of reliability in on-line testing with high error detection probability becomes possible as it is shown in Figure 1, b.

Other way of increase in reliability of the on-line testing methods is realized for the most widespread case of low probability $P_E < 0.5$ under condition of error detection with low probability $P_D < 0.5$. This way can be considered by the example of a known on-line testing method which checks iterative array squarer using the forbidden values of the result residue by modulo. Iterative array squarer uses operand A for calculating result $Q = A^2$. The error detection circuit executed by this method is shown in Figure 2 (Drozd 2008).

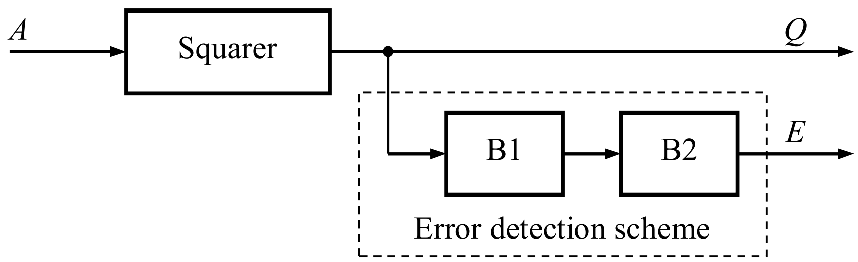

Figure 2. Error detection scheme of a squarer

The circuit contains two blocks. The block B1 calculates a result residue by modulo. The block 2 forms check code E, which identifies the forbidden values of residue R.

Let's consider the checking for modulo M = 15. Then on first half of values of the modulo 15 number A accepts values from 0 up to 7 for which values of squares and their residues by modulo accept values 0, 1, 4, 9, 16, 25, 36, 49 and 0, 1, 4, 9, 1, 10, 6, 4, accordingly. On the following half of modulo values the residue accept the same values only upside-down, as $A^2\ mod\ M = (M - A^2)\ mod\ M$. The calculated residues make set of the allowed values 0, 1, 4, 6, 9 and 10. Other values of the module are forbidden. Accepting equal probability of occurrence of any value of number A frequency of occurrence of the allowed residues is defined. The zero meets only once on set of the result residues, both residues 1 and 4 meet twice in half of this set, and the residues 6, 9 and 10 once. Hence, the allowed values meet with frequency 1, 4, 4, 2, 2 and 2, accordingly.

Typical faults of iterative array squarer deform result on weight of any one bit, determining a kind of errors as $\pm 2^W$, where W – number of the result bit. Amount N of such errors by modulo is final. Errors by modulo M = 15 accept values ±1, ±2, ±4 and ±8. The error is detected, if the sum of its and the allowed value is equal to the forbidden value.

Frequency of error detection in case of receiving a forbidden value is shown in Table 1.

Table 1. Frequency of error detection

Z	*Y*								*S*
	1	2	4	8	-1	-2	-4	-8	
2	4	1		2		4	2	2	15
3		4		2	4				10
5	4		4		2		2		12
7	2					2		1	5
8		2	4	1	2	2		4	15
11	2	2					1	4	9
12		2		4			4		10
13			2			1		2	5
14			2	2	1	4			9

Lines and columns of table contain forbidden values Z and values Y of errors by modulo 15, accordingly. Last column contains sums S_r of elements in lines.

The probability of error detection can be appreciated as $P_D = S / (M \cdot N)$. The maximal value of probability P_D is determined in case of the check of all forbidden values when $S = 90$ and $P_D = 0.75$. The minimal value of probability P_D is calculated at the minimal sum of values S_r for lines which nonzero elements cover all errors. Minimal probability P_D is achieved checking only two forbidden values 11 and 14. It is equal to $P_D = 0.15$.

In case of checking of the exact data all errors are essential, $P_E = 1$. It determines reliability by the formula (1) as $R = P_D = 0.75$. For the approximated calculations which are executed with probability $P_E = 0.1$ the reliability is estimated for cases of maximal and minimal probability P_D as $R = 0.30$ and $R = 0.78$, accordingly.

Thus, the increase in amount of checks from 2 forbidden values up to 9 reduces reliability of on-line testing method from 0.78 down to 0.30 in 2.6 times.

The important role in natural development of resources plays a method of the results preparation. Thus method allows starting to solve a task before obtaining of all initial data, simultaneously (in parallel) their formation. This determines approximate way for solving a task firstly receiving a set of possible results. One result is selected from set of possible results on receipt of the missing data (by using of them).

Efficiency of the method is explained by structuring in particularity of the parallel and approximated Universe. Comparing its parallelism with iterative array and pipeline structures, it is necessary to note, that paralleling of calculations two types of dependences interfere: on the data and on control. In the first case operation is executed consistently to formation process of initial data as results of the previous operations. In the second case the branching of algorithms obstructs to paralleling of calculations. Matrix parallelism is realized at absence of both types of dependences. Pipeline parallelism removes dependence on the data. A method of results preparation (and only it) removes dependence on control. The method of results preparation not only reduces time of calculations, but simultaneously reduces expenses of the equipment, traditional opposed to speed.

The most simple and high-speed (on half of bits of the address) the realization of a memory which is carried out on architecture 2.5 D (Ugryumov 2004), also is an example of using a method of results preparation. In blocks of memory with structure 2.5 D hardware expenses for decoding of the 16-digit address are reduced 85 times (Drozd 2012).

Efficiency of a method considerably grows at a choice of the several prepared results. So libraries where sets of results continuously are prepared and get out are constructed. All modern co-design of digital systems and components is based on a method of the results preparation. For example, every FPGA chip is initially preform for set of projects, and the chip programmed under one project is preparation of results (for various input data) in the tables which have been written down in memory LUT (Altera Corporation, 2004).

Due to structuring into a reality of Universe development FPGA-projects receive the features allowing to provide at a high level a set of characteristics: productivity of calculations and reliability of their results, universality, efficiency of designing, adaptability to manufacture, flexibility of decisions, and the most important advantage which is the combination of achievable levels testifying to their mutual consistency.

Development of resources by the way of elimination of contradictions can be taking into account for predicting such development in concrete applications. For example, now there is a contradiction between checkability, playing a special role in digital components of safety-critical systems (Drozd 2011), and power-saving. For maintenance of checkability it is necessary to train points of digital circuits by their switchings making the basic part of a dynamic component of power consumption. Therefore it is possible to assume, that in digital circuitry will win the principle of the accumulator used, for example, in movement of electric trains when at their dispersal the electric power is consumed, and in a mode of braking comes back. Similarly, switching of a voltage in points of the circuit from a low level up to high should consume energy, and in case of return switching to return. Then the current consumption will be influenced not with the sum of amounts of direct and return switchings, and their difference which is coming nearer to zero.

4 CONCLUSIONS

The resources used for the decision of problems of both synthesis and the analysis, pass a way of natural development, being structured under features of the Universe. Such development most clearly is shown in the Computer World which is artificial created by the human. Models, methods and means which are target resources for the decision of problems of co-design and testing of computer systems, permanently raise a level of parallelism and the fuzziness inherent in the Universe.

Creation of the Computer World is natural as well as formalizations of human experience is an obligatory stage during accumulation of the information.

Development of resources by the natural way is motivated by the method of "carrot and stick" using natural selection on the one hand and gifts as realization of natural resources on the other hand.

Natural resources can be considered as particularities of the target resources used at previous stages of the decision of a problem. Use of natural resources at the following stages can considerably simplify the decision of a problem and increase parameters of result.

Structurization of target resources under the same features of the Universe grow together them, showing natural resources in elimination of traditional contradictions between target resources. The traditional contradiction between speed and expenses of the equipment is eliminated at development of arithmetic operation up to a level of the truncated operation which are executed in single-cycle devices with single accuracy. Simplification of the device and reduction of operating time is achieved simultaneously with increase of reliability of on-line testing methods using high probability of error detection. The method of results preparation which realizes a high level of parallelism and fuzziness shows elimination of many contradictions.

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A NEW APPROACH TO PERFORM ACCELERATED RELIABILITY TESTING OF LOW TENSION MOTORS

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ABSTRACT

Most of the engineers finds it very tedious to do accelerated reliability test of any motor product if it has to be mechanically loaded as it involves lot of difficulties in motor's loading capabilities and its size, which also directly impacts the development time and cost. This is why all the sequences in accelerated reliability test are done on no-load condition which gives results but are not accurate since in field motors are always under loading condition. To overcome all these difficulties, the paper proposes a new approach to perform accelerated reliability testing of low tension 3 phase motors under full load without actually coupling mechanical load to the motor shaft. The approach is simple and easy to implement. In this approach, instead of mechanical loading motor is equivalently loaded just by giving a supply voltage approximately 20-25% greater than the rated voltage to get same load current similar to fully loaded motor across all the three phases. Therefore, the application of proposed approach to perform accelerated reliability testing is experimented on low tension motor and results were discussed in detailed in the paper. From our practical work, the proposed approach has proven more accurate results compared to the results of previously conducted Reliability analysis done on no-load condition. Hence, the approach will definitely leads to get faster and better results in a simple and cost effective way.

Keywords: Accelerated testing, HALT, motor loading schemes, motor test set ups

1. INTRODUCTION

Reliability has continued to be prime concern of the motor designers and manufacturers since induction motor is the workhorse for modern manufacturing and process plants. It has been clearly seen that some form of "accelerated reliability testing" is required to detect latent failures modes in the laboratory which can pose reliability problems during field, since large scale manufacturers cannot wait for the field results to establish the reliability of their products. An accelerated Reliability test like HALT (Highly accelerated life testing) is an advance testing technique used to identify the design weaknesses, failure modes and product design margins (operating limits). In such test an aging/invoked deterioration of an item will induce to produce normal failures in a very short time by operating at stress levels much higher than would be expected in normal use. But conducting "accelerated testing" on motors would be a challenging task for the engineers as it involves many difficulties in motor loading capabilities and motor sizes. Also, to do so one should have large reliability test facilities this requires high development cost and time. To avoid these difficulties, in general, most of the engineers perform these accelerated tests at no-load conditions. However, it would be inadequate as we aren't simulating field conditions so that test results may not be as same as field results.

From literature survey it is observed that a number of loading schemes have been developed without actually loading the motor such as dual frequency method, phantom loading method, synthetic loading method, variable inertia test [1]–[4] but these methods have their corresponding pros and cons. Recently Metwally [5] proposed the concept of "Loadless full load method" for equivalent loading of 3-phase induction which is simple and cost effective. The method is simply to connect the motor under test while it is unloaded to a supply of voltage higher than the rated voltage of the motor by about 20-25%. It has been found that a voltage of 120% of the rated voltage of the motor under test is a suitable value for motors of different sizes and speeds. This value of voltage circulates a current which exceeds the full load current for small size motors and stays less than the full load current for large size motors. This method does not require any mechanical load to be coupled to the motor shaft. The motor draws only the full load losses from the electrical supply. Hence, there is no need for some arrangement to either dump an electrical power equal to the rated power of the motor under test or to return it to the supply. To validate Loadless full load method, an experimental study has been carried out and the results have provided confidence to use this method.

Therefore, in this work, the concept of "Loadless full load method" is adopted for equivalent loading of 3 phase motor and conducting accelerated reliability testing on them. This approach helps us to simulate the field conditions i.e., full load condition instead of no-load condition, and also avoids motor loading setup with high development cost and time.

2. PROPOSED ACCELERETED RELIABILITY TEST SET-UP FOR 3-PHASE MOTORS

Figure 1 represents a schematic diagram of proposed laboratory layout of Accelerated Reliability test set up. The proposed test setup is very simple, cost effective and easy to implement. This is because as the setup completely avoids mechanical load set up. The set up includes a test motor; 3-phase power source to run motor; auto transformer to get required over voltages; power instrumentation with data acquisition system to monitor and store line to line voltages and line currents at motor terminal during the entire test; thermocouples and a data logger to monitor temperature rise in and around test motors; and a environmental chamber to create temperature and humidity stresses on motor at much elevated stress levels.

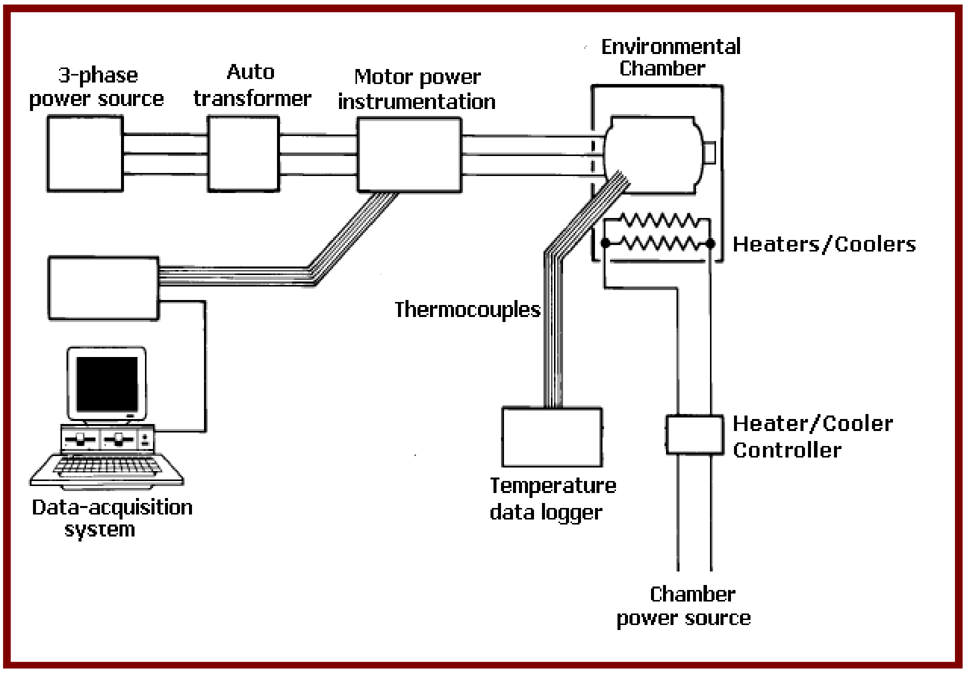

Figure1: Schematic diagram of proposed laboratory layout of Accelerated Reliability test setup

3. EXPERIMENTAL WORK TO VALIDATE "LOADLESS FULL-LOAD METHOD"

Tests have been conducted on Crompton Greaves make 7.5 hp 4P TEFC SCR Motor to validate the method i.e., achieving full load current by increasing input voltage by approx. 20-25% without actually loading the motor. Both the full load tests at rated voltage (415V) and no-load test at 23% above the rated voltage (510V) were performed with the help of motor testing equipment i.e., RDS®, M.E.A. Testing System for a duration of three hours each. Using motor test system performance test data has been captured for every 5 min of operation in both tests as shown in Appendix-I, Table 1-2. Table 1 present the motor performance test data conducted at rated voltage and Table 2 present the motor performance test conducted at above 23% above the rated voltage. Respectively the temperature rise graphs for both the cases are also captured as shown in Appendix-II, Figure 2.

These experimental results were compared each other and the results confirmed that the full load current is same in both the cases. Hence this method is used for equivalent loading of motor during accelerated reliability testing of motors.

4. PROPOSED APPROACH TO EXECUTE ACCELERATED RELIABILITY TESTING – A CASE STUDY ON 20HP MOTORS

An accelerated Reliability test like HALT is conducted on Crompton Greaves make 15 kW 4P TEFC SCR Motor (ND160L) to determine its operating limits and to identify design weaknesses/weak components. A step-by-step approach to execute HALT is explained as below:

Step1: Conducting design verification test to confirm product specifications

Before doing HALT, it is necessary that the motors to be tested should pass design verification tests which performed within the specified limits. This is because, in general HALT stress levels will start above specified stress levels.

Step2: Development of HALT test plan

To develop a HALT test plan one should have good understanding of product like its functionality, specifications, applications, applicable field stresses, critical parameters to reliability (CTR), failure criterion etc. Since motor is not a new product, it is known that in the field motors experienced gradual/rapid deterioration is due to combined effect of electrical (like voltage fluctuations), mechanical (like vibrations) and environmental stresses (like temperature, humidity. Therefore all these stresses are considered in HALT test cycle as shown in Table 3 and the test cycle repeats in a stepped manner.

Cycle	Voltage (V)	Current (amps)	Duration	Load / No load	Combined Stresses		
					Temperature	Humidity	Switching (On/ Off)
1	510	27.5	3 hrs	Full load	50ºC	95% RH	50 min On & 10 min Off
Total duration per Cycle = **3 hrs** No. of Switching per cycle = **3** No. of samples to be tested = **3**							

Table 3: Test cycle plan

In general, HALT is a series of tests. The following typical tests were performed during HALT on motors and corresponding stress profiles are shown in Fig. 3-6.

1. Vibration step stress test
2. Low temperature step stress test

3. High temperature step stress test with humidity
4. Thermal Cycling test

Step3: Design and development of HALT test up

Proposed Accelerated Reliability test setup (explained in section 3) was designed and developed to execute the above series of tests

Step4: Execution of HALT

Motor to be tested is kept inside the environmental chamber to create control environment and connected to a power source which supplies input voltage of 23% greater than rated voltage to get full load current on motor winding. To measure/monitor temperature rise of the motor, thermocouples were positioned at different locations of motor body and stator windings. Before starting the test, major motor parameters are measured at room temperature under operation condition and recorded. After that the following tests are performed in a series until the operating limits were found or it reaches to fundamental limit of the technology.

1. Vibration step stress test

   The parameters for random and sinusoidal Vibration test are the frequency limits, the 'g' or 'Grms' level and the duration of exposure. Uniformly for each vibration step rise level, 3 hours of exposure is applied in both sinusoidal and random vibration tests. The frequency band is chosen as 10 Hz to 250 Hz. The test has been carried out as shown in Fig. 3 until an operating limit has been found.

2. Low temperature step stress test

   The low temperature test steps started from -10°C. Depending on the failure yield at each step, the advancing steps are decided. The test has been carried out as shown in Fig. 4 until an operating limit has been found

3. High temperature step stress test with humidity

   The high temperature & humidity steps were started from 60°C & 95 %RH. Depending on the failure yield at each step, the advancing steps are decided. The test has been carried out as shown in Fig. 5 until an operating limit has been found.

4. Thermal cycling test

   The thermal cycling is carried out within the observed thermal operating limits as shown in Fig. 6. The dwell time in each of the temperature extremes (high and low) is chosen as 3hrs. The transition time from high to low & low high is taken as approx. 30mins and total test duration for one cycle is 7hrs. Uniformly, 7 numbers of thermal cycles are successfully carried out.

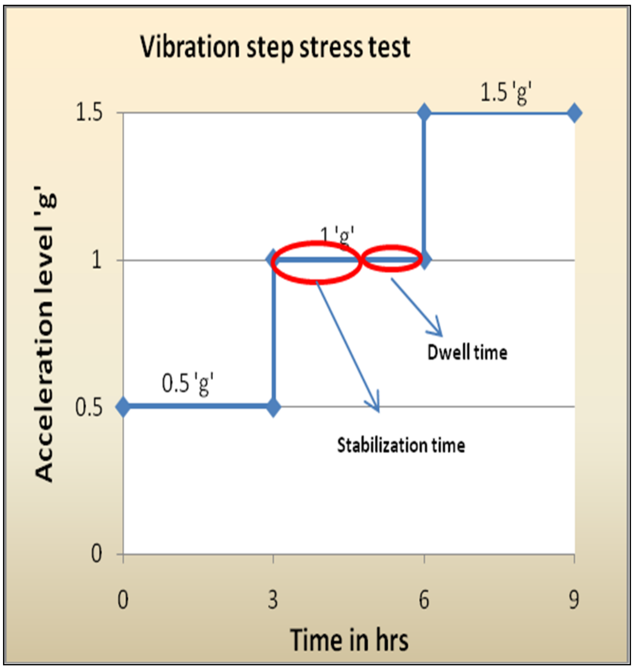

Fig. 3: Vibration step stress test profile

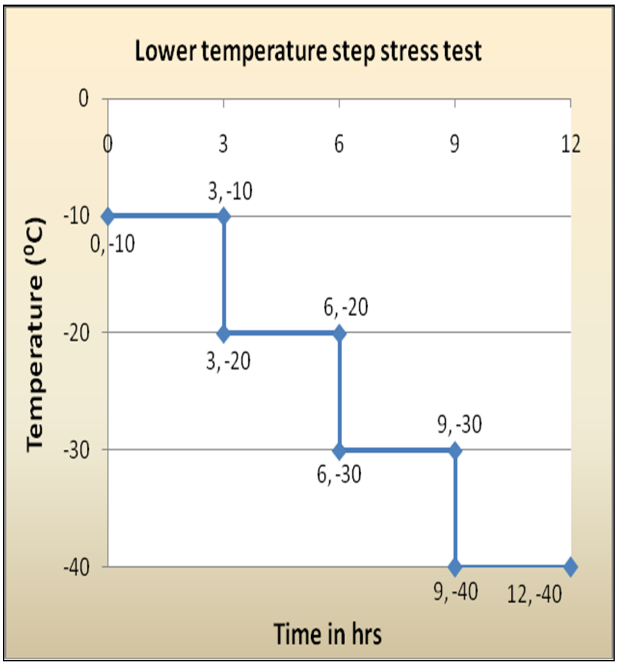

Fig. 4: Low temperature step stress test profile

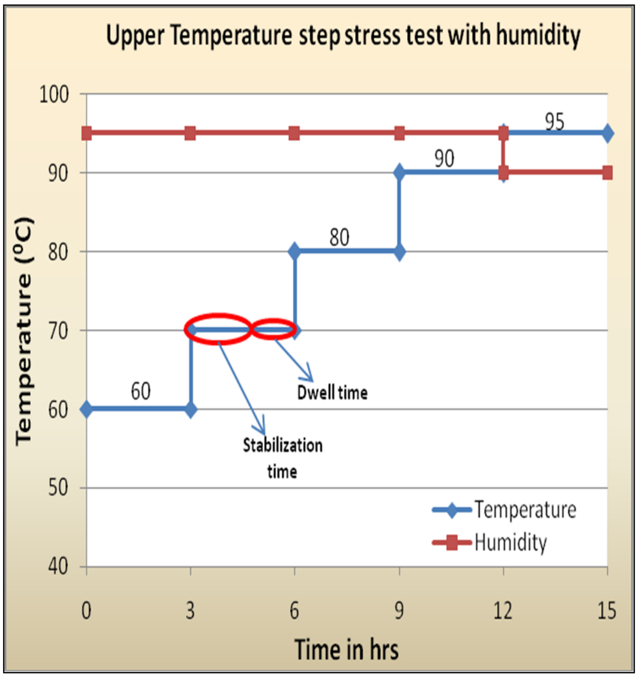

Fig. 5: High temperature step stress test profile

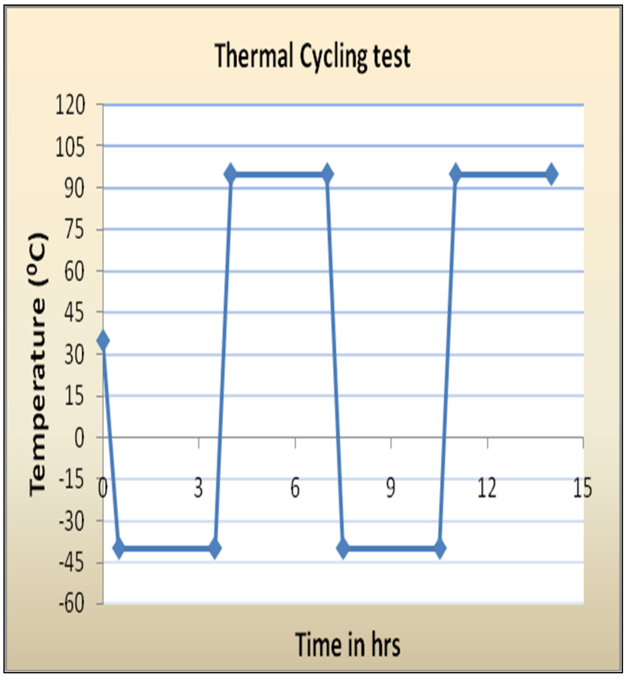

Fig. 6: Thermal cycling test profile

Step5: HALT test results

Series of tests conducted	HALT Test Findings		
	Sample 1	Sample 2	Sample 3
1. Vibration step stress test	No abnormality is observed	No abnormality is observed	At the end of 4th step rise, heavy noise is observed and the reason was found to be as **rotor touch failure (soft failure)**. The problem was corrected and test continued.
2. Low temperature step stress test	No abnormality is observed at the end of test	No abnormality is observed at the end of test	No abnormality is observed at the end of test
3. High temperature step stress test with humidity	At the start of 5th step rise, sudden rise in temperature is observed. No failure is observed at the end of 7th step rise.	**FAILED** at 7th step rise **(hard failure)** It is observed that sample drew more current before it fails. It is concluded as **winding failure** due to **inter-turn short**.	No abnormality is observed at the end of 7th step rise.
4. Thermal cycling test	No failure is observed at the end of 7 thermal cycles	Test stopped as motor failed	Sudden rise in temperature was observed at the end of 4th cycle. No failure was observed at the end of 7 no. of thermal cycles.
5. High temperature step stress test with humidity	**FAILED** at the start of 11th step rise **(hard failure).** It is concluded as **winding failure** due to **inter-turn short**.	Test stopped as motor failed	**FAILED** at the end of 9th step rise **(hard failure).** Additional sudden rise in winding temperature was observed before it fails. It is concluded as **winding failure** due to **phase to phase short**

Step6: HALT conclusion

The observed failure modes and their failure mechanisms, operating limits etc. from HALT results are compared with the previously conducted motor reliability analyses. From the results it is found that an average of 25% of deviation in thermal limits and 40% of deviation in vibration limits are observed. It means that the operating limits are much lower compare to previous results. In addition, the analysis also identified CTR (Critical to Reliability) components/parameters like insulation paper, location of failure zone etc for reliability improvement to further motor life extension.

5. CONCLUSION

This paper presented an approach to perform accelerated reliability testing on 3- phase motors using the proposed test set up. The approach is very simple, cost effective and easy to implement. This is because; the proposed test set up completely avoids mechanical load setup since motor is equivalently loaded just by giving a supply voltage approximately 20-25% greater than the rated voltage to get same load current similar to fully loaded motor. This in turn helps to eliminate the complexities involved with motor loading capabilities, motor sizes, and requirement of large reliability test facilities. Therefore, the proposed setup directly saves development cost and lots of time. Therefore, using the proposed approach a practical work of HALT on 20hp three phase motor is conducted and results of work is explain in-detailed in the paper. The results of the proposed work provided good results when compared with previous history of reliability test results conducted at no-load conditions in terms of its operating limits determined for the same failure modes. In addition, the analysis provided information on critical to reliability parameters to further motor life extension. Hence, this approach will lead any engineer to get faster and better results in a simple and cost effective way.

6. ACKNOWLEDGMENT

The authors would like to express their gratitude to the management of Crompton Greaves Ltd., for providing the necessary authorization and opportunity to publish this paper.

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APPENDIX-I Motor performance test data

Motor Test Results – Full load test @ 415V										
Model.		s1				Test No.		1		
Serial No.		1832J				Date		12/5/2011		
Speed [rpm]	Torque [Nm]	Current_ L1 [A]	Current_ L2 [A]	Current_ L3 [A]	Voltage_U RS [V]	Voltage_U ST [V]	Power In [W]	Power_O ut [W]	Efficiency [%]	Power Factor
1455	35.48	10.61	10.38	10.54	417.4	415	6366	5406	84.9	0.84
1453	35.48	10.65	10.42	10.56	417	414.7	6387	5399	84.5	0.84
1452	35.6	10.65	10.41	10.56	417.2	414.7	6408	5412	84.5	0.84
1450	35.6	10.62	10.38	10.53	417.1	414.6	6397	5405	84.5	0.85
1449	35.48	10.58	10.35	10.49	416.9	414.3	6379	5384	84.4	0.85
1448	35.48	10.65	10.41	10.55	417.1	414.8	6432	5380	83.7	0.85
1447	35.6	10.68	10.45	10.58	416.5	414.4	6454	5394	83.6	0.85
1447	35.6	10.66	10.43	10.56	417	414.3	6462	5394	83.5	0.85
1446	35.6	10.67	10.43	10.57	417.1	414.7	6465	5390	83.4	0.85
1445	35.48	10.63	10.39	10.52	417.1	414.5	6440	5369	83.4	0.85
1445	35.48	10.65	10.41	10.54	417.1	414.5	6457	5369	83.1	0.85
1444	35.48	10.64	10.41	10.54	417.1	414.3	6460	5365	83.1	0.85
1444	35.6	10.66	10.42	10.55	417.1	414.6	6472	5383	83.2	0.85
1443	35.6	10.67	10.43	10.56	416.9	414.3	6480	5379	83	0.85
1443	35.48	10.69	10.45	10.58	417	414.5	6499	5362	82.5	0.85
1443	35.6	10.66	10.43	10.55	416.9	414.3	6476	5379	83.1	0.85
1443	35.48	10.66	10.42	10.55	416.9	414.6	6476	5362	82.8	0.85
1443	35.48	10.67	10.44	10.56	416.9	414.3	6492	5362	82.6	0.85
1442	35.48	10.85	10.62	10.75	416.7	414.4	6623	5358	80.9	0.86
1442	35.6	10.67	10.44	10.57	416.9	414.5	6496	5375	82.7	0.85
1442	35.6	10.63	10.41	10.53	417.2	414.4	6472	5375	83.1	0.85
1442	35.6	10.66	10.44	10.56	416.8	414.2	6489	5375	82.8	0.85
1442	35.6	10.65	10.43	10.55	416.7	414.1	6484	5375	82.9	0.86
1442	35.48	10.7	10.48	10.6	416.8	414.1	6518	5358	82.2	0.86
1441	35.48	10.65	10.42	10.55	417	414.2	6479	5354	82.6	0.85
1441	35.48	10.65	10.43	10.55	416.8	414.3	6477	5354	82.7	0.85
1441	35.48	10.65	10.43	10.54	416.7	414.2	6492	5354	82.5	0.86
1441	35.48	10.68	10.45	10.57	416.9	414.4	6495	5354	82.4	0.85
1441	35.6	10.67	10.45	10.57	416.7	414.3	6491	5371	82.8	0.85
1441	35.6	10.66	10.44	10.56	416.9	414.3	6495	5371	82.7	0.86
1441	35.48	10.65	10.42	10.54	416.9	414.4	6482	5354	82.6	0.86
1441	35.48	10.7	10.47	10.59	417	414.6	6507	5354	82.3	0.85
1441	35.48	10.68	10.45	10.57	416.8	414.3	6502	5354	82.3	0.86
1441	35.48	10.67	10.44	10.56	416.7	414.2	6494	5354	82.5	0.86
1441	35.48	10.7	10.47	10.59	416.8	414.2	6511	5354	82.2	0.85
1441	35.48	10.67	10.44	10.56	416.7	414.1	6492	5354	82.5	0.86
Generated by RDS®, M.E.A. Testing Systems Ltd. http://www.meatesting.com										

Table 1: Full load motor performance test data conducted at rated voltage (415V)

Motor Test Results – No load test @ 510V										
Model.		S1				Test No.		2		
Serial No.		1832J				Date		12/6/2011		
Speed [rpm]	Torque [Nm]	Current_L1 [A]	Current_L2 [A]	Current_L3 [A]	Voltage_URS [V]	Voltage_UST [V]	Power_In [W]	Power_Out [W]	Efficiency [%]	Power Factor
1495	0.68	10.65	10.37	10.48	510.6	507.4	1216	106	8.7	0.13
1495	0.68	10.66	10.37	10.47	510.4	507.3	1239	106	8.6	0.13
1495	0.68	10.64	10.35	10.46	510.4	507.6	1227	106	8.6	0.13
1495	0.57	10.59	10.31	10.41	510.2	507	1222	88.5	7.2	0.13
1495	0.68	10.57	10.29	10.38	510	506.7	1218	106	8.7	0.13
1495	0.68	10.58	10.31	10.41	510.4	507.2	1240	106	8.6	0.14
1495	0.68	10.57	10.3	10.39	509.9	507.1	1244	106	8.5	0.14
1495	0.57	10.58	10.31	10.4	510.4	507.3	1253	88.5	7.1	0.14
1495	0.68	10.55	10.28	10.37	510.3	506.8	1224	106	8.7	0.13
1495	0.57	10.51	10.24	10.33	510	506.8	1242	88.5	7.1	0.14
1495	0.68	10.61	10.33	10.42	510.3	507.5	1249	106	8.5	0.14
1495	0.57	10.54	10.25	10.35	510.3	507	1244	88.5	7.1	0.14
1494	0.79	10.59	10.31	10.38	510.4	507.3	1244	124	10	0.14
1495	0.79	10.57	10.29	10.36	510	507	1241	124	10	0.14
1495	0.79	10.56	10.28	10.35	510.3	506.8	1235	124	10	0.14
1495	0.68	10.58	10.3	10.37	510.5	507.6	1253	106	8.5	0.14
1495	0.68	10.56	10.28	10.35	510.3	507.2	1255	106	8.5	0.14
1495	0.68	10.51	10.23	10.34	509.9	506.9	1240	106	8.6	0.14
1495	0.79	10.51	10.23	10.33	509.6	506.5	1232	124	10	0.14
1495	0.68	10.55	10.27	10.38	510.4	507.5	1252	106	8.5	0.14
1495	0.79	10.51	10.24	10.35	510.2	507.1	1236	124	10	0.14
1495	0.79	10.54	10.26	10.36	510.2	507.3	1228	124	10.1	0.13
1495	0.79	10.51	10.23	10.34	509.8	506.6	1228	124	10.1	0.14
1495	0.79	10.58	10.3	10.42	510.6	507.9	1278	124	9.7	0.14
1495	0.79	10.49	10.23	10.33	509.9	506.7	1226	124	10.1	0.14
1495	0.79	10.53	10.28	10.37	510.1	506.9	1259	124	9.8	0.14
1495	0.79	10.56	10.29	10.41	510.6	507.4	1273	124	9.7	0.14
1495	0.79	10.57	10.31	10.42	510.4	507.6	1248	124	9.9	0.14
Generated by RDS®, M.E.A. Testing Systems Ltd. http://www.meatesting.com										

Table 2: No-load motor performance test data conducted at 23% above rated voltage (510V)

APPENDIX-II Motor temperature rise graphs

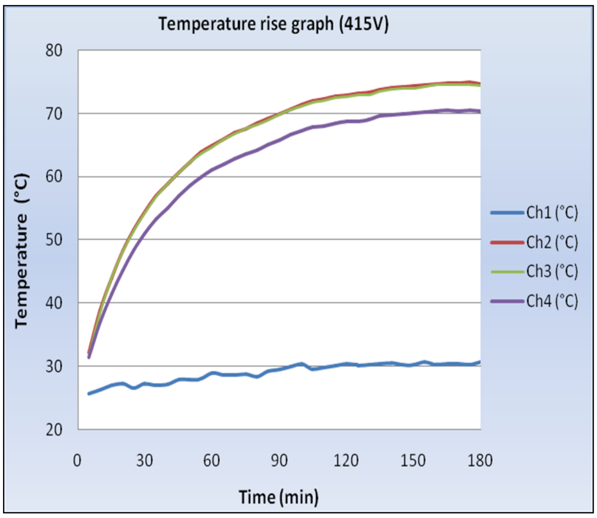

Fig. 2(a): Temperature rise graph @ 415V

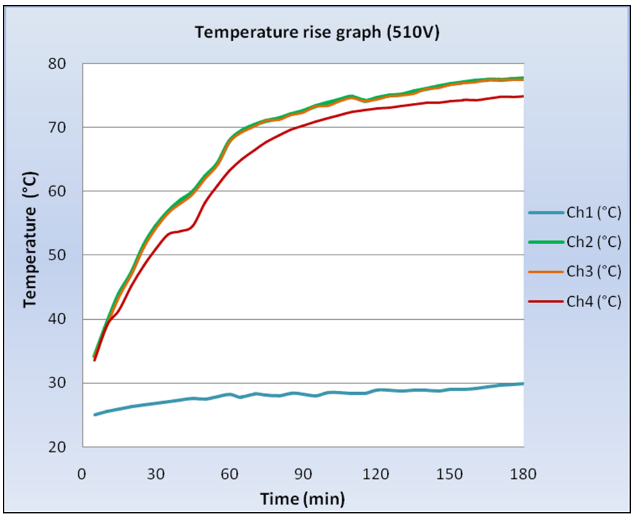

Fig. 2(b): Temperature rise graph @ 510V

REGRESSION EQUATIONS FOR MARSHALL-OLKIN TRIVARIATE EXPONENTIAL DISTRIBUTION AND RELIABILITY MEASURES OF RELATED THREE-UNIT SYSTEMS

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ABSTRACT

Marshall and Olkin(1967) proposed a multivariate exponential distribution and derived some properties including the moment generating function. Proschan and Sullo(1976) provide the probability density function which involves some tedious notations. In this paper, we provide an explicit expression for the probability density function in the trivariate case and derive the conditional distributions, regression equations and the moment generating function. By considering four three unit systems with trivariate exponential failure time distribution, we derive the reliability measures of the systems.

Key words: Conditional distribution, moment generating function, multiple regression, performance measure, trivariate exponential.

1 Introduction

Marshall-Olkin(1967) proposed a multivariate exponential distribution as a model arising out of Poisson shocks. The distribution is not absolutely continuous and so the distribution received considerable attention among the researchers. Bemis et al(1972) provide a probability distribution which is not absolutely continuous with respect to the Lebesgue measure in R^2. Inference for bivariate exponential distribution was discussed by Arnold(1968), Bemis et al(1972), Bhattacharya and Johnson(1973) among others. For the trivariate case with equal marginals, Samanta(1983) discussed the problem of testing independence. Proschan and Sullo(1976) discussed parameter estimation for multivariate case and proposed the probability density function(pdf) involving tedious notations.

The aim of this paper is to provide an explicit expression for the pdf in the trivariate case and derive the conditional distributions, regression equations and the moment generating function. Further, by considering four three unit systems with trivariate exponential failure time distribution, we derive the reliability measures of the systems. Section 2 proposes the pdf whose bivariate marginal is the one given by Bemis et al(1972). Section 3 derives the conditional distributions, both univariate and bivariate. Further the multiple regression equations are obtained and shown that they are not linear. The moment generating function is obtained in Section 4. Finally in Section 5, three component standby, parallel, series and relay systems with trivariate exponential failure times are discussed and the performance measures are obtained.

2 The probability density function

The survival function of the Marshall-Olkin trivariate exponential distribution (MOTVE) is of the form

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$$\overline{F}(x_1,x_2,x_3)=\exp\left\{-\sum_{i=1}^{3}\lambda_i x_i-\sum_{i<}\sum_{j}\lambda_{ij}\left(x_i\vee x_j\right)-\lambda_{123}\left(x_1\vee x_2\vee x_3\right)\right\},\ x_1,x_2,x_3\geq 0.$$

Here $\lambda_1,\lambda_2,\lambda_3>0$, $\lambda_{12},\lambda_{13},\lambda_{23},\lambda_{123}\geq 0$ and $x_1\vee x_2\vee .x_3=\max(x_1,x_2,x_3)$.

Let $\lambda_1^*=\lambda_1+\lambda_{12}+\lambda_{13}+\lambda_{123}$, $\lambda_2^*=\lambda_2+\lambda_{12}+\lambda_{23}+\lambda_{123}$, $\lambda_3^*=\lambda_3+\lambda_{13}+\lambda_{23}+\lambda_{123}$, $\lambda_{ji}=\lambda_{ij}$, $i<j$ and $\lambda=\lambda_1+\lambda_2+\lambda_3+\lambda_{12}+\lambda_{13}+\lambda_{23}+\lambda_{123}$.

We propose the following pdf for the MOTVE distribution

$$f(x_1,x_2,x_3)=\begin{cases}\lambda_i(\lambda_j+\lambda_{ij})\lambda_k^*\exp\left\{-\lambda_i x_i-\left(\lambda_j+\lambda_{ij}\right)x_j-\lambda_k^* x_k\right\}, & x_i<x_j<x_k\\ \lambda_i(\lambda_{jk}+\lambda_{123})\exp\left\{-\lambda_i x_i-\left(\lambda-\lambda_i\right)x_j\right\}, & x_i<x_j=x_k\\ \lambda_{jk}\lambda_i^*\exp\left\{-(\lambda_j+\lambda_k+\lambda_{jk})x_j-\lambda_i^* x_i\right\}, & x_j=x_k<x_i\\ \lambda_{123}\exp\left(-\lambda x_1\right), & x_1=x_2=x_3\end{cases}\qquad\ldots(2.1)$$

Remark 2.1 It can be verified that the total integral is one. While integrating the pdf, there are 6 triple integrals, 6 double integrals and one single integral. It may be noted that we have the same number of cases as in Samanta(1983) and (2.1) reduces to the one given in the paper for equal marginals situation.

2.1 Bivariate marginals

It is known that the bivariate marginals are bivariate exponential. We use the pdf approach to derive the bivariate marginals.

Theorem 2.1 The pdf of (X_1, X_2) at (x_1, x_2) is given by

$$f_{12}(x_1,x_2)=\begin{cases}\left(\lambda_1+\lambda_{13}\right)\lambda_2^* e\text{xp}\left\{-\left(\lambda_1+\lambda_{13}\right)x_1-\lambda_2^* x_2\right\}, & x_1<x_2\\ \left(\lambda_2+\lambda_{23}\right)\lambda_1^*\exp\left\{-\left(\lambda_2+\lambda_{23}\right)x_2-\lambda_1^* x_1\right\}, & x_1>x_2\\ \left(\lambda_{12}+\lambda_{123}\right)\exp\left\{-\left(\lambda-\lambda_3\right)x_1\right\}, & x_1=x_2.\end{cases}$$

Proof The pdf of (X_1, X_2) at (x_1, x_2) is $f_{12}(x_1,x_2)=\int_0^\infty f(x_1,x_2,x_3)\,dx_3$.

Three cases arise according as $x_1<x_2$, $x_1>x_2$ and $x_1=x_2$.

Case 1: $x_1<x_2$

In this case

$$f(x_1,x_2,x_3)=\begin{cases}\lambda_3\left(\lambda_1+\lambda_{13}\right)\lambda_2^*\exp\left\{-\lambda_3 x_3-\left(\lambda_1+\lambda_{13}\right)x_1-\lambda_2^* x_2\right\}, & x_3<x_1<x_2\\ \lambda_1\left(\lambda_3+\lambda_{13}\right)\lambda_2^*\exp\left\{-\lambda_1 x_1-\left(\lambda_3+\lambda_{13}\right)x_3-\lambda_2^* x_2\right\}, & x_1<x_3<x_2\\ \lambda_1\left(\lambda_2+\lambda_{12}\right)\lambda_3^*\exp\left\{-\lambda_1 x_1-\left(\lambda_2+\lambda_{12}\right)x_2-\lambda_3^* x_3\right\}, & x_1<x_2<x_3\\ \lambda_1\left(\lambda_{23}+\lambda_{123}\right)\exp\left\{-\lambda_1 x_1-\left(\lambda+\lambda_1\right)x_2\right\}, & x_1<x_2=x_3\\ \lambda_{13}\lambda_2^*\exp\left\{-\left(\lambda_1+\lambda_3+\lambda_{13}\right)x_1-\lambda_2^* x_2\right\}, & x_2>x_1=x_3\end{cases}$$

Thus

$$f_{12}(x_1,x_2)=\int_0^{x_1} f(x_1,x_2,x_3)dx_3+\int_{x_1}^{x_2} f(x_1,x_2,x_3)dx_3+\int_{x_2}^{\infty} f(x_1,x_2,x_3)dx_3+f(x_1,x_2,x_1)+f(x_1,x_2,x_2)$$

$$=\int_0^{x_1}\lambda_3(\lambda_1+\lambda_{13})\lambda_2^*\exp\{-(\lambda_1+\lambda_{13})x_1-\lambda_2^*x_2-\lambda_3x_3\}dx_3+$$

$$\int_{x_1}^{x_2}\lambda_1(\lambda_3+\lambda_{13})\lambda_2^*\exp\{-\lambda_1x_1-\lambda_2^*x_2-(\lambda_3+\lambda_{13})x_3\}dx_3+$$

$$\int_{x_2}^{\infty}\lambda_1(\lambda_2+\lambda_{12})\lambda_3^*\exp\{-\lambda_1x_1-(\lambda_2+\lambda_{12})x_2-\lambda_3^*x_3\}dx_3+$$

$$\lambda_{13}\lambda_2^*\exp\{-(\lambda_1+\lambda_3+\lambda_{13})x_1-\lambda_2^*x_2\}+\lambda_1(\lambda_{23}+\lambda_{123})\exp\{-\lambda_1x_1-(\lambda-\lambda_1)x_2\}$$

$$=(\lambda_1+\lambda_{13})\lambda_2^*\exp\{-(\lambda_1+\lambda_{13})x_1-\lambda_2^*x_2\}+(1-\exp\{-\lambda_3x_1\})$$

$$\lambda_1\lambda_2^*\exp\{-\lambda_1x_1-\lambda_2^*x_2\}(\exp\{-(\lambda_3+\lambda_{13})x_1-(\lambda_1+\lambda_{13})x_2\})$$

$$+\lambda_1(\lambda_2+\lambda_{12})\exp\{-\lambda_1x_1-(\lambda_2+\lambda_{12})x_2\}.\exp\{-\lambda_3^*x_2\}+$$

$$\lambda_{13}\lambda_2^*\exp\{-(\lambda_1+\lambda_3+\lambda_{13})x_1-\lambda_2^*x_2\}+\lambda_1(\lambda_{23}+\lambda_{123})\exp\{-\lambda_1x_1-(\lambda-\lambda_1)x_2\}$$

$$=(\lambda_1+\lambda_{13})\lambda_2^*\exp\{-(\lambda_1+\lambda_{13})x_1-\lambda_2^*x_2\}.$$

Case 2: $x_1>x_2$

As in Case 1, we can show that

$$f_{12}(x_1,x_2)=(\lambda_2+\lambda_{23})\lambda_1^*\exp\{-(\lambda_2+\lambda_{23})x_2-\lambda_1^*x_1\}.$$

Case 3: $x_1=x_2$

In this case

$$f(x_1,x_2,x_3)=\begin{cases}\lambda_3(\lambda_{12}+\lambda_{123})\exp\{-\lambda_3x_3-(\lambda-\lambda_3)x_1\}, & x_3<x_1=x_2\\ \lambda_{12}\lambda_3^*\exp\{-(\lambda_1+\lambda_2+\lambda_{12})x_1-\lambda_3^*x_3\}, & x_3>x_1=x_2\\ \lambda_{123}\exp(-\lambda x_1), & x_1=x_2=x_3.\end{cases}$$

$$\textit{Thus}\quad f_{12}(x_1,x_2)=\int_0^{x_1}\lambda_3(\lambda_{12}+\lambda_{123})\exp\{-\lambda_3x_3-(\lambda-\lambda_3)x_1\}dx_3$$

$$+\int_{x_1}^{\infty}\lambda_{12}\lambda_3^*\exp\{-(\lambda_1+\lambda_2+\lambda_{12})x_1-\lambda_3^*x_3\}+\lambda_{123}\exp(-\lambda x_1)$$

$$=(\lambda_{12}+\lambda_{123})\exp\{-(\lambda-\lambda_3)x_1\}$$

Hence the theorem.

Remark 2.2 Thus $(X_1,X_2)\sim BVE(\lambda_1+\lambda_{13},\lambda_2+\lambda_{23},\lambda_{12}+\lambda_{123})$, in view of Bemis et al(1972). Similarly, it can be shown that the other two bivariate distributions are also bivariate exponential.

3 Conditional distributions and regressions

In this section, we determine the univariate and bivariate conditional distributions.

3.1 Univariate conditional distributions

The conditional pdf of X_1 given $\left(X_2, X_3\right)=\left(x_2, x_3\right)$ is

$$f_{1.23}\left(x_1 \mid x_2, x_3\right)=\frac{f\left(x_1, x_2, x_3\right)}{f_{23}\left(x_2, x_3\right)},\ 0<x_1<\infty .$$

Note that $f\left(x_1, x_2, x_3\right)$ is given in (2.1) and

$$f_{23}\left(x_2, x_3\right)=\begin{cases}\left(\lambda_2+\lambda_{12}\right)\lambda_3^* \exp\left\{-\left(\lambda_2+\lambda_{12}\right)x_2-\lambda_3^* x_3\right\}, & x_2<x_3 \\ \left(\lambda_3+\lambda_{13}\right)\lambda_2^* \exp\left\{-\lambda_2^* x_2-\left(\lambda_3+\lambda_{13}\right)x_3\right\}, & x_2>x_3 \\ \left(\lambda_{23}+\lambda_{123}\right)\exp\left\{-\left(\lambda-\lambda_1\right)x_2\right\}, & x_2=x_3 .\end{cases}$$

Three cases arise according as $x_2<x_3, x_2>x_3, x_2=x_3$.

Case 1: $x_2<x_3$

$$\begin{aligned} f_{1.23}\left(x_1 \mid x_2, x_3\right)= & \lambda_1 \exp\left(-\lambda_1 x_1\right), & x_1<x_2 \\ & \frac{\lambda_2\left(\lambda_1+\lambda_{12}\right)}{\left(\lambda_2+\lambda_{12}\right)} \exp\left\{-\left(\lambda_1+\lambda_{12}\right)x_1+\lambda_{12} x_2\right\}, & x_2<x_1<x_3 \\ & \frac{\lambda_2\left(\lambda_3+\lambda_{23}\right)\lambda_1^*}{\left(\lambda_2+\lambda_{12}\right)\lambda_3^*} \exp\left\{-\lambda_1^* x_1+\lambda_{12} x_2+\left(\lambda_{13}+\lambda_{123}\right)x_3\right\}, & x_3<x_1 \\ & \frac{\lambda_{12}}{\left(\lambda_2+\lambda_{12}\right)} \exp\left\{-\lambda_1 x_2\right\}, & x_1=x_2 \\ & \frac{\lambda_2\left(\lambda_{13}+\lambda_{123}\right)}{\left(\lambda_2+\lambda_{12}\right)\lambda_3^*} \exp\left\{-\left(\lambda_1+\lambda_{12}\right)x_3+\lambda_{12} x_2\right\}, & x_1=x_3 .\end{aligned}$$

Case 2: $x_2>x_3$

$$\begin{aligned} f_{1.23}\left(x_1 \mid x_2, x_3\right)= & \lambda_1 \exp\left\{-\lambda_1 x_1\right\}, & x_1<x_3 \\ & \frac{\lambda_3\left(\lambda_1+\lambda_{13}\right)}{\left(\lambda_3+\lambda_{13}\right)} \exp\left\{-\left(\lambda_1+\lambda_{13}\right)x_1+\lambda_{13} x_3\right\}, & x_3<x_1<x_2 \\ & \frac{\lambda_3\left(\lambda_2+\lambda_{23}\right)\lambda_1^*}{\left(\lambda_3+\lambda_{13}\right)\lambda_2^*} \exp\left\{-\lambda_1^* x_1+\lambda_{13} x_3+\left(\lambda_{12}+\lambda_{123}\right)x_2\right\}, & x_2<x_1 \\ & \frac{\lambda_{13}}{\left(\lambda_3+\lambda_{13}\right)} \exp\left(-\lambda_1 x_3\right), & x_1=x_3 \\ & \frac{\lambda_3\left(\lambda_{12}+\lambda_{123}\right)}{\left(\lambda_3+\lambda_{13}\right)\lambda_2^*} \exp\left\{-\left(\lambda_1+\lambda_{13}\right)x_2+\lambda_{13} x_3\right\}, & x_1=x_2 .\end{aligned}$$

Case 3: $x_2=x_3$

$$\begin{aligned} f_{1.23}\left(x_1 \mid x_2, x_3\right)= & \lambda_1 \exp\left(-\lambda_1 x_1\right), & x_1<x_2 \\ & \frac{\lambda_{23}\lambda_1^*}{\left(\lambda_{23}+\lambda_{123}\right)} \exp\left\{-\lambda_1^* x_1+\left(\lambda_{12}+\lambda_{13}+\lambda_{123}\right)x_2\right\}, & x_2<x_1 \\ & \frac{\lambda_{123}}{\left(\lambda_{23}+\lambda_{123}\right)} \exp\left(-\lambda_1 x_2\right), & x_1=x_2 .\end{aligned}$$

Remark 3.1 Similarly, one can find the other two univariate conditional distributions.

Theorem 3.1 The regression equation of X_1 on $\left(X_2, X_3\right)$ is

$$X_1 = \begin{cases} \dfrac{1}{\lambda_1} + \left\{ \dfrac{\lambda_3}{(\lambda_1+\lambda_{13})(\lambda_3+\lambda_{13})} - \dfrac{1}{\lambda_1} \right\} \exp(-\lambda_1 X_3) \\ + \left\{ \dfrac{\lambda_3(\lambda_2+\lambda_{23})}{(\lambda_3+\lambda_{13})\lambda_1^*\lambda_2^*} - \dfrac{\lambda_3}{(\lambda_1+\lambda_{13})(\lambda_3+\lambda_{13})} \right\} \exp\{\lambda_{13}X_3 - (\lambda_1+\lambda_{13})X_2\}, & X_2 > X_3 \\ \dfrac{1}{\lambda_1} + \left\{ \dfrac{\lambda_2}{(\lambda_1+\lambda_{12})(\lambda_2+\lambda_{12})} - \dfrac{1}{\lambda_1} \right\} \exp(-\lambda_1 X_2) \\ + \left\{ \dfrac{\lambda_2(\lambda_3+\lambda_{23})}{(\lambda_2+\lambda_{12})\lambda_1^*\lambda_3^*} - \dfrac{\lambda_2}{(\lambda_1+\lambda_{12})(\lambda_2+\lambda_{12})} \right\} \exp\{\lambda_{12}X_2 - (\lambda_1+\lambda_{12})X_3\}, & X_2 < X_3 \\ \dfrac{1}{\lambda_1} + \left\{ \dfrac{\lambda_{23}}{(\lambda_{23}+\lambda_{123})\lambda_1^*} - \dfrac{1}{\lambda_1} \right\} \exp(-\lambda_1 X_2), & X_2 = X_3. \end{cases}$$

Proof For $x_2 > x_3$,

$$E\left[X_1 \middle| (X_2, X_3) = (x_2, x_3)\right] = \int_0^{x_3} x_1 \lambda_1 \exp\{-\lambda_1 x_1\} dx_1 + \int_{x_3}^{x_2} x_1 \frac{\lambda_3(\lambda_1+\lambda_{13})}{(\lambda_3+\lambda_{13})} \exp\{\lambda_{13}x_3 - (\lambda_1+\lambda_{13})x_1\} dx_1 +$$

$$\int_{x_2}^{\infty} x_1 \frac{\lambda_3(\lambda_2+\lambda_{23})\lambda_1^*}{(\lambda_3+\lambda_{13})\lambda_2^*} \exp\{-\lambda_1^* x_1 + (\lambda_{12}+\lambda_{123})x_2 + \lambda_{13}x_3\} dx_1$$

$$+ x_3 \frac{\lambda_{13}}{(\lambda_3+\lambda_{13})} \exp(-\lambda_1 x_3) + x_2 \frac{\lambda_3(\lambda_{12}+\lambda_{123})}{(\lambda_3+\lambda_{13})\lambda_2^*} \exp\{-(\lambda_1+\lambda_{13})x_2\}$$

$$= \frac{1}{\lambda_1} + \left\{ \frac{\lambda_3}{(\lambda_1+\lambda_{13})(\lambda_3+\lambda_{13})} - \frac{1}{\lambda_1} \right\} \exp(-\lambda_1 x_3) +$$

$$\left\{ \frac{\lambda_3(\lambda_2+\lambda_{23})}{(\lambda_3+\lambda_{13})\lambda_2^*\lambda_1^*} - \frac{\lambda_3}{(\lambda_1+\lambda_{13})(\lambda_3+\lambda_{13})} \right\} \exp\{-(\lambda_1+\lambda_{13})x_2 + \lambda_{13}x_3\}$$

Similarly one can derive the regression equation for $x_2 < x_3$.

For $x_2 = x_3$,

$$E\left[X_1 \middle| (X_2, X_3) = (x_2, x_3)\right] = \int_0^{x_3} x_1 \lambda_1 \exp\{-\lambda_1 x_1\} dx_1 +$$

$$x_2 \frac{\lambda_{123}}{(\lambda_{23}+\lambda_{123})} \exp\{-\lambda_1 x_2\} \int_{x_2}^{\infty} x_1 \frac{\lambda_{23}\lambda_1^*}{(\lambda_{23}+\lambda_{123})} \exp\{-\lambda_1^* x_1 + (\lambda_{12}+\lambda_{13}+\lambda_{123})x_2\} dx_1$$

$$= \frac{1}{\lambda_1} + \left\{ \frac{\lambda_{23}}{(\lambda_{23}+\lambda_{123})\lambda_1^*} - \frac{1}{\lambda_1} \right\} \exp(-\lambda_1 x_2)$$

Hence the theorem.

Remark 3.2 The other two regression equations can be derived in a similar manner. Thus the multiple regression equations are non-linear.

3.2 Bivariate Conditional Distributions

Let us find the conditional distribution of $\left(X_1, X_2\right)$ given $X_3 = x_3$.

Note that $f_3(x_3) = \lambda_3^* \exp(-\lambda_3^* x_3), x_3 > 0$

The conditional pdf of (X_1, X_2) given $X_3 = x_3$ is

$$f_{12.3}(x_1, x_2 | x_3) = \frac{f(x_1, x_2, x_3)}{f_3(x_3)},\ x_1, x_2 > 0.$$

Here three cases arise.

For $x_1 < x_2$

$$f_{12.3}(x_1, x_2 | x_3) = \begin{cases} \dfrac{\lambda_3(\lambda_1 + \lambda_{13})\lambda_2^*}{\lambda_3^*} \exp\{-(\lambda_1 + \lambda_{13})x_1 - \lambda_2^* x_2 + (\lambda_{12} + \lambda_{23} + \lambda_{123})x_3\}, & x_3 < x_1 \\ \dfrac{\lambda_1(\lambda_3 + \lambda_{13})\lambda_2^*}{\lambda_3^*} \exp\{-\lambda_1 x_1 - \lambda_2^* x_2 + (\lambda_{23} + \lambda_{123})x_3\}, & x_1 < x_3 < x_2 \\ \dfrac{\lambda_{13}\lambda_2^*}{\lambda_3^*} \exp\{-\lambda_2^* x_2 - (\lambda_1 - \lambda_{23} - \lambda_{123})x_3\}, & x_1 = x_3 \\ \dfrac{\lambda_1(\lambda_{23} + \lambda_{123})}{\lambda_3^*} \exp\{-\lambda_1 x_1 - (\lambda_2 + \lambda_{12})x_3\}, & x_2 = x_3. \end{cases}$$

For $x_1 > x_2$

$$f_{12.3}(x_1, x_2 | x_3) = \begin{cases} \dfrac{\lambda_3(\lambda_2 + \lambda_{23})\lambda_1^*}{\lambda_3^*} \exp\{-(\lambda_2 + \lambda_{23})x_2 - \lambda_1^* x_1 + (\lambda_{12} + \lambda_{13} + \lambda_{123})x_3\}, & x_3 < x_2 \\ \dfrac{\lambda_2(\lambda_3 + \lambda_{23})\lambda_1^*}{\lambda_3^*} \exp\{-\lambda_1^* x_1 - \lambda_2 x_2 + (\lambda_{13} + \lambda_{123})x_3\}, & x_2 < x_3 < x_1 \\ \lambda_2(\lambda_1 + \lambda_{12}) \exp\{-(\lambda_1 + \lambda_{12})x_1 - \lambda_2 x_2\}, & x_1 < x_3 \\ \dfrac{\lambda_{23}\lambda_1^*}{\lambda_3^*} \exp\{-\lambda_1^* x_1 - (\lambda_2 - \lambda_{13} - \lambda_{123})x_3\}, & x_2 = x_3 \\ \dfrac{\lambda_2(\lambda_{13} + \lambda_{123})}{\lambda_3^*} \exp\{-\lambda_2 x_2 - (\lambda_1 + \lambda_{12})x_3\}, & x_1 = x_3 \end{cases}$$

For $x_1 = x_2$

$$f_{12.3}(x_1, x_2 | x_3) = \begin{cases} \dfrac{\lambda_3(\lambda_{12} + \lambda_{123})}{\lambda_3^*} \exp\{-(\lambda - \lambda_3)x_1 + (\lambda_{12} + \lambda_{23} + \lambda_{123})x_3\}, & x_3 < x_1 \\ \lambda_{12} \exp\{-(\lambda_1 + \lambda_2 + \lambda_{12})x_1\}, & x_1 < x_3 \\ \dfrac{\lambda_{122}}{\lambda_3^*} \exp\{-(\lambda_1 + \lambda_2 + \lambda_{12})x_3\}, & x_1 = x_3 \end{cases}$$

Remark 3.3 It can be verified with routine but tedious integration, that $f_{12.3}(x_1, x_2 | x_3)$ is a pdf. Similarly one can derive the other two bivariate conditional distributions.

4 Moment generating function

Theorem 4.1 The mgf of (X_1 , X_2 , X_3) at (t_1 , t_2 , t_3) is given by

$$M(t_1, t_2, t_3) = = \frac{\lambda_1 \lambda_2^* \lambda_3^*}{(\lambda_2^* - t_2)(\lambda_3^* - t_3)(\lambda - t_1 - t_2 - t_3)} + \frac{\lambda_1(\lambda_{23} + \lambda_{123})t_2 t_3}{(\lambda_2^* - t_2)(\lambda_3^* - t_3)(\lambda - t_1 - t_2 - t_3)(\lambda - \lambda_1 - t_2 - t_3)}$$
$$+ \frac{\lambda_2 \lambda_1^* \lambda_3^*}{(\lambda_1^* - t_1)(\lambda_3^* - t_3)(\lambda - t_1 - t_2 - t_3)} + \frac{\lambda_2(\lambda_{13} + \lambda_{123})t_1 t_3}{(\lambda_1^* - t_1)(\lambda_3^* - t_3)(\lambda - t_1 - t_2 - t_3)(\lambda - \lambda_2 - t_1 - t_3)}$$

$$+\frac{\lambda_3\lambda_1^*\lambda_2^*}{(\lambda_1^*-t_1)(\lambda_2^*-t_2)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_3(\lambda_{12}+\lambda_{123})t_1t_2}{(\lambda_1^*-t_1)(\lambda_2^*-t_2)(\lambda-t_1-t_2-t_3)(\lambda-\lambda_3-t_1-t_2)}$$
$$+\frac{\lambda_{12}t_3}{(\lambda-t_1-t_2-t_3)(\lambda_3^*-t_3)}+\frac{\lambda_{13}t_2}{(\lambda-t_1-t_2-t_3)(\lambda_2^*-t_2)}+\frac{\lambda_{23}t_1}{(\lambda-t_1-t_2-t_3)(\lambda_1^*-t_1)}$$
$$+\frac{\lambda_{12}+\lambda_{13}+\lambda_{23}+\lambda_{123}}{(\lambda-t_1-t_2-t_3)}$$

Proof $M(t_1,t_2,t_3)=E\{\exp(t_1X_1+t_2X_2+t_3X_3)\}$

$$=\frac{\lambda_1(\lambda_2+\lambda_{12})\lambda_3^*}{(\lambda_3^*-t_3)(\lambda-\lambda_1-t_2-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_1(\lambda_3+\lambda_{13})\lambda_2^*}{(\lambda_2^*-t_2)(\lambda-\lambda_1-t_2-t_3)(\lambda-t_1-t_2-t_3)}$$
$$+\frac{\lambda_2(\lambda_1+\lambda_{12})\lambda_3^*}{(\lambda_3^*-t_3)(\lambda-\lambda_2-t_2-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_2(\lambda_3+\lambda_{23})\lambda_1^*}{(\lambda_1^*-t_1)(\lambda-\lambda_2-t_2-t_3)(\lambda-t_1-t_2-t_3)}$$
$$+\frac{\lambda_3(\lambda_1+\lambda_{13})\lambda_2^*}{(\lambda_2^*-t_2)(\lambda-\lambda_3-t_1-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_3(\lambda_2+\lambda_{23})\lambda_1^*}{(\lambda_1^*-t_1)(\lambda-\lambda_3-t_2-t_3)(\lambda-t_1-t_2-t_3)}$$
$$+\frac{\lambda_1(\lambda_{23}+\lambda_{123})}{(\lambda-\lambda_1-t_2-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_2(\lambda_{13}+\lambda_{123})}{(\lambda-\lambda_2-t_1-t_3)(\lambda-t_1-t_2-t_3)}$$
$$+\frac{\lambda_3(\lambda_{12}+\lambda_{123})}{(\lambda-\lambda_3-t_1-t_2)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_{12}\lambda_3^*}{(\lambda-t_1-t_2-t_3)(\lambda_3^*-t_3)}+$$
$$+\frac{\lambda_{13}\lambda_2^*}{(\lambda-t_1-t_2-t_3)(\lambda_2^*-t_2)}+\frac{\lambda_{23}\lambda_1^*}{(\lambda-t_1-t_2-t_3)(\lambda_1^*-t_1)}+\frac{\lambda_{123}}{(\lambda-t_1-t_2-t_3)}$$

$$=\left\{\frac{\lambda_1(\lambda_2+\lambda_{12})\lambda_3^*}{(\lambda_3^*-t_3)(\lambda-\lambda_1-t_2-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_1(\lambda_3+\lambda_{13})\lambda_2^*}{(\lambda_2^*-t_2)(\lambda-\lambda_1-t_2-t_3)(\lambda-t_1-t_2-t_3)}\right.$$
$$\left.+\frac{\lambda_1(\lambda_{23}+\lambda_{123})}{(\lambda-\lambda_1-t_2-t_3)(\lambda-t_1-t_2-t_3)}\right\}+\left\{\frac{\lambda_2(\lambda_1+\lambda_{12})\lambda_3^*}{(\lambda_3^*-t_3)(\lambda-\lambda_2-t_1-t_3)(\lambda-t_1-t_2-t_3)}\right.$$
$$\left.+\frac{\lambda_2(\lambda_3+\lambda_{23})\lambda_1^*}{(\lambda_1^*-t_1)(\lambda-\lambda_2-t_1-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_2(\lambda_{13}+\lambda_{123})}{(\lambda-\lambda_2-t_1-t_3)(\lambda-t_1-t_2-t_3)}\right\}$$
$$+\left\{\frac{\lambda_3(\lambda_2+\lambda_{23})\lambda_1^*}{(\lambda_1^*-t_1)(\lambda-\lambda_3-t_1-t_2)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_3(\lambda_1+\lambda_{13})\lambda_2^*}{(\lambda_2^*-t_2)(\lambda-\lambda_3-t_1-t_2)(\lambda-t_1-t_2-t_3)}\right.$$
$$\left.+\frac{\lambda_3(\lambda_{12}+\lambda_{123})}{(\lambda-\lambda_3-t_1-t_2)(\lambda-t_1-t_2-t_3)}\right\}+\frac{\lambda_{12}\lambda_3^*}{(\lambda_3^*-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_{13}\lambda_2^*}{(\lambda_2^*-t_2)(\lambda-t_1-t_2-t_3)}$$
$$+\frac{\lambda_{23}\lambda_1^*}{(\lambda_1^*-t_1)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_{123}}{(\lambda-t_1-t_2-t_3)}$$

$$=\frac{\lambda_1\lambda_2^*\lambda_3^*}{(\lambda_2^*-t_2)(\lambda_3^*-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_1(\lambda_{23}+\lambda_{123})t_2t_3}{(\lambda_2^*-t_2)(\lambda_3^*-t_3)(\lambda-t_1-t_2-t_3)(\lambda-\lambda_1-t_2-t_3)}$$
$$+\frac{\lambda_2\lambda_1^*\lambda_3^*}{(\lambda_1^*-t_1)(\lambda_3^*-t_3)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_2(\lambda_{13}+\lambda_{123})t_1t_3}{(\lambda_1^*-t_1)(\lambda_3^*-t_3)(\lambda-t_1-t_2-t_3)(\lambda-\lambda_2-t_1-t_3)}$$
$$+\frac{\lambda_3\lambda_1^*\lambda_2^*}{(\lambda_1^*-t_1)(\lambda_2^*-t_2)(\lambda-t_1-t_2-t_3)}+\frac{\lambda_3(\lambda_{12}+\lambda_{123})t_1t_2}{(\lambda_1^*-t_1)(\lambda_2^*-t_2)(\lambda-t_1-t_2-t_3)(\lambda-\lambda_3-t_1-t_2)}$$

$$+\frac{\lambda_{12}t_3}{(\lambda-t_1-t_2-t_3)(\lambda_3^*-t_3)}+\frac{\lambda_{13}t_2}{(\lambda-t_1-t_2-t_3)(\lambda_2^*-t_2)}+\frac{\lambda_{23}t_1}{(\lambda-t_1-t_2-t_3)(\lambda_1^*-t_1)}$$
$$+\frac{\lambda_{12}+\lambda_{13}+\lambda_{23}+\lambda_{123}}{(\lambda-t_1-t_2-t_3)}$$

Remark 4.1 The mgf of (X_1, X_2) at (t_1, t_2) is M(t_1, t_2, 0) and reduces to the mgf of BVE $(\lambda_1+\lambda_{13},\lambda_2+\lambda_{23},\lambda_{12}+\lambda_{123})$ at (t_1, t_2) in view of Barlow and Proschan(1975).

5 Reliability measures of systems with MOTVE components

In this section we derive the performance measures associated with four three component systems, assuming that the component failure times have a joint MOTVE distribution.

5.1 Standby system

Consider a three unit standby system with component failure times X_1, X_2, X_3 respectively. Then the system failure time is $T=\sum_{i=1}^{3}X_i$. Assume that the component failure times are identically distributed. As in Samantha(1983), take $\lambda_1=\lambda_2=\lambda_3=\beta_1$, $\lambda_{12}=\lambda_{13}=\lambda_{23}=\beta_2$, $\lambda_{123}=\beta_3$. Let us first find the mgf of T.

Define $\alpha_1=\beta_1+2\beta_2+\beta_3, \alpha_2=2\beta_1+3\beta_2+\beta_3$ and $\alpha_3=3\beta_1+3\beta_2+\beta_3$. From Theorem 4.1,

$$M(t_1,t_2,t_3)=\beta_1(\beta_1+\beta_2)\alpha_1$$
$$\left\{\frac{1}{(\alpha_1-t_1)(\alpha_2-t_1-t_2)(\alpha_3-t_1-t_2-t_3)}+\frac{1}{(\alpha_1-t_1)(\alpha_2-t_1-t_3)(\alpha_3-t_1-t_2-t_3)}\right\}$$
$$+\left\{\frac{1}{(\alpha_1-t_2)(\alpha_2-t_1-t_2)(\alpha_3-t_1-t_2-t_3)}+\frac{1}{(\alpha_1-t_2)(\alpha_2-t_2-t_3)(\alpha_3-t_1-t_2-t_3)}\right\}$$
$$+\left\{\frac{1}{(\alpha_1-t_3)(\alpha_2-t_1-t_3)(\alpha_3-t_1-t_2-t_3)}+\frac{1}{(\alpha_1-t_3)(\alpha_2-t_2-t_3)(\alpha_3-t_1-t_2-t_3)}\right\}$$
$$+\beta_1(\beta_2+\beta_3)\left\{\frac{1}{(\alpha_2-t_1-t_2)(\alpha_3-t_1-t_2-t_3)}+\frac{1}{(\alpha_2-t_1-t_3)(\alpha_3-t_1-t_2-t_3)}+\right.$$
$$\left.\frac{1}{(\alpha_2-t_2-t_3)(\alpha_3-t_1-t_2-t_3)}\right\}+\frac{\beta_3}{(\alpha_3-t_1-t_2-t_3)}$$

Thus the mgf of $X_1 + X_2 + X_3$ at t is,

$$M^*(t)=\frac{6\beta_1(\beta_1+\beta_2)\alpha_1}{(\alpha_1-t)(\alpha_2-2t)(\alpha_3-3t)}+\frac{3\beta_1(\beta_2+\beta_3)}{(\alpha_2-2t)(\alpha_3-3t)}+\frac{3\beta_2\alpha_1}{(\alpha_1-t)(\alpha_3-3t)}+\frac{\beta_3}{(\alpha_3-3t)}$$

Note that the mgf exists for $t<\min\left\{\alpha_1,\frac{\alpha_2}{2},\frac{\alpha_3}{3}\right\}$

Resolving the first three terms into partial fractions and simplifying we get

$$M^*(t)=\frac{6\beta_1(\beta_1+\beta_2)\alpha_1}{(\beta_2+\beta_3)(3\beta_2+2\beta_3)(\alpha_1-t)}-\frac{24\beta_1(\beta_1+\beta_2)\alpha_1}{(\beta_2+\beta_3)(3\beta_2+\beta_3)(\alpha_2-2t)}$$
$$+\frac{54\beta_1(\beta_1+\beta_2)\alpha_1}{(3\beta_2+2\beta_3)(3\beta_2+\beta_3)(\alpha_3-3t)}+\frac{-6\beta_1(\beta_2+\beta_3)}{(3\beta_2+\beta_3)(\alpha_2-2t)}+\frac{9\beta_1(\beta_2+\beta_3)}{(3\beta_2+\beta_3)(\alpha_3-3t)}$$
$$+\frac{-3\beta_2\alpha_1}{(3\beta_2+2\beta_3)(\alpha_2-2t)}+\frac{9\beta_2\alpha_1}{(3\beta_2+2\beta_3)(\alpha_3-3t)}+\frac{\beta_3}{(\alpha_3-3t)}$$

Let us express the mgf as the weighted average of three exponential mgfs.

Define $M_1(t)=\left(1-\frac{t}{\alpha_1}\right)^{-1}, M_2(t)=\left(1-\frac{2t}{\alpha_2}\right)^{-1}$ and $M_3(t)=\left(1-\frac{3t}{\alpha_3}\right)^{-1}$

Then $M^*(t)=w_1M_1(t)+w_2M_2(t)+w_3M_3(t)$, where

$$w_1=\frac{6\beta_1(\beta_1+\beta_2)}{(\beta_2+\beta_3)(3\beta_2+2\beta_3)}-\frac{3\beta_2}{(3\beta_2+2\beta_3)}$$
$$w_2=-\frac{24\beta_1(\beta_1+\beta_2)\alpha_1}{(\beta_2+\beta_3)(3\beta_2+\beta_3)\alpha_2}+\frac{-6\beta_1(\beta_2+\beta_3)}{(3\beta_2+\beta_3)\alpha_2} \text{ and}$$
$$w_3=\frac{54\beta_1(\beta_1+\beta_2)\alpha_1}{(3\beta_2+2\beta_3)(3\beta_2+\beta_3)\alpha_3}+\frac{9\beta_1(\beta_2+\beta_3)}{(3\beta_2+\beta_3)\alpha_3}+\frac{9\beta_2\alpha_1}{(3\beta_2+2\beta_3)\alpha_3}+\frac{\beta_3}{\alpha_3}$$

It can be verified that $w_1+w_2+w_3=1$.

Therefore, the reliability function $R(t)=\sum_{i=1}^{3}w_i\exp\{-(\alpha_i/i)t\}$, t > 0, and the MTBF $=\sum_{i=1}^{3}i\,w_i/\alpha_i$.

5.2 Parallel system

Consider a three unit parallel system with component failure times X_1, X_2, X_3 respectively. Then the system failure time is $T=\underset{1\le i\le 3}{Max}\ X_i$.

The distribution function of T at x is

$$G(x)=1-\overline{F}(x,0,0)-\overline{F}(0,x,0)-\overline{F}(0,0,x)+\overline{F}(x,x,0)+\overline{F}(x,0,x)+\overline{F}(0,x,x)-\overline{F}(x,x,x)$$

Therefore the reliability function of the system is,

$$R(t)=\sum_{i=1}^{3}\exp(-\lambda_i^*t)-\sum_{i=1}^{3}\exp\{-(\lambda-\lambda_i)t\}+\exp(-\lambda t),\ t>0.$$

The MTBF is

$$MTBF=\sum_{i=1}^{3}\frac{1}{\lambda_i^*}-\sum_{i=1}^{3}\frac{1}{(\lambda-\lambda_i)}+\frac{1}{\lambda}.$$

5.3 Series system

Consider a three unit series system with component failure times X_1, X_2, X_3 respectively. Then the system failure time is $T=\underset{1\le i\le 3}{Min}\ X_i$.

The reliability function is $R(t)=\overline{F}(t,t,t)$
$=\exp\{-\lambda t\}$, t > 0.

$$MTBF = \frac{1}{\lambda t}.$$

5.4 Relay system

A three component relay system operates if and only if component 1 and at least one of the remaining two components operate. The system failure time is T = $X_1 \wedge (X_2 \vee X_3)$. (Barlow and Proschan, 1975). It seems natural to assume that X_2 and X_3 are identically distributed. Thus we assume that

$$\overline{F}(x_1, x_2, x_3) = \exp\{-\lambda_1 x_1 - \lambda_2(x_2 + x_3) - \lambda_{12}(x_1 \vee x_2 + x_1 \vee x_3) - \lambda_{23}(x_2 \vee x_3) - \lambda_{123}(x_1 \vee x_2 \vee x_3)\}$$

Note that (X_1, X_2) and (X_1, X_3) are identically distributed.

The reliability function is

$$R(t) = \overline{F}(t,t,0) + \overline{F}(t,0,t) - \overline{F}(t,t,t)$$

$$= 2\exp\{-(\lambda_1 + \lambda_2 + 2\lambda_{12} + \lambda_{23} + \lambda_{123})t\} - \exp\{-(\lambda_1 + 2\lambda_2 + 2\lambda_{12} + \lambda_{23} + \lambda_{123})t\},\ t > 0.$$

The MTBF is

$$MTBF = \frac{2}{(\lambda_1 + \lambda_2 + 2\lambda_{12} + \lambda_{23} + \lambda_{123})} - \frac{1}{(\lambda_1 + 2\lambda_2 + 2\lambda_{12} + \lambda_{23} + \lambda_{123})}$$

$$= \frac{(\lambda_1 + 3\lambda_2 + 2\lambda_{12} + \lambda_{23} + \lambda_{123})}{(\lambda_1 + \lambda_2 + 2\lambda_{12} + \lambda_{23} + \lambda_{123})(\lambda_1 + 2\lambda_2 + 2\lambda_{12} + \lambda_{23} + \lambda_{123})}.$$

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RELIABILITY OPTOMIZATION OF COMPLEX SYSTEMS

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ABSTRACT

The method based on the results of the joint model linking a semi-Markov modelling of the system operation process with a multistate approach to system reliability and the linear programming are proposed to the operation and reliability optimization of complex technical systems at the variable operation conditions. The method consists in determining the optimal values of limit transient probabilities at the system operation states that maximize the system lifetimes in the reliability state subsets. The proposed method is applied to the operation and reliability optimization of the exemplary technical multistate non-homogeneous system composed of a series-parallel and a series-"*m* out of *l*" subsystems linked in series that is changing its reliability structure and its components reliability parameters at its variable operation conditions.

1 INTRODUCTION

The complex technical systems reliability improvement and decreasing the risk of exceeding a critical reliability state are of great value in the industrial practice (Kołowrocki, Soszyńska-Budny, 2011; Kuo, Prasad, 2000; Kuo, Zuo 2003; Vercellis, 2009). In everyday practice, there are needed the tools that could be applied to improving the reliability characteristics of the multistate systems operating at variable conditions. There are needed the tools allowing for finding the distributions and the expected values of the optimal times until the exceeding by the system the reliability critical state, the optimal system risk function and the moment when the system risk function exceeds a permitted level and allowing for changing their operation processes after comparing the values of these characteristics with their values before their operation processes optimization in order to improve their reliability (Klabjan, Adelman, 2008; Kołowrocki, Soszyńska-Budny, 2009, 2010, 2011; Lisnianski, Levitin 2003, Tang, Yin, Xi, 2007).

2 COMPLEX SYSTEM RELIABILITY AND OPERATION PROCESS OPTIMIZATION

Considering the equation (25) (Kołowrocki, Soszyńska-Budny, 2013), it is natural to assume that the system operation process has a significant influence on the system reliability. This influence is also clearly expressed in the equation (26) (Kołowrocki, Soszyńska-Budny, 2013) for the mean values of the system unconditional lifetimes in the reliability state subsets.

From the linear equation (26) (Kołowrocki, Soszyńska-Budny, 2013), we can see that the mean value of the system unconditional lifetime $M(u)$, $u = 1,2,...,z,$ is determined by the limit values of transient probabilities p_b, $b = 1,2,...,v,$ of the system operation process at the operation states given by (8) (Kołowrocki, Soszyńska-Budny, 2013) and the mean values $M_b(u)$, $b = 1,2,...,v,$ $u = 1,2,...,z,$ of the system conditional lifetimes in the reliability state subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ given by (27) (Kołowrocki, Soszyńska-Budny, 2013). Therefore, the system lifetime optimization approach based on the linear programming (Klabjan, Adelman, 2008; Kołowrocki, Soszyńska-Budny, 2009, 2010, 2011).

can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities p_b, $b = 1,2,...,\nu$, of the system operation process at the operation states to maximize the mean value $M(u)$ of the unconditional system lifetimes in the reliability state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z$, under the assumption that the mean values $M_b(u)$, $b = 1,2,...,\nu$, $u = 1,2,..., z$, of the system conditional lifetimes in the reliability state subsets are fixed. As a special and practically important case of the above formulated system lifetime optimization problem, if r, $r = 1,2,..., z$, is a system critical reliability state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities p_b, $b = 1,2,...,\nu$, of the system operation process at the system operation states to maximize the mean value $M(r)$ of the unconditional system lifetime in the reliability state subset $\{r, r+1, ,..., z\}$, $r = 1,2,..., z$, under the assumption that the mean values $M_b(r)$, $b = 1,2,...,\nu$, $r = 1,2,..., z$, of the system conditional lifetimes in this reliability state subset are fixed. More exactly, we may formulate the optimization problem as a linear programming model with the objective function of the following form

$$M(r) = \sum_{b=1}^{\nu} p_b M_b(r) \tag{1}$$

for a fixed $r \in \{1,2,..., z\}$ and with the following bound constraints

$$\breve{p}_b \le p_b \le \hat{p}_b, \ b = 1,2,...,\nu, \tag{2}$$

$$\sum_{b=1}^{\nu} p_b = 1, \tag{3}$$

where

$$M_b(r), \ M_b(r) \ge 0, \ b = 1,2,...,\nu, \tag{4}$$

are fixed mean values of the system conditional lifetimes in the reliability state subset $\{r, r+1,..., z\}$ and

$$\breve{p}_b, \ 0 \le \breve{p}_b \le 1 \text{ and } \hat{p}_b, \ 0 \le \hat{p}_b \le 1, \ \breve{p}_b \le \hat{p}_b, \ b = 1,2,...,\nu, \tag{5}$$

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,...,\nu$, respectively.
Now, we can obtain the optimal solution of the formulated by (1)-(5) the linear programming problem, i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,...,\nu$, that maximize the objective function given by (1).
First, we arrange the system conditional lifetime mean values $M_b(r)$, $b = 1,2,...,\nu$, in non-increasing order

$$M_{b_1}(r) \ge M_{b_2}(r) \ge . . . \ge M_{b_\nu}(r), \text{ where } b_i \in \{1,2,...,\nu\} \text{ for } i = 1,2,...,\nu.$$

Next, we substitute

$$x_i = p_{b_i}, \ \breve{x}_i = \breve{p}_{b_i}, \ \hat{x}_i = \hat{p}_{b_i} \text{ for } i = 1,2,...,\nu \tag{6}$$

and we maximize with respect to x_i, $i = 1,2,...,\nu$, the linear form (1) that after this transformation takes the form

$$M(r) = \sum_{i=1}^{\nu} x_i M_{b_i}(r)$$
(7)

for a fixed $r \in \{1,2,...,z\}$ with the following bound constraints

$$\breve{x}_i \le x_i \le \hat{x}_i,\ i = 1,2,...,\nu, \quad (8)$$

$$\sum_{i=1}^{\nu} x_i = 1,$$
(9)

where

$$M_{b_i}(r),\ M_{b_i}(r) \ge 0,\ i = 1,2,...,\nu,$$

are fixed mean values of the system conditional lifetimes in the reliability state subset $\{r, r+1,...,z\}$ arranged in non-increasing order and

$$\breve{x}_i,\ 0 \le \breve{x}_i \le 1 \text{ and } \hat{x}_i,\ 0 \le \hat{x}_i \le 1,\ \breve{x}_i \le \hat{x}_i,\ i = 1,2,...,\nu, \quad (10)$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,...,\nu$, respectively.
To find the optimal values of x_i, $i = 1,2,...,\nu$, we define

$$\breve{x} = \sum_{i=1}^{\nu} \breve{x}_i,\ \hat{y} = 1 - \breve{x} \quad (11)$$

and

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0 \text{ and } \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i,\ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \text{ for } I = 1,2,...,\nu. \quad (12)$$

Next, we find the largest value $I \in \{0,1,...,\nu\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y}$$
(13)

and we fix the optimal solution that maximize (7) in the following way:
i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 2,3,...,\nu; \quad (14)$$

ii) if $0 < I < \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,...,I,\ \dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1} \text{ and } \dot{x}_i = \breve{x}_i$$

for $i = I+2, I+3, ..., \nu$; (15)

iii) if $I = \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,...,\nu. \quad (16)$$

Finally, after making the inverse to (6) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \text{ for } i = 1,2,...,\nu, \quad (17)$$

that maximize the system mean lifetime in the reliability state subset $\{r, r+1, ..., z\}$, defined by the linear form (1), giving its maximum value in the following form

$$\dot{M}(r) = \sum_{b=1}^{\nu} \dot{p}_b M_b(r) \quad (18)$$

for a fixed $r \in \{1,2,...,z\}$.

From the expression (18) for the maximum mean value $\dot{M}(r)$ of the system unconditional lifetime in the reliability state subset $\{r, r+1, ..., z\}$, replacing in it the critical reliability state r by the reliability state u, $u = 1,2,...,z$, we obtain the corresponding optimal solutions for the mean values of the system unconditional lifetimes in the reliability state subsets $\{u, u+1, ..., z\}$ of the form

$$\dot{M}(u) = \sum_{b=1}^{\nu} \dot{p}_b M_b(u) \text{ for } u = 1,2,...,z. \quad (19)$$

Further, according to (24)-(25) (Kołowrocki, Soszyńska-Budny, 2013), the corresponding optimal unconditional multistate reliability function of the system is the vector

$$\dot{\boldsymbol{R}}(t,\cdot) = [1, \dot{\boldsymbol{R}}(t,1), ..., \dot{\boldsymbol{R}}(t,z)], \quad (20)$$

with the coordinates given by

$$\dot{\boldsymbol{R}}(t,u) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{R}(t,u)]^{(b)} \text{ for } t \geq 0,\ u = 1,2,...,z. \quad (21)$$

And, by (29) (Kołowrocki, Soszyńska-Budny, 2013), the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states are

$$\dot{\overline{M}}(u) = \dot{M}(u) - \dot{M}(u+1),\ u =,1,...,z-1,\ \dot{\overline{M}}(z) = \dot{M}(z).$$
(22)

Moreover, considering (30) and (31) (Kołowrocki, Soszyńska-Budny, 2013), the corresponding optimal system risk function and the optimal moment when the risk exceeds a permitted level δ, respectively are given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}(t,r),\ t \geq 0,$$
(23)

and

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta), \quad (24)$$

where $\dot{\boldsymbol{R}}(t,r)$ is given by (21) for $u = r$ and $\dot{\boldsymbol{r}}^{-1}(t)$, if it exists, is the inverse function of the optimal risk function $\dot{\boldsymbol{r}}(t)$.

Replacing in (8) (Kołowrocki, Soszyńska-Budny, 2013) the limit transient probabilities p_b of the system operation process at the operation states by their optimal values $\dot{p}_b$, maximizing the mean value $M(r)$ of the system lifetime in the reliability states subset $\{r, r+1,...,z\}$ defined by (1) and the mean values m_b of the unconditional sojourn times at the operation states by their corresponding unknown optimal values $\dot{m}_b$, we get the system of equations

$$\dot{p}_b = \frac{\pi_b \dot{m}_b}{\sum_{l=1}^{v} \pi_l \dot{m}_l}, \quad b = 1,2,...,v. \quad (25)$$

After simple transformations the above system takes the form

$$(\dot{p}_1 - 1)\pi_1 \dot{m}_1 + \dot{p}_1 \pi_2 \dot{m}_2 + ... + \dot{p}_1 \pi_v \dot{m}_v = 0$$

$$\dot{p}_2 \pi_1 \dot{m}_1 + (\dot{p}_2 - 1)\pi_2 \dot{m}_2 + ... + \dot{p}_2 \pi_v \dot{m}_v = 0$$

$$...$$

(26)

$$\dot{p}_v \pi_1 \dot{m}_1 + \dot{p}_v \pi_2 \dot{m}_2 + ... + (\dot{p}_v - 1)\pi_v \dot{m}_v = 0,$$

where $\dot{m}_b$ are unknown variables we want to find, $\dot{p}_b$ are optimal transient probabilities determined by (17) and π_b are steady probabilities determined by (9) (Kołowrocki, Soszyńska-Budny, 2013).

Since the system of equations (26) is homogeneous and it can be proved that the determinant of its main matrix is equal to zero, then it has nonzero solutions and moreover, these solutions are ambiguous. Thus, if we fix some of the optimal values $\dot{m}_b$ of the mean values m_b of the unconditional sojourn times at the operation states, for instance by arbitrary fixing one or a few of them, we may find the values of the remaining once and this way to get the solution of this equation.

Having this solution, it is also possible to look for the optimal values $\dot{m}_{bl}$ of the mean values m_{bl} of the conditional sojourn times at the operation states using the following system of equations

$$\sum_{l=1}^{v} p_{bl} \dot{m}_{bl} = \dot{m}_b, \quad b = 1,2,...,v, \quad (27)$$

obtained from (7) (Kołowrocki, Soszyńska-Budny, 2013) by replacing m_b by $\dot{m}_b$ and m_{bl} by $\dot{m}_{bl}$, were p_{bl} are known probabilities of the system operation process transitions between the operation states z_b i z_l, $b,l = 1,2,...,v$, $b \neq l$, defined by (2) (Kołowrocki, Soszyńska-Budny, 2013).

Another very useful and much easier to be applied in practice tool that can help in planning the operation processes of the complex technical systems are the system operation process optimal mean values of the total system operation process sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b = 1,2,...,v,$ during the fixed system operation time θ, that can be obtain by the replacing in the formula (10) (Kołowrocki, Soszyńska-Budny, 2013) the transient probabilities p_b at the operation states z_b by their optimal values $\dot{p}_b$ and resulting in the following expession

$$\dot{\hat{M}}_b = \dot{E}[\hat{\theta}_b] = \dot{p}_b\theta, \ b = 1,2,...,v. \tag{28}$$

The knowledge of the optimal values $\dot{m}_b$ of the mean values of the unconditional sojourn times and the optimal values $\dot{m}_{bl}$ of the mean values of the conditional sojourn times at the operation states and the optimal mean values $\dot{\hat{M}}_b$ of the total sojourn times at the particular operation states during the fixed system operation time may by the basis for changing the complex technical systems operation processes in order to ensure these systems operation more reliable.

3 APPLICATION

We consider a series system S composed of the subsystems S_1 and S_2, with the scheme showed in Figures 1-3 (Kołowrocki, Soszyńska-Budny, 2013). This system reliability structure and its components reliability parameters depend on its changing in time operation states with arbitrarily fixed the number of the system operation process states $v = 4$ and their influence on the system reliability indicated in Sections 2-3 (Kołowrocki, Soszyńska-Budny, 2013) where its main reliability characteristics are predicted.

To find the optimal values of those system reliability characteristics, we conclude that the objective function defined by (1), in this case, as the exemplary system critical state is $r = 2$, according to (89) (Kołowrocki, Soszyńska-Budny, 2013), takes the form

$$M(2) = p_1 \cdot 25.00 + p_2 \cdot 14.88 + p_3 \cdot 13.04 + p_4 \cdot 7.04. \tag{29}$$

Arbitrarily assumed, the lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown optimal values of transient probabilities p_b, $b = 1,2,3,4,$ respectively are:

$$\breve{p}_1 = 0.201, \ \breve{p}_2 = 0.03, \ \breve{p}_3 = 0.245. \ \breve{p}_4 = 0.309;$$

$$\hat{p}_1 = 0.351, \ \hat{p}_2 = 0.105, \ \hat{p}_3 = 0.395, \ \hat{p}_4 = 0.459.$$

Therefore, according to (2)-(3), we assume the following bound constraints

$$0.201 \le p_1 \le 0.351, \ 0.030 \le p_2 \le 0.105,$$

$$0.245 \le p_3 \le 0.395, \ 0.309 \le p_4 \le 0.459. \tag{30}$$

$$\sum_{b=1}^{4} p_b = 1, \tag{31}$$

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,3,4,$ that maximize the objective function (29), w arrange the system conditional lifetime mean values $M_b(2)$, $b = 1,2,3,4,$ in non-increasing order

$$M_1(2) \geq M_2(2) \geq M_3(2) \geq M_4(2).$$

Further, according to (6), we substitute

$$x_1 = p_1, \ x_2 = p_2, \ x_3 = p_3, \ x_4 = p_4, \tag{32}$$

and

$$\breve{x}_1 = \breve{p}_1 = 0.201, \ \breve{x}_2 = \breve{p}_2 = 0.030, \ \breve{x}_3 = \breve{p}_3 = 0.245, \ \breve{x}_4 = \breve{p}_4 = 0.309; \tag{33}$$

$$\hat{x}_1 = \hat{p}_1 = 0.351, \ \hat{x}_2 = \hat{p}_2 = 0.105, \ \hat{x}_3 = \hat{p}_3 = 0.395, \ \hat{x}_4 = \hat{p}_4 = 0.459, \tag{34}$$

and we maximize with respect to x_i, $i = 1,2,3,4,$ the linear form (29) that according to (7)-(9) takes the form

$$M(2) = x_1 \cdot 25.00 + x_2 \cdot 14.88 + x_3 \cdot 13.04 + x_4 \cdot 7.04, \tag{35}$$

with the following bound constraints

$$0.201 \leq x_1 \leq 0.351, \ 0.030 \leq x_2 \leq 0.105,$$

$$0.245 \leq x_3 \leq 0.395, \ 0.309 \leq x_4 \leq 0.459. \tag{36}$$

$$\sum_{i=1}^{4} x_i = 1. \tag{37}$$

According to (11), we calculate

$$\breve{x} = \sum_{i=1}^{4} \breve{x}_i = 0.785, \ y = 1 - \breve{x} = 1 - 0.785 = 0.215 \tag{38}$$

and according to (12), we determine

$$\breve{x}^0 = 0, \ \hat{x}^0 = 0, \ \hat{x}^0 - \breve{x}^0 = 0,$$

$$\breve{x}^1 = 0.201, \ \hat{x}^1 = 0.351, \ \hat{x}^1 - \breve{x}^1 = 0.150,$$

$$\breve{x}^2 = 0.231, \ \hat{x}^2 = 0.456, \ \hat{x}^2 - \breve{x}^2 = 0.225,$$

$$\breve{x}^3 = 0.476. \ \hat{x}^3 = 0.851, \ \hat{x}^3 - \breve{x}^3 = 0.375,$$

$$\breve{x}^4 = 0.785 \ \ \hat{x}^4 = 1.31 \ \ \hat{x}^4 - \breve{x}^4 = 0.525. \tag{39}$$

From the above, as according to (38), the inequality (13) takes the form

$$\hat{x}^I - \breve{x}^I < 0.215, \tag{40}$$

it follows that the largest value $I \in \{0,1,2,3,4\}$ such that this inequality holds is $I = 1$.
Therefore, we fix the optimal solution that maximize linear function (35) according to the rule (15). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.351,$$

$$\dot{x}_2 = \hat{y} - \hat{x}^1 + \breve{x}^1 + \breve{x}_2 = 0.215 - 0.351 + 0.201 + 0.030 = 0.095,$$

$$\dot{x}_3 = \breve{x}_3 = 0.245, \ \ \dot{x}_4 = \breve{x}_4 = 0.309. \tag{41}$$

Finally, after making the inverse to (32) substitution, we get the optimal transient probabilities

$$\dot{p}_1 = \dot{x}_1 = 0.351, \ \ \dot{p}_2 = \dot{x}_2 = 0.095, \ \ \dot{p}_3 = \dot{x}_3 = 0.245, \ \ \dot{p}_4 = \dot{x}_4 = 0.309,$$
(42)

that maximize the exemplary system mean lifetime $M(2)$ in the reliability state subset $\{2,3\}$ expressed by the linear form (29) giving, according to (18) and (42), its optimal value

$$\dot{M}(2) = \dot{p}_1 \cdot 25.00 + \dot{p}_2 \cdot 14.88 + \dot{p}_3 \cdot 13.04 + \dot{p}_4 \cdot 7.04$$

$$= 0.351 \cdot 25.00 + 0.095 \cdot 14.88 + 0.245 \cdot 13.04 + 0.309 \cdot 7.07 \cong 15.56. \tag{43}$$

Substituting the optimal solution (42) into the formula (19), we obtain the optimal solution for the mean values of the exemplary system unconditional lifetimes in the reliability state subsets $\{1,2,3\}$ and $\{3\}$, that are as follows

$$\dot{M}(1) = \dot{p}_1 \cdot 27.78 + \dot{p}_2 \cdot 16.27 + \dot{p}_3 \cdot 14.82 + \dot{p}_4 \cdot 7.72$$

$$= 0.351 \cdot 27.78 + 0.095 \cdot 16.27 + 0.245 \cdot 14.82 + 0.309 \cdot 7.72 \cong 17.31, \tag{44}$$

$$\dot{M}(3) = \dot{p}_1 \cdot 22.73 + \dot{p}_2 \cdot 13.71 + \dot{p}_3 \cdot 11.48 + \dot{p}_4 \cdot 6.47$$

$$= 0.351 \cdot 22.73 + 0.095 \cdot 13.71 + 0.245 \cdot 11.48 + 0.309 \cdot 6.47 \cong 14.09 \tag{45}$$

and according to (22), the optimal values of the mean values of the system unconditional lifetimes in the particular reliability states 1, 2 and 3, respectively are

$$\dot{\bar{M}}(1) = \dot{M}(1) - \dot{M}(2) = 1.75, \ \dot{\bar{M}}(2) = \dot{M}(2) - \dot{M}(3) = 1.47,$$

$$\dot{\bar{M}}(3) = \dot{M}(3) = 14.09. \tag{46}$$

Moreover, according to (20)-(21), the corresponding optimal unconditional multistate reliability function of the system is of the form

$$\dot{\boldsymbol{R}}(t,\cdot) = [1, \dot{\boldsymbol{R}}(t,1), \ \dot{\boldsymbol{R}}(t,2), \ \dot{\boldsymbol{R}}(t,3)], \ t \geq 0, \tag{47}$$

with the coordinates given by

$$\dot{\boldsymbol{R}}(t,1) = 0.351 \cdot [\boldsymbol{R}(t,1)]^{(1)} + 0.095 \cdot [\boldsymbol{R}(t,1)]^{(2)} + 0.245 \cdot [\boldsymbol{R}(t,1)]^{(3)}$$

$$+ 0.309 \cdot [\boldsymbol{R}(t,1)]^{(4)} \text{ for } t \geq 0, \quad (48)$$

$$\dot{\boldsymbol{R}}(t,2) = 0.351 \cdot [\boldsymbol{R}(t,2)]^{(1)} + 0.095 \cdot [\boldsymbol{R}(t,2)]^{(2)} + 0.245 \cdot [\boldsymbol{R}(t,2)]^{(3)}$$

$$+ 0.309 \cdot [\boldsymbol{R}(t,2)]^{(4)} \text{ for } t \geq 0, \quad (49)$$

$$\dot{\boldsymbol{R}}(t,3) = 0.351 \cdot [\boldsymbol{R}(t,3)]^{(1)} + 0.0095 \cdot [\boldsymbol{R}(t,3)]^{(2)} + 0.245 \cdot [\boldsymbol{R}(t,3)]^{(3)}$$

$$+ 0.309 \cdot [\boldsymbol{R}(t,3)]^{(4)} \text{ for } t \geq 0, \quad (50)$$

where $[\boldsymbol{R}(t,1)]^{(b)}$, $[\boldsymbol{R}(t,2)]^{(b)}$, $[\boldsymbol{R}(t,3)]^{(b)}$, $b = 1,2,3,4$, are fixed in Section 3 (Kołowrocki, Soszyńska-Budny, 2013).
The graph of the exemplary system optimal reliability function $\dot{\boldsymbol{R}}(t,\cdot)$ given by (47)-(50) is presented in Figure 1.

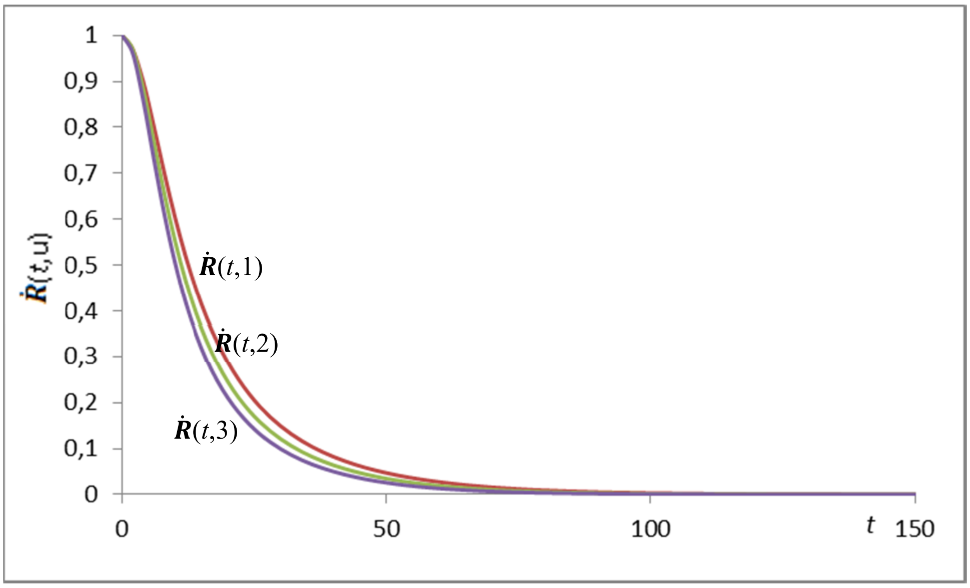

Fig. 1. The graph of the exemplary system optimal reliability function $\dot{\boldsymbol{R}}(t,\cdot)$ coordinates

As the critical reliability state is r =2, then the exemplary system optimal system risk function, according to (23), is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}(t,2) \text{ for } t \geq 0, \quad (51)$$

where $\dot{\boldsymbol{R}}(t,2)$ is given by (49).
Hence and considering (24), the moment when the optimal system risk function exceeds a permitted level, for instance δ = 0.025, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 2.55. \quad (52)$$

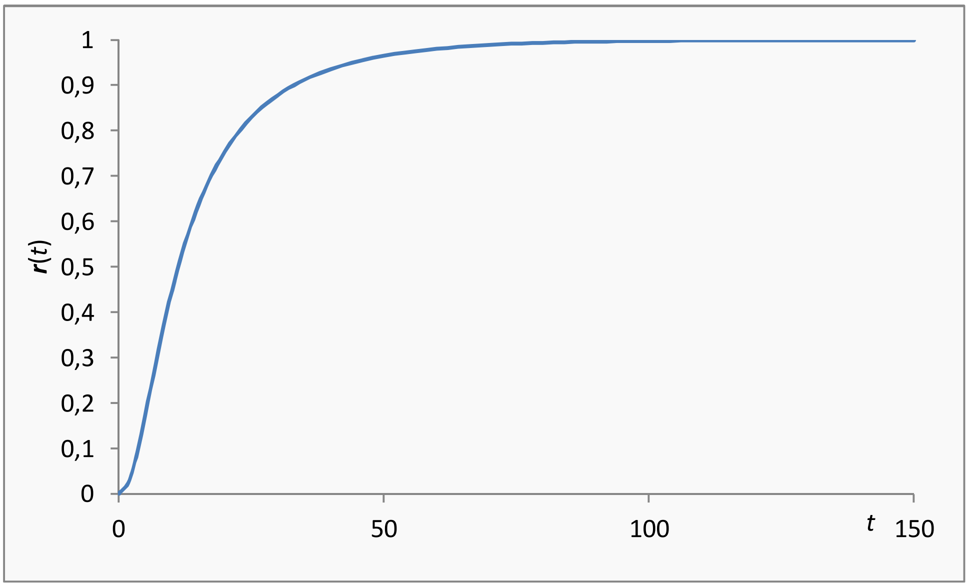

Fig. 2. The graph of the exemplary system optimal risk function $\dot{r}(t)$

Substituting the exemplary operation process optimal transient probabilities at operation states

$$\dot{p}_1 = 0.351,\ \dot{p}_2 = 0.095,\ \dot{p}_3 = 0.245,\ \dot{p}_4 = 0.309,$$

determined by (42) and the steady probabilities

$$\pi_1 \cong 0.236,\ \pi_2 \cong 0.169,\ \pi_3 \cong 0.234,\ \pi_4 \cong 0.361,$$

determined by (17) in Section 2 (Kołowrocki, Soszyńska-Budny, 2013) into (26), we get the following system of equations with the unknown optimal mean values $\dot{m}_b$ of the exemplary system operation process unconditional sojourn times at the operation states we are looking for

$$-0.153164\dot{m}_1 + 0.059319\dot{m}_2 + 0.082134\dot{m}_3 + 0.126711\dot{m}_4 = 0$$

$$0.02242\dot{m}_1 - 0.152945\dot{m}_2 + 0.02223\dot{m}_3 + 0.034295\dot{m}_4 = 0$$

$$0.05782\dot{m}_1 + 0.041405\dot{m}_2 - 0.17667\dot{m}_3 + 0.088445\dot{m}_4 = 0$$

$$0.072924\dot{m}_1 + 0.052221\dot{m}_2 + 0.072306\dot{m}_3 - 0.249451\dot{m}_4 = 0. \quad (53)$$

The determinant of the main matrix of the above homogeneous system of equations is equal to zero and therefore there are non-zero solutions of this system of equations that are ambiguous and dependent on one or more parameters. Thus, we may fix some of them and determine the remaining ones. To show the way of solving this system of equations, we may suppose that we are arbitrarily interested in fixing the value of $\dot{m}_4$ and we put

$$\dot{m}_4 = 400.$$

Substituting the above value into the system of equations (53), we get

$$-0.153164\dot{m}_1 + 0.059319\dot{m}_2 + 0.082134\dot{m}_3 = -50.6844$$

$$0.02242\dot{m}_1 - 0.152945\dot{m}_2 + 0.02223\dot{m}_3 = -13.7180$$

$$0.05782\dot{m}_1 + 0.041405\dot{m}_2 - 0.17667\dot{m}_3 = -35.3780$$

$$0.072924\dot{m}_1 + 0.052221\dot{m}_2 + 0.072306\dot{m}_3 = 99.7804$$

and we solve it with respect to $\dot{m}_1$, $\dot{m}_2$ and $\dot{m}_3$, after omitting its last equation. This way obtained solutions that satisfy (53), are

$$\dot{m}_1 \cong 689, \ \dot{m}_2 \cong 261, \ \dot{m}_3 \cong 487, \ \dot{m}_4 = 400. \quad (54)$$

It can be seen that these solution differ much from the values m_1, m_2, m_3 and m_4 estimated in Section 2 (Kołowrocki, Soszyńska-Budny, 2013) and given by (13)-(16) (Kołowrocki, Soszyńska-Budny, 2013).
Having these solutions, it is also possible to look for the optimal values $\dot{m}_{bl}$ of the mean values m_{bl} of the exemplary system operation process conditional sojourn times at operation states. Namely, substituting the values $\dot{m}_b$ instead of m_b, the probabilities

$$[p_{bl}] = \begin{bmatrix} 0 & 0.22 & 0.32 & 0.46 \\ 0.20 & 0 & 0.30 & 0.50 \\ 0.12 & 0.16 & 0 & 0.72 \\ 0.48 & 0.22 & 0.30 & 0 \end{bmatrix}$$

of the exemplary system operation process transitions between the operation states given by (11) in Section 2 (Kołowrocki, Soszyńska-Budny, 2013) and replacing m_{bl} by $\dot{m}_{bl}$ in (27), we get the following system of equations

$$0.22\dot{m}_{12} + 0.32\dot{m}_{13} + 0.46\dot{m}_{14} = 689$$

$$0.20\dot{m}_{21} + 0.30\dot{m}_{23} + 0.50\dot{m}_{24} = 261$$

$$0.12\dot{m}_{31} + 0.16\dot{m}_{32} + 0.72\dot{m}_{34} = 487$$

$$0.48\dot{m}_{41} + 0.2\dot{2}_{42} + 0.30\dot{m}_{14} = 400 \quad (55)$$

with the unknown optimal values $\dot{m}_{bl}$ we want to find.
As the solutions of the above system of equations are ambiguous, then we fix some of them, say that because of practically important reasons, and we find the remaining ones. For instance:

- we fix in the first equation $\dot{m}_{12} = 200$, $\dot{m}_{13} = 500$ and we find $\dot{m}_{14} \cong 1054$;
- we fix in the second equation $\dot{m}_{21} = 100$, $\dot{m}_{23} = 100$ and we find $\dot{m}_{24} \cong 422$;

- we fix in the third equation $\dot{m}_{31} = 900,\ \dot{m}_{32} = 500$ and we find $\dot{m}_{34} \cong 415$;
- we fix in the fourth equation $\dot{m}_{41} = 300,\ \dot{m}_{42} = 500$ and we find $\dot{m}_{43} \cong 487$. (56)

It can be seen that these solutions differ much from the mean values of the exemplary system conditional sojourn times at the particular operation states before its operation process optimization given by (12) (Kołowrocki, Soszyńska-Budny, 2013).
Another very useful and much easier to be applied in practice tool that can help in planning the operation process of the exemplary system are the system operation process optimal mean values of the total sojourn times at the particular operation states during the system operation time that by the same assumpion as in Section 2 (Kołowrocki, Soszyńska-Budny, 2013) is equel to $\theta = 1$year = 365 days. Under this assumption, after aplying (28), we get the optimal values of the exemplary system operation process total sojourn times at the particular operation states during 1 year

$$\dot{\hat{m}}_1 = \dot{E}[\hat{\theta}_1] = \dot{p}_1\theta = 0.341 \cdot 365 \cong 124.5,$$

$$\dot{\hat{m}}_2 = \dot{E}[\hat{\theta}_2] = \dot{p}_2\theta = 0.105 \cdot 365 \cong 38.3,$$

$$\dot{\hat{m}}_3 = \dot{E}[\hat{\theta}_3] = \dot{p}_3\theta = 0.245 \cdot 365 \cong 89.4,$$

$$\dot{\hat{m}}_4 = \dot{E}[\hat{\theta}_4] = \dot{p}_4\theta = 0.309 \cdot 365 \cong 112.8, \quad (57)$$

that differ much from the values of $\hat{m}_1$, $\hat{m}_2$, $\hat{m}_3$, $\hat{m}_4$, determined by (19) in Section 2 (Kołowrocki, Soszyńska-Budny, 2013).
In practice, the knowledge of the optimal values of $\dot{m}_b$ $\dot{m}_{bl}$ and $\dot{\hat{m}}_b$ given respectively by (54), (56), (57), can be very important and helpful for the system operation process planning and improving in order to make the system operation more reliable.

4 CONCLUSION

Presented in this paper tool is useful in reliability and operation optimization of a very wide class of real technical systems operating at the varying conditions that have an influence on changing their reliability structures and their components reliability parameters. The results can be interesting for reliability practitioners from various industrial sectors.

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MODELLING RELIABILITY OF COMPLEX SYSTEMS

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ABSTRACT

Modelling and prediction of the operation and reliability of technical systems related to their operation processes are presented. The emphasis is on multistate systems composed of ageing components and changing their reliability structures and their components reliability parameters during their operation processes that are called the complex systems. The integrated general model of complex systems' reliability, linking their reliability models and their operation processes models and considering variable at different operation states their reliability structures and their components reliability parameters is constructed. This theoretical tool is applied to modelling and prediction of the operation processes and reliability characteristics of the multistate non-homogeneous system composed of a series-parallel and a series-"m out of l" subsystems linked in series, changing its reliability structure and its components reliability parameters at variable operation conditions.

1 INTRODUCTION

Most real technical systems are very complex and it is difficult to analyze their reliability. Large numbers of components and subsystems and their operating complexity cause that the identification, evaluation and prediction of their reliability are complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability parameters are very often met in real practice. Thus, the practical importance of an approach linking the system reliability models and the system operation processes models into an integrated general model in reliability assessment of real technical systems is evident.

The convenient tools for analyzing these problems are semi-Markov modelling the systems' operation processes (Ferreir, Pacheco, 2007; Glynn, Hass, 2006; Habibullah et al. 2009; Kołowrocki, Soszyńska, 2009; Mercier, 2008; Soszyńska et al. 2010; Grabski, 2002; Kołowrocki, Soszyńska-Budny, 2011; Limnios, Oprisan, 2001; Kołowrocki 2008) multistate approach to the systems' reliability evaluation (Kołowrocki, Soszyńska, 2009; Xue, 1985; Xue, Yang 1995b; Kołowrocki, 2008). The common usage of the multistate systems' reliability models and the semi-Markov model for the systems' operation processes in order to construct the joint general system reliability model related to its operation process (Kołowrocki, 2006; Kołowrocki, 2007a; Kołowrocki 2007b; Kołowrocki, Soszyńska, 2006; Kołowrocki, Soszyńska, 2010, Soszyńska, 2007a; Soszyńska, 2007b; Kołowrocki, Soszyńska-Budny, 2011; Soszyńska 2007c; Kołowrocki et all 2008) and to apply it to the reliability analysis of complex technical systems is this paper main idea.

2 COMPLEX SYSTEM OPERATION PROCESS MODELLING

We assume that the system during its operation process is taking $\nu, \nu \in N$, different operation states $z_1, z_2, \dots, z_\nu$. Further, we define the system operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \dots, z_\nu\}$. Moreover, we assume that the system operation process $Z(t)$ is a semi-Markov process (Kołowrocki, Soszyńska, 2009; Kołowrocki, Soszyńska, 2010; Grabski, 2002; Soszyńska, 2007b) with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,...,v$, $b \neq l$. Under these assumptions, the system operation process may be described by:

- the vector of the initial probabilities $p_b(0) = P(Z(0) = z_b)$, $b = 1,2,...,v$, of the system operation process $Z(t)$ staying at particular operation states at the moment $t = 0$

$$[p_b(0)]_{1 \times v} = [p_1(0), p_2(0),..., p_v(0)]; \quad (1)$$

- the matrix of probabilities p_{bl}, $b, l = 1,2,...,v$, $b \neq l$, of the system operation process $Z(t)$ transitions between the operation states z_b and z_l

$$[p_{bl}]_{v \times v} = \begin{bmatrix} p_{11} \ p_{12} \ \cdots \ p_{1v} \\ p_{21} \ p_{22} \ \cdots \ p_{2v} \\ \cdots \\ p_{v1} \ p_{v2} \ \cdots \ p_{vv} \end{bmatrix}, \quad (2)$$

where by a formal agreement

$$p_{bb} = 0 \text{ for } b = 1,2,...,v;$$

- the matrix of conditional distribution functions $H_{bl}(t) = P(\theta_{bl} < t)$, $b, l = 1,2,...,v$, $b \neq l$, of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states

$$[H_{bl}(t)]_{v \times v} = \begin{bmatrix} H_{11}(t) \ H_{12}(t) \ldots H_{1v}(t) \\ H_{21}(t) \ H_{22}(t) \ldots H_{2v}(t) \\ \cdots \\ H_{v1}(t) \ H_{v2}(t) \ldots H_{vv}(t) \end{bmatrix}, \quad (3)$$

where by formal agreement

$$H_{bb}(t) = 0 \text{ for } b = 1,2,...,v.$$

We introduce the matrix of the conditional density functions of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states corresponding to the conditional distribution functions $H_{bl}(t)$

$$[h_{bl}(t)]_{v \times v} = \begin{bmatrix} h_{11}(t) \ h_{12}(t) \ldots h_{1v}(t) \\ h_{21}(t) \ h_{22}(t) \ldots h_{2v}(t) \\ \cdots \\ h_{v1}(t) \ h_{v2}(t) \ldots h_{vv}(t) \end{bmatrix}, \quad (4)$$

where

$$h_{bl}(t) = \frac{d}{dt}[H_{bl}(t)] \text{ for } b, l = 1,2,...,v, \ b \neq l,$$

and by formal agreement

$$h_{bb}(t) = 0 \text{ for } b = 1,2,...,v.$$

As the mean values $E[\theta_{bl}]$ of the conditional sojourn times θ_{bl} are given by

$$m_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t) = \int_0^\infty t h_{bl}(t) dt, \ b, l = 1,2,...,v, \ b \neq l, \tag{5}$$

then from the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ_b, $b = 1,2,...,v$, of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,...,v$, are given by (Grabski, 2002; Kołowrocki, Soszyńska-Budny, 2011; Soszyńska, 2007b; Limnios, Oprisan, 2001)

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ b = 1,2,...,v. \tag{6}$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,...,v$, at the operation states are given by

$$m_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} m_{bl}, \ b = 1,2,...,v, \tag{7}$$

where m_{bl} are defined by the formula (5).
The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b), \ t \in <0, +\infty), \ b = 1,2,...,v,$$

are given by (Grabski, 2002; Kołowrocki, Soszyńska-Budny, 2011; Soszyńska, 2007b; Limnios, Oprisan, 2001)

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b m_b}{\sum_{l=1}^{v} \pi_l m_l}, \ b = 1,2,...,v, \tag{8}$$

where m_b, $b = 1,2,...,v$, are given by (7), while the steady probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \tag{9}$$

In the case of a periodic system operation process, the limit transient probabilities p_b, $b = 1,2,...,v$, at the operation states defined by (8), are the long term proportions of the system operation process $Z(t)$ sojourn times at the particular operation states z_b, $b = 1,2,...,v$.
Other interesting characteristics of the system operation process $Z(t)$ possible to obtain are its total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b = 1,2,...,v$, during the fixed system opetation time. It is well known (Grabski, 2002; Kołowrocki, Soszyńska-Budny, 2011; Soszyńska, 2007b; Limnios, Oprisan, 2001) that the system operation process total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}_b = E[\hat{\theta}_b] = p_b \theta, \quad b = 1,2,...,v, \tag{10}$$

where p_b are given by (8).

Example
We consider a series system S composed of the subsystems S_1 and S_2, with the scheme showed in Figure 1.

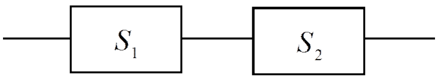

Fig. 1. The scheme of the exemplary system S reliability structure

We assume that the subsystem S_1 is a series-parallel system with the scheme given in Figure 2 and the subsystem S_2 illustrated in Figure 3 is either a series-parallel system or a series-"2 out of 4" system.

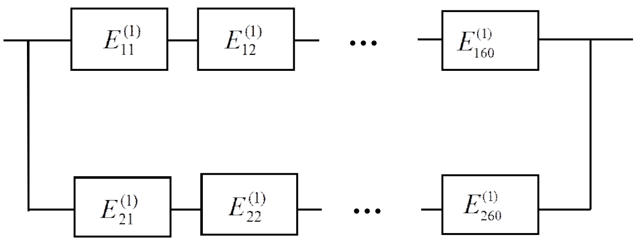

Fig. 2. The scheme of the subsystem S_1 reliability structure

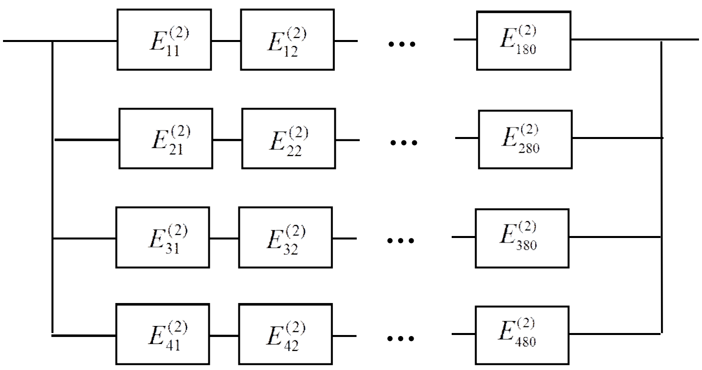

Fig. 3. The scheme of the subsystem S_2 reliability structure

The subsystems S_1 and S_2 are forming a general series reliability structure of the system presented in Figure 1. However, this system reliability structure and its subsystems and components reliability depend on its changing in time operation states (Kołowrocki, Soszyńska, 2009; Soszyńska, 2007b).
Under the assumption that the system operation conditions are changing in time, we arbitrarily fix the number of the system operation process states $\nu = 4$ and we distinguish the following as its operation states:

- an operation state z_1 – the system is composed of the subsystem S_1 with the scheme showed in Figure 2 that is a series-parallel system,
- an operation state z_2 – the system is composed of the subsystem S_2 with the scheme showed in Figure 3 that is a series-parallel system,
- an operation state z_3 – the system is a series system with the scheme showed in Figure 1 composed of the subsystems S_1 and S_2 that are series-parallel systems with the schemes respectively given in Figure 2 and Figure 3,
- an operation state z_4 – the system is a series system with the scheme showed in Figure 1 composed of the subsystem S_1 and S_2, while the subsystem S_1 is a series-parallel system with the scheme given in Figure 2 and the subsystem S_2 is a series-"2 out of 4" system with the scheme given in Figure 3.

The influence of the above system operation states changing on the changes of the exemplary system reliability structure is indicated in these operation states above definitions and illustrated in Figures 1-3. Its influence on the system components reliability will be defined in this example continuation in Section 3.
We arbitrarily assume that the probabilities p_{bl} of the exemplary system operation process transitions from operation state z_b into the operation state z_l are given in the matrix below

$$[p_{bl}] = \begin{bmatrix} 0 & 0.25 & 0.30 & 0.45 \\ 0.20 & 0 & 0.25 & 0.55 \\ 0.15 & 0.20 & 0 & 0.65 \\ 0.40 & 0.25 & 0.35 & 0 \end{bmatrix}. \tag{11}$$

We also arbitrarily fix the conditional mean values $m_{bl} = E[\theta_{bl}],\ b,l = 1,2,3,4,$ of the exemplary system sojourn times at the particular operation states as follows:

$$\begin{aligned} &m_{12} = 190,\ m_{13} = 480,\ m_{14} = 200,\\ &m_{21} = 100,\ m_{23} = 80,\ m_{24} = 60,\\ &m_{31} = 870,\ m_{32} = 480,\ m_{34} = 300,\\ &m_{41} = 320,\ m_{42} = 510,\ m_{43} = 440. \end{aligned} \tag{12}$$

This way, the exemplary system operation process is defined and we may find its main characteristics. Namely, applying (7), (11) and (12), the unconditional mean sojourn times at the particular operation states are given by:

$$m_1 = E[\theta_1] = p_{12}m_{12} + p_{13}m_{13} + p_{14}m_{14} = 0.25\cdot 190 + 0.30\cdot 480 + 0.45\cdot 200 = 281.5, \tag{13}$$

$$m_2 = E[\theta_2] = p_{21}m_{21} + p_{23}m_{23} + p_{24}m_{24} = 0.20\cdot 100 + 0.25\cdot 80 + 0.55\cdot 60 = 73.00, \tag{14}$$

$$m_3 = E[\theta_3] = p_{31}m_{31} + p_{32}m_{32} + p_{34}m_{34} = 0.15 \cdot 870 + 0.20 \cdot 480 + 0.65 \cdot 300 = 421.5, \quad (15)$$

$$m_4 = E[\theta_4] = p_{41}m_{41} + p_{42}m_{42} + p_{43}m_{43} = 0.40 \cdot 320 + 0.25 \cdot 510 + 0.35 \cdot 440 = 409.5. \quad (16)$$

Further, according to (9), the system of equations

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4] = [\pi_1, \pi_2, \pi_3, \pi_4][p_{bl}]_{4x4} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1, \end{cases}$$

after considering (11), takes the form

$$\begin{cases} \pi_1 = 0.20\pi_2 + 0.15\pi_3 + 0.40\pi_4 \\ \pi_2 = 0.25\pi_1 + 0.20\pi_3 + 0.25\pi_4 \\ \pi_3 = 0.30\pi_1 + 0.25\pi_2 + 0.35\pi_4 \\ \pi_4 = 0.45\pi_1 + 0.55\pi_2 + 0.65\pi_3 \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1. \end{cases}$$

The approximate solutions of the above system of equations are:

$$\pi_1 \cong 0.216, \ \pi_2 \cong 0.191, \ \pi_3 \cong 0.237, \pi_4 \cong 0.356. \quad (17)$$

After considering the result (17) and (13)-(16), we have

$$\sum_{l=1}^{4} \pi_l m_l \cong 0.216 \cdot 281.5 + 0.191 \cdot 73.0 + 0.237 \cdot 421.5 + 0.356 \cdot 409.5 = 320.4245,$$

and according to (8), the limit values of the exemplary system operation process transient probabilities $p_b(t)$ at the operation states z_b are given by

$$p_1 = \frac{0.216 \cdot 281.5}{320.4245} \cong 0.190, \ \ p_2 = \frac{0.191 \cdot 73.0}{320.4245} \cong 0.043,$$
$$p_3 = \frac{0.237 \cdot 421.5}{320.4245} \cong 0.312, \ \ p_4 = \frac{0.356 \cdot 409.5}{320.4245} \cong 0.455. \quad (18)$$

Hence, the expected values of the total sojourn times $\hat{\theta}_b$, $b = 1,2,3,4,$ of the exemplary system operation process at the particular operation states z_b, $b = 1,2,3,4,$ during the fixed operation time $\theta = 1$ year = 365 days, after applying (9.10), amount:

$$\hat{m}_1 = E[\hat{\theta}_1] = 0.190 \cdot 1 = 0.190 \text{ year} = 69.3 \text{ days},$$

$$\hat{m}_2 = E[\hat{\theta}_2] = 0.043 \cdot 1 = 0.043 \text{ year} = 15.7 \text{ days},$$

$$\hat{m}_3 = E[\hat{\theta}_3] = 0.312 \cdot 1 = 0.312 \text{ year} = 113.9 \text{ days},$$

$$\hat{m}_4 = E[\hat{\theta}_4] = 0.455 = 0.455 \text{ year} = 166.1 \text{ days}. \quad (19)$$

3 COMPLEX SYSTEM RELIABILITY MODELLING

We assume that the changes of the operation states of the system operation process $Z(t)$ have an influence on the system multistate components E_i, $i = 1,2,...,n,$ reliability and the system reliability structure as well. Consequently, we denote the system multistate component E_i, $i = 1,2,...,n,$ conditional lifetime in the reliability state subset $\{u, u+1,...,z\}$ while the system is at the operation state z_b, $b = 1,2,...,v,$ by $T_i^{(b)}(u)$ and its conditional reliability function by the vector

$$[R_i(t,\cdot)]^{(b)} = [1, [R_i(t,1)]^{(b)}, ..., [R_i(t,z)]^{(b)}], \tag{20}$$

with the coordinates defined by

$$[R_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b) \tag{21}$$

for $t \in < 0, \infty)$, $u = 1,2,...,z,$ $b = 1,2,...,v.$

The reliability function $[R_i(t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the reliability state subset $\{u, u+1,...,z\}$ is greater than t, while the system operation process $Z(t)$ is at the operation state z_b.

Similarly, we denote the system conditional lifetime in the reliability state subset $\{u, u+1,...,z\}$ while the system is at the operation state z_b, $b = 1,2,...,v,$ by $T^{(b)}(u)$ and the conditional reliability function of the system by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(b)} = [1, [\boldsymbol{R}(t,1)]^{(b)}, ..., [\boldsymbol{R}(t,z)]^{(b)}], \tag{22}$$

with the coordinates defined by

$$[\boldsymbol{R}(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b) \tag{23}$$

for $t \in < 0, \infty)$, $u = 1,2,...,z,$ $b = 1,2,...,v.$

The reliability function $[\boldsymbol{R}(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the reliability state subset $\{u, u+1,...,z\}$ is greater than t, while the system operation process $Z(t)$ is at the operation state z_b.

Further, we denote the system unconditional lifetime in the reliability state subset $\{u, u+1,...,z\}$ by $T(u)$ and the unconditional reliability function of the system by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), ..., \boldsymbol{R}(t,z)], \tag{24}$$

with the coordinates defined by

$$\boldsymbol{R}(t,u) = P(T(u) > t) \text{ for } t \in < 0, \infty), \; u = 1,2,...,z.$$

In the case when the system operation time θ is large enough, the coordinates of the unconditional reliability function of the system defined by (24) are given by

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}(t,u)]^{(b)} \text{ for } t \geq 0,\ u = 1,2,...,z, \tag{25}$$

where $[\boldsymbol{R}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,v$, are the coordinates of the system conditional reliability functions defined by (23) and p_b, $b = 1,2,...,v$, are the system operation process limit transient probabilities given by (9).

Thus, the mean value $\mu(u) = E[T(u)]$ of the system unconditional lifetime $T(u)$ in the reliability state subset $\{u, u+1,..., z\}$ is given by (Kołowrocki, Soszyńska-Budny, 2011; Soszyńska, 2007b),

$$M(u) \cong \sum_{b=1}^{v} p_b M_b(u),\ u = 1,2,...,z, \tag{26}$$

where $M_b(u) = E[T^{(b)}(u)]$ are the mean values of the system conditional lifetimes $T^{(b)}(u)$ in the reliability state subset $\{u, u+1,..., z\}$ at the operation state z_b, $b = 1,2,...,v$, given by

$$M_b(u) = \int_0^{\infty} [\boldsymbol{R}(t,u)]^{(b)} dt,\ u = 1,2,...,z, \tag{27}$$

$[\boldsymbol{R}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,v$, are defined by (23) and p_b are given by (9). Since the relationships between the system unconditional lifetimes $\overline{T}(u)$ in the particular reliability states and the system unconditional lifetimes $T(u)$ in the reliability state subsets can be expressed by

$$\overline{T}(u) = T(u) - T(u+1),\ u = 0,1,..., z-1,\ \overline{T}(z) = T(z), \tag{28}$$

then we get the following formulae for the mean values of the unconditional lifetimes of the system in particular reliability states

$$\overline{M}(u) = M(u) - M(u+1),\ u = 0,1,..., z-1,\ \overline{M}(z) = M(z), \tag{29}$$

where $M(u)$, $u = 0,1,..., z$, are given by (27).

Moreover, if $s(t)$ is the system reliability state at he moment t, $t \in <0,\infty)$, and r, $r \in \{1,2,..., z\}$, is the system critical reliability state, then the system risk function

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \leq t),\ t \in <0,\infty),$$

defined as the probability that the system is in the subset of states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the state z at the moment $t = 0$ is given by (Kołowrocki, Soszyńska-Budny, 2011)

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,r),\ t \in <0,\infty), \tag{30}$$

where $\boldsymbol{R}(t,r)$ is the coordinate of the system unconditional reliability function given by (25) for $u = r$ and if τ is the moment when the system risk function exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{31}$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (30).
Further, we assume that the system components E_i, $i = 1,2,...,n,$ at the system operation states z_b, $b = 1,2,...,v,$ have the exponential reliability functions, i.e. their coordinates are given by

$$[R_i\ (t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b) = \exp[-[\lambda_i\ (u)]^{(b)}\, t] \tag{32}$$

for $t \in < 0, \infty)$, $u = 1,2,...,z$, $b = 1,2,...,v$.
The reason for this strong assumption on the system components is that the exponential distribution has "no memory" (Kołowrocki, Soszyńska-Budny, 2011). Both of them, the assumption about the exponential reliability functions of the system components and this property, justify the following form of the formula (25) (Kołowrocki, Soszyńska-Budny, 2011)

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}(t,u)]^{(b)}$$
$$= \sum_{b=1}^{v} p_b [\boldsymbol{R}(\exp[-[\lambda_1(u)]^{(b)} t], \exp[-[\lambda_2(u)]^{(b)} t], ..., \exp[-[\lambda_n(u)]^{(b)} t])]^{(b)} \tag{33}$$

for $t \geq 0$, $u = 1,2,...,z$.

The application of the above formula and the results given in Chapter 3 of (Kołowrocki, Soszyńska-Budny, 2011) yield the following results formulated in the form of the following proposition.

Proposition 1
If components of the multi-state system at the operation state z_b, $b = 1,2,...,v,$ have the exponential reliability functions given by

$$[R_{ij}(t,\cdot)]^{(b)} = [1, [R_{ij}(t,1)]^{(b)}, \ldots, [R_{ij}(t,z)]^{(b)}],\ t \in (-\infty,\infty),\ b = 1,2,...,v,$$

where

$$[R_{ij}(t,u)]^{(b)} = \exp[-[\lambda_{ij}(u)]^{(b)} t] \text{ for } t \geq 0,\ [\lambda_{ij}(u)]^{(b)} > 0, i = 1,2,...,k, j = 1,2,...,l_i,\ u = 1,2,\ldots,z,$$
$$b = 1,2,...,v,$$

then its multistate unconditional reliability function is given by the vector:
i) for a series-parallel system with the structure shape parameters $k^{(b)}$, $l_i^{(b)}$, $i = 1,2,..., k^{(b)}$, at the operation state z_b, $b = 1,2,...,v$,

$$\boldsymbol{R}(t,\cdot) = [1,\ \boldsymbol{R}(t,1), ..., \boldsymbol{R}(t,z)], \tag{34}$$

where

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{v} \boldsymbol{p_b R}_{k^{(b)};l_1^{(b)},l_2^{(b)},...,l_{k^{(b)}}^{(b)}}(t,u),\ u = 1,2,...,z, \tag{35}$$

$$\boldsymbol{R}_{k^{(b)};l_1^{(b)},l_2^{(b)},...,l_{k^{(b)}}^{(b)}}(t,u) = 1 - \prod_{i=1}^{k^{(b)}} [1 - \prod_{j=1}^{l_i^{(b)}} [R_{ij}(t,u)]^{(b)}]$$

$$= 1 - \prod_{i=1}^{k^{(b)}} [1 - \exp[-\sum_{j=1}^{l_i^{(b)}} [\lambda_{ij}(u)]^{(b)} t]], \ t \geq 0, \ u = 1,2,...,z, \ b = 1,2,...,\nu; \tag{36}$$

ii) for a series-"*m* out of *k*" system with the structure shape parameters $m^{(b)}$, $k^{(b)}$, $l_i^{(b)}$, $i = 1,2,...,k^{(b)}$, at the operation state z_b, $b = 1,2,...,\nu$,

$$\boldsymbol{R}(t,\cdot) = [1, \ \boldsymbol{R}(t,1),..., \boldsymbol{R}(t,z)], \tag{37}$$

where

$$\boldsymbol{R}(t,u) \cong \sum_{\boldsymbol{b=1}}^{\nu} \boldsymbol{p_b} \boldsymbol{R}^{m^{(b)}}_{k^{(b)}; l_1^{(b)}, l_2^{(b)}, ..., l_{k^{(b)}}^{(b)}} (t,u), \ u = 1,2,...,z, \tag{38}$$

$$\boldsymbol{R}^{m^{(b)}}_{k^{(b)}; l_1^{(b)}, l_2^{(b)}, ..., l_{k^{(b)}}^{(b)}} (t,u) = 1 - \sum_{\substack{r_1, r_2, ..., r_k = 0 \\ r_1 + r_2 + ... + r_k \leq m^{(b)} - 1}}^{1} \prod_{i=1}^{k^{(b)}} [\prod_{j=1}^{l_i^{(b)}} [R_{ij}(t,u)]^{(b)} t]]^{r_i} \ [1 - \prod_{j=1}^{l_i^{(b)}} R_{ij}(t,u)]^{(b)}]]^{1-r_i}$$

$$= 1 - \sum_{\substack{r_1, r_2, ..., r_k = 0 \\ r_1 + r_2 + ... + r_k \leq m^{(b)} - 1}}^{1} \prod_{i=1}^{k^{(b)}} [\prod_{j=1}^{l_i^{(b)}} \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{r_i}$$

$$\cdot [1 - \prod_{j=1}^{l_i^{(b)}} \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{1-r_i}, \ t \geq 0, \ u = 1,2,...,z, \ b = 1,2,...,\nu. \tag{39}$$

- ***Example*** (continuation)
- In Section 2, it is fixed that the exemplary system reliability structure and its subsystems and components reliability depend on its changing in time operation states. Considering the assumptions and agreements of these sections, we assume that its subsystems S_υ, $\upsilon = 1,2$, are composed of four-state, i.e. z = 3, components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2$, having the conditional reliability functions given by the vector

$$[R_{ij}^{(\upsilon)}(t,\cdot)]^{(b)} = [1, \ [R_{ij}^{(\upsilon)}(t,1)]^{(b)}, [R_{ij}^{(\upsilon)}(t,2)]^{(b)}, [R_{ij}^{(\upsilon)}(t,3)]^{(b)}], \ b = 1,2,3,4,$$

with the exponential co-ordinates

$$[R_{ij}^{(\upsilon)}(t,1)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}],$$
$$[R_{ij}^{(\upsilon)}(t,2)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}],$$
$$[R_{ij}^{(\upsilon)}(t,3)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(3)]^{(b)}],$$

different at various operation states z_b, $b = 1,2,3,4,$ and with the intensities of departure from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, respectively

$$[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}, \ [\lambda_{ij}^{(\upsilon)}(2)]^{(b)}, \ [\lambda_{ij}^{(\upsilon)}(3)]^{(b)}, \ b = 1,2,3,4.$$

The influence of the system operation states changing on the changes of the system reliability structure and its components reliability functions is as follows.

At the system operation state z_1, the system is composed of the series-parallel subsystem S_1 with the structure showed in Figure 2, containing two identical series subsystems ($k^{(1)} = 2$), each composed of sixty components ($l_1^{(1)} = 60$, $l_2^{(1)} = 60$) with the exponential reliability functions. In both series subsystems of the subsystem S_1 there are respectively:

- the components $E_{ij}^{(1)}$, $i = 1,2$, $j = 1,2,...,40,$ with the conditional reliability function coordinates

$$[R_{ij}^{(1)}(t,1)]^{(1)} = \exp[-0.0008t],\ [R_{ij}^{(1)}(t,2)]^{(1)} = \exp[-0.0009t],$$
$$[R_{ij}^{(1)}(t,3)]^{(1)} = \exp[-0.0010t],\ i = 1,2,\ j = 1,2,...,40;$$

- the components $E_{ij}^{(1)}$, $i = 1,2$, $j = 41,42,...,60,$ with the conditional reliability function coordinates

$$[R_{ij}^{(1)}(t,1)]^{(1)} = \exp[-0.0011t],\ [R_{ij}^{(1)}(t,2)]^{(1)} = \exp[-0.0012t],$$
$$[R_{ij}^{(1)}(t,3)]^{(1)} = \exp[-0.0013t],\ i = 1,2,\ j = 41,42,...,60.$$

Thus, at the operational state z_1, the system is identical with the subsystem S_1 that is a four-state series-parallel system with its structure shape parameters , $l_1^{(1)} = 60$, $l_2^{(1)} = 60$, and according to the formulae appearing after Definition 3.11 in (Kołowrocki, Soszyńska-Budny, 2011) and (34)-(36), its conditional reliability function is given by

$$[\boldsymbol{R}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}(t,1)]^{(1)}, [\boldsymbol{R}(t,2)]^{(1)}, [\boldsymbol{R}(t,3)]^{(1)}],\ t \geq 0, \tag{40}$$

where

$$\begin{aligned}[\boldsymbol{R}(t,1)]^{(1)} &= \boldsymbol{R}_{2;60,60}(t,1) = 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{60}[R_{ij}^{(1)}(t,1)]^{(1)}] \\ &= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{60}[\lambda_{ij}^{(1)}(1)]^{(1)}t]] \\ &= 1 - [1 - \exp[-[0.0008 \cdot 40 + 0.0011 \cdot 20]t]]^2 \\ &= 1 - [1 - \exp[-0.054t]]^2 \\ &= 2\exp[-0.054t] - \exp[-0.108t], \end{aligned} \tag{41}$$

$$\begin{aligned}[\boldsymbol{R}(t,2)]^{(1)} &= \boldsymbol{R}_{2;60,60}(t,2) = 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{60}[R_{ij}^{(1)}(t,2)]^{(1)}] \\ &= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{60}[\lambda_{ij}^{(1)}(2)]^{(1)}t]] \\ &= 1 - [1 - \exp[-[0.0009 \cdot 40 + 0.0012 \cdot 20]t]]^2 \\ &= 1 - [1 - \exp[-0.060t]]^2 \\ &= 2\exp[-0.060t] - \exp[-0.120t], \end{aligned} \tag{42}$$

$$\begin{aligned}[\boldsymbol{R}(t,3)]^{(1)} &= \boldsymbol{R}_{2;60,60}(t,3) = 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{60}[R_{ij}^{(1)}(t,3)]^{(1)}] \\ &= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{60}[\lambda_{ij}^{(1)}(3)]^{(1)}t]] \end{aligned}$$

$$= 1-[1-\exp[-[0.0010\cdot 40+0.0013\cdot 20]t]]^2$$
$$= 1-[1-\exp[-0.066t]]^2$$
$$= 2\exp[-0.066t]-\exp[-0.132t].$$

(43)

The expected values and standard deviations of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_1, calculated from the results given by (40)-(43), according to (27), respectively are:

$$M_1(1) = \int_0^\infty [\boldsymbol{R}(t,1)]^{(1)}\,dt = 2/0.054 - 1/0.108 \cong 27.78, \quad (44)$$

$$M_1(2) = \int_0^\infty [\boldsymbol{R}(t,2)]^{(1)}dt = 2/0.060 - 1/0.120 = 25.00, \quad (45)$$

$$M_1(3) = \int_0^\infty [\boldsymbol{R}(t,3)]^{(1)}dt = 2/0.066 - 1/0.132 \cong 22.73.$$

(46)

At the system operation state z_2, the system is composed of the series-parallel subsystem S_2 with the structure showed in Figure 3, containing four identical series subsystems ($k^{(2)} = 4$), each composed of eighty components ($l_1^{(2)} = 80$, $l_2^{(2)} = 80$, $l_3^{(2)} = 80$, $l_4^{(2)} = 80$) with the exponential reliability functions. In all series subsystems of the subsystem S_2 there are respectively:

- the components $E_{ij}^{(2)}$, $i = 1,2,3,4$, $j = 1,2,...,40,$ with the conditional reliability function co-ordinates

$$[R_{ij}^{(2)}(t,1)]^{(2)} = \exp[-0.0014t],\ [R_{ij}^{(2)}(t,2)]^{(2)} = \exp[-0.0015t],$$
$$[R_{ij}^{(2)}(t,3)]^{(2)} = \exp[-0.0016t],\ i = 1,2,3,4,\ \ j = 1,2,...,40;$$

- the components $E_{ij}^{(2)}$, $i = 1,2,3,4$, $j = 21,22,...,40,$ with the conditional reliability function co-ordinates

$$[R_{ij}^{(2)}(t,1)]^{(2)} = \exp[-0.0018t],\ [R_{ij}^{(2)}(t,2)]^{(2)} = \exp[-0.0020t],$$
$$[R_{ij}^{(2)}(t,3)]^{(2)} = \exp[-0.0022t],\ i = 1,2,3,4,\ j = 41,42,...,80.$$

Thus, at the operation state z_2, the system is identical with the subsystem S_2 that is a four-state series-parallel system with its structure shape parameters $k^{(2)} = 4$), $l_1^{(2)} = 80$, $l_2^{(2)} = 80$, $l_3^{(2)} = 80$, $l_4^{(2)} = 80$, and according to the formulae appearing after Definition 3.11 in (Kołowrocki, Soszyńska-Budny, 2011) and (34)-(36), its conditional reliability function is given by

$$[\boldsymbol{R}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}(t,1)]^{(2)}, [\boldsymbol{R}(t,2)]^{(2)}, [\boldsymbol{R}(t,3)]^{(2)}],\ t \geq 0, \quad (47)$$

where

$$[\boldsymbol{R}(t,1)]^{(2)} = \boldsymbol{R}_{4;80,80,80,80}(t,1) = 1-\prod_{i=1}^{4}[1-\prod_{j=1}^{80}[R_{ij}^{(2)}(t,1)]^{(2)}]$$
$$= 1-\prod_{i=1}^{4}[1-\exp[-\sum_{j=1}^{80}[\lambda_{ij}^{(2)}(1)]^{(2)}t]]$$
$$= 1-[1-\exp[-[0.0014\cdot 40+0.0018\cdot 40]t]^{4}$$
$$= 1-[1-\exp[-0.128t]]^{4}$$
$$= 4\exp[-0.128t]-6\exp[-0.256t]+4\exp[-0.384t]-\exp[-0.512t], \quad (48)$$

$$[\boldsymbol{R}(t,2)]^{(2)} = \boldsymbol{R}_{4;80,80,80,80}(t,2) = 1-\prod_{i=1}^{4}[1-\prod_{j=1}^{80}[R_{ij}^{(2)}(t,2)]^{(2)}]$$
$$= 1-\prod_{i=1}^{4}[1-\exp[-\sum_{j=1}^{80}[\lambda_{ij}^{(2)}(2)]^{(2)}t]]$$
$$= 1-[1-\exp[-[0.0015\cdot 40+0.0020\cdot 40]t]^{4}$$
$$= 1-[1-\exp[-0.140t]]^{4}$$
$$= 4\exp[-0.140t]-6\exp[-0.280t]+4\exp[-0.420t]-\exp[-0.560t], \quad (49)$$

$$[\boldsymbol{R}(t,3)]^{(2)} = \boldsymbol{R}_{4;80,80,80,80}(t,3) = 1-\prod_{i=1}^{4}[1-\prod_{j=1}^{80}[R_{ij}^{(2)}(t,3)]^{(2)}]$$
$$= 1-\prod_{i=1}^{4}[1-\exp[-\sum_{j=1}^{80}[\lambda_{ij}^{(2)}(3)]^{(2)}t]]$$
$$= 1-[1-\exp[-[0.0016\cdot 40+0.0022\cdot 40]t]^{4}$$
$$= 1-[1-\exp[-0.152t]]^{4}$$
$$= 4\exp[-0.152t]-6\exp[-0.304t]+4\exp[-0.456t]-\exp[-0.608t]. \quad (50)$$

The expected values and standard deviations of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_1, calculated from the results given by (47)-(50), according to (27), respectively are:

$$M_2(1) = \int_0^\infty [\boldsymbol{R}(t,1)]^{(2)}dt = 4/0.128 - 6/0.256 + 4/0.384 - 1/0.512 \cong 16.27, \quad (51)$$

$$M_2(2) = \int_0^\infty [\boldsymbol{R}(t,2)]^{(2)}dt = 4/0.140 - 6/0.280 + 4/0.420 - 1/0.560 \cong 14.88, \quad (52)$$

$$M_2(3) = \int_0^\infty [\boldsymbol{R}(t,3)]^{(2)}dt = 4/0.152 - 6/0.304 + 4/0.456 - 1/0.608 \cong 13.71. \quad (53)$$

At the system operation state z_3, the system is a series system with the structure showed in Figure 1, composed of two series-parallel subsystems S_1 and S_2 illustrated respectively in Figure 2 and Figure 3.

The subsystem S_1 with the structure showed in Figure 2, consists of two identical series subsystems ($k^{(3)} = 2$), each composed of sixty components ($l_1^{(3)} = 60$, $l_2^{(3)} = 60$) with the exponential reliability functions. In both series subsystems of the subsystem S_1 there are respectively:

- the components $E_{ij}^{(1)}$, $i = 1,2$, $j = 1,2,...,40$, with the conditional reliability function co-ordinates

$[R_{ij}^{(1)}(t,1)]^{(3)} = \exp[-0.0009t], [R_{ij}^{(1)}(t,2)]^{(3)} = \exp[-0.0010t],$
$[R_{ij}^{(1)}(t,3)]^{(3)} = \exp[-0.0011t],\ i = 1,2,\ j = 1,2,...,40;$

- the components $E_{ij}^{(1)}$, $i = 1,2,\ j = 41,42,...,60,$ with the conditional reliability function co-ordinates

$[R_{ij}^{(1)}(t,1)]^{(3)} = \exp[-0.0012t],\ [R_{ij}^{(1)}(t,2)]^{(3)} = \exp[-0.0014t],$
$[R_{ij}^{(1)}(t,3)]^{(3)} = \exp[-0.0016t],\ i = 1,2,\ j = 41,42,...,60.$

Thus, at the operation state z_3, the subsystem S_1 is a four-state series-parallel system with its structure shape parameters $k^{(3)} = 2$, $l_1^{(3)} = 60$, $l_2^{(3)} = 60$, and according to the formulae appearing after Definition 3.11 in [18] and (34)-(36), its conditional reliability function is given by

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(3)} = [1,\ [\boldsymbol{R}^{(1)}(t,1)]^{(3)},\ [\boldsymbol{R}^{(1)}(t,2)]^{(3)},\ [\boldsymbol{R}^{(1)}(t,3)]^{(3)}],\ t \geq 0,$$
(54)

where

$$[\boldsymbol{R}^{(1)}(t,1)]^{(3)} = \boldsymbol{R}_{2;60,60}(t,1) = 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{60}[R_{ij}^{(1)}(t,1)]^{(3)}]$$
$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{60}[\lambda_{ij}^{(1)}(1)]^{(3)}t]]$$
$$= 1 - [1 - \exp[-[0.0009 \cdot 40 + 0.0012 \cdot 20]t]^2$$
$$= 1 - [1 - \exp[-0.060t]]^2$$
$$= 2\exp[-0.060t] - \exp[-0.120t], \quad (55)$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(3)} = \boldsymbol{R}_{2;60,60}(t,2) = 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{60}[R_{ij}^{(1)}(t,2)]^{(3)}]$$
$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{630}[\lambda_{ij}^{(1)}(2)]^{(3)}t]]$$
$$= 1 - [1 - \exp[-[0.0010 \cdot 40 + 0.0014 \cdot 20]t]^2$$
$$= 1 - [1 - \exp[-0.068t]]^2$$
$$= 2\exp[-0.068t] - \exp[-0.136t], \quad (56)$$

$$[\boldsymbol{R}(t,3)]^{(1)} = \boldsymbol{R}_{2;60,60}(t,3) = 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{60}[R_{ij}^{(1)}(t,3)]^{(1)}]$$
$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{60}[\lambda_{ij}^{(1)}(3)]^{(1)}t]]$$
$$= 1 - [1 - \exp[-[0.0011 \cdot 40 + 0.0016 \cdot 20]t]^2$$
$$= 1 - [1 - \exp[-0.076t]]^2$$
$$= 2\exp[-0.076t] - \exp[-0.152t]. \quad (57)$$

The subsystem S_2 with the structure showed in Figure 3, consists of four identical series subsystems ($k^{(3)} = 4$), each composed of eighty components ($l_1^{(3)} = 80$, $l_2^{(3)} = 80$, $l_3^{(3)} = 80$,

$l_4^{(3)} = 80$) with the exponential reliability functions given below. In all series subsystems of the subsystem S_2 there are respectively:

- the components $E_{ij}^{(2)}$, $i = 1,2,3,4,$ $j = 1,2,...,40,$ with the conditional reliability function co-ordinates

$$[R_{ij}^{(2)}(t,1)]^{(3)} = \exp[-0.0010t],\ [R_{ij}^{(2)}(t,2)]^{(3)} = \exp[-0.0011t],$$
$$[R_{ij}^{(2)}(t,3)]^{(3)} = \exp[-0.0012t],\ i = 1,2,3,4,\ \ j = 1,2,...,40;$$

- the components $E_{ij}^{(2)}$, $i = 1,2,3,4,$ $j = 41,42,...,80,$ with the conditional reliability function co-ordinates

$$[R_{ij}^{(2)}(t,1)]^{(3)} = \exp[-0.0014t],\ [R_{ij}^{(2)}(t,2)]^{(3)} = \exp[-0.0016t],$$
$$[R_{ij}^{(2)}(t,3)]^{(3)} = \exp[-0.0018t],\ i = 1,2,3,4,\ j = 41,42,...,80.$$

Thus, at the operation state z_3, the subsystem S_2 is a four-state series-parallel system with its structure shape parameters $k^{(3)} = 4$, $l_1^{(3)} = 80$, $l_2^{(3)} = 80$, $l_3^{(3)} = 80$, $l_4^{(3)} = 80$, and according to the formulae appearing after Definition 3.11 in (Kołowrocki, Soszyńska-Budny, 2011) and (34)-(36), its conditional reliability function is given by

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(3)} = [1,\ [\boldsymbol{R}^{(2)}(t,1)]^{(3)}, [\boldsymbol{R}^{(2)}(t,2)]^{(3)}, [\boldsymbol{R}^{(2)}(t,3)]^{(3)}],\ t \ge 0,$$
(58)

where

$$[\boldsymbol{R}^{(2)}(t,1)]^{(3)} = \boldsymbol{R}_{4;80,80,80,80}(t,1) = 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{80}[R_{ij}^{(2)}(t,1)]^{(3)}]$$
$$= 1 - \prod_{i=1}^{4}[1 - \exp[-\sum_{j=1}^{80}[\lambda_{ij}^{(2)}(1)]^{(3)}t]]$$
$$= 1 - [1 - \exp[-[0.0010 \cdot 40 + 0.0014 \cdot 40]t]^4$$
$$= 1 - [1 - \exp[-0.096t]]^4$$
$$= 4\exp[-0.096t] - 6\exp[-0.192t] + 4\exp[-0.288t] - \exp[-0.384t],$$
(59)

$$[\boldsymbol{R}^{(2)}(t,2)]^{(3)} = \boldsymbol{R}_{4;80,80,80,80}(t,2) = 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{80}[R_{ij}^{(2)}(t,2)]^{(3)}]$$
$$= 1 - \prod_{i=1}^{4}[1 - \exp[-\sum_{j=1}^{80}[\lambda_{ij}^{(2)}(2)]^{(3)}t]]$$
$$= 1 - [1 - \exp[-[0.0011 \cdot 40 + 0.0016 \cdot 40]t]^4$$
$$= 1 - [1 - \exp[-0.108t]]^4$$
$$= 4\exp[-0.108t] - 6\exp[-0.216t] + 4\exp[-0.324t] - \exp[-0.432t], \quad (60)$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(3)} = \boldsymbol{R}_{4;80,80,80,80}(t,3) = 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{80}[R_{ij}^{(2)}(t,3)]^{(3)}]$$
$$= 1 - \prod_{i=1}^{4}[1 - \exp[-\sum_{j=1}^{80}[\lambda_{ij}^{(2)}(3)]^{(3)}t]]$$

$$= 1-[1-\exp[-[0.0012\cdot 40+0.0018\cdot 40]t]^4$$
$$=1-[1-\exp[-0.120t]]^4$$
$$= 4\exp[-0.120t]-6\exp[-0.240t]+4\exp[-0.360t]-\exp[-0.480t]. \quad (61)$$

Considering that the system at the operation state z_3 is a four-state series system composed of subsystems S_1 and S_2, after applying the formulae appearing after Definition 3.4 in (Kołowrocki, Soszyńska-Budny, 2011) and (54)-(57) and (58)-(61), its conditional reliability function is given by

$$[\boldsymbol{R}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}(t,1)]^{(3)}, [\boldsymbol{R}(t,2)]^{(3)}, [\boldsymbol{R}(t,3)]^{(3)}],\ t \geq 0, \quad (62)$$

where

$$[\boldsymbol{R}(t,1)]^{(3)} = \overline{\boldsymbol{R}}_2(t,1) = [\boldsymbol{R}^{(1)}(t,1)]^{(3)}\,[\boldsymbol{R}^{(2)}(t,1)]^{(3)}$$
$$= 8\exp[-0.156t]-12\exp[-0.252t]+8\exp[-0.348t]$$
$$-2\exp[-0.424t]-4\exp[-0.216t]+6\exp[-0.312t]$$
$$-4\exp[-0.408t]+\exp[-0.504t],$$
(63)

$$[\boldsymbol{R}(t,2)]^{(3)} = \overline{\boldsymbol{R}}_2(t,2) = [\boldsymbol{R}^{(1)}(t,2)]^{(3)}\,[\boldsymbol{R}^{(2)}(t,2)]^{(3)}$$
$$= 8\exp[-0.176t]-12\exp[-0.284t]+8\exp[-0.392t]$$
$$-2\exp[-0.500t]-4\exp[-0.236t]+6\exp[-0.344t]$$
$$-4\exp[-0.452t]+\exp[-0.560t], \quad (64)$$

$$[\boldsymbol{R}(t,3)]^{(3)} = \overline{\boldsymbol{R}}_2(t,3) = [\boldsymbol{R}^{(1)}(t,3)]^{(3)}\,[\boldsymbol{R}^{(2)}(t,3)]^{(3)}$$
$$= 8\exp[-0.196t]-12\exp[-0.316t]+8\exp[-0.436t]$$
$$-2\exp[-0.556t]-4\exp[-0.256t]+6\exp[-0.376t]$$
$$-4\exp[-0.496t]+\exp[-0.616t].$$
(65)

The expected values and standard deviations of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_3, calculated from the results given by (62)-(65), according to (27), respectively are:

$$M_3(1) = \int_0^\infty [\boldsymbol{R}(t,1)]^{(3)}dt = 8/0.156 - 12/0.252 + 8/0.348 - 2/0.424 - 4/0.216$$
$$+ 6/0.312 - 4/0.408 + 1/0.504 \cong 14.82, \quad (66)$$

$$M_3(2) = \int_0^\infty [\boldsymbol{R}(t,2)]^{(3)}dt = 8/0.176 - 12/0.284 + 8/0.392 - 2/0.500 - 4/0.236$$
$$+ 6/0.344 - 4/0.452 + 1/0.560 \cong 13.04, \quad (67)$$

$$M_3(3) = \int_0^\infty [\boldsymbol{R}(t,3)]^{(3)}dt = 8/0.196 - 12/0.316 + 8/0.436 - 2/0.556 - 4/0.256$$
$$+ 6/0.376 - 4/0.496 + 1/0.616 \cong 11.48. \quad (68)$$

At the system operation state z_4, the system is a series system with the scheme showed in Figure 1, composed of the subsystem S_1 and S_2 illustrated respectively in Figure 2 and Figure 3, whereas the subsystem S_1 is a series-parallel system and the subsystem S_2 is a series-"2 out of 4" system.
The subsystem S_1 consists of two identical series subsystems ($k^{(4)} = 2$), each composed of sixty components ($l_1^{(4)} = 60$, $l_2^{(4)} = 60$) with the exponential reliability functions the same as at the operation state z_1. Thus, according to (54)-(57), the subsystem S_1 conditional reliability function at the operation state z_4, is given by

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}^{(1)}(t,1)]^{(4)}, [\boldsymbol{R}^{(1)}(t,2)]^{(4)}, [\boldsymbol{R}^{(1)}(t,3)]^{(4)}],\ t \ge 0, \tag{69}$$

where

$$[\boldsymbol{R}^{(1)}(t,1)]^{(4)} = 2\exp[-0.054t] - \exp[-0.108t], \tag{70}$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(4)} = 2\exp[-0.060t] - \exp[-0.120t],$$
(71)

$$[\boldsymbol{R}(t,3)]^{(4)} = 2\exp[-0.066t] - \exp[-0.132t]. \tag{72}$$

The subsystem S_2 consists of four identical series subsystems ($k^{(4)} = 4$), each composed of eighty components ($l_1^{(4)} = 80$, $l_2^{(4)} = 80$, $l_3^{(4)} = 80$, $l_4^{(4)} = 80$) with the exponential reliability functions the same as at the operation state z_2 and is a series-"2 out of 4" system ($m = 2$). Thus, at the operation state z_4, the subsystem S_2 is a four-state series-"2 out of 4" system, with its structure shape parameters ($k^{(4)} = 4$), each composed of eighty components $l_1^{(4)} = 80$, $l_2^{(4)} = 80$, $l_3^{(4)} = 80$, $l_4^{(4)} = 80$, and according to the formulae appearing after Definition 8.1 in (Kołowrocki, Soszyńska-Budny, 2011) and (37)-(39), its conditional reliability function is given by

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(4)} = [1,\ [\boldsymbol{R}^{(2)}(t,1)]^{(4)}, [\boldsymbol{R}^{(2)}(t,2)]^{(4)}, [\boldsymbol{R}^{(2)}(t,3)]^{(4)}],\ t \ge 0,$$
(73)

where

$$[\boldsymbol{R}^{(2)}(t,1)]^{(4)} = \boldsymbol{R}^2_{4;80,80,80,80}(t,1) = 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4} [\prod_{j=1}^{80} [R_{ij}^{(2)}(t,1)]^{(4)}]^{r_i} [1 - \prod_{j=1}^{80} [R_{ij}^{(2)}(t,1)]^{(4)}]^{1-r_i}$$

$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4} \exp[-r_i \sum_{j=1}^{80} [\lambda_{ij}^{(2)}(1)]^{(4)} t][1 - \exp[-\sum_{j=1}^{80} [\lambda_{ij}^{(2)}(1)]^{(4)} t]]^{1-r_i}$$

$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4} \exp[-r_i[0.0014 \cdot 40 + 0.0018 \cdot 40]t]$$

$$\cdot [1 - \exp[-[0.0014 \cdot 40 + 1.0018 \cdot 40]t]]^{1-r_i}$$

$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4} \exp[-r_i 0.128t][1 - \exp[-0.128t]]^{1-r_i}$$

$$= 1 - \sum_{i=0}^{1} \binom{4}{i} \exp[-i \cdot 0.128t][1 - \exp[-0.128t]]^{4-i}$$

$$= 1 - \exp[-0 \cdot 0.128\,t]\,[1-\exp[-0.128t]^4 - 4\exp[-1 \cdot 0.128\,t]\,[1-\exp[-0.128t]^3$$
$$= 6\exp[-0.256t] - 8\exp[-0.384t] + 3\exp[-0.512t], \quad (74)$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(4)} = \boldsymbol{R}^{2}_{4;80,80,80,80}(t,2) = 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4}[\prod_{j=1}^{80}[R^{(2)}_{ij}(t,2)]^{(4)}]^{r_i}[1-\prod_{j=1}^{80}[R^{(2)}_{ij}(t,2)]^{(4)}]^{1-r_i}$$
$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4}\exp[-r_i\sum_{j=1}^{80}[\lambda^{(2)}_{ij}(2)]^{(4)}t][1-\exp[-\sum_{j=1}^{80}[\lambda^{(2)}_{ij}(2)]^{(4)}t]]^{1-r_i}$$
$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4}\exp[-r_i[0.0015\cdot 40 + 0.0020\cdot 40]t]$$
$$\cdot[1-\exp[-[0.0015\cdot 40 + 1.0020\cdot 40]t]]^{1-r_i}$$
$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4}\exp[-r_i 0.140t][1-\exp[-0.140t]]^{1-r_i}$$
$$= 1 - \sum_{i=0}^{1}\binom{4}{i}\exp[-i\cdot 0.140t][1-\exp[-0.140t]]^{4-i}$$
$$= 1 - \exp[-0 \cdot 0.140\,t]\,[1-\exp[-0.140t]^4 - 4\exp[-1 \cdot 0.140\,t]\,[1-\exp[-0.140t]^3$$
$$= 6\exp[-0.280t] - 8\exp[-0.420t] + 3\exp[-0.560t], \quad (75)$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(4)} = \boldsymbol{R}^{2}_{4;80,80,80,80}(t,3) = 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4}[\prod_{j=1}^{80}[R^{(2)}_{ij}(t,3)]^{(4)}]^{r_i}[1-\prod_{j=1}^{80}[R^{(2)}_{ij}(t,3)]^{(4)}]^{1-r_i}$$
$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4}\exp[-r_i\sum_{j=1}^{80}[\lambda^{(2)}_{ij}(3)]^{(4)}t][1-\exp[-\sum_{j=1}^{80}[\lambda^{(2)}_{ij}(3)]^{(4)}t]]^{1-r_i}$$
$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4}\exp[-r_i[0.0016\cdot 40 + 0.0022\cdot 40]t]$$
$$\cdot[1-\exp[-[0.0016\cdot 40 + 1.0022\cdot 40]t]]^{1-r_i}$$
$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1} \prod_{i=1}^{4}\exp[-r_i 0.152t][1-\exp[-0.152t]]^{1-r_i}$$
$$= 1 - \sum_{i=0}^{1}\binom{4}{i}\exp[-i\cdot 0.152t][1-\exp[-0.152t]]^{4-i}$$
$$= 1 - \exp[-0 \cdot 0.152\,t]\,[1-\exp[-0.152t]^4 - 4\exp[-1 \cdot 0.152\,t]\,[1-\exp[-0.152t]^3$$
$$= 6\exp[-0.304t] - 8\exp[-0.456t] + 3\exp - 0.608t]. \quad (76)$$

Considering that the system at the operation state z_4 is a four-state series system composed of subsystems S_1 and S_2, after applying the formulae appearing after Definition 3.4 in (Kołowrocki, Soszyńska-Budny, 2011) and (69)-(72) and (73)-(76), its conditional reliability function is given by

$$[\boldsymbol{R}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}(t,1)]^{(4)}, [\boldsymbol{R}(t,2)]^{(4)}, [\boldsymbol{R}(t,3)]^{(4)}],\ t \ge 0, \quad (77)$$

where

$$[\boldsymbol{R}(t,1)]^{(4)} = \overline{\boldsymbol{R}}_2(t,1) = [\boldsymbol{R}^{(1)}(t,1)]^{(4)}\,[\boldsymbol{R}^{(2)}(t,1)]^{(4)}$$
$$= 12\exp[-0.310t] - 6\exp[-0.364t] - 16\exp[-0.438t]$$

$$+8\exp[-0.492t]+6\exp[-0.566t]-3\exp[-0.620t], \quad (78)$$

$$[\boldsymbol{R}(t,2)]^{(4)} = \overline{\boldsymbol{R}}_2(t,2) = [\boldsymbol{R}^{(1)}(t,2)]^{(4)}\,[\boldsymbol{R}^{(2)}(t,2)]^{(4)}$$
$$= 12\exp[-0.340t]-6\exp[-0.400t]-16\exp[-0.480t]$$
$$+8\exp[-0.540t]+6\exp[-0.620t]-3\exp[-0.680t], \quad (79)$$

$$[\boldsymbol{R}(t,3)]^{(4)} = \overline{\boldsymbol{R}}_2(t,3) = [\boldsymbol{R}^{(1)}(t,3)]^{(4)}\,[\boldsymbol{R}^{(2)}(t,3)]^{(4)}$$
$$= 12\exp[-0.370t]-6\exp[-0.436t]-16\exp[-0.522t]$$
$$+8\exp[-0.588t]+6\exp[-0.674t]-3\exp[-0.740t]. \quad (80)$$

The mean values of the system sojourn times $T(u)$ in the reliability state subsets after applying the formula (77)-(80) and (27), are:

$$M_4(1) = \int_0^\infty [\boldsymbol{R}(t,1)]^{(4)}\,dt = 12/0.310 - 6/0.364 - 16/0.438 + 8/0.492 + 6/0.566 - 3/0.620 \cong 7.72, \quad (81)$$

$$M_4(2) = \int_0^\infty [\boldsymbol{R}(t,2)]^{(4)}\,dt = 12/0.340 - 6/0.400 - 16/0.480 + 8/0.540 + 6/0.620 - 3/0.680 \cong 7.04, \quad (82)$$

$$M_4(3) = \int_0^\infty [\boldsymbol{R}(t,3)]^{(4)}\,dt = 12/0.370 - 6/0.436 - 16/0.522 + 8/0.588 + 6/0.674 - 3/0.740 \cong 6.47. \quad (83)$$

In the case when the system operation time is large enough its unconditional four-state reliability function is given by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \boldsymbol{R}(t,2), \boldsymbol{R}(t,3)],\ t \ge 0, \quad (84)$$

where according to (25) and considering the exemplary system operation process transient probabilities at the operation states determined by (18), the vector co-ordinates are given respectively by

$$\boldsymbol{R}(t,1) = p_1[\boldsymbol{R}(t,1)]^{(1)} + p_2[\boldsymbol{R}(t,1)]^{(2)} + p_3[\boldsymbol{R}(t,1)]^{(3)} + p_4[\boldsymbol{R}(t,1)]^{(4)}$$
$$= 0.190\cdot[\boldsymbol{R}(t,1)]^{(1)} + 0.043\cdot[\boldsymbol{R}(t,1)]^{(2)} + 0.312\cdot[\boldsymbol{R}(t,1)]^{(3)} + 0.455\cdot[\boldsymbol{R}(t,1)]^{(4)} \quad (85)$$
for $t \ge 0$,

$$\boldsymbol{R}(t,2) = p_1[\boldsymbol{R}(t,2)]^{(1)} + p_2[\boldsymbol{R}(t,2)]^{(2)} + p_3[\boldsymbol{R}(t,2)]^{(3)} + p_4[\boldsymbol{R}(t,2)]^{(4)}$$
$$= 0.190\cdot[\boldsymbol{R}(t,2)]^{(1)} + 0.043\cdot[\boldsymbol{R}(t,2)]^{(2)} + 0.312\cdot[\boldsymbol{R}(t,2)]^{(3)} + 0.455\cdot[\boldsymbol{R}(t,2)]^{(4)} \quad (86)$$
for $t \ge 0$,

$$\boldsymbol{R}(t,3) = p_1[\boldsymbol{R}(t,3)]^{(1)} + p_2[\boldsymbol{R}(t,3)]^{(2)} + p_3[\boldsymbol{R}(t,3)]^{(3)} + p_4[\boldsymbol{R}(t,3)]^{(4)}$$
$$= 0.190\cdot[\boldsymbol{R}(t,3)]^{(1)} + 0.043\cdot[\boldsymbol{R}(t,3)]^{(2)} + 0.312\cdot[\boldsymbol{R}(t,3)]^{(3)} + 0.455\cdot[\boldsymbol{R}(t,3)]^{(4)} \quad (87)$$
for $t \ge 0$,

where coordinates $[\boldsymbol{R}(t,1)]^{(1)}$, $[\boldsymbol{R}(t,1)]^{(2)}$, $[\boldsymbol{R}(t,1)]^{(3)}$, $[\boldsymbol{R}(t,1)]^{(4)}$ are given by (41), (48), (62), (76), $[\boldsymbol{R}(t,2)]^{(1)}$, $[\boldsymbol{R}(t,2)]^{(2)}$, $[\boldsymbol{R}(t,2)]^{(3)}$, $[\boldsymbol{R}(t,2)]^{(4)}$ are given by (42), (49), (63), (77) and $[\boldsymbol{R}(t,3)]^{(1)}$, $[\boldsymbol{R}(t,3)]^{(2)}$, $[\boldsymbol{R}(t,3)]^{(3)}$, $[\boldsymbol{R}(t,3)]^{(4)}$ are given by (43), (50), (64), (80).
The graph of the four-state exemplary system reliability function is illustrated in Figure 4.

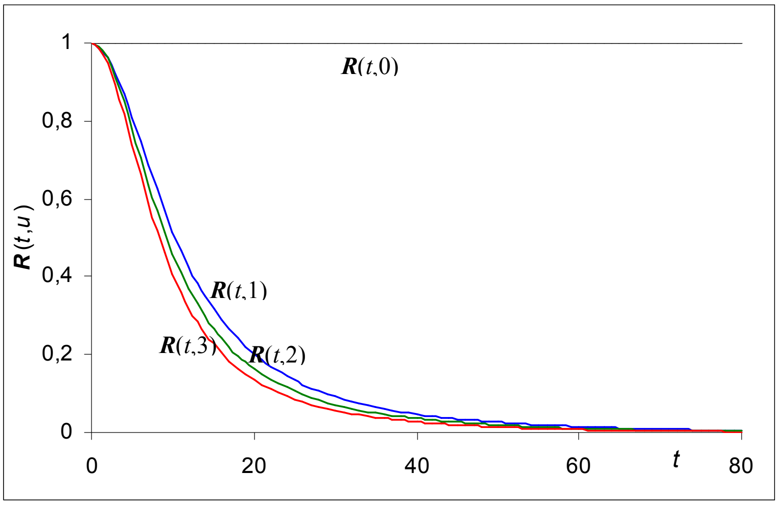

Fig. 4. The graph of the exemplary system reliability function $\boldsymbol{R}(t,\cdot)$ coordinates

The expected values and standard deviations of the system unconditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, calculated from the results given by (84)-(87), according to (27) and considering (18), (44)-(46), (51)-(53), (66)-(68) and (81)-(83), respectively are:

$$M(1) = p_1 M_1(1) + p_2 M_2(1) + p_3 M_3(1) + p_4 M_4(1)$$
$$= 0.190 \cdot 27.78 + 0.043 \cdot 16.27 + 0.312 \cdot 14.82 + 0.455 \cdot 7.72 \cong 14.11, \quad (88)$$

$$M(2) = p_1 M_1(2) + p_2 M_2(2) + p_3 M_3(2) + p_4 M_4(2)$$
$$= 0.190 \cdot 25.00 + 0.043 \cdot 14.88 + 0.312 \cdot 13.04 + 0.455 \cdot 7.04 \cong 12.66, \quad (89)$$

$$M(3) = p_1 M_1(3) + p_2 M_2(3) + p_3 M_3(3) + p_4 M_4(3)$$
$$= 0.190 \cdot 22.73 + 0.043 \cdot 13.71 + 0.312 \cdot 11.48 + 0.455 \cdot 6.47 \cong 11.43. \quad (90)$$

Farther, considering (29) and (88), (89) and (90), the mean values of the system unconditional lifetimes in the particular reliability states 1, 2, 3, respectively are:

$$\overline{M}(1) = M(1) - M(2) = 1.45, \ \overline{M}(2) = M(2) - M(3) = 1.23, \ \overline{M}(3) = M(3) = 11.43. \quad (91)$$

Since the critical reliability state is $r = 2$, then the system risk function, according to (30), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,2) \text{ for } t \geq 0, \quad (92)$$

where $\boldsymbol{R}(t,2)$ is given by (86).

Hence, by (31), the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 2.255. \tag{93}$$

The graph of the risk function $\boldsymbol{r}(t)$ of the exemplary four-state system operating at the variable conditions is given in Figure 5.

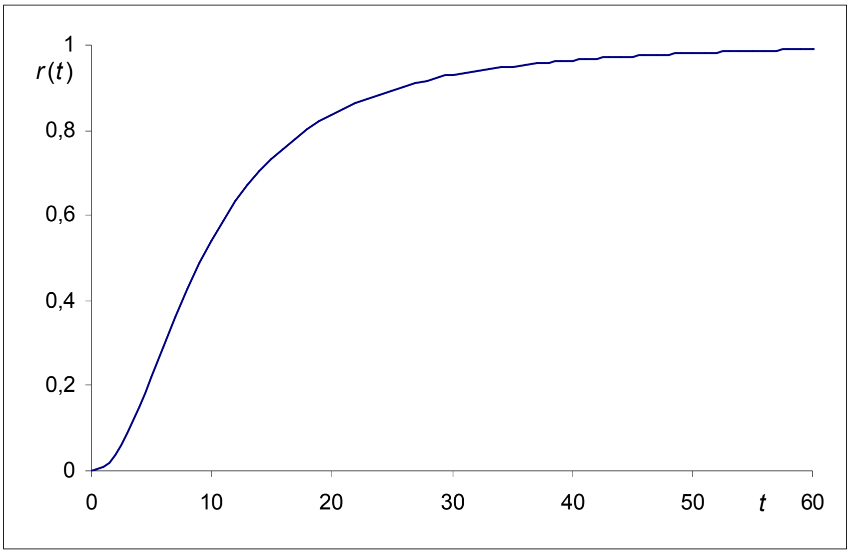

Fig. 5. The graph of the exemplary system risk function $r(t)$

4 CONCLUSION

The integrated general model of complex systems' reliability, linking their reliability models and their operation processes models and considering variable at different operation states their reliability structures and their components reliability parameters is constructed and applied to the reliability evaluation of the exemplary system composed of a series-parallel and a series-"*m* out of *l*" subsystems linked in series. The predicted reliability characteristics of the exemplary system operating at the variable conditions are different from those determined for this system operating at constant conditions. This fact justifies the sensibility of considering real systems at the variable operation conditions that is appearing out in a natural way from practice. This approach, upon the good accuracy of the systems' operation processes and the systems' components reliability parameters identification, makes their reliability prediction more precise.

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ISSN 1932-2321