

## COMPARISON PARAMETERS AVERAGE AND INDIVIDUAL RELIABILITY EQUIPMENT OF ELECTROPOWER SYSTEMS

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### ABSTRACT

Existing comparison criteria average values of random variables of general population cannot be used at comparison of average random variables of multivariate data. The method, algorithm and an example of calculation of critical values recommended statistics offered.

### I. INSTRUCTION

One of the most representative group parameters reliability of the equipment and devices of electro power systems (EPS) are the parameters calculated as an average arithmetic random variables. To them at the characteristic of non-failure operation concern a time between failures and between refusals, at the characteristic of maintainability – averages value of duration of non-working conditions (emergency idle time, being in a reserve, emergency repair at automatic switching-off owing to damage and switching-off under the emergency application owing to defect, capital, average and current scheduled repairs), and at the characteristic of a shelf life– an average idle time at restoration of the adjacent equipment.

Assume, that population is known  $\{\tau\}_\Sigma$  from  $n_\Sigma$  multivariate continuous random  $\tau$  variables, the average arithmetic which is equal  $M_\Sigma^*(\tau)$ . On some version of an attribute (VA) these data (for example type of the equipment, a class of a voltage, service life and so forth) sample is lead  $\{\tau\}_V$  from  $n_V$  random variables, an average which arithmetic realizations equally  $M_V^*(\tau)$ . The expediency of classification of data is defined by probability distinction  $M_\Sigma^*(\tau)$  and  $M_V^*(\tau)$ .

Two methods of the decision of similar problems [1] known. Both of method assumes normal distribution of random variables. In the first method the hypothesis about a casual divergence of average value of random variables is checked  $\tau$  samples  $M_V^*(\tau)$  from a population mean of general population of  $M_\Sigma(\tau)$ . The dispersion of general population is unknown. The criterion looks like:

$$|t| > t_{1-\alpha/2; n_V-1}$$

$$t = \frac{M_\Sigma(\tau) - M_V^*(\tau)}{\frac{S}{\sqrt{n_V}}}$$

where

$$S = \sqrt{\frac{\sum_{i=1}^{n_V} [\tau_i - M_V^*(X)]^2}{(n_V - 1)}}$$

$t_{1-\alpha/2; (n_V-1)}$  - a random variable distributed under law Student with  $(n-1)$  by degrees of freedom and a significance value  $\alpha$ .

In the second method the hypothesis about equality of average values  $M_{V,1}^*(\tau_1)$  and  $M_{V,2}^*(\tau_2)$  two samples normally distributed random variables which dispersions are equal is checked, but are unknown. The criterion looks like:

$$|t| > t_{1-\alpha/2; (n_{v1}+n_{v2}-2)},$$

where

$$t = \frac{[M_{V1}^*(\tau_1) - M_{V2}^*(\tau_2)]}{S_{[M_{V1}^*(\tau_1) - M_{V2}^*(\tau_2)]}}$$

$$S_{[M_{V1}^*(X) - M_{V2}^*(X)]} = \frac{S}{(n_1 + n_2)}$$

$$S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$$

$$S_1^2 = \frac{\sum_{i=1}^{n_{v1}} [\tau_{1,i} - M_{v,1}^*(\tau_1)]^2}{(n_{v1} - 1)}$$

$$S_2^2 = \frac{\sum_{i=1}^{n_{v2}} [\tau_{2,i} - M_{v,2}^*(\tau_2)]^2}{(n_{v2} - 1)}$$

Thus, initial preconditions of these methods are the normal law of distribution of random variables of sample of general population. Population of statistical data of maintenance service, repair equipment and devices of EES concerns to group of multivariate data. They depend on set of attributes and their versions, and their distribution, as a rule, dissymmetrical. Therefore, to apply at classification of statistical data the methods noted above certainly it is possible, but reliability of result definitely will not correspond to the set significance  $\alpha$  value. Below the algorithm of the decision of a problem of comparison of parameters of average and individual reliability on the basis imitating modeling and the theory of check of statistical hypotheses is resulted.

## 2. ALGORITHM OF COMPARISON $M_{\Sigma}^*(\tau)$ and $M_V^*(\tau)$

For an illustration of recommended algorithm of comparison  $M_{\Sigma,e}^*(\tau)$  and  $M_{V,e}^*(\tau)$  (the index «e» allocates the estimation  $M_V^*(\tau)$  calculated according to operation) enter following designations:

- $F_{\Sigma}^*(\tau)$  - statistical function of distribution (s.f.d.) random variables of population of multivariate data  $\{\tau\}_{\Sigma}$ ;
- $F_V^*(\tau)$  - s.f.d. not casual sample of random variables  $\{\tau\}_V$ ;
- $H_1$  and  $H_2$  – assumptions, accordingly, about casual and not casual distinction  $M_{\Sigma,e}^*(\tau)$  and  $M_{V,e}^*(\tau)$ ;
- $F^*[M_V^*(\tau)]$  - s.f.d. realizations of average value of sample from  $n_V$  modeled random variables  $\tau$  provided that sample  $\{\tau\}_V$  concerning a data population  $\{\tau\}_{\Sigma}$  it is representative  

$$F^*[M_V^*(\tau)] = 1 - R^*[M_V^*(\tau)]$$
- $F^*[M_V^{**}(\tau)]$  - s.f.d. realizations of average values of sample from  $n_V$  the random variables modeled on s.f.d.  $F_V^*(\tau)$

Functions of distribution  $R^*[M_V^*(\tau)]$  and  $F^*[M_V^{**}(\tau)]$  are necessary, first of all, for an estimation of critical values, accordingly,  $M_{V,\alpha}^*(\tau)$  for a preset value of a error I type  $\alpha$  and  $M_{V,\beta}^*(\tau)$  for a preset value of a error II type  $\beta$ .

Algorithm of calculation  $M_{V,\alpha}^*(\tau)$  reduced to following sequence of calculations:

1. On a data population  $\{\tau\}_\Sigma$  pays off  $M_\Sigma^*(\tau)$ ;
2. For set VA from  $\{\tau\}_\Sigma$  sample  $n_v$  random variables is spent  $\tau$  and further pays off  $M_{V,e}^*(\tau)$
3. It is modeled s.f.d.  $F^*[M_V^*(\tau)]$ . For what:
  - 3.1. Method of imitating modeling on s.f.d.  $F_\Sigma^*(\tau)$  it is modeled  $n_v$  random variables  $\tau$ . Calculations are spent under the formula recommended in [2]:
 
$$\tau = \tau_i + (\tau_{i+1} - \tau_i)[n_\Sigma \cdot \xi - (i-1)] \quad (1)$$
 where  $\xi$  - a random variable with uniform distribution in an interval  $[0,1]$ ;
  - 3.2. The estimation  $M_V^*(\tau)$  pays;
  - 3.3. Items 3.1 and 3.2 repeat N time, where N – number of iterations of modeling  $\{\tau\}_v$  and calculation of realizations  $M_V^*(\tau)$ . The number of iterations N is defined as follows:
    - First are modeled N=500 realizations of casual values  $M_V^*(\tau)$  and placed in ascending order;
    - Realization corresponding  $F^*[M_V^*(\tau)] = 0,5$  is defined;
    - Relative deviation is calculated:
 
$$\delta M_{V,e}^*(\tau) = \left| \frac{M_{V;0,5}^*(\tau) - M_{V,e}^*(\tau)}{M_{V,e}^*(\tau)} \right| \quad (2)$$
    - If  $\delta M_{V,e}^*(\tau) > 0,01$ , the next sample from N=500 random variables  $\{M_V^*(\tau)\}_N$  is modeled and on 2N to realizations  $M_V^*(\tau)$  the next value  $\delta M_V^*(\tau)$  is calculated
    - Modeling  $M_V^*(\tau)$  comes to the end at  $\delta M_{V,e}^*(\tau) < 0,01$ ;
  - 3.4. N realizations  $M_V^*(\tau)$  are placed in ascending order and to each value  $M_V^*(\tau)$  the probability  $F^*[M_V^*(\tau)] = \frac{i}{N}$  with  $i=1, N$  is appropriated;
  - 3.5. For the fixed value of a error I type  $\alpha=0,05$  on s.f.d.  $R^*[M_V^*(\tau)] = \{1 - F^*[M_V^*(\tau)]\}$  Critical value  $M_{V,\alpha}^*(\tau)$  is defined;
4. If  $M_{V,0,05}^*(\tau) < M_{V,e}^*(\tau)$ ,  $H \Rightarrow H_2$ , i.e. sample  $\{\tau\}_v$  it is unrepresentable. Process of classification proceeds with that distinction that as population of multivariate data unrepresentable sample is accepted. If  $M_{V,e}^*(\tau) \leq M_{V,0,05}^*(\tau)$ , we pass to check of assumption  $H_2$ ;
5. Modeling s.f.d.  $F^*[M_V^{**}(\tau)]$ . For what:
  - 5.1. Method of imitating modeling on s.f.d.  $F_V^*(\tau)$  modeled  $n_v$  random variables  $\tau$ . Calculations are spent under the formula (1);
  - 5.2. Average value  $n_v$  realizations pays off  $\tau$ , which we shall designate as  $M_V^{**}(\tau)$ ;
  - 5.3. Items 5.1 and 5.2 repeat N time;
  - 5.4. N realizations  $M_V^{**}(\tau)$  are placed in ascending order and to each value of some  $M_V^{**}(\tau)$  the probability  $F^*[M_V^{**}(\tau)] = \frac{i}{N}$  with  $i=1, N$  is appropriated;
  - 5.5. On s.f.d.  $F^*[M_V^{**}(\tau)]$  critical value  $M_V^{**}(\tau)$  for a error II type is calculated  $\beta=0,05$ .
6. If it will appear, that  $M_{V,e}^*(\tau) < M_{V,0,05}^{**}(\tau)$ ,  $H \Rightarrow H_2$ , i.e. classification is expedient. Otherwise we pass to comparison of risk of the erroneous decision, accordingly, assumptions  $H_1$  and  $H_2$ , i.e. sizes  $Ri^*(H_1)$  and  $Ri^*(H_2)$ ;

7. Risk of the erroneous decision  $Ri^*(H_1) = R^*[M_{V,e}^*(\tau)]$ ;  $Ri^*(H_2) = F^*[M_{\Sigma,e}^{**}(\tau)]$ . Remind, that  $Ri^*(H_2) > \beta$ , and  $Ri^*(H_1) > \alpha$
8. If 
$$\left. \begin{aligned} \frac{Ri^*(H_1) - \alpha}{Ri^*(H_2) - \beta} > (1 + \alpha), \quad \text{to } H \Rightarrow H_1 \\ \frac{Ri^*(H_2) - \beta}{Ri^*(H_1) - \alpha} > (1 + \beta), \quad \text{to } H \Rightarrow H_2 \end{aligned} \right\} \quad (3)$$

### 3. REALIZATION OF ALGORITHM

In the illustrative purposes practical realization of algorithm of modeling of distribution  $F^*[M_{V,m}^*(X)]$  consider on an example of pseudo-random numbers  $\xi$  with uniform distribution in an interval  $[0,1]$ . Model  $n_v$  pseudo-random numbers, define their average statistical value  $M_v^*(\xi)$  and absolute value of relative change under the formula

$$\delta M_v^*(\xi) = \frac{|M(\xi) - M_v^*(\xi)|}{M(\xi)} = \left| 1 - \frac{M_v^*(\xi)}{M(\xi)} \right| = |1 - 2M_v^*(\xi)|$$

Further dependence is required to us

$$M_v^*(\xi) = \frac{1 - \delta M_v^*(\xi)}{2} \quad (4)$$

Calculate N realizations  $\delta M_v^*(\xi)$ , and ranging  $\delta M^*(\xi)$  in ascending order, we build s.f.d.  $F^*[\delta M_v^*(\xi)]$ . Transition from realizations  $M_v^*(\xi)$  to realizations  $\delta M_v^*(\xi)$  allows to compare distributions  $F^*[\delta M_v^*(\xi)]$  not only for different  $n_v$ , but also for various  $F_{\Sigma}(\xi)$ , for example, for uniform distribution in an interval  $[0.5; 1]$ . In table 1 are resulted quantile s.f.d.  $F^*[\delta M_v^*(\xi)]$  for of some  $n_v$  and discrete values of probabilities of these distributions. Laws of change s.f.d.  $R^*[\delta M^*(\xi)] = 1 - F^*[\delta M^*(\xi)]$  for  $n_v=4; 22$  and  $150$  are resulted on fig.1, and on fig.2. of some critical values quantile distributions  $R^*[\delta M^*(\xi)]$  depending on  $n_v$ . Regression analysis of these dependences has shown-laws of change, that laws of change  $M_{V,m,\alpha}^*(\xi) = f(n_v)$ , with high accuracy (factor of determination  $> 0.999$ ) correspond to following dependence:

$$\delta M_{V,m,\alpha}^*(\xi) = \frac{A}{n_v^{-0,5}} \quad (5)$$

Values of factor A depending on a error I type are resulted in table 2

Under the standard program at  $\alpha \leq 0,2$  greatest convergence ( $R^2=0.994$ ) the equations of regress  $A=f(\alpha)$  took place for a polynom

$$A = -18,5\alpha^2 - 7,33\alpha + 1,48 \quad (6)$$

Table 2

Experimental estimations of constant factor the equations of regress (6)

Error I type	0,01	0,05	0,1	0,2
Value of A factor	1,42	1,13	0,95	0,75

Thus, dependence  $\delta M_{V,\alpha}^*(\xi) = f(\alpha, n_v)$  looks like

$$\delta M_{V,\alpha}^*(\xi) = \frac{(18.5\alpha^2 - 7.33\alpha + 1.48)}{\sqrt{n_v}} \quad (7)$$

Table 3

Quantile distributions  $M^*[\delta M^*(\xi)]$

	2	3	4	5	6	7	11	22	29	40	90	150
0.001	0.001	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01	0.006	0.004	0.004	0.003	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001
0.05	0.026	0.022	0.018	0.017	0.015	0.014	0.011	0.008	0.007	0.006	0.004	0.003
0.1	0.05	0.044	0.037	0.033	0.031	0.027	0.023	0.015	0.014	0.011	0.008	0.006
0.15	0.077	0.066	0.056	0.049	0.046	0.041	0.034	0.024	0.02	0.017	0.012	0.009
.02	0.105	0.088	0.076	0.066	0.061	0.056	0.046	0.031	0.027	0.023	0.016	0.012
0.25	0.136	0.111	0.095	0.083	0.076	0.071	0.057	0.040	0.035	0.03	0.02	0.015
0.3	0.166	0.134	0.116	0.101	0.092	0.085	0.069	0.048	0.042	0.036	0.024	0.018
0.35	0.196	0.158	0.136	0.119	0.109	0.101	0.08	0.056	0.05	0.042	0.028	0.022
0.4	0.226	0.182	0.158	0.138	0.125	0.116	0.093	0.065	0.057	0.049	0.032	0.025
0.45	0.259	0.207	0.180	0.158	0.142	0.134	0.106	0.074	0.065	0.055	0.037	0.028
0.5	0.293	0.233	0.202	0.178	0.16	0.151	0.12	0.083	0.073	0.062	0.041	0.032
0.55	0.33	0.259	0.225	0.199	0.179	0.168	0.134	0.093	0.081	0.07	0.046	0.036
0.6	0.369	0.288	0.25	0.223	0.20	0.186	0.15	0.103	0.091	0.077	0.052	0.040
0.65	0.409	0.320	0.277	0.246	0.223	0.206	0.165	0.115	0.101	0.086	0.057	0.045
0.7	0.453	0.353	0.307	0.271	0.247	0.228	0.182	0.127	0.112	0.095	0.064	0.049
0.75	0.5	0.393	0.339	0.301	0.274	0.253	0.202	0.142	0.123	0.105	0.07	0.055
0.8	0.554	0.435	0.374	0.334	0.304	0.282	0.225	0.158	0.137	0.117	0.079	0.061
0.85	0.614	0.489	0.417	0.372	0.339	0.317	0.252	0.177	0.154	0.131	0.088	0.068
0.9	0.687	0.555	0.474	0.423	0.385	0.361	0.287	0.202	0.176	0.15	0.101	0.078
0.95	0.776	0.639	0.557	0.498	0.453	0.425	0.341	0.241	0.210	0.179	0.12	0.093
0.99	0.905	0.785	0.71	0.638	0.59	0.545	0.439	0.311	0.276	0.235	0.157	0.121
0.999	0.969	0.901	0.839	0.769	0.71	0.667	0.56	0.386	0.351	0.296	0.198	0.151
1	0.995	0.958	0.917	0.909	0.872	0.816	0.656	0.467	0.455	0.342	0.244	0.139

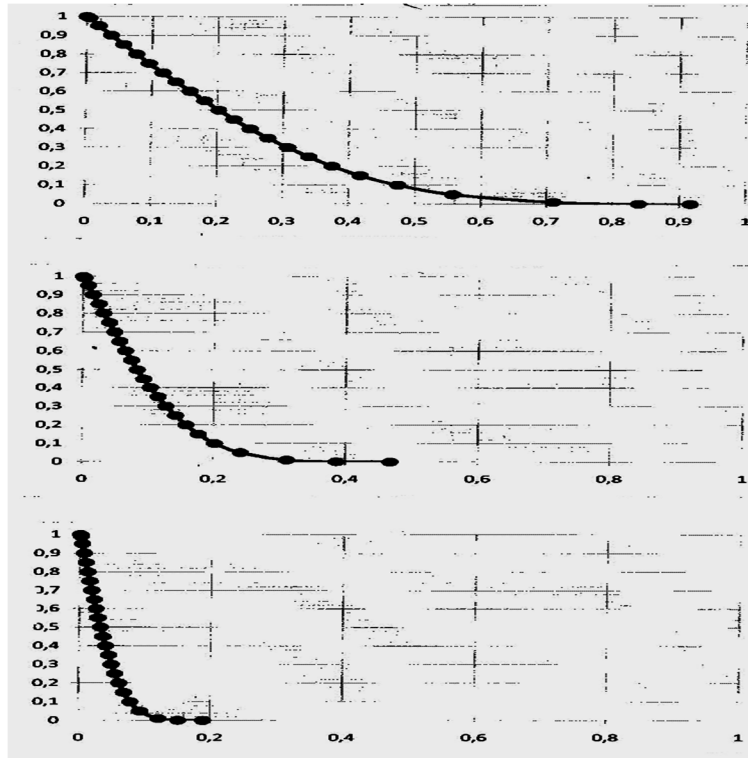


Fig.1. Laws of change s.f.d.  $R^*[\delta M^*(\xi)]$  for  $n_v=4; 22$  and  $150$

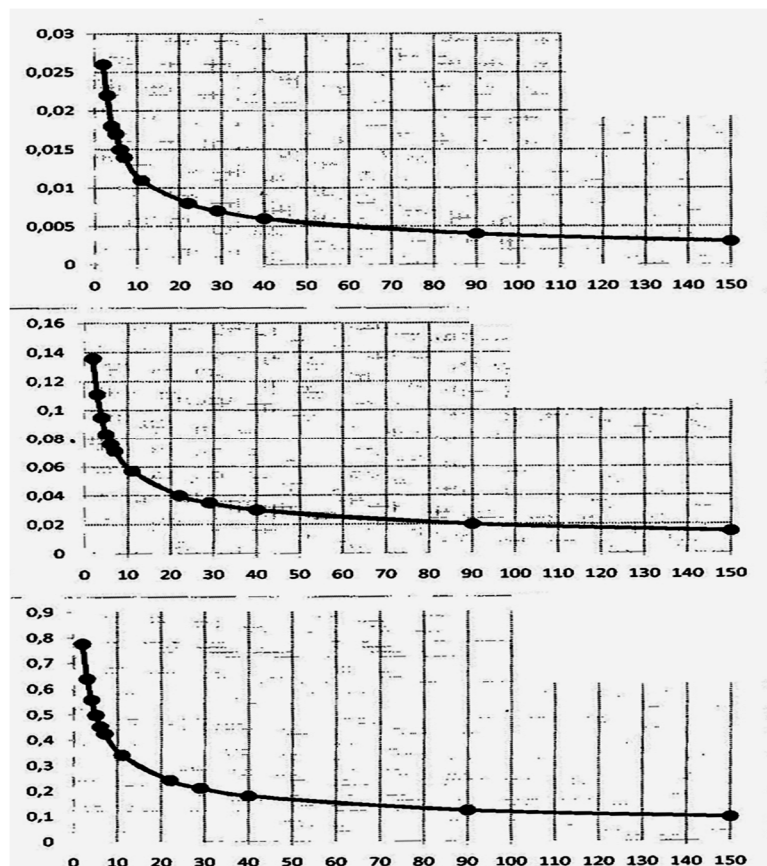


Fig.2. Laws of change of critical values quantile distributions  $R^*[\delta M^*(\xi)]$  depending on  $n_v$  and a significance value  $\alpha=0.05; 0.25$  and  $0.95$

#### 4. EXAMPLE OF CALCULATION.

In the present example on the basis of the lead researches analytically we shall confirm intuitively clear conclusion: casual character of  $M_{\Sigma}(X)$  and  $M_{\nu}^*(X)$  it is possible at an essential divergence of corresponding functions of distribution  $F_{\Sigma}(X)$  and  $F_{\nu}^*(X)$ . It is easy to notice, that the example corresponds to the approach at the decision of «return problem», when the result known and efficiency of the approach is checked.

Let  $F_{\Sigma}(X)$  is a function with uniform distribution in an interval  $[0,1]$ . Random variables ( $y$ ) samples in volume  $n_{\nu}$  are calculated under the formula:

$$y = a + F_{\Sigma}(y) \cdot (b-a) \tag{8}$$

where  $F_{\Sigma}(y)$  – function of distribution of a random variable  $y$ .

$F_{\Sigma}(y)$  corresponds to the uniform law of distribution in an interval  $[a, b]$ , where  $a=0,5$ , and  $b=1$ . Hence

$$y = 0.5 [1 + F_{\Sigma}(y)] \tag{9}$$

As realizations  $F_{\Sigma}(y)$  take advantage to first four ( $n_{\nu}=4$ ) of random numbers of table 9.1 [3] with uniform distribution in an interval  $[0,1]$ . This: 0,1009; 0,3754; 0,0842 and 0,9901. Having substituted them in the equation (10), receive accordingly 0,55; 0,688; 0,542 and 0,995. Check of hypothesis about casual divergence  $F_{\Sigma}(X)$  and  $F_{\nu}^*(Z)$  lead according to table 5 [4], and a hypothesis about a casual divergence of  $M_{\Sigma}(X)$  and  $M_{\nu}^*(y)$  according to table 1. Results of calculations are resulted in table 3.

Table 3

The data checks of statistical hypotheses

i	$X_i$	$Y_i$	$Z_i$	$F_{\nu}^*(Z_i)$	$ \Delta_i $	
1	0.1009	0.550	0.550	0.25	0.300	$M_{\Sigma}(=0,5; M_{\Sigma}(y) =0.75$ $M_{\nu}^*(Z)=0.694; \delta M_{\nu}^*(Z)=0.075$ $M_{\nu}^*(\Delta)=0.103$
2	0.3754	0.688	0.542	0.50	0.042	
3	0.0842	0.542	0.688	0.75	0.062	
4	0.9901	0.995	0.995	1.0	0.005	
Total			2,775	-	0,409	$R^*[M_{\nu}^*(Z)=0.80$ $R^*[M_{\nu}^*(\Delta)<0.01$

As the hypothesis about casual divergence follows from table 3  $F_{\Sigma}(X)$  and  $F_{\nu}^*(y)$  does not prove to be true  $R^*[M_{\nu}^*(\Delta)] \ll \alpha$ , that completely corresponds to valid parity  $F_{\Sigma}(X)$  and  $F_{\nu}^*(y)$ . A hypothesis about a casual divergence of  $M_{\Sigma}(X)$  and  $R^*[M_{\nu}^*(\Delta)] \gg \alpha$ . In other words, sample  $\{y\}_{n_{\nu}}$  from the point of view of distinction of  $M(X)$  and  $M_{\nu}^*(y)$  proves to be true homogeneous, and classification of data inexpedient.

#### CONCLUSION.

1. For the parameters of reliability calculated as an average arithmetic multivariate of random variables which distribution differs from the normal law, the method, algorithm and criterion of comparison of estimations of parameters of reliability are developed at classification of multivariate data;
2. Application for these purposes of a method of comparison s.f.d. to multivariate population of initial data and samples of this population leads to unjustified decrease in a significance

value owing to the unreasonable account of characteristics of disorder of random variables.

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