OPTIMUM LIKELIHOOD QUANTIZATION OF THE INFORMATION IN SPACE WITH RESTRICTION OF ZONES OF INFLUENCE OF QUANTA

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ABSTRACT

The model of optimum three-dimensional likelihood quantization of the determined or casual volume space is offered by set of equal quanta at which the probability of representation quantized reaches spaces of the maximum value. The size of optimum quantum is defined by distribution of its zone of influence, values of boundary probability and influence parameter. The model of estimation of quantity of the information or other product in quantization space is entered.

Keywords: volume space, optimum quantization, a zone of influence of quantum, distribution, probabilities, boundary value, influence parameter, estimation of quantity of the information.

1. INTRODUCTION

The first basic results of the decision of problems of optimum quantization of the casual have been received in [1, 2]. In article [1] the problem of the theory of value of the information with the account of its cost is solved. Minimised function is presented in a kind:

$$\Psi(x)=\int_0^\infty [x(E(z/x)+1-z]dF(z)+cJ+const,\quad J=\int_0^\infty E(z/x)dF(z).$$

The first composed - average losses for the account of distortion of value of size as a result of the quantization, the second composed - losses for the account for the account of use of quantity of the information.

Authors [2], quoting [1], and a minimized function represent in a kind:

$$\Psi(x)=(x+c)\int_0^\infty (E(z/x)+1)dF(z),$$

where x,c size of quantum and interval size between quanta, $E(\alpha)$ the whole part of number α, $F(z)$ distribution function quantized a random variable $\hat{Z}$, having a final population mean.

A distinctive advantage [2] in comparison with [1] is that authors have offered a simple physical illustration and have developed strict algorithm of the integer decision of a problem.

In article [3] the idea of work [2] for estimation of probability of successful transfer quantized information in the conditions of adversary counteraction is used. Expression for probability of the successful message of full quantity quantized information in not deformed kind, duration transferred in the quanta t with pauses of length dividing them c, is equal:

$$R(t)=\int_0^\infty [\frac{\int_0^\infty \overline{B}(t+u)dA(u)}{\int_0^\infty \overline{B}(u)dA(u)}\cdot \overline{P}(c)]^{E(\frac{z}{t+c})+1}dF(z).$$

In this formula $B(t)$ – function of distribution of length of quantum of the information, transferred by the first party. $A(t)$ – function of distribution of duration of quantum of the information, deformed by the second party in the course of transfer of quantum of the information by the first party. $F(t)$ – function of distribution of full quantity of the transferred information. $\overline{P}(c)$ – probability of successful realisation of a pause in length between quanta.

The maximum value $R(t)$ and the corresponding size of quantum are numerically. In conclusion of article the example of a game situation and a choice of strategy of the contradictory parties is resulted.

In article [4] simple problems of quantization of the syntactic and semantic information were considered.

In the present article the following problem of volume quantization with likelihood criterion function is considered.

In three-dimensional information field placing of the identical centers of gathering of the information is supposed. Each centre has the volume review of gathering of the information. The density of probability of size of the review of the centre is defined by some distribution. In средине the centre of gathering the density has the maximum value, and in process of removal to its periphery value of density monotonously decreases. Law of decrease of density is identical on three axes. The density of probability except constant maximum value also is characterized by the average quadratic deviation which size can change depending on some constructive conditions. All centers of influence in space of gathering of the information settle down strictly horizontally and vertically on parallel lines, adjoining to each other. Zones of influence of the centers are crossed with zones of influence of the next centers on horizontals and verticals. The size of degree of crossing is that, that between six next zones located round a separate volume zone, should the part of space captured by their influence. It means that each border of a separate zone of influence adjoins to two next zones of other zones of influence as the crow flies. Scope of zones of influence on three axes is identical. The scope size is defined by some boundary probability defining possibility of gathering of the information by the centre in it кубичной of area. This probability depends on constructive technical characteristics of the centre of gathering. Being set by value of boundary probability of the centre and changing size of parameter of its influence - an average quadratic deviation (aqd) - it is possible to find such density of probability of influence of the centre at which the probability of gathering of the information in the limited space reaches the maximum value.

The purpose of given article is search of density of probability of influence of the centre, number of the centers of influence and values of their parameters at which the maximum value of probability of gathering of the information in the set volume space is reached.

2. THE REASON OF EXISTENCE OF THE MAXIMUM OF PROBABILITY

Let's assume that the size of parameter of influence - an average quadratic deviation of three-dimensional density of probability - is small enough. Provided that in space enough considerable quantity of the centers of influence is necessary to arrange. It will lead to that total losses of all centers of influence from not used volumes of zones of influence because of introduction at them borders will be great enough. On the other hand, losses from not used volumes of in addition entered centers of influence for information gathering on space periphery will be rather small.

Now we will assume that the size of parameter of influence (aqd) will be great enough. Arguing how it is stated above, we come to a conclusion that the size of total losses will be defined basically by losses of not used volumes of influence of in addition entered centers.

This inconsistent situation in definition of total volume losses at gathering of the information also is the basis for existence of the optimum decision. At it there is such number of the volume centers of influence, when probability of coverage of the space defined on size will reach the maximum value.

3. MATHEMATICAL FORMALIZATION OF THE PROBLEM OF QUANTIZATION

We assume that we know family of density of probabilities of zones of influence of the centre $f(x,y,z,\sigma_x,\sigma_y,\sigma_z)$, where $\sigma_x,\sigma_y,\sigma_x$ the parameters of influence defining degree of disorder of influence concerning the centre on we rub to its axes. Further for the purpose of simplicity of a statement of a problem we will believe that three-dimensional density of probability it is represented in the form of product of three identical one-dimensional density with population means m and average quadratic deviations σ. It means that one-dimensional random variables are independent and possess identical parameters. Then our formalization will be reduced to consideration only one density of probability $f(x,\sigma)$. We will provide the general boundary probability q ,defining size of volume of a zone of the centre of influence – quantum. The there will be a size less q, the it is more size of a zone of influence of the centre, but, on the other hand, the it is less degree of influence of the centre on quantum border. The quantum is represented a cube with an edge $2(m-r)$, where m population mean of an initial zone of influence, and r – an interval from the beginning of co-ordinates to border of a zone of influence. Otherwise it is possible to name it an interval, the unilateral limiting size of a zone of influence of quantum on any of three coordinates. Value of size r is at the set probability q and conditions that six next quanta cover one quantum at which internal borders densely adjoin quanta of six next quanta. Formally value of size r should be in result of the decision of the equation:

$$q = (\int_0^r C_\omega f(z,\sigma)dz)^3 . \tag{1}$$

The right part (1) is probability of hit of a one-dimensional random variable in a unilateral restrictive zone of quantum. For a restrictive zone on any one co-ordinate it is accordingly received $\int_0^r C_\sigma f(z,\sigma)dz - \sqrt[3]{q} = 0$. In the equation (1) and its consequence a $C_\sigma -$ constant of rationing of density of probability at the accepted value of parameter of influence σ. Then the probability of hit of a random variable in the field of quantum is as:

$$p(r,\sigma) = (\int_r^{2m-r} C_\sigma f(z,\sigma)dz)^3 . \tag{2}$$

Expression (2) is fair only provided that distribution in the field of quantum is spherical. Expression and for эллипсоидного distributions can be if necessary written down.

Further we will present the sizes of space, on which gathering of the necessary information should be made by the centers of influence. For simplicity of the analysis we assume that it is a rectangle with the parties X,Y,Z, presented in determined sizes. Then the necessary number of the quanta, which had along the parties X,Y,Z, will be accordingly equal:

$$E(\frac{X}{2(m-r)})+1,\ E(\frac{Y}{2(m-r)})+1,\quad E(\frac{Z}{2(m-r)})+1, \tag{3}$$

where the symbol E means the integer approximated with a lack from the number presented in brackets.

The probability of a covering of volume of space in quanta of the centers of gathering of the information can be defined in the form of following expression:

$$P_{XYZ}(q,\sigma) = (\int_r^{2m-r} C_\sigma f(z,\sigma)dz)^{3\{E[\frac{X}{2(m-r)}]+1\}\{E[\frac{Y}{2(m-r)}]+1\}\{E[\frac{Z}{2(m-r)}]+1\}} . \tag{4}$$

In that case when sizes are casual, the probability (4) is represented in a kind:

$$P_{XYZ}(q,\sigma)=\int_0^\infty\int_0^\infty\int_0^\infty\Big(\int_r^{2m-r} C_\sigma f(u,\sigma)du\Big)^{3\{E[\frac{X}{2(m-r)}]+1\}\{E[\frac{Y}{2(m-r)}]+1\}\{E[\frac{Z}{2(m-r)}]+1\}} dF_X(x)dF_Y(y)dF_Z(z), \qquad (5)$$

where $F_X(x), F_Y(y), F_Z(z)$ – functions of distributions of the sizes of the parties of volume space of gathering of the information.

Expressions (4) and (5) can be presented and in other kinds if areas of influence of quanta represent not spheres, and ellipsoids. In them dependences of random variables on all coordinate axes, besides, can be considered.

4. THE EXAMPLE 1

As one objective realization of an example it is possible to present a reservoir of the limited size in which there can be a subject dangerous to stay. It is required to define, how many the detection centers it is necessary to provide and as them to arrange, that with the maximum probability to receive data on a place of its finding. Centre technical characteristics are known and can change in some limits. All three one-dimensional coordinates of a zone of the centre of influence are described by density of probability of normal distribution:

$$f(x,\sigma)=(C_\sigma/\sqrt{2\pi}\sigma)\exp(-(x-m)^2/2\sigma^2, \qquad (6)$$

$m=30$ *ед.*, $\sigma-$parameter of influence which can change in some limits. We will accept size of boundary probability $q=0{,}00000001$, and space of gathering of the information we will present a cube with the party $X=100$ *ед.*. It is required to find values of all parameters of quantization provided that the probability of gathering of the information in specified the cubic space reaches the maximum value.

The problem decision. We are set by some value σ from area of admissible values. From the equation (1) we find value for the least border r of quantum. We define value of size of an edge of quantum under the formula $2(m-r)$ under condition of symmetry of curve density of probability to a population mean. We find probability of hit of a random variable of influence in the field of quantum under the formula $p=(\int_r^{2m-r} C_\sigma f(z,\sigma)dz)^3$. We substitute the received probability p in expression (4) and we calculate value of the received probability $P_{XYZ}(q,\sigma)$.

Arriving in a similar way for other values σ from a range of possible values, we find values of all set of probabilities $P_{XYZ}(q,\sigma)$. Definitively, we choose from this set of values that value which is maximum. Then we define values of all necessary parameters accompanying the decision of a problem of optimization.

In the conditions of our example (the index 0 at symbols of parameters means an accessory of values of parameters to the optimum decision) it is received for $q=0{,}00000001$ $\max_\sigma P_{XYZ}(q,\sigma)=P_0=0{,}893,\ \sigma_0=8{,}92-8{,}98$ *ед.*, $r_0=4{,}98$ *ед.*, $C_0=1,\ n_0=8,$ probability of hit of a random variable of influence in the field of quantum $p_0=0{,}986,$ length of an edge of quantum $l=50{,}04$ *ед.*.

Check of correctness of the decision of an example was carried out by comparison of the total volume covered n_0 in quanta, with the set volume of quantization.

So, as a result of the problem decision the answer is received: it is necessary to have eight quanta - the centers in the form of a cube, forming cubic structure with two quanta in its any edge, completely covering the set volume of supervision and providing performance of the purpose with the specified probability P_0.

5. THE EXAMPLE 2

In the conditions of the optimum quantization, considered in an example 1, suppose, that in volume кубичном space with length of edges some product is distributed in the form of a spatial suspension. The quantity of a product in a co-ordinate grid of volume is defined by expression:

$$W(x,y,z) = xyz + [\frac{(x-2)^2}{2} + \frac{(y-3)^2}{3} + \frac{(z-4)^2}{4}]. \quad (7)$$

According to the results received in an example 1, quantization maximizations accompany following numerical values: influence parameter $\sigma_0 = 8{,}92 - 8{,}98$ *ед.*, boundary value of quantum $r_0 = 4{,}98$ *ед.*, a constant of rationing of density $C_0 = 1$, number of quanta $n_0 = 8$, probability of hit of a random variable in volume of quantum $p_0 = 0{,}986$, length of an edge of quantum $l_0 = 50{,}04$ *ед.*, the maximum value of probability of quantization (a covering of volume $X \times Y \times Z$ with the party 100 *ед.*) $P_0 = 0{,}893$.

Using necessary values of some of the resulted parameters, we will define average sizes of the product getting to area of each of eight quanta, and also average total size of a product in the field of quantization.

Let's enter a designation of size of an average product for quantum ijk a symbol D_{ijk}. Indexes i,j,k can accept only values 0 or 1 . For example, the size D_{101} needs to be defined from following expression:

$$D_{101} = \int_{2m-r_0}^{4m-3r_0} f_Z(z) \int_{r_0}^{2m-r_0} f_Y(y) \int_{2m-r_0}^{4m-3r_0} W(x,y,z) f_X(x) dxdydz. \quad (8)$$

All density of probability are identical and equal in the given expression

$$f(x) = \frac{C_0}{\sqrt{2\pi}\sigma_0} e^{-\frac{(x-m)^2}{2\sigma_0^2}}.$$

Calculating everything D_{ijk}, we receive following numerical values of an average product:

$D_{000} = 2{,}746 \times 10^4$; $D_{001} = 136{,}764$; $D_{010} = 3{,}395 \times 10^{-3}$; $D_{011} = 0{,}681$; $D_{100} = 136{,}764$; $D_{101} = 0{,}684$; $D_{110} = 0{,}681$; $D_{111} = 3{,}395 \times 10^{-3}$.

Total value of an average product is defined as $D = \sum_{i,j,k=0}^{1,1,1} D_{i,j,k}$, and it will be equal $2{,}787 \times 10^4$. For check of this value we will calculate D under the formula:

$$D = \int_{r_0}^{4m-3r_0} f_Z(z) \int_{r_0}^{4m-3r_0} f_Y(y) \int_{r_0}^{4m-3r_0} W(x,y,z) f_X(x) dxdydz = 2{,}774 \times 10^4. \quad (9)$$

The error of calculation of previous value with value (9) will be equal $\approx 0{,}5$ %.

6. THE CONCLUSION

Task in view of realization of optimum likelihood quantization in sense of filling in three-dimensional space. The important factors defining existence of the given decision, are use of boundary probability and influence parameter. The example of the decision of a problem of quantization is resulted.

Expression for an indicator of criterion function of quantization – probabilities of a covering quantized volume in space is received. The example of estimation of average quantity of the product placed in the set volume of quantization, both separate quanta, and full quantized is resulted by volume.

The considered questions of quantization can find various practical appendices in three-dimensional space.

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