COMPARATIVE RELIABILITY ANALYSIS OF FIVE REDUNDANT NETWORK FLOW SYSTEMS

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ABSTRACT

In this paper, probabilistic models for five repairable redundant network flow systems have been developed to analyze and compare their availability and profit. Explicit expressions for steady-state availability, busy period of repairman and profit function for the five redundant network flow systems are developed. Furthermore, we compare the five redundant network flow systems based on their availability and profit and found that configuration II is more reliable and profitable than the remaining configurations.

1. INTRODUCTION

Reliability connection between networks can be usually achieved through a number of redundant paths/units, thus making the connection reliable. The reliability of these network systems is of increasing importance since the failure of some components may lead to disastrous results. Example of such systems include water distribution, oil and gas supply, power generation and transmission, transport by rail and by road, communication system consisting of a transmitter, relay stations and a receiver, where a signal from transmitter is received by two consecutive relay and distributed to other relay stations before it finally arrived at the receiver for consumptions. Availability and profit of an industrial system are becoming an increasingly important issue. Where the availability of a system increases, the related profit will also increase. High system reliability and availability plays a vital role towards industrial growth as the profit is directly dependent on production volume which depends upon system performance. Because of their prevalence in power plants, manufacturing systems, and industrial systems, many researchers have studied reliability comparison of different systems, a great number of models have been introduced to describe the behaviour and performance of the systems. Evaluation of reliability of network flows with stochastic capacity and cost constraint was studied by Fathabadi and Khodaei (2012). Ke and Chu (2007) performed comparative analysis of availability of redundant system. Wang and Chen (2009) performed comparative analysis of availability of three systems with general repairs, reboot delay and switching failure. Wang et al. (2012) performed comparison of availability between two systems with warm standby units and different imperfect coverage. Wang et al. (2006) performed comparison of reliability and availability between four systems with warm standby components standby switching failures. Yusuf (2013) performed comparative analysis of some reliability characteristics between two systems requiring supporting devices for operation. Yusuf (2014) performed comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation.

The present paper is devoted to modelling and analysis steady-state availability, busy period and profit of five redundant network flow systems. The contributions of this paper are twofold. Based on the first order linear differential equations, explicit expressions of steady-state availability, busy period and profit function for the five redundant network flow systems are developed. Comparisons are performed based on assumed numerical values given to system parameters to determine the optimal system using MATLAB.

2. DESCRIPTION OF THE CONFIGURATIONS

We consider five dissimilar redundant network flow systems as follows. The first configuration consists of two subsystems A and B arranged in parallel has two units each. The second configuration consists of three subsystems A, B and C. With subsystems A and B in series and parallel to subsystem C. Subsystem A has one unit while subsystems B and C two units each. The third configuration consists of three subsystems A, B and C with subsystem A and B in parallel and series to subsystem C with two units each. The fourth configuration is parallel-series system with two units in series in subsystem A and parallel to subsystems B and C. Subsystems B and C are in series and have two units each. The fifth configuration consists of the three subsystems A, B and C in series. Subsystem A has two units in cold standby, subsystem B consists of 2-out-of -3 units while subsystem C consists of one unit.
It is assume that switching from standby to operation is perfect and instantaneous. We also assume that two or more units cannot fail simultaneously. Each active unit fails independent of the state of others. Whenever a unit fails with failure rate α, it is immediately sent to service station for repair with service rate β and the standby unit/subsystem is immediately switched into operation.

3. MODELS FORMULATION

3.1 Availability, Busy period and Profit of Configuration I

For the analysis of availability case of configuration I, we define $P_i(t)$ to be the probability that the system at time $t \geq 0$ is in state S_i. Also let $P(t)$ be the probability row vector at time t. The initial condition for this problem is: $P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \ldots, P_{11}(0)]$ = $[1,0,0,0,0,0,0,0,0,0,0,0]$

The steady-state equations for configuration 1 can be expressed as follows:

$$\frac{dP(t)}{dt} = Q_1 P \tag{1}$$

This can be written in the matrix form as

$$\dot{P} = Q_1 P \tag{2}$$

where

$$Q_1 = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -Y & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -X & 0 & 0 & 0 & \beta & \beta & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & -Y & 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & -X & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & \alpha & 0 & 0 & 0 & -2\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta \end{pmatrix}$$

$X = (2\alpha + \beta)$, $Y = (\alpha + \beta)$

The steady-state availability and busy period are given by

$$A_{V1}(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) \quad (3)$$

$$B_{P1}(\infty) = 1 - P_0(\infty) \quad (4)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (2) become

$$Q_1 P = 0 \quad (5)$$

which is in matrix form

$$\begin{pmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \\ \dot{P}_{10} \\ \dot{P}_{11} \end{pmatrix} = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -Y & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -X & 0 & 0 & 0 & \beta & \beta & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & -Y & 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & -X & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & \alpha & 0 & 0 & 0 & -2\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \\ P_{11}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + ... + P_{11}(\infty) = 1 \quad (6)$$

Substituting (6) in the last row of (5) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V1}(\infty) = \frac{\beta^3 + 2\alpha\beta + 4\alpha^2\beta}{5\alpha^3 + 4\alpha^2\beta + 2\alpha\beta^2 + \beta^3} \quad (7)$$

$$B_{P1}(\infty) = \frac{\alpha^3 + 4\alpha^2 b + 2\alpha\beta^2}{5\alpha^3 + 4\alpha^2\beta + 2\alpha\beta^2 + \beta^3} \quad (8)$$

Let C_0 and C_1 be the revenue generated when the system is in working state and no income when in failed state, cost of each repair respectively. The expected total profit per unit time incurred to the system in the steady-state is

Profit=total revenue generated – cost incurred when repairing the failed units.

$$PF_1 = C_0 A_{V1}(\infty) - C_1 B_{P1}(\infty) \quad (9)$$

where PF_1 is the profit incurred to the system.

3.2 Availability, Busy period and Profit Analysis of Configuration II

For the analysis of availability case of configuration II, the same initial conditions are

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \ldots, P_{10}(0)] = [1,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P} = Q_2 P \quad (10)$$

Where

$$Q_2 = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -Y & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & \beta & \beta & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -Y & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & -Y & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & -Y & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -Y & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -Y & 0 & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta \end{pmatrix}$$

The steady-state availability and busy period are given by

$$A_{V2}(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) + P_8(\infty) \quad (11)$$

$$B_{P2}(\infty) = 1 - P_0(\infty) \quad (12)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (10) become

$$Q_2 P = 0 \quad (13)$$

which is in matrix form

$$\begin{pmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \\ \dot{P}_{10} \end{pmatrix} = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -Y & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & \beta & \beta & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -Y & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & -Y & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & -Y & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -Y & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -Y & 0 & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + \ldots + P_{10}(\infty) = 1 \quad (14)$$

Substituting (14) in the last row of (13) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V2}(\infty) = \frac{\alpha_2(2\alpha\beta^4 + 5\alpha^2\beta^3 + 5\alpha^3\beta^2 + 2\alpha^4\beta) + \alpha_1(\beta^4 + \alpha^2\beta^2)}{(4\alpha^5 + 7\alpha^4\beta + 9\alpha^3\beta^2 + 3\alpha\beta^4 + \beta^5)(4\alpha^4 + 3\alpha^3\beta + 6\alpha^2\beta^2 + 2\alpha\beta^3 + \beta^4)} \quad (15)$$

$$B_{P2}(\infty) = \frac{2\alpha^4 + 3\alpha^3\beta + 3\alpha^2\beta^2 + 2\alpha\beta^3}{2\alpha^4 + 3\alpha^3\beta + 3\alpha^2\beta^2 + 2\alpha\beta^3 + \beta^4} \quad (16)$$

Using the procedure described in configuration I above**,** expected profit is

$$PF_2 = C_0 A_{V2}(\infty) - C_1 B_{P2}(\infty) \tag{17}$$

3.3 Availability, Busy period and Profit Analysis of Configuration III

For the analysis of availability case of configuration III, the same initial conditions are

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \ldots, P_{10}(0)] \ = [1,0,0,0,0,0,0,0,0,0,0]$

The differential equations are expressed in the form

$$\dot{P} = Q_3 P \tag{18}$$

Where

$$Q_3 = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & 0 & \beta & \beta & \beta & 0 & 0 \\ 0 & \alpha & 0 & -X & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & -\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -X & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & & 0 & \alpha & 0 & 0 & -\beta \end{pmatrix}$$

The steady-state availability and busy period are given by

$$A_{V3}(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) P_7(\infty) \tag{19}$$

$$B_{P3}(\infty) = 1 - P_0(\infty) \tag{20}$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (18) become

$Q_3 P = 0$

(21)

which is in matrix form

$$\begin{pmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \\ \dot{P}_{10} \end{pmatrix} = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & 0 & \beta & \beta & \beta & 0 & 0 \\ 0 & \alpha & 0 & -X & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & -\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -X & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & & 0 & \alpha & 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+...+P_{10}(\infty)=1 \quad (22)$$

Substituting (22) in the last row of (21) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V3}(\infty)=\frac{\beta^3+\alpha\beta^2+\alpha^2\beta}{4\alpha^3+4\alpha^2\beta+2\alpha\beta^2+\beta^3} \quad (23)$$

$$B_{P3}(\infty)=\frac{4\alpha^3+4\alpha^2\beta+2\alpha\beta^2}{4\alpha^3+4\alpha^2\beta+2\alpha\beta^2+\beta^3} \quad (24)$$

Using the procedure described in configuration I above**,** expected profit is

$$PF_3=C_0A_{V3}(\infty)-C_1B_{P3}(\infty) \quad (25)$$

3.4 Availability, Busy period and Profit Analysis of Configuration IV

For the analysis of availability case of configuration II, the same initial conditions are

$$P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),...,P_{11}(0)]\ =[1,0,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P}=Q_4P \quad (26)$$

Where

$$Q_4=\begin{pmatrix}
-\alpha & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\alpha & -X & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \alpha & -X & 0 & \beta & 0 & \beta & 0 & 0 & 0 & 0 & 0\\
0 & \alpha & 0 & -X & 0 & \beta & 0 & \beta & 0 & 0 & 0 & 0\\
0 & 0 & \alpha & 0 & -X & 0 & 0 & 0 & \beta & \beta & 0 & 0\\
0 & 0 & 0 & \alpha & 0 & -X & 0 & 0 & 0 & 0 & \beta & \beta\\
0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0\\
0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta
\end{pmatrix}$$

The steady-state availability and busy period are given by

$$A_{V4}(\infty)=P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty) \quad (27)$$

$$B_{P4}(\infty)=1-P_0(\infty) \quad (28)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (26) become

$$Q_4P=0 \quad (29)$$

which is in matrix form

$$\begin{pmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \\ \dot{P}_{10} \\ \dot{P}_{11} \end{pmatrix} = \begin{pmatrix} -\alpha & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & -X & 0 & \beta & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -X & 0 & \beta & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & -X & 0 & 0 & 0 & \beta & \beta & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & -X & 0 & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \\ P_{11}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + \ldots + P_{11}(\infty) = 1 \tag{30}$$

Substituting (30) in the last row of (29) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V4}(\infty) = \frac{\beta^4 + \alpha\beta^3 + 2\alpha^2\beta^2 + 2\alpha^3\beta}{4\alpha^4 + 4\alpha^3\beta + 2\alpha^2\beta^2 + \alpha\beta^3 + \beta^4} \tag{31}$$

$$B_{P4}(\infty) = \frac{4\alpha^4 + 4\alpha^3\beta + 2\alpha^2\beta^2 + \alpha\beta^3}{4\alpha^4 + 4\alpha^3\beta + 2\alpha^2\beta^2 + \alpha\beta^3 + \beta^4} \tag{32}$$

Using the procedure described in configuration I above**,** expected profit is

$$PF_4 = C_0 A_{V4}(\infty) - C_1 B_{P4}(\infty) \tag{33}$$

3.5 Availability, Busy period and Profit Analysis of Configuration V

For the analysis of availability case of configuration II, the same initial conditions are

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \ldots, P_{11}(0)] = [1,0,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P} = Q_5 P \tag{34}$$

Where

$$Q_5 = \begin{pmatrix} -3\alpha & \beta & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -W & 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -W & 0 & 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 \\ \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & \alpha & 0 & 0 & 0 & -Z & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta \end{pmatrix}$$

$W = (3\alpha + \beta)\,, Z = (3\alpha + 2\beta)$

The steady-state availability and busy period are given by

$$A_{V5}(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_6(\infty) \quad (35)$$

$$B_{P5}(\infty) = 1 - P_0(\infty) \quad (36)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (34) become

$$Q_5 P = 0 \quad (37)$$

which is in matrix form

$$\begin{pmatrix} -3\alpha & \beta & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -W & 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -W & 0 & 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 \\ \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & \alpha & 0 & 0 & 0 & -Z & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \\ P_{11}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + \ldots + P_{11}(\infty) = 1 \quad (38)$$

Substituting (38) in the last row of (37) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V5}(\infty) = \frac{\beta^3 + 2\alpha\beta^2 + \alpha^2\beta}{3\alpha^3 + 5\alpha^2\beta + 3\alpha\beta^2 + \beta^3} \quad (39)$$

$$B_{P5}(\infty)=\frac{3\alpha^3+5\alpha^2\beta+3\alpha\beta^2}{3\alpha^3+5\alpha^2\beta+3\alpha\beta^2+\beta^3} \tag{40}$$

Using the procedure described in configuration I above**,** expected profit is

$$PF_5=C_0A_{V5}(\infty)-C_1B_{P5}(\infty) \tag{41}$$

4. GRAPHICAL ANALYSIS OF THE NETWORKS

In this section, the main purpose of this section is to present specific numerical comparisons for the configurations for steady-state availability and profit. For each model the following set of parameters values are fixed throughout the simulations for consistency:

Case I: We fix $\beta=0.3$ **,** $C_1=500,000$**,** $C_2=80,000$ and vary the values of α for from 0 to 1 Figures 1 and 4.

Case II: We fix $\alpha=0.4$ **,** $C_1=500,000$**,** $C_2=80,000$ and vary the values of β for from 0 to 1 Figures 2 and 3.

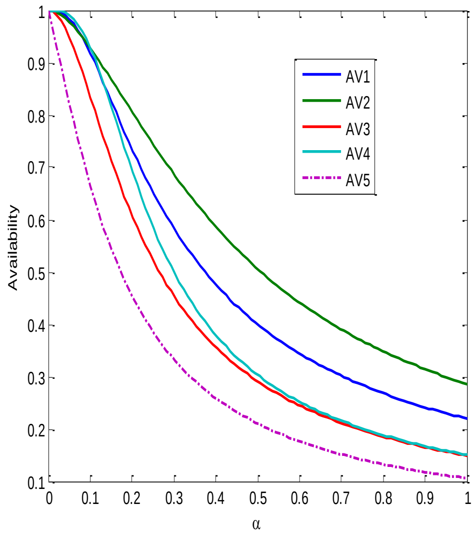

Figure 1 Availability against α

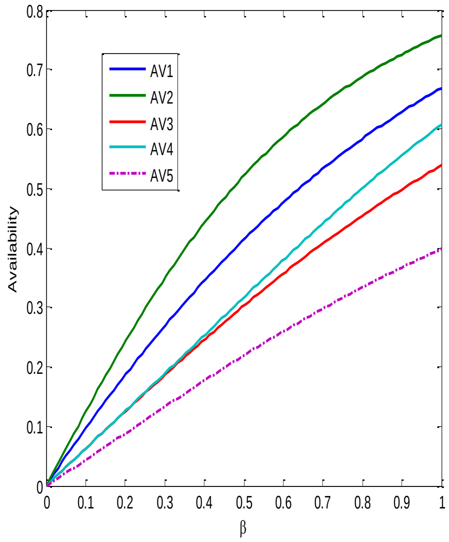

Figure 2 Availability against β

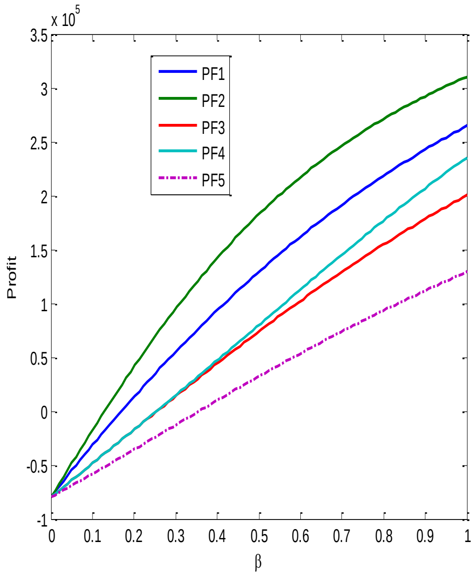

Figure 3: Profit against β

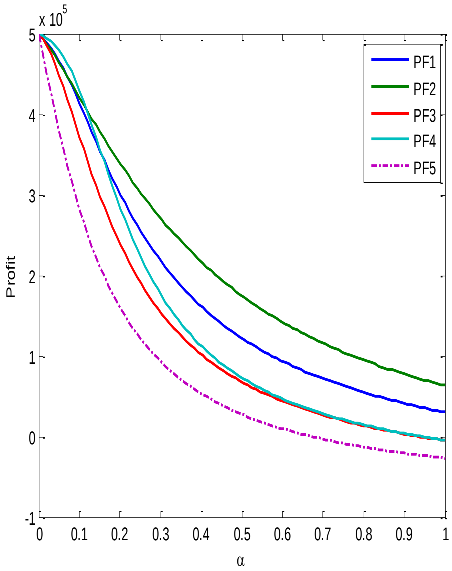

Figure 4: Profit against α

Figures 1 and 4 depict the availability and profit results for the five systems being studied against the failure rate α . The steady-state availability decrease as α increases for any configuration. It is clear from the Figures that configuration II has higher availability with respect to α as compared with the other three configurations. These tend to suggest that configuration II is better than the other configurations. On the other hand, Figures 2 and 3 depict the availability and profit calculations for the five configurations against repair rate β . The steady-state availability and profit increase as β increases for any configuration. The observations that can be made here are much similar to those made from Figures 1 and 4. It is evident that from Figures 2 and 3 that configuration II is better than the other configurations. Thus,

$$A_{V2}(\infty) > A_{V1}(\infty) > A_{V4}(\infty) > A_{V3}(\infty) > A_{V5}(\infty)$$

$$PF_2(\infty) > PF_1(\infty) > PF_4(\infty) > PF_3(\infty) > PF_5(\infty)$$

5. CONCLUSION

In this paper, we analysed five different redundant communication networks with standby units to study the availability and profit analysis of five configurations. For each configuration, we present the explicit expressions for steady-state availability, busy period of repairman and profit and performed comparative analysis numerically to determine the optimal configuration. It is evident from Figures 1-4 that configuration II is optimal configuration using steady-state availability and profit. The present study will help the engineers and designers to develop sophisticated models and to design more critical system in interest of human kind.

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