MODIFIED CONCEPT OF DANIELS' SEQUENCE AND ITS APPLICATION TO ANALYSIS OF THE RELIABILITY OF SERIES-PARALLELS SYSTEM AND FATIGUE LIFE OF UNIDIRECTIONAL FIBROUS COMPOSITE

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ABSTRACT

This is short review of previous publications and new development of the concept of Daniels' sequence (DS) which can be used to consider the association of distribution of the static strength of unidirectional fiber composite (UFC) component with distributions of its fatigue life and residual strength of the UFC itself. A generalization of the model of series-parallel system as description of structure of UFC is given. A new description of the defected structural longitudinal element(LE) of UFC, new condition of failure of UFC, a definition of residual Daniels' function is introduced. Numerical examples of processing of experimental data (data of fatigue test of glass-fiber composite) are presented. The estimates of the parameters of the corresponding nonlinear regression model for simultaneous processing of result of the fatigue test and test of residual tensile strength are obtained (parameters of the distribution of the local strength parameters of structural LE).

1. INTRODUCTION
1.1 Background

Usually the reliability of some system is described only by the distribution of its time to failure. Only the two states of the system (intact, failure) are considered. The use of the concept of Daniels' sequences (DSs) for explanation the reliability of some parallel system makes it possible to take into account the parameters of the process of loading of the system (for example, the tensile or cycling test of a unidirectional fibrous composite (UFC)) and explain the quantitative relation between the distribution of the static strength of longitudinal elements (LEs), taking up the basic load, and the static strength, fatigue life and residual strength of a UFC itself considered as series-parallel system. Here the processing of the result of fatigue test of UFC is considered. It is of particular importance, that the use of Daniels' Sequence (DS) concept allows to explain the existence of an fatigue-limit (maximum of cycling stress at which the fatigue life is equal to infinity). The history of the question on the relation between distributions of the static strength of a UFC and its components has been discussed in detail in our preceding papers, therefore, here we mention only several authors who have related works : Peirce (1926), Daniels (1945), Gucer & Gurland (1962), Smith (1982) and others. The use of the theory of Markov processes for the problems of static and cyclic fatigue life is rather thoroughly considered in Bogdanov et all (1989) and for the tests on the tensile strength in Paramonov at all (2006-2012a). The corresponding models of relation between the static strength and the fatigue life of composite materials is suggested in Paramonov at all (2006,2008-2012b). Some concluding results are reported in Paramonov at all (2011). The concept of the Daniels' sequence, with reference to the description of the process of fatigue failure, was first introduced in Paramonov at all (2006). Its successful application to describing the relation between the strength of LE and the fatigue life of UFC is discussed in Cimanis at all (2012) and Paramonov at all (2013a). In Paramonov at all (2013b) a more general definition of DS, which can be applied to processing of result of testing UFC specimens both on the fatigue life and static strength. It is shown that the use of DS concept gives the explanation of existence of fatigue-limit. In this paper a new development of the DS concept is offered. Definition of residual Daniels' function is introduced. It is taken into account that the event " residual strength becomes lower than the maximum level of cycle stress" is only necessary but is not enough condition. Some energy must be accumulated to get the failure of specimen. Numerical examples of processing of experimental data are presented, and estimates of the parameters of the corresponding nonlinear regression model for simultaneous processing of result of the fatigue test and test of residual tensile strength are obtained (parameters of the distribution of the local strength of structural longitudinal elements).

1.2. The Simple Daniels' sequence

The Simple Daniels' sequence (SDS) (here we use this name for initial version of Daniels' sequence initially introduced in [12]) for description of fatigue phenomenon is based on the model studied by Daniels (1945). Let $X_{(1)}, X_{(2)},..., X_{(n)}$ be the ordered values of the random strengths of the LEs forming the parallel system. Assuming the independence of the random variabels (RVs) $X_1, X_2,..., X_n$ with the same cumulative distribution function (CDF) $F_X(x)$, Daniels showed that the RV $Y = \max\{X_{(k)}(n-k+1)/n : 1 \le k \le n\}$ has an asymptotically normal distribution with the average and standard deviation

$$\mu_D = \max x(1-F_X(x)) = x^*(1-F_X(x^*)), \quad \sigma_D = (\mu_D x^* F_X(x^*)/n)^{1/2}. \tag{1}$$

By "unwrapping" this model in time, we obtain a sequence of local (in specific link where the fatigue damage develops) stresses $\{s_0, s_1, s_2, ...\}$, which we name the realization of *simple Daniels's sequence* (*SDS*) for fatigue test, described by the equation

$$s_{i+1} = s/(1-\nu(s_i)/n) = s/(1-\hat{F}_X(s_i)), \quad i = 0,1,2,..., \quad s_0 = s \tag{2}$$

where s is the parameter of the initial cycling nominal stress (for example, by s is implied the maximum of the cyclic stress); $\nu(s_i)$ is the number of LE with strength lower than or equal to s_i; the function $\hat{F}_X(.)$ is the estimate of the CDF (empirical CDF) of the local strength of LE in framework of specific parallel system using the sample $x_{1:n} = (x_1,...,x_n)$ (the realization of the vector ($X_1, X_2,..., X_n$)) . We suppose that in the last part of this equation another definition of the estimate of CDF can be used. For example, if $F(x) = G(x,\theta)$ then the estimate $\hat{F}(x) = G(x,\hat{\theta})$ can be used where $\hat{\theta}$ is the estimate of the parameter θ.

The SDS is completely determined by the CDF estimation method and by the structure $(s, n, F_X(x))$ and its realization determined by the pair (s, $x_{1:n}$). It has the following properties (see Paramonov at all (2013a):

(1) Since $\hat{F}_X(x) \le 1$ and it is a no decreasing, then $s_{i+1} \ge s_i$; i.e., the SDS is a nondecreasing sequence also. If $s > \max x(1-\hat{F}_X(x))$, the SDS increases up to infinite. We call this sequence the first-kind DS and designate the pair that generates it by the symbol $(s, x_{1:n})^*$.

(2) Now, assume that $0 < s \le \max x(1-\hat{F}_X(x))$. Then, there is the solution to the equation

$$s = x(1-\hat{F}_X(x)), \tag{3}$$

and there is such first $i = i^{**}$ that $s_{i^{**}+1} = s_{i^{**}}$, and the SDS growth process stops. We call this sequence the second-kind SDS.

WE assume, that the failure of the parallel system (link) under the alternating cyclic loading takes place if the share of the operable LEs is reduced to the value f_C, where f_C is the function of s. Then, the critical local stress that correspond to this event, s_C, is determined from the equation

$$f_C = 1-\hat{F}_X(s_C): \quad s_C = \hat{F}_X^{-1}(1-f_C).$$

The transition from s_i to s_{i+1} is called the SDS step. It corresponds to the damage of all the LEs with strength in the interval of $(s_i, s_{i+1}]$. The maximum number of steps at which the local stress is still smaller than the critical one we call the *SDS life* (SDSLf) of the parallel system at the nominal stress s:

$$N_D = \max(i : s_i < s_C,\ s_i \in \{s_0, S_1, S_2 ...\},\ s_0 = s) + 1, \qquad (4)$$

where $\{s_0, S_1, S_2 ...\}$ is the SDS for $s_0 = s$; i.e., N_D is equal to one plus the number of those SDS elements (or the maximum number of SDS elements among those elements), which do not exceed s_C. For the second-kind SDS, N_D is equal to infinity. Examples of the SDS are shown in Fig.2.

For some materials, there is the maximum cyclic stress s for which the number of before-damage cycles is equal to infinity. This stress is called the fatigue-limit or altimate fatigue strength. If the solution to equation (3) exists, then, as it was already stated, the increase of the SDS components stops after some step number. The critical stress s_C will never be reached, and the SDSLf will be equal to infinity. The maximum stress at which this phenomenon takes place

$$S_D = \max x(1 - \hat{F}_X(x)) \qquad (5)$$

should be considered as SDS fatigue-limit (SDSLt).

So the use of SDS can explain the existence of fatigue-limit. The dependence of the distribution of RV N_D on the stress s is quite similar to the dependence of the fatigue life on s in fatigue tests of the composite. However, the values of N_D for $s > S_D$ are too small for as compared with the real data. The value of S_D is too large and itself coincides with Daniels' definition of tensile strength of bundle of fibers. So some additional assumptions should be made in order to describe the real result of fatigue test. The development of this initial definition of SDS is needed .

2. UFC SPECIMENS FOR TENSION AND FATIGUE TESTS AS A SERIES – PARALLEL SYSTEM

Here we offer some modified description of the structure of UFC as a series-parallel system (SPS), Fig. 1.

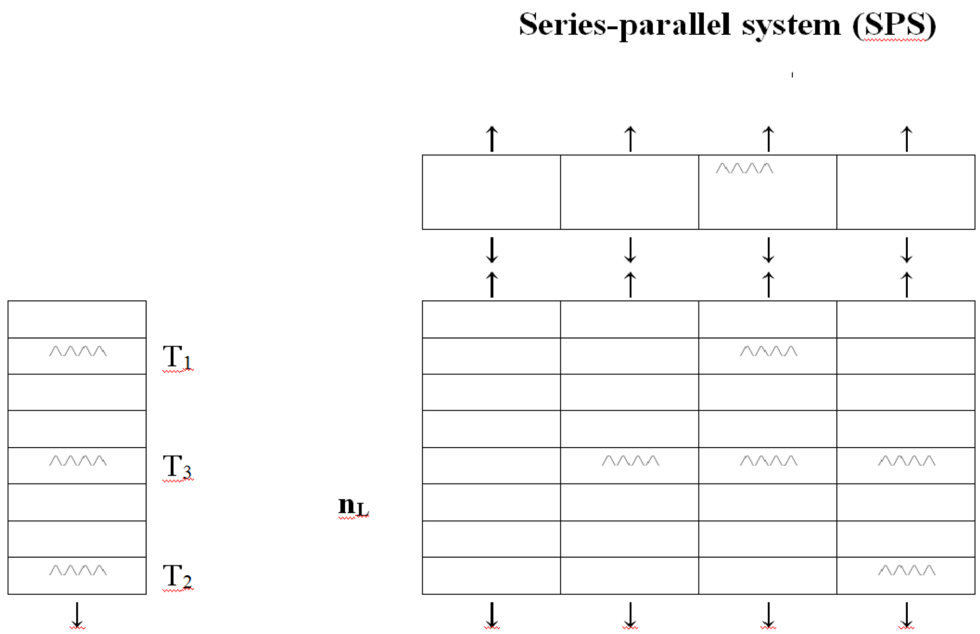

Fig. 1. Structures of UFC as a series, a parallel and a series-parallel system.

This SPS is considered as a sequence (chain) of n_L links which are the parallel systems. In K_L links the defects already exists or can appear. The value K_L is a RV, $1 \le K_L \le n_L$. In ($n_L - K_L$) links, the defects cannot appear. In flawless links, there are n_C elements. Unlike the previous definition now we suppose that in defected link there are two types of LEs: there are K_C, $1 \le K_C \le n_C$, random number of defected (weak) LEs with CDF of strength $F_W(x)$ and $M_C = n_C - K_C$ LEs without defects with CDF of strength $F_U(x)$.

3. MODIFIED DANIELS' SEQUENCE

Let us mention three salient features of Daniels' model:

(1) a continuous increase in the external load is assumed, but its rate is not taken into account;

(2) it is assumed that destructions of one by one fibers with a successively growing strength are accumulated;

(3) by the strength of a fiber bundle is meant the external load at the instant when it becomes higher than the load-carrying ability of the specimen tested. Thereafter, the destruction, i.e., rupture, of the bundle is presumed.

In the present study, which is the development of Paramonov at all (2013b), we make the following assumptions.

(1) The process of the external loading is described by discrete sequence $s^+_{i:\infty} = \{s^+_i, i = 0,1,2,...\}$. For tension test $s^+_{i+1} \ge s^+_i$, $\lim s_i = \infty$, for cycling loading $s^+_i = s^+ = Const$ for all $i = 0,1,2,...$.

(2) There is a local stress concentration, defined by a concentration factor k_C for cycling loading or concentration factor k_T for tension test (it is supposed that in general case different type of stress distribution for tensile and cycling test takes place). And there is a local decreasing of the strength in a weak link. For cyclic load the modified Daniels' sequence (MDS_CL) is defined by the equation

$$s_i = k_C s^+ / (1 - \nu(s_{i-1}) / n) = k_C s^+ / (1 - \hat{F}_{X_L}(s_{i-1})),$$

where $\hat{F}_{X_L}(s)$ is the estimate of the CDF of the local tensile strength of a link in which there are the defected LEs with $F_W(x)$ and LEs without defects with $F_U(x)$ (in the following numerical example we suppose that $F_{X_L}(s)$ is a mixture of $F_W(x)$ and $F_U(x)$, $F_W(x) = F_U(k_F x)$, k_F, $k_F \ge 1$, is some parameter of the model).

For increasing load at the tensile strength test MDS_IL it is defined by the equation

$$s_i = k_T s^+_i / (1 - \nu(s_{i-1}) / n) = k_T s^+_i / (1 - \hat{F}_{X_L}(s_{i-1}))$$

where $s_0 = s^+_1$, $s^+_{i+1} \ge s^+_i$, $i = 1,2,...$, $\lim s_i = \infty$.

So now the random Daniels' fatigue limit, S_D is defined by maximum value of s^+ for which there is a solution of the equation $x = k_C s^+ / (1 - \hat{F}_L(x)$.

So we have

$$S_D = \max x(1 - \hat{F}_{X_L}(x)) / k_C \quad .$$

(3) The failure of the link (parallel system) takes place at kth step if the *Daniels' residual tensile strength*, $R_D(s)$, becomes lower than the applied load

$$R_D(s_i) > s^+_i \text{ for all } i = 1,2,...,k-1 \text{ but } R_D(s_k) \le s^+_k$$

where Daniels residual tensile strength, $R_D(s)$, is defined by equation

$$R_D(s) = \max_{z>s} d(z),$$

Daniels' function, $d(z)$, is defined by equation

$$d(z) = k_T z(1 - \hat{F}_{X_L}(z)),$$

where k_T is the stress concentration factor for tensile.

In this paper we consider only the case of cycling loading. We are interested only in fatigue life and residual strength of a link (as a parallel system) and of UFC (as a series-parallel system (SPS)).

It can be seen that for MDS as for SDS the number of steps to failure is very small if $s^+ > S_D$ or is equal to infinity if $s^+ \le S_D$. So two additional assumptions are made.

(4) The process of transition from some state of MDS s_i to next state s_{i+1} is described by Markov chain (MCh) (see detales in following section).

(5) For failure of some set of LEs the accumulation of some energy is necessary. In some cycle with parameter s^+ this energy is a function of s^+ (example of approximation of this function see in section (5)).

3. LONGEVITY OF A PARALLEL SYSTEM UNDER A CYCLIC LOAD. APPLICATION OF MARKOV CHAIN THEORY

There are two types MDS for cycling loading. MDS_CL1 corresponds to the failure of a link (if some condition of a failure is achieved) . MDS_CL2 corresponds to the infinite number of the steps without failure (if the nominal stress is lower or equal to Daniels' fatigue-limit).The random process MDS_CL can be described by Markov chain with random transition probability matrix, P, with two corresponding absorbing states. So we have the following type of matrix P:

		j_B	*1*			*2*		
		j_A	*1*	...	j^*	*1*	...	j^{**}
i_B	i_A	i \ j	1	…	j^*	j^*+1	…	j^*+j^{**}
1	*1*	1	p_{11}	…	0	0	…	0
	...	…	0	…	…			
	i^*	i^*	0	0	1	0	…	0
2	*1*	i^*+1	0	0	0	$p_{(j^*+1)(j^*+1)}$	…	0
	...	…	0	0	0	0	…	…
	i^{**}	i^*+i^{**}	0	0	0	0	0	1

In fact, the matrix P is complex of two submatrices. The first (n_D+1) states, $(1,2,\ldots,i^*)$, of submatrix in left upper corner are connected with the components of MDS_CL1, $\{s_0, s_1, ..., s_{n_D}\}$. The absorbing ($n_D+2$)-th state corresponds to the condition of failure of a link. Here, $\{s_0, s_1, ..., s_{n_D}\}$ is the realization of the random MDS_CL1 , $\{s_0, S_1, S_2 ...\}$, generated by sample $x_{L,1:n}$ (the realization of the vector ($X_{L,1}, X_{L,2}, ..., X_{L,n}$)) and nominal stress s^+, where $s^+ > S_D$. We denote the corresponding pair by the symbol $(s, x_{1:n})^*$. The value n_D is the realization of RV N_D. The submatrix in right-hand bottom corner with states $(1,2,\ldots,i^{**})$ is connected with components of MDS_CL2 , $\{s_0, s_1, ..., s^{**}, s^{**}, s^{**}, ...\}$. Absorbing state, i^{**}, corresponds to stopping of the process of destruction (to the achievement by the local stress of the final level s^{**}). The random process is defined by the upper matrix if $s^+ > S_D$ and by the lower one if $s^+ \le S_D$.

Let us denote by the symbol $R_{(s,x_{L,1:n_C})^*}$ the space of pairs $(s, x_{L,1:n_C})^*$ corresponding to the MDS_CL1 and by $F_{X_L}(s,\hat{\theta})$ the estimate of CDF itself of RV X_L where $\hat{\theta}$ is the estimate of parameter of CDF.

We define the random transition-probability submatrix $\hat{P}_I$ corresponding to MDS_CL1 in following way:

$\hat{p}_{i,i+1} = (F_{X_L}(s_i,\hat{\theta}) - F_{X_L}(s_{i-1},\hat{\theta}))/(1 - F_{X_L}(s_{i-1},\hat{\theta}))$ is the the conditional probability of the failure of all the longitudinal elements having strength in the interval (s_{i-1}, s_i); $\hat{p}_{ii} = 1 - \hat{p}_{i,i+1}$, $i = 1,...,n_D + 1$;

$\hat{p}_{01} = F_{X_L}(s_0,\hat{\theta})$, $\hat{p}_{00} = 1 - \hat{p}_{01}$; $\hat{p}_{(n_D+2)(n_D+2)} = 1$. All the remaining probabilities are equal to 0.

In the same way the submatrix corresponding to MDS_CL2 can be defined. But really we are interested only to know the probability of the event $s^+ \le S_D$. So in the following let us use $\hat{P}$ instead of $\hat{P}_1$. Note ones again that the matrix $\hat{P}$ is the realization of the random matrix, since it is a function of the random sample $x_{L,1:n_C}$, which was used for estimation of the parameter $\hat{\theta}$ of CDF $F_{X_L}(x,\theta)$ and development of the corresponding MDS.

Let us designate the transition time from s_{i-1} to s_i by $\hat{T}_i$ in the MCh with the realization of the random transition-probability matrix $\hat{P}$. For the RV $\hat{T} = \hat{T}_1 + \hat{T}_2 + ... + \hat{T}_{n_D}$, i.e., the corresponding number of steps before the absorption (now this is the longevity of one link (one parallel system)), the estimate of the corresponding random CDF is as follows:

$$\hat{F}_T(t,\eta;(s,x_{L,1:n_C})^*) = \pi \hat{P}^t b, \quad t = 1,2,3,...,$$

where vector $\pi = (\pi_0, \pi_1, ..., \pi_{n_D+1})$ is the vector of the prior probabilities in the possible initial states (in the simplest case, $\pi = (1,0,...,0)$, i.e., all n_C of the LE are active), the vector b is the vector-column in the form $(0,\dots0,1)'$, the vector η is the vector of parameters of the model (f_C, n_C,…), and s^+ is the initial nominal stress level. The average, $E(\hat{T}_i)$, and variance , $V(\hat{T}_i)$ of the RV realization $\hat{T}$ is easily calculated bearing in mind that the RVs $\hat{T}_i$, $i = 1,2,...,n_D + 1$ have a geometric distribution: $E(\hat{T}_i) = 1/p_i$, $V(\hat{T}_i) = (1 - p_i)/p_i^2$.

Owing to the randomness of the sample $x_{L,1:n_C}$, it is necessary to average (e.g., using the Monte Carlo method) the obtained random CDF over the space of the samples included into $R_{(s,x_{L,1:n_C})^*}$ and then to calculate in the average value T and the quantiles we are interested in as functions of the initial cyclic stress s^+.

The example of supposed connection between the time of absorption $\hat{T}$ and corresponding number of cycles is shown in section 5.

4. THE CONNECTION BETWEEN THE RELIABILITY OF THE SEPARATE LINK AND THE SPS

Defected links can be present in the considered system even before its tests start or can appear in the process of the system's operation. Let us designate the number of defects in the *i*-th link at the time moment t (after t cycles) by $K_{Ci}(t)$, $0 \le K_{Ci}(t) \le n_C$. It is postulated that $K_{Ci}(t)$ does not decrease with time.

Two hypotheses for the connection between the longevity of the separate link and the SPS can be put forward if defected links are presented in the considered system before its tests start.

(a) The process of accumulation of damage of separate LEs occurs in all the links of the system, in which defects can appear; in addition, these processes are mutually independent and identically distributed.

(b) This process develops only in the link with the minimum residual strength (or maximum initial number of defected LEs).

In the first case (it can be called the RMinMax hypothesis), the failure moment of the ith link is the first moment when the event $R_{Di}(t) \le s^+$ takes place. So fatigue life of the whole SPS

$$T = \min_{1\le i\le K_L} T_i = \min_{1\le i\le K_L} \max_t (t : R_{Di}(t) > s^+)$$

where $R_{Di}(t)$ is the Daniels' residual strength in ith link, $i = 1,2,...,n_L$. In simplest case, when the strength of the defected LE is equal to zero, in Paramonov at all (2013a) the MinMax hypothesis was introduced with

$$T = \min_{1\le i\le K_L} T_i = \min_{1\le i\le K_L} \max_t (t : n_C - K_{Ci}(t) \ge 0) .$$

The corresponding CDF is as follows:

$$F_T(x) = \sum_{k=1}^{n_L} p_{K_L}(k)(1-(1-F_{T_1}(x))^k) ,$$

where $F_{T_1}(x)$ is the CDF of the random time to failure of some random link, T_1, and $p_{K_L}(k)$, $k = 1,2,...$ is the prior distribution of the RV K_L. If, e.g., $K_L = 1 + K$ and, when $n_L \to \infty$, K has Poisson's distribution with the parameter λ, then

$$F_T(x) = 1-(1-F_{T_1}(x)) \exp(-\lambda F_{T_1}(x)) .$$

If $P(K_L = n_L) = 1$ then $F_T(x) = 1-(1-F_{T_1}(x))^{n_L}$.

As applied to the second hypothesis, we designate the initial Daniels' residual strength in ith link before beginning of test by $R_{Di}(0)$ and the smallest one in a weakest link by $R_{Di^*}(0) = \min_{0\le i\le K_L} R_{Di}(0)$. The assumption can be made that fatigue damage accumulation takes place only in the weakest link i^*. So the time to failure of SPS

$$T = \max_t (t : R_{Di^*}(t) > s^+) .$$

This hypothesis can be called RMaxMin. If the strength of defected LEs is equal to zero then

$$T = \max (t : M_{Ci^*}(0) - K_{Ci^*}(t) \ge 0)$$

where $M_{Ci^*}(0) = n_C$ - $K_{Ci^*}(0)$, $K_{Ci^*}(t)$ corresponds to the weakest link.

In this case, we obtain the CDF of RV T by averaging the CDF of RV $T_1(k)$, corresponding to the presence in one link random number intact LEs: $F_T(x) = \sum_{k=1}^{n_C-1} F_{T_1(k)}(x) p_{M_{C^*}}(k)$

where

$$p_{M_{C^*}}(k) = P(M_{Ci^*}(0) = k) = F_{M_{Ci^*}}(k) - F_{M_{Ci^*}}(k-1) , \quad k = 1,2,...,n_C - 1;$$

$$F_{M_{C^*}}(k) = \sum_{k_L=1}^{n_L} p_{K_L}(k_L)(1-(1-F_{M_{C1}}(k))^{k_L}) .$$

Besides, if $K_L = 1 + K$, the RV K has Poisson's distribution with the parameter λ then,

$$F_{M_{C^*}}(k) = 1-\left(1-F_{M_{C1}}(k)\right)\exp\left(-\lambda F_{M_{C1}}(k)\right) .$$

Links with defects can also appear during cyclic loading. Assume that X_i, $i = 1,2,...,$ are the random intervals between the moments of the appearance of these links; X_1, $X_1 + X_2$, $X_1 + X_2 + X_3$,... are the time moments of their appearance; and T_i are the lives of the corresponding links. Then, if the failure of the system can occur only due to the failure of the links with defects, for $n_C = \infty$, we have $T = \min(T_1, T_2 + X_1, T_3 + X_1 + X_2,...)$ or

$$T = \min(T_1, T^+ + X_1) , \qquad (6)$$

where the RV T^+ has the same CDF as the RV *T*. It is appropriate to name the hypothesis for the model at which the CDF of the RV T meets equation (6) the MinTime hypothesis.

Equation (6) is true for arbitrary positive random variables. In the particular case of initiating faulty links in accordance with the Poisson process with the intensity μ for CDF of RV T, we obtain

$$F_T(y) = 1-(1-F_{T_1}(y))\exp(-\mu\int_0^y F_{T_1}(t)dt). \quad (7)$$

The three considered hypotheses can find application in solving problems of investigating the longevity of composites with various physical properties (more or less fragile and more or less viscous). However, all these hypotheses are closed in the end for the problem of determining the CDF of the longevity of one link $F_{T_1}(t)$, the determination of which is the main problem considered in this paper. Later, to reduce the writing, we will write T instead of T_1.

5. EXAMPLE OF PROCESSING THE DATA ON THE FATIGUE LIFE AND RESIDUAL STRENGTH

To verify the examined model, the fatigue tests of specimens made of a unidirectional composite (Udo UD ES 500/300 — SGL epo GmbH with LH 160 in "Composites HAVEL"; structure [0/45/0]; effective length, *l* = 60 mm) were carried out at the Riga Institute of Polymer Mechanics. The average static strength of specimen was 487.56 MPa. The fatigue tests was performed at stress ratio *R* = 0.1. The maximum stresses of the cycle of fatigue tests were 292.5, 341.3, and 390.5 MPa . It was assumed that the number of LEs in a "link" (it was assumed that n_L =1) n_C =1000, the number of defected ("weak") LEs is the RV with binomial distribution $b(p,n_C)$, $p=0.1$. The tensile strength of both LEs has the lognormal distribution with CDF of the type : $F(x)=\Phi((\log(x)-\theta_0)/\theta_1)$. The following parameters was found for fitting the test data: for undamaged LE θ_{0U} =6.757 (exp (θ_{0U})=860); for weak LE $\theta_{0W}=\theta_{0U}$ - log(k_F), k_F =3; for both cases θ_1 =0.3. It was assumed additionally that there is a stress concentration defined by the coefficient $k_C=1.7$.

In Fig.2 the nominal cycling stress level (-*- S0), examples of Daniels' sequences (-▲-DS), random residual Daniels' function (-▼- Resid) and a part (multiplied by 100) of already destroyed LEs (-- Pfailure*100) as function of DS step number are shown for cycling nominal stress levels 292.5and 390.5 MPa .

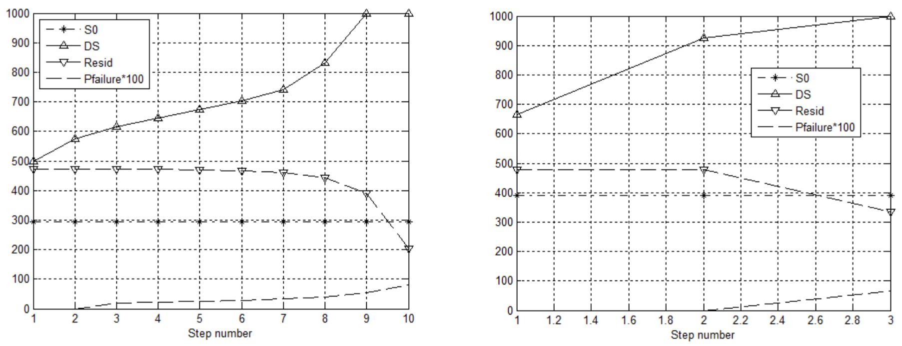

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Fig.2. The nominal cycling stress level (-*- S0), the examples of Daniels' sequences (-▲-DS), random residual Daniels' function (-▼- Resid) and part (multiplied by 100) of already destroyed LEs (-- Pfailure*100) as the function of DS steps for cycling nominal stress levels 292.5 (a) and 390.5 MPa (b).

The nominal cycling stress level (-*- S0), the 10 examples of random Daniels' residual strength (-▼- Resid) and result of test of residual stress as function of cycles number for the same cycling nominal stress levels 292.5and 390.5 MPa are shown in Fig.3 .

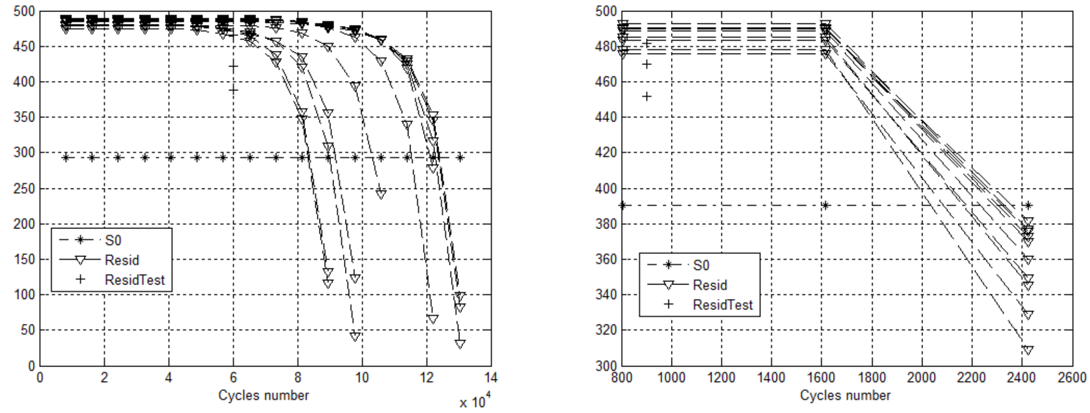

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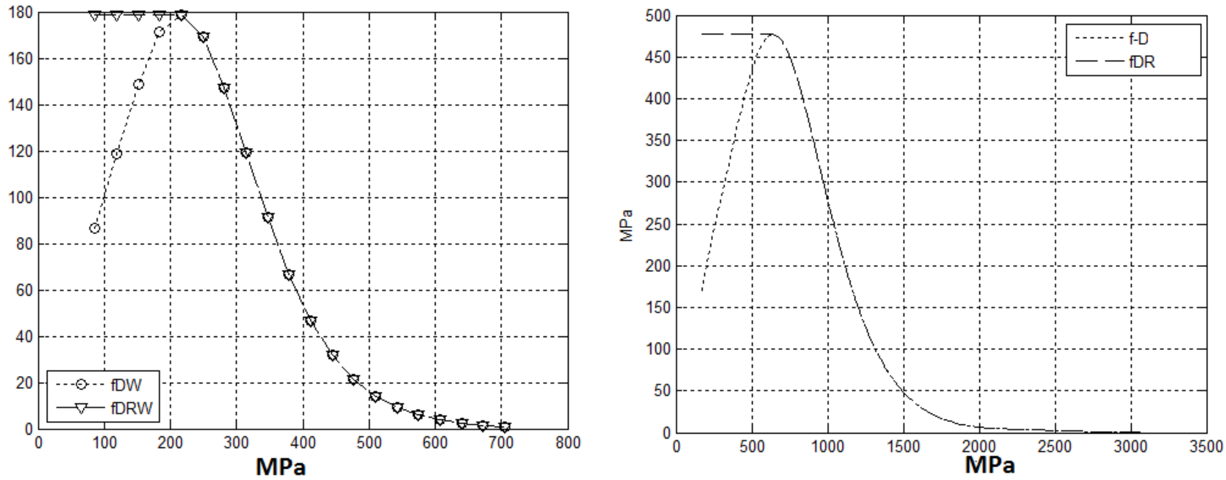
Fig.3 The nominal cycling stress level (-*- S0), the 10 examples of random residual Daniels' function (-▼- Resid) and result of test of residual stress (+ ResidTest) as function of cycles number.

In Fig.2 and 3 we see result of Monte Carlo calculation. The 'teoretical" Daniels' function and residual Daniels' function are shown in Fig.4 for damaged LEs (a) and for mixture of undamaged and damaged LEs (b).



a b

Fig.4. The residual (-▼- fDRW) and 'teoretical" (-o- fDW) Daniels' function for damaged LEs (a) and the same (-- fDR) and (… fD) for mixture of undamaged and damaged LEs (b).

The result of Monte Carlo calculation of fatigue curve (►is a smallest value,* is a mean value,◄ is a largest value) and the result of fatigue test (+) are shown in Fig. 5. For every Monte Carlo trial (n_W values of strength of weak (damaged) LEs and $n_U = n_C - n_W$ values of strength of undamaged LEs) the estimate of CDF $F_{X_L}(x,\hat{\theta})$, the realization of MDS, the items of matrix $\hat{P}$, the values of time to absorption $\hat{T}$ and the corresponding number of cycles N was calculated. The connection between $\hat{T}$ and N was approximated in following way : $N = k_m (R/s^+)^\gamma T$, where k_m, R, γ are parameter of nonlinear regression model. It is assumed that this approximation in some way takes into account that for failure of some set of LEs the accumulation of some energy is necessary (in first approximation, it is assumed that in every cycle this energy is proportional to $(s^+)^\gamma$, the corresponding number of cycles to failure is proportional to $1/(s^{+})^{\gamma}$). For fitting of considered in this paper data (Fig. 3 and 5) the estimates $k_m = 1.46$, $R = 860$, $\gamma = 8$ are used.

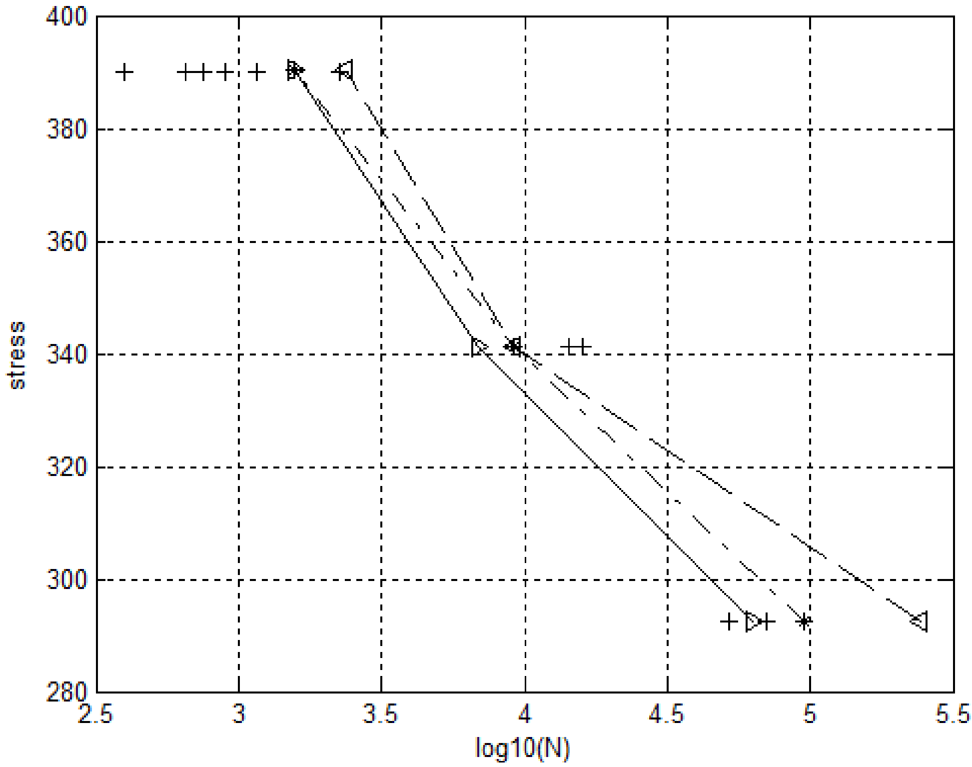


Fig. 5. The result of Monte Carlo calculation of fatigue curve (►is the smallest value,* is the mean value,◄ is the largest value) and the result of fatigue test (+) .

It can be sad that we have enough reasonable fitting of test data.

SUMMARY AND DEVELOPMENT LINES OF THE PROBLEM

1. The model of reliability of series-parallel system (SPS) under tension and cycling load can be considered as a model of reliability of UFC. MinMax, MaxMin, MinTime hypotheses explain the size effect and the connection of CDF of one link (weak micro volume) and the UFC itself.
2. The use of the MDS concept allows to build the model of reliability of a link as a part of SPS.
3. The numerical results of the use the MDS concept are very pure: the fatigue limit is too high, the fatigue life is very small.
4. The developed regression model based of MDS concept together with the use of Markov chains theory and Monte Carlo method allows to get simultaneously the reasonable fitting of the result of tests of fatigue life and residual strength:

(i) to get connection of CDF of tensile strength of component of UFC and the CDFs of the tensile strength, the fatigue life of composite itself and fatigue limit;

(ii) to explain the “strange” behavior of residual strength as function of the number of cycles of fatigue loading (weak dependence at the initial period of loading and sudden decreasing at the last one).

It is shown that it is of great importance to know the moment of drastic decreasing of residual strength. The special study of corresponding CDF is needed.

Parameters of considered regression model can be interpreted as parameters of CDF of the local strength of components of UFC

It is necessary to underline that the offered MDS concept is only the base for different type of regression models which can be based on it. The analysis of the numerical examples considered in this paper and previous publications show that this concept deserves to be studied more intensively. The main problem is the increasing of the number of steps of MDS. Probably it can be done using another approximation of distribution of local strength of LE. In framework of composite it has specific support

and it is not clear the "length" (or the number) of links in corresponding UFC. It (or n_L) can be considered as an additional parameter. Next important problem is to take into account the matrix of real composite material. Some possible approach was considered in Paramonov at all (2011) but it can not explain the existence of fatigue limit. Much more sophisticated hypothesis should be offered to take into account the accumulation of some energy necessary for fatigue failure.

The event " residual strength becomes lower than the maximum level of cycle stress" is only the necessary but not enough condition of the failure of UFC. Much more sophisticated hypothesis should be offered to take into account the accumulation of some energy necessary for fatigue failure. The theory of semi-Markov process with rewords can be used for solution of considered problem. Very important problem is to develop appropriate method of model parameter estimation in order to get better fitting of test result.

The search of the parameters of nonlinear regression for simultaneosly processing tensile, fatigue test and residual strength test result is a difficult task, but we think that, in due course, the structure of models suggested will be of interest not only for graduation theses of students, but also for engineering applications, in particular, for predicting variations in the parameters of strength and durability of UFCs upon changes in the parameters of their components.

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