ESTIMATION AND OPTIMAL DESIGN OF CONSTANT STRESS PARTIALLY ACCELERATED LIFE TEST FOR GOMPERTZ DISTRIBUTION WITH TYPE I CENSORING

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ABSTRACT

This study deals with simple Constant Stress Partially Accelerated life test (CSPALT) using type-I censoring. The lifetime distribution of the test item is assumed to follow Gompertz distribution. The Maximum Likelihood (ML) Estimation is used to estimate the distribution parameters and acceleration factor. Asymptotic confidence interval estimates of the model parameters are also evaluated by using Fisher information matrix. Statistically optimal PALT plans are developed such that the Generalized Asymptotic Variance (GAV) of the Maximum Likelihood Estimators (MLEs) of the model parameters at design stress is minimized. In the last, to illustrate the statistical properties of the parameters, a simulation study is performed

KEYWORDS: Reliability; Partially Accelerated Life Tests; Acceleration factor; constant stress; maximum likelihood estimation; Fisher information matrix; generalized asymptotic variance; optimum test plans; time censoring

1 INTRODUCTION

The life test of the Products having high reliability under normal use conditions often requires a long period of time. In such problems, accelerated life tests (ALTs) are often used to quickly obtain information on the life time distribution of products by testing them at accelerated conditions than normal use conditions to induce early failures**.**

In ALT , the mathematical model relating to the lifetime of an item and stress is known or can be assumed.But,in some cases these relationships are not known and can not be assumed, i.e the data obtained from ALT can not be extrapolated to use condition.So, partially accelerated life test can be used in such cases in which the test items are run at both normal and higher than normal stress conditions. PALT can be carried out using constant-stress, step-stress, or Progressive-stress (linearly increasing stress). In constant stress PALT products are tested at either usual or higher than usual condition only until the test is terminated. The approach to accelerate failures is the step stress which increases the load applied to the products in a specified discrete sequence. A sample of test items is first run at use condition and, if it does not fail for a specified time, then it is run at accelerated condition until a pre specified numbers of failures are obtained or a pre specified time has reached.

There is an amount of literature on PALT which has been studied by many authors. Bai and Chung (1) discussed the optimall designing constant strss PALT or the test item having exponential distribution under Type I censoring.Bai et.al (2) discussed the PALT plan for lognormal distribution under time censored data.after that Bai et al (3) also considered the problemof failure-censored accelerated life-test sampling plans for lognormal and Weibull distributions. Abdel-Ghani (4) investigated some lifetime models under partially accelerated life tests. Ghaly et al. (5) discussed the PALT problem of parameter estimation for Pareto using Type I censoring and after that Ghaly et al. (6) considered the same problem under Type II censoring. Ismail (7) used the maximum likelihood method to estimate the acceleration factor and parameters of the Pareto distribution under PALT. Ismail (8) discuss the constant stress PALT for the Weibull failure distribution under failure censored case. Ismail (9) considered the problem of optimally designing a simple time-step-stress PALT which terminates after a pre-specified number of failures and developed optimum test plans for products having a two-parameter Gompertz lifetime distribution. Zarrin et al. (10) considered constant stress PALT with type-I censoring. Assuming Rayleigh distribution as the underlying lifetime distribution, the MLEs of the distribution parameter and acceleration factor were obtained. More recent Saxena et al (11) consider the PALT design for extreme value distribution using type I censoring and Kamal et al. (12) discuss the same problem for Inverted Weibull distribution.

This work was conducted for constant-stress PALT under type II censored sample. the problems of estimation in constant stress PALT are considered under Rayleigh distribution. Maximum likelihood estimates and confidence intervals for parameters and acceleration factor are obtained.

2 THE MODEL AND TEST METHOD

2.1 The Gompertz Distribution

The lifetimes of the test items are assumed to follow a Gompertz distribution. The probability density function (pdf) of the Gompertz distribution is given by

$$f(t) = \theta\, e^{\alpha t} \exp^{\left\{-\theta/\alpha\left[e^{\alpha t}-1\right]\right\}} \qquad t > 0, \theta > 0, \alpha > 0. \tag{1}$$

where θ is the scale parameter and α is the shape parameter of the distribution.

And the cumulative distribution function is given by

$$F(t) = 1 - \exp^{\left\{-\theta/\alpha\left[e^{\alpha t}-1\right]\right\}} \qquad t > 0, \theta > 0, \alpha > 0. \tag{2}$$

The reliability function of the Gompertz distribution is given by

$$R(t) = \exp^{\left\{-\theta/\alpha\left[e^{\alpha t}-1\right]\right\}} \qquad t > 0, \theta > 0, \alpha > 0. \tag{3}$$

And the corresponding hazard rate is given by

$$h(t) = \theta\, e^{\alpha t}$$

When $\alpha \to 0$, the Gompertz distribution will tend to an exponential distribution, see Wu et al. (13).

The two-parameter Gompertz model is a commonly used survival time distribution in actuarial science, reliability and life testing. There are several forms for the Gompertz distribution given in the literature. Some of these are given in Johnson et al. (14, 15).The pdf formula given in Equation (1) is the commonly used form and it is unimodal. It has positive skewness and an increasing hazard rate function.

2.2 Constant stress PALT procedure

i. Total n items are divided randomly into two samples of sizes $n(1-s)$ and ns respectively where s is sample proportion. First sample is allocated to normal use condition and other is assigned to accelerated conditions.
ii. Each test item of every sample is run until the censoring time τ and the test condition is not changed.

2.3 Assumptions

i. The life time of the test product at use condition follows the Gompertz distribution given in (1).
ii. The life time of the test product at accelerated condition is obtained by using the relation $X = \beta^{-1}T$, where $\beta > 1$ is an acceleration factor. Therefore, the pdf at accelerated condition is given by equation (4) as follows

$$f(x) = \theta\beta\, e^{\alpha\beta x} \exp^{\left\{-\theta/\alpha\left[e^{\alpha\beta x}-1\right]\right\}} \qquad x > 0, \theta > 0, \alpha > 0. \tag{4}$$

iii. The lifetimes T_i , $i = 1,2,............n(1-s)$ of items allocated to normal use condition, are i.i.d. random variables.
iv. The lifetime X_j , $j = 1,2,............ns$ of items allocated to accelerated condition, are i.i.d random variables.
v. The lifetimes T_i and X_j are mutually statistically-independent.

3 MAXIMUM LIKELIHOOD ESTIMATION

The maximum likelihood parameter estimation is used to determine the estimates of the parameter that maximizes the likelihood of the sample data. Also the MLEs have the desirable properties of being consistent and asymptotically normal for large samples
Since, the type-I censoring test terminates after a pre specified time is reached, so, the observed lifetimes $t_{(1)} \le .... \le t_{(n_u)} \le \tau$ and $t_{(1)} \le .... \le t_{(n_a)} \le \tau$ are ordered failure times at normal use and accelerated conditions respectively, where τ is the pre specified time at which the test is terminated, n_u and n_a are the numbers of items failed at normal use and accelerated use conditions, respectively which are given by

$$n_u = \sum_{i=1}^{n(1-s)} \delta_{ui} \text{ and } n_a = \sum_{j=1}^{ns} \delta_{aj}$$

and $n_u + n_a = r$, r is the total number of the failed items.
Let δ_{ui} and δ_{aj} be the indicator functions such that

$$\delta_{ui} = \begin{cases} 1 & t_i \le \tau \\ 0 & otherwise \end{cases} \qquad i = 1,2,..........n(1-s)$$

And

$$\delta_{aj} = \begin{cases} 1 & x_j \le \tau \\ 0 & otherwise \end{cases} \qquad j = 1,2,..........ns$$

Then the likelihood function for (t_i, δ_{ui}), the likelihood function for (x_j, δ_{aj}) and the total likelihood function for $(t_1; \delta_{u1}, \ldots\ldots, t_{n(1-s)}; \delta_{un(1-s)}$, $x_1; \delta_{a1}, \ldots\ldots\ldots, x_{ns}; \delta_{ans})$ are respectively given by

$$L_{ui}(t_i, \delta_{ui} | \alpha, \theta) = \prod_{i=1}^{n(1-s)} \left[\theta e^{\alpha t_i} \exp\left\{ -\left(\frac{\theta}{\alpha}\right) \left[e^{\alpha t_i} - 1\right] \right\} \right]^{\delta_{ui}} \left[\exp\left\{ -\left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\tau} - 1\right] \right\} \right]^{\bar{\delta}_{ui}} \tag{5}$$

$$L_{aj}(x_j, \delta_{aj} | \alpha, \theta) = \prod_{i=1}^{ns} \left[\beta\theta e^{\alpha\beta x_{j_i}} \exp\left\{ -\left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\beta x_j} - 1\right] \right\} \right]^{\delta_{aj}} \left[\exp\left\{ -\left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\beta\tau} - 1\right] \right\} \right]^{\bar{\delta}_{aj}} \tag{6}$$

$$\begin{aligned} L(t, x | \alpha, \beta, \theta) = & \prod_{i=1}^{n(1-s)} \left[\theta e^{\alpha t_i} \exp\left\{ -\left(\frac{\theta}{\alpha}\right) \left[e^{\alpha t_i} - 1\right] \right\} \right]^{\delta_{ui}} \left[\exp\left\{ -\left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\tau} - 1\right] \right\} \right]^{\bar{\delta}_{ui}} \\ & \prod_{i=1}^{ns} \left[\beta\theta e^{\alpha\beta x_{j_i}} \exp\left\{ -\left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\beta x_j} - 1\right] \right\} \right]^{\delta_{aj}} \left[\exp\left\{ -\left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\beta\tau} - 1\right] \right\} \right]^{\bar{\delta}_{aj}} \end{aligned} \tag{7}$$

where $\bar{\delta}_{ui} = 1 - \delta_{ui}$ and $\bar{\delta}_{aj} = 1 - \delta_{aj}$

Taking log of above equation

$$\begin{aligned} l = \ln L = & \sum_{i=1}^{n(1-s)} \delta_{ui} \left[\ln\theta + \alpha t_i - \left(\frac{\theta}{\alpha}\right) \left[e^{\alpha t_i} - 1\right] \right] - \left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\tau} - 1\right] \sum_{i=1}^{n(1-s)} (1 - \delta_{ui}) \\ & + \sum_{i=1}^{ns} \delta_{aj} \left[\ln\theta + \ln\beta + \alpha\beta x_j - \left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\beta x_j} - 1\right] \right] - \left(\frac{\theta}{\alpha}\right) \left[e^{\alpha\beta\tau} - 1\right] \sum_{j=1}^{ns} (1 - \delta_{aj}) \end{aligned} \tag{8}$$

MLEs of α, β and θ are obtained by solving the equations $\frac{\partial l}{\partial \alpha} = 0, \frac{\partial l}{\partial \beta} = 0$ and $\frac{\partial l}{\partial \theta} = 0$.

$$\frac{\partial l}{\partial \theta} = \frac{r}{\theta} - \frac{1}{\alpha} \left[-n + \sum_{i=1}^{n(1-s)} \delta_{ui} e^{\alpha t_i} + \sum_{j=1}^{ns} \delta_{aj} e^{\alpha\beta x_j} + e^{\alpha\tau} \{ n(1-s) - n_u \} + e^{\alpha\beta\tau} (ns - n_a) \right] \tag{9}$$

$$\begin{aligned} \frac{\partial l}{\partial \alpha} = & \sum_{i=1}^{n(1-s)} \delta_{ui} t_i + \beta \sum_{j=1}^{ns} \delta_{aj} x_j - \frac{\theta}{\alpha} \left[\sum_{i=1}^{n(1-s)} \delta_{ui} t_i e^{\alpha t_i} + \beta \sum_{j=1}^{ns} \delta_{aj} x_j e^{\alpha\beta x_j} + \tau e^{\alpha\tau} \{ n(1-s) - n_u \} + \beta\tau e^{\alpha\beta\tau} (ns - n_a) \right] \\ & + \frac{\theta}{\alpha^2} \left[-n + \sum_{i=1}^{n(1-s)} \delta_{ui} e^{\alpha t_i} + \sum_{j=1}^{ns} \delta_{aj} e^{\alpha\beta x_j} + e^{\alpha\tau} \{ n(1-s) - n_u \} + e^{\alpha\beta\tau} (ns - n_a) \right] \end{aligned} \tag{10}$$

$$\frac{\partial l}{\partial \beta} = \frac{n_a}{\beta} + \alpha \sum_{j=1}^{ns} \delta_{aj} x_j - \theta \sum_{j=1}^{ns} \delta_{aj} x_j e^{\alpha\beta x_j} - \theta\tau e^{\alpha\beta\tau} (ns - n_a) \tag{11}$$

It is difficult obtain a closed form solution to nonlinear equations to (9), (10) and (11). Newton-Raphson method is used to solve these equations simultaneously to obtain $\hat{\alpha}, \hat{\beta}$ and $\hat{\theta}$.

The asymptotic variance-covariance matrix of α, β and θ is obtained by numerically inverting the Fisher-information matrix composed of the negative second derivatives of the natural logarithm of the likelihood function evaluated at the ML estimates. The asymptotic Fisher-information matrix can be written as:

$$\frac{\partial^2 l}{\partial\theta^2}=-\frac{r}{\theta^2}$$

$$\begin{aligned}\frac{\partial^2 l}{\partial\alpha^2}=&-\frac{\theta}{\alpha}\left[\sum_{i=1}^{n(1-s)}\delta_{ui}t_i^2e^{\alpha t_i}+\beta^2\sum_{j=1}^{ns}\delta_{aj}x_j^2e^{\alpha\beta x_j}+\tau^2e^{\alpha\tau}\{n(1-s)-n_u\}+\beta^2\tau^2e^{\alpha\beta\tau}(ns-n_a)\right]\\&+\frac{2\theta}{\alpha^2}\left[\sum_{i=1}^{n(1-s)}\delta_{ui}t_ie^{\alpha t_i}+\beta\sum_{j=1}^{ns}\delta_{aj}x_je^{\alpha\beta x_j}+\tau e^{\alpha\tau}\{n(1-s)-n_u\}+\beta\tau e^{\alpha\beta\tau}(ns-n_a)\right]\\&-\frac{2\theta}{\alpha^3}\left[\sum_{i=1}^{n(1-s)}\delta_{ui}e^{\alpha t_i}+\sum_{j=1}^{ns}\delta_{aj}e^{\alpha\beta x_j}+e^{\alpha\tau}\{n(1-s)-n_u\}+e^{\alpha\beta\tau}(ns-n_a)\right]\end{aligned}$$

$$\frac{\partial^2 l}{\partial\beta^2}=-\frac{n_a}{\beta^2}-\alpha\theta\sum_{j=1}^{ns}\delta_{aj}x_j^2e^{\alpha\beta x_j}-\alpha\theta\tau^2e^{\alpha\beta\tau}(ns-n_a)$$

$$\begin{aligned}\frac{\partial^2 l}{\partial\theta\partial\alpha}=&\frac{1}{\alpha^2}\left[-n+\sum_{i=1}^{n(1-s)}\delta_{ui}e^{\alpha t_i}+\sum_{j=1}^{ns}\delta_{aj}e^{\alpha\beta x_j}+e^{\alpha\tau}\{n(1-s)-n_u\}+e^{\alpha\beta\tau}(ns-n_a)\right]\\&-\frac{1}{\alpha}\left[\sum_{i=1}^{n(1-s)}\delta_{ui}t_ie^{\alpha t_i}+\beta\sum_{j=1}^{ns}\delta_{aj}x_je^{\alpha\beta x_j}+\tau e^{\alpha\tau}\{n(1-s)-n_u\}+\beta\tau e^{\alpha\beta\tau}(ns-n_a)\right]\end{aligned}$$

$$\frac{\partial^2 l}{\partial\theta\partial\beta}=-\sum_{j=1}^{ns}\delta_{aj}x_je^{\alpha\beta x_j}+\tau e^{\alpha\beta\tau}(ns-n_a)$$

$$\frac{\partial^2 l}{\partial\alpha\partial\beta}=\sum_{j=1}^{ns}\delta_{aj}x_j-\theta\beta\sum_{j=1}^{ns}\delta_{aj}x_j^2e^{\alpha\beta x_j}-\theta\beta\tau^2e^{\alpha\beta\tau}(ns-n_a)$$

The variance covariance and covariance matrix of the parameter can be written as

$$\Sigma=\begin{bmatrix}-\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\beta} & -\frac{\partial^2 l}{\partial\alpha\partial\theta}\\ -\frac{\partial^2 l}{\partial\beta\partial\alpha} & -\frac{\partial^2 l}{\partial\beta^2} & -\frac{\partial^2 l}{\partial\beta\partial\theta}\\ -\frac{\partial^2 l}{\partial\theta\partial\alpha} & -\frac{\partial^2 l}{\partial\theta\partial\beta} & -\frac{\partial^2 l}{\partial\theta^2}\end{bmatrix}^{-1}=\begin{bmatrix}AVar(\hat\alpha) & ACov(\hat\alpha\hat\beta) & ACov(\hat\alpha\hat\theta)\\ ACov(\hat\beta\hat\alpha) & AVar(\hat\beta) & ACov(\hat\beta\hat\theta)\\ ACov(\hat\theta\hat\alpha) & ACov(\hat\theta\hat\beta) & AVar(\hat\theta)\end{bmatrix}$$

4 INTERVAL ESTIMATES FOR MODEL PARAMETER

To construct a confidence interval for a population parameter σ, assume that $L_\sigma=L_\sigma(y_1..........y_n)$ and $U_\sigma=U_\sigma(y_1.............y_n)$ are functions of the sample data $y_1...............y_n$ then a confidence interval for a population parameter σ is given by

$$P[L_\sigma\le\sigma\le U_\sigma]=\xi \tag{12}$$

where the interval $[L_\sigma,U_\sigma]$ is called a two sided $\xi 100\%$ confidence interval for σ. L_σ and U_σ are the lower and upper confidence limits for σ, respectively.

For large sample size, the MLEs, under appropriate regularity conditions, are consistent and asymptotically normally distributed.

Therefore, the two sided approximate $\xi 100\%$ confidence limits for the MLE $\hat\sigma$ of a population parameter σ can be constructed, such that

$$P\left[-z \le \frac{\hat{\sigma}-\sigma}{\phi(\hat{\sigma})} \le z\right] = \xi \tag{13}$$

where z is the $\left[100\left(1-\xi/2\right)\right]^{th}$ standard normal percentile.

Therefore, the two sided approximate $\xi 100\%$ confidence limits for α, β and θ are given respectively as follows

$$L_\alpha = \hat{\alpha} - z\phi(\hat{\alpha}) \qquad U_\alpha = \hat{\alpha} + z\phi(\hat{\alpha})$$
$$L_\beta = \hat{\beta} - z\phi(\hat{\beta}) \qquad U_\beta = \hat{\beta} + z\phi(\hat{\beta})$$
$$L_\theta = \hat{\theta} - z\phi(\hat{\theta}) \qquad U_\theta = \hat{\theta} + z\phi(\hat{\theta})$$

5 OPTIMUM SIMPLEC ONSTANT-STRESS TEST PLAN

In this section the problem of optimally designing a simple constant stress PALT, which terminates after a pre specified time is discussed. Optimum test plan for the items having Gompertz distribution is developed.

Most of the test plans allocate the same number of test units at each stress i.e. they are equally spaced test stresses. Such test plans are usually inefficient for estimating the mean life at design stress, see Yang (16). To decide the optimal sample proportion allocated to each stress, statistically optimum test plans are developed. Therefore, to determine the optimal sample proportion s^* allocated to accelerated condition, s is chosen such that the GAV of the ML estimators of the model parameters is minimized. The GAV of the ML estimators of the model parameters as an optimality criterion is defined as the reciprocal of the determinant of the Fisher-Information matrix F (Bai, Kim and Chun [3]). That is

$$GAV\left(\hat{\theta}, \hat{\alpha}, \hat{\beta}\right) = \frac{1}{|F|}$$

The minimization of the GAV over s solves the following equation

$$\frac{\partial GAV}{\partial s} = 0$$

The solution to the above equation is not in the closed form, so the Newton- Raphson method is applied to determine s^* which minimize the GAV. Accordingly, the corresponding expected optimal numbers of items failed at normal use and accelerated use conditions can be respectively as follows

$n_u^* = n\left(1-s^*\right)P_u$ and $n_a^* = ns^*P_a$

where,

$P_u =$ Probability that an item tested only use condition fails by τ.

$P_a =$ Probability that an item tested only accelerated condition fails by τ

6 SIMULATION STUDY

In order to obtain MLEs of β, α and θ and to study the properties of these estimates through Mean squared errors (MSEs), variance of the estimators and confidence limits for 95% and 99% asymptotic confidence interval, a simulation study is performed. Furthermore, optimum test plans are developed.

For this purpose, several data sets generated from Gompertz distribution under type-I censored data are considered with sample sizes 100, 200, 300, 400 and 500 using 500 replications for each sample size. Under Type I censoring choose a proportion of sample units allocated to accelerated condition to be $s = 30\%$ and censoring time of a PALT to be $\tau = 55$ .The combinations (β, α, θ) of values of the parameters are chosen to be (1.4,0.4,3) and (1.2,0.6,5).Computer programs are prepared and the Newton-Raphson method is used for the practical application of the ML estimators of α, β and θ. Table (1) and Table (3) give the MSE, variance of the estimators and the two sided approximate confidence limits at 95% and 99% level of significance. Tables (2) and (4) represent the results of the test design in which, the optimal sample-proportion s^* allocated to accelerated use condition, the expected fraction failing at each stress, represented by n_u^* and n_a^* and the optimal GAV of the MLEs of the model parameters are obtained numerically for each sample size.

Table 1: Simulation result for the parameters (β, α, θ) set as (1.4, 0.4, 3) respectively, given as s=0.30 and $\tau = 55$ for different sized samples under type-I censoring in constant-stress PALT

n	Parameters $\begin{pmatrix}\hat\beta\\ \hat\alpha\\ \hat\theta\end{pmatrix}$	MSE	Variance	95%		99%	
				LCL	UCL	LCL	UCL
100	1.4921	0.0248	1.9972	1.1429	1.9733	1.1356	1.9334
	0.5193	0.0433	0.9341	0.1933	0.8153	0.1842	0.8615
	3.8653	0.0297	2.6411	2.4549	3.7177	2.4426	3.6961
200	1.4755	0.0212	1.6546	1.2684	1.9454	1.2569	1.9831
	0.5014	0.0398	0.7857	0.2408	0.6842	0.2371	0.6648
	3.6411	0.0172	2.1981	2.5113	3.6883	2.5049	3.6749
300	1.4592	0.0170	1.3427	1.3102	1.8931	1.2994	1.8912
	0.4822	0.1669	0.3942	0.3889	0.5978	0.3313	0.5410
	3.3983	0.0241	1.9428	2.6906	3.5917	2.6769	3.5236
400	1.4204	0.0043	1.2099	1.3863	1.7394	1.2312	1.7661
	0.4185	0.0136	0.2082	0.3862	0.4604	0.3042	0.4018
	3.2864	0.0109	1.1839	2.7694	3.3019	2.7526	3.3924
500	1.4112	0.0015	1.1478	1.3900	1.6876	1.3757	1.7019
	0.4023	0.0049	0.2143	0.3911	0.4395	0.3817	0.4962
	3.1482	0.0068	1.1017	2.8114	3.1198	2.8939	3.2991

Table 2: The results of optimal design of the life test for different sized samples under type-I censoring in constant-stress PALT

n	s^*	n_u^*	n_a^*	Optimal GAV
100	0.3521	38	42	1.0941
200	0.3832	45	115	0.0834
300	0.4645	56	184	0.0210
400	0.5137	73	247	0.0026
500	0.5592	88	312	0.0019

Table 3: Simulation result for the parameters (β, α, θ) set as (1.2, 0.6, 5) respectively, given as s=0.30 and $\tau = 55$ for different sized samples under type-I censoring in constant-stress PALT

n	Parameters $\begin{pmatrix}\hat{\beta}\\ \hat{\alpha}\\ \hat{\theta}\end{pmatrix}$	MSE	Variance	95%		99%	
				LCL	UCL	LCL	UCL
100	1.3162	0.0639	3.5956	0.7998	1.8944	0.5783	1.7603
	0.8673	0.0911	1.8960	0.2674	0.9032	0.2386	0.8172
	7.8018	0.0315	1.8555	3.7793	7.4439	3.4491	6.9927
200	1.3096	0.0389	2.8720	1.0260	1.7503	1.0168	1.6118
	0.7791	0.0617	0.8472	0.4055	0.8541	0.4302	0.8017
	6.5409	0.0276	1.3879	4.6841	6.8617	3.5169	6.8005
300	1.2873	0.0406	1.5612	1.0704	1.5633	1.1027	1.4997
	0.6371	0.0235	0.7395	0.4559	0.8878	0.4399	0.7548
	5.7365	0.0227	0.7258	4.7018	6.5609	4.5918	6.7541
400	1.2415	0.0209	1.3291	1.1712	1.3093	1.1626	1.2963
	0.6079	0.0195	0.2959	0.5874	0.7040	0.5032	0.6817
	5.6671	0.0185	0.3940	4.5084	6.6012	4.7203	6.6019
500	1.2055	0.0186	1.1089	1.1831	1.2994	1.1291	1.3022
	0.5909	0.0122	0.2468	0.5937	0.6991	0.5135	0.6530
	5.2841	0.0096	0.1874	4.9163	6.8278	4.8021	5.9879

Table 4: The results of optimal design of the life test for different sized samples under type-I censoring in constant-stress PALT

n	s^*	n_u^*	n_a^*	Optimal GAV
100	0.3988	36	44	3.0849
200	0.4503	62	88	1.8692
300	0.5844	86	144	0.0672
400	0.5994	98	262	0.0151
500	0.6348	104	316	0.0039

7 Conclusions

This study deals with the problem of estimation and optimally designing simple constant stress PALT for the Gompertz distribution under type-I censored data. From the table (1) and (2), it is observed that that the ML estimates approximate the true values of the parameters as the sample size n increases. Also, we find that, for a fixed α, β and θ the mean squared errors and asymptotic variances of the estimators are decreasing with the increasing value of *n*. It is also noticed that when the sample size increases, the interval of the estimators are decreases.

Tables (2) and (4) present the optimal GAV of the ML estimators of the model parameters which is obtained numerically with *s** in place of *s* for different sized samples. As expected, the optimal GAV decreases as the sample size *n* increases. The minimization of the GAV of the MLEs of model parameters was adopted as an optimality criterion. It may be concluded that the PALT model is an appropriate plan. In practice, the optimum test plans are important for improving the level of precision in parameter estimation and thus improving the quality of the inference. So, statistically, optimum plans are needed, and the experimenters are advised to use it for estimating the life distribution at design stress because it enables us to save time and money in a limited time without necessarily using a high stress to all test units.

As a result , it is right to say that the proposed model work well which helps to save time and money considerably without using a high stress to all test units.

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