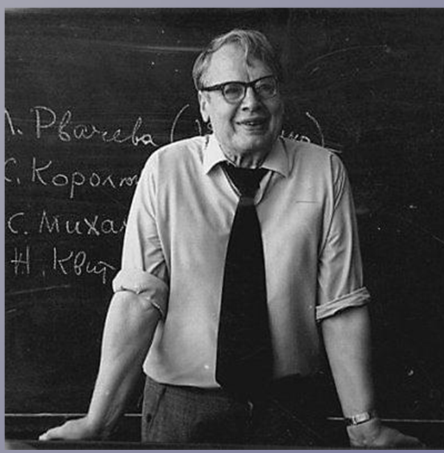
ELECTRONIC JOURNAL
OF INTERNATIONAL GROUP
ON RELIABILITY

Gnedenko Forum Publications



JOURNAL IS REGISTERED
IN THE LIBRARY
OF THE U.S. CONGRESS

•

ISSN 1932-2321

•

VOL.9 NO.3 (34)
SEPTEMBER, 2014

RELIABILITY:
THEORY&APPLICATIONS

RELIABILITY

RISK ANALYSIS

MAINTENANCE

SAFETY

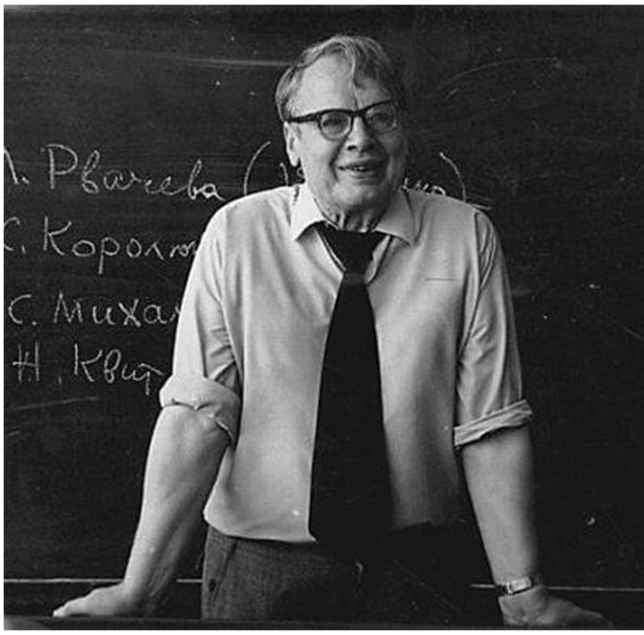
Vol.9
No.3 (34)
•
September
2014

San Diego

ISSN 1932-2321

http://www.gnedenko-forum.org/Journal/index.htm

Journal of International Group on Reliability



RELIABILITY:

THEORY & APPLICATIONS

Vol.9 No.3 (34),
September, 2014

San Diego
2014

Send your paper

e-Journal ***Reliability: Theory & Applications*** publishes papers, reviews, memoirs, and bibliographical materials on Reliability, Quality Control, Safety, Survivability and Maintenance.

Theoretical papers have to contain new problems, finger practical applications and should not be overloaded with clumsy formal solutions.

Priority is given to descriptions of case studies.

General requirements for presented papers

1. Papers have to be presented in English in MSWord format. (Times New Roman, 12 pt , 1.5 intervals).
2. The total volume of the paper (with illustrations) can be up to 15 pages.
3. A presented paper has to be spell-checked.
4. For those whose language is not English, we kindly recommend to use professional linguistic proofs before sending a paper to the journal.

* * *

The Editor has the right to change the paper title and make editorial corrections.

The authors keep all rights and after the publication can use their materials (re-publish it or present at conferences).

Publication in this e-Journal is equal to publication in other International scientific journals.

Papers directed by Members of the Editorial Boards are accepted without referring.

The Editor has the right to change the paper title and make editorial corrections.

The authors keep all rights and after the publication can use their materials (re-publish it or present at conferences).

Send your papers to

the Editor-in-Chief ,
Igor Ushakov
igusha22@gmail.com

or

the Deputy Editor,
Alexander Bochkov
a.bochkov@gmail.com

Table of Contents

The article has been withdrawn by the authors 7
Who is retracting the article: Vicente González Prida
The reason(s) for retraction: David Teira did not consent to the publication

G. Tsitsiashvili, M. Osipova
"COLORING" OF MAP BY FINITE NUMBER OF COLORED POINTS USING FUZZY RECTANGLES 9

In this paper an algorithm of a map "coloring" by a finite number of color points is constructed. This algorithm is based on the procedure of the interval images recognition and on the algorithm of a construction of a fuzzy rectangle. It is significantly simpler and compact than the triangulation procedure using in the mapping.

Dmitry A. Maevsky, Elena J. Maevskaya, Oleg P. Jekov, Ludmila N. Shapa
VERIFICATION OF THE SOFTWARE RELIABILITY MODELS 14

The paper concerns the verification of the existing Software reliability models and their comparison to a new one based on the theory of Software system dynamics. A statistically significant number of observations over the process of fault detecting in the fifty different Software systems has been used for the verification. The results of comparison of estimation correctness of the nine most widely used reliability models to the new one based on the theory of Software system dynamics are represented. It has been proven the Software system dynamics model provided 2,7 times higher correctness of reliability estimation than the existing reliability models.

Mark P. Kaminskiy
CONFIDENCE LIMITS ON PERCENTILES BASED ON TEST RESULTS WITH
A FEW FAILURES – NON-PARAMETRIC VERSUS EXPONENTIAL 23

The non-parametric lower confidence limits on percentiles in the classes of continuous and increasing failure rate distributions are compared to their parametric exponential counterpart for the Type II censored data. In opposite to the common belief that non-parametric estimation procedures are always less effective than analogous parametric procedures, in the considered case, it turns out that the non-parametric procedures provide either better or the same confidence estimates. In a particular case, when the data include only one uncensored observation (failure) and all three estimates (in the classes of continuous distributions, increasing failure rate distributions and the exponential distribution) exist, the respective three lower confidence limits on percentiles coincide.

G. Tsitsiashvili
ASYMPTOTIC OF CYCLY EXISTENCE IN ORIENTED GRAPH WITH LOW RELIABLE EDGES 29

In this paper a power asymptotic of a probability that there is a cycle in a random oriented graph with n nodes and low reliable edges is constructed. An accelerated algorithm for a calculation of asymptotic coefficients with $O(s(n)\ln n)$ products, where $s(n)$ is an amount of products in a multiplication of two matrixes with a size $n \times n$, is constructed.

G. Unnikrishnan, Shrihari, Nihal A. Siddiqui
APPLICATION OF BAYESIAN METHODS TO EVENT TREES WITH CASE STUDIES 32

Event trees are used extensively to analyze accident scenarios in several domains. The tree and its branching structures are used to represent the failure of successive barriers to an initiating event. The end positions of the branches indicate the outcome of progression of each scenario. When probabilities are assigned to success and failure of each barrier the end probabilities can be calculated fairly easily. The logical sequence of the events is clear from the tree structure. However Bayesian networks (BN) are directed acyclic graphs with the nodes indicating events and connected arcs indicating the relationships between the nodes. Initial probabilities are assigned to the parent nodes and conditional probabilities of child nodes are worked out using Bayes Theorem. Bayesian network is a probabilistic modeling technique. Event trees can be mapped into Bayesian networks. Once an event tree is mapped as a Bayesian network, forward (same as in event trees) and backward analysis (possible but involved in event trees) can be performed. Additionally BN has the flexibility for adding causal factors that influence the events. It offers a different perspective of probabilities and better understanding of the incident scenarios. This paper will present mapping of event trees typically found in process industries to Bayesian networks with case studies.

G.Sh. Tsitsiashvili
ORIENTED GRAPHS WITH UNRELIABLE NODES 46

In applications there are networks with unreliable nodes. To construct models of such networks and effective algorithms of their analysis in this paper some elements from the graph theory, the algebra, the sets theory and the probability theory are combined. These constructions allow to investigate oriented graphs with unreliable nodes using as accuracy so asymptotic formulas. All constructed algorithms have a linear complexity by a number of graph nodes.

Farzaliyev Y.Z.
COMPARISON OF WAYS OF NORMALIZATION AT CLASSIFICATION OF INITIAL DATA 50

Normalization at classification of initial data is an indispensable condition of an opportunity of comparison of technical and economic characteristics of the equipment and devices of power supply systems. In work, results of comparison of efficiency of ways of normalization of quantitative estimations of analyzed characteristics used in practice are resulted.

V. M. Chacko
TOTAL TIME ON TEST TRANSFORMS ORDERING OF SEMI-MARKOV SYSTEM.......................................57

First passage time of semi-Markov performance process of a multistate system are considered. TTT (Total time on Test) transform ordering is discussed.

M.Uspensky, I.Kyzrodev
A SOLUTION CONVERGENCE IN A NEURAL NETWORK, AND AN ACCOUNTING
OF LOAD PRIORITIES AT A POWER SYSTEM RESTORATION 64

Uniqueness of solution at a scheme choice for the restorable power system on the artificial neural network (ANN) base is shown. The elementary scheme of a power network is used for this purpose and the subsequent distribution of its results is applied on any network configuration. The way of priority accounting for loads is developed at creation of the restorable power system on the base of the proof on solution uniqueness for ANN.

“COLORING” OF MAP BY FINITE NUMBER OF COLORED POINTS USING FUZZY RECTANGLES

*G. Tsitsiashvili, **M. Osipova
IAM FEB RAS, FEFU,
Vladivostok, Russia,

e-mails: *guram@iam.dvo.ru, mao1975@list.ru

ABSTRACT

In this paper an algorithm of a map “coloring” by a finite number of color points is constructed. This algorithm is based on the procedure of the interval images recognition and on the algorithm of a construction of a fuzzy rectangle. It is significantly simpler and compact than the triangulation procedure using in the mapping.

Keywords: a fuzzy rectangle, an interval images recognition, a polygon triangulation, a classification algorithm, a mapping problem.

1. INTRODUCTION

In this paper an algorithm of a map “coloring” by a finite number of color points is constructed. This algorithm is based on a concept of fuzzy rectangles [1-7]. For any color point surrounding internal and external rectangles are built. The rectangles containing points with same colors coincide or do not intersect. If these points have different colors then their internal rectangles do not intersect and their external rectangles may intersect only if they correspond to points with different colors. But membership functions connected with intersecting rectangles are conformed. Conformation algorithm is based on the interval images recognition procedure [8]. It is much simpler and compact than the triangulation procedure using in the mapping.

2. INTERNAL AND EXTERNAL RECTANGLES

A main idea of [1] in a construction of multidimensional segments (rectangles) is a dividing of recognized class of objects into subclasses. An each subclass is accorded to some external and embedded in it internal multidimensional segments. Segments accorded to different subclasses of a same class do not intersect (or intersect on a set with zero Lebesgue measure). Internal segments accorded to different classes also do not intersect (or intersect on a set with zero Lebesgue measure).

Assume that there is a finite set of vectors on a plane

$$Z_1 = \big((x_{11},\ x_{21}), j_1\big), \dots, Z_n = \big((x_{1n},\ x_{2n}), j_n\big),$$

$$-\infty < x_{ik} < \infty,\ i = 1,2,\ j_k \in \{1, \dots, m\},\ k = 1, \dots, n.$$

If real numbers c, d coincide then the interval $(c, d) = \emptyset$. The coordinates x_{ik} characterize a displacement of the point Z_k on the plane and the coordinate j_k is the number of the class to which this point belong (the color of the point). Assume that the set $\{x_{i1}, \dots, x_{in}\}$, $i = 1,2,$ consists of different numbers. Denote $P_s = \{k\colon\ j_k = s\}$, $Q_s = \{k\colon\ j_k \neq s\}$.

Internal and external one dimensional segments. Fix the index i and put $k \in P_s$, denote

$$a_{ik} = \min\big(x_{it}\colon\ x_{it} \leq x_{ik}\ ,\ \ t \in P_s\ ,\ x_{it} \leq x_{iq} \leq x_{ik} \Longrightarrow q \in P_s\big),$$

$$b_{ik} = \max\left(x_{it}:\ x_{it} \geq x_{ik}\ ,\ \ t \in P_s\ ,\ x_{it} \geq x_{iq} \geq x_{ik} \Longrightarrow q \in P_s\right),\ a_{ik} \leq b_{ik},$$

$$A_{ik} = \max(x_{it}:\ t \in Q_s\ ,\ x_{it} < a_{ik}),\ B_{ik} = \min(x_{it}:\ t \in Q_s\ ,\ x_{it} > b_{ik}),\ A_{ik} \leq B_{ik}\ . \qquad (1)$$

Among the segments which contain the point x_{ik} and do not contain points x_{it}, $t \in Q_s$, the segment $[a_{ik}\ ,b_{ik}]$ is maximal. Call it the internal segment.

Theorem 1. The segments $[a_{ik}\ ,b_{ik}]$, $[a_{ik'}\ ,b_{ik'}]$, $k \neq k'$, coincide or do not intersect.

The number A_{ik} can not be defined by the formula (1) if $x_{ik} = \min(x_{it}: 1 \leq t \leq n)$. Then put $A_{ik} = \min(x_{it}:\ 1 \leq t \leq n) = A_i$. Analogously the number B_{ik} cannot be defined by the formula (1) if $x_{ik} = \max(x_{it}:\ 1 \leq t \leq n)$. Then put $B_{ik} = \max(x_{it}:\ 1 \leq t \leq n) = B_i$.

Among segments which contain the point x_{ik} and do not contain points x_{it}, $t \in Q_s$, the segment $[A_{ik}\ ,B_{ik}]$ is maximal. Call $[A_{ik}\ ,B_{ik}]$ the external segment containing x_{ik} and $[a_{ik}\ ,b_{ik}] \subseteq [A_{ik}\ ,B_{ik}]$.

Theorem 2. If $j_k = j_{k'}$, $k \neq k'$ then the segments $[A_{ik}\ ,B_{ik}]$, $[A_{ik'}\ ,B_{ik'}]$ coincide or have general boundary point. If $j_k \neq j_{k'}$ then these segments cannot coincide but may intersect and each point of this intersection contains no more than in two different segments.

Proofs of Theorems 1, 2 are based on elementary logic-geometric considerations.

Rectangles surrounding dedicated points. Define internal and external rectangles surrounding the point x_{ik} by the equalities

$$[a_{1k}\ ,b_{1k}]\otimes[a_{2k}\ ,b_{2k}],\ [A_{1k}\ ,b_{1k}]\otimes[A_{2k}\ ,B_{2k}].$$

Theorem 3. The rectangles $[a_{1k}\ ,b_{1k}]\otimes[a_{2k}\ ,b_{2k}]$, $[a_{1k'}\ ,b_{1k'}]\otimes[a_{2k'}\ ,b_{2k'}]$, $k \neq k'$, coincide or do not intersect.

Theorem 4. The rectangles $[A_{1k}\ ,B_{1k}]\otimes[A_{2k}\ ,B_{2k}]$, $[A_{1k'}\ ,B_{1k'}]\otimes[A_{2k'}\ ,B_{2k'}]$, $j_k = j_{k'}$, $k \neq k'$ coincide or intersect on a set with zero Lebesgue measure (by pieces of their boundaries).

The statements of Theorems 3, 4 directly follow Theorems 1, 2.

Theorem 5. The rectangles $[A_{1k}\ ,B_{1k}]\otimes[A_{2k}\ ,B_{2k}]$, $[A_{1k'}\ ,B_{1k'}]\otimes[A_{2k'}\ ,B_{2k'}]$, $j_k \neq j_{k'}$, $k \neq k'$, cannot coincide but may intersect. An each point of such intersections may belong no more than to two different rectangles.

Proof. Fix k and put $s = j_k$. By a definition in the sets

$$[a_{1k}\ ,b_{1k}]\otimes[a_{2k}\ ,b_{2k}],\ [A_1\ ,A_{1k}]\otimes[a_{2k}\ ,b_{2k}],\ [B_{1k}\ ,B_1]\otimes[a_{2k}\ ,b_{2k}],$$
$$[a_{1k}\ ,b_{1k}]\otimes[A_2\ ,A_{2k}],\ [a_{1k}\ ,b_{1k}]\otimes[B_{2k}\ ,B_2]$$

only points $X_t = (x_{1t}, x_{2t})$ satisfying the equality $j_t = s$, may contain. In the sets

$$(A_{1k}, a_{1k}) \otimes\ [A_2, B_2],\ (b_{1k}, B_{1k}) \otimes\ [A_2, B_2],\ [A_1, B_1] \otimes (A_{2k}, a_{2k}),\ [A_1, B_1] \otimes (b_{2k}, B_{2k})$$

there are not points X_t, $1 \leq t \leq n$. Consequently the rectangles

$$[A_{1k}\ ,a_{1k}]\otimes[a_{2k}\ ,b_{2k}],\ [b_{1k}\ ,B_{1k}]\otimes[a_{2k}\ ,b_{2k}],\ [a_{1k}\ ,b_{1k}]\otimes[A_{2k}\ ,a_{2k}],\ [a_{1k}\ ,b_{1k}]\otimes[b_{2k}\ ,B_{2k}]$$

contain in a single external rectangle $[A_{1k}\ ,B_{1k}]\otimes[A_{2k}\ ,B_{2k}]$.

Define the sets

$$C_{1k}^- = [A_1\ ,A_{1k}]\otimes\{A_{2k}\},\ C_{1k}^+ = [B_{1k}\ ,B_1]\otimes\{A_{2k}\},\ C_{2k}^- = [A_1\ ,A_{1k}]\otimes\{B_{2k}\},$$
$$C_{2k}^+ = [B_{1k}\ ,B_1]\otimes\{B_{2k}\},\ \ C_{3k}^- = \{A_{1k}\}\otimes[A_2\ ,A_{2k}],\ C_{3k}^+ = \{A_{1k}\}\otimes[B_{2k}\ ,B_2],$$
$$C_{4k}^- = \{B_{1k}\}\otimes[A_2\ ,A_{2k}],\ C_{4k}^+ = \{B_{1k}\}\otimes[B_{2k}\ ,B_2],\ \ C_{pk} = C_{pk}^+ \cup C_{pk}^-,\ \text{p=1,…,4}.$$

In any set $C_{pk}, p = 1, \dots, 4$ there is only single point $X_{t_{pk}}$ from the set $\{X_1, \dots, X_n\}$. And all these points (some of them may coincide) satisfy inequalities $j_{t_{pk}} \neq s$. Construct now rectangles

$$R_{1k} = [A_1, a_{1k}]\otimes[A_2, a_{2k}],\ L_{1k} = [A_1, A_{1k}]\otimes[A_2, A_{2k}],\ R_{2k} = [A_1, a_{1k}]\otimes[b_{2k}, B_2],$$

$$L_{2k} = [A_1, A_{1k}]\otimes[B_{2k}, B_2],\ R_{3k} = [b_{1k}, B_1]\otimes[A_2, a_{2k}],\ L_{3k} = [B_{1k}, B_1]\otimes[A_2, A_{2k}],$$

$$R_{4k} = [b_{1k}, B_1]\otimes[b_{2k}, B_2],\ L_{4k} = [B_{1k}, B_1]\otimes[B_{2k}, B_2],\ L_{pk} \subseteq R_{pk},\ p=1,\dots,4,$$

$$S_{1k} = [A_{1k}, a_{1k}]\otimes[A_{2k}, a_{2k}],\ S_{2k} = [A_{1k}, a_{1k}]\otimes[b_{2k}, B_{2k}],$$

$$S_{3k} = [b_{1k}, B_{1k}]\otimes[A_{2k}, a_{2k}],\ S_{4k} = [b_{1k}, B_{1k}]\otimes[b_{2k}, B_{2k}],$$

By the definition of the external rectangle we obtain that for any point $X_t \in L_{pk}$, $j_t \neq s$, the inclusion $[A_{1t}, B_{1t}]\otimes[A_{2t}, B_{2t}] \subseteq R_{pk}$ is true. And the external rectangle $[A_{1t}, B_{1t}]\otimes[A_{2t}, B_{2t}]$ which has some internal point of the rectangle S_{pk} contains S_{pk} completely.

Prove now that internal points of the rectangle S_{pk} may belong besides of $[A_{1k}, B_{1k}]\otimes[A_{2k}, B_{2k}]$ to no more than single another external rectangle $[A_{1t}, B_{1t}]\otimes[A_{2t}, B_{2t}]$, $j_t \neq s$. Consider the case $p = 1, k = 1$ because in all other cases this statement may be verified similar.

If for all t so that $X_t \in L_{1k}$, $j_t \neq s$, we have that all j_t coincide. then last statement is obvious. Assume now that $X_t \in L_{1k}$, $j_t \neq s$, $S_{1k} \subseteq [A_{1t}, B_{1t}]\otimes[A_{2t}, B_{2t}]$ and there is $X_{t'} \in L_{1k}$, so that $j_{t'} \neq s$, $j_{t'} \neq j_t$. Then it is clear that $[A_{1t'}, B_{1t'}]\otimes[A_{2t'}, B_{2t'}] \cap S_{1k} = \emptyset$ because $x_{t'1} < x_{t1}$, $x_{t'2} < x_{t2}$ and $(A_{1,k}, A_{2,k}) \in [A_{1t}, B_{1t}]\otimes[A_{2t}, B_{2t}]$. Similar statements may be proved for internal points of the rectangles

$$[A_{1,k}, a_{1,k}]\otimes[b_{2,k}, B_{2,k}],\qquad [b_{1,k}, B_{1k}]\otimes[A_{2,k}, a_{2,k}],\ [b_{1,k}, B_{1k}]\otimes[b_{2,k}, B_{2,k}].$$

So Theorem 5 is proved.

3. CONSTRUCTION OF FUZZY SET FOR POINTS WITH IDENTICAL COLOR

Without a restriction of a generality suppose that there are numbers $0 = J_0 < J_1 < J_2 < \dots < J_m = n$ so that $P_s = \{k: J_{s-1} < k \leq J_s\}$, $1 \leq s \leq m$. From Theorems 3, 4 for fixed s the set of indexes $\{k \in P_s\}$ is divided into equivalence classes with elements which belong to coincident internal and external rectangles.

Consider the case $s = 1$ and suppose that appropriate equivalence classes are indexes sets $\{1, \dots, k_1\}$, $\{k_1 + 1, \dots, k_2\}$,…, $\{k_{l-1} + 1, \dots, k_l = J_1\}$. Denote $\gamma_{iq} = a_{ik_q}$, $\Gamma_{i,q} = A_{ik_q}$, $\delta_{iq} = b_{ik_q}$, $\Delta_{iq} = B_{ik_q}$. It is clear that $\Gamma_{iq} \leq \gamma_{iq} \leq \delta_{iq} \leq \Delta_{iq}$, $i = 1,2$, $1 \leq q \leq l$.

For fixed q, $1 \leq q \leq l$, define the function $\mu_q(X)$, $X \in E^2$, by conditions:

a) $X \in \mathcal{B}_q$, $\mathcal{B}_q = [\gamma_{1q}, \delta_{1q}]\otimes[\gamma_{2q}, \delta_{2q}] \Rightarrow \mu_q(X) = 1$,

b) $X \notin \mathcal{A}_q$, $\mathcal{A}_q = [\Gamma_{1q}, \Delta_{1q}]\otimes[\Gamma_{2q}, \Delta_{2q}] \Rightarrow \mu_q(X) = 0$,

c) assume that for $0 \leq \lambda \leq 1$ the inclusion $X = (x_1, x_2) \in \partial G_\lambda$ is true where ∂G_λ is the set

$$G_\lambda = \otimes_{i=1}^{2} [\gamma_{iq} + \lambda(\Gamma_{iq} - \gamma_{iq}),\ \delta_{iq} + \lambda(\Delta_{iq} - \delta_{iq})],$$

boundary then $\mu_q(X) = 1 - \lambda$, and so for $X \in \mathcal{A}_q \backslash \mathcal{B}_q = \bigcup_{0 \leq \lambda \leq 1} \partial G_\lambda$

$$\mu_q(X) = 1 - \max_{1 \leq i \leq 2} \max\left[\frac{x_i - \gamma_{iq}}{\Gamma_{iq} - \gamma_{iq}}, \frac{x_i - \delta_{iq}}{\Delta_{iq} - \delta_{iq}}\right]. \qquad (2)$$

From the equality (2) obtain

$$\mu_q(X) \leq 1 - \frac{x_i - \gamma_{iq}}{\Gamma_{iq} - \gamma_{iq}},\ \Gamma_{iq} \leq x_i \leq \gamma_{iq}\ ;\ \mu_q(X) \leq 1 - \frac{x_i - \delta_{iq}}{\Delta_{iq} - \delta_{iq}},\ \delta_{iq} \leq x_i \leq \Delta_{iq}. \quad (3)$$

Define now the fuzzy set which denotes an inclusion of the point $X = (x_1, x_2)$ into one of constructed external rectangles $[\Gamma_{1q}, \Delta_{1q}] \otimes [\Gamma_{2q}, \Delta_{2q}]$, $1 \leq q \leq m$ [2]. As these external rectangles intersect only on pieces of their boundaries where appropriate functions equal zero then it is possible to define the membership function of this fuzzy set by the equality $\mu(X) = \sum_{q=1}^{l} \mu_q(X)$.

4. CONSTRUCTION OF FUZZY SETS FOR POINTS WITH DIFFERENT COLORS USING MAP BACKGROUND

For any s, $s = 1, \dots, m$, we constructed a fuzzy set with a membership function $\mu^s(X)$ of the point X .

Theorem 6. The following inequality

$$\sum_{s=1}^{m} \mu^s(X) \leq 1\ ,\ X \in E^2, \quad (4)$$

takes place.

Proof. To prove the inequality (4) it is necessary to use Theorem 5 with its designations and proof and to estimate the function $\sum_{s=1}^{m} \mu^s(X)$ for $X \in S_{1k}$. In this case

$$\sum_{s=1}^{m} \mu^s(X) = \mu^s(X) + \mu^{j_t}(X)$$

where from the formula (3) we have

$$\mu^s(X) \leq 1 - \frac{x_1 - a_{1k}}{A_{1k} - a_{1k}},\ \mu^{j_t}(X) \leq 1 - \frac{x_1 - A_{1k}}{a_{1k} - A_{1k}}.$$

So the inequality (4) is true. Theorem 6 is proved.

At the end of this considerations denote $\mu^0(X) = 1 - \sum_{s=1}^{m} \mu^s(X)$ and call this nonnegative difference the membership function of the fuzzy set which describes a background of the map. Consequently on a base of an information about the finite set of colored points on a plane we construct fuzzy sets which define coloring of the map.

REFERENCES

1. Zadeh L.A.1965. Fuzzy sets. Information and Control. Vol. 8, no. 3. P. 338-353.
2. Kaufmann A. 1973. Introduction a la theorie des sous-ensembles flous. Paris. Masson.
3. Robbins H. E. 1944/45. On the measure of a random set. Ann. Math. Statist. Vol. 15/16. P. 70-74/342-347.
4. Ambartzumian R.V., Mecke J., Shtoyan D. 1989. Introduction to stochastic geometry. Moscow: Nauka. (In Russian).
5. Chakraborty D., Ghosh D. 2014. Analytical fuzzy plane geometry II. Fuzzy Sets and Systems. Vol. 243. P. 84–109.
6. Efremov A.A. 2013. New operations with fuzzy numbers and intervals. Dokladi TUSURa. Vol. 1 (27). P. 95-99. (In Russian).
7. Tsitsiashvili G.Sh. 2013. Image recognition by multidimensional intervals. Reliability: Theory and Applications. Vol. 8, no. 9. P. 21-23.

8. Tsitsiashvili G.Sh., Kharchenko Yu.N., Losev A.S. and Osipova M.A. 2014. Interval images recognition and fuzzy sets. International Mathematical Forum. Vol. 9, no. 19, 917 - 921.
9. Bolotin E.I., Tsitsiashvili G.Sh. 2003. Spatial-temporal prognostication in functioning of center of mite encephalitis infection. Bulletin of FEB RAS. 2003. Vol. 1. P. 5-19. (In Russian).
10. Tsitsiashvili G.Sh., Shatilina T.A., Kulik V.V., Nikitina M.A., Golycheva I.V. 2002. Modification of method of interval mathematics applicable to the forecast of extreme ice cover in the Okhotsk Sea. Bulletin of FEB RAS. Vol. 4. P. 111-118. (In Russian)

VERIFICATION OF THE SOFTWARE RELIABILITY MODELS

Dmitry A. Maevsky, Elena J. Maevskaya, Oleg P. Jekov, Ludmila N. Shapa
•
Odessa National Polytechnic University, Odessa, Ukraine
e-mail: Dmitry.A.Maevsky@gmail.com

ABSTRACT

The paper concerns the verification of the existing Software reliability models and their comparison to a new one based on the theory of Software system dynamics. A statistically significant number of observations over the process of fault detecting in the fifty different Software systems has been used for the verification. The results of comparison of estimation correctness of the nine most widely used reliability models to the new one based on the theory of Software system dynamics are represented. It has been proven the Software system dynamics model provided 2,7 times higher correctness of reliability estimation than the existing reliability models.

1 INTRODUCTION

The problem of providing and forecasting the Software reliability of informational systems is one of the most up to date in the modern program engineering. Nowadays the cost of program failure can be measured not only in million dollars but also in million human lives. Since the modern informational technologies have extensively entered all spheres of our life the majority of mankind becomes in a sense a hostage of its own creature – Software systems. These systems are relied on a considerable amount of functions of control in traffic, communications, power, economy, defense and other areas control. In order to immobilize a big city activity a failure in a system of control dealing with the work of traffic lights in its main transport lines is enough. The consequences of computer errors in mobile communications and power systems are much more dramatic. And a nuclear reactor management system failure at the nearby power station can be catastrophic for all the continent.

So the creation of the reliable computer systems and further keeping their reliability during operation is of vital importance. A new and comparatively young trend in the reliability theory – Software reliability (SR) – deals with estimation and forecasting the Software system reliability. Its task is to develop the theoretical base of software reliability as well as models, methods and practical technology for Software resources reliability determination.

2 THE PROBLEM NOWADAYS

At present there exist about twenty different Software reliability models (SRM). Such an abundance conditions the necessity to classify them, and now we have got several schemes of classification. The most popular one has been proposed by J.D.Musa and Okumoto [1]. It distinguishes such characteristics as:

— Model time. It determines a time counting system applied to the Model — either actual astronomic (calendar) time or processor time, spent on the work with the given Software by the moment of fault detection.

— Model category. It determines the amount of faults which can be detected when investigation time is infinite. According to this characteristic all models are classified into finite and infinite.

— Model type. It determines the probability distribution of random event occurrence, fault detection in our case. Two types of distribution are used in the models of reliability: the Poisson distribution and binomial distribution.

— Model class. This characteristic is only used for finite category models and determines a type of function describing the law of intensity change of fault appearing.

— Family. This characteristic is only used for infinite models and possesses the same meaning as the characteristic "class" for finite ones.

In the above classification a particular attention should be focused on such a characteristic as "model time" because it is a principal factor. First of all a time counting system is different for an individual analyzing Software systems and for an analyzed Software itself. A human lives in his (her) own time counting system breaking the stream of time into habitual time intervals – years, months, days, hours, etc. For a human being the time is uninterrupted. From the viewpoint of a Software system – if we are trying to imagine ourselves a program system – all events happen in absolutely another way. Assume a researcher detected and eliminated a program fault in the evening of November, 25, 2011 at 20 o'clock according to the local time. We see it is a natural way of time counting for a human. Assume the 25th of November is Friday and after fault detection and elimination the computer has been off over Saturday and Sunday. So the analyzed system was run and started operating only at 8 o'clock in the morning on Monday. The next fault was detected by the same researcher at 9 o'clock in the morning on Monday, November, 28. What is the time interval between these two sequential detections? From the human viewpoint it is 61 hours. And what is it for a Software system (SS)? While the computer was off the Software system was not downloaded in the memory and executed. It is possible to say that over sixty hours the system did not exist in general and all the processes in it were stopped! It "immersed itself in suspended animation" at 20 o'clock on Friday and "raised again" at 8 o'clock on Monday morning. So from the viewpoint of the system (you understand here we are trying to make the system "human" and suppose that it is able to have its own opinion) the period of time between two fault detections is an hour. We have proven that SS time was discontinuous and consisted of separate intervals during which it was active.

We can ask the question: what time is more correct in estimating the hazard rate? Certainly, the system operation time, i.e. the processor time. But when using for astronomical time modeling, the cumulative curve has gaps on the axis of abscess. The data of the axis of abscess (time) have changed, and the ones of the ordinate axis (cumulative fault number) have not. It makes a false impression of dissimilar fault detection rate which distorts the modeling results and decreases the prediction ability of the model. It can be clearly seen in the diagram represented in Fig.1.

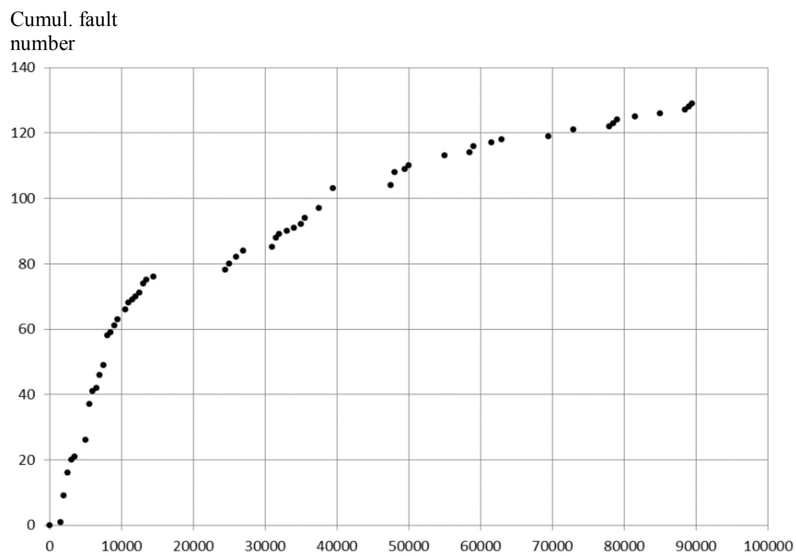


Fig. 1. The cumulative curve gaps in using the astronomical time

The data for the diagram have been taken from the appendix for [14] (Chapter 4, file Csr2.dat). Along the axis of abscess the moments of time are represented (the scale is not mentioned in the literature), along the ordinate axis – the cumulative fault number.

In the diagram we can see several gaps of mentioned type. For example, take the beginning site of the curve before the first gap. According to the diagram the initial point of the curve has got the coordinates (0, 0), а конечная – (14500, 76). Thus for 14500 time units 76 faults are detected, the average rate of detection is $\frac{76}{14500} = 0{,}0052$ fault per time unit. Then the curve demonstrates another distinct gap – the point has got the coordinates (24500, 78), i.e. for the next 10000 time units only two faults are detected. The fault detection rate in this site equals $\frac{2}{10000} = 0{,}0002$ fault per time unit, i.e. 26 times less than in the previous one! As we see in the diagram the fault detection rate is abruptly restored to its original value in the next third site again. It is obvious that the jumps of detection rate of such kind cannot be explained by anything but the ordinary interruptions in observation.

If the processor time were used the mentioned gap would not appear in spite of the interruption conditioned by some subjective factors. It is necessary to note though the instruction provides the astronomical time application in the model, the processor time can be used as well. And all the mathematical formula and equations remains the same.

In order to estimate the correctness of reliability index modeling the examination of the most popular models belonging to different branches of the described classification has been carried out. Let us consider the main characteristics of these models.

1. Jelinski-Moranda's Model [2]. Time – astronomical; category – finite; type – binomial; class – exponential. Assumptions: failure intensity is proportional to the actual fault number in the program and remains constant in the time interval between any two neighboring moments of fault detecting; detection of all the faults in the program is equiprobable and independent; all the faults have got similar degree of importance; time until the detection of the next Software fault is distributed exponentially.

2. Goel-Okumoto's Model [3]. Time – astronomical; category – finite; type – Poisson; class – exponential. Assumptions: all the SS faults are mutually independent; the detected faults are eliminated immediately; the fault detection process is a stream of homogeneous events and has got the Poisson distribution.

3. Schneidewind's Model [4]. Time – astronomical; category – finite; type – Poisson; class – exponential. The main distinctive peculiarity of this model – the failure intensity determining in the later time is supposed to be more correct for prediction of the further process development than the one measured at the earlier stages. Assumptions: the fault number in the given time interval is independent on the fault number in the other intervals; the detected fault number decreases from interval to interval; the failure intensity is proportional to the fault number detecting at that exact moment.

4. Musa's Model [5]. Time –processor; category – finite; type – Poisson; class – exponential. Assumptions: the fault detection process is the Poisson process; the fault detection is proportional to the number of faults which were not detected yet.

5. Weibull's Model [6]. Time – processor; category – finite; type –binomial; class – exponential. Assumptions: at the initial moment of observation there is a finite number of faults in SS; time before the fault detection is a stochastic value having probability subjected to Weibull distribution.

6. S-form Model [7]. Time – processor; category – finite; type – Poisson; class – gamma distribution. Assumptions: the fault detection process is the Poisson process; in detecting a fault it is immediately eliminated without entering new ones.

7. Duan's Model [8]. Time – astronomical; category – infinite; type – Poisson; family – gamma distribution. Assumptions: the cumulative fault detection is the Poisson process with the function of distribution $\mu(t) = \alpha \cdot t^{\beta}$, where α and β – are positive numbers.

8. Moranda's Geometrical Model [9]. Time – astronomical; category – infinite; type – Poisson; family – exponential. Assumption: the failure intensity is a geometric progression $\lambda(t) = D \cdot \phi^{i-1}$ with denominator $0 < \phi < 1$; the probability of detection of every certain fault is subjected to the exponential distribution law.

9. Musa-Okumoto's Logarithmic Model [10]. Time – astronomical; category – infinite; type – Poisson; family – exponential. Assumption: the failure intensity is decreased over time according to the exponential law; the fault detection process is the Poisson process.

10. Software System Dynamics (SSD) Model. The SSD theory fundamentals have been developed in [11] and [12] as an absolutely new deterministic approach to formulating the reliability parameters taking into account the secondary fault influence. SSD is different from the existing Software reliability theory because it is not based on the probability theory but on the non-equilibrium process theory, and it does not consider the fault appearing in Software system as an occasional event but as a result of deterministic fault flow impact.

SSD is based on the following assumptions:

1. SS is an open non-equilibrium system that interacts with its subject area ac-cording to the laws of the non-equilibrium processes. This is a new point of view on the program system. It is assumed that the properties of a software system are similar ones of other open systems.
2. The state of the SS is characterized by a special state function f(t) – the number of the defects containing in it. Here it means the number of primary or secondary defects.
3. Disappearing and appearing the defects in the SS is the result of the joint action of the direct (outcoming) and reverse (incoming) defect flows. It is implied that the primary defects are removed from the system by the direct flow and secondary defects are appeared in the system as a result of the reverse flow.
4. The intensity of each flow is proportional to the number of defects that this flow forms. This is a basic principle of the non-equilibrium processes theory. For a software system, this principle means that the reduction the number of defects causes the decrease of their detection rate.
5. All defects are equivalent, and participate in the formation of the flow in the same way, regardless of the causes, location, and type of defect (the principle of equivalence).
6. Function f(t) is differentiable on the whole domain (the principle of continuity).

The basic concept SSD is the one of software defect flows. Each defect is considered as an integral part of the total flow, which obeys not the laws of the probability theory but the laws of identification and evolution of flows in non-equilibrium systems. The identification of the defect flows in the SS is shown in Fig. 2.

In the SS operation defects are the causes that the result which is produced by SS does not correspond to the result expected by the subject area. This discrepancy is detected by the user which is in contact with the SS on the one hand and with its subject area on the other. Thus, firstly the user acts as an error detector, and secondly – a kind of "contact surface" between the SS and its subject area. We assume that the user is ideal, that is, he detects and records each defect at the time of its identification.

In the process of correcting the defect disappears from the SS due to changes made in its code. This disappearance can be supposed as a result of the of defects removal from the SS. Considering this process in time, we obtain the flow of defects from the SS through the "contact surface", i.e. the user. This flow is shown by arrows "Detection" and "Correction" in Fig. 2.

It is possible to insert additional "secondary" defects in the process of correcting defects in the SS. The process of inserting the secondary defect may be regarded as the second, counter-flow of defects, which operates in the direction from the subject area to the SS.

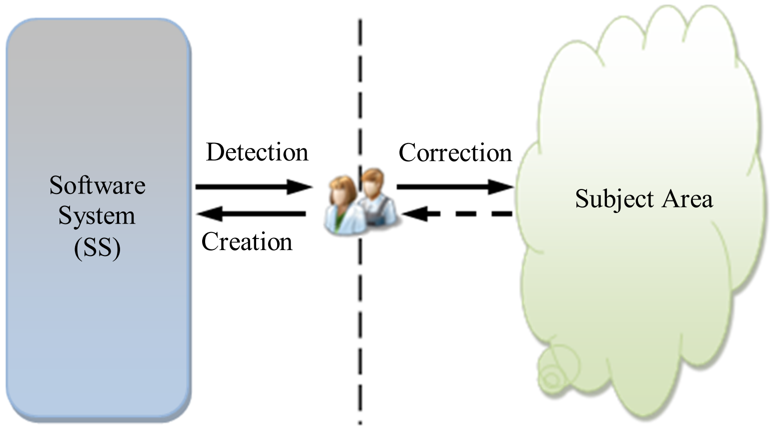


Fig. 2. Defect emergence in the SS

We will numerically characterize the flow of defects by the rate (intensity) of the flow, which can be determined by hypothesis 6 (principle of continuity). Taking into account the outcoming flow only, SS is characterized by the number of defects, which are contained in the system – coordinate f1(t). The defects leave the system for the subject area. It has just one degree of freedom, and is described by the differential equation of first order. In the case of taking into account of the second process (insertion of secondary defects), its coordinate is their current number – f2(t). Thus we obtain two coordinates – f1(t) and f2(t). SS in this case is a system with two degrees of freedom and described by differential equations of second order.

SSD is describes by the following autonomous system of differential equations:

$$\begin{cases} \frac{df_1}{dt} = -A_1 \cdot f_1 + A_2 \cdot f_2 \\ \frac{df_2}{dt} = A_2 \cdot f_1 - A_1 \cdot f_2 \end{cases}$$

She's solution allows to determine the time variation of the primary and secondary defects existing in SS:

$$f_1 = F_0 \cdot e^{-A_1 t} \cdot cosh(A_2 t)$$
$$f_2 = F_0 \cdot e^{-A_1 t} \cdot sinh(A_2 t)$$

The presented mathematical equations give the opportunity to estimate the number of the primary and secondary faults in the Software system and carry out a comparative analysis of correctness of the described reliability models.

3 THE RESULTS OF INVESTIGATION

As the initial data for the described reliability model verification we have used the data of the fault detection process analyzed in fifty different-type Software system. All of them were scripted in different computer languages and have got different functions. The information about these systems are given in Table 1. All the used series are inhomogeneous: because of the changes making in the system the law of fault detection curve changes is different over time. That is why in order to increase the correctness each of the series have been divided into sites with the help of the unchangeable law of fault detection.

Table 1

№	Data Sources	Information	Number of points	Numb er of inter vals
1	[13]	OS «Android», version 2.3	765	
2	[14], Chapter 4, file Csr1.dat	No data	397	
3	[14], Chapter 4, file Csr2.dat	No data	129	
4	[14], Chapter 4, file Csr3.dat	No data	104	
5	[14], Chapter 4, file SS3.dat	No data	278	
6	[14], Chapter 4, file Sys1.dat	No data	136	
7	[14], Chapter 7, file Sys1.dat	No data	136	
8	[14], Chapter 7, file Sys2.dat	No data	86	
9	[14], Chapter 7, file Sys3.dat	No data	207	
10	[14], Chapter 7, file J1.dat	No data	62	
11	[14], Chapter 7, file J2.dat	No data	181	
12	[14], Chapter 7, file J3.dat	No data	41	
13	[14], Chapter 7, file J4.dat	No data	114	
14	[14], Chapter 7, file J5.dat	No data	73	
15	[14], Chapter 8, file 8.txt	Multiprocessor System	186	
16	[14], Chapter 9, file Odc1.dat	Large IBM Project	1207	
17	[14], Chapter 9, file Odc3.dat	No data	400	
18	[14], Chapter 10, file S2.dat	No data	54	
19	[14], Chapter 10, file S27.dat	No data	41	
20	[14], Chapter 10, file SS4.dat	No data	197	
21	[14], Chapter 10, file SS1.dat	The Language of Assembler	81	
22	https://github.com /AArnott/dotnetopenid	C#	55	5
23	https://github.com /activescaffold/active_scaffold	Ruby	97	7
24	https://github.com /adamzap/landslide	Python	89	9
25	https://github.com /addyosmani/backbone-fundamentals	JavaScript	25	3
26	https://github.com /AFNetworking/AFNetworking	Objective-C	155	12
27	https://github.com /ai/r18n	Ruby	36	5
28	https://github.com /akzhan/jwysiwyg	JavaScript	194	18
29	https://github.com /alankligman/gladius	JavaScript	43	6
30	https://github.com /AlanQuatermain/AQGridView	Objective-C	88	11
31	https://github.com /alecgorge/jsonapi	Java	111	15
32	https://github.com /alohaeditor/Aloha-Editor	JavaScript	391	27
33	https://github.com /amatsuda/kaminari	Ruby	191	11
34	https://github.com /andreasgal/B2G	Rust	136	13
35	https://github.com /andreasronge/neo4j	Ruby	112	14
36	https://github.com /andrewplummer/Sugar	JavaScript	88	15
37	https://github.com /andris9/Nodemailer	JavaScript	50	6
38	https://github.com /andymatuschak/Sparkle	Objective-C	132	5
39	https://github.com /antirez/hiredis	C	70	11
40	https://github.com /apneadiving/Google-Maps-for-Rails	Ruby	138	12
41	https://github.com /Araq/Nimrod	Nimrod	91	10
42	https://github.com /arsduo/koala	Ruby	160	19
43	https://github.com /asual/jquery-address	C#	126	18
44	https://github.com /away3d/away3d-core-fp11	JavaScript	195	28
45	https://github.com /bartaz/impress.js	Java	65	12
46	https://github.com /BaseXdb/basex	JavaScript	271	23
47	https://github.com /Baystation12/Baystation12	No data	302	39
48	https://github.com /bbatsov/ruby-style-guide	Ruby	73	12
49	https://github.com /benbarnett/jQuery-Animate-Enhanced	JavaScript	67	14
50	https://github.com /bengottlieb/Twitter-OAuth-iPhone	Objective-C	102	17

We have counted 522 intervals with the same law of change of detected fault amount over time. Along all these intervals for the ten analyzed models both 5220 estimations of reliability are made and their correctness determined. The correctness is obtained according to standard deviation (SD) criterion observed and calculated with the help of the fault value model and SD value dispersion for different Software systems. SD values are calculated on the formula:

$$SD = \frac{\sum_{i=1}^{n}\left(f_{io} - f_{ic}\right)^2}{n}$$

where n is the number of points in series, f_{io} is the observed value and f_{ic} is the calculated value.
The results of comparing the reliability estimation correctness by different models are represented in Table 2. Besides this Table demonstrates the number of series (in percentage to the total number of series) unprocessed by each of the model, minimum and maximum SD values obtained for a model, the average value according to the model as well as the logarithm for dispersion. The analysis of the Table has shown that only two of the investigated nine models – SSD and S-form model – are able to carry out the reliability estimation for all of 522 time intervals.

Table2. The Results of Verification

Model	SSD	Jel.-Mor	NHPP	Schneiderw.	Musa	Weib.	S-form.	Duan	Moranda's Geom	Musa-Okum.
Unprocessed %	0,00	54,2	15,9	36,8	55,6	0,96	0,00	28,2	9,4	70,9
Min. SD	0,00	0,23	0,02	0,00	0,01	0,00	0,00	0,09	0,14	0,00
Max. SD	54,5	379	800	928	246	559,7	245,9	996	556,3	417,8
Aver. SD	1,5	12,1	18,7	26,5	4,1	5,3	4,5	25,3	7,8	10,3
$\lg\left(\sigma^2\right)$	3,37	7,59	8,47	8,75	5,91	6,86	5,75	9,09	7,27	7,72

According to this index the worst is Musa-Okumoto's Logarithmic Model that was not able to process almost 71 % intervals. According to the SD and dispersion value the best turns out SSD Model. It shows 2,7 times more correct results than Musa's Model which takes the second place as to the correctness. According to the dispersion value SSD Model demonstrates two and more orders of magnitude less than the value of the other analyzed models. The results of verification are represented graphically in Fig. 3.

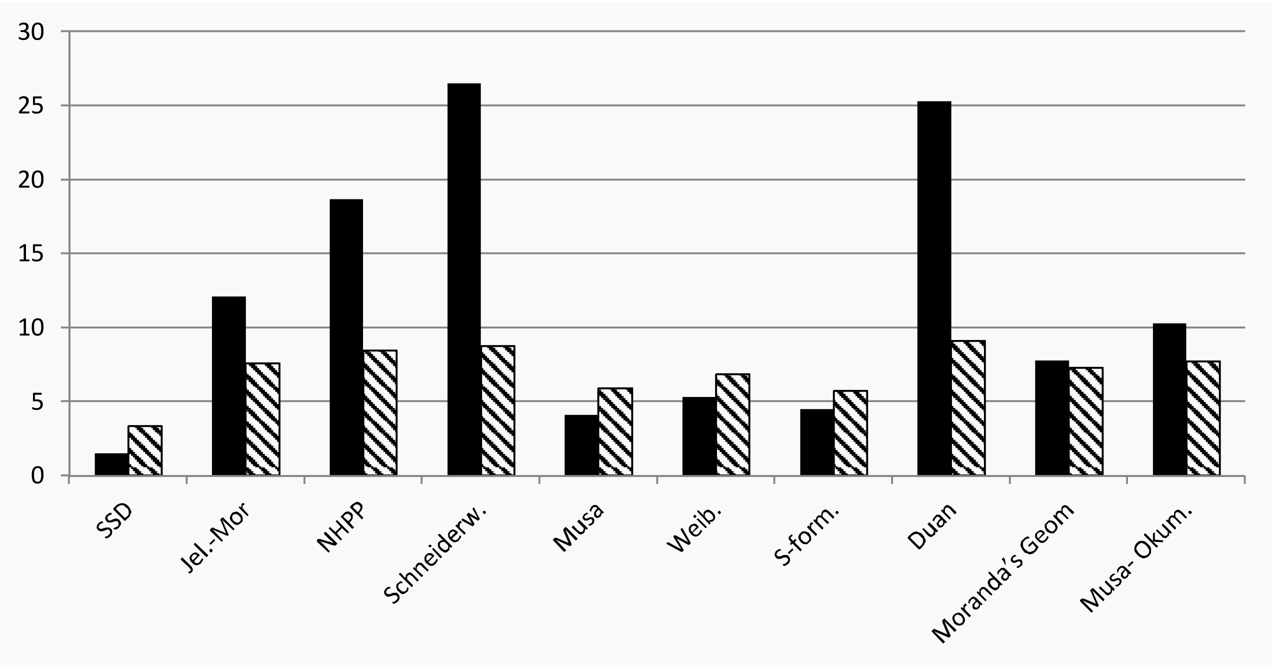


Fig. 3. The results of verification

In this diagram the solid part shows the SD values, and the hatch – the dispersion value logarithms. We have used the logarithmic scale for dispersion because its absolute values vary in orders from model to model.

4 CONCLUSIONS

Thus the verification results show that nowadays SSD Model is the best as to the correctness of reliability estimation by a model. The low dispersion values demonstrate the fact that SSD Model constantly shows the most correct results in all the analyzed Software and can be considered as a universal model. Besides a doubtless advantage of SSD Model lies in its capability to predict the appearing of secondary faults in a Software system. At present SSD is the only model which can not only take into account the secondary fault impact but also foresee their number.

5 REFERENCES

1. Musa, J. D. Software Reliability Models: Con-cepts, Classification, Comparisons, and Practice / J. D. Musa, K. Okumoto // Electronic Systems Effectiveness and Life Cycle Costing, NATO ASI Series, F3, Springer-Verlag, Heidelberg, p. 395 – 424.
2. Moranda P.B. Final Report of Software Relia-bility Study. — / P.B. Moranda, J. Jelinski // McDonnel Douglas Astronautics Company. MDC Report № 63921, dec. – 1972. – 51 c.
3. Goel, A.L., Time-Dependent Error-Detection Rate Model for Software and Other Performance Measures / A.L. Goel, K. Okumoto // IEEE Transactions on Reliability, v. R-28, № 5, August. – 1979. – P. 206 – 211.
4. Schneidewind, N.F. Software Reliability Model with Optimal Selection of Failure Data / N.F. Schneidewind //IEEE Transactions on Software Engi-neering. – Vol. 19. – No. 11. Nov. – 1993. P. 1095 – 1104
5. Musa J.D. Validity of Execution time theory of software reliability // IEEE Trans. on reliability. – 1979. – № 3. – P.199–205.
6. Quadri, S. M. K. Software Reliability Growth Modeling with New Modified Weibull Testing–effort and Optimal Release Policy / S. M. K. Quadri, N. Ahmad // International Journal

of Computer Applications. – Vol. 6. – 2010. – № 12. – C. 1 – 10.

7. Yamada, S. S-Shaped Reliability Growth Mod-eling for Software Error Detection / S. Yamada, M. Ohba, S. Osaki //IEEE Trans-actions on Reliability. Vol. R-32. No. 5, Dec. – 1983. – P. 475 – 478.

8. Duan J.T. Learning Curve Approach to Relia-bility Monitoring // IEEE Trans. on Aero-space. – 1964. – Vol. 2. – P. 563 – 566.

9. Moranda, P.B. Event-Altered Rate Models for General Reliability Analysis / P.B. Moranda // IEEE Transactions on Reliability. Vol.R-28. – No. 5. – 1979. – C.376 – 381

10. Musa, J.D. A Logarithmic Poisson Time Model for Software Reliability Measurement / J.D.Musa, K. Okumoto //Proc. Sevent International Conference on Software Engineering. – Orlando, Florida: – 1984. – P. 230 – 238.

11. Maevsky D. A. A New Approach to Software Reliability / Dmitry A. Maevsky // Lecture Notes in Computer Science: Software Engineering for Resilient Systems. – № 8166. – Berlin: Springer, 2013. – pp. 156 – 168.

12. Maevsky, D. A. Fundamentals of software stability theory [Electronic resource] / D. A. Maevsky // Reliability: Theory & Applications. – 2012. – Vol.7. – № 4(27). – p. 31 – 40. Access mode: http://www.gnedenko-forum.org/Journal/2012/ RTA_4_2012.pdf

13. Android – An Open Handset Alliance Project [Electronic resource]. Access mode: http://code. google. com/p/android/issues/list

14. Lyu, M. R. Handbook of Software Reliability Engineering / M. R. Lyu. – New York: McGraw-Hill Company. – 1996. – 805 p.

CONFIDENCE LIMITS ON PERCENTILES BASED ON TEST RESULTS WITH A FEW FAILURES – NON-PARAMETRIC VERSUS EXPONENTIAL

Mark P. Kaminskiy

ABSTRACT

The non-parametric lower confidence limits on percentiles in the classes of continuous and increasing failure rate distributions are compared to their parametric exponential counterpart for the Type II censored data. In opposite to the common belief that non-parametric estimation procedures are always less effective than analogous parametric procedures, in the considered case, it turns out that the non-parametric procedures provide either better or the same confidence estimates. In a particular case, when the data include only one uncensored observation (failure) and all three estimates (in the classes of continuous distributions, increasing failure rate distributions and the exponential distribution) exist, the respective three lower confidence limits on percentiles coincide.

Index Terms -- Confidence limits on percentiles, parametric estimation, non-parametric estimation, Type II censoring, reliability test planning

Acronym[1]

CDF	cumulative distribution function
IFR	increasing failure rate

Notation

$F(t)$	time to failure CDF
γ	confidence probability
p	quantile level, $100p$ percentile level
t_p	$100p$ percentile of time to failure
$\tau_p(\gamma)$	lower γ confidence limit on t_p
$I_x(a, b)$	incomplete beta function
$n_{min}(p, \gamma)$	minimal sample size needed to estimate $\tau_p(\gamma)$
$E_n(x)$	function rounding x up to the closest integer
$F(x, k)$	CDF of χ^2 distribution with k degrees of freedom
$\chi^2_\gamma(k)$	γth quantile of Chi-square distribution with k degrees of freedom
$\Gamma(m)$	Gamma function
$gamma(l, z)$	lower incomplete Gamma function

1. INTRODUCTION

The lower confidence limit on the $100p$th percentile (or pth *quantile*, or *quantile of level p*) is one of the most popular reliability measures. The random variable $\tau_p(\gamma)$ is the lower γ confidence limit ($\gamma = 1 - \alpha$) on the $100p$th percentile t_p, if these quantities satisfy the following relationship:

$$P_r\left[\int_{\tau_p(\gamma)}^{\infty} d\big(F(t)\big) \geq 1 - p\right] \geq \gamma \qquad (1)$$

where $F(t)$ is the time to failure cumulative distribution function (CDF), and $F(t_p) = p$.

[1] The singular and plural of an acronym are always spelled the same.

The statistical procedures for constructing the lower confidence limits on percentiles are now available for the most popular lifetime distributions e.g., exponential, Weibull, and Lognormal. These procedures can be found in the popular books on statistical reliability engineering and lifetime data analysis; see for instance, Nelson [1], Lawless [2] , Kapur and Lamberson [3].

Many modern hardware products are so reliable that reliability engineers often deal with the test data having a few distinct (uncensored) failure times, which makes the problem of reliability estimation under strong censoring an important practical matter.

Another closely related problem is the reliability demonstration test planning. Based on the corresponding estimation procedures, the demonstration test planning related to the lifetime percentiles can be performed using the non-parametric as well as parametric approaches. The respective software tools are realized in commercially available software systems, e.g. Weibull++ developed by Reliasoft.

There is a common belief among reliability statisticians and engineers that non-parametric estimation procedures are always less effective than analogous parametric procedures. In this paper, we are going to show that in the case of strong Type II censoring (failure terminated testing), the non-parametric procedures for the percentiles estimation provide either better or the same results as their exponential counterparts. Note that the exponential distribution is still the most popular lifetime distribution in reliability engineering.

2. NONPARAMETRIC LIMITS IN CLASS OF CONTINUOUS DISTRIBUTIONS

The Type II censoring (the failure terminated testing) case is considered. Let $t_{(r)}$ be the time to the rth failure observed during a test of a sample of n identical items. The rth failure time (order statistic) $t_{(r)}$ is the lower γ confidence limit on the 100pth percentile t_p in the class of continuous distributions, if its order number, r, satisfies the following inequality (Wilks [4]):

$$I_p(r, n-r+1) \geq \gamma \tag{2}$$

where $\gamma = 1 - \alpha$, and $I_x(a,b)$ is the incomplete beta function, given by

$$I_x(a,b) = \frac{\int_0^x t^{a-1}(1-t)^{b-1}\,dt}{\int_0^1 t^{a-1}(1-t)^{b-1}\,dt}$$

and $0 \leq x \leq 1$.

In other words, if p, r, n, and γ satisfy inequality (2), then the lower γ confidence limit $\tau_p(n, r, \gamma)$ on the 100pth percentile t_p is equal to the rth order statistic, i.e.

$$\tau_p(n, r, \gamma) = t_{(r)} \tag{2-1}$$

It should be noted that for practical applications, the left side of (2) must be as close to the right side as possible.

Note also, that for given γ and p, there exists a minimal necessary sample size, $n_{\min}(p, \gamma)$, for which the time to the first failure $t_{(1)}$ is the lower γ confidence limit on the 100pth percentile t_p, i.e.,

$$\tau_p(n_{\min}, 1, \gamma) = t_{(1)}. \tag{2-2}$$

Using relationship (2), this minimal sample size can be evaluated as

$$n_{\min}(p,\gamma) = E_n\left(\frac{\ln(1-\gamma)}{\ln(1-p)}\right) \tag{3}$$

where $E_n(x)$ is the function rounding x up to the closest integer. The $E_n(x)$ function usually makes the confidence probability γ a little higher than one needs, which is illustrated by the following table.

Table1. Actual Confidence Probabilities for γ = 0.9 in Equation (3)

Quantile level p	$n_{\min}$	Actual γ (left side of (2))
0.1	22	0.902
0.05	45	0.901
0.01	229	0.900

Note that, Equation (3) is often used in reliability demonstration test planning.

3. PARAMETRIC LIMITS FOR EXPONENTIAL DISTRIBUTION

Now let us assume that the TTF distribution is exponential. Under this assumption, for the same Type II censored data, one can also estimate the lower confidence limit on percentile, using the same sample of $n_{\min}(p, \gamma)$ identical items with the first and only failure at time $t_{(1)}$.

Consider the well-known lower confidence limit on percentile $\tau_p(\gamma)$ of exponential distribution for a Type II censored sample of size n with r uncensored failure times $t_{(1)} < t_{(2)} < \ldots < t_{(r)}$. This lower confidence limit is given by:

$$\tau_p(n,r,\gamma) = \frac{2T_{nr}(-\ln(1-p))}{\chi^2_\gamma(2r)} \tag{4}$$

where $T_{nr} = \sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}$ is the total failure-free operation time accumulated by all items of the sample (total time on test), and $\chi^2_\gamma(2r)$ is the γth quantile of Chi-square distribution with $2r$ degrees of freedom.

In the particular case of $r = 1$, the lower γ confidence limit (4) takes on the following form:

$$\tau_p(n,1,\gamma) = \frac{-2nt_{(1)}\ln(1-p)}{\chi^2_\gamma(2)} \tag{4-1}$$

Now, we are going to show that, if the sample size n in the confidence estimate (4-1) is given by Equation (3), the confidence estimate (4-1) is reduced to $t_{(1)}$, i.e., $\tau_p(n, 1, \gamma) = t_{(1)}$. In other words, in this case, the nonparametric lower γ confidence limit on percentile coincides with its parametric (exponential) counterpart.

Let $F(x, k)$ be the CDF of χ^2 distribution with k degrees of freedom, which is given by

$$F(x;k) = \frac{gamma(k/2, x/2)}{\Gamma(k/2)} \tag{5}$$

where $\Gamma(m)$ denotes the Gamma function, and $gamma(l, z)$ is the lower incomplete Gamma function, which is defined as

$$gamma\ (l,x) = \int_0^x t^{l-1} e^{-t} dt \ .$$

In our case of $k = 2$ (see Equation (4-1)), the CDF (5) is reduced to:

$$F(x;2) = gamma\ (1, x/2) = 1 - e^{-x/2} \tag{5-1}$$

Using (5-1), the γth quantile of the Chi-square distribution with 2 degrees of freedom $\chi^2_\gamma(2)$ can be written as

$$\chi^2_\gamma(2) = -2\ln(1-\gamma) \tag{6}$$

Replacing n and $\chi^2_\gamma(2)$ in (4-1) by the right sides of (3) and (6) respectively, one gets

$$\tau_p(n, 1, \gamma) = t_{(1)}, \tag{4-2}$$

which proves that in the considered case, the exponential lower γ confidence limit on percentile (4-1) coincides with its nonparametric counterpart in the class of continuous distributions.

3. NONPARAMETRIC ESTIMATION IN CLASS OF IFR DISTRIBUTIONS

The mentioned above minimal required sample size $n_{\min}(p, \gamma)$ can be a serious limitation to applying the non-parametric lower confidence limit in the class of continuous distribution (2). This limitation stimulated obtaining the lower confidence limit in the narrower class of increasing failure rate (IFR) distributions by Barlow and Proschan [5, 6].

For the Type II censored sample of size n with r uncensored failure times $t_{(1)} < t_{(2)} < \ldots < t_{(r)}$, the Barlow-Proschan lower confidence limit on the 100pth percentile of IFR distribution is given as

$$\tau_p(n, r, \gamma) = T_{nr} \min\left[\frac{-2\ln(1-p)}{\chi^2_\gamma(2r)}, \frac{1}{n}\right] \tag{7}$$

It is important to note that, if

$$\min\left[\frac{-2\ln(1-p)}{\chi^2_\gamma(2r)}, \frac{1}{n}\right] = \frac{-2\ln(1-p)}{\chi^2_\gamma(2r)} \quad , \tag{8}$$

the lower confidence limit for IFR distributions (7) coincides with the lower confidence limit for the exponential distribution (4). On the other hand, it can be shown that, if

$$\min\left[\frac{-2\ln(1-p)}{\chi^2_\gamma(2r)}, \frac{1}{n}\right] = \frac{1}{n} \tag{9}$$

then the statistic $t_{(r)}$ order number r satisfies Inequality (2) with the same parameters n, p and γ (Kaminsky, [7]). Thus, if condition (9) is satisfied, one has two competing non-parametric confidence estimates – one in the class of continuous distribution and another in the class of IFR distributions.

At this point, an expected question is "which estimate is better, if Equation (9) holds?" Below, we are going to show that the better lower confidence limit is the one given by Equation (2-1), which is applicable to any continuous time to failure distribution.

If condition (9) is satisfied, according Equation (2-1), the lower γ confidence limit on the percentile t_p in the class of continuous distributions exists as the rth order statistic $t_{(r)}$. Under the same condition, according to Equation (7), the respective lower γ confidence limit on the percentile t_p in the class of IFR distribution is given by

$$\tau_p(n,r,\gamma) = \frac{T_{nr}}{n} = \frac{\sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}}{n} \tag{10}$$

Comparing the right side of (2-1) with the right side of (10), one comes to conclusion that under condition (9) the lower γ confidence limit on percentile t_p in the class of continuous distributions is better, as the one taking on a greater value compared to its IFR counterpart. Finally, it is easily to show that under condition (9), the exponential confidence estimate (given by Equation (4)) is worse compared even to the respective lower γ confidence limit on the percentile of IFR distributions (Equation (7)).

If Equation (8) holds, the lower γ confidence limit on the percentile t_p in the class of continuous distributions does not exist, i.e. the order number r of $t_{(r)}$ does not satisfy Inequality (2). Nevertheless, the respective IFR confidence limit on t_p does exist and coincides with its exponential counterpart (4).

We still have one case unconsidered. It is the case when the sample size n is too small to construct the low γ confidence limit in the class of continuous distributions, and there is one failure in the sample, i.e. $r = 1$ and $n < n_{min}(p, \gamma)$. In this case, Equation (8) holds, and the lower confidence limit for IFR distributions (7) coincides with the lower confidence limit for the exponential distribution (4).

5. CONCLUSIONS

We have compared the procedure for constructing lower confidence limits for percentile of the exponential distribution with its non-parametric alternatives in the classes of continuous distributions and IFR distributions.

Table 2 on next page displays a summary of the above discussion. Analyzing the table helps one to come to the following conclusions.

1. In all the cases when both non-parametric estimates are available, the estimates in the class of continuous distribution provide either better or the same results as the IFR estimates and the parametric estimates for the exponential distribution.
2. In some cases, the non-parametric estimates for IFR distribution coincide with the respective exponential estimates, which comes with no surprise, if one recalls that the exponential distribution belongs to the IFR class (Barlow and Proschan [5]).
3. In an important from practical standpoint case, when the sample size is minimal needed to get the non-parametric estimate in the class of continuous distribution based on the first and only failure, it is shown that all three considered estimation procedures provide the same result.
4. For the given sample size n, number of uncensored failure times r, percentage $100p$, and confidence probability γ, Table 1 helps to choose the non-parametric estimation procedure yielding the same or better result than the one based on the assumption of exponentially distributed failure times. This can be especially helpful in the situations when the samples are strongly censored and applying goodness-of-fit tests is not very useful.

Table 2. Non-parametric and Exponential Lower γ Confidence Limits τ_p for 100*p*th Percentile for Type II Censored Sample of Size *n* with *r* Uncensored Failure Times

Sample Size, *n*	Number of uncensored failure times, *r*	Distribution or Class of Distributions			$\min\left[\frac{-2\ln(1-p)}{\chi^2_\gamma(2r)}, \frac{1}{n}\right]$
		Continuous $\tau_{p\ cont}$	IFR $\tau_{p\ IFR}$	Exponential $\tau_{p\ Exp}$	
$n < n_{min}$[2]	1	Does not exists	Eq. (7) $\tau_{p\ IFR} = \tau_{p\ Exp}$	Eq. (4) $\tau_{p\ Exp} = \tau_{p\ IFR}$	$\frac{-2\ln(1-p)}{\chi^2_\gamma(2r)}$
$n = n_{min}$	1	Eq. (2-2) $\tau_{p\ cont} = t_{(1)}$	Eq. (7) $\tau_{p\ IFR} = t_{(1)}$	Eq. (4-2) $\tau_{p\ Exp} = t_{(1)}$	$\frac{1}{n}$
$n < n_{min}$	> 1	Does not exists as $t_{(r)}$	Eq. (7) $\tau_{p\ IFR} = \tau_{p\ Exp}$	Eq. (4) $\tau_{p\ IFR} = \tau_{p\ Exp}$	$\frac{-2\ln(1-p)}{\chi^2_\gamma(2r)}$
$n > n_{min}$	> 1	Eq. (2-1) $\tau_{p\ cont} = t_{(r)}$ $\tau_{p\ cont} > \tau_{p\ IFR}$	Eq. (7) $\tau_{p\ cont} > \tau_{p\ IFR} > \tau_{p\ Exp}$	Eq. (4) $\tau_{p\ Exp} < \tau_{p\ IFR}$	$\frac{1}{n}$

REFERENCES

1. Nelson, W. Applied life data analysis. New York, Wiley, 1982
2. Lawless, J. Statistical models and methods for lifetime data. 2nd edition. New York, Wiley-Interscience, 2003
3. Kapur, K. and Lamberson, L. Reliability in Engineering Design. New York, Wiley, 1997
4. Wilks, S. Mathematical statistics, Wiley, New York, 1962
5. Barlow, R. and Proschan, F. Mathematical theory of reliability. Philadelphia, SIAM, 1996
6. Barlow, R. and Proschan, F. "Tolerance and Confidence Limits for Classes of Distributions Based on Failure Rates," *Annals of Mathematical Statistics*, 37, 6, pp. 1593−1601, 1966
7. Kaminsky, M. "Non-parametric Confidence Estimation of Quantiles of Duration of Failure-Free Operation in Classes of Continuous and Aging Distributions," *Engineering Cybernetics*, 22, 1, pp. 10 – 14, 1984

Mark P. Kaminskiy is the chief statistician at the Center of Technology and Systems Management of the University of Maryland (College Park), USA. Dr. Kaminskiy is a researcher & consultant in reliability engineering, risk analysis and life data analysis. He has conducted numerous research & consulting projects funded by the government & industrial companies, such as Department of Transportation, Coast Guards, Army Corps of Engineers, Navy, Nuclear Regulatory Commission, American Society of Mechanical Engineers, Ford Motor Company, General Dynamics, Qualcomm Inc, and several other engineering companies. He has taught several graduate courses on Reliability Engineering at the University of Maryland. He is a coauthor of *Reliability Engineering and Risk Analysis* book, the second edition of which was published in 2009.
Dr. Kaminskiy is the author or coauthor of over 50 publications in journals, conference proceedings, and reports.

[2] n_{min} is given by Equation (3)

ASYMPTOTIC OF CYCLY EXISTENCE IN ORIENTED GRAPH WITH LOW RELIABLE EDGES

G. Tsitsiashvili
Russia, Vladivostok, IAM FEB RAS, FEFU

e-mail: guram@iam.dvo.ru

ABSTRACT

In this paper a power asymptotic of a probability that there is a cycle in a random oriented graph with n nodes and low reliable edges is constructed. An accelerated algorithm for a calculation of asymptotic coefficients with $O(s(n)\ln n)$ products, where $s(n)$ is an amount of products in a multiplication of two matrixes with a size $n \times n$, is constructed.

Keywords: a cycle, an oriented graph, an edge, a probability.

1. INTRODUCTION

We consider a problem of power asymptotic construction for a probability of a cycle existence in a random graph with low reliable edges. A presence of cycles in a deterministic oriented graph allows to factorize it by a relation of a cycle equivalence [1], [2]. A calculation of an amount of cycles with minimal length may be applied in an investigation of free scale networks which receive large spread last years. [3, Theorems 10 - 12]. An algorithm of a calculation of power asymptotic coefficients with products amount $O(s(n)\ln n)$, where $s(n)$ is an amount of products for a multiplication of two matrixes with a size $n \times n$, is constructed.

2. ASYMPTOTIC OF CYCLE EXISTENCE

Consider an oriented graph G with nodes $1, \dots, n$, without loops and fold edges. Denote $A = \|a_{ij}\|_{i,j=1}^{n}$ its adjacency matrix, D - minimal cycle length, C - an amount of cycles with minimal length in the graph G. Construct a model of an oriented random graph G_* with nodes $1, \dots, n$ in which only edges of the graph G may enter. The edge (i, j) enters with the probability $p_{ij} = h$, $h \to 0$ (it is low reliable). Random events that different edges enter the graph G_* are independent. Denote S the event that there is a cycle in the graph G_* and put $P(S)$ its probability.

Theorem 1. The limit relation $P(S) \sim Ch^D,\ h \to 0$ is true.

Proof. As $= \bigcup_{1<k\le n} S_k$, where S_k is the event that there is simple (without repetitions of nodes) cycle with the length k in the graph G_* so *P(S)* satisfies the relation

$$P(S) = P(\bigcup_{1<k\le n} S_k) = P(\bigcup_{D\le k\le n} S_k) \sim P(S_D) \sim C(h^D),\ h \to 0.$$

Theorem 1 is proved.
Define $c_k = trA^k$ and calculate asymptotic constants *D, C*.
Theorem 2. If $\min(k\colon c_k > 0) \le n$ then $D = \min(k\colon c_k > 0)$, $C = \frac{c_D}{D}$.

Proof. It is well known that the element $a_{ii}^{(k)}$ of the matrix A^k equals the amount of ways $(i = i_1, \dots, i_{k-1}, i_k, i)$ with the length k in the oriented graph G. If $k = D$ then all cycles with the length k

contain k different nodes. Indeed if not there is a cycle with the length k passes through some node more than one time. So this cycle has length smaller than k.

Consequently the equality $D = \min(k: c_k > 0)$ is true and all cycles with the length D are simple. So the cycle $(i = i_1, \dots, i_{k-1}, i_k, i)$ adds units in k diagonal elements of the matrix A^k and the equality $C = \frac{c_D}{D}$ takes place. Theorem 2 is proved.

Assume that the constant D is known and $k_1 = \min(k: 2^k > n)$. Represent the constant D in the binary-number system and write it in the form

$$D = 2^{l_1} + 2^{l_2} + \dots + 2^{l_r},\ 0 \le l_1 < l_2 < \dots < l_r \le k_1.$$

Calculate now the matrixes $A^{2^1} = A \times A,\ A^{2^2} = A^{2^1} \times A^{2^1}, \dots,\ A^{2^{k_1}} = A^{2^{k_1-1}} \times A^{2^{k_1-1}}$, using $k_1 s(n) = O(s(n)\ln n)$ products. Then the constant C may be calculated by the formula

$$C = \frac{tr\left(A^{l_1} \cdot A^{l_2} \cdot \ldots \cdot A^{2^{l_r}}\right)}{D} \tag{1}$$

using $O(s(n)\ln n)$ products. The constant $D = \min(k: trA^k > 0)$ may be found by a sequential calculation of the matrixes A^k, $1 < k \le n$, using $O(s(n)n)$ products. So there is a problem to accelerate an algorithm of the constant D calculation.

3. ACCELERATED ALGORITHM OF CONSTANT *D* CALCULATION

Put $B = A + I$ where I is the unit matrix and denote $d_k = trB^k - n$.

Theorem 3. If Theorem 2 condition is true then

$$D = \min(k: b_k > 0),\ 0 = b_1 < b_2 < \dots < b_n\ . \tag{2}$$

Proof. The relation (2) is a corollary of the equality

$$b_k = tr(A + I)^k - n = \sum_{j=1}^{k} C_k^j\, trA^j\ ,$$

where C_k^j is a number of combination from k by j.

Using Theorem 3 and an idea of a dichotomy dividing for a search of a root of monotonically increasing and continuous function construct the following algorithm of the constant D definition. Using the formulas $B^{2^{t+1}} = B^{2^t} \cdot B^{2^t}$, $t > 0$, calculate by $s(n)$ products. If $d_{2^{k_1}} = 0$ then we stop calculation and put formally $D = \infty$, $C = 0$. If not define $q_1 = \min\left(k: d_{2^k} > 0\right)$, $q_1 < [\log_2 n] + 1$, where $[a]$ is an integer part of a real number a.

Denote $P = 2^{q_1}$, $Q = 2^{q_1-1}$ and construct the following recurrent procedure: if $d_{Q+2^{q_1-2}} > 0$ then $P := Q + 2^{q_1-2}$, else $Q := Q + 2^{q_1-2}$, if $d_{Q+2^{q_1-3}} > 0$ then $P := Q + 2^{q_1-3}$, else $Q := Q + 2^{q_1-3}$ and so on. This procedure continues $q_1 - 1$ steps till we obtain the equality $P - Q = 1$. Then the relation $D = P$ is true. To fulfill this recurrent procedure it is necessary to make $O(s(n)\ln n)$ products. Theorem 3 is proved.

Consequently the asymptotic constants D, C may be calculated by $O(s(n)\ln n)$ products.

4. CONCLUSION REMARKS

For the standard algorithm of the multiplication of two matrixes with the size $n \times n$ $s(n) = O(n^3)$, for F. Strassen algorithm $s(n) = O(n^{2.81})$, for *D*. Coppersmith and Sh. Winograd algorithm $s(n) = O(n^{2.3755})$ and for V. Williams algorithm $s(n) = O(n^{2.3727})$ [4]. But main part of calculators consider that the F. Strassen algorithm is the most applicable among algorithms accelerated in a comparison with the standard one.

Assume that elements of the matrix $V = \|v_{ij}\|_{i,j=1}^{n}$, $v_{ij} \geq 0$, characterize weights of the graph G edges and in the model of the random graph G_* the probability $p_{ij} \sim v_{ij}h$, $h \to 0$. Then it is not complicated to obtain that the probability of the cycle existence in the graph G_* satisfies the relation $P(S) \sim \frac{trV^D}{D}$,. $h \to 0$. And the matrix V^D is calculated similar to the matrix A^D by $O(s(n) \ln n)$ products.

REFERENCES

1. Mirzayanov M.R. 2006 . On minimal strongly connected congruencies of a directed path. Izvestiiia Saratovskogo Universiteta. Mathematika. Mekhanika. Informatika. Vol. 6 (1/2) P. 91-95 (In Russian).
2. Karmanova E.O. 2012. Congruences of Paths: Some combinatory properties. Prikladnaya diskretnaya mathematika. Vol. 2 (16). P. 86-89. (In Russian).
3. Raigorodskiy A.M. 2010. Models of random graphs and their application. Trudi MFTI. Vol. 2:4. P. 130-140. (In Russian).
4. Cohn H., Kleinberg R., Szegedy B. and Umans Ch. Group-theoretic algorithms for matrix multiplication. Arhiv: math. Gr/0511460. Proceedings of the 46th Annual symposium on foundations of Computer Science, 23-25 October 2005, Pittsburgh, PA, IEEE Computer Society. P 379-388.

APPLICATION OF BAYESIAN METHODS TO EVENT TREES WITH CASE STUDIES

G. Unnikrishnan[1]*, Shrihari[2], Nihal A. Siddiqui[3]
[1,2,3]Department of Health, Safety & Environment, University Of Petroleum
and Energy Studies, Dehradun, India

*email: ukrishnan77@yahoo.com

ABSTRACT

Event trees are used extensively to analyze accident scenarios in several domains. The tree and its branching structures are used to represent the failure of successive barriers to an initiating event. The end positions of the branches indicate the outcome of progression of each scenario. When probabilities are assigned to success and failure of each barrier the end probabilities can be calculated fairly easily. The logical sequence of the events is clear from the tree structure. However Bayesian networks (BN) are directed acyclic graphs with the nodes indicating events and connected arcs indicating the relationships between the nodes. Initial probabilities are assigned to the parent nodes and conditional probabilities of child nodes are worked out using Bayes Theorem. Bayesian network is a probabilistic modeling technique. Event trees can be mapped into Bayesian networks. Once an event tree is mapped as a Bayesian network, forward (same as in event trees) and backward analysis (possible but involved in event trees) can be performed. Additionally BN has the flexibility for adding causal factors that influence the events. It offers a different perspective of probabilities and better understanding of the incident scenarios. This paper will present mapping of event trees typically found in process industries to Bayesian networks with case studies

1. INTRODUCTION

Event trees are used widely in the process industries to represent incident scenarios. They are used to show the probabilities of success and failure of protective barriers and progression of an initiating event to several potential scenarios. Guidelines for Hazard Evaluation Procedures by Center for Chemical Process Safety (2008) [1] describes the method in detail. Bearfield & Marsh (2005) [2] argued that event tree and Bayesian network are complimentary and both models can be used together to have a better understanding of the potential incident scenarios. They presented event tree for a train derailment initiating event and mapped it in to a Bayesian network.

Other authors namely Bobbio et al (2001) [3], Khakzad et al (2012-1) [4] have explained how to map fault trees and bow ties into Bayesian network respectively. In the latter paper Khakzad et al (2012-1) [4] explained mapping of an event tree which is a part of bow tie, to a Bayesian net. They mapped an event tree for an initiating event of gasoline release followed by possible ignition and consequences of vapor cloud or pool fire. Kalantarina et al (2010) [5] presented part of an event tree for outcome of failure of ISOM unit at BP Texas City Refinery accident in connection with their paper on modeling of BP Texas City refinery accident using dynamic risk assessment approach. Khakzad et al (2012-2) [6] discussed dynamic risk assessment using bow tie approach specifically stating that usefulness of Bayesian approach in updating generic information with site specific data. The paper presented a case study of a dust explosion at a sugar manufacturing facility using bow tie model consisting of Fault Tree and Event Tree. The event tree consisted of 3 barriers namely; high concentration barrier, primary explosion barrier and venting barrier. However details of how the event trees have been mapped into Bayesian network is not discussed in the above two papers.

This paper will describe the method of mapping event trees into Bayesian networks and its usefulness in getting a different picture and better understanding the incident probabilities of process facilities with examples.

Section 2 gives brief introduction to Bayesian networks, section 3 presents event trees and corresponding equivalent Bayesian Networks (BN) with case studies for process industry incidents. Section 4 will present discussion on the potential use of Bayesian networks mapped from event trees for process industry applications.

2. BAYESIAN NETWORKS

A Bayesian Network (BN) is a directed acyclic graph (DAG) in which the nodes represent the system variables and the arcs symbolize the dependencies or the cause–effect relationships among the variables. A BN is defined by a set of nodes and a set of directed arcs. Probabilities are associated with each state of the node. The probability is defined, a priori for a root (parent) node and computed in the BN by inference for the others (child nodes). Each child node has an associated probability table called conditional probability table (CPT).

The computation of the net is based on the Bayes Theorem which states that if P (B) is probability of B happening, then P (A/B) is probability of A happening given that B has happened, given P (B) not equal to zero.

This is equal to:

From fundamental rule of conditional probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A, B)}{P(B)} \qquad 1$$

Using the rules of probability to rewrite P (A, B) as P (B/A) (A), we get the common form of Bayes equation

$$P(A/B) = \frac{P(B/A) \times P(A)}{P(B)} \qquad 2$$

In Bayesian terminology, the right hand side represents the prior situation –which when computed gives the left hand side –called posterior values. The value P (A) is the prior probability and P (B/A) is the likelihood function –which is data specific to the situation. P (B) is the unconditional probability of B- which is calculated from the rule

$$P(B) = P(B/A) \times P(A) + P B/A' \times P(A') \qquad 3$$

Where A' stands for A not happening
The Bayes equation (2) can be applied to several nodes using laws of probability.
The above concept is used to represent typical conditional probability, namely cause & effect or hypothesis & evidence as shown in the form of a simple Bayes nets in Figure 1 a & b

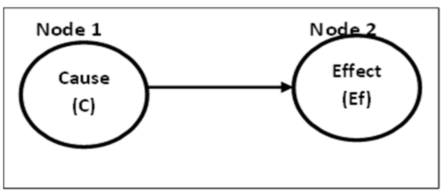


Figure 1 a: Bayes Net for cause and effect

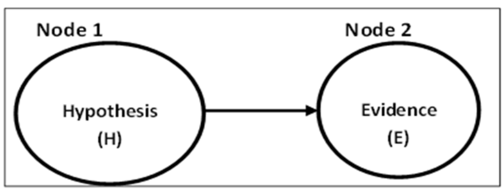


Figure 1 b: Bayes Net for Hypothesis and Evidence

Corresponding equations are:
For cause & effect

$$P(C/Ef) = \frac{\underline{P(Ef/C) \times P(C)}}{P(Ef)} \qquad 4$$

For Hypothesis & Ev

$$P(H/E) = \frac{\underline{P(E/H) \times P(H)}}{P(E)} \qquad 5$$

Extension of the pri

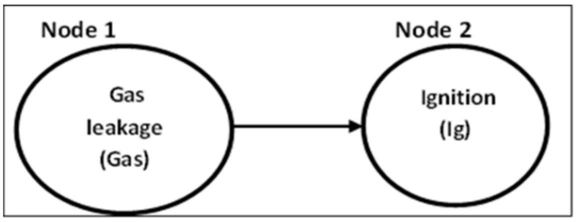


Figure 1c: Bayes net for gas leakage & Ignition

Corresponding equation will be:

$$P(Gas/Ig) = \frac{\underline{P(Ig/Gas) \times P(Gas)}}{P(Ig)} \qquad 6$$

3. EVENT TREES AND BAYESIAN NETWORKS: PROCESS INDUSTRY APPLICATIONS

3.1 The flexibility of bayesian network can be demonstrated by a simple example.

Let us assume that the probability of an Emergency Shut Down Valve (ESDV) working is 0.85. Conversely probability of ESDV not working is 0.15. If ESDV works the probability of Safe Shutdown is 0.97. If ESDV does not work the probability of Safe Shut down is only 0.02. (The probability values are hypothetical and not from any database). The situation can be represented as an Event tree given in Figure 2 below.

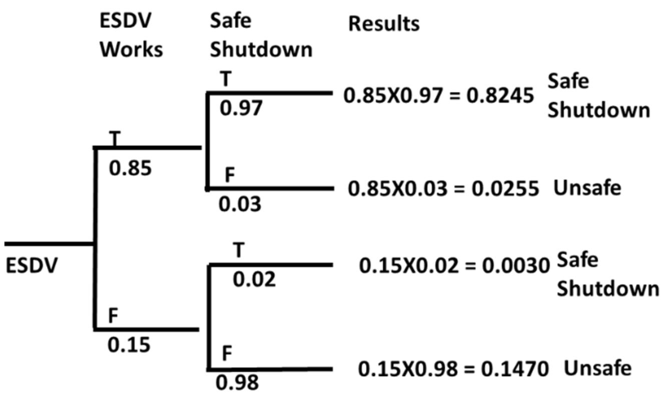


Figure 2: Event tree for ESDV action and Safe shutdown

From the Event Tree the following can be calculated:

Probability of Safe Shutdown = 0.8245 + 0.0030 = 0.8275
Probability of Unsafe situation= 0.0255 + 0.1470 = 0.1725

The Even tree can be converted to a Bayesian Network shown below in Figure 3.

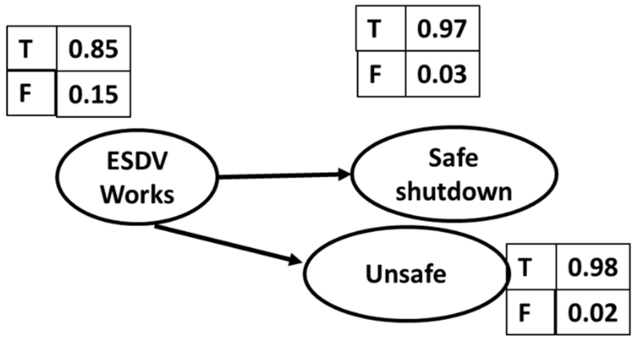


Figure 3: BN for Event tree for ESDV action and Safe shutdown

Further, the conditional probability statements are expressed as a table below:

Table1: Conditional probabilities for Safe Shutdown

ESDV works	Safe shutdown	
	T	F
T (0.85)	0.97	0.03
F (0.15)	0.02	0.98

In order to fully appreciate the flexibility of BN, it has to be modeled in suitable software. Author has used Netica^{TM} from Norsys Corporation. Other modeling software are also available. Model of the above scenario is shown in Figure 4 below.

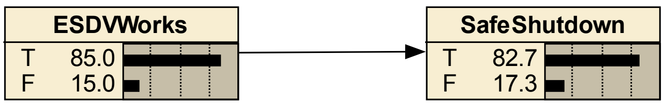


Figure 4: Bayesian Network model for ESDV and Safe shutdown

The model has calculated the forward probabilities which are same as the results from Event Tree. Now we have a situation where we know that Safe Shutdown has occurred. Then what is the probability that ESDV has worked?

In order to calculate the same, Bayes theorem has to be used which is illustrated below:

Probabilities of Safe Shutdown and No Safe Shutdown, given that ESDV has worked

$$P\,\frac{\text{Safe Shutdown}}{\text{ESDV works}-\text{True}} = 0.97 \quad (7)$$

$$P\ \frac{\text{Safe Shutdown}}{\text{ESDV works– False}} = 0.02 \qquad 8$$

Applying Bayes theorem for finding the probability ESDV working given there is Safe Shutdown:

$$P\ \frac{\text{ESDV Works–True}}{\text{Safe Shutdown}} = \frac{P\frac{\text{Safe Shutdown}}{\text{ESDV works– True}}\ \text{X}\ \ P(\text{ESDV Works} - \text{True})}{P\ (Safe\ Shutdown)} \qquad 9$$

In the above expression, right hand side numerator values are known. The unconditional probability of Safe Shutdown P (Safe Shutdown) in the denominator needs to be calculated.

$$P\ (Safe\ Shutdown) = P(\text{ESDV Works} - \text{True})\ \text{X}\ P\ \frac{\text{Safe Shutdown}}{\text{ESDV works–True}} + P(\text{ESDV Works} - \text{False})\ \text{X}\ P\ \frac{\text{Safe Shutdown}}{\text{ESDV works– False}}$$

$$= 0.85\ \text{X}\ 0.97 + 0.15\ \text{X}\ 0.02 = 0.8275 \qquad 10$$

Substituting the above value in the equation 9

$$P\ \frac{\text{Safe Shutdown}}{\text{ESDV works– False}} = \frac{0.97\ \text{X}\ 0.85}{0.8275} = 0.9963$$

The above computation can be readily achieved in the Bayesian simulation by changing the Safe Shutdown True to 100%. The computation is propagated backwards using the Bayes theorem to give the result as 0.9963 as shown in Figure 5 below

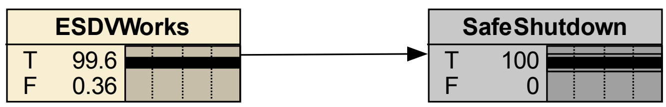


Figure 5: Bayesian Network model for Safe shutdown

3.2 Further case studies are given in the following sections

3.2.1 Case study No.1: Flammable & toxic gas leak

3.2.1.1 Event Tree

Event trees model an incident as a sequence of events. Each event has success or failure probability. The event tree branching is created from left to right, starting from an initiating event and continuing to the sequence of events (failure of barriers) till a logical consequence is obtained. As example of event tree for flammable and toxic gas leak is given in Figure 6.

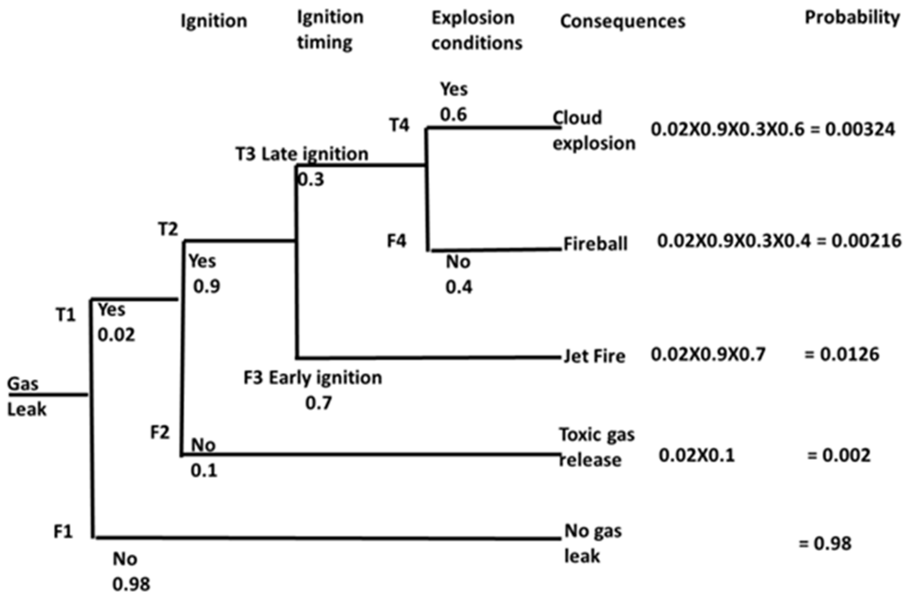


Figure 6: Event tree for Flammable & toxic gas leak

If flammable & toxic gas release is the initiating event (probability of which is given) then four types of incident scenarios are possible, depending on the probabilities of ignition, early or late ignition and explosion conditions.

Once the probabilities of all branches of the event trees are known, the consequence probabilities are worked out by multiplying the initiating probability with the probabilities in the corresponding branches of the event tree as shown in Figure 6. Sum of probabilities at any branching point should be equal to 1. Binary branching like success or failure is the most common branching used. More than two branches are also possible.

With an initiating event probability of gas leak as 0.02, the probabilities of the consequence are calculated and shown on right hand side of the Figure 2. The probabilities used are for illustration only and have not been taken from any database.

3.2.1.2 Bayesian Network (BN)

Figure 7 shows the corresponding BN for the flammable and toxic gas leak scenario shown in event tree in Figure 6.

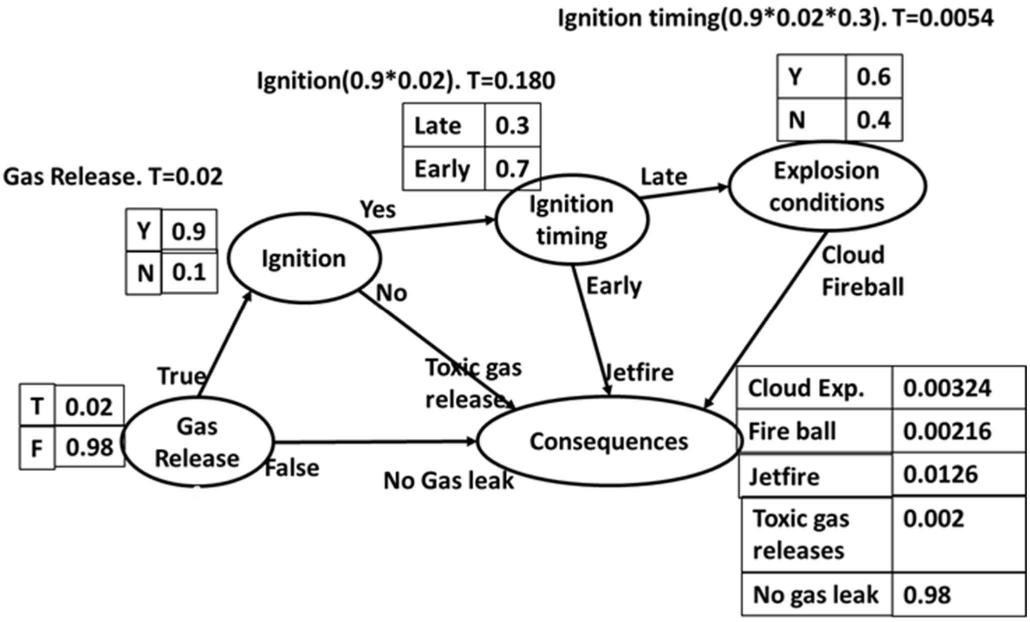


Figure 7: Bayesian Network (BN) for flammable & toxic gas leak

When translating to Bayesian Network (BN), following transformations have been done:

- Branching point in Event Tree → Node in BN
- Branches in Event Tree → Connecting arcs in BN showing relationship between nodes
- Branching conditions in Event Tree →Node states in BN

For example, the two braches for condition 'Ignition' in Figure 6 is captured in node states Y (yes) and N (No) with probabilities of 0.9 (yes) and 0.1(no) respectively in Figure 7 of the corresponding BN. Node states are given in the tables shown adjacent to the node with the probability values. The conditional probability is given above the node state, which is the precondition for the event to reach the node

3.2.1.3 Bayesian Network (BN) simulation for gas leak.

The BN simulation model for flammable & toxic gas leak event tree is given in Figure 8.

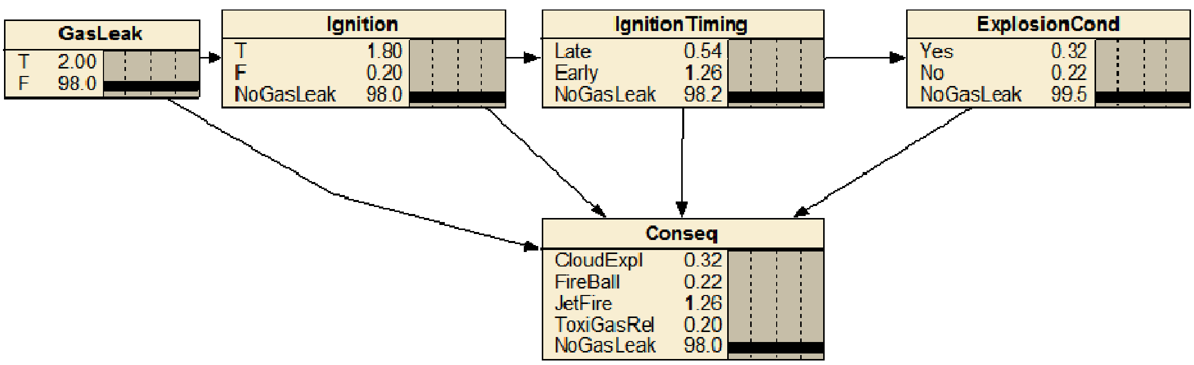


Figure 8: Bayesian Network (BN) simulation model for flammable & toxic gas leak

Table 2 e: CPT for Consequences

				Conseq:				
GasLeak	Ignition	IgnitionTiming	ExplosionCond	CloudExpl	FireBall	JetFire	ToxiGasRel	NoGasLeak
T	T	Late	Yes	**1**	0	0	0	0
T	T	Late	No	0	**1**	0	0	0
T	T	Late	NoGasLeak	0	0	0	0	0
T	T	Early	Yes					
T	T	Early	No					
T	T	Early	NoGasLeak	0	0	**1**	0	0
T	T	NoGasLeak	Yes					
T	T	NoGasLeak	No					
T	T	NoGasLeak	NoGasLeak					
T	F	Late	Yes					
T	F	Late	No					
T	F	Late	NoGasLeak					
T	F	Early	Yes					
T	F	Early	No					
T	F	Early	NoGasLeak					
T	F	NoGasLeak	Yes					
T	F	NoGasLeak	No					
T	F	NoGasLeak	NoGasLeak	0	0	0	**1**	0
T	NoGasLeak	Late	Yes					
T	NoGasLeak	Late	No					
T	NoGasLeak	Late	NoGasLeak					
T	NoGasLeak	Early	Yes					
T	NoGasLeak	Early	No					
T	NoGasLeak	Early	NoGasLeak					
T	NoGasLeak	NoGasLeak	Yes					
T	NoGasLeak	NoGasLeak	No					
T	NoGasLeak	NoGasLeak	NoGasLeak					
F	T	Late	Yes					
F	T	Late	No					
F	T	Late	NoGasLeak					
F	T	Early	Yes					
F	T	Early	No					
F	T	Early	NoGasLeak					
F	T	NoGasLeak	Yes					
F	T	NoGasLeak	No					
F	T	NoGasLeak	NoGasLeak					
F	F	Late	Yes					
F	F	Late	No					
F	F	Late	NoGasLeak					
F	F	Early	Yes					
F	F	Early	No					
F	F	Early	NoGasLeak					
F	F	NoGasLeak	Yes					
F	F	NoGasLeak	No					
F	F	NoGasLeak	NoGasLeak					
F	NoGasLeak	Late	Yes					
F	NoGasLeak	Late	No					
F	NoGasLeak	Late	NoGasLeak					
F	NoGasLeak	Early	Yes					
F	NoGasLeak	Early	No					
F	NoGasLeak	Early	NoGasLeak					
F	NoGasLeak	NoGasLeak	Yes					
F	NoGasLeak	NoGasLeak	No					
F	NoGasLeak	NoGasLeak	NoGasLeak	0	0	0	0	**1**

Node probabilities are entered in the Conditional Probability Table (CPT) as indicated in Tables 2 a, b, c & d. The state 'NoGasLeak' has to be continued in from the second CPT for 'Ignition' to all subsequent CPTs in order to meet the criteria that the sum of probabilities at every branch point has to be equal to 1.

Table 2 a: CPT for gas leak

GasLeak:	
T	F
0.02	0.98

Table 2 b: CPT for Ignition

	Ignition:		
GasLeak	T	F	NoGasLeak
T	0.9	0.1	0
F	0	0	1

Table 2 c: CPT for Ignition timing

	IgnitionTiming:		
Ignition	Late	Early	NoGasLeak
T	0.3	0.7	0
F	0	0	1
NoGasLeak	0	0	1

The above model can be further presented in a more compact form. The simulation diagram and the conditional probability tables for the compact form are given below in Figure 9 and Tables 3 a, b, c, d:

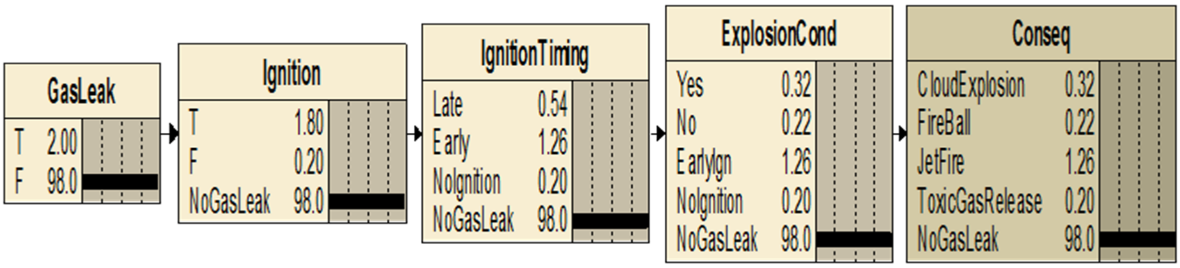


Figure 9: Compact Bayesian network simulation model for flammable & toxic gas leak

CPT tables for the compact model are given below:

Table 3 a: CPT for GasLeak

GasLeak	
T	F
0.02	0.98

Table 3 b: CPT for Ignition

	Ignition		
GasLea	T	F	NoGasLea
T	0.9	0.1	0
F	0	0	1

Table 3 c: CPT for Ignition Timing

	IgnitionTiming			
Ignition	Lat	Earl	NoIgnitio	NoGasLea
T	0.3	0.7	0	0
F	0	0	1	0
NoGasLe	0	0	0	1

Table 3 d: CPT for ExplosionCond

	ExplosionCond				
IgnitionTim	Yes	No	Earl	NoIgnitio	NoGasLea
Late	0.6	0.4	0	0	0
Early	0	0	1	0	0
NoIgnition	0	0	0	1	0
NoGasLeak	0	0	0	0	1

Following are to be noted in this context:

a. The software has the capability to set up the equations when the nodes, its connectivity's and node probability tables are established. Normally there is no need to enter the equations manually when using discrete values. Failure distribution can be used. Then the equations will be entered in the appropriate input box.
b. What is required is careful consideration of the dependencies(cause & effect), its probabilities and node states
c. The node states have to match the number of branches under each event.
d. The probability values are entered for each node state in the conditional probability (CP) table as given in Tables 2 a, b, c & d and 3 a, b, c, and d. For a node the sum of state probabilities should be equal to 1.
e. The initiating event tree branches can have probability values associated with it instead of having a probability of 1 & 0. In such cases, the unsuccessful (F) branch has to be included as a node state in the BN.
f. In an event tree the sum of probabilities at a branching point is 1. Similarly in the conditional probability table, the sum of probabilities of each state has to be 1. The sum of the computed conditional probabilities for each node in the BN (as shown in Figure 4) also has to be 1. Therefore to take care of the requirement of sum of conditional probabilities to be 1 at a node, the unsuccessful (F) state at the initiating node has to be continued in the Conditional Probability Table (CPT) for the successive nodes. Thus the state called 'NoGasLeak' state has to be indicated in all the CP tables as given in the above example for Gas Leak (F=0.98). 'NoGasLeak' state is indicated in all CP tables of nodes continued up to the 'Consequences'. When this unsuccessful (F) state is included in the CP table, the sum of the conditional probabilities at each node will sum to 1. In essence it is state of the nodes and its probability values that capture the branching points of an event tree.
g. The software does not display the CP tables in the nodes as such, but give the calculated conditional probability values for each state. The CP values given as inputs are hidden from the normal view. For example the 'Ignition' node in Figure 4 & 6 (which is a screen shot) displays the conditional probability for ignition when there is a gas leak in percentage. (100* 0.02 X 0.9 =1.8)

The last node 'Consequences' is a function node for summing up the consequences. (In this case each outcome is different).

Once the BN is set up, it can be used for predictions as well as diagnostics. Predictions are forward calculations from left to right; for example if there is a definitive gas leak, the probability of gas leak goes up to 1.0. In the event tree the consequences can be worked out by revising the value for initiating event. In the BN model this can be done by changing the probability of gas leak to 100 as shown in Figure 10.

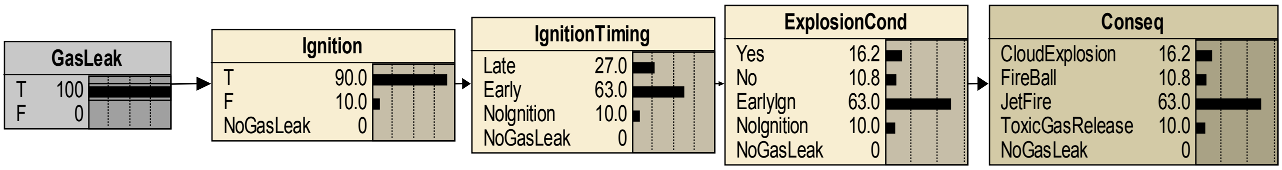


Figure 10: BN simulation model for prediction of consequences for gas leak

While event tree can also calculate this forward calculation easily, it is comparatively difficult to do the reverse, which is diagnostics.

Diagnostics involve finding out the most probable causes for occurrence of an event. In this case BN is flexible. BN uses equation 2 for calculating the probability of causes. For example, with all probabilities remaining the same, we can enter the state for the actual consequence scenario, Jet fire -which has happened as 1, then the BN model will recalculate the probabilities of all precursor events in the tree. Here in this case, the BN shows that, given there is a jet fire, there has been a gas leak and ignition with early timing. See Figure 9.

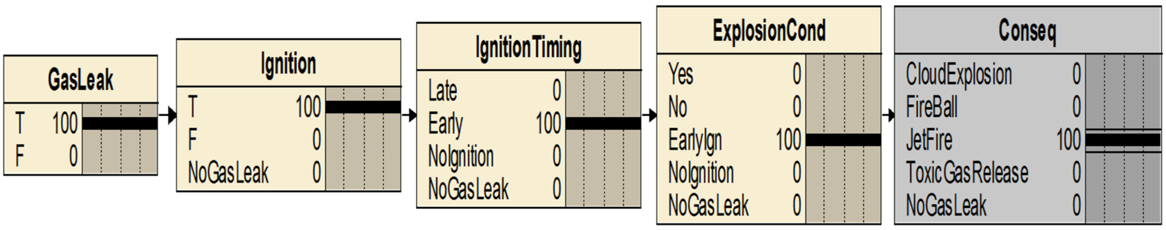


Figure 11: Bayesian network simulation for Jet fire scenario

The above presents a simple case. The event tree logic and corresponding BN can be set up to any level of complexity and used for forward (predictions) and backward (diagnostics).

3.2.2 Case study No.2 Tank high level
3.2.2.1 Event Tree

The second example is the event tree for tank high level shown in Figure 10. The probabilities have been worked out the same way as in Figure 2. (Probabilities used are only for illustration) Corresponding BNs are described in the next section.

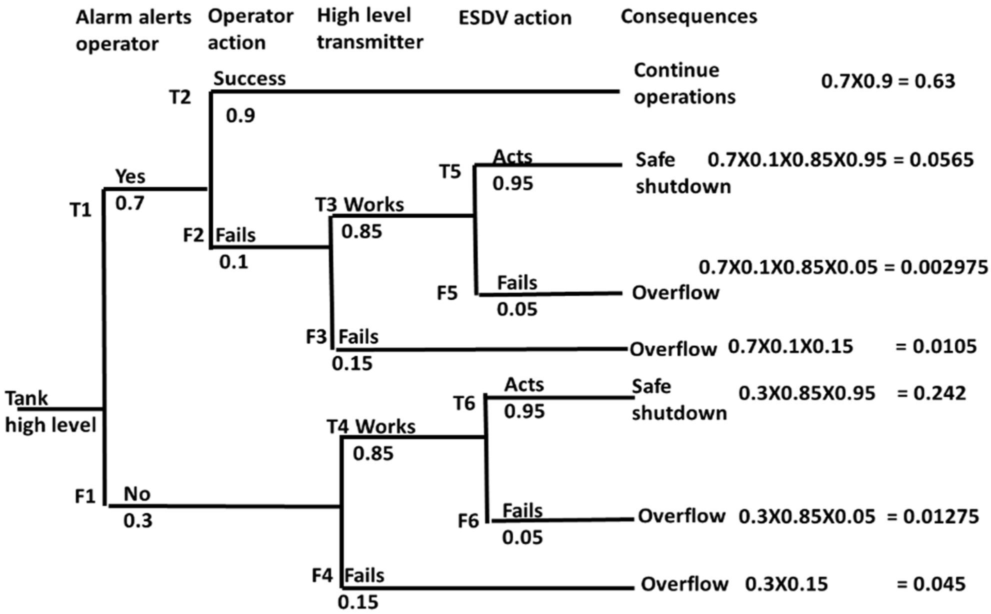


Figure 10: Event tree for tank high level

3.2.2.2 Bayesian Network for tank high level

BN for the tank overflow event tree in Figure 10 is given in Figure 11. Here high level is considered as occurred and so probability of 'Tank high level' is 1.

Node probability tables have been entered the same way as given in Tables 3 a, b, c, d & e
BN Simulation diagram is given in Figure 12

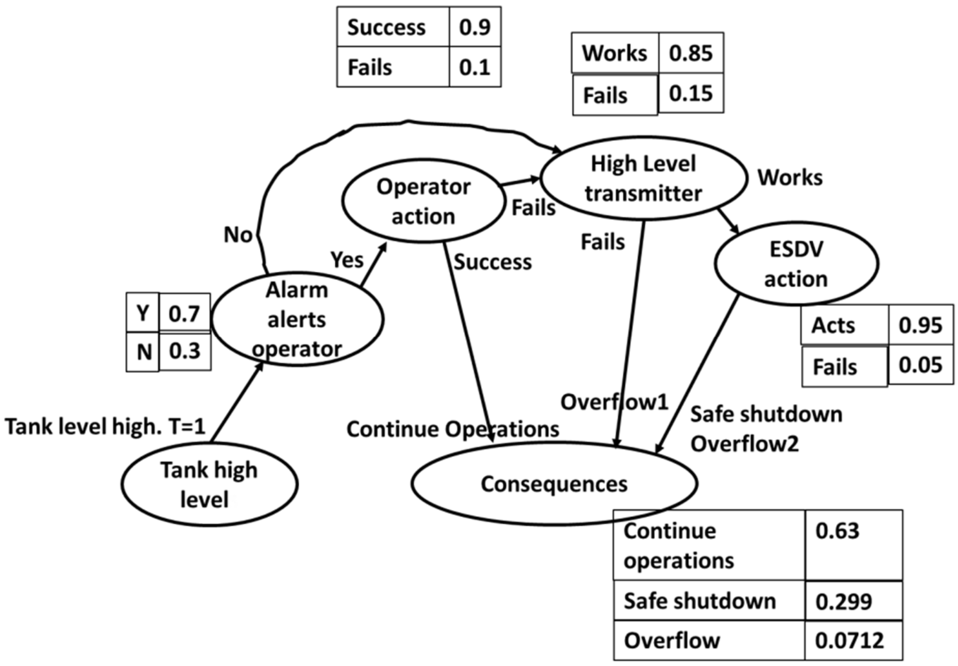


Figure 11: BN for tank high level

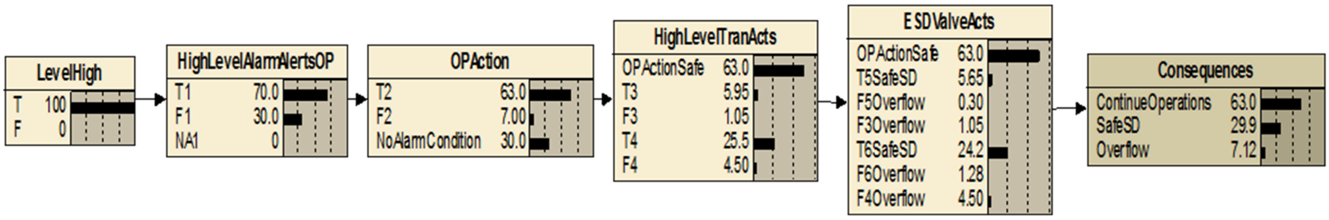


Figure 12: Bayesian network simulation diagram for tank high level scenario

3.2.2.3 Additional factors in BN

Another possibility in BN is the inclusion of additional casual factors that contributes to the probabilities of events.

We can include the testing of the Emergency Shut Down Valve (ESDV) as a causal factor connected to the node ESDVValveActs and give an improved probability of ESDV acting (0.97) instead of 0.95 in the event tree in Figure 10, if the testing is on schedule. While such an addition will require an additional branching in event tree, it can be easily implemented in BN with clear representation as a cause influencing ESDV action.

One more casual factor is added next; namely the type of sensor used for High level detection. For conventional float type the same probabilities (0.85) given in the event tree (Figure 11) is used. But for Radar type an improved probability for action (0.96) has been assigned. Both the causal factors are shown in Figure 13.

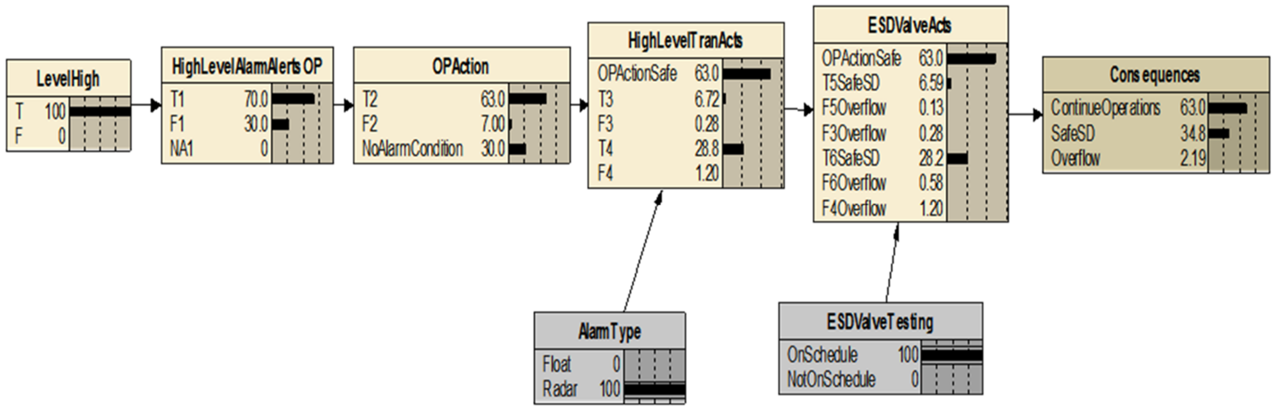


Figure 13: Bayesian network simulation diagram for tank high level scenario with addition of causal factors: ESD valve testing (On schedule) & type of high level detection (Radar type)

Figure 13 shows that:
When ESDV testing is as per schedule and with Radar type transmitter for alarm, there is improved performance of ESDV action (probability of safe shutdown has gone up to 0.348), and the possibility for overflow can be reduced from 7.12 to 2.19.

4. DISCUSSION

Event trees and its corresponding BN offer different perspectives. While event tree indicates the logical sequences of event progression (barrier success & failures) to incident scenarios, BN depicts the probabilities in a flexible manner offering easy ways for prediction (forward calculations) and diagnostics (backward calculations) of entire tree structure. BN allows additional casual factors to be included and modeled which can show influences of the factors on events directly. The two case studies have demonstrated the flexibility and power of the Bayesian methods. Updating of information can be based on site data. In the examples given, if the organization has records of the actual testing of ESDV or type of level transmitters, the same can be used to predict the incident outcome probabilities of the system with more confidence. Organizational and human factors can also be included which is not normally available in an event tree. The capability for doing predictions and diagnostics of a process system, starting from generic data and updating with site specific data is the main advantage of the Bayesian methods. Since each system is unique, event tree and corresponding BN has to be developed for each. Work is to be done in this area to make the incident probabilities available to decision makers and operational staff.

REFERENCES

[1] Center for Chemical Process Safety., "Guidelines for Hazard Evaluation Procedures, Wiley-AIChE, New York. (2008)
[2] Bearfield, G. and Marsh W., "Generalizing event trees using Bayesian networks with a case study of train derailment", SAFECOMP 2005, LNCS 3688, pp. 52-66 (2005).
[3] Bobbio, A., Portinale, L., Minichino, M., and Caincamerla, E., "Improving the analysis of dependable systems by mapping FTs into Bayesian networks", Reliability Engineering and System Safety, 71, pp. 249-260 (2001).
[4] Khakzad, N., Khan, F., and Amyotte, P., "Dynamic safety analysis of process systems by mapping bow tie into Bayesian network", Process Safety and Environmental Protection, 91, pp. 46-53 (2012)
[5] Kalantarnia, M., Khan, F., and Hawbolt, K., "Modeling of BP Texas city Refinery accident using dynamic risk assessment approach", Process Safety and Environmental Protection, 88, pp. 191-199 (2010)
[6] Khakzad, N., Khan, F., and Amyotte, P., "Dynamic risk analysis using bow-tie approach", Reliability Engineering and System Safety, 104, pp. 36-44 (2012)

ORIENTED GRAPHS WITH UNRELIABLE NODES

G.Sh. Tsitsiashvili

IAM, FEB RAS, Vladivostok, Russia,
Far Eastern Federal University

e-mail: guram@iam.dvo.ru

ABSTRACT

In applications there are networks with unreliable nodes. To construct models of such networks and effective algorithms of their analysis in this paper some elements from the graph theory, the algebra, the sets theory and the probability theory are combined. These constructions allow to investigate oriented graphs with unreliable nodes using as accuracy so asymptotic formulas. All constructed algorithms have a linear complexity by a number of graph nodes.

Keywords: an oriented graph, an unreliable node, relations of equivalence and partial order.

INTRODUCTION

In the reliability theory usually graphs with unreliable edges are investigated [1], [2]. But networks represented by graphs with unreliable nodes appear in modern applications. [3]. In this paper a model of random graph in which a failure of some node stops work of all other nodes to which there are ways in the graph is considered. [4]. A similar model is a model of a random network controllability [5].

In a problem of the random network controllability an important role play a minimal nodes cover in the graph. A nodes cover is a set of nodes that each edge touches with this set. The nodes cover is minimal if it has a minimal number of nodes. The Konig theorem establishes an equivalence of the minimal nodes cover and the maximal matching [6]. Here the maximal matching is a set of edges which have not common nodes. A matching is maximal if it has a maximal number of edges. The Konig theorem states that in a bipartite graph a number of edges in the maximal matching equals a number of nodes in the minimal nodes cover. It gives a large number of effective algorithms of the graph theory [6] – [13].

But an analysis of stochastic networks demands not only to define a number of nodes in the minimal nodes cover but to enumerate all such covers or at least to find their amount. This problem becomes NP problem. So in a consideration of networks with unreliable nodes it is necessary to change a model of a network using some specific of the oriented graph. In this paper such a problem is solved using concepts of an equivalence and a partial order between graph nodes. This approach allows to obtain algorithms of a calculation some failures events in the graph with the complexity $O(n^2)$ by the number n of the graph nodes.

1. FAILURES IN ORIENTED GRAPHS WITH UNRELIABLE NODES

Assume that deterministic and oriented graph G have the nodes set U and the edges set V. Each node v of the graph графа G with the probability q_v fails. These failures of different nodes are independent random events. Each failure of some node leads to stops of all other nodes to which there are ways from failed node in the graph G. In such a way the random graph G_* is defined. We shall calculate probabilities that some sets of nodes in the graph G_* stop.

On the nodes set V define the binary relation $v_1 \sim v_2$ if in the graph G there are ways from the node v_1 to the node v_2 and from the node v_2 to the node v_1. The relation " $\sim$" is symmetric, transitive and reflexive. So it is the equivalence relation. As a result the nodes set V may be divided into the classes of the equivalence (the clusters) $w(v) = \{v' : v' \sim v\}$. On the set of the clusters $W = \{w(v) : v \in V\}$ introduce the binary relation $w(v_1) \succcurlyeq w(v_2)$ if in the graph G there is a way from the node v_1 to the node v_2. So for any $v_1' \in w(v_1)$, $v_2' \in w(v_2)$ in the graph G there is a way from the node v_1' to the node v_2'. The relation " $\succcurlyeq$ " is ant symmetric, transitive and reflexive. So this relation is the partial order relation.

From the definition of the random graph G_* we have that in the cluster $w \in W$ all nodes fail with the probability

$$q(w) = 1 - \prod_{v \in w} (1 - q_v)$$

Or work with the probability $p(w) = 1 - q(w)$. Further denote

$$A(T) = \prod_{w \in T} p(w)\ , B(T) = \prod_{w \in T} q(w)\ , T \subseteq W$$

with $A(\emptyset) = B(\emptyset) = 1$.

Later we shall consider the set of clusters W in which any cluster w fails with the probability $q(w)$ or does not fail with the probability $p(w)$. The cluster w failure leads to a stop of all clusters $w' \in W$ which satisfy the condition $w \succcurlyeq w'$ independently that they fail or do not fail.

Assume that $\widetilde{W}$ is the set of maximal (in a sense of the partial order " $\succcurlyeq$ ") elements from W. Calculate the probability Q that all clusters from the set W stop. It is not complicated to obtain that this event in a stop of all clusters $w \in \widetilde{W}$. Consequently we have:

$$Q = B(\widetilde{W}). \tag{1}$$

Consider the probability $Q(w)$ that the cluster $w \in W$ stops. For this aim define the set of clusters $F(w) = \{w' : w' \succcurlyeq w\}$. It is not difficult to check that

$$Q(w) = 1 - A(F(w))\ . \tag{2}$$

Calculate the probability $Q(R)$ that at least a single cluster from the set R stops. Define the sets $F_i = F(w_i)$, $i = 1, \dots, r$ and put

$$F(R) = \bigcup_{i=1}^{r} F_i$$

then

$$Q(R) = 1 - A(F(R)). \tag{3}$$

Calculate now the probability $S(R, L)$ that in the random graph G_* all clusters from the set L work and there is at least a single cluster in the set R which stops. Introduce the sets $W_1 = F(L)$, $W_2 = F(R) \backslash W_1$ then

$$S(R, L) = A_1(W_1)\big(1 - A_1(W_2)\big). \tag{4}$$

2. ASYMPTOTIC ANALYSIS OF FAILURES PROBABILITY FOR SET OF CLUSTERS

Consider a problem of a calculation of the probability $Q'(R)$ that all clusters from the set $R = \{w_1, \dots, w_r\} \subseteq W$ stop their work. Accuracy calculation of the probability $Q'(R)$ demands an amount of arithmetical operations which geometrically increases by the number n of the graph G nodes. So this problem will be solved in an assumption that there are positive numbers $c(w)$ which satisfy the relation

$$p(w) = \exp(-c(w)h)\,,\, h \longrightarrow 0\,,\, w \in W. \tag{5}$$

Theorem 1. Assume that $c(w) \equiv 1$ then for any integer r, for $h \longrightarrow 0$ and for $n(1, \dots, r)$ equal to a number of clusters in the intersection T of the sets $F_i, i = 1, \dots, r$, we have:

$$Q'(R) = hn(1, \dots, r) + o(h). \tag{6}$$

Proof. Assume that $p = \exp(-h)$ and denote $D(k)$ the random event that all clusters of the set F_k .work. Then from the known formula for a probability of a finite set of events aggregation we have

$$Q'(R) = 1 - P(\cup_{k=1}^{r} D(k)) = 1 - \sum_{k=1}^{r} (-1)^{k-1} \sum_{1 \le i(1) < \dots < i(k) \le r} P\left(D\big(i(1)\big) \dots D\big(i(k)\big)\right). \tag{7}$$

A number of clusters in the set $G\big(i(1), \dots, i(k)\big) = F_{i(1)} \cup \dots \cup F_{i(k)}$ similar to Formula (7) equals

$$\sum_{s=1}^{k} (-1)^{s-1} \sum_{1 \le j(1) < \dots, j(s) \le k} n\left(i\big(j(1)\big) \dots i\big(j(s)\big)\right)$$

Consequently

$$P\left(D\big(i(1)\big) \dots D\big(i(k)\big)\right) = \exp\left(-h\left(\sum_{s=1}^{k} (-1)^{s-1} \sum_{1 \le j(1) < \dots < j(s) \le k} n\left(i\big(j(1)\big), \dots, i\big(j(s)\big)\right)\right)\right). \tag{8}$$

Substituting the relations (8) into Formula (7) and using the exponent expansion into the Taylor series we obtain:

$$\begin{aligned} Q'(R) &= 1 - \sum_{k=1}^{r} (-1)^{k-1} \sum_{1 \le i(1) < \dots < i(k) \le r} \\ &\times \left(1 - h\left(\sum_{s=1}^{k} (-1)^{s-1} \sum_{1 \le j(1) < \dots < j(s) \le k} n\left(i\big(j(1)\big), \dots, i\big(j(s)\big)\right)\right)\right) + o(h) = \\ &= l_1 + \sum_{k=1}^{r} \sum_{1 \le i(1) < \dots < i(k) \le r} n\big(i(1), \dots, i(k)\big)\, l\left(i(1), \dots, i(k)\right) + o(h),\ h \longrightarrow 0\,. \end{aligned} \tag{9}$$

As any commutation of the indexes $1, \dots, r$ in Formula (9) does not change $Q'(R)$ so the following equalities are true:

$$l\big(i(1), \dots, i(k)\big) = l(1, \dots, k)\ ,\ 1 \le i(1) < \dots < i(k) \le r,\ 1 \le k \le r. \tag{10}$$

From Formula (9) we have:

$$l_1 = 1 - \sum_{k=1}^{r} (-1)^{k-1} = 1 + \left(\sum_{k=0}^{r} (-1)^{k} - 1\right) = 0\,. \tag{11}$$

$$\begin{gathered} l(1) = \sum_{k=1}^{r} (-1)^{k-1} C_{r-1}^{k-1} = \sum_{k=0}^{r-1} (-1)^{k} C_{r-1}^{k} = 0\ ,\ l(1,2) = -\sum_{k=1}^{r} (-1)^{k-1} C_{r-2}^{k-2} = 0, \dots \\ l(1, \dots, r-1) = (-1)^{r-2} \sum_{k=r-1}^{r} (-1)^{k-1} C_{1}^{k-r+1} = 0\,, \qquad l(1, \dots, r) = 1. \end{gathered} \tag{12}$$

Theorem 1 is proved.

Corollary 1. Denote $c(1, \dots, r)$ the sum of $c(w)$ by clusters w from the set T then for $h \longrightarrow 0$ we have:

$$Q'(R) = hc(1, \dots, r) + o(h)\,. \tag{13}$$

Proof. Indeed in Corollary 1 assumption Formula (9) may be rewritten as follows:

$$Q'(R) = l_1 + \sum_{k=1}^{r} \sum_{1 \le i(1) < \dots < i(k) \le r} c\big(i(1), \dots, i(k)\big)\, l\left(i(1), \dots, i(k)\right) + o(h)$$

Using Formulas (11), (12) we obtain Formula (13). Corollary 1 is proved.

Remark 1. Corollary 1 allows to analyze essential cooperative effects in the oriented graph with unreliable nodes because the characteristic $c(1, \dots, r)$ may differ significantly from the sum

$\sum_{k=1}^{r}$ c(k). The formulas (1) – (4), (13) show that procedures to calculate probabilities in these formulas have the complexity $O(n^2)$ by the number n of the graph nodes.

REFERENCES

1. Barlow R.E., Proshan F. 1965. Mathematical theory of reliability. London and New York: Wiley.
2. Ushakov I.A. et al. Reliability of technical systems. Handbook. Moscow: Radio and communication. (In Russian).
3. Popkov V.K. 2006 Mathematical models of connectivity. Institute for Computing Mathematics and Mathematical Geophysics, Siberian Branch of RAS. (In Russian).
4. Yastrebenetskiy M.A., Ivanova G.M.1989. Reliability and automated systems for technology processes control. Moscow: Energoatomizdat. (In Russian).
5. Liu Y.Y., Slotine J.J., Barabasi A.L.V. 2011. Controllability of complex networks// Nature. Vol. 473. P. 167-173.
6. Konig D. 1931. Grafok es matrixok// Matematikai es Fizikai Lapok. Tom 38. P. 116-119.
7. Lovasz L., Plummer M. D. 2009. Matching Theory. American Mathematical Soc.
8. Rabin M.O., Vazirani V.V. 1989. Maximum matchings in general graphs through randomization// Journal of algorithms. Vol.10. P. 557-567.
9. Tutte W.T. 1946. The Factorization of Linear Graphs// J. London Math. Soc. Vol. 22. P. 107-111.

COMPARISON OF WAYS OF NORMALIZATION AT CLASSIFICATION OF INITIAL DATA

Farzaliyev Y.Z.

Azerbaijan Scientific-Research and Design-Prospecting Institute of Energetic
AZ1012, Ave. H.Zardabi-94,

e-mail:yuszey2002@yahoo.com

ABSTRACT

Normalization at classification of initial data is an indispensable condition of an opportunity of comparison of technical and economic characteristics of the equipment and devices of power supply systems. In work, results of comparison of efficiency of ways of normalization of quantitative estimations of analyzed characteristics used in practice are resulted.

Keywords. Normalization, classification, the equipment, characteristics, the importance attributes.

I. INSTRUCTION

The automated classification of the equipment, its nameplate data and data on conditions of operation, refusals, test and repair on the set versions of attributes (VA) is widely used at the decision of some exploitation problems, volume number of problems about expediency of carrying out of maintenance service and repair. For example, it is necessary to establish most both the least reliable and economic power units. Value of the first is necessary at the decision of problems on distribution of loading, switching-off in a reserve, emergency switching-off at system failures and so forth. For the least reliable and economic power units carrying out of corresponding maintenance service and restoration of deterioration is required.

Thus, problems about the importance of attributes solved, as a rule, at an intuitive level, and ranging VA on the importance - is subjective, that is why it is often erroneous. Difficulties of an estimation of importance VA is caused, first, by a natural variety of scales and scale of measurement of attributes, their interrelation and a various orientation of influence.

According to the accepted division of a scale happen quantitative (for continuous random variables, for example, residual service life, resistance to a direct current, loading, extent of transmission lines, number of short circuits, etc.), interval (for example, month of year, rated voltage and capacity, etc.), nominal, establishing quality standards (for example, quality of repair, type and conditions of the equipment, etc.).

Methodology of ranking of importance VA with interval and nominal scales of measurement has been considered by us in [1]. If for interval and nominal scales number VA fixed, at a quantitative scale of measurement classification demands development of special approaches. In practice, the quantitative scale, as a rule, transformed in interval. The number of intervals (i.e. VA) established subjectively, proceeding from features of a solved problem. Character of a divergence of average values of realizations VA among themselves and all data set does not stipulate, though divergences can have casual character. At classification of data it leads to loss of the information and finally to growth of risk of the erroneous decision. For prevention of these consequences, it is necessary to develop a method of classification of data, which would consider expediency of classification and overcame noted above difficulty.

II. TRANSFORMATION ALGORITHM OF INITIAL DATA.

Provides following sequence of calculations:

1. Maintenance of a uniform orientation of influence. As «an orientation of influence», understand character of change of properties of the equipment (for example, reliability and profitability of power units) at increase or reduction of their quantitative characteristics (we shall agree to name their parameters). For example, the increase in the specific charge of conditional fuel (C_F) testifies to decrease in profitability of work owing to deterioration of a technical condition of the capital equipment of power station. The same can be approved at increase in the charge of the electric power at own needs (P_{ON}). But the more operating ratio of the established capacity (R_C), the reliability and profitability of work of the power unit above. In other words, C_F and R_C have a various orientation. The automated comparison of such parameters leads to erroneous decisions. Recognition of an orientation of influence at a small number of parameters is often accessible manually. With increase in number of parameters, the probability of a mistake increases. In the automated mode, recognition of distinction of an orientation of influence offered to spend by construction of a correlation matrix. As a whole, this matrix will appear interrelation of parameters necessary at estimation. For maintenance of a uniform orientation of parameters, it is enough to choose as the first (control) parameter of a matrix a parameter with known character of change at change of properties of object. Then the first line (column) of a matrix on a negative sign on factors of correlation will allow establishing number and conditional number of the parameters having other orientation, than a control parameter.

2. Classification of data to the interconnected attributes is inexpedient, since also leads to loss of the information and growth of risk of the erroneous decision. Association of the interconnected versions of attributes allows to lower essentially their number and to simplify calculations. The quantitative characteristic of interrelation of attributes can be calculated in the form of estimations of factors of correlation $(K^*_{i,j})$. Than number of classified objects (m_Σ) It is less, that casual character $K^*_{i,j}$, where i≠j; i=1, m_Σ; j=1, m_Σ, It is shown in a greater degree, both on size, and on a sign. In this connection dependence between attributes with probability not smaller than (1-α), where α - the significance value, can be confirmed by the control of performance of following inequalities: $K^*_{i,j} < \underline{K}_{i,j,\alpha}$ at $K^*_{i,j} < 0$ and $K_{i,j} > \overline{K}^*_{i,j,\alpha}$ at $K_{i,j} > 0$, where $\underline{K}_{i,j}$ and $\overline{K}_{i,j}$ - critical values of factor of correlation at $F^*(K_{i,j})$, according to equal α and (1-α), where $F^*(K_{i,j})$ – statistical function of distribution $K_{i,j}$. The estimation $\underline{K}_{i,j}$ also $\overline{K}_{i,j}$ is spent in following sequence:

2.1. Two samples from m_Σ are modeled random variables ξ with uniform distribution in an interval [0,1];

2.2. Count factor of correlation between realizations samples;

2.3. Items 1 and 2 repeat N time;

2.4. On N to realizations $K^*_{1,2}$ is under construction $F^*(K_{1,2})$;

2.5. Are defined $\underline{K}_{1,2}$ and $\overline{K}_{1,2}$ for set α.

If to assume, that difficulties with various orientations VA eliminated, i.e. all realizations $K^*_{i,j}$ will be positive, the size $\overline{K}_{i,j}$ is necessary for practical calculations only. Some results of calculations $\overline{K}_{i,j}$ for of some α and m are resulted in table 1, and in figure 1 experimental and theoretical dependences $\overline{K}_{i,j} = f(m)$ for most often used value are resulted α=0,05.

Table 1

Critical values of factors of correlation independent samples

n	Significance value			
	0,95	0,975	0,99	0,995
	$\overline{K}$	$\overline{K}$	$\overline{K}$	$\overline{K}$
3	0,989	0,998	0,999	0,999
5	0,778	0,864	0,937	0,959
8	0,632	0,712	0,726	0,847
10	0,514	0,608	0,689	0,730
20	0,393	0,452	0,506	0,534
30	0,301	0,357	0,424	0,446
50	0,220	0,274	0,320	0,377

It is established, that for $m_\Sigma \geq 3$ size $\overline{K}_{1,2}$ with reliability not less than 0,99 can be calculated under the formula $\overline{K}_{1,2} = 1.79/\sqrt{m_\Sigma}$. For example, at $m_\Sigma=20$ sizes $\overline{K}_{1,2} = 1.79/\sqrt{20} = 0.40$, and a divergence with result of modeling do not exceed 2,5 %

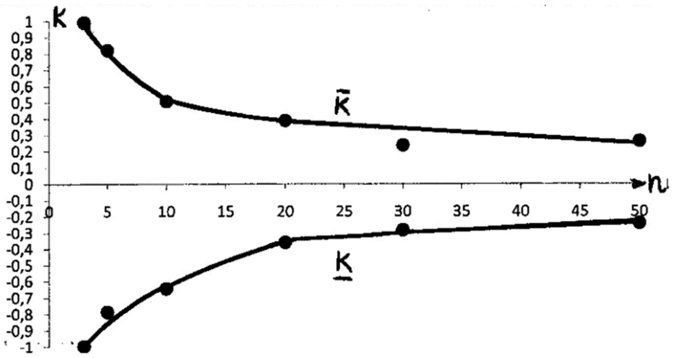


Fig.1. Laws of change of critical value $\overline{K}_{1,2}$ in function m_Σ For α=0,05.

3. Overcoming of influence of various units of measure and scale of parameters is reached by normalization (standardization) of quantitative estimations

Normalization spent on one of following formulas [2]:

$$Z = \frac{X}{\overline{\overline{X}}};\ \frac{X}{X_{max}};\ \frac{X-\overline{X}}{\sigma};\ \frac{X-\overline{X}}{L}$$

where $\overline{X} = m^{-1}\sum_{i=1}^{m} X_i$; X_{max}=max $\{X_1; X_2;..., X_m\}$; X_{min}=min $\{X_1; X_2;..., X_m\}$, $\sigma = \sqrt{\frac{(X-\overline{X})^2}{m-1}}$;

$L=X_{max}-X_{min}$;.

The Importance of these formulas does not stipulate.

As it has noted been above, normalization of initial data is necessary for comparison of attributes. We shall specify, about what comparison there is a speech. For this purpose, it is enough to consider only the first stage of recommended algorithm of classification of initial data [3]. The essence of the first a stage consists in the following. Let given m_Σ objects (for example, power units or transformers, or switches) and their basic characteristics. It is required to range these objects by way of increase of reliability and profitability of their work, for what:

- Realizations of each of quantitative characteristics we shall consider as set of random variables;
- We calculate a number of their statistical parameters. Namely: average arithmetic value $M^*_\Sigma(\Pi_i)$, minimal $\Pi_{i,min}$ and maximal $\Pi_{i,max}$ values, disorder $L^*_\Sigma(\Pi_i)$ and an average quadratic deviation $\sigma^*_\Sigma(\Pi_i)$ under formulas:

$$M^*_\Sigma(\Pi_i) = m_\Sigma^{-1}\sum_{j=1}^{m_\Sigma}\Pi_{i,j}; \quad \Pi_{i.min} = \min\{\Pi_i\}_{m_\Sigma}; \quad \Pi_{i,max} = \max\{\Pi_i\}_{m_\Sigma}; \quad L^*_\Sigma(\Pi_i) = (\Pi_{i,max} - \Pi_{i,min});$$

$$\sigma^*_\Sigma(\Pi_i) = \sqrt{\frac{[\Pi_i - M^*_\Sigma(\Pi_i)]^2}{(m_\Sigma - 1)}}$$

- Realizations for which $\Pi_i > M^*_\Sigma(\Pi_i)$ carry to the first sample (to the first version i- th an attribute). Realizations, for which $\Pi_i < M^*_\Sigma(\Pi_i)$ with $i=1,n_\Sigma$ – to the second sample (accordingly to the second version i- th an attribute). Such classification of data is widely used in practice, physically proved;
- For both samples (v) each set average arithmetic values $M^*_{v,1}(\Pi_i)$ and $M^*_{v,2}(\Pi_i)$ with $i=1,n_\Sigma$ are calculated. We shall notice, that essential distinction $M^*_{v,1}(\Pi_i)$ and $M^*_{v,2}(\Pi_i)$ is caused by distinction of number of both elements samples ($m_{v,1,i} \neq m_{v,2,i}$);
- For everyone ($i=1,n_\Sigma$) data sets we define sample, for which divergence $\Delta M^*_v(\Pi_i)$ the greatest, i.e. $\Delta M^*_v(\Pi_i) = \max\{\Delta M^*_{v,1}(\Pi_i); \Delta M^*_{v,2}(\Pi_i)\}$, where $\Delta M^*_{v,1}(\Pi_i) = [M^*_{v,1}(\Pi_i) - M^*_\Sigma(\Pi_i)]$, and $\Delta M^*_{v,2}(\Pi_i) = [M^*_\Sigma(\Pi_i) - M^*_{v,2}(\Pi_i)]$;
- Define the greatest value $\Delta M^*_v(\Pi)$ among n_Σ values $\Delta M^*_v(\Pi_i)$. Here we collide with distinction of dimension and scale of attributes.

In the subsequent, we shall consider efficiency of following transformations

$$\delta\Pi_{i,1} = \frac{\Pi_i}{M^*_\Sigma(\Pi_i)}; \quad \delta\Pi_{i,2} = \frac{\Pi_i}{L^*_\Sigma(\Pi_i)}; \quad \delta\Pi_{i,3} = \frac{\Pi_i - M^*_\Sigma(\Pi_i)}{M^*_\Sigma(\Pi_i)}; \quad \delta\Pi_{i,4} = \frac{\Pi_i - M^*_\Sigma(\Pi_i)}{L^*_\Sigma(\Pi_i)}$$

III. RESULTS OF COMPARISON WAYS OF NORMALIZATION INITIAL DATA

If to consider the above-stated it is easy to conclude, that without normalization, ranging of objects on the importance at number of attributes $n_\Sigma>1$ it is labour consuming and with growth n_Σ Labour input increases. In the illustrative purposes in table 2 two characteristics of power, units are resulted and it is required to range these power units on the importance.

Table 2

Monthly average data on work PU

Parameter	Conditional number PU							
	1	2	3	4	5	6	7	8
Specific charge of conditional fuel	374,6	371	368,4	369,7	336,7	373,9	363	374,2
Charge of the electric power on own needs	4,1	4,4	4,0	3,9	3,5	4	3,7	3,5

For normalization we shall define, corresponding statistical parameters of the sets resulted in table 2. Results of calculations given in table 3

Table 3

Results of calculations of sample parameters

Parameter	$M_{\Sigma}^{*}(\Pi_i)$	$\Pi_{i,\,min}$	$\Pi_{i,max}$	$L_{\Sigma}^{*}(\Pi_i)$
Specific charge of conditional fuel	366,4	336,7	374,6	37,9
Charge of the electric power on own needs	3,89	3,5	4,4	0,9

In tables 4 and 5 results of normalization, accordingly, estimations of the specific charge of conditional fuel (C_F^*) and the charge of the electric power on own needs P_{ON}^*) are resulted.

Table 4

Results of transformation of estimations of the specific charge of conditional fuel

Conditional PU	Type of transformation			
	$\delta C_{F,\,1}$	$\delta C_{F,\,2}$	$\delta C_{F,\,3}$	$\delta C_{F,\,4}$
1	1,021	9,88	0,021	0,21
2	1,012	9,79	0,012	0,12
3	1,005	9,72	0,005	0,05
4	1,009	9,75	0,009	0,09
5	0,919	8,88	-0,081	-0,79
6	1,020	9,87	0,020	0,20
7	0,991	9,58	-0,009	-0,09
8	1,021	9,87	0,021	0,21

Table 5

Results of transformation of estimations of the charge of the electric power for own needs

Conditional PU	Type of transformation			
	$\delta P_{ON\,1}$	$\delta P_{ON.,\,2}$	$\delta P_{ON.,\,3}$	$\delta P_{ON.,\,4}$
1	1,054	4,58	0,054	0,24
2	1,131	4,9	0,131	0,58
3	1,028	4,44	0,028	0,12
4	1,003	4,33	0,003	0,01
5	0,900	3,89	-0,10	-0,43
6	1,028	4,44	0,028	0,12
7	0,951	4,11	-0,049	-0,21
8	1,900	3,89	-0,10	-0,43

According to the sequence of classification of initial data stated above, we shall divide samples with identical transformation of realizations of random variables on two groups:

- for the first transformation the first group includes realizations $\delta\Pi_{i,1}>1$, and the second group – realizations $\delta\Pi_{i,1}<1$
- for the second transformation the first group includes realizations $\delta\Pi_{i,2} > \Pi_i / L_{\Sigma}^{*}(\Pi_i)$, and the second group – realizations $\delta\Pi_{i,2} < \Pi_i / L_{\Sigma}^{*}(\Pi_i)$
- for the third and fourth transformation is, accordingly, positive (+) and negative (-) values $\delta\Pi_i$.

In semantic aspect, the first group is a group of "bad" power units, and the second – from "good".

Let's define average value of realizations of each of samples for both parameters and four kinds of transformation $\delta\Pi_i$ and divergences $M^*_\Sigma(\delta\Pi_i)$ with $M^*_{v,j}(\delta\Pi_i)$, i.e. we shall define $\Delta M^*_{\Sigma,j}(\delta\Pi_i) = M^*_\Sigma(\delta\Pi_i) - M^*_{v,j}(\delta\Pi_i)$. Results of calculations are resulted in table 6.

It is obvious, that the more differs $M^*_\Sigma[\Pi_i]$ from $M^*_v[\Pi_i]$, the importance of sample above. This parity accepted to criterion of the importance of sample.

Table 6

Estimations of a deviation of average value of realizations samples from average value of population $\Delta M^*_{\Sigma,j}(\delta\Pi_i)$

Parameter	Groups	Samples of random numbers $\delta_{(\Pi i)}$			
		j=1	j=2	j=3	j=4
Specific charge of conditional fuel	1 2	0,015 0,045	0,15 0,44	0,015 0,045	0,15 0,44
Charge of the electric power on own needs	1 2	0,045 0,083	0,21 0,36	0,045 0,083	0,21 0,36

Analysis of given tables 4, 5 and 6 allows to conclude:

1. Tables 4 and 5 testify that transformations of estimations of the specific charge of conditional fuel and the charge of the electric power on own needs $\delta\Pi_{i,1}$ and $\delta\Pi_{i,2}$ though demand less calculations, but do not solve one of the main tasks of normalization – distinction of scale of measurement.
2. The greatest value of a divergence $\Delta M^*_{\Sigma,1}(\delta\Pi_i) = \Delta M^*_{\Sigma,3}(\delta\Pi_i)$ takes place for sample of realizations of the charge of the electric power for own needs of power units (0.083), and at j=2 and j=4 – for sample of realizations of the specific charge of conditional fuel of power units. Such divergence speaks distinction in reflection of statistical parameters of samples. At j=1 and j=3 average value $M^*_\Sigma(\Pi_i)$ is considered only, and at j=2 and j=4 – average value and disorder. With increase, $M^*_\Sigma(\Pi_i)$ the relative size of a deviation $\delta\Pi_i$ depends on type of a parameter. For example, for the specific charge of conditional fuel the size of a deviation is measured in terms of percent, and for average capacity – in tens percent. With increase in scope $L^*_\Sigma(\Pi_i)$ the relative size of a deviation $\Delta M^*_{\Sigma,j}(\delta\Pi_i)$ increases. The factor of correlation here is significant.
3. Hence, for recommended algorithm from the considered four variants of transformation it is expedient to use only a variant with $\delta\Pi_i = [\Pi_i - M^*_\Sigma(\Pi_i)]/L^*_\Sigma(\Pi_i)$

CONCLUSION

1. In the practice for the classification of multivariate data, are used various methods of normalization of quantitative estimations of the attributes describing object of research.
2. Among transformations used in practice by the most effective it is necessary to consider transformation, for which $\delta\Pi_i = [\Pi_i - M_\Sigma(\Pi_i)]/L_\Sigma(\Pi_i)$

REFERENCES

1. Farhadzadeh E.M., Farzaliyev Y.Z., Muradaliyev A.Z. Method and algorithm of the choice optimum number attributes describing reliability of the equipment of electro installations.

Journal: « Reliability: Theory&applications+. R&RATA (Vol.9 No.1 (32 2014, May, USA, p.73-80

2. Bureeva N.N. Multivariate the statistical analysis with use ПППП "STATISTICA". The methodical material under the program of improvement of professional skill. «Application of software in scientific researches and teaching of mathematics and mechanics». Nizhniy Novgorod, 2007, 112 p.

TOTAL TIME ON TEST TRANSFORMS ORDERING OF SEMI-MARKOV SYSTEM

V. M. Chacko

Department of Statistics
St. Thomas' College, Thrissur, Kerala-680001

ABSTRACT

First passage time of semi-Markov performance process of a multistate system are considered. TTT (Total time on Test) transform ordering is discussed.

1. INTRODUCTION

First passage times of appropriate stochastic process have often been used to represent *times to failure* of devices or systems which are subject to shocks and wear, random repair time and random interruptions during their operations. The life distribution properties of these processes have therefore been widely investigated in reliability and maintenance literature. When the performance process of a multistate reliability system is Markov or semi-Markov process, we need to study the ageing properties of the first passage time distribution from up states to down states. Identification of failure rate model in statistical lifetime data analysis is major problem in the field of reliability and survival analysis. The total time on test (TTT) transform is used as a tool for identification of failure distribution model in binary system. When a system is modeled by Markov or semi-Markov process it is quiet interesting to get a procedure for the ordering of the failure distribution using its TTT which based on transition probability function.

Use of TTT transform for the identification of failure rate models (IFR/DFR/ Bathtub shaped/constant) in the binary system case is discussed by Barlow and Campo (1975). Later, Klefsjo (1982) presented some relationship between the TTT transform and other ageing properties (with their duals) of random variable. Barlow and Proschan (1996) discussed a wide application of IFR/DFR distributions in maintenance and replacement policies of a binary reliability system. Nair et al. (2008) studied the properties of TTT transforms of order n and examined their applications in reliability analysis.

But when we consider a complex system whose performance process is Markov or semi-Markov, we need the knowledge of ordering for applying suitable maintenance and repair/replacement policies. The ordering of distribution of performance process is Markov/semi-Markov will be helpful to the engineers and designers for applying suitable maintenance and repair or replacement policies.

In this paper, we consider a semi-Markov system whose first passage time distribution and the reliability function based on the transition probability function in the up states. We define the TTT using transition probability function. The conditions of ordering are discussed.

This paper is arranged as follows. Section 2 recall the existing results for identification of failure rate model of random variables based on TTT. In section 3, we discuss TTT transform ordering based on transition probability function of a semi-Markov process. In Section 4, we introduce some conditions for ordering properties of the semi-Markov system based on TTT built from transition probability function. Conclusions are given at the last section.

2. TTT TRANSFORM OF A LIFETIME RANDOM VARIABLE

Many parametric lifetime models such as Gamma, Weibull, and Truncated Normal distributions have monotone failure rate. The failure rate function $\lambda(t)$ will be continuous and twice differentiable for all $t > 0$ with the exception of the exponential distribution. Total time on test (TTT) transform is a fundamental tool in reliability investigation.

Given a random sample of size n from a nonnegative random variable with distribution F, let $X_{n1}, X_{n2}, ..., X_{nr}, ..., X_{nn}$ be the order statistic corresponding to the sample. The total time on test to the r-th failure is

$$T(X_{nr}) = nX_{n1} + (n-1)(X_{n2} - X_{n1}) + ... + (n-r+1)(X_{nr} - X_{n(r-1)}).$$

It is the sum of all observed lifetimes that can be expressed as $T(X_{nr}) = \sum_{k=1}^{r} X_{nk} + (n-r)X_{nr}$.

If F is exponential with mean θ and we observe the first r ordered values, then it is well known that the maximum likelihood, minimum variance, unbiased estimator of θ is, $\hat{\theta}(r,n) = \frac{1}{r} T(X_{nr})$.

Define $H_n^{-1}(r/n) = \frac{1}{n} T(X_{rn})$, then $H_n^{-1}(r/n) = \int_0^{F_n^{-1}(r/n)} [1 - F_n(u)]du$, where $F_n(u)$ is the empirical distribution function defined as

$$F_n(u) = \begin{cases} 0, & u < X_{n1} \\ i/n, & X_{in} \le u < X_{i+1,n} \\ 1, & u \ge X_{nn} \end{cases} \quad and \quad F_n^{-1}(u) = \inf\{x : F_n(x) \ge u\}.$$

The fact that $F_n(x) \to F(x)$ uniformly at continuity points of F, by Glivenko Cantelli theorem, implies $\lim_{n\to\infty, r/n \to t} \int_0^{F_n^{-1}(r/n)} [1 - F_n(u)]du = \int_0^{F^{-1}(t)} [1 - F(u)]du$ uniformly in $t \in [0,1]$.

Define $H_F^{-1}(t) = \int_0^{F^{-1}(t)} [1 - F(u)]du, \quad t \in [0,1]$ to be the TTT of the distribution F.

Barlow and Campo (1975) proved the following result.

Theorem 2.1 *There is a 1-1 correspondence between distributions F and their transform* H_F^{-1}.

Note that H_F is a distribution with support on $[0, \mu]$, where μ is the mean of F, since $H_F^{-1}(1) = \int_0^{F^{-1}(1)} [1 - F(u)]du = \mu$ when $F(0^-) = 0$.

Then $\phi(t) = \frac{H_F^{-1}(t)}{H_F^{-1}(1)}$ is a continuous increasing function on [0; 1], which is 0 at $t = 0$ and 1 at $t = 1$.

Model Identification

Let $G(x) = 1 - \exp(-x/\theta), \quad x, \theta \ge 0$ be the exponential distribution with mean μ. Then $H_G^{-1}(t) = \int_0^{G^{-1}(t)} e^{-x/\theta} dx = \int_0^{G^{-1}(t)} \theta dG(x) = \theta t$ and scaled TTT,

$$\phi(t) = \frac{H_G^{-1}(t)}{H_G^{-1}(1)} = t, \quad t \in [0,1]. \qquad (3.1)$$

The scaled TTT of the Exponential distribution is a 45^0 line on $[0,1]$. The normalized total time on test is the boundary between the corresponding transforms of IFR and DFR distributions. TTT that permits to classify distributions according to their failure rate is that its slope evaluated at *t* = *F*(*x*) is the reciprocal of the failure rate at *X*.

$$\frac{d}{dt}H_F^{-1}(t)|_{t=F(x)} = \frac{(1-t)}{f[F^{-1}(t)]}|_{t=F(x)} = \frac{1-F(x)}{f(x)} = \frac{1}{\lambda(x)}, \qquad (3.2)$$

where λ is the failure rate of *F*.

Semi-Markov system

We are concerned with a multistate system (MSS) having $M + 1$ states 0,1,…,M where '0' is the best state and '*M*' is the worst state, see Barlow and Wu (1978) for details of MSSs. At time zero the system begins at its best state and as time passes the system begins to deteriorate. It is assumed that the time spent by the system in each state is random with arbitrary sojourn time distribution. The system stays in some acceptable states for some time and then it moves to unacceptable (down) state. The first time at which the MSS enters the down state after spending a random amount of time in acceptable states is termed as the first passage time (failure time) to the down state of the MSS. We study the aging properties of the first passage time distribution of the MSS modeled by the semi-Markov process $\{Y_t, t \geq 0\}$. In the MSS with states *{0,1,...,k,k+1,...,M}* where *{0,1,...,k}* is the acceptable states, the sojourn time between state '*i*' to state '*j*' is assumed to be distributed with arbitrary distribution F_{ij}.

First Passage Time And Reliability Function

Let *E* = *{0,1,...,M}* be a set representing the state of the MSS and probability space with probability function *P*, on which we define a bivariate time homogeneous Markov chain $(X,T) = \{X_n, T_n, n \in \{0,1,2,...\}\}$, X_n takes values of *E* and T_n on the half real line $R^+ = [0,\infty)$, with $0 \leq T_1 \leq T_2 \leq ... \leq T_n \leq ...$ Put $U_n = T_n - T_{n-1}$ for all $n \geq 1$. This Markov process is called a Markov renewal process (MRP) with transition function, the semi-Markov kernel, Q = [Q_{ij}], where $Q_{ij}(t) = P[X_{n+1} = j, U_n \leq t \mid X_n = i], i, j \in E, t \geq 0$ and $Q_{ii}(t) = 0, i \in E, t \geq 0$.

Now we consider the semi-Markov process (SMP), as defined in Pyke (1961). It is the generalization of Markov process with countable state space. SMP is a stochastic process which moves from one state to another of a countable number of states with successive states visiting form a Markov chain, and that the process stays in a given state a random length of time, the distribution of which may depend on this state as well as on the one to be visited in the next. Define $Z_t = X_{N_t}, N_t = \sup\{n, T_n = U_1 + U_2 + ... + U_n \leq t\}$, it is the semi-Markov process associated with the MRP defined above. In terms of *Z*, the times $T_1, T_2, ...$ are successive times of transitions for *Z*, and $X_0, X_1, ...$ are successive states visited. If Q has the form $Q_{ij}(t) = P[X_{n+1} = j \mid X_n = i][1 - e^{-\lambda(i)t}], i, j \in E, t \geq 0,$ for some function λ(*i*), $j \in E$ then the process Z_t is a Markov process. That is, in a Markov process, the distributions of the sojourn times are all exponential independent of the next state. The word *semi*-Markov comes from the somewhat limited Markov property which *Z* enjoys, namely, that the future of *Z* is independent of its past given the present state provided the "present" is the time of jump. Let I_{ij} =indicator function of $\{i = j\}$. Define the transition probability that system occupied state $j \in E$ at time *t* > 0, given that it is started at state *i* at time zero, as, $i, j \in E, t \geq 0$

$$p_{ij}(t)=P[Z_t=j\mid Z_0=i]=P[X_{N_t}=j\mid X_0=i]=h_i(t)I_{ij}+Q*P(t)(i,j),$$

where $h_i(t)=1-\sum_k Q_{ik}(t),\quad P(t)=[p_{ij}(t)]$ and $Q*P(t)(i,j)=\sum_{k\in E}\int_0^t Q_{ik}(dx)p_{kj}(t-x)$

To obtain the reliability function of the semi-Markov system described above, we must define a new process, *Y* with state space $U\cup\nabla$, where *U* denotes set of all up states *{*0*;* 1*; :::; k}* and ∇ is the absorbing state in which all the states *{k* + 1*, ...;M}* of the system is united. Let T_D denote the time of first entry to the down states of *Z* process.

That is, $Y_t=Z_t(\omega)$ if $t<T_D(\omega)$ and $Y_t=\nabla$ if $t\ge T_D(\omega)$.

Let $1=(1,1,...,1)^1$, a unit row vector with appropriate dimension. The process Y_t is a semi-Markov process with semi-Markov kernel

$$\begin{bmatrix}\overbrace{Q_{11}(t)}^{Up} & \overbrace{Q_{12}(t)}^{Down}\\ 0 & 0\end{bmatrix}$$

We denote $\alpha=(\overbrace{\alpha(0),...,\alpha(k)}^{Up,\alpha_1},\overbrace{\alpha(k+1),...,\alpha(M)}^{Down,\alpha_2})$ where $\alpha(i)=P(Y_0=i)$.

The reliability function is

$$\begin{aligned}R(t)&=P[\forall u\in[0,t],Z_u\in U]=P[Y_t\in U]=\sum_{j\in U}P[Y_t=j]\\&=\sum_{i\in U}\sum_{j\in U}P[Y_t=j,Y_0=i]\\&=\sum_{i\in U}\sum_{j\in U}p_{ij}(t)\alpha(i).\end{aligned}\qquad(4.3)$$

3. TTT ORDERING OF SEMI-MARKOV SYSTEM

In order to identify the failure rate behavior of a semi-Markov system based on the transition probability function, we define the TTT based on transition probability function in up states as follows. Let *F* be the first passage time distribution of a semi-Markov system, define

$$H_{p_{ij}}^{-1}(t)=\int_0^{F^{-1}(t)}p_{ij}(u)du,\forall i,j\in U,t\in[0,1]\qquad(4.4)$$

where

$$\begin{aligned}F^{-1}(t)&=\inf\{(1-R(t))\ge t\}\\&=\inf\left\{x:\left(1-\sum_{i\in U}\sum_{j\in U}p_{ij}(x)\alpha(i)\right)\ge t\right\}\\&=\inf\left\{x:\left(\sum_{i\in U}\sum_{j\in U}p_{ij}(x)\alpha(i)\right)\le 1-t\right\}\end{aligned}$$

But

$$\begin{aligned}H_F^{-1}(t)&=\int_0^{F^{-1}(t)}\sum_{i\in U}\sum_{j\in U}p_{ij}(u)du,\forall i,j\in U,t\in[0,1]\\&=\sum_{i\in U}\sum_{j\in U}\alpha(i)\int_0^{F^{-1}(t)}p_{ij}(x)dx\\&=\sum_{i\in U}\sum_{j\in U}\alpha(i)H_{p_{ij}}^{-1}(t)\end{aligned}$$

Then

$$H_F^{-1}(1)=\sum_{i\in U}\sum_{j\in U}\alpha(i)\int_0^{F^{-1}(1)} p_{ij}(x)dx=\sum_{i\in U}\sum_{j\in U}\alpha(i)H_{p_{ij}}^{-1}(1)\quad \frac{H_F^{-1}(t)}{H_F^{-1}(1)}=\frac{\sum_{i\in U}\sum_{j\in U}\alpha(i)\int_0^{F^{-1}(t)} p_{ij}(x)dx}{\sum_{i\in U}\sum_{j\in U}\alpha(i)\int_0^{F^{-1}(1)} p_{ij}(x)dx},\quad t\in[0,1].$$

Let F and G be the distributions of first passage time of two semi-Markov processes with transition probability functions $p_{ij}(u)$ *and* $q_{ij}(u)\ \forall i,j\in U, t\in[0,1]$

Theorem 3.1. IF $F^{-1}(t)\le G^{-1}(t),\forall i,j\in U, t\in[0,1]$ and $p_{ij}(u)\ge q_{ij}(u)\forall i,j\in U, t\in[0,1]$ then $H_{p_{ij}}^{-1}(t)=\int_0^{F^{-1}(t)} p_{ij}(u)du\le \int_0^{G^{-1}(t)} q_{ij}(u)du = H_{q_{ij}}^{-1}(t)$.

When interarrival time of semi-Markov p-system is less that of q-system, we have TTT of F is less than that of G.

Let $H_F^{-1}(t)=\sum_{i\in U}\sum_{j\in U}\alpha(i)H_{p_{ij}}^{-1}(t)\le\sum_{i\in U}\sum_{j\in U}\beta(i)H_{q_{ij}}^{-1}(t)=H_G^{-1}(t), t\in[0,1]$. When this holds we say that the first passage time of semi-Markov p-system is less that that of q-system in TTT order. But this is possible only when $F^{-1}(t)\le G^{-1}(t),\forall i,j\in U, t\in[0,1]$ and $p_{ij}(u)\ge q_{ij}(u)\forall i,j\in U, t\in[0,1]$.

As the rate of transition from state *i* to state *j* is greater in p-sysetm than in q-system then the failure of system occur rapidly. So that TTT of the p-system will be smaller than that of q-system.

Also $F^{-1}(t)\le G^{-1}(t)$ indicate that chance of occurrence of p-system early failure is greater than that of the q-system.

In a semi-Markov system it is very difficult to compute mean or variance or expected value of any convex function of first passage time random variable. In such situation TTT is found to be good ordering tool.

4. TTT ORDERING OF CONTINUOUS TIME MARKOV PROCESSES

Consider a Markov process in continuous time and discrete state space {1,2,…,M}, Doob (1953), p.241. The system starts in state '1' at time zero and as it enters 'M', it remains there. Consider the intensity matrix, $H=[h_{ij}]$ with entries

$$h_{ij}=0,\quad i\in\{1,2,...,M-1\}, j\ne i+1, h_{ii+1}=h,\quad h_{iM}=0.$$

The Kolmogorov's system of differential equation becomes,
for $p_{ij}(t-u)=P[Y_t=j\,|\,Y_u=i], 0\le u<t$ and we take $u=0$,

$$p_{ik}^1(t)=-hp_{ik}(t)+hp_{i+1k}(t), i<M,\quad p_{Mk}(t)=0$$

with initial conditions, $p_{ik}(0)=\delta_{ik}$, the indicator of *{i=k}*. Then,

$$p_{Mk}(t)=0,\quad k\ne M, p_{MM}(t)=1$$

and it is easily verified that the solution is

$$p_{ik}(t)=\begin{cases}0, & k<i\\ \dfrac{(ht)^{k-i}e^{-ht}}{\Gamma(k)}, & i\le k<M\\ e^{-ht}[e^{ht}-1-ht-\ldots-\dfrac{(ht)^{M-i-1}}{\Gamma(M-i)}], & k=M.\end{cases}$$

Here the process is of monotone paths. Now consider $\forall i,j\in\{0,1,\ldots,M-1\}$

$$H^{-1}_{p_{ij}}(1)\quad=\int_0^\infty\frac{(ht)^{k-i}e^{-ht}}{\Gamma(k)}dt=\frac{h^{k-i}}{\Gamma(k)}\int_0^\infty t^{k-i}e^{-ht}dt=\frac{\Gamma(k-i+1)}{\Gamma(k)h}.$$

Therefore

$$\frac{H^{-1}_{p_{ij}}(t)}{H^{-1}_{p_{ij}}(1)}=\frac{\Gamma(k)h}{\Gamma(k-i+1)}\int_0^{F^{-1}(t)}\frac{(hu)^{k-i}e^{-hu}}{\Gamma(k)}du$$
$$=\frac{h^{k-i+1}}{\Gamma(k-i+1)}\int_0^{F^{-1}(t)}u^{k-i}e^{-hu}du.$$

It is the scaled TTT transform for q-system with *r* and *k*.
Similarly, let

$$\frac{H^{-1}_{q_{ij}}(t)}{H^{-1}_{q_{ij}}(1)}=\frac{\Gamma(k)r}{\Gamma(k-i+1)}\int_0^{G^{-1}(t)}\frac{(ru)^{k-i}e^{-ru}}{\Gamma(k)}du$$
$$=\frac{r^{k-i+1}}{\Gamma(k-i+1)}\int_0^{G^{-1}(t)}u^{k-i}e^{-ru}du.$$

be the scaled TTT transform for q-system with *r* and *k*.

Since $u^{k-i}e^{-ru}$ is unimodel curve, $\dfrac{H^{-1}_{p_{ij}}(t)}{H^{-1}_{p_{ij}}(t)}\le\dfrac{H^{-1}_{q_{ij}}(t)}{H^{-1}_{q_{ij}}(1)}$ if $\int_0^{F^{-1}(t)}u^{k-i}e^{-hu}\le\int_0^{G^{-1}(t)}u^{k-i}e^{-ru}du, t\in[0,1], h\le r.$

This is possible only when $F^{-1}(t)\le G^{-1}(t), t\in[0,1]$. But $\overline{F}(x)\le\overline{G}(x)$ imply $F^{-1}(t)\le G^{-1}(t), t\in[0,1]$. Thus $Z_p\le_{st}Z_q$ would imply $Z_p\le_{TTT}Z_q$ order of semi-Markov systems with restrictions.

5. CONCLUSION

The TTT transform ordering of first passage time distribution of a semi-Markov system is discussed. This ordering is applicable for Multi-state systems whose performance process is modeled using semi-Markov process.

REFERENCES

1. Barlow, R. E. and Wu, A. S. (1978) Coherent system with multistate components, *Math.Oper. Res.*, 3, 275-281.
2. Barlow, R. E. and Proschan, F. (1975) Statistical Theory of Reliability and Life Testing, Holt, Rinehart and Winston, New York.

3. R. E. Barlow and Campo, R. A. (1975) Total Time On Test Processes And Application To Failure Data Analysis, reserch report No. University of California.
4. Brown, M. and Chaganty, N. R. (1983) On the first passage time distributions for the class of Markov chains, *Ann. Prob.*, 11, 1000-1008.
5. Bryson, M. C. and Siddiqui, M. M. (1969) Some criteria for aging, *J. Amer. Stat. Asso.*, 64, 1472-1483.
6. Belzunce, F., Ortega, E. M. and Ruizon, J. M. (2002) Ageing Properties of First-Passage Times of Increasing Markov Processes, *Adv. Appl. Prob.*, 34, 241-259.
7. Klefsjo, B. (1981) On Ageing Properties And Total Time on Test Transforms
8. Chacko, V. M. and Manoharan, M. (2009) Ageing properties of first passage time distribution of multistate reliability system, *ProbStat Forum: e-journal*, 2, 22-30.
9. Cinlar, E. (1975) *Introduction to stochastic processes*, Prentice-Hall, Englewood Cliff, N.J.
10. Deshpande, J. V., Kochar, S. C. and Singh, H. (1986) Aspects of positive aging, *J. Appl. Prob.*,23, 748-758.
11. Doob, J. L. (1953) *Stochastic Process*, John Wiley and sons, New York.
12. El.Neweihi, E. and Proschan, F. (1984) Degradable systems: a survey of multistate reliability theory, *Comm. Stat. Theory and Methods*, 13, 403-432.
13. El-Neweihi, E., Proschan, F., and Sethuraman, J. (1978) Multistate coherent systems, *J. Appl. Prob.*, 15, 675-688.
14. Pyke, R. (1961) Markov renewal processes: definitions and preliminary properties, *Ann. Math.Stat.*, 32, 1231-1242.
15. Unnikrishnan. N., P. G. Sankaran and B. Vinesh Kumar (2008) TTT transforms of order n and their implications in reliability analysis, Cochin University of Science and Technology 18.

A SOLUTION CONVERGENCE IN A NEURAL NETWORK, AND AN ACCOUNTING OF LOAD PRIORITIES AT A POWER SYSTEM RESTORATION

M.Uspensky, I.Kyzrodev

●

ISE&EPN Komi SC UrB RAN, Syktyvkar, Russia
uspensky@energy.komisc.ru

ABSTRACT

Uniqueness of solution at a scheme choice for the restorable power system on the artificial neural network (ANN) base is shown. The elementary scheme of a power network is used for this purpose and the subsequent distribution of its results is applied on any network configuration. The way of priority accounting for loads is developed at creation of the restorable power system on the base of the proof on solution uniqueness for ANN.

Keywords: power system restoration, network configuration search, artificial neural network, load priority.

1. INTRODUCTION

Power supply restoration with mode restrictions after blackouts is an important component of operational reliability for power networks [1-3]. This task is difficult for finding of an acceptable solution in the conditions of rigid time constraint because of a large number of sources, consumers and the switching devices. The use of the neural network technique together with graph processing algorithm for its solution was discussed earlier [4-6]. Such combination allows significant accelerating for a solution search, and self-training using of the artificial neural networks (ANN) expands a mode variation area.

Without stopping in details on the basic provisions [4], we want to note, the combination selection of a line breaker state for a power network, offered by ANN, is executed by the mode calculation block (MCB). It checks validity of the received combination on mode conditions and the generalized error vector, which controls ANN solution search. It is obvious; such process should not lead to a situation, when the last offered combination: a) repeats one of the previous ones, b) is situated farther of a required solution, than previous ones. The satisfaction of these constraints defines stability and convergence of the search.

On basis of back propagation algorithm for ANN training [7], it is possible to state, at the specified input data the ANN will aim to configure the weighting coefficients, to get the required response on an output, which set a difference of the current ANN response and an error in our case [5]. Thus, for the proof of solution stability it is required to show convergence of an mismatch error for the current and required ANN responses.

2. A SOLUTION CONVERGENCE

We begin the convergence proof with consideration of an elementary case for three circuits united in the triangular scheme (fig. 1). We admit there are no sources and loadings in a node x_3 for simplification of situation. We construct circuit states by search method of all possible options in the mode of ANN self-training on the scheme base. Since the proof is carried out for a circuit breaker

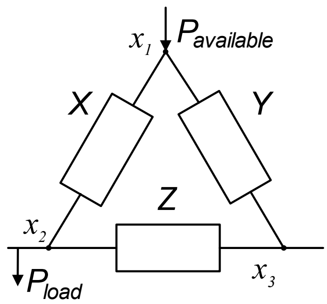


Figure 1. The elementary cell of a power network.

state, target parameter considers the consumer providing with active power. Voltage level restrictions in nodes and currents in circuit are assigned to MCB.

Reaction of the error forming algorithm to all possible breaker state combinations for a distribution network and ANN responses to them is shown in tab. 1. Here ORS – operational restrictions on breakers of a distribution network: 1 – operations are allowed with the breaker, 0 – operations are prohibited with the breaker. ANN – the breaker state is offered ANN: 1 – the breaker is switched on, 0 – the breaker is switched off. MCB – a network state is after MCB work: 1 – the breaker is switched on, 0 – the breaker is switched off. The *n*-th approximation error: 1 – the unallowable switching on because of the mode revealed by the error shaping block in MCB after its work. *X,Y,Z* – circuit breakers (fig. 1).

Table 1. Forming stages of an error vector for all possible breaker state combinations of a network and an ANN response

ANN *XYZ*	ORS *XYZ*	MCB *XYZ*	An error of 1-st approx.	An error of 2-nd approx.	An error of 3-rd approx.	Combination #
000	000	000	000	–	–	0,0
000	001	000	001	–	–	0,1
000	010	000	010	–	–	0,2
000	011	000	011	–	–	0,3
000	100	000	000	–	–	0,4
000	101	000	001	100	000	0,5
000	110	000	010	100	000	0,6
000	111	000	011	000	–	0,7
001	000	000	001	000	–	1,0
001	001	001	001	000	–	1,1
001	010	000	011	000	–	1,2
001	011	001	011	000	–	1,3
001	100	000	001	000	–	1,4
001	101	001	101	000	–	1,5
001	110	000	011	100	000	1,6
001	111	001	011	000	–	1,7
010	000	000	010	000	–	2,0
010	001	000	011	000	–	2,1
010	010	010	000	–	–	2,2
010	011	010	001	000	–	2,3
010	100	000	010	000	–	2,4
010	101	000	011	000	–	2,5
010	110	010	100	000	–	2,6
010	111	010	001	000	–	2,7
011	000	000	011	000	–	3,0
011	001	001	010	000	–	3,1
011	010	010	001	000	–	3,2
011	011	011	000	–	–	3,3
011	100	000	011	000	–	3,4
011	101	001	110	000	–	3,5
011	110	010	101	000	–	3,6

continuation of table 1

ANN	ORS	MCB	An error of 1-st approx.	An error of 2-nd approx.	An error of 3-rd approx.	Combination #
XYZ	*XYZ*	*XYZ*				
011	111	011	000	–	–	3,7
100	000	000	100	000	–	4,0
100	001	000	101	000	–	4,1
100	010	000	110	000	–	4,2
100	011	000	111	000	–	4,3
100	100	000	100	–	–	4,4
100	101	000	101	100	000	4,5 (!)
100	110	000	110	100	000	4,6 (!)
100	111	000	111	000	–	4,7
101	000	000	101	000	–	5,0
101	001	001	100	000	–	5,1
101	010	000	111	000	–	5,2
101	011	001	110	000	–	5,3
101	100	000	101	000	–	5,4
101	101	101	000	–	–	5,5
101	110	000	111	100	–	5,6 (!)
101	111	101	000	–	–	5,7
110	000	000	110	000	–	6,0
110	001	000	111	000	–	6,1
110	010	010	100	000	–	6,2
110	011	010	101	000	–	6,3
110	100	000	110	000	–	6,4
110	101	000	111	100	–	6,5 (!)
110	110	110	000	–	–	6,6
110	111	110	000	–	–	6,7
111	000	000	111	000	–	7,0
111	001	001	110	000	–	7,1
111	010	010	101	000	–	7,2
111	011	011	100	000	–	7,3
111	100	000	111	000	–	7,4
111	101	101	010	000	–	7,5
111	110	110	001	000	–	7,6
111	111	011	100	000	–	7,7

Since the error is equal to zero after several iterations for all considered cases, the assumption of a solution convergence for the power network scheme (fig. 1) is established. It is necessary to consider separately the cases noted by a sign "(!)". Here at the initial stage the error indicates circuit switch on, inadmissible on mode conditions. This circuit will be switched on nevertheless, but at other admissible state combination of breakers. It occurs because the error-shaping block forbids circuit switch on for the studied combination in this case because of mode restrictions, but search of the correct solution remains for ANN.

Now we will consider a case of a power network from any number of circuits, sources and consumers. Let's, $V_k = 1$, if a voltage isn't in the node k, and $V_k = 0$, if the node k is connected correctly. Function f_{mod} is determined as the sum of all nodes V_k, i.e. $f_{mod} = \sum_{k=1}^{n} V_k$, where n is number of nodes, and $\min(f_{mod})$ is the sum of all V_k, which can't be provided with the electric power in the current network configuration because of mode and other restrictions, for example, for single line that is impossible to switch on.

When the first ANN response $(\bar{I})$ is received), it is checked by MCB, then the vector of the generalized error $(\bar{E})$ is formed for the current response. As a result, some solution (a required response) $(\bar{R})$ is formed, to which it is necessary to lead the current response, i.e. $\bar{R} = \bar{I} - \bar{E}$. The $\bar{R}$ definition initializes BP algorithm. ANN weighting coefficients change so that $\bar{I} \to \bar{R}$. When $\bar{I}$ will change (the state at least of one network line will change), training activity stops, a new ANN response $\bar{I}_1$ forms $\bar{E}_1$ and $\bar{R}_1$, then the training algorithm is started again. The training cycle is repeated, while $f_{mod} > \min(f_{mod})$.

Really when forming $\bar{E}$ there is a continuous updating $\bar{R}$, until $\bar{R}$ become a valid solution, i.e. equality will be executed $f_{mod}(\bar{R}) = \min\left(f_{mod}(\bar{R})\right)$. If to prove that f_{mod} constantly aims to $\min(f_{mod})$, it will be proved that any problem can be solved for a finite number of iterations, i.e. computation process is stable.

It is obvious from forming of an error vector algorithm [5] that only one line is considered at the same time. Let's designate it as *X* line from x_1 top to x_2 top. Then all other power network graph can simplify for forming time of an error vector for the considered line as the next ways:

1) if *X* line (switching on) has a available power source $P_{available.}$ (it is admissible in the x_1 node), and some available / consumed power $P_{\pm}$ is available in the x_3 node, the graph is minimized to presented it on fig. 2. Thus, all other network can be minimized to relative *Y* line, which characterizes possibility to give power from x_3 to x_1 (operations with *Y* are prohibited in the presence $P_{available.}$ and P_{+}), and to relative *Z* line, which considering possibility to provide the x_2 loading through the x_3 node, and also $P_{\pm}$, which can be to receive/deliver through *Z* or *Y*. Here $P_{\pm}$ is meant that the node can be a source, a consumer or their combination. P_{+} and $P_{available}$ can have both one source and different ones. Then the graph convergence of a power network is considered in tab. 1 (a combination 2, 7). If *Z* line is absent / is prohibited to switching on, the combination 2, 3 corresponds from tab. 1 to this case. If *Y* line is absent / is prohibited to switching on, the combination 2, 6 corresponds from tab. 1 to this case. If *Z* and *Y* lines are absent / are prohibited to switching on, the combination 2, 2 corresponds from tab. 1 to this case.

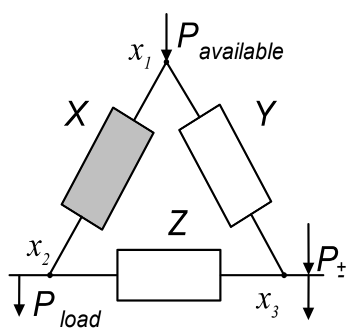


Figure 2. Scheme of network with three lines and available power in x_1 node.

2) If $P_{available.}$ is in the x_1 node and the x_2 node is provided with the power through *Z* line, the network can be minimized to presented it on fig. 3. In this case *Y* line characterizes possibility of power transfer to the x_1 or x_2 nodes by a different way. It is considered in tab. 1 (a combination 4, 7). If *Z* line is absent / is prohibited to switching on, from tab. 1 the combination 4, 6 corresponds from tab. 1 to this case. If *Y* line is absent / is prohibited to switching on, from tab. 1 the combination 4, 3 corresponds from tab. 1 to this case. If *Z* and *Y* lines are absent / are prohibited to switching on, the combination 4, 2 corresponds from tab. 1 to this case.

It is obvious that two described above a case are almost identical and easily pass one in other replacement of *X* line by *Z* and vice versa, but they show in couple as ANN training will behave, when finding the best option of power transfer to the consumer, which is already provided with energy.

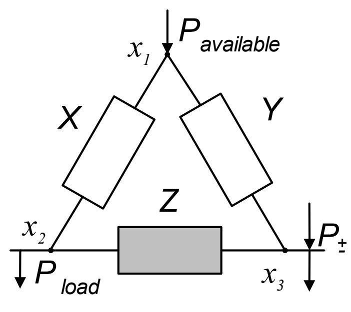


Figure 3. Scheme of network with available power in x_1 and x_3 nodes.

3) If the x_1 and x_2 nodes are not connected to a source, the simplified network graph corresponds to it on fig. 4 where Y line allows connecting the x_1 node, and Z line allows connecting the x_2 node. This case is provided to tab. 1 (a combination 0, 7). If Z line is absent / is prohibited to switching on, the combination 0, 6 corresponds from tab 1 to this case. If Y line is absent / is prohibited to switching on, the combination 0, 3 corresponds from tab. 1 to this case. If Z and Y lines are absent / are prohibited to switching on, the combination 0, 2 corresponds from tab. 1 to this case.

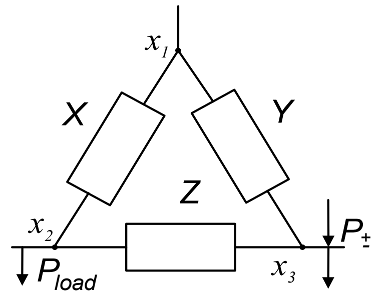


Figure 4. Scheme of network without available power in x_1 and x_2

After forming of an error for X line $=X_i$ the program passes to forming of an error for other lines $X=X_k$. There are considered the simplified network graph representations, and X_i can belong now both Y line, and Z line for new minimized graphs.

The similar is executed for all lines of the network graph then forming process of an error comes to an end, ANN training procedure is executed to change of its response. Thus, function f_{mod} changes towards reduction. The further forming procedure of the generalized error and ANN training repeats necessary number of times, until the condition $f_{mod} = \min(f_{mod})$ will be satisfied.

Thus, it is proved that each subsequent solution does not increase the search function of combinations f_{mod}, therefore, a solution is stabile.

3. AN ACCOUNTING OF LOAD PRIORITIES

The restorable generators start giving out power and gradually increase it in restoration process of power system. At equal load importance, their providing on the ANN basis is carried out according to above the stated technique. However, different consumers define requirements to load restoration urgency in different degree. In this case, operational restriction on the breaker is prohibition on its switch on (because of repair, audit, etc.). Nevertheless, it is obvious that first of all it is necessary to provide auxiliary of the generator. Generally, degree of urgency is defined by loading priority. How there is such providing for loadings?

If ANN is used, a task complexity is connected with a parallel search of the scheme for load providing. Generally, operational restriction on the breaker defines prohibition on change of the breaker state by the scheme search. Such approach allows using of ANN for load providing taking into account their priority.

If the highest priority loads are provided, the their values, the available powers in the corresponding network nodes and data on the prohibited to change of breaker states are used as basic data (here, prohibition on switch on), if those are available, and zero loadings in all other nodes. The scheme of load providing for the highest priority is defined and is remembered in the training set of the highest priority level. Further, all breakers of network which are switched on at this stage are put under restriction "prohibition on change of state". Loadings of the second priority level are added in node data, and scheme search of load providing is carried out, including the second priority level. Transition to the following priority level is made on power availability of sources after the scheme solution for level with the highest priority. The found new solution is remembered in the training set of the second priority level. These operations repeat for loadings of each priority and stop, if the available power of sources will be settled or if the level of the lowest priority is provided.

Thus, own training set is formed at each priority level, which allows putting in unambiguous compliance the basic data with the received decision, as it has been shown for case with one priority level at assessment of solution convergence on the ANN basis.

4. CONCLUSIONS

A power system restoration can be automated by an ANN use for increase of its reliability. It allows reducing the recovery time of a power supply especially for consumers of a high priority.

It is important to convince for network scheme search of load providing on the ANN basis that the solution is only for the proper data set. It is proved on the analysis basis of the elementary scheme and options of reduction of the network scheme to the elementary scheme that each combination of a breaker state does not repeat in the scheme search process of a network restoration, and each subsequent solution does not increase combination search function f_{mod}, therefore, a solution is stabile and converges to $\min(f_{mod})$.

For accounting of load priority in network scheme search by ANN means, it is necessary to impose condition "prohibition on the change of breaker state" for the breakers, which prohibited for switching on and for the breakers, which have connected loadings with higher priority. Own training set is formed at each priority level, which allows putting in unambiguous compliance the basic data with the received decision. The uniqueness of the solution has been proved earlier for the single priority level.

5. REFERENCES

[1] Adibi M.M., Fink L.H. Restoration after cascading failures // IEEE power & energy magazine, September/October, 2006. – P. 68-77.
[2] Barkans J., Zalostiba D. Protection against Blackouts and Self-Restoration of Power Systems / Riga, Latvija RTU Publishing House. 2009. – 142 p.
[3] Bretas A. S., Phadke A. G. Artificial Neural Networks in Power System Restoration // IEEE Trans. on PWRD. – 2003. Vol. 18. No. 4.– P. 1181-1191.
[4] Uspensky M., Kyzrodev I. Power supply restoration in distributive networks Reliability // Theory & Applications Vol.2. No. 2, issue of June, 2011 – P. 72-84.
[5] Uspensky M., Kyzrodev I. Combined Method of a Distribution Network Reconfiguration for Power Supply Restoration // Proceeding of the IEEE PowerTech 2005, St. Petersburg, Russia, 27-30.06, Ref. 33.
[6] Uspensky M., Smirnov S. The Occurrence Reasons and Countermeasures to Power System Blackouts // The International Journal of Energy Engineering, No. 1, 2014. – P. 1-8.
[7] Rumelhart D.E., Hinton G.E., Williams R.J. Learhing integral representations by error propagation // Parallel Distributed Processing, vol. 1, No. 8, 1986. – P. 318-362

ISSN 1932-2321