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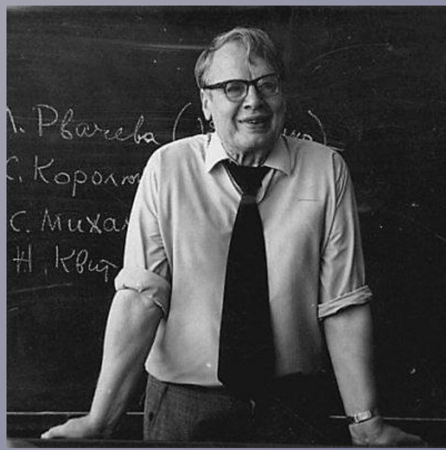

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RELIABILITY:
THEORY&APPLICATIONS

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RISK ANALYSIS
MAINTENANCE
SAFETY

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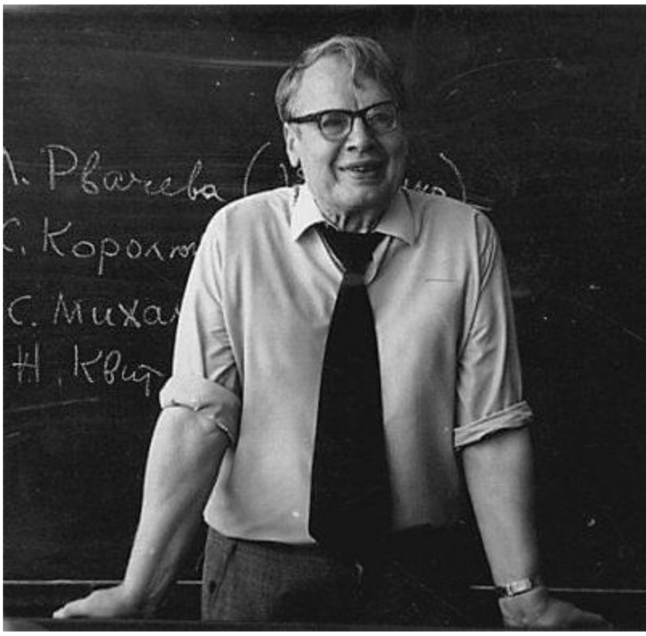

RELIABILITY:

THEORY & APPLICATIONS

Vol.9 No.4 (35),
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San Diego
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Table of Contents

In this paper, probabilistic models for five repairable redundant network flow systems have been developed to analyze and compare their availability and profit. Explicit expressions for steady-state availability, busy period of repairman and profit function for the five redundant network flow systems are developed. Furthermore, we compare the five redundant network flow systems based on their availability and profit and found that configuration II is more reliable and profitable than the remaining configurations.

This is short review of previous publications and new development of the concept of Daniels' sequence (DS) which can be used to consider the association of distribution of the static strength of unidirectional fiber composite (UFC) component with distributions of its fatigue life and residual strength of the UFC itself. A generalization of the model of series-parallel system as description of structure of UFC is given. A new description of the defected structural longitudinal element (LE) of UFC, new condition of failure of UFC, a definition of residual Daniels' function is introduced. Numerical examples of processing of experimental data (data of fatigue test of glass-fiber composite) are presented. The estimates of the parameters of the corresponding nonlinear regression model for simultaneous processing of result of the fatigue test and test of residual tensile strength are obtained (parameters of the distribution of the local strength parameters of structural LE).

This study deals with simple Constant Stress Partially Accelerated life test (CSPALT) using type-I censoring. The lifetime distribution of the test item is assumed to follow Gompertz distribution. The Maximum Likelihood (ML) Estimation is used to estimate the distribution parameters and acceleration factor. Asymptotic confidence interval estimates of the model parameters are also evaluated by using Fisher information matrix. Statistically optimal PALT plans are developed such that the Generalized Asymptotic Variance (GAV) of the Maximum Likelihood Estimators (MLEs) of the model parameters at design stress is minimized. In the last, to illustrate the statistical properties of the parameters, a simulation study is performed

USE OF A DISCRETE SIMULATION FOR AN ASSESSMENT OF THE MISSION COMPLETION FROM A VIEWPOINT OF MAINTENANCE COSTS

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•

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ABSTRACT

An anticipated simulation is an experimental method of study, using probability dynamic approaches using experimentation with a computer model generated in Matlab. Simulation is seen as a process of a gradual building up of a mathematical and logical model described with computer algorithms of a simulation program representing a model of a reviewed system and subsequently experimentation with such a model aiming to obtain estimates of results of the system activity. There are many positive reasons why to perform a simulation study, as for example a missing analytic description of a system or an assessment of a capacity value of an existing system within other operational conditions. Major problems, rising in respect to a model implementation in a program, are recording the model structure, obtaining random parameter values, recording the dynamic features of models. Additional issues are added as experimentation with a model or development of resulting output in a suitable form.

1 ANALYSIS OF COSTS AND A RATE OF THEIR RISK

We used the distribution parameters obtained for simulation of the same numbers of values as it is for number of data we had processed and assessed. Through comparing we can see that results from simulation of an exponential distribution and Weibull distribution of probability match with hypotheses. Of course, they do not significantly differ with regard to the parameter of a shape of the Weibull distribution being close to 1 value.

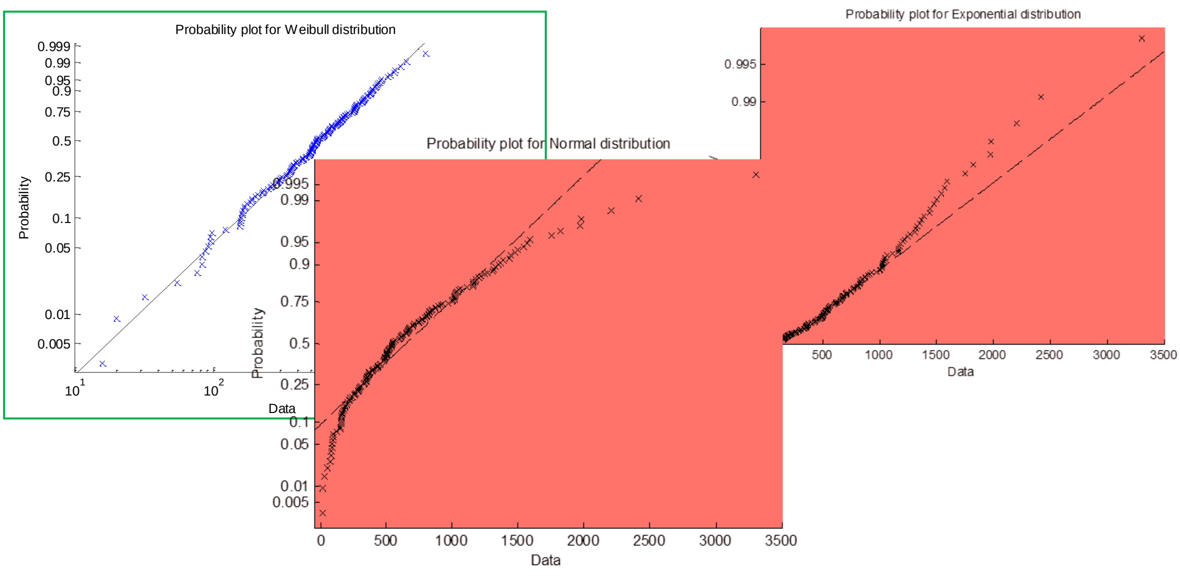

Figure 1. Verification of hypotheses for an exponential distribution from generated data.

Simulated values of a normal distribution with calculated parameters proved an assumption, that a standard deviation approaching to a mean value will cause an occurrence of negative values.

Hypothesis of a non-acceptation of a normal distribution was validated. We chose the most suitable hypothesis of the Weibull distribution of probability we had verified and stated parameters of probability distribution of input parameters for failure-less operation with a 99.9%, level of reliability

Maintenance costs for particular groups of vehicle statistically processed and they show the distribution parameters illustrated in the Figure 4 and Figure 5.
and courses of function for a probability of density and distribution function in the Figure 2 and Figure 3.

From a course of costs distribution functions we can conclude a range in which the costs would occur. The costs for material, assessing a mean of probability 0.5, define an increasing order for costs in cost groups as electric installation, steering, body, and a frame, braking system, gear system, engine with systems Figure 6 and Figure 7.

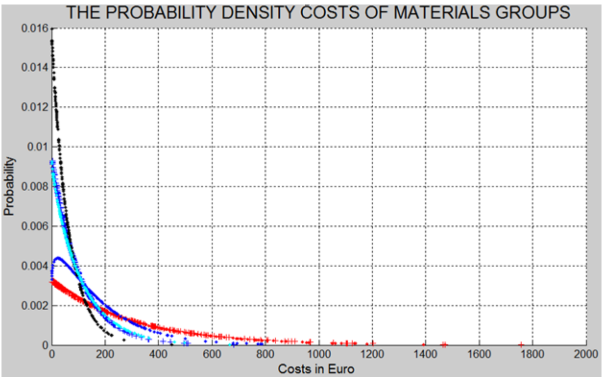

Figure 2. The probability of density costs of material groups

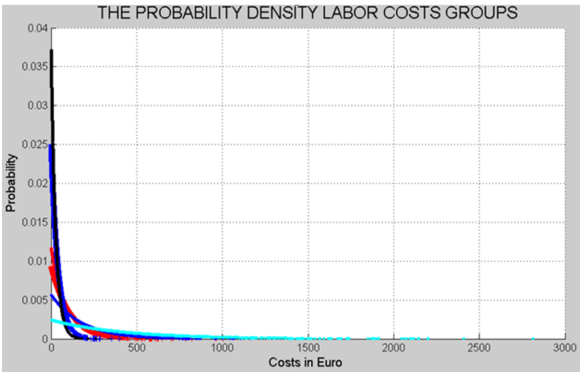

Figure 3. Probability of density of labor costs groups

At labor costs the order is electric installation, steering, gear system, an engine with systems, a braking system, body and a frame Figure 5.

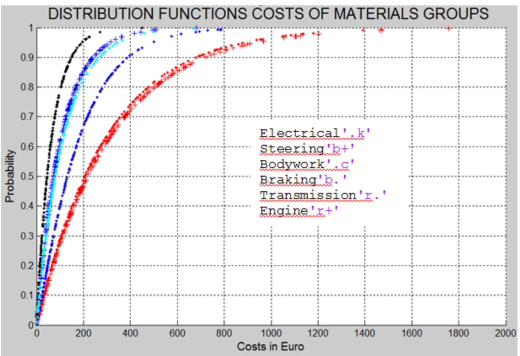

Figure 4. Distribution functions of costs of material groups.

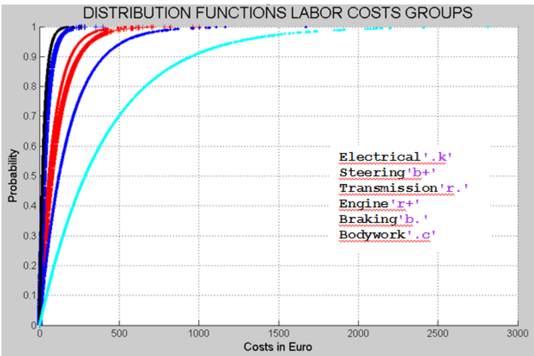

Figure 5. Distribution functions of labor costs groups

We use a function of density of a failure probability as a rise of a negative event – a failure and an amount of total costs as a result of an unfavourable event.

Visual expression of an intersection of these events gives us a notion about a rate of rise of critical situation. We can quantify this fact and to express it by probability of risk matrix.

We will use a distribution of a failure probability to generate a rise of a negative event – a failure and a distribution of a probability of costs to generate the amount of costs resulted from an unwanted event.

Graphic expression of an intersection of these events in a point of costs matrix and operation in hours provides us with a perception relating with quantification if a risk situation rises. Aggregations of their occurrence and their quantification on the legs enable comparing the risks from costs for maintenance of objects being assessed.

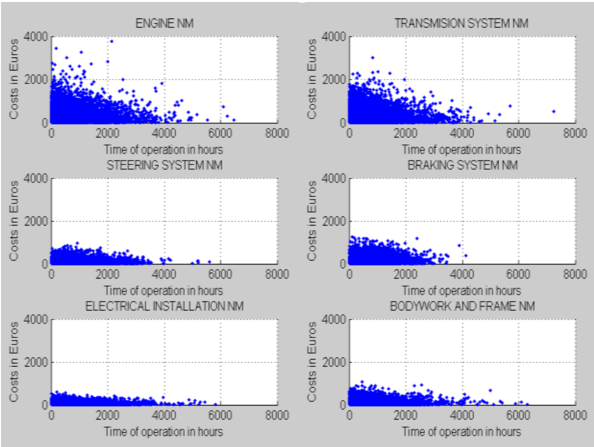

Figure 6. Probable risk matrix of costs of materials

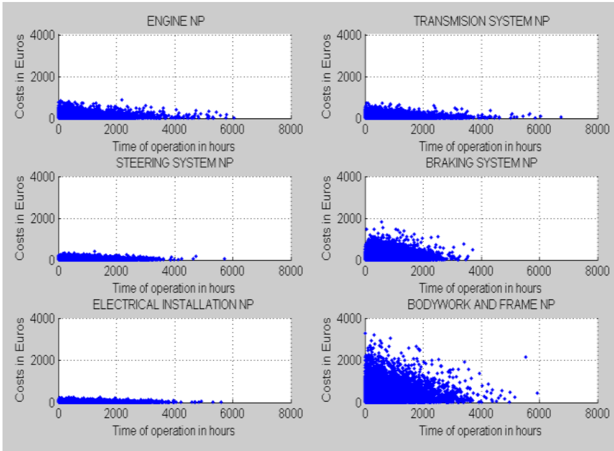

Figure 7. Probable risk matrix of labor costs groups

We can quantify this probability and to define it with a probability of elements, lines or columns of the risk matrix.

2. DISCRETE SIMULATION WITH A VARIABLE TIME STEP IN ASSESSING COSTS TO MISSION

A previous way of risk assessment resulting from costs to maintenance shows that it was made a provision for a whole life cycle phase. In some way it takes a time factor into consideration. In a practical risk definition within a defined time period, expressed by consumed operational units and limited funds for maintenance, it is more suitable to use a discrete simulation with a variable time step.

Essence of simulation consists in performing of following activities:

1. Substitution of initial terms and specification of values of variables in an initial simulation time period.
2. Generating the intervals of rise of failures, maintenance period and amount of particular costs from probabilities distribution.
3. Shift of a time axis by a failure interval and a maintenance interval. Increase of costs amount by a generated value.
4. To collect and to process statistical methods on input and output parameters.
5. Testing of terms of simulation course completion. If a value of a simulation time reaches a TEND value having been defined in advance, a simulation course stops. Otherwise the activities in 2-4 points repeat.
6. Outputs of results on a display and a printer. Completion of a simulation experiment after having completed all simulation courses matching a number of vehicles being asessed.

In models there are used the parameters of input data obtained from previous experiments and from articles having been published before. Course and a way of displaying the simulation experiments is obvious from illustration of 5 simulations. Course and a way of displaying the simulation experiments is obvious from illustration of 10 simulations.

Parameter, distribution, input parameters
TBF Weibull *a= 676.1854287560616* *b= 1.0655900608402*
TTR Exponential *l= 6.974398734177213*
NM Weibull *a= 158.4948195963805* *b= 0.6430315567794*
NP Weibull *a= 52.33794187645434* *b= 0.7037168761996*

Figure.8 Parameters of input data

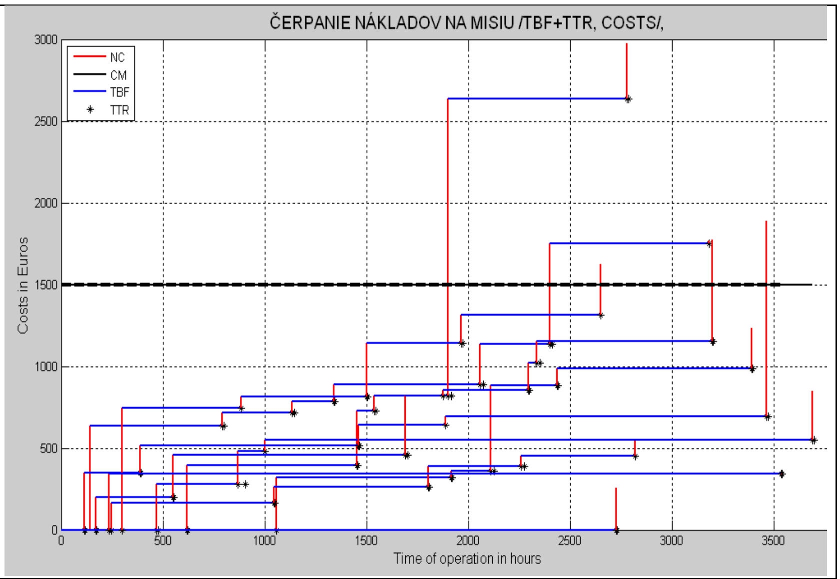

Figure.9 Course of simulation of 10 vehicles, probability of a mission completion is 0.8838

Total costs are illustrated as an addition of generated costs – labor costs and material costs. The intervals in the TTR maintenance implementation are short, we marked them with an /*/ asterisk. The assigned funds in Euros for a maintenance of a vehicle during a mission are *CM=1500* Euros. An expected number of operational hours of the vehicle, Time of operation = 2500 hours, i.e. 150000 km illustrated with a completion of simulation when an event exceeding this time period has been completed. The numerical statistical values of probability that the assigned funds will not overrun the funds assigned for all mission vehicles are processed in addition to a graphical illustration. A simple case of a statistical processing of data is shown in a following table.

Table. 1 Probabilities of mission completion with 6 vehicles in 10 simulation courses

SIM. COURSE	Probabilities of a mission completion with vehicles						
	1	2	3	4	5	6	MEAN
1	0.8472	0.6591	0.0891	1.1328	2.6407	0.8636	1.0388
2	1.0379	3.5237	0.7768	1.1540	1.3789	0.2427	1.3523
3	0.5262	2.4834	2.0140	0.4064	0.3243	0.1223	0.9794
4	1.0966	0.3014	1.0631	0.3395	0.3297	0.9167	0.6745
5	0.6773	0.3837	0.6154	0.5474	0.5136	1.0200	0.6262
6	1.6672	2.4367	0.5955	1.4356	2.6404	1.5050	1.7134
7	0.7552	0.4496	0.4994	2.4372	0.6153	1.8709	1.1046
8	0.5868	0.7583	0.7027	0.1376	2.4066	0.4078	0.8333
9	1.3435	0.7022	0.7442	0.7951	1.8390	1.3646	1.1314
10	2.6011	1.1555	0.3043	0.8724	0.2607	1.3646	0.8772

Table.2 Total statistical indicators of a mission simulation courses

SIM. COURSE	TOTAL STAT INDICATORS				
	MAX	MIN	MEAN	STD	MED
1	2.6407	0.0891	1.0388	0.8589	0.8554
2	3.5237	0.2427	1.3523	1.1328	1.0960
3	2.4834	0.1223	0.9794	1.0030	0.4663
4	1.0966	0.3014	0.6745	0.3894	0.6281
5	1.0200	0.3837	0.6262	0.2170	0.5814
6	2.6404	0.5955	1.7134	0.7423	1.5861
7	2.4372	0.4496	1.1046	0.8390	0.6853
8	2.4066	0.1376	0.8333	0.8030	0.6448
9	1.8390	0.7022	1.1314	0.4577	1.0693
10	2.6011	0.0692	0.8772	0.9392	0.5883
MEAN S	2,2688	0,3093	1,033	0,7556	0,7613

A more complicated case is illustrated in the following picture. It illustrates a course of a simulation experiment for 56 vehicles.

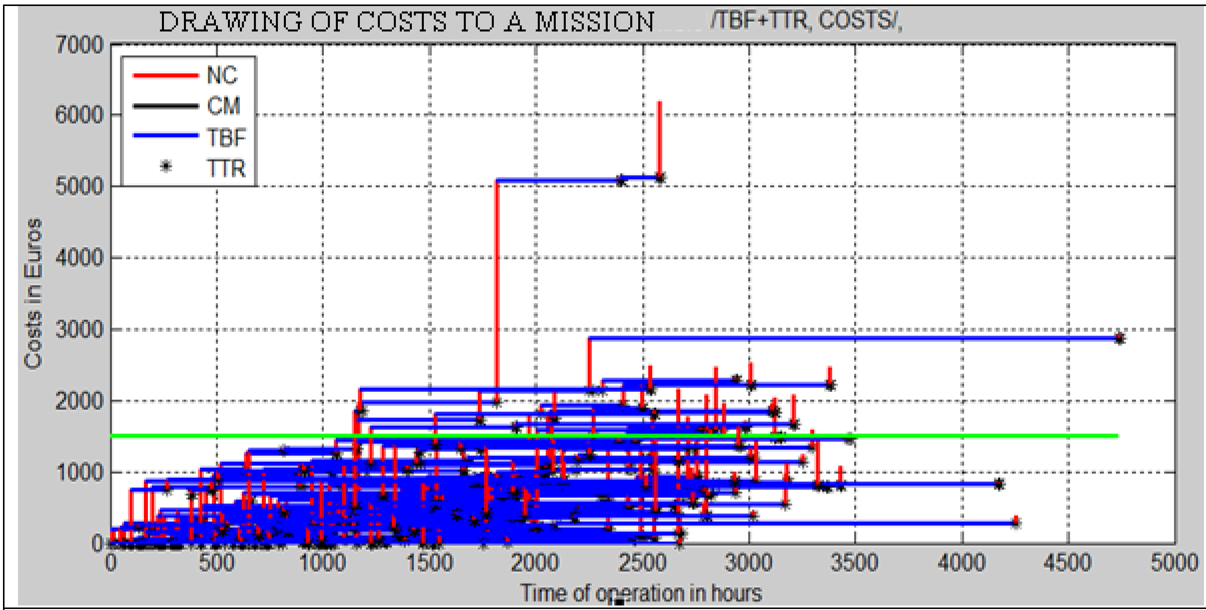

Figure. 10 Flowchart of a simulation experiment.

Probability that the vehicles will not exceed an assigned limit per a vehicle for a mission amounting 1500 Euros is 0.9894. Graphical illustration requires computer equipment with a large memory and a swift processor. If we remove the graphic we can assess and statistically process more simulation courses or to use other graphical means.

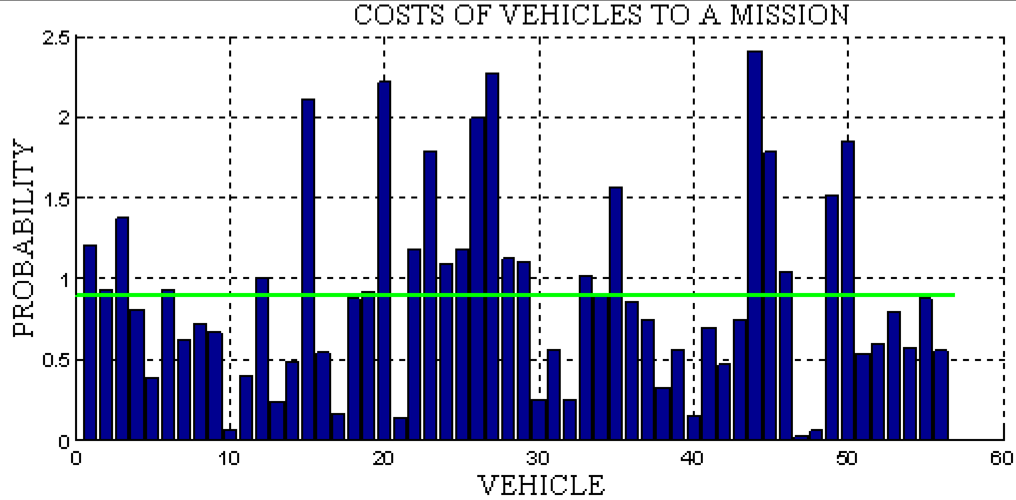

Figure. 11 Illustration of a simulation experiment with a histogram for 56 vehicles.

Probability that the vehicles will not exceed an assigned limit per a vehicle for a mission amounting of 1500 Euros is 0.8960. The largest possible number of simulation courses is needed for a serious assessment of results; the graphics is unable to provide suitable predicative information.

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In Table there are shown statistical results for an increasing number of simulation courses for assessment of 56 vehicles. Previous way of expression with tables as in a previous simple case is not possible. We are processing only aggregate statistical indicators.

Table.3 Processing of aggregate statistical indicators

Num.of sim. courses	AGGREGATE STAT. INDICATORS				
	MAX	MIN	MEAN	STD	MED
10	2.8555	0.0675	0.8561	0.6055	0.7102
100	2.9676	0.0725	0.8835	0.6191	0.7438
1000	2.9841	0.0717	0.8891	0.6221	0.7520
10000	3.0097	0.0720	0.8908	0.6239	0.7537

We can see that values of aggregate statistical indicators usually increase. It means and it proves that an affected area enlarges with an increased number of simulations as well as values of risk indicators. In taking the MEAN value into consideration ranging from 10 up to 10000 simulations by a value of 0,0347 i.e. 3.47 %.

3. CONCLUSIONS

The above mentioned outputs and data processing from the performed experiments result in the following conclusions. Statistical characteristics of a failure-free operation of vehicles, particular groups and statistic characteristics of costs are more suitable for an application of risk theory and solution of tasks related with maintenance, logistic problems than quantitative assessment or semi-quantitative methods of risk assessment. Mathematical modeling and simulation is for an analysis, modeling and prediction of random events in operation, maintenance, logistics, and risk assessment very favorable, first of all for a possible visualization and monitoring through graphical outputs providing better perception and display of stochastic processes.

In practice it is very difficult to take all system characteristics into consideration in a model. It can be caused by a fact, that there are many random variables entering into simulation or there exists incomplete knowledge and information about these events. Therefore we have to count for a certain rate of uncertainty, which is always present in defining a preliminary hypothesis. The hypothesis, being a base of a model and values of its parameters can lead to uncertainty in a general model of output that must be quantified on a realistic system statement. Expressed in a mathematic way, a probability of a system failure can be expressed as a multi-dimensional integral. However a risk assessment requires realistic modeling of structures and mechanical parts of a system and comprehensive modeling of particular states representing behavior, loading conditions and mechanisms of defects and failures, we anticipate, that will rise during system lifecycles.

Thanks to performance, flexibility and a relatively easy use the simulation program, or software is a popular and an efficient tool in a wide range of areas. A next advantage is a possible application of mathematical and logical data expressing flowcharts and activities of elements of a system being modelled (with respect to a simulation goal) or substitution of complicated and unknown relations and procedures. Random influences are included in form of probability characteristics, time included of course. The model enables repeated calculations and adjustment of input data. We believe, that this paper will inspire for improvement of standard methods of research in a submitted issue.

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RELIABILITY ASSESSMENT MODEL OF ELECTRIC POWER SYSTEMS IN LONG-TERM OPERATION PLANNING

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ABSTRACT

The paper presents a conceptual statement of the problem of determining the reliability indices of electric power systems of random configuration, mathematical formulation of the problem, and problem-solving methods. The authors provide the reference data on the computer program developed for solving the given problem and recommend possible application areas for the program.

References: 9 sources.
Keywords: reliability, electric power industry, individual reliability characteristics, indices, models, probability, factors.

1. INTRODUCTION

One of the indices of effective development and operation control for the electric power system is the reliability. Specialists from different countries continue the development of methods and tools for analysis and support of reliability at all levels of territorial and temporal control.

In the process of its operation the electric power system, as any other technical system, experiences various disturbances: internal disturbances conditioned by failures of components, mistakes made by the operating personnel, etc.; and external ones caused by changes in the level of demand, conditions of resource supply to the system, influence of the environment [0]. The consequences of the indicated disturbances can be represented by the interruptions to power supply to consumers.

These consequences are partially or fully compensated for by enhancing the equipment reliability; creating the redundant production capacities, transfer capability margins of transmission lines and resource reserves; as well as improving the control systems; and organizing the operation process more effectively, etc.

Modern electric power system reliability theory based on the postulates of the general reliability theory has some specific differences determined by the characteristics of expansion and operation control of the electric power system that are not inherent in other technical systems. First of all, electric power industry has to deal with large systems spread over vast territories; therefore, one of the important technological constituents is the electricity transport. Secondly, the physical nature of electric power system components is quite specific and diverse (a variety of power equipment, types of power plants, transmission lines, etc.). Thirdly, the structure of energy production, conversion, transportation, distribution and consumption is such that the reliability assessment requires the special operability criteria and particular reliability and efficiency indices to be introduced. This, in turn, needs the development of special mathematical models and methods of study.

The electric power system reliability is characterized by a number of so called "individual" properties: failure-free operation, longevity, maintainability (restorability), stabilability, survivability, controllability, and storability [2]. The assessment of these properties within one model is still impossible, that is why when constructing the computational electric power system models intended for the determination of indices describing certain individual reliability characteristics, the developers confine themselves to one or several most compatible individual characteristics.

Below we present our calculation model that assesses the reliability in terms of failure-free operation and restorability of large complex electric power systems represented by any (radial, ring) multi-node calculation scheme with constrained transfer capabilities of ties among the nodes. Each node of the scheme is a concentrated subsystem, which, in the general case, is represented by the generation unit commitment and the total load and electricity consumption. The problem is solved for national, integrated regional and local electric power systems. The reliability indices determined using this model consider the rest of the individual properties to the extent to which they are reflected in the indices of failure-free operation and maintainability of the system equipment (components) that represent the input data in this model. In fact, this model is a model for assessing the reliability of power supply to consumers as a set of electric power system reliability properties that determine the output performance of the system.

In the market environment with a large number of energy market participants modern electric power systems are characterized by an increase in generating capacities, improvement in their structure, and development of backbone and interstate electric ties due to the construction of super high-voltage transmission lines. This makes it possible not only to satisfy the needs of local consumers, but also to enter the wholesale electricity and capacity markets. Therefore, when solving the problem of electric power system reliability assessment, we should take into account the market conditions and introduce technical and economic indices to the problem statement. It is known that the electricity and capacity markets require extra investment in reserves due to greater uncertainty of development as compared to the planned economy. Additional redundancy is also necessary for the competitiveness among energy companies and for extra profit. Elevated redundancy can be connected to legislative and contractual obligations of energy companies to provide reliable power supply to consumers. The reliability model should consider different types of mutual aid of power companies, depending on their contractual relations.

The current stage of development of electric power systems is also characterized by a more intensive integration of large systems for their simultaneous operation within the Unified Power System of Russia and creation of interstate and even intercontinental interconnections. The integration of electric power systems requires a feasibility study, which should take into consideration the system effects closely connected to reliability.

The reliability models should take into account the extent to which the electric power system is provided with resources of different kinds: financial, human, physical. It is especially important to provide power plants with primary energy resources (fuel at thermal power plants and water at hydro power plants). In the planned economy fuel was first of all supplied to power plants, whereas under the market conditions the fuel transportation costs can be overestimated, fuel supplies can fail due to crisis phenomena in the society and the economy, etc. Moreover, it is very difficult to select a strategy and tactics of energy resource consumption, which should be reflected in the algorithm of the model. This is conditioned by the fact that there are no clear principles (except certain cases), and there is a wide variety of strategies and tactics in practice. Here we have to take into account a substantial number of different characteristics of the electric power system operation. This variety is explained not only by objective reasons (for instance, actual share of hydro power plants in the system, great uncertainty of forecasting the subsequent operating conditions of the electric power system), but by subjective factors as well (values pursued by the management of energy companies and rules of regional energy commissions, instant benefits and ways to limit the consumers: frequency and voltage decrease, "special" constraints, rolling (rotating) blackouts, etc.).

It is known that under the conditions of market and competition any economic activity is associated with risks due to impossibility of accurately predicting and considering the conditions under which the planned activity is going to be carried out. In the electric power industry, with the transition to the market economy the economic (commercial) risk has started to dominate the technological risk of failure to perform the main functions related to the equipment failures.

Therefore, it is crucial to find the means and ways to neutralize or reduce the negative consequences of the commercial risk. To prevent such consequences it is suggested to create an extra reserve in the electric power system (in addition to the technological one) conventionally called "commercial reserve" [3].

Let us present the problem statement and a method of solving the problem of reliability assessment of the central electric power system segment, i.e. the main structure, which, along with the distribution segment and schemes of power supply to certain plants of consumers, significantly determines the power supply reliability.

By the main structure we mean the unit commitment and parameters of generating equipment and the backbone network equipment of the electric power system. Thus, assessing the reliability of the main structure we do not take into consideration the reliability of the electric power system distribution network and the reliability of schemes for power supply to consumers (their reliability should be assessed by other software packages, but in terms of reliability of the main electric power system structure).

There are a lot of methods for assessing the reliability of the main electric power system structure (analytical, statistical, etc.) [4, 5]. The models are commonly constructed using different combinations of methods, each of which is applied to solving the subproblems. Combined models are the most suitable in terms of accuracy and calculation speed.

Let us present the statement of the electric power system reliability assessment problem. Its mathematical form is based on a variety of methods. We will consider the properties and specific features of the developed software package and the areas of its application.

2. CONCEPTUAL STATEMENT, INITIAL PROPOSITIONS, MAIN ASSUMPTIONS, SIMPLIFICATIONS, AND CALCULATION CONDITIONS

Solution to the problem is of an assessment character. It is necessary to determine the selected reliability indices for the specified variant of the main electric power system structure operation. The corresponding calculation scheme is considered as a set of load and generation nodes and ties among them. The components of the system are represented by the main electric power system equipment (generating units and transmission lines). Each node in such a scheme is "concentrated", i.e. a network of transmission lines provides the power flows at the node under all possible operating conditions. The ties among the nodes of the calculation scheme represent a set of all transmission lines among the corresponding subsystems represented by the given nodes.

The problem of determining the reliability indices is formulated in the following way:

For the specified levels and structure of power consumption at the concentrated nodes (subsystems), configuration of backbone ties, unit commitment and parameters of electric power system equipment (generating units and backbone transmission lines), and the availability of different resources, determine the reliability indices of individual nodes and the entire system for the considered period and for the specified intervals of the period.

The developed model considers the most significant factors of the electric power system operation that influence its reliability. These are first of all failures, emergency and scheduled maintenance of the system equipment. The model also takes into account seasonal unevenness of random processes in the electric power system (for instance, equipment failure flows) and the changes in the unit commitment and parameters of equipment during the year; power consumption in the form of characteristic daily load curves in terms of time zone shifts for different areas of the electric power system; and random deviations of load and supply of primary energy resources. In the case of calculated failures the calculated states are optimized with respect to the strategy assumed for the limitation of consumers in individual subsystems and possible mutual assistance among the subsystems.

The reliability of complex electric power systems in terms of physical and technical characteristics is assessed in accordance with the following initial considerations. The adequacy of the electric power system viewed here as a degree to which consumers are supplied with electricity [2] is characterized by frequency, duration, and amount of possible power deficit in the system. In the determination of power supply reliability with respect to the buses of nodal substations feeding the load, the operating conditions of consumers are taken as an external factor specified by the corresponding equivalent load curves. In this case the states of the electric power system can be quite fully determined using the curves of electricity consumption, energy parameters of the main equipment, and its reliability indices. In the latter case it is necessary to take into account any downtimes of the main equipment (complete and partial), including the downtimes caused by unreliable operation of auxiliary equipment of the electric power system (auxiliary needs of the plants and substations; switching equipment; protection, automation, and control devices), as well as the downtimes caused by water deficit at hydro power plants or fuel deficit at thermal power plants.

According to the initial propositions, the general state of loads and the main equipment determines a set of the main possible states of the electric power system in space and time.

The electric power system operation model based on the indicated initial propositions is typified by the following specific features, assumptions, and simplifications.

1. By the system downtime we mean a transition of the electric power system [2] to any operating conditions characterized by power deficit. It is assumed that the automation and personnel made the deficit conditions feasible by rationally using all the possibilities of reducing the deficit and limiting the consumers by the minimum possible value.

Here we do not consider the operation failures of consumers in emergency transient processes, which can be severer than under the steady-state postemergency conditions, yet their duration is by several orders less than the duration of the postemergency conditions. Therefore, some underestimation of the power undersupply value can be considered as negligibly small compared to the power undersupply in the whole period of the system operation under postemergency (deficit) conditions.

With this assumption we take into consideration the system operating conditions sufficient for practical calculations, when some of the components are in a non-operating state. In this case it becomes much easier to make an analysis, since there is no need to assess many sudden emergencies and prepared disconnections, transient processes that correspond to them and conditions that ensure their optimality, duration, and scale of short-term emergency power dips, which depend on various factors.

2. Scheduled maintenance of generating equipment is modeled according to the standards provided it is definitely performed. Instead of the standards it is possible to specify a schedule of current and overhaul maintenance of the equipment. It is suggested that the scheduled maintenance of transmission lines either should not be considered or should be considered in the corresponding calculation intervals in combination with emergency maintenance (by increasing the relative duration of maintenance downtime).

3. Equipment failures in the model are not divided into sudden and predicted. It is considered that all types of reserves (spinning and standing) are used and the system failure is determined by a general or local excess of load of the whole generated power. It is assumed that the reserve available in the system is divided into spinning and standing in accordance with the ratio of sudden failures to predicted ones. In this case sudden failures are eliminated by the primary spinning reserve, and predicted failures with different lead time – by the spinning reserve of the following orders and the standing reserve until they are completely exhausted.

4. Power deficit (and therefore, power undersupply) is determined by the global or local

deficit of generating capacity. Such forms of deficit as frequency decrease in the system or voltage decrease in consumer buses are not analyzed.

5. In the developed model the specific of operating conditions of hydro power plants, pumped-storage power plants and thermal power plants are taken into account by specifying the initial data on these facilities (corresponding distribution functions of their states and probabilities of resource supply, including energy resources.

6. Maximum transfer capabilities of individual transmission lines are taken as constant and independent of the system operation, however different (if necessary) for each calculation interval. The total transfer capabilities of ties between the nodes are determined additively as the functions of transmission line states (operable and inoperable). However, it is possible to specify the relationship between the transfer capabilities of ties and the state of constituent transmission lines more accurately by the corresponding distribution functions.

7. To optimize the calculated states we apply the interior point method [6], and suggest using one of the four models [7] which provide the sought results on the basis of:

- the first Kirchhoff law,
- power losses in the networks,
-technical and economic characteristics of electricity production and transmission,
- functioning of the wholesale electricity and capacity markets, and – the specified strategies to limit consumers under the given bilateral constraints on the flows in transmission lines.

Let us enumerate the main features of these models for optimization of the calculated states:

1) a model which makes it possible to minimize power deficit in terms of only the first Kirchhoff law and distribute the total system power deficit among the nodes in proportion to their loads (estimation of system deficit only);

2) a model which provides unique distribution of the total power deficit among nodes in terms of power losses in ties between the nodes (also estimation of system deficit only);

3) a model which considers technical and economic indices of electricity production and transmission costs and provides the distribution of the total power deficit among nodes in terms of power losses in ties among the nodes (estimation of deficit and deficit-free states of the system);

4) a model which is analogous to the third model, but additionally considers the specific features of electric power system operation in the wholesale market environment.

Depending on the model used, we consider either the sets of post-emergency deficit conditions only or, in order to take into account fuel consumption and the impact of wholesale markets on the electric power system operation, all possible operating conditions (statistically, of course). The consideration of all the electric power system states (deficit and deficit-free) provides additional possibilities for the estimation of the load distribution functions in transmission lines. These distribution functions represent the data necessary for the reasonable selection of transfer capability of ties among the subsystems. In the course of optimization of the calculated states, the reliability can be studied in terms of different ways of mutual assistance among the power companies, depending on their contractual relations.

It has to be mentioned that practically all the operating conditions should be calculated for the case of the reliability analysis within the studies of electric power system survivability and energy security, since in this case major disturbances are considered (global fuel undersupply, low-water years, large-scale failures of plant capacities and large intersystem ties, etc.).

8. The developed model suggests using a simple principle effective from the viewpoint of the authors. The principle considers the constraints on all the resources, and first of all, the constraints on primary energy resources for different power plants (water in hydro power plant storages, fuel at thermal power plants). To this end, it is necessary to have reported or statistical data on the probabilities of certain supply of the resources. The multiplication of a generating capacity state distribution series of a power plant by a resource supply distribution series at the plant gives us the resultant series of operable plant capacities provided with resources. The use of these resultant

series for all the plants makes it possible to determine the probability distribution function of load supply, depending on the distribution functions of resources supplied and capacities available.

9. When choosing the type of backup generating capacity and electric energy we take into account that, on the one hand, each individual power company should have a certain minimum level of their own backup, and on the other hand, the total level of backup should exceed the one obtained in the calculations for technological reserves (reserve for current, overhaul, and medium maintenance, reserve for modernization and equipment reconstruction, and operating reserve).

The minimum level of technological backup is determined by the specified standard of power supply reliability for a power company in the case of its isolated operation. Such standards are known for the USA, for example ($\mathcal{P}$ not less than 0.9, where $\mathcal{P}$ – probability of deficit-free operation). There is no such a standard for Russia, nevertheless the model is adapted to the evaluation of the required power reserve level in each electric power system for its isolated operating conditions at a specified $\mathcal{P}$.

Here we present the most significant and specific assumptions and possibilities. The reliability theory has some other assumptions and propositions as well (for instance, equipment failure independence, disregard for different failure rates of operating and repaired equipment, etc.).

Required initial data*: the calculation scheme of the electric power system (equivalent subsystems and ties among them); probabilities of emergency downtimes of generating units or transmission lines; annual duration of overhaul and medium maintenance of the indicated electric power system components (standard); scope of current maintenance of the electric power system components during the year (standard); unit commitment and parameters of generating units for each node and each calculation interval during the year (the calculation period equals 1 year); transfer capabilities of ties between the nodes in both directions; characteristic daily load curves for each node, duration (number of working days) of the corresponding load periods, into which the interval is divided (for example, the load on working and non-working days); mean square deviation of loads from the projected curves; probabilities of the necessary resource supply to generating capacities; electricity rates, total and variable costs of production and transmission, if the calculated operating conditions are optimized using a corresponding model that considers the electricity markets. To optimize the calculated states of electric power system in terms of reliability with the model which takes into consideration the wholesale market conditions we need the data on specific discounted costs of the electric power system equipment at nodes and in ties and compensation costs (specific damages)* $\mathbf{y}_0$ *in case of power undersupply to consumers at nodes.*

In the formation of a calculation scheme of the electric power system, to determine reliability we should take into account the probability of random division of the system into subsystems (nodes). The scheme should be composed so that selecting the nodes we could detect the "weak" ties and take into consideration the territorial and organizational hierarchy of the electric power system.

The accuracy of the calculations depends on how fully and accurately the model considers the factors of the electric power system operation which influence its reliability. The software can provide almost any extent of factor consideration and accuracy of calculations within the accepted assumptions.

Sought information. As a result of the calculations we determine the following reliability indices in subsystems and in the whole system for each calculation interval (months, quarters) and for the year: probability of failure-free operation of the system (node) $\mathcal{P}$; mean value of power undersupply to consumers W^{und}; power availability ratio[1] π; damages caused by electricity

[1] Relative satisfaction of demand for electricity (the ratio of supply to demand)

undersupply D. The values of calculated technological reserves at nodes and in the entire system are: reserve for overhaul (medium) maintenance; reserve for current maintenance; operating reserve; total value of reserves of all types. The model calculates the distribution functions of power deficit and the energy reliability characteristics of the ties.

Here the energy reliability characteristic of the tie is the function of power flow distribution in the given tie under the electric power system operating conditions and with the specified equipment reliability characteristics. The indicated function determines the character of an interconnection between the two adjacent subsystems and the possible mutual assistance in terms of the rest of the system. The considered energy reliability characteristic is a quantitative reliability characteristic that determines the actual efficiency of using the corresponding tie in the given conditions. Therefore, the energy reliability characteristic can be applied in the mutual coordination of reliability calculations at different territorial levels of electric power system.

The dual (objectively reasonable) estimates of deficit of the main resources (generating capacity at nodes and transfer capabilities of ties) are also calculated to provide the reliability of power supply to consumers. The estimates characterize the "contribution" of the generating capacities of each node and the transfer capabilities of each tie to the electric power system reliability and allow the optimization of reliability under the scheduled operating conditions of the system. Apart from the reliability indices the software calculates their mean square deviations σW^{und} and $\sigma\pi$.

3. MATHEMATICAL STATEMENT OF THE PROBLEM

In accordance with the given statement of the problem we represent the calculation scheme of the electric power system as a connected graph, whose vertices (nodes) correspond to the equivalent calculation subsystems, and edges - to the transmission lines. The considered calculation period T_{p}, which usually equals one year, is divided into S intervals, where the given electricity consumption curves and unit commitment and parameters of the equipment are fixed. Let us determine the reliability indices in the following way:

for nodes $(m=\overline{1,M})$:

a) in interval $(s=\overline{1,S})$

- probability of failure-free operation

$$\mathcal{P}_{ms} = 1 - Q_{ms} = 1 - \frac{1}{\tau_s} \sum_{\varphi=1}^{\Phi_s} \sum_{\eta=1}^{L} \sum_{k=1}^{K_s} q_{m\varphi\eta k}^{\text{def}} \cdot \tau_\varphi \; ; \tag{1}$$

- average power undersupply to consumers (kWh)

$$W_{ms}^{\text{und}} = \sum_{\varphi=1}^{\Phi_s} \sum_{\eta=1}^{L} \sum_{k=1}^{K_s} P_{m\varphi\eta k}^{\text{def}} \cdot q_{m\varphi\eta k}^{\text{def}} \cdot \tau_\varphi \; ; \tag{2}$$

- power availability ratio

$$\pi_{ms} = 1 - W_{ms}^{\text{und}} / W_{ms} = 1 - W_{ms}^{\text{und}} / \sum_{\varphi=1}^{\Phi_s} P_{m\varphi}^{\text{L}} \cdot \tau_\varphi \; ; \tag{3}$$

b) period $T_{\text{p}} = \sum_{s=1}^{S} \tau_s$ has the same indices (averaged)

$$\mathcal{P}_m = 1 - Q_m = 1 - \frac{1}{T_{\text{p}}} \sum_{s=1}^{S} Q_{ms} \cdot \tau_s \; ; \tag{4}$$

$$W_m^{\text{und}} = \sum_{s=1}^{S} W_{ms}^{\text{und}} \; ; \tag{5}$$

$$\pi_m = 1 - W_m^{\text{und}} / W_m = 1 - W_m^{\text{und}} / \sum_{s=1}^{S} W_{ms} \; ; \tag{6}$$

for the system:

a) at each interval *s* indices $\mathcal{P}_{cs}$, W_{cs}^{und} and π_{cs} are calculated by analogy with (1)–(3);

b) for period T_{p} indices $\mathcal{P}_{\text{c}}$, $W_{\text{c}}^{\text{und}}$ and π_{c} are calculated by analogy with (4)–(6).

To calculate the system indices we should determine the values $q_{\text{c}\varphi\eta k}^{\text{def}}, P_{\text{c}\varphi\eta k}^{\text{def}}, Q_{\text{c}s}$ and Q_{c}, analogous to the values $q_{m\varphi\eta k}^{\text{def}}, P_{m\varphi\eta k}^{\text{def}}, Q_{ms}$ and Q_m in (1)–(6).

In (1)–(6) $Q_{ms}, Q_m, Q_{\text{cs}}, Q_{\text{c}}$ are the relative probabilities of power supply interruption; $\tau_s = \sum_{\varphi=1}^{\Phi_s} \tau_\varphi$ – length of the interval *s*; φ, Φ_s – current number and quantity of calculation periods at interval *s*, which are determined by constant values of average load $P_{m\varphi}^{\text{L}}$ at all nodes (in terms of consumption for the system auxiliaries and losses in distribution networks); τ_φ – length of the φ-th subperiod in hours; η, H – current number and quantity of calculated random values of irregular load components; k, K_s – current number and quantity of random system states, determined by the k_{ms}-th random values of generating capacities at nodes and the k_{ns}-th random states of transmission lines in ties; $q_{m\varphi\eta k}^{\text{def}}, q_{\text{c}\varphi\eta k}^{\text{def}}$ – probabilities of power deficit $P_{m\varphi\eta k}^{\text{def}}$ and $P_{\text{c}\varphi\eta k}^{\text{def}}$ in the $\varphi\eta k$-th calculated conditions; W_{ms}, W_m, W_{cs}, W_{c} – required electricity outputs at the *m*-th node and in the entire system (c), at the *s*-th interval and in the calculation period T_{p}, respectively.

Probabilities of power deficit are determined as:

$$q_{m\varphi\eta k}^{\text{def}} = \begin{cases} q_{\varphi\eta} \cdot q_k, & if \;\; \Delta P_{m\varphi\eta k}^{\text{o}} > 0, \\ 0, & if \;\; \Delta P_{m\varphi\eta k}^{\text{o}} \le 0; \end{cases} \tag{7}$$

$$q_{\text{c}\varphi\eta k}^{\text{def}} = \begin{cases} q_{\varphi\eta} \cdot q_k, & if \;\; \Delta P_{\text{c}\varphi\eta k}^{\text{o}} > 0, \\ 0, & if \;\; \Delta P_{\text{c}\varphi\eta k}^{\text{o}} \le 0; \end{cases} \tag{8}$$

where $q_{\varphi\eta}$ – probability of the η-th random load deviation at nodes in the φ-th period; q_k – probability of the k-th state of the system equipment.

The optimal (minimized) value of load $\Delta P_{\text{c}\varphi\eta\kappa}^{\text{o}}$, which is not supplied throughout the entire system, is calculated by the corresponding values $\Delta P_{m\varphi\eta k}^{\text{o}}$ at each of *M* nodes:

$$\Delta P_{\text{c}\varphi\eta\kappa}^{\text{o}} = \sum_{m=1}^{M} \Delta P_{m\varphi\eta\kappa}^{\text{o}} \; ; \tag{9}$$

$$P_{m\varphi\eta k}^{\text{def}} = \begin{cases} \Delta P_{m\varphi\eta k}^{\text{o}}, & if \;\; \Delta P_{m\varphi\eta k}^{\text{o}} > 0, \\ 0, & if \;\; \Delta P_{m\varphi\eta k}^{\text{o}} \le 0; \end{cases} \tag{10}$$

$$P_{\text{c}\varphi\eta k}^{\text{def}} = \sum_{m=1}^{M} P_{m\varphi\eta k}^{\text{def}} \; . \tag{11}$$

The values required for the calculation of $\Delta P_{m\varphi\eta k}^{\text{o}}$ such as the values of the calculated random deviations of loads $P_{\text{rand}\varphi\eta_m}^{\text{L}}$ from their average values $P_{m\varphi}^{\text{L}}$ and generation $P_{k_{ms}}^{\text{G}}$ at nodes, as well as the values of transfer capabilities of ties $\overline{P}k_{ns}$, $\underline{P}k_{ns}$ $(n=\overline{1,N})$, where n, N – number of the tie and quantity of ties among the nodes in the calculation scheme, respectively) are determined using the corresponding distribution functions by the Monte-Carlo method.

The indicated distribution functions are specified or calculated on the basis of the given law of random load distribution, unit commitment and parameters of the equipment at nodes and in ties for each calculation interval s:

$$q_{\varphi\eta_m}(P^{\mathrm{L}}_{\mathrm{rand}\,\varphi\,\eta_m}) = F(P^{\mathrm{L}}_{m\varphi}, \sigma^{\mathrm{L}}_m), \qquad m=\overline{1,\,M}; \tag{12}$$

$$q_{k_{ms}}(P^{\mathrm{G}}_{k_{ms}}) = F(P^{\mathrm{G}}_{i_{ms}},\, q_{i_{ms}} \,|\, i_{ms}=\overline{1,\, I}_{ms}), \quad m=\overline{1,\, M}; \tag{13}$$

$$q_{k_{ns}}(\underline{P}k_{ns}, \overline{P}k_{ns}) = F(\underline{P}_{i_{ns}}, \overline{P}i_{ns},\, q_{i_{ns}} \,|\, i_{ns}=\overline{1, I}_{ns}), \;\; n=\overline{1,N}, \tag{14}$$

where σ^{L}_m – mean-square deviation of load in per unit from values $P^{\mathrm{L}}_{m\varphi}$ (the distribution law is taken as normal); $P^{\mathrm{G}}_{k_{ms}} = P^{\mathrm{G}}_{\mathrm{av}\;ms} - P^{\mathrm{G}}_{\mathrm{em}\;k_{ms}}$ – the calculated k_{ms}-th value of the total generating capacity, which is not under the conditions of emergency downtime; $P^{\mathrm{G}}_{i_{ms}}$, $q_{i_{ms}}$ – available capacity and probability of the emergency downtime of the i_{ms}-th power unit; i_{ms}, I_{ms} – current number and quantity of power units (or calculated stages of the generating capacity); $\overline{P}i_{ns}$, $\underline{P}_{i_{ns}}$ – limits of transfer capability of the i_{ns}-th transmission line in the directions assumed as positive and negative (direct and inverse), respectively; $q_{i_{ns}}$ – probability of downtime of the i_{ns}-th transmission line:

$$q_{i_{ns}} = \begin{cases} q_{\mathrm{em}\,i_{ns}} & \text{scheduled maintenance of transmission line is not considered;} \\ q_{\mathrm{em}\,i_{ns}} + q_{\mathrm{sch}\;i_{ns}} & \text{scheduled maintenance of transmission line is considered,} \end{cases}$$

where $q_{\mathrm{em}\;i_{ns}}$, $q_{\mathrm{sch}\;i_{ns}}$ – probabilities of outages of the i_{ns}-th transmission line in emergency and scheduled maintenance, respectively; i_{ns}, I_{ns} – current number and total quantity of transmission lines in the n-th tie in the s-th interval.

Optimal values of power deficit $P^{\mathrm{def}}_{m\varphi\eta k}(\Delta P^{\mathrm{o}}_{m\varphi\eta k})$ are determined by optimizing the calculated states of the electric power system, depending on the adopted strategy to constrain consumers, electricity market conditions, etc. [6, 7]. In the simplest case, when power deficit is distributed among nodes in proportion to the load power the function:

$$\sum_{m=1}^{M} C_m \Delta P^{\mathrm{L}}_{m\varphi\eta k} \Rightarrow \min\,, \tag{15}$$

where

$$\Delta P^{\mathrm{L}}_{m\varphi\eta k} = P^{\mathrm{L}}_{m\varphi} + \Delta P^{\mathrm{L}}_{\mathrm{rand}\,\varphi\,\eta_m} - \Delta P^{\mathrm{G}}_{m\varphi\eta k} > 0 \tag{16}$$

provided the following conditions and constraints are met in order to obtain physically and technically feasible solutions

$$P^{\mathrm{L}}_{m\varphi\eta k} - P^{\mathrm{G}}_{m\varphi\eta k} + \sum_{n=1}^{N_m} P_{n\varphi\eta k} = 0 \tag{17}$$

for $m=\overline{1,M}$; $\varphi=\overline{1,\Phi}_s$; $\eta=\overline{1,\mathrm{H}}$; $k=\overline{1,K}_s$; $s=\overline{1,S}$.

Here C_m – coefficients determining the significance of loads at nodes; $P^{\mathrm{L}}_{m\varphi\eta k}$, $\Delta P^{\mathrm{L}}_{m\varphi\eta k}$ –

values of supplied and unsupplied load, respectively; $P^{\mathrm{G}}_{m\varphi\eta k}$ – value of the used generating capacity; $P_{n\varphi\eta k}$ – power flow in tie (here the positive direction is taken as the power flow from the given node to the neighboring ones; and the negative direction is represented by the power flow from the neighboring nodes to the given one); N_m – number of ties adjacent to the *m*-th node.

The components of balance equations (17) are determined as follows:

$$\begin{gathered} 0 \le P^{\mathrm{G}}_{m\varphi\eta k} \le P^{\mathrm{G}}_{\mathrm{op}\,k_{ms}} \\ 0 \le P^{\mathrm{L}}_{m\varphi\,\eta k} \le P^{\mathrm{L}}_{m\varphi} + \Delta P^{\mathrm{L}}_{\mathrm{rand}\,\varphi\,\eta_m} \end{gathered}, \tag{18}$$

where

$$P^{\mathrm{G}}_{\mathrm{op}\,k_{ms}} = P^{\mathrm{G}}_{k_{ms}} - P^{\mathrm{G}}_{\mathrm{sch}\,ms}, \qquad k_{ms} = \overline{1, K}_{ms}; \tag{19}$$

$$\underline{P}^{\mathrm{L}}_{k_{ns}} \le P^{\mathrm{L}}_{n\varphi\eta k} \le \overline{P}^{\mathrm{L}}_{k_{ns}}, \qquad k_{ns} = \overline{1, K}_{ns}\ . \tag{20}$$

Here $\Delta P^{\mathrm{L}}_{\mathrm{rand}\ \varphi\,\eta_m}$ – from expression (16), $P^{\mathrm{G}}_{\mathrm{op}\,k_{ms}}$ – operating generating capacity; $P^{\mathrm{G}}_{\mathrm{sch}\ ms}$ – generating capacity under scheduled maintenance.

The available capacity at each node $P^{\mathrm{G}}_{\mathrm{av}\ ms}$ (for the calculation of $P^{\mathrm{G}}_{k_{ms}}$) at each interval *s* is determined by the given initial data on the equipment:

$$P^{\mathrm{G}}_{\mathrm{av}\ ms} = \sum_{i_{ms}=1}^{I_{ms}} P^{\mathrm{G}}_{i_{ms}}\ . \tag{21}$$

The scheduled maintenance of generating equipment $P^{\mathrm{G}}_{\mathrm{sch}\ ms}$ is taken into consideration in the following way[2]:

$$P^{\mathrm{G}}_{\mathrm{sch}\ ms} = P^{\mathrm{G}}_{\mathrm{cur}\ ms} + P^{\mathrm{G}}_{\mathrm{ov}\ ms}; \tag{22}$$

$$P^{\mathrm{G}}_{\mathrm{cur}\ ms} = \sum_{i_{ms}=1}^{I_{ms}} \bar{\alpha}_{\mathrm{cur}\,i_{ms}} \cdot P^{\mathrm{G}}_{i\ ms}; \tag{23}$$

$$P^{\mathrm{G}}_{\mathrm{ov}\ ms} = f_1(V_{\mathrm{ov}\ m},\ F_{\mathrm{dip}m}); \tag{24}$$

$$V_{\mathrm{ov}\ m} = \Big(\sum_{i_m=1}^{I_m} \bar{\tau}_{ov\ i_m} \cdot P^{\mathrm{G}}_{i_m}\Big)/k_{\mathrm{dip}}; \tag{25}$$

$$F_{\mathrm{dip}m} = f_2(P^{\mathrm{L}}_{\max\ ms} \,/\, s = \overline{1, S}). \tag{26}$$

The current $P^{\mathrm{G}}_{\mathrm{cur}\ ms}$, and overhaul and medium $P^{\mathrm{G}}_{\mathrm{ov}\ ms}$ (22)–(26) maintenances are considered independently, using different techniques [8]. The common requirement for the consideration of these types of maintenance in the model is that the maintenance should definitely be carried out to the necessary degree.

In formulas (23)–(26):

$\bar{\alpha}_{\mathrm{cur}\ i_{ms}}$ – norm (relative total duration) of scheduled current maintenance of the i_{ms}-th unit; $V_{\mathrm{ov}m}$ – required “area” of overhaul maintenance of generating equipment at node (MW per day); $F_{\mathrm{dip}m}$ –

[2] The software for this model realized Russian principles of scheduled equipment repairs.

dip area in the curve of the monthly node peak loads (MW per day); $\bar{\tau}_{\text{ov}i_m}$ – norm of scheduled overhaul and medium maintenance of the i_m-th unit (day/year); k_{dip} – coefficient of using the dip of the annual curve of monthly maximums for overhaul maintenance, in per units [8]; $P^{\text{L}}_{\max ms}$ – maximum load at the m-th node in the s-th interval.

The additional data obtained by solving the stated problem includes: probabilities of power deficit of different degrees (formulas (7), (8)); probabilities of power flows $P_{n\varphi\eta\kappa}$ (energy reliability characteristics) in ties, obtained while solving problem (15)–(20); dual estimates as a result of power deficit optimization in the electric power system.

Such data allow us to calculate mean square deviations of indices W^{und}, π; series of power deficit distribution among nodes and in the entire system for each s-th interval and for the whole calculation period T_{p}: $q^{\text{def}}_{ms}(P^{\text{def}}_{ms})$, $q^{\text{def}}_{m}(P^{\text{def}}_{m})$, $q^{\text{def}}_{\text{c}s}(P^{\text{def}}_{\text{c}s})$, $q^{\text{def}}_{\text{c}}(P^{\text{def}}_{\text{c}})$. These indices can change, depending on the duration of electric power system operation at different frequencies less than 50 Hz, if the coefficient of frequency-regulating effect of load is known: $\tau_f = F(f)$ for the entire system and its individual nodes at each s-th interval and within T_{p}. Also, using these data we can determine energy reliability characteristics of ties $q_{ns}(P_{ns})$, $q_n(P_n)$, and integral dual estimates for each node and tie.

The damage caused by power undersupply D at nodes and in the system for intervals s and for the whole calculation period are determined by multiplying the corresponding values of power undersupply W^{und}, calculated for the nodes and for the whole system, for individual calculation intervals and calculation period T_{p}, by the value of specific damage due to electricity undersupply d_0.

The extent to which power plants are provided with primary energy resources should be considered in the way described above in the paper (point 8 – specific features, assumptions and simplifications). If the corresponding kind of resource is not limited, the distribution series of the resource availability degenerates into the probability equal to 1. Using the obtained values $P^{\text{G}}_{m\varphi\eta k}$ and $q_{m\varphi\eta k}$ and knowing the specific consumption b_ν of the ν-th resource, we can estimate the required amount of the resource.

4. METHODS AND TECHNIQUES FOR SOLVING THE PROBLEM.

The problem of reliability assessment of a large scheme represents a complex "tree" of subproblems. By solving these subproblems we determine the values $q^{\text{def}}_{m\varphi\eta k}$ and $P^{\text{def}}_{m\varphi\eta k}$ as the basis for the calculation of the reliability indices.

The electric power system reliability indices are calculated using the following successive modules:

I. Module of the initial data preparation:

1. Input, analysis, and processing of the initial data.
2. Calculation of complex characteristics of the factors that determine the reliability of nodes and the whole system.

II. Probability module

3. Construction of load curves and the curves of scheduled maintenance of generating equipment for each node.
4. Calculation of distribution functions of the generating equipment states at each node.
5. Determination of possible states of electric ties in terms of the transmission line maintenance downtime.
6. Formation of a set of calculated states of generating equipment, transmission lines, and loads (in terms of random load variations) in the system.

III. Module of calculated state optimization:

7. Selection of an optimization model of calculated states
8. Optimization of a calculated state, including minimization of power deficit (determination of power deficit at nodes, economic indices, and flows in ties, and check of the resource availability).

IV. Module of calculation of reliability indices:

9. Calculation of energy reliability characteristics of ties at time intervals and within the calculation period T_p.
10. Calculation of reliability indices of nodes at time intervals and within the calculation period T_p.
11. Calculation of reliability indices of the system at time intervals and within the calculation period T_p.
12. Calculation of economic indices of the system at time intervals and within the calculation period T_p (depending on the chosen model in module III).
13. Processing of the calculation results.

Some subproblems are included in the general problem in the form of repeatedly used algorithms. For instance, stages 3–12 are calculated for all s intervals.

The different character of subproblems solved in the calculation of reliability indices explains the use of a variety of methods:

– method for calculating the distribution series of random states of components in the calculation scheme of a system on the basis of a generating function of the general replication theorem, and the addition and multiplication theorems of probabilities of different events;

– statistical testing method for the formation of the calculated system states;

– combinatorial methods of dividing the system states in terms of a given criterion;

– methods of linear and nonlinear programming for the optimization problem of calculated states in the electric power system (an interior point method [6]).

The problem is solved by the method of simulation modeling of electric power system operation during the calculation period. The states of loads and system equipment are played out by the Monte-Carlo method. The power distribution series of units at nodes and transmission lines in ties under emergency downtime conditions are calculated beforehand.

The advantages of the implemented algorithm over the analogous algorithms [4] are as follows:

1. The possibility of studying the systems of complex configuration with the ties of limited transfer capability.

2. The calculations do not require preliminary equivalenting of the unit commitment. This

reduces the efforts necessary for the initial data preparation and improves the accuracy of calculations, especially for the electric power system with numerous units of different types.

3. The reliability calculations can use both the given functions of equipment state distribution among nodes and ties and the functions calculated with the software on the basis of data on the reliability of certain system components.

4. The software allows the user to specify the initial data in different forms and with different accuracy. Apart from reliability indices and parameters of their dispersion, it is possible to calculate complex indices characterizing the specific properties and conditions of the electric power system operation that determine its reliability. The latter is especially important for the study of reliability properties of the electric power system.

5. The optimization of calculated states in terms of the operational strategy for the limitation of consumers makes it possible to obtain the indices of power supply reliability both for the entire system and for individual nodes.

6. The possibility of assessing the reliability indices within the whole calculation period and within individual intervals.

7. Energy reliability characteristics of the ties along with dual estimates at nodes and in ties make up the data for technical and economic analysis of the system performance.

8. The software consists of modules, which allows the user to easily change it in the course of improvement or modification.

5. REFERENCE DATA ON THE SOFTWARE AND ITS APPLICATION

The software package YANTAR is intended for the determination of reliability indices of the electric power system represented by a calculation scheme. The software is written in FORTRAN and operates in batch mode.

The software YANTAR can be considered a computational tool that implements a most complete model for the calculation of the selected reliability indices of electric power system. The solutions can be used for the engineering analysis and reliability optimization of the electric power system and constituent subsystems under the scheduled conditions of their operation.

The software can also be used as a standard for checking and correcting the applied and newly developed simplified models for considering the reliability factor when solving the control problems.

This software package can be used to solve the following problems:

- selection of all types of reserves of the electric power system generating capacity: reserves for scheduled (overhaul, medium, current) maintenance, operating reserve, commercial reserve;
- rational allocation of the chosen amount of total reserve across subsystems and power plants of the system in terms of transfer capabilities of the networks;
- calculation of power supply reliability by subsystem within the system (estimation of deficit-free operation probability, relative degree to which the consumers are provided with electricity, average annual value of electricity undersupply to consumers);
- power supply reliability assessment for a certain consumer in the given electric power system;
- assessment of the system-wide benefit in terms of the reliability factor in different variants on the expansion of the main electric power system structure (generation and network segments of the system, the resource supply segment);
- optimization of a commissioning period of new equipment in the electric power system in terms of the reliability factor.

Apart from the main factors, we can take into account the following aspects of the expansion and operation of the electric power system: the burn-in and aging of equipment (by correspondingly specifying the statistical values of equipment failure rate for certain calculation periods); special schemes of equipment operation (two-boiler single-turbine units, special connection schemes of transmission lines and influence of the inner electric power system ties on the transfer capabilities of intersystem lines); seasonal and other types of unit commitment and characteristics of the system components, including the reliability characteristics; time zone shifts for different nodes of the electric power system, etc.

The estimation of reliability indices for different variants of the electric power system operation in combination with the cost estimation makes it possible to select the best variant.

The comparative analysis of the electric power system reliability allows us not only to choose the best variant on the system expansion, but also to solve the second indicated problem. The latter is possible owing to the software-based calculation of nodal reliability indices both for the whole calculation period and for individual intervals. This makes it possible to assess the operating conditions at different nodes of the system and the nature of changes in their reliability in time. In the course of the analysis we identify the nodes of the system with insufficient power supply reliability and then, using the dual estimates and various engineering computations, improve the reliability of both specified nodes and the whole system, since the system reliability is conditioned by the reliability of its nodes.

In the last decade the software package YANTAR has been used to make a great number of reliability calculations for different schemes. These are the schemes of the Unified Power System of the USSR; Unified Power System of Russia; Integrated Power System of the East; interstate interconnections between the Electric Power Systems of the Far East of Russia and Japan; Integrated Power Systems of Siberia, North China and the East; Integrated Power Systems of the Far East of Russia, North Korea, and South Korea, etc. The reliability of the Unified Power System of Russia was studied in the course of construction of a 1150 kV transmission line in the territory of Russia, from the Integrated Power System of Siberia to the Ural Federal District. The survivability of the Unified Power System of Russia under the conditions of large-scale and long-term disturbances (gas undersupply, moratorium on nuclear power plants, isolated electric power system operation, low-water years for Russian rivers, etc.) was assessed. The performed calculations allowed us to consider the reliability factor when studying the schemes of electric power system expansion and operation [9 etc.].

6. CONCLUSIONS

1. The paper presents a conceptual problem statement and the main principles of constructing the algorithm and calculation procedures intended for the reliability assessment of modern complex electric power systems.
2. The research shows the differences between the developed software package YANTAR and other analogous software packages. Attention is given to the great practical experience in the application of this software to the calculations of the electric power system reliability of different levels in the analysis of various projects for the expansion of modern systems.
3. The YANTAR allows online synthesis of electric power system reliability, taking into account the technical and economic characteristics of the system equipment.

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THE EXCITATION CONTROL AND STABILITY OF SMALL HYDROELECTRIC POWER STATIONS SYNCHRONOUS GENERATORS WHEN OPERATION IN POWER SYSTEM

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ABSTRACT

The article discusses the minimization of active power losses depending on the mode of operation of the generator and the load profile. The optimal law of control of synchronous generator excitation in small hydroelectric power plants, at which the energy losses are minimized due to the optimal reactive power interchange, was determined.

In addition to solar and wind power the power of micro and small hydroelectric power stations (SHPS), as an unconventional renewable power sources, is widely used.

These, as a rule, are the diversion hydro power stations, using the enery of mountain rivers and channels. Small HPS are also used in Azerbaijan; their installed capacity is 10-13 MW. Power supply circuits of load nodes with small HPSs availability are various, however when broad electrification the electrical networks cover practically all any significant human settlements (in any case in Azerbaijan), therefore, the circuit, shown in Fig.1, can be taken as the most frequently used one of power supply.

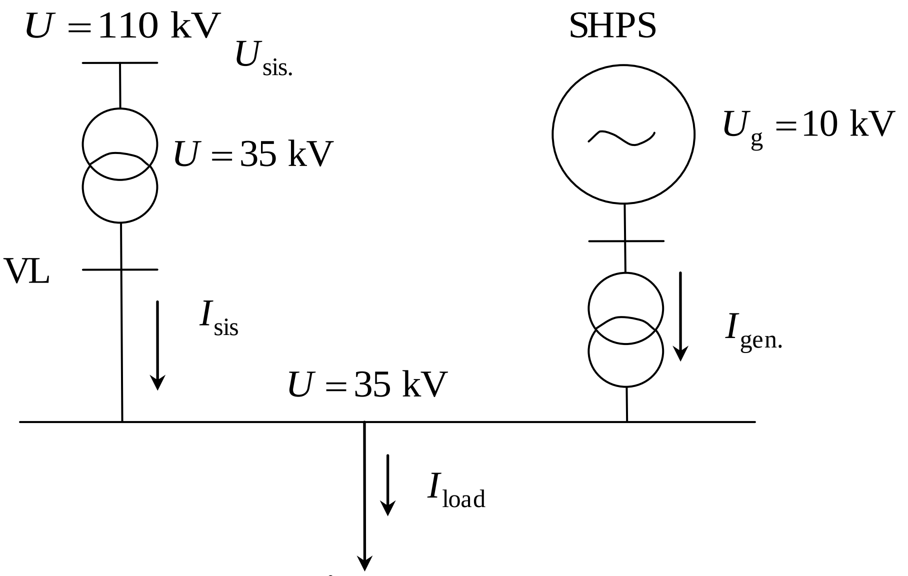

Figure 1.

In this case the load (it could be small settlement with some production, etc.), is supplied simultaneously from the generator of small HPS and the system on VL line. As for the active power, the strategy of its control is evident: the consumer's load-curve on active power must be first of all met by the generation of active power by small HPS, and in case of shortage by power interchange from the system on VL line. In this simple power supply circuit the electricity (power) losses from active power interchange are entirely determined by consumer's load curve on active power and by the curve of active power output by small HPS.

As for the reactive power, so far as there is a source of controlled reactive power in the circuit, and it is the synchronous generator of small HPS, it is possible to determine the optimal control law of small HPS's synchronous generator excitation, where the minimum power losses in mentioned circuit is achieved only at the expense of optimal reactive power interchange.

Control of synchronous machines' excitation current at a constant shaft load results in a change of power losses in stator copper and excitation system.

The total losses are shown in [1] and presented in the form of:

$$\Delta p_{\mathrm{g}} = \frac{\left(\beta^2 \cdot P_n^2 + \alpha^2 \cdot Q_n^2\right) \cdot R}{U^2} + \frac{R_f}{x_{ad}^2}\left[\frac{x_d \cdot x_q}{U^2}\left(\beta^2 \cdot P_n^2 + \alpha^2 Q_n^2\right) + \alpha\left(x_d + x_q\right) \cdot Q_n + U^2\right] \cdot \left[1 + R(Q)\right] \qquad (1),$$

where $\alpha = \frac{Q}{Q_n}$ – load factor on reactive power; $\beta = \frac{P}{P_n}$ – load factor on active power; P_n, Q_n – rated active and reactive power of generator; U – phase-to-phase voltage of generator; x_d, x_{ad} – synchronous inductive impedance and mutual induction reactance along the direct axis *d*; x_q – synchronous inductive impedance along the quadrature axis *q*; R, R_f - accordingly active resistances of armature winding and excitation winding reduced to stator; $R(Q)$ – a factor, taking into account a saliency of synchronous machine. The value $R(Q)$ is determined by the correlation:

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$$R(Q) = \frac{\left(x_d - x_q\right) \cdot \left(\frac{\beta^2 \cdot P_n^2 + \alpha^2 \cdot Q_n^2}{U^2} \cdot x_q + \alpha \cdot Q_n\right)}{\frac{x_q^2}{U^2} \cdot \left(\beta^2 \cdot P_n^2 + \alpha^2 \cdot Q_n^2\right) + 2\alpha \cdot x_q \cdot Q_n + U^2} \qquad (2)$$

The last expression is linearized by [1] authors, and determined that, it can be replaced with 10% accuracy by the expression of following view:

$$R(Q) = R_0 + \left(R_i - R_0\right)\frac{Q}{S_{baz}} \qquad (3)$$

where, R_0 – is a value of (2) function with $Q = 0$, and R_i – is a value of (2) function with $Q = S_{baz}$; S_{baz} – is a base power equal to apparent power of synchronous machine.

The (1) expression with consideration for (3) expression can be reduced to the form of:

$$\Delta p_{\text{g}} = a \cdot Q^3 + b \cdot Q^2 + c \cdot Q + d \tag{4}$$

where, $a = \dfrac{(R_i - R_o) \cdot R_f \cdot x_d \cdot x_q}{S_{baz} \cdot x_{ad}^2 \cdot U^2}$; $b = \dfrac{R}{U^2} + (1 + R_0)\dfrac{x_d \cdot x_q \cdot R_f}{x_{ad}^2 \cdot U^2} + \dfrac{R_f \cdot (R_i - R_0)}{S_{baz} \cdot x_{ad}^2} \cdot (x_d + x_q)$

$$c = \frac{R_f}{x_{ad}^2} \cdot (x_d + x_q) \cdot (1 + R_0) + \frac{(R_i - R_0) \cdot R_f}{S_{baz} \cdot x_{ad}^2} \left(\frac{x_d \cdot x_q}{U^2} \cdot \beta^2 \cdot P_i^2 + U^2 \right);$$

$$d = \frac{R}{U^2} \cdot \beta^2 \cdot Đ_n^2 + \frac{R_f}{x_{ad}^2} \left(\frac{x_d \cdot x_q}{U^2} \cdot \beta^2 \cdot P_n^2 + U^2 \right) \cdot (1 + R_0).$$

In (4) expression the first term is by some orders less than the rest ones, so they can be neglected. Then the dependence of Δp losses on reactive power of small HPS's generator will be expressed in the form of:

$$\Delta p_{\text{g}} = b \cdot Q_{\text{gen}}^2 + \tilde{n} \cdot Q_{\text{gen}} + d \tag{5}$$

When supplying according to the circuit in fig. 1, the load or part of it can also be supplied from the system on VL line, loss in which will be:

$$\Delta p_L = I_L^2 \cdot R_L = \left(\frac{P_L^2}{U^2} + \frac{Q_L^2}{U^2} \right) \cdot R_L = \frac{P_L^2}{U^2} \cdot R_L + \frac{R_L}{U^2} \cdot Q_L^2 = m + n \cdot Q_L^2 \tag{6}$$

where $m = \dfrac{P_L^2}{U^2} \cdot R_L$, $n = \dfrac{R_L}{U^2}$, $Đ_L$, Q_L – are the active and reactive powers, transmitted via VL. R_L – active resistance of VL, U – phase-to-phase voltage of VL.

Thus, the reactive power of load in general case is equal to:

$$Q_{load} = Q_{\text{gen}} + Q_L \tag{7}$$

The total power losses in line and generator are equal to:

$$\Sigma \Delta p = \Delta p_L + \Delta p_{\text{gen}} = m + n \cdot Q_L^2 + b \cdot Q_{\text{gen}}^2 + \tilde{n} \cdot Q_{\text{gen}} + d \tag{8}$$

Determining Q_L from (7) and substituting into (8) we will obtain:

$$\Sigma \Delta p = m + n \cdot \left(Q_{\text{load}}^2 - 2 \cdot Q_{\text{load}} \cdot Q_{\text{g}} + Q_{\text{g}}^2 \right) + b \cdot Q_{\text{g}}^2 + \tilde{n} \cdot Q_{\tilde{a}} + d \tag{9}$$

Taking the derivative of $\Sigma \Delta p$ with respect to Q_{g} and equating it to zero we will get:

$$-2 \cdot Q_{\text{load}} \cdot n + 2 \cdot n \cdot Q_{\text{g}} + 2 \cdot b \cdot Q_{\text{g}} + c = 0 \tag{10}$$

And, finally, solving (10) relative to Q_{g} will find:

$$Q_{\text{g}} = \frac{2 \cdot Q_{\text{load}} \cdot n - c}{2 \cdot (n + b)} = \frac{n}{n + b} \cdot Q_{\text{load}} - \frac{c}{2(n + b)} \tag{11}$$

If to minimize the losses only in the generator itself, the (11) expression is transformed into the form of:

$$Q_{\text{g}} = -\frac{c}{2 \cdot b} \tag{12}$$

This expression is given in [1]. It should be noted that for cylindrical rotor synchronous machines b and c factors do not depend on active power, i.e., when changing the motive torque on the generator's shaft, the minimum losses are achieved at a constant reactive power.

In that way, it follows from the expression (11) that, the reactive power of generator (and consequently the excitation current) needs to be controlled in proportion to value of load's reactive power with taking into account the parameters of transmission line and generator.

Let's illustrate the above algorithm in a specific example.

Generator's parameters: S_{rat} = 11,8 MVA; P_{rat} = 10 MW; Q_{rat} = 5,15MVAr; U_{rat} = 10KV; x_d = 0,987(relative unit); $R_{G\text{-}T}$ = 0,02(relative unit) (a resistance of generator's stator winding is combined with active resistance of transformer T_R. 10/35), R_f = 0,045 (relative unit); x_f = 1,1 (relative unit); x_q = 0,633 (relative unit); x_{ad} = 0,787 (relative unit); x_{aq} = 0,433 (relative unit). The calculated factors of (5) equation are: $b = 5{,}3\cdot10^{-10}$; $c = 3{,}7\cdot10^{-3}$; $d = 37\cdot10^{3}$; $R_0 = 0$; $R_i = 0{,}16$.

The daily load curve (Fig.1) of 35 kV section according to the readings of Indigo counters is adopted from [2] and presented in Fig.2. Let's take the length of 35 kV transmission line (Fig.1) equal to 15 km, the specific resistance and inductance, of which, are accordingly equal to $r_0 = 0{,}3$ ohm/km; x_0 = 0,4 ohm/km).

Then $r_0 = 4{,}5$ ohm and the factor n in (11) formula will be equal to:

$$n = \frac{4{,}5}{35^2\cdot10^6} = 3{,}69\cdot10^{-9} = 36{,}7\cdot10^{-10}$$

In accordance with (11) formula:

$$Q_g = \frac{36{,}7\cdot10^{-10}}{36{,}7\cdot10^{-10}+5{,}3\cdot10^{-10}}\cdot Q_{load} - \frac{36{,}7\cdot10^{-10}}{2\cdot\left(36{,}7\cdot10^{-10}+5{,}3\cdot10^{-10}\right)} = 0{,}87\cdot Q_{load} - 0{,}44 \text{ (MVAr)} \quad (13)$$

The maximum value of reactive load on load curve is equal to $Q_{load\,\boldsymbol{max}} = 4$ MVAr = 4000 kVAr; the average value of $Q_{load\,\boldsymbol{md}} = 3{,}5$ MVAr and the minimum value of $Q_{load\,\boldsymbol{min}} = 3{,}0$ MVAr=3000 kVAr. Let's calculate the losses in line and generator for these values Q_g, Q_L, as well as the total losses $\Sigma\Delta p = \Delta p_g + \Delta p_L$.

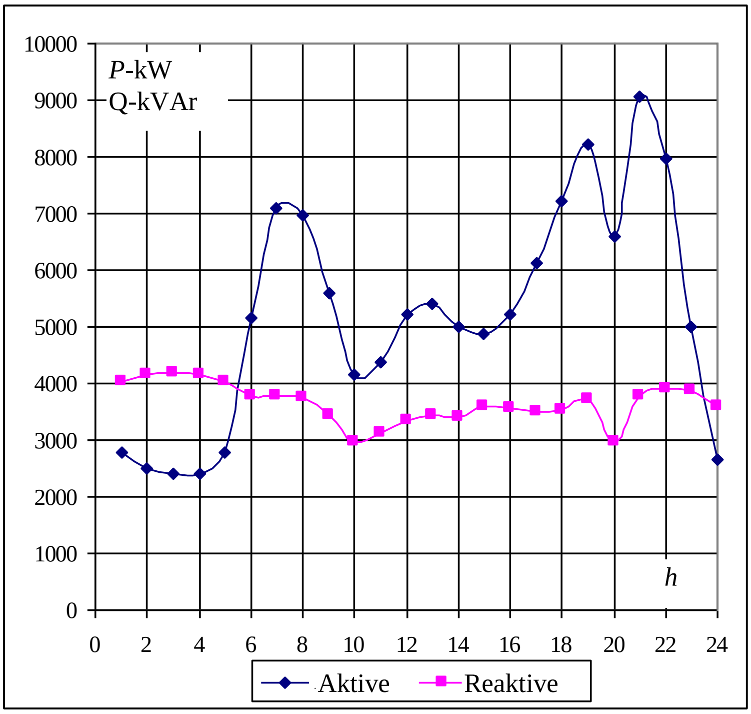

Figure 2.

<u>For $Q_{load.max}$=4000 kVAr</u> $Q_g = 3040$ kVAr (according to formula 12); $Q_L = 960$ kVAr;

$\Delta p_L = 3{,}38$ kW; $\Delta p_g = b\cdot Q_g^2 + c\cdot Q_g + d = 53{,}14$ kW; $\Sigma\Delta p = 56{,}52$ kW

For $Q_{\text{load.av.}}$=3500 kVAr $Q_{\text{g}} = 2605$ kVAr, $Q_L = 895$ kVAr, $\Delta p_L = 2{,}94$ kW, $\Delta p_{\text{g}} = 50{,}23$ kW; $\Sigma\Delta p = 53{,}17$ kW

For $Q_{\text{load min}}$=3000 kVAr $Q_{\text{g}} = 2170$ kVAr, $Q_L = 830$ kVAr, $\Delta p_L = 2{,}53$ kW, $\Delta p_{\text{g}} = 47{,}52$ kW, $\Sigma\Delta p = 50{,}05$ kW

Thus, the obtained expressions for total power losses are the most minimal for indicated consumer's loads.

It is not difficult to convince in this, carrying out a check. Let for $Q_{\text{load.max}}$=4000 kVAr, Q_{g}=2000 kVAr and Q_L =2000 kVAr. Then $\Sigma\Delta p$=61,2 kW, for Q_{g}=4000 kVAr Q_L=0, $\Sigma\Delta p$=60,28kW and finally for Q_{g}=0 and Q_L=4000 kVAr $\Sigma\Delta p$=95 kW.

With such relatively small turndowns of load's reactive power it is reasonable not to control the reactive power of generator as a functions of load's reactive power, but to find such value of generator's reactive power, which remaining changeless, for maximum and minimum values of load's reactive power would give, the total power losses not more than 5 %, for example. Naturally such mode is quasi-optimal.

For given calculating example such value of generator's reactive power is equal to Q_{g}=2800kVAr=const. In this case the error in comparison with the optimal diagram when determining the total losses at maximum load (Q=4000 kVAr) constitutes Δ=0,46 %, and at minimal load it is Δ=3,2 %. That is quite acceptable for engineering calculations.

Thus, with a relatively small fluctuations values of load's reactive power about the average value, it is possible to restrict to quasi-optimal mode, at which the reactive power of generator remains changeless, the errors are minimal in this mode and do not exceed 5 %.

When reducing the output reactive power of generator, its value becomes smaller than the rated one, and is determined by a minimum of electric power losses, it is necessary to check a generator's operation from stability point of view. As for static stability without excitation control, it will, naturally, be for smaller excitation current (reactive power) value less than for the rated one. It is known that to ensure a dynamic stability, each generator is supplied with relay field forcing when short-circuits in generator's external circuit or a sharp increase of motive torque on a generator's shaft. To check this it needs to use the full mathematical model of generator, which, for reduced generator parameters is presented in [3] form:

$$\left.\begin{aligned}
&\rho\psi_{ds} = -U_s \cdot sin\theta + \Psi_{qs} \cdot (1-s) - 0{,}02 \cdot i_{ds} \\
&\rho\psi_{qs} = U_s \cdot cos\theta - \Psi_{ds} \cdot (1-s) - 0{,}02 \cdot i_{qs} \\
&\rho\psi_{dr} = -0{,}028 \cdot \Psi_{dr} + 0{,}022 \cdot i_{ds} + 0{,}022 \cdot i_{df} \\
&\rho\psi_{qr} = -0{,}027 \cdot \Psi_{qr} + 0{,}0117 \cdot i_{qr} \\
&\rho\psi_{qr} = -0{,}057 \cdot U^{*}_{df} - 0{,}045 \cdot i_{df} \\
&\rho s = 0{,}001 \cdot m_{HT} - 0{,}001 \cdot m_{EM} \\
&\rho\theta = s \\
&m_{EM} = \Psi_{ds} \cdot i_{qs} - \Psi_{qs} \cdot i_{ds} \\
&i_{ds} = 3{,}23 \cdot \psi_{ds} - 1{,}12 \cdot \psi_{df} - 1{,}65 \cdot \psi_{dr} \\
&i_{qs} = 2{,}745 \cdot \psi_{qs} - 1{,}7 \cdot \psi_{qr} \\
&i_{df} = 2{,}47 \cdot \psi_{df} - 1{,}12 \cdot \psi_{ds} \cdot 1{,}07 \cdot \psi_{dr}
\end{aligned}\right\} \qquad (14)$$

For $cos\varphi = -0{,}94$ of rated active power of generator equal in relative units to p = 0,784, the reactive power will be equal to q = 0,373, this mode is corresponded to the excitation voltage equal to $U^{*}_{f} = 1{,}58$. These and other operation parameters are presented on fluktogrammas in Fig. 3 (*a*, *b*, *c*), where the images of torque and powers are the functions of time $m_{EM} = f(\tau)$, $p = f(\tau)$, and $q = f(\tau)$, and up to 500 radian the motive torque $m_{HT} = 0$ and $U_f^* = 0$. From 500 up to 12000 radian when $m_{HT} = 0$, $U_f^* = 1$ (generator locked in synchronism), further from 12000 radian up to 20000 radian the motive torque of generator became equal to $m = -0{,}8$ (Fig.3,*a*) and excitation voltage $U_f^* = 1{,}58$, corresponding to that the active power value is equal to $p = -0{,}784$ (рис.3,b), and the reactive one $q = -0{,}373$ (Fig. 3c), thus, the mode equal to $cos\varphi = -0{,}9$ (leading) was reproduced. From 20000 up to 20157 radian a motive torque jump on generator's shaft is formed up to the value of $2{,}4 \cdot m_{st.}$, i.e. $m_{HÒ} = -1{,}9$, with duration of action $\tau = 157$ radian. (t = 0,5 sec.). In Fig. 3 (*d*, *e*, *f*, *g*) are accordingly shown the slip, angle *θ*, excitation current and electromagnetic torque in the range of $2 \cdot 10^4$ up to $2{,}08 \cdot 10^4$ radian (i.e., in the range of 800 radian). It is seen from the fluktogrammas, that the generator is dynamically stable. For example, load angle *θ* varies from 0,363 radian (20,7°) for $m = -0{,}8$ by 2,1 radian (119,7°), i.e., the total angle leading constitutes 140,4°.

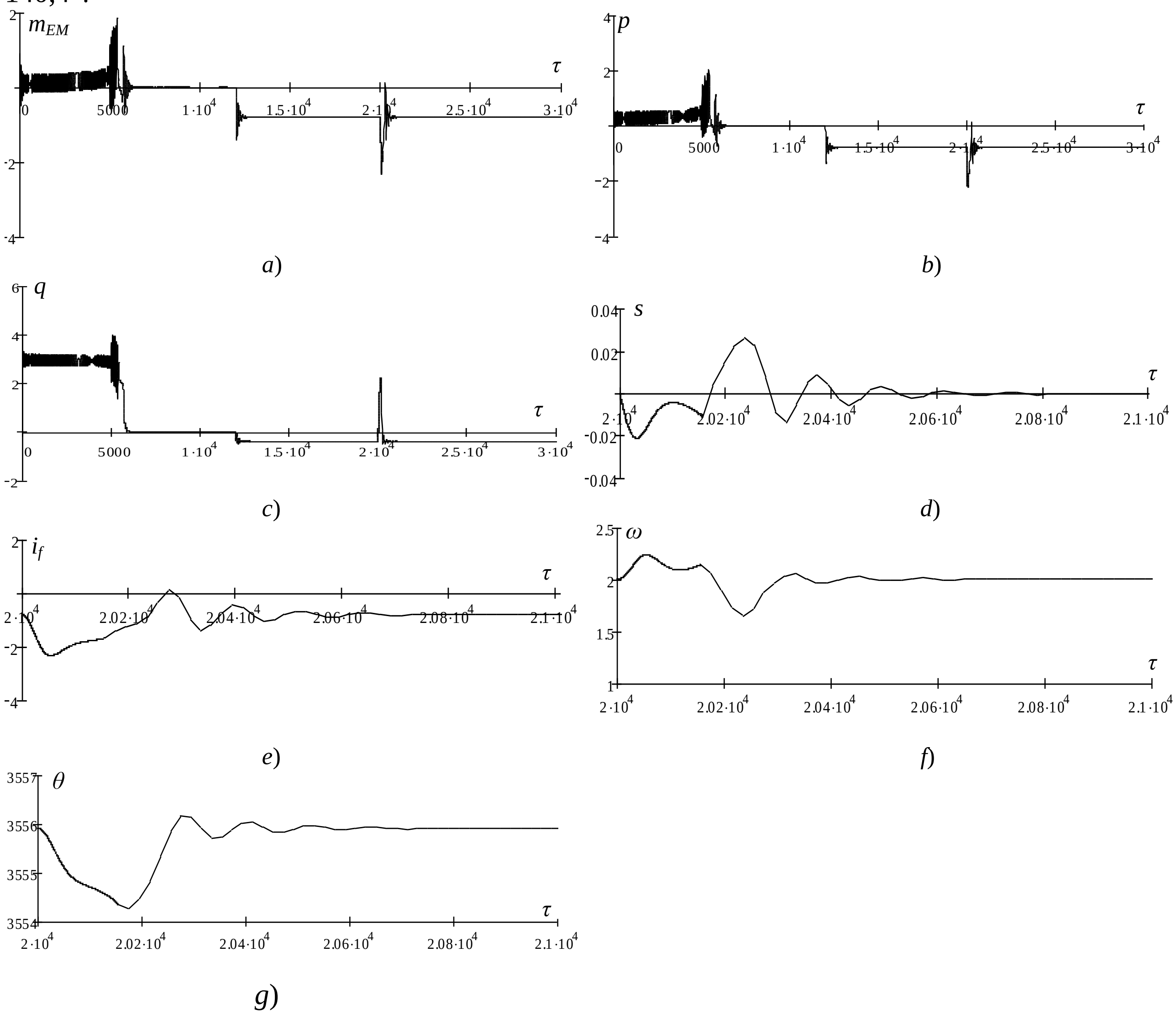

Figure 3.

In fig.4 (*a* ,*b*, *c*) the fluktogrammas of generator's m_{EM} , p and q mode parameters changes are presented when $q = -0{,}207$, which corresponds to the optimal value of reactive power for ensuring the minimal losses in electric power supply system, Fig.1. As it is seen from the fluktogrammas, when attempting to jump a torque up to $m_{HT} = -1{,}9$ value the system from $2 \cdot 10^4$ radian falls out of step.

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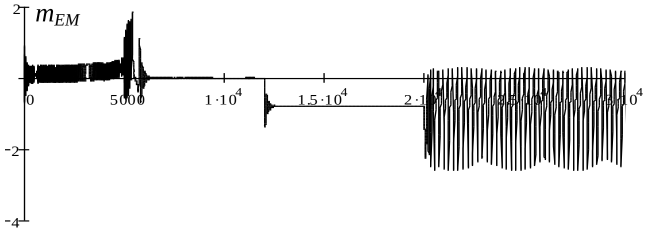
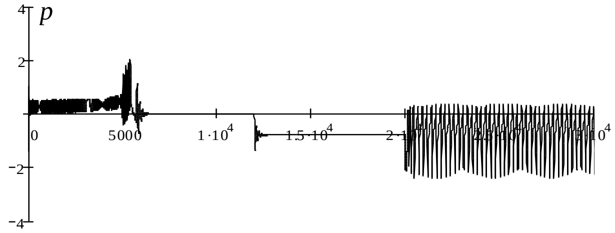
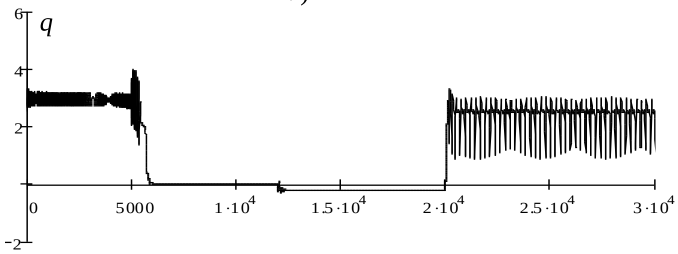


a)



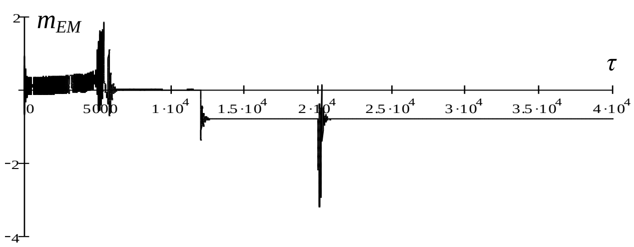
b)



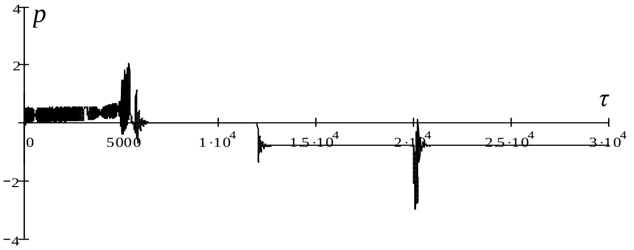
c)

Figure 4.

As it has been noted, the generators are equipped with field enhancing system. In Fig.5 the fluktogrammas of generator's m_{EM} , p and q (Fig.5, *a* , *b* ,*c*) and *s*, *θ*, i_f and m_{EM} mode parameters changing are presented in the time range from $2\ 10^4$ up to $2{,}08 \cdot 10^4$ radian (Fig.5, *d*, *e*, *f*, *g*) for the same conditions, that in Fig.3.4, but with turning on the relay field forcing. Twofold field forcing $U_{fd} \approx 3$ (from $U_{fd_{nom}} = 1{,}58$) with 50 radian delay (0,16 sec.) turned on after giving $m_{H\grave{O}} = -1{,}9$. As it is seen from the fluktogrammas the system has remained dynamically stable.



a)



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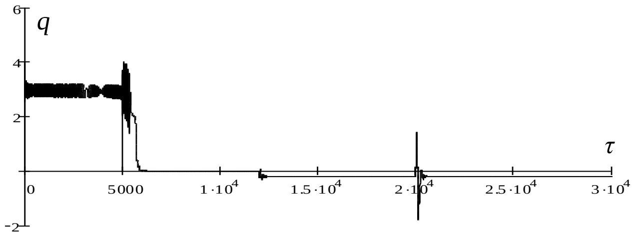

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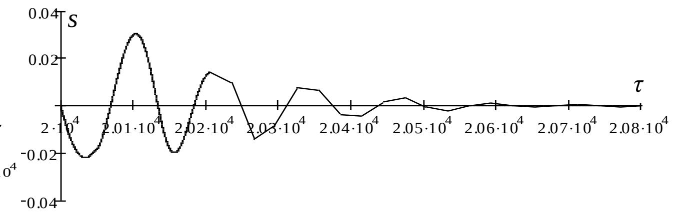

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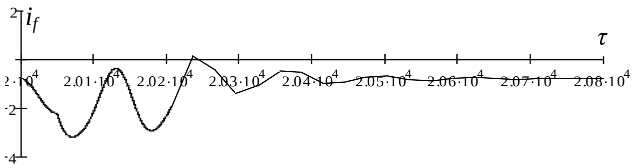

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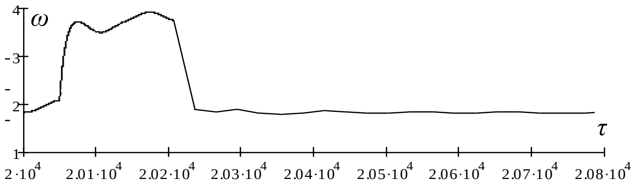

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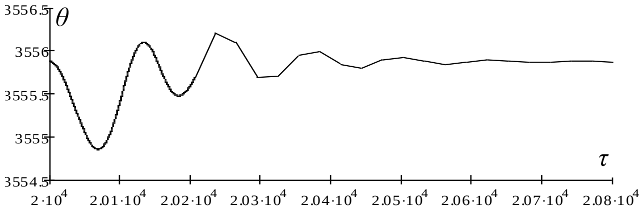

g)

Figure 5.

Thus, the generator can successfully operate from dynamic stability point of view with excitation less than the rated one, returning in this process to the network the optimal reactive power, which in Fig.1 circuit is approximately 40 % less than the rated one.

CONCLUSION

For generator of small power HPS, supplying the load simultaneously with the system power transmission line, the optimal value of generated reactive power is determined, at which the minimization of electric power losses from reactive power interchange is achieved in considered load node.

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APPLICATION OF THE FUZZY-SET THEORY TO THE TASKS OF AZERBAIJAN ELECTROENERGETICS SECURITY FOR SHORT-TERM PERIODS

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ABSTRACT

Electric power industry for short-term tasks of security is presented by the combination of four subsystems, namely: fueling subsystem, electricity production subsystem, transmission and distribution of electricity subsystem, export and import of electricity. To evaluate the security of each subsystem the linguistic variables, rules and membership functions have been defined. The resultant level of short-term security of Azerbaijan republic's electric power industry is determined on the basis of each subsystem's security by developed table.

Energy security is a component of power trilemma both for estimation of power industry functioning efficiency and for estimation of its stability. The continuously growing complexity of power engineering systems, their mutual influence and interconnectivity makes it difficult to estimate definitely the energy security level, and therefore it has to decompose the energy security problem by considering it at different time and space levels of power engineering systems. In this process it often has to handle the fuzzy and incomplete information, which doesn't define clearly the condition of power engineering [1].

This article deals with the application of fuzzy-set theory to electric power industry tasks for short-term periods. As it has been shown in [2], when studying the tasks of electroenergetics security for short-term periods the electric power industry is presented in the form of four interconnected subsystems: fueling subsystem, electricity production subsystem, electric power transmission and distribution subsystem and connections with the neighboring power systems and electricity import subsystem. Electroenergetics security is understood as the immunity of state, society and individual citizens from the threats of deficit when providing their needs with economically affordable electricity of acceptable quality and the threats of disturbance of continuous power supply. The most characteristic indexes (indicators) are selected for each subsystem, and external and internal risks and stabilities are separately grouped.

According to the classification of International Energy Agency the letter symbols from A to *E* can be used when estimating the energy security, as it is shown in Fig. 1, where *A* corresponds to the lowest risks and maximum stability, and *E* to the highest risks and lowest stability [3]. Applying the linguistic variables to subsystems' security classification, the following compliances can be obtained: *A* - "excellent", *B* - "normal", *C* - "not bad", *D* -"bad" and *E* -"very bad". The selected indexes of each subsystem take one of the three values: low, medium and high.

Studies show, that the limited by stroke lines areas appear because of both fuzziness of security indexes' values and their dynamics' change, as it is shown in Fig. 1. Different quantities of estimate layers and indicators are used for the estimation of security of above-named subsystems. The indicators often take the range of values with crossing borders, and sometimes they are distant from each other, that makes an ambiguity when determining the security on them.

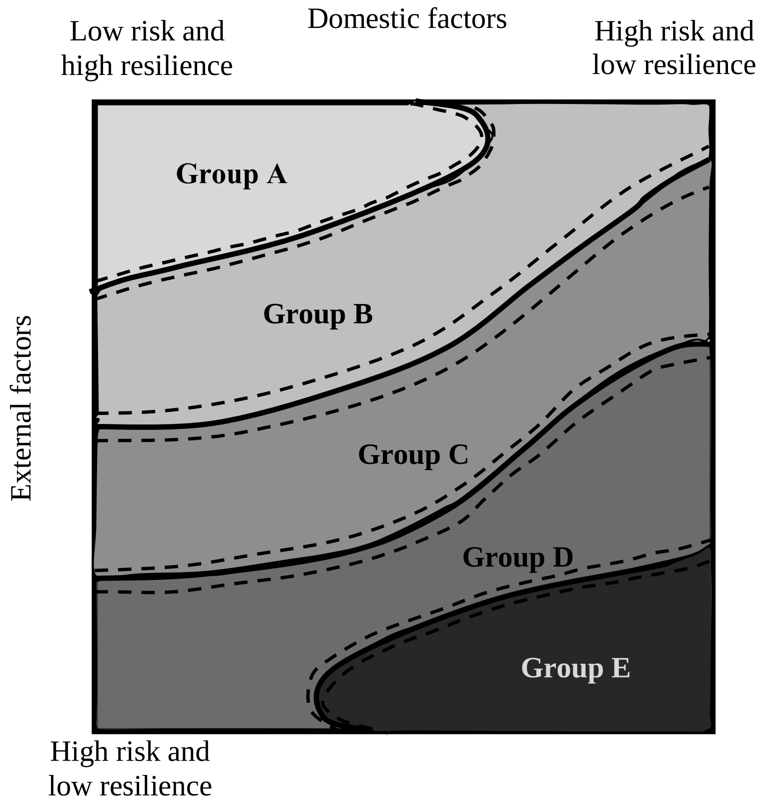

Figure 1.

When determining the security on Fig.1, the obtained result turns out to be sooner qualitative, than quantitative. For example, receiving an estimate of *C* - "not bad", it is difficult to judge, whether this result is closer to the border of B - "normal" or to the border of D - "bad", or it applies strictly to C. Reducing area of each state can be achieved by increasing the number of states with "almost perfect", "almost normal", "not so bad" and so on. But in this case by reducing the inaccuracy of estimate, the scheme of adequate response complicates many times, and subjectivism increases when making a decision about matching the indicators' values to one or another range.

To solve the problem of fuzziness of indicators values, take into account their dynamics' changes and obtain the quantitative value of security on the basis of linguistic information, the clauses of fuzzy-set theory and fuzzy logic can be used.

Fuzzy logical conclusion is carried out on the basis of fuzzy knowledge base, expressed by the linguistic statements of "if-then" type and the operation with fuzzy sets, as it is shown in Fig.2 [4].

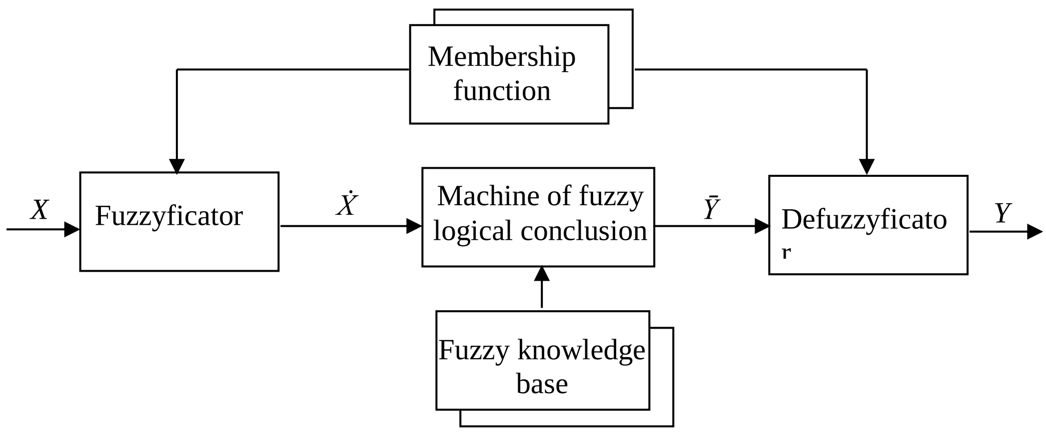

Figure 2.

Fuzzy model contains the following blocks:

- *fuzzyficator,* which transforms a fixed vector of influencing factors X into the vector of fuzzy sets $\tilde{X}$, required for performing the fuzzy logical conclusion;
- fuzzy knowledge base, containing the information about dependence $Y = f(X)$ in the form of a linguistic rules of "*FI- THEN*" type;
- machine of fuzzy logical conclusion, which on the basis of knowledge base rules defines a value of output variable in the form of a fuzzy set $\tilde{Y}$, corresponding to fuzzy values of input variables $\tilde{X}$;
- *defuzzyficator*, transforming the output fuzzy set $\tilde{Y}$ into a clear number Y.

Mathlab program has the package of Fuzzy Logic Toolbox, in which two types of fuzzy models of Mamdany and Sygeno type are realized. For our case the fuzzy model of Mamdany type is preferable.

The relationship between inputs $X = (x_1, x_2, ...x_n)$ and output y in the model of Mamdany type is determined by the fuzzy knowledge base of following format:

IF $(x_1 = a_{1,j1})$ AND $(x_2 = a_{2,j1})$ AND...AND $(x_n = a_{n,j1})$

OR $(x_1 = a_{1,j2})$ AND $(x_2 = a_{2,j2})$ AND...AND $(x_n = a_{n,j2})$

.....

OR $(x_1 = a_{1,jk_j})$ AND $(x_2 = a_{2,jk_j})$ AND...AND $(x_n = a_{n,jk_j})$

THEN $y = d_j$, $i = \overline{1,m},$

where $a_{i,jp}$ is linguistic term by which the variable x_i is estimated in the line with jp $(p = \overline{1,k_j})$ number; k_j -is a number of lines-conjunction, in which the output y is estimated by linguistic term d_j; m - is the number of terms, used for linguistic estimation of output variable y.

All linguistic terms in the knowledge base are presented as the fuzzy sets, specified by the relevant membership functions, as it is shown in Fig. 2:

$\mu_{jp}(x_i)$ - membership function of input x_i to a fuzzy term $a_{i,jp}$, $i = \overline{1,n},$ $j = \overline{1,m},$ $p = \overline{1,k_j},$ i.e.

$$a_{i,jp} = \int_{\underline{x}_i}^{\overline{x}_i} \mu_{jp}(x_i) / x_i, \quad x_i \in [\underline{x}_i, \overline{x}_i];$$

$\mu_{d_j}(y)$ - membership function of output y to a fuzzy term d_j, $j = \overline{1,m},$ i.e.

$$d_i = \int_{\underline{y}}^{\overline{y}} \mu_{d_j}(y) / y, \; y \in [\underline{y}, \overline{y}].$$

The membership degree of input vector $X^* = (x_1^*, x_2^*, ...x_n^*)$ to fuzzy terms d_j from fuzzy knowledge base is determined by the following system of fuzzy logical equations:

$$\mu_{d_j}(X^*) = \underset{p=\overline{1,k_j}}{\vee} \; \underset{i=\overline{1,n}}{\wedge} \left[\mu_{jp}(x_i^*)\right], \; j = \overline{1,m}, \qquad (2)$$

where $\vee$ ($\wedge$) - is the operation of s -norm (t -norm), i.e. from a variety of implementation of *OR (AND)* logical operations. The following implementations are used most often: for *OR* operation - finding a maximum, for *AND* operation - finding a minimum.

The fuzzy set $\tilde{y}$, corresponding to input vector X^*, is determined as follows:

$$\tilde{y} = \underset{j=1,m}{agg}\left(\int_{\underline{y}}^{\overline{y}} imp\left(\mu_{d_j}\left(X^*\right), \mu_{d_j}\left(y\right)\right) / y \right),$$

Where imp -is an implication, usually implemented as the operation of minimum finding; agg - is an aggregation of fuzzy sets, which is most often implemented by the operation of maximum finding.

A clear value of output y, corresponding to input vector X^*, is defined as a result of defuzzyfication of fuzzy set $\tilde{y}$. A defuzzyfication by the method of centre of gravity is most often used:

$$y = \frac{\int_{\underline{y}}^{\overline{y}} y \cdot \mu_{\tilde{y}}(y)dy}{\int_{\underline{y}}^{\overline{y}} \mu_{\tilde{y}}(y)dy}$$

In our model the defuzzyfication is also carried out by this method.

Using the above, the model of fuzzy conclusion for evaluation of each subsystem's security of electric power industry separately is drawn. To evaluate the fueling level of electric power industry, it needs to evaluate the fueling of country, the fuzzy output of which is one of inputs of electric power industry fuelling subsystem.

Natural gas supply of the country

For this subsystem the input subsystem's parameters and their turndowns are shown in Table 1.

Table 1.

Natural gas supply-*SNGS*			
Input parameters	Terms' meanings		
	L-low	M-medium	H-high
DI -dependence on import	<10%	30–40%	>70%
II -infrastructure of import	>60%	30–60%	<30%
RP -variety of suppliers	>60%	30–60%	<30%
PQ -power of delivery from gas storage	<50%	50–100%	>100%

Given in Table 1 the membership function of input parameters, are shown in Fig. 3.

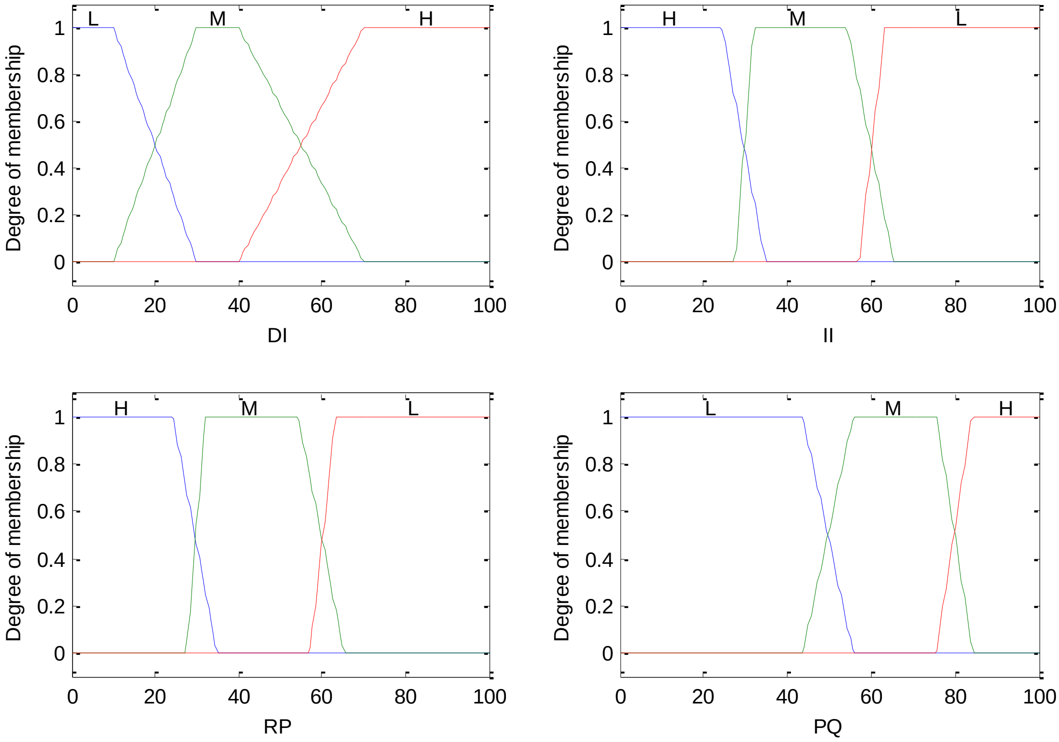

Figure 3.

Security of all subsystems is estimated in Table 2, where the compliances of output value, expressed in percentage terms, with the letter symbols are shown.

Table 2.

Output	*A*	*B*	*C*	*D*	*E*
	85–100	63–85	39–63	18–39	0–18

The membership function of output parameter for "Natural gas supply of the country" subsystem is shown in Fig. 4.

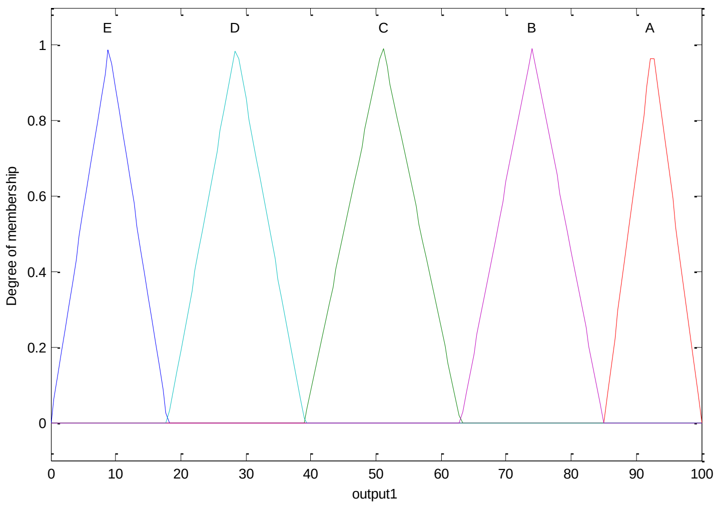

Figure 4.

It needs to note, that the membership function of output parameter for all subsystems is the same.

To estimate the security level of "Natural gas supply of the country" subsystem it needs to use the fuzzy knowledge base, drawing up in the form of table of rules, as it is shown in Table 3.

Table 3.

№	DI	II	RP	PQ	O
1	H	H	H	H	B
2	H	H	H	M	B
3	H	H	H	L	C
4	H	H	M	H	B
5	H	H	M	M	C
6	H	H	M	L	D
7	H	H	L	H	B
8	H	H	L	M	C
9	H	H	L	L	D
10	H	M	H	H	C
11	H	M	H	M	C
12	H	M	H	L	D
13	H	M	M	H	C
14	H	M	M	M	C
15	H	M	M	L	D
16	H	M	L	H	C
17	H	M	L	M	D
18	H	M	L	L	E
19	H	L	–	H	C
20	H	L	–	M	D
21	H	L	–	–	E
22	M	H	–	–	A
23	M	M	H	H	A
24	M	M	H	H	A
25	M	M	H	L	B
26	M	M	M	H	A
27	M	M	M	M	B
28	M	M	M	L	B
29	M	M	L	H	B
30	M	M	L	M	C
31	M	M	L	L	D
32	M	L	–	H	B
33	M	L	–	M	C
34	M	L	–	L	D
35	L	–	–	–	A

Security of "Natural gas supply of the country" subsystem becomes for Azerbaijan after defuzzyfication of the output value equal to 92.5 %, which well corresponds to *A* level.

Electric power industry fuelling

One of the input values of "Electric power industry fueling" subsystem- *PFE* is the output of "Natural gas supply of the country" subsystem- *SNGS*. Two another inputs are "Variety of fuel types" and "Diversification of delivery ways", as it is shown in Table 4 [5].

Table 4.

Electric power industry fueling - *PFE*			
Input parameters	Terms' meanings		
	L-low	M-medium	H-high
SNGS -output of "Natural gas supply of the country" subsystem	60–100%	40–60%	0–40%
VF -variety of fuel types	>64%	33–64%	<33%
DPD-diversification of delivery ways	>64%	33–64%	<33%

The membership function of input parameters for "Electric power industry fuelling" subsystem are shown in Fig. 5.

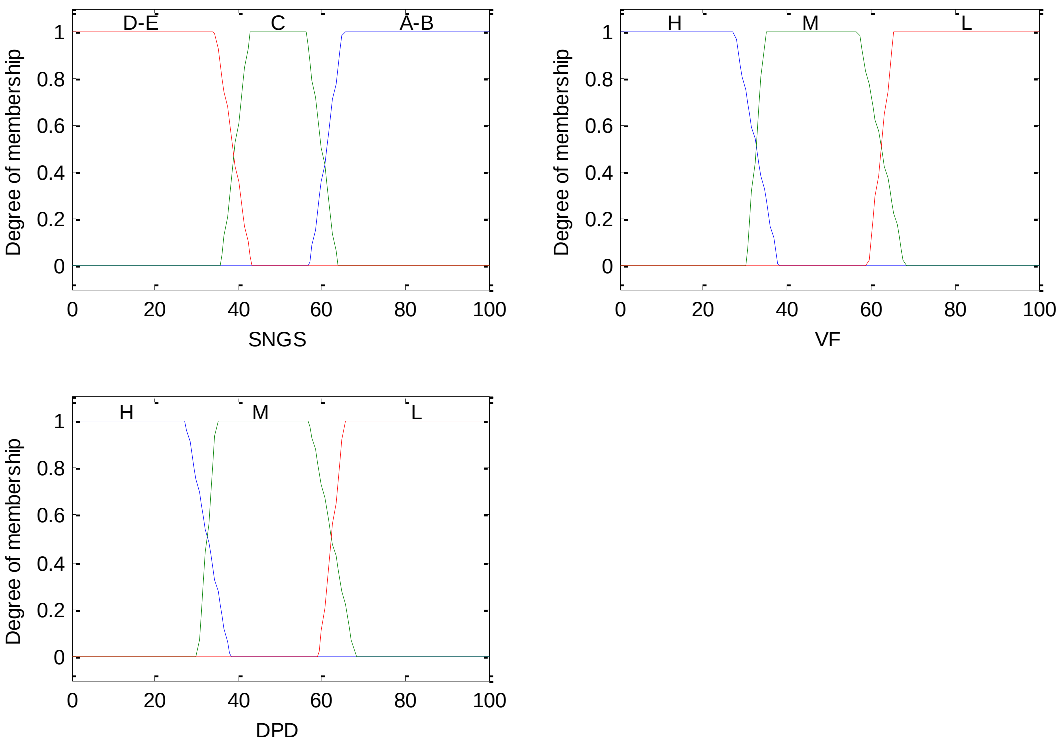

Figure 5.

The fragment of fuzzy knowledge base for evaluation of security of "Electric power industry fueling" subsystem is given in Table 5. It should be noted that, for making the knowledge base the central indicators' values are used.

Table 5.

№	SNGS	VF	DPD	O
1	H	H	H	C
2	H	H	M	C
3	H	H	L	D
4	H	M	H	C
5	H	H	M	D
6	H	H	L	E
7	H	L	H	D
8	H	H	M	E
9	H	H	L	E
10	M	H	H	B

If to use the current values of input parameters of Azerbaijan "Electric power industry fueling" subsystem and the knowledge base, then the security of this subsystem will be 74 %, which corresponds *to B* "normal".

Electricity production

The most important indicators and their ranges for evaluation of this subsystem's security are presented in Table 6.

Table 6.

Electricity production-*EP*			
Input parameters	Terms' meanings		
	L-low	M-medium	H-high
G -electric power generation by own sources	<80%	80–90%	>90%
R -reserve level	<15%	15–25%	>30%
CI - wear degree of capital equipment	<15%	15–30%	>40%
MP -maneuver and distributed generation	<15%	15–30%	>40%

The functions of input parameters' belonging of "Electricity production" subsystem are shown in Fig. 6.

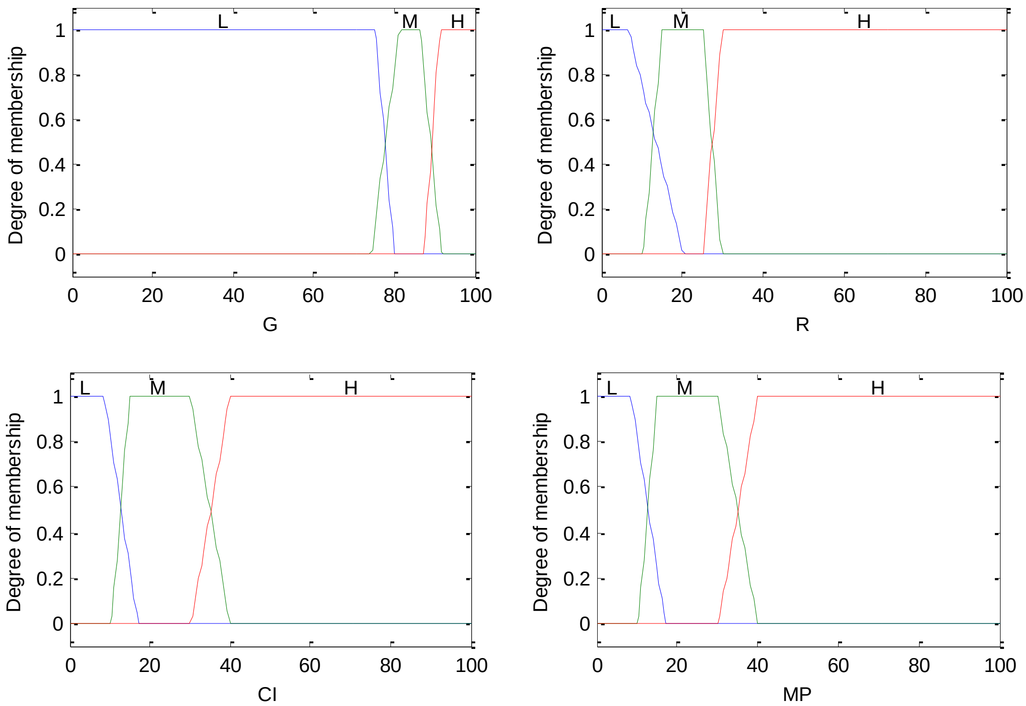

Figure 6.

Table 7 shows a fragment of fuzzy knowledge base for evaluation of security of "Electricity production" subsystem.

Table 7.

№	G	R	S	MP	O
1	H	H	L	L	B
2	H	H	L	M	A
3	H	H	L	H	A
4	H	H	M	L	B
5	H	H	M	M	A
6	H	H	M	H	A
7	H	H	H	L	D
8	H	H	H	M	C
9	H	H	H	H	B
10	H	M	L	L	C

The output parameter of "Electricity production" subsystem just as the outputs of all subsystems is estimated in accordance with Table 2. After defuzzyfication of output parameter, and with taking the input parameters *G*-100 %, *R*-20 %, *CI*-25 %, *MP*–25%, this subsystem's security for Azerbaijan turns out to be 72.8 %, which also corresponds to "normal" level.

Transmission and distribution of electricity

The input parameters for evaluation of this subsystem's security-*TDE* and their values are shown in Table 8.

Table 8.

Transmission and distribution of electricity-*TDE*			
Input parameters	Terms' meanings		
	L-low	M-medium	H-high
WS -wear level of substations	<25%	30–50%	>60%
WT- wear of transformers	<25%	30–50%	>60%
WL -wear of air lines	<25%	30–50%	>60%
SBR - balance degree of regions	<40%	40–70%	>70%

The functions of input parameters' membership of "Transmission and distribution of electricity" subsystem are shown in Fig. 7.

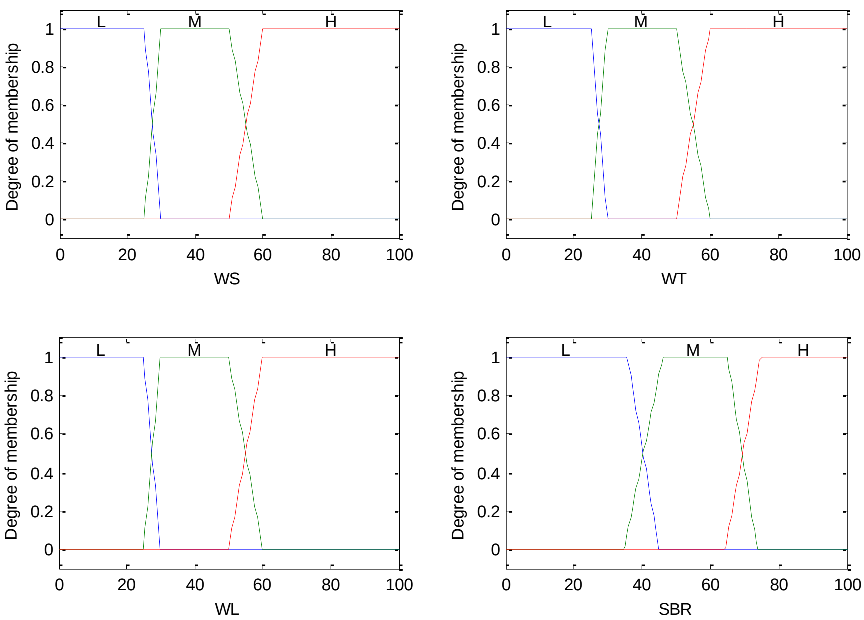

Figure 7.

The fragment of fuzzy knowledge base for evaluation of security of "Transmission and distribution of electricity" subsystem is shown in Table 9.

Table 9.

№	WS	WT	WL	SBR	O
1	H	H	H	H	D
2	H	H	H	M	D
3	H	H	H	L	E
4	H	H	M	H	D
5	H	H	M	M	D
6	H	H	M	L	E
7	H	M	H	H	D
8	H	M	H	M	D
9	H	M	H	L	E
10	H	M	M	H	D

Calculating the security of "Transmission and distribution of electricity" subsystem at input parameters of *WS*-67 %, *WT*-62 %, *WL*-60 %, *SBR*-60 % we shall receive 28.5 %, which corresponds to *D*-"poor" security level.

Connections with the neighboring power systems and electricity import

Input parameters for evaluation of security of this *CEI*-subsystem and their values are shown in Table 10.

Table 10.

Connections with the neighboring power systems and electricity import-*CEI*			
Input parameters	Terms' meanings		
	L-low	M-medium	H-high
LI -level of import	<10%	10–30%	>50%
II-infrastructure of import	>64%	33–64%	<33%
RMC -reserve of transfer capability of intersystem connections	<20%	20–40%	>50%

Fig. 8 shows the functions of input parameters’ belonging of "Connections with the neighboring power systems and electricity import" subsystem.

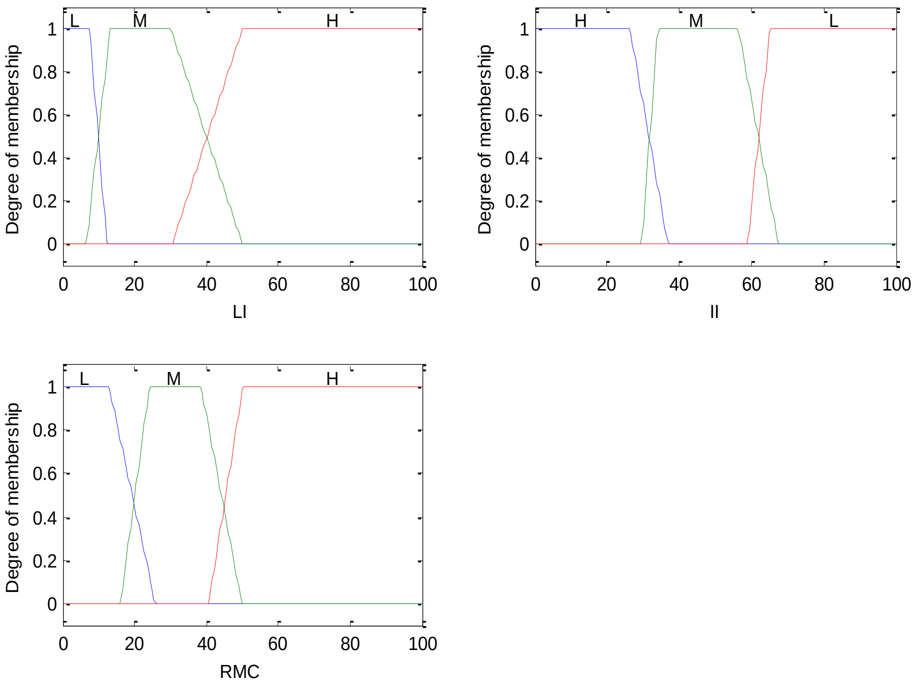

Figure 8.

The fragment of fuzzy knowledge base for evaluation of security of "Connections with neighboring power systems and electricity import" subsystem is shown in Table 11.

Table 11.

№	LI	II	RMC	O
1	H	H	H	C
2	H	H	M	C
3	H	H	L	D
4	H	M	H	C
5	H	M	M	D
6	H	M	L	E
7	H	L	H	D
8	H	L	M	E
9	H	L	L	E
10	M	H	H	B

Security of "Connections with neighboring power systems and electricity import" subsystem with input parameter *LI*-0.5 % turns out to be 92.5 %, which corresponds to *A* -"excellent" security level.

Electroenergetics security of the country

Electroenergetics security of the country is estimated with the help of fuzzy values of subsystems' security, constituting the electric power industry, as it is shown in Fig.9.

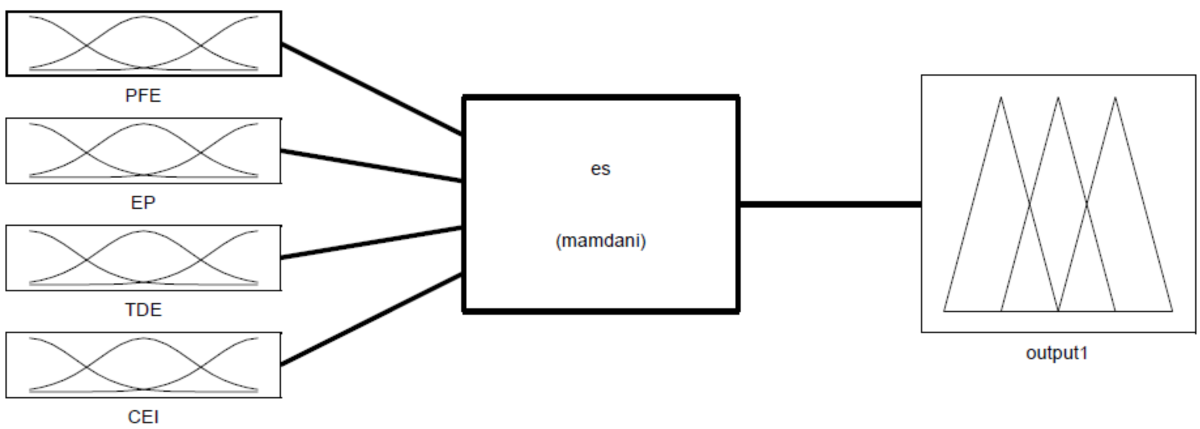

Figure 9.

The Input values of evaluation system of electroenergetics security and their values are shown in Table 12.

Table 12.

Electroenergetics security of the country			
Input parameters	Terms' meanings		
	L-low	M-medium	H-high
PFE-Fueling of electric power industry	0–39%	39–63%	63–100%
EP -Electricity production	0–39%	39–63%	63–100%
TDE- Transmission and distribution of electricity	0–39%	39–63%	63–100%
CEI- Connection with neighboring power systems and electricity import	0–39%	39–63%	63–100%

The membership function of inputs and output for evaluation of electroenergetics security are shown in Fig. 10.

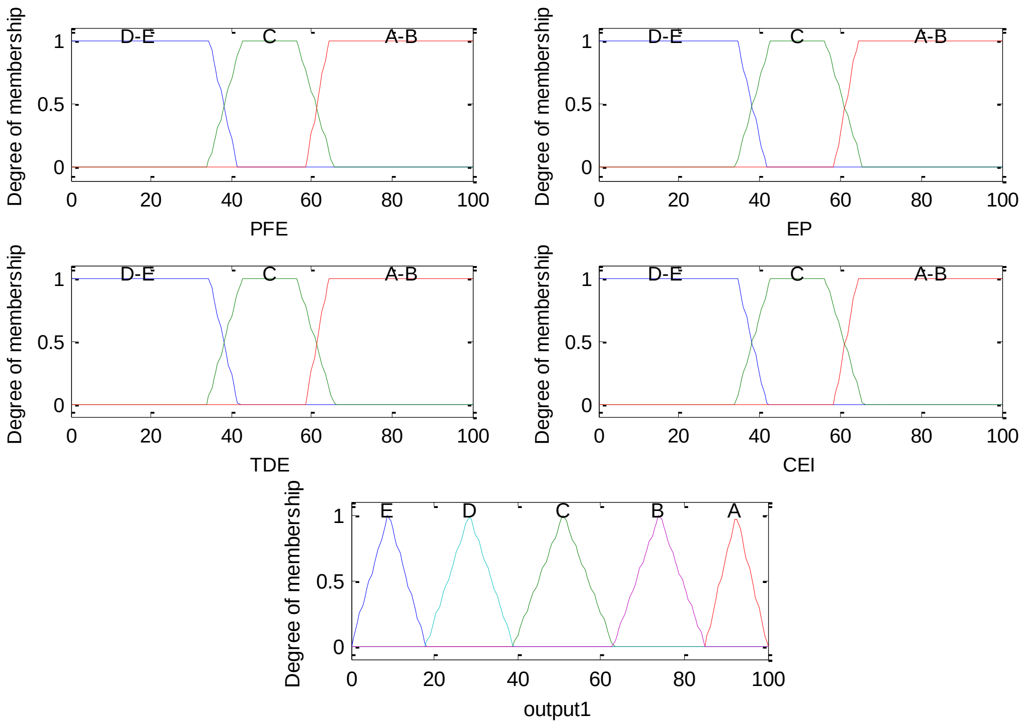

Figure 10.

Table 13 reflects the fragment of fuzzy knowledge base for evaluation of security of "Connection with neighboring power systems and electricity import" subsystem.

Table 13.

№	Electric power industry fuelling	Electricity production	Transmission and distribution of electricity	Electricity import	Result
1	A-B	A-B	A-B	A-B	A
2	A-B	A-B	A-B	C	B
3	A-B	A-B	A-B	D-E	B
4	A-B	A-B	C	A-B	B
5	A-B	A-B	C	C	B
6	A-B	A-B	C	D-E	C
7	A-B	A-B	D-E	A-B	B
8	A-B	A-B	D-E	C	C
9	A-B	A-B	D-E	D-E	D
10	A-B	C	A-B	A-B	B

With the obtained calculating values of electroenergetics security of subsystems: electric power industry fueling-74 %, electricity production-72.8 %, transfer and distribution of electricity-28.5 %, connections with neighboring power systems and electricity import - 92.5 %, the electroenergetics security of Azerbaijan will constitute 74 %, which corresponds to firm "normal" value, as it is shown in Fig. 11.

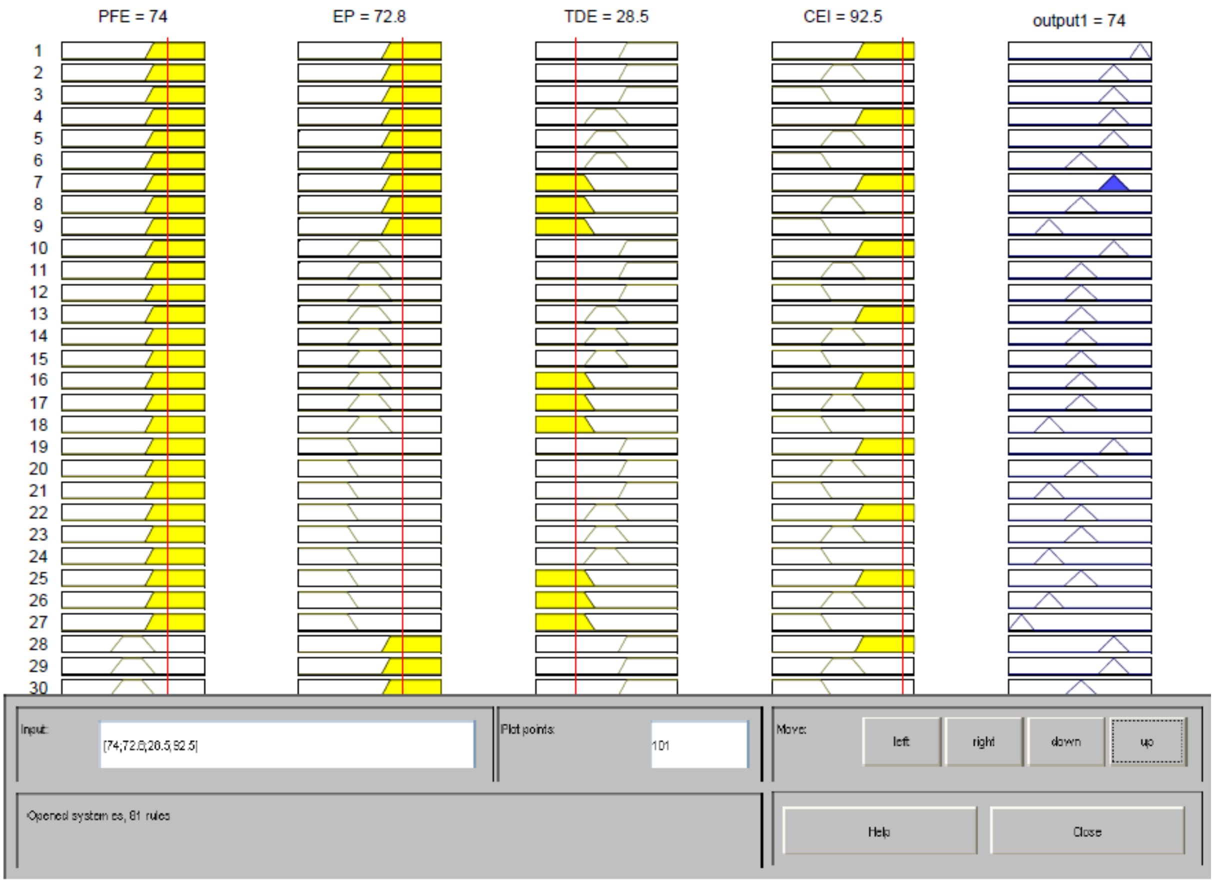

Figure 11.

CONCLUSIONS

1. The fuzziness and incompleteness of indicators' values as well as the dynamics of their change make uncertainty for determining the electroenergetics security.
2. Electroenergetics security for short-term periods can be studied with using 4 interconnected subsystems.
3. Electroenergetics security can be determined by the security of subsystems' components with using the theory of fuzzy sets and fuzzy logic.
4. Applying the fuzzy sets theory the following values have been obtained for electroenergetics security and subsystems' components of Azerbaijan: electric power industry fueling -74 %, which corresponds to *B* security level; production of electricity-72.8 %, the security level-*B;* transmission and distribution of electricity-28.5 %, the level of security-*D*; connections with neighboring power systems and electricity import-92.5 %, the level of security-*A*, the electroenergetics security of Azerbaijan will constitute 74 %, which corresponds to firm "normal" *B*-value.
5. Developed method allows evaluating the security quantitatively, and therefore gives an opportunity to implement the time-based monitoring of energy security level changes of electric power industry and to estimate the effectiveness of policy in electric power industry field in terms of energy security.

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COMPARATIVE RELIABILITY ANALYSIS OF FIVE REDUNDANT NETWORK FLOW SYSTEMS

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ABSTRACT

In this paper, probabilistic models for five repairable redundant network flow systems have been developed to analyze and compare their availability and profit. Explicit expressions for steady-state availability, busy period of repairman and profit function for the five redundant network flow systems are developed. Furthermore, we compare the five redundant network flow systems based on their availability and profit and found that configuration II is more reliable and profitable than the remaining configurations.

1. INTRODUCTION

Reliability connection between networks can be usually achieved through a number of redundant paths/units, thus making the connection reliable. The reliability of these network systems is of increasing importance since the failure of some components may lead to disastrous results. Example of such systems include water distribution, oil and gas supply, power generation and transmission, transport by rail and by road, communication system consisting of a transmitter, relay stations and a receiver, where a signal from transmitter is received by two consecutive relay and distributed to other relay stations before it finally arrived at the receiver for consumptions. Availability and profit of an industrial system are becoming an increasingly important issue. Where the availability of a system increases, the related profit will also increase. High system reliability and availability plays a vital role towards industrial growth as the profit is directly dependent on production volume which depends upon system performance. Because of their prevalence in power plants, manufacturing systems, and industrial systems, many researchers have studied reliability comparison of different systems, a great number of models have been introduced to describe the behaviour and performance of the systems. Evaluation of reliability of network flows with stochastic capacity and cost constraint was studied by Fathabadi and Khodaei (2012). Ke and Chu (2007) performed comparative analysis of availability of redundant system. Wang and Chen (2009) performed comparative analysis of availability of three systems with general repairs, reboot delay and switching failure. Wang et al. (2012) performed comparison of availability between two systems with warm standby units and different imperfect coverage. Wang et al. (2006) performed comparison of reliability and availability between four systems with warm standby components standby switching failures. Yusuf (2013) performed comparative analysis of some reliability characteristics between two systems requiring supporting devices for operation. Yusuf (2014) performed comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation.
The present paper is devoted to modelling and analysis steady-state availability, busy period and profit of five redundant network flow systems. The contributions of this paper are twofold. Based on the first order linear differential equations, explicit expressions of steady-state availability, busy period and profit function for the five redundant network flow systems are developed. Comparisons are performed based on assumed numerical values given to system parameters to determine the optimal system using MATLAB.

2. DESCRIPTION OF THE CONFIGURATIONS

We consider five dissimilar redundant network flow systems as follows. The first configuration consists of two subsystems A and B arranged in parallel has two units each. The second configuration consists of three subsystems A, B and C. With subsystems A and B in series and parallel to subsystem C. Subsystem A has one unit while subsystems B and C two units each. The third configuration consists of three subsystems A, B and C with subsystem A and B in parallel and series to subsystem C with two units each. The fourth configuration is parallel-series system with two units in series in subsystem A and parallel to subsystems B and C. Subsystems B and C are in series and have two units each. The fifth configuration consists of the three subsystems A, B and C in series. Subsystem A has two units in cold standby, subsystem B consists of 2-out-of -3 units while subsystem C consists of one unit.
It is assume that switching from standby to operation is perfect and instantaneous. We also assume that two or more units cannot fail simultaneously. Each active unit fails independent of the state of others. Whenever a unit fails with failure rate α , it is immediately sent to service station for repair with service rate β and the standby unit/subsystem is immediately switched into operation.

3. MODELS FORMULATION

3.1 Availability, Busy period and Profit of Configuration I

For the analysis of availability case of configuration I, we define $P_i(t)$ to be the probability that the system at time $t \geq 0$ is in state S_i . Also let $P(t)$ be the probability row vector at time t . The initial condition for this problem is: $P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),...,P_{11}(0)]$ =
$[1,0,0,0,0,0,0,0,0,0,0,0]$
The steady-state equations for configuration 1 can be expressed as follows:

$$\frac{dP(t)}{dt}=Q_1P \tag{1}$$

This can be written in the matrix form as

$$\dot{P}=Q_1P \tag{2}$$

where

$$Q_1=\begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -Y & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -X & 0 & 0 & 0 & \beta & \beta & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & -Y & 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & -X & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & \alpha & 0 & 0 & 0 & -2\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta \end{pmatrix}$$

$X=(2\alpha+\beta)$, $Y=(\alpha+\beta)$

The steady-state availability and busy period are given by

$$A_{V1}(\infty)=P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty) \quad (3)$$

$$B_{P1}(\infty)=1-P_0(\infty) \quad (4)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (2) become

$$Q_1P=0 \quad (5)$$

which is in matrix form

$$\begin{pmatrix}\dot{P}_0\\ \dot{P}_1\\ \dot{P}_2\\ \dot{P}_3\\ \dot{P}_4\\ \dot{P}_5\\ \dot{P}_6\\ \dot{P}_7\\ \dot{P}_8\\ \dot{P}_9\\ \dot{P}_{10}\\ \dot{P}_{11}\end{pmatrix}=\begin{pmatrix}-2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \alpha & -X & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \alpha & 0 & -X & 0 & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0\\ 0 & \alpha & 0 & -Y & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0\\ 0 & \alpha & 0 & 0 & -X & 0 & 0 & 0 & \beta & \beta & 0 & 0\\ 0 & 0 & \alpha & 0 & 0 & -Y & 0 & 0 & 0 & 0 & \beta & 0\\ 0 & 0 & \alpha & 0 & 0 & 0 & -X & 0 & 0 & 0 & \beta & \beta\\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \alpha & \alpha & 0 & 0 & 0 & -2\beta & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta\end{pmatrix}\begin{pmatrix}P_0(\infty)\\ P_1(\infty)\\ P_2(\infty)\\ P_3(\infty)\\ P_4(\infty)\\ P_5(\infty)\\ P_6(\infty)\\ P_7(\infty)\\ P_8(\infty)\\ P_9(\infty)\\ P_{10}(\infty)\\ P_{11}(\infty)\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+...+P_{11}(\infty)=1 \quad (6)$$

Substituting (6) in the last row of (5) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V1}(\infty)=\frac{\beta^3+2\alpha\beta+4\alpha^2\beta}{5\alpha^3+4\alpha^2\beta+2\alpha\beta^2+\beta^3} \quad (7)$$

$$B_{P1}(\infty)=\frac{\alpha^3+4\alpha^2b+2\alpha\beta^2}{5\alpha^3+4\alpha^2\beta+2\alpha\beta^2+\beta^3} \quad (8)$$

Let C_0 and C_1 be the revenue generated when the system is in working state and no income when in failed state, cost of each repair respectively. The expected total profit per unit time incurred to the system in the steady-state is

Profit=total revenue generated – cost incurred when repairing the failed units.

$$PF_1=C_0A_{V1}(\infty)-C_1B_{P1}(\infty) \quad (9)$$

where PF_1 is the profit incurred to the system.

3.2 Availability, Busy period and Profit Analysis of Configuration II

For the analysis of availability case of configuration II, the same initial conditions are

$$P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),...,P_{10}(0)]\ =[1,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P}=Q_2P \quad (10)$$

Where

$$Q_2 = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -Y & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & \beta & \beta & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -Y & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & -Y & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & -Y & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -Y & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -Y & 0 & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta \end{pmatrix}$$

The steady-state availability and busy period are given by

$$A_{V2}(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) + P_8(\infty) \tag{11}$$

$$B_{P2}(\infty) = 1 - P_0(\infty) \tag{12}$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (10) become

$$Q_2 P = 0 \tag{13}$$

which is in matrix form

$$\begin{pmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \\ \dot{P}_{10} \end{pmatrix} = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -Y & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & \beta & \beta & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -Y & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & -Y & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & -Y & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -Y & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -Y & 0 & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + \ldots + P_{10}(\infty) = 1 \tag{14}$$

Substituting (14) in the last row of (13) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V2}(\infty) = \frac{\alpha_2(2\alpha\beta^4 + 5\alpha^2\beta^3 + 5\alpha^3\beta^2 + 2\alpha^4\beta) + \alpha_1(\beta^4 + \alpha^2\beta^2)}{(4\alpha^5 + 7\alpha^4\beta + 9\alpha^3\beta^2 + 3\alpha\beta^4 + \beta^5)(4\alpha^4 + 3\alpha^3\beta + 6\alpha^2\beta^2 + 2\alpha\beta^3 + \beta^4)} \tag{15}$$

$$B_{P2}(\infty) = \frac{2\alpha^4 + 3\alpha^3\beta + 3\alpha^2\beta^2 + 2\alpha\beta^3}{2\alpha^4 + 3\alpha^3\beta + 3\alpha^2\beta^2 + 2\alpha\beta^3 + \beta^4} \tag{16}$$

Using the procedure described in configuration I above**,** expected profit is

$$PF_2 = C_0 A_{V2}(\infty) - C_1 B_{P2}(\infty) \tag{17}$$

3.3 Availability, Busy period and Profit Analysis of Configuration III

For the analysis of availability case of configuration III, the same initial conditions are

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), ..., P_{10}(0)] \quad = [1,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P} = Q_3 P \tag{18}$$

Where

$$Q_3 = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & 0 & \beta & \beta & \beta & 0 & 0 \\ 0 & \alpha & 0 & -X & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & -\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -X & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & & 0 & \alpha & 0 & 0 & -\beta \end{pmatrix}$$

The steady-state availability and busy period are given by

$$A_{V3}(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty)P_7(\infty) \tag{19}$$

$$B_{P3}(\infty) = 1 - P_0(\infty) \tag{20}$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (18) become

$Q_3 P = 0$

(21)

which is in matrix form

$$\begin{pmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \\ \dot{P}_{10} \end{pmatrix} = \begin{pmatrix} -2\alpha & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & 0 & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -X & 0 & 0 & 0 & \beta & \beta & \beta & 0 & 0 \\ 0 & \alpha & 0 & -X & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & -\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -X & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & & 0 & \alpha & 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+...+P_{10}(\infty)=1 \tag{22}$$

Substituting (22) in the last row of (21) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V3}(\infty)=\frac{\beta^3+\alpha\beta^2+\alpha^2\beta}{4\alpha^3+4\alpha^2\beta+2\alpha\beta^2+\beta^3} \tag{23}$$

$$B_{P3}(\infty)=\frac{4\alpha^3+4\alpha^2\beta+2\alpha\beta^2}{4\alpha^3+4\alpha^2\beta+2\alpha\beta^2+\beta^3} \tag{24}$$

Using the procedure described in configuration I above**,** expected profit is

$$PF_3=C_0A_{V3}(\infty)-C_1B_{P3}(\infty) \tag{25}$$

3.4 Availability, Busy period and Profit Analysis of Configuration IV

For the analysis of availability case of configuration II, the same initial conditions are

$$P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),...,P_{11}(0)]\ =\left[1,0,0,0,0,0,0,0,0,0,0,0\right]$$

The differential equations are expressed in the form

$$\dot{P}=Q_4P \tag{26}$$

Where

$$Q_4=\begin{pmatrix}
-\alpha & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\alpha & -X & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \alpha & -X & 0 & \beta & 0 & \beta & 0 & 0 & 0 & 0 & 0\\
0 & \alpha & 0 & -X & 0 & \beta & 0 & \beta & 0 & 0 & 0 & 0\\
0 & 0 & \alpha & 0 & -X & 0 & 0 & 0 & \beta & \beta & 0 & 0\\
0 & 0 & 0 & \alpha & 0 & -X & 0 & 0 & 0 & 0 & \beta & \beta\\
0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0\\
0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta
\end{pmatrix}$$

The steady-state availability and busy period are given by

$$A_{V4}(\infty)=P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty) \tag{27}$$

$$B_{P4}(\infty)=1-P_0(\infty) \tag{28}$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (26) become

$$Q_4P=0 \tag{29}$$

which is in matrix form

$$\begin{pmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \\ \dot{P}_{10} \\ \dot{P}_{11} \end{pmatrix} = \begin{pmatrix} -\alpha & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -X & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & -X & 0 & \beta & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & -X & 0 & \beta & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & -X & 0 & 0 & 0 & \beta & \beta & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & -X & 0 & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \\ P_{11}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + \ldots + P_{11}(\infty) = 1 \tag{30}$$

Substituting (30) in the last row of (29) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V4}(\infty) = \frac{\beta^4 + \alpha\beta^3 + 2\alpha^2\beta^2 + 2\alpha^3\beta}{4\alpha^4 + 4\alpha^3\beta + 2\alpha^2\beta^2 + \alpha\beta^3 + \beta^4} \tag{31}$$

$$B_{P4}(\infty) = \frac{4\alpha^4 + 4\alpha^3\beta + 2\alpha^2\beta^2 + \alpha\beta^3}{4\alpha^4 + 4\alpha^3\beta + 2\alpha^2\beta^2 + \alpha\beta^3 + \beta^4} \tag{32}$$

Using the procedure described in configuration I above**,** expected profit is

$$PF_4 = C_0 A_{V4}(\infty) - C_1 B_{P4}(\infty) \tag{33}$$

3.5 Availability, Busy period and Profit Analysis of Configuration V

For the analysis of availability case of configuration II, the same initial conditions are

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \ldots, P_{11}(0)] = [1,0,0,0,0,0,0,0,0,0,0,0]$

The differential equations are expressed in the form

$$\dot{P} = Q_5 P \tag{34}$$

Where

$$Q_5 = \begin{pmatrix} -3\alpha & \beta & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -W & 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -W & 0 & 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 \\ \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & \alpha & 0 & 0 & 0 & -Z & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta \end{pmatrix}$$

$W = (3\alpha + \beta)\,, Z = (3\alpha + 2\beta)$

The steady-state availability and busy period are given by

$$A_{V5}(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_6(\infty) \quad (35)$$

$$B_{P5}(\infty) = 1 - P_0(\infty) \quad (36)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (34) become

$$Q_5 P = 0 \quad (37)$$

which is in matrix form

$$\begin{pmatrix} -3\alpha & \beta & \beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -W & 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & -W & 0 & 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 \\ \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & \alpha & 0 & 0 & 0 & -Z & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \\ P_{11}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing conditions:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + \ldots + P_{11}(\infty) = 1 \quad (38)$$

Substituting (38) in the last row of (37) to compute the steady-state probabilities, the expression for steady-state Availability and Busy period are given by

$$A_{V5}(\infty) = \frac{\beta^3 + 2\alpha\beta^2 + \alpha^2\beta}{3\alpha^3 + 5\alpha^2\beta + 3\alpha\beta^2 + \beta^3} \quad (39)$$

$$B_{P5}(\infty)=\frac{3\alpha^3+5\alpha^2\beta+3\alpha\beta^2}{3\alpha^3+5\alpha^2\beta+3\alpha\beta^2+\beta^3} \tag{40}$$

Using the procedure described in configuration I above**,** expected profit is

$$PF_5=C_0A_{V5}(\infty)-C_1B_{P5}(\infty) \tag{41}$$

4. GRAPHICAL ANALYSIS OF THE NETWORKS

In this section, the main purpose of this section is to present specific numerical comparisons for the configurations for steady-state availability and profit. For each model the following set of parameters values are fixed throughout the simulations for consistency:

Case I: We fix $\beta=0.3$ **,** $C_1=500,000$**,** $C_2=80,000$ and vary the values of α for from 0 to 1 Figures 1 and 4.

Case II: We fix $\alpha=0.4$ **,** $C_1=500,000$**,** $C_2=80,000$ and vary the values of β for from 0 to 1 Figures 2 and 3.

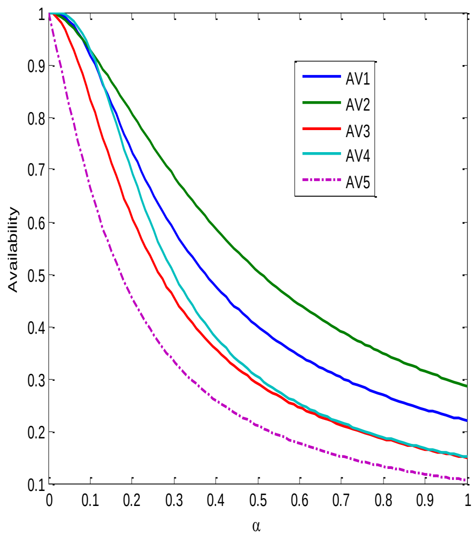

Figure 1 Availability against α

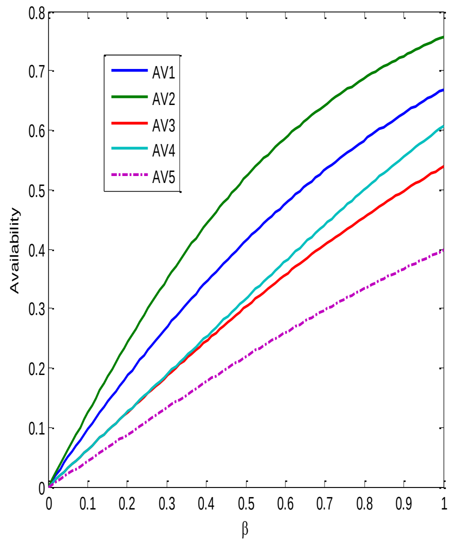

Figure 2 Availability against β

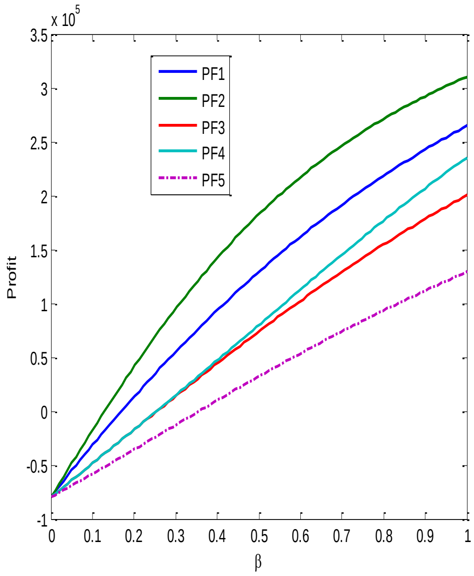

Figure 3: Profit against β

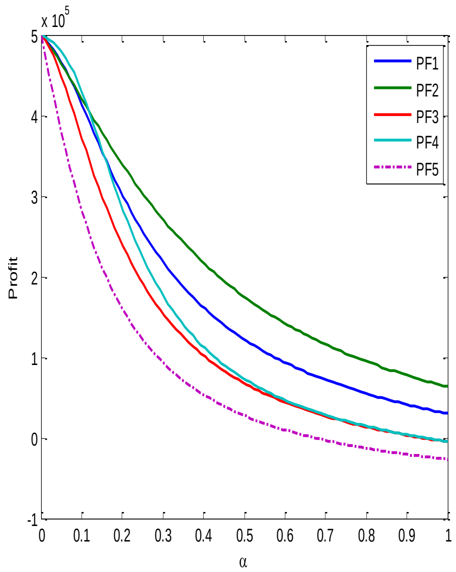

Figure 4: Profit against α

Figures 1 and 4 depict the availability and profit results for the five systems being studied against the failure rate α . The steady-state availability decrease as α increases for any configuration. It is clear from the Figures that configuration II has higher availability with respect to α as compared with the other three configurations. These tend to suggest that configuration II is better than the other configurations. On the other hand, Figures 2 and 3 depict the availability and profit calculations for the five configurations against repair rate β . The steady-state availability and profit increase as β increases for any configuration. The observations that can be made here are much similar to those made from Figures 1 and 4. It is evident that from Figures 2 and 3 that configuration II is better than the other configurations. Thus,

$$A_{V2}(\infty) > A_{V1}(\infty) > A_{V4}(\infty) > A_{V3}(\infty) > A_{V5}(\infty)$$

$$PF_2(\infty) > PF_1(\infty) > PF_4(\infty) > PF_3(\infty) > PF_5(\infty)$$

5. CONCLUSION

In this paper, we analysed five different redundant communication networks with standby units to study the availability and profit analysis of five configurations. For each configuration, we present the explicit expressions for steady-state availability, busy period of repairman and profit and performed comparative analysis numerically to determine the optimal configuration. It is evident from Figures 1-4 that configuration II is optimal configuration using steady-state availability and profit. The present study will help the engineers and designers to develop sophisticated models and to design more critical system in interest of human kind.

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MODIFIED CONCEPT OF DANIELS' SEQUENCE AND ITS APPLICATION TO ANALYSIS OF THE RELIABILITY OF SERIES-PARALLELS SYSTEM AND FATIGUE LIFE OF UNIDIRECTIONAL FIBROUS COMPOSITE

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ABSTRACT

This is short review of previous publications and new development of the concept of Daniels' sequence (DS) which can be used to consider the association of distribution of the static strength of unidirectional fiber composite (UFC) component with distributions of its fatigue life and residual strength of the UFC itself. A generalization of the model of series-parallel system as description of structure of UFC is given. A new description of the defected structural longitudinal element(LE) of UFC, new condition of failure of UFC, a definition of residual Daniels' function is introduced. Numerical examples of processing of experimental data (data of fatigue test of glass-fiber composite) are presented. The estimates of the parameters of the corresponding nonlinear regression model for simultaneous processing of result of the fatigue test and test of residual tensile strength are obtained (parameters of the distribution of the local strength parameters of structural LE).

1. INTRODUCTION

1.1 Background

Usually the reliability of some system is described only by the distribution of its time to failure. Only the two states of the system (intact, failure) are considered. The use of the concept of Daniels' sequences (DSs) for explanation the reliability of some parallel system makes it possible to take into account the parameters of the process of loading of the system (for example, the tensile or cycling test of a unidirectional fibrous composite (UFC)) and explain the quantitative relation between the distribution of the static strength of longitudinal elements (LEs), taking up the basic load, and the static strength, fatigue life and residual strength of a UFC itself considered as series-parallel system. Here the processing of the result of fatigue test of UFC is considered. It is of particular importance, that the use of Daniels' Sequence (DS) concept allows to explain the existence of an fatigue-limit (maximum of cycling stress at which the fatigue life is equal to infinity). The history of the question on the relation between distributions of the static strength of a UFC and its components has been discussed in detail in our preceding papers, therefore, here we mention only several authors who have related works : Peirce (1926), Daniels (1945), Gucer & Gurland (1962), Smith (1982) and others. The use of the theory of Markov processes for the problems of static and cyclic fatigue life is rather thoroughly considered in Bogdanov et all (1989) and for the tests on the tensile strength in Paramonov at all (2006-2012a). The corresponding models of relation between the static strength and the fatigue life of composite materials is suggested in Paramonov at all (2006,2008-2012b). Some concluding results are reported in Paramonov at all (2011). The concept of the Daniels' sequence, with reference to the description of the process of fatigue failure, was first introduced in Paramonov at all (2006). Its successful application to describing the relation between the strength of LE and the fatigue life of UFC is discussed in Cimanis at all (2012) and Paramonov at all (2013a). In Paramonov at all (2013b) a more general definition of DS, which can be applied to processing of result of testing UFC specimens both on the fatigue life and static strength. It is shown that the use of DS concept gives the explanation of existence of fatigue-limit. In this paper a new development of the DS concept is offered. Definition of residual Daniels' function is introduced. It is taken into account that the event " residual strength becomes lower than the maximum level of cycle stress" is only necessary but is not enough condition. Some energy must be accumulated to get the failure of specimen. Numerical examples of processing of experimental data are presented, and estimates of the parameters of the corresponding nonlinear regression model for simultaneous processing of result of the fatigue test and test of residual tensile strength are obtained (parameters of the distribution of the local strength of structural longitudinal elements).

1.2. The Simple Daniels' sequence

The Simple Daniels' sequence (SDS) (here we use this name for initial version of Daniels' sequence initially introduced in [12]) for description of fatigue phenomenon is based on the model studied by Daniels (1945). Let $X_{(1)}, X_{(2)},..., X_{(n)}$ be the ordered values of the random strengths of the LEs forming the parallel system. Assuming the independence of the random variabels (RVs) $X_1, X_2,..., X_n$ with the same cumulative distribution function (CDF) $F_X(x)$, Daniels showed that the RV $Y = \max\{X_{(k)}(n-k+1)/n : 1 \le k \le n\}$ has an asymptotically normal distribution with the average and standard deviation

$$\mu_D = \max x(1-F_X(x)) = x^*(1-F_X(x^*)), \quad \sigma_D = (\mu_D x^* F_X(x^*)/n)^{1/2}. \tag{1}$$

By "unwrapping" this model in time, we obtain a sequence of local (in specific link where the fatigue damage develops) stresses $\{s_0, s_1, s_2,...\}$, which we name the realization of *simple Daniels's sequence* (*SDS*) for fatigue test, described by the equation

$$s_{i+1} = s/(1-\nu(s_i)/n) = s/(1-\hat{F}_X(s_i)), \quad i = 0,1,2,..., \quad s_0 = s \tag{2}$$

where s is the parameter of the initial cycling nominal stress (for example, by s is implied the maximum of the cyclic stress); $\nu(s_i)$ is the number of LE with strength lower than or equal to s_i; the function $\hat{F}_X(.)$ is the estimate of the CDF (empirical CDF) of the local strength of LE in framework of specific parallel system using the sample $x_{1:n} = (x_1,..., x_n)$ (the realization of the vector ($X_1, X_2,..., X_n$)) . We suppose that in the last part of this equation another definition of the estimate of CDF can be used. For example, if $F(x) = G(x,\theta)$ then the estimate $\hat{F}(x) = G(x,\hat{\theta})$ can be used where $\hat{\theta}$ is the estimate of the parameter θ.

The SDS is completely determined by the CDF estimation method and by the structure $(s, n, F_X(x))$ and its realization determined by the pair (s, $x_{1:n}$). It has the following properties (see Paramonov at all (2013a):

(1) Since $\hat{F}_X(x) \le 1$ and it is a no decreasing, then $s_{i+1} \ge s_i$; i.e., the SDS is a nondecreasing sequence also. If $s > \max x(1-\hat{F}_X(x))$, the SDS increases up to infinite. We call this sequence the first-kind DS and designate the pair that generates it by the symbol $(s, x_{1:n})^*$.

(2) Now, assume that $0 < s \le \max x(1-\hat{F}_X(x))$. Then, there is the solution to the equation

$$s = x(1-\hat{F}_X(x)), \tag{3}$$

and there is such first $i = i^{**}$ that $s_{i^{**}+1} = s_{i^{**}}$, and the SDS growth process stops. We call this sequence the second-kind SDS.

WE assume, that the failure of the parallel system (link) under the alternating cyclic loading takes place if the share of the operable LEs is reduced to the value f_C, where f_C is the function of s.Then, the critical local stress that correspond to this event, s_C, is determined from the equation

$$f_C = 1-\hat{F}_X(s_C): \quad s_C = \hat{F}_X^{-1}(1-f_C).$$

The transition from s_i to s_{i+1} is called the SDS step. It corresponds to the damage of all the LEs with strength in the interval of $(s_i, s_{i+1}]$. The maximum number of steps at which the local stress is still smaller than the critical one we call the *SDS life* (SDSLf) of the parallel system at the nominal stress s:

$$N_D = \max(i : s_i < s_C,\ s_i \in \{s_0, S_1, S_2 ...\},\ s_0 = s) + 1, \qquad (4)$$

where $\{s_0, S_1, S_2 ...\}$ is the SDS for $s_0 = s$; i.e., N_D is equal to one plus the number of those SDS elements (or the maximum number of SDS elements among those elements), which do not exceed s_C. For the second-kind SDS, N_D is equal to infinity. Examples of the SDS are shown in Fig.2.

For some materials, there is the maximum cyclic stress s for which the number of before-damage cycles is equal to infinity. This stress is called the fatigue-limit or altimate fatigue strength. If the solution to equation (3) exists, then, as it was already stated, the increase of the SDS components stops after some step number. The critical stress s_C will never be reached, and the SDSLf will be equal to infinity. The maximum stress at which this phenomenon takes place

$$S_D = \max x(1 - \hat{F}_X(x)) \qquad (5)$$

should be considered as SDS fatigue-limit (SDSLt).

So the use of SDS can explain the existence of fatigue-limit. The dependence of the distribution of RV N_D on the stress s is quite similar to the dependence of the fatigue life on s in fatigue tests of the composite. However, the values of N_D for $s > S_D$ are too small for as compared with the real data. The value of S_D is too large and itself coincides with Daniels' definition of tensile strength of bundle of fibers. So some additional assumptions should be made in order to describe the real result of fatigue test. The development of this initial definition of SDS is needed .

2. UFC SPECIMENS FOR TENSION AND FATIGUE TESTS AS A SERIES – PARALLEL SYSTEM

Here we offer some modified description of the structure of UFC as a series-parallel system (SPS), Fig. 1.

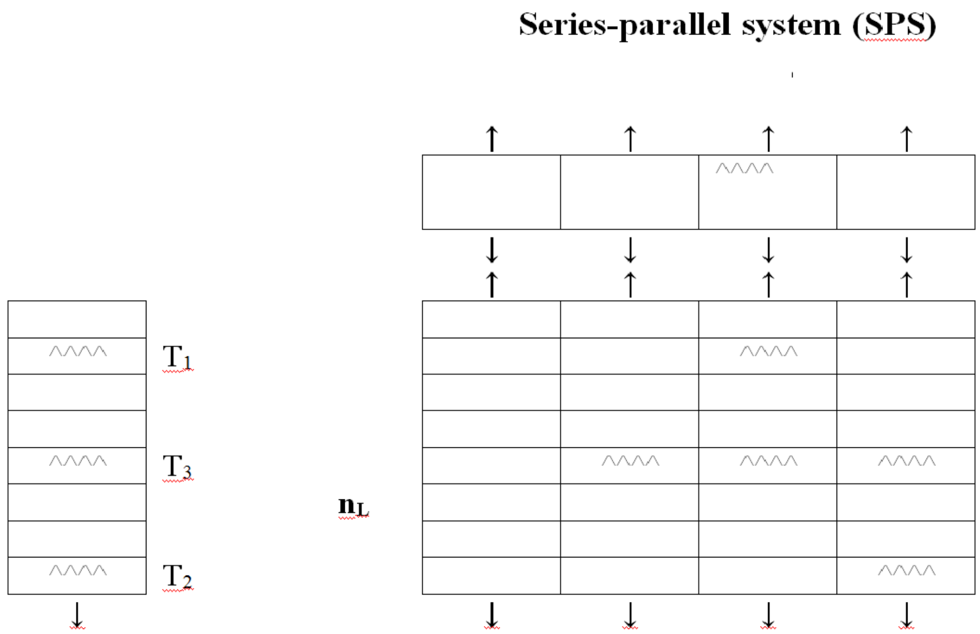


Fig. 1. Structures of UFC as a series, a parallel and a series-parallel system.

This SPS is considered as a sequence (chain) of n_L links which are the parallel systems. In K_L links the defects already exists or can appear. The value K_L is a RV, $1 \le K_L \le n_L$. In ($n_L - K_L$) links, the defects cannot appear. In flawless links, there are n_C elements. Unlike the previous definition now we suppose that in defected link there are two types of LEs: there are K_C, $1 \le K_C \le n_C$, random number of defected (weak) LEs with CDF of strength $F_W(x)$ and $M_C = n_C - K_C$ LEs without defects with CDF of strength $F_U(x)$.

3. MODIFIED DANIELS' SEQUENCE

Let us mention three salient features of Daniels' model:

(1) a continuous increase in the external load is assumed, but its rate is not taken into account;
(2) it is assumed that destructions of one by one fibers with a successively growing strength are accumulated;
(3) by the strength of a fiber bundle is meant the external load at the instant when it becomes higher than the load-carrying ability of the specimen tested. Thereafter, the destruction, i.e., rupture, of the bundle is presumed.

In the present study, which is the development of Paramonov at all (2013b), we make the following assumptions.
(1) The process of the external loading is described by discrete sequence $s^+_{i:\infty} = \{s^+_i, i = 0,1,2,...\}$. For tension test $s^+_{i+1} \ge s^+_i$, $\lim s_i = \infty$, for cycling loading $s^+_i = s^+ = Const$ for all $i = 0,1,2,...$.
(2) There is a local stress concentration, defined by a concentration factor k_C for cycling loading or concentration factor k_T for tension test (it is supposed that in general case different type of stress distribution for tensile and cycling test takes place). And there is a local decreasing of the strength in a weak link. For cyclic load the modified Daniels' sequence (MDS_CL) is defined by the equation

$$s_i = k_C s^+ / (1 - \nu(s_{i-1})/n) = k_C s^+ / (1 - \hat{F}_{X_L}(s_{i-1})),$$

where $\hat{F}_{X_L}(s)$ is the estimate of the CDF of the local tensile strength of a link in which there are the defected LEs with $F_W(x)$ and LEs without defects with $F_U(x)$ (in the following numerical example we suppose that $F_{X_L}(s)$ is a mixture of $F_W(x)$ and $F_U(x)$, $F_W(x) = F_U(k_F x)$, k_F, $k_F \ge 1$, is some parameter of the model).

For increasing load at the tensile strength test MDS_IL it is defined by the equation

$$s_i = k_T s^+_i / (1 - \nu(s_{i-1})/n) = k_T s^+_i / (1 - \hat{F}_{X_L}(s_{i-1}))$$

where $s_0 = s^+_1$, $s^+_{i+1} \ge s^+_i$, $i = 1,2,...$, $\lim s_i = \infty$.

So now the random Daniels' fatigue limit, S_D is defined by maximum value of s^+ for which there is a solution of the equation $x = k_C s^+ / (1 - \hat{F}_L(x))$.

So we have

$$S_D = \max x(1 - \hat{F}_{X_L}(x)) / k_C \quad .$$

(3) The failure of the link (parallel system) takes place at kth step if the *Daniels' residual tensile strength*, $R_D(s)$, becomes lower than the applied load

$$R_D(s_i) > s^+_i \text{ for all } i = 1,2,...,k-1 \text{ but } R_D(s_k) \le s^+_k$$

where Daniels residual tensile strength, $R_D(s)$, is defined by equation

$$R_D(s) = \max_{z > s} d(z),$$

Daniels' function, $d(z)$, is defined by equation

$$d(z) = k_T z(1 - \hat{F}_{X_L}(z)),$$

where k_T is the stress concentration factor for tensile.

In this paper we consider only the case of cycling loading. We are interested only in fatigue life and residual strength of a link (as a parallel system) and of UFC (as a series-parallel system (SPS)).

It can be seen that for MDS as for SDS the number of steps to failure is very small if $s^+ > S_D$ or is equal to infinity if $s^+ \le S_D$. So two additional assumptions are made.

(4) The process of transition from some state of MDS s_i to next state s_{i+1} is described by Markov chain (MCh) (see detales in following section).

(5) For failure of some set of LEs the accumulation of some energy is necessary. In some cycle with parameter s^+ this energy is a function of s^+ (example of approximation of this function see in section (5)).

3. LONGEVITY OF A PARALLEL SYSTEM UNDER A CYCLIC LOAD. APPLICATION OF MARKOV CHAIN THEORY

There are two types MDS for cycling loading. MDS_CL1 corresponds to the failure of a link (if some condition of a failure is achieved) . MDS_CL2 corresponds to the infinite number of the steps without failure (if the nominal stress is lower or equal to Daniels' fatigue-limit).The random process MDS_CL can be described by Markov chain with random transition probability matrix, P, with two corresponding absorbing states. So we have the following type of matrix P:

		j_B	*1*			*2*		
		j_A	*1*	...	j^*	*1*	...	j^{**}
i_B	i_A	i \ j	1	…	j^*	j^*+1	…	j^*+j^{**}
1	*1*	1	p_{11}	…	0	0	…	0
	...	…	0	…	…			
	i^*	i^*	0	0	1	0	…	0
2	*1*	i^*+1	0	0	0	$p_{(j^*+1)(j^*+1)}$	…	0
	...	…	0	0	0	0	…	…
	i^{**}	i^*+i^{**}	0	0	0	0	0	1

In fact, the matrix P is complex of two submatrices. The first $(n_D + 1)$ states, $(1,2,\ldots,i^*)$, of submatrix in left upper corner are connected with the components of MDS_CL1, $\{s_0, s_1, ..., s_{n_D}\}$. The absorbing ($n_D + 2$)-th state corresponds to the condition of failure of a link. Here, $\{s_0, s_1, ..., s_{n_D}\}$ is the realization of the random MDS_CL1 , $\{s_0, S_1, S_2 ...\}$, generated by sample $x_{L,1:n}$ (the realization of the vector ($X_{L,1}, X_{L,2}, ..., X_{L,n}$)) and nominal stress s^+, where $s^+ > S_D$. We denote the corresponding pair by the symbol $(s, x_{1:n})^*$. The value n_D is the realization of RV N_D. The submatrix in right-hand bottom corner with states $(1,2,\ldots,i^{**})$ is connected with components of MDS_CL2 , $\{s_0, s_1, ..., s^{**}, s^{**}, s^{**}, ...\}$. Absorbing state, i^{**}, corresponds to stopping of the process of destruction (to the achievement by the local stress of the final level s^{**}). The random process is defined by the upper matrix if $s^+ > S_D$ and by the lower one if $s^+ \le S_D$.

Let us denote by the symbol $R_{(s,x_{L,1:n_C})^*}$ the space of pairs $(s, x_{L,1:n_C})^*$ corresponding to the MDS_CL1 and by $F_{X_L}(s,\hat{\theta})$ the estimate of CDF itself of RV X_L where $\hat{\theta}$ is the estimate of parameter of CDF.

We define the random transition-probability submatrix $\hat{P}_I$ corresponding to MDS_CL1 in following way:

$\hat{p}_{i,i+1} = (F_{X_L}(s_i,\hat{\theta}) - F_{X_L}(s_{i-1},\hat{\theta}))/(1 - F_{X_L}(s_{i-1},\hat{\theta}))$ is the the conditional probability of the failure of all the longitudinal elements having strength in the interval (s_{i-1}, s_i); $\hat{p}_{ii} = 1 - \hat{p}_{i,i+1}$, $i = 1,...,n_D + 1$;

$\hat{p}_{01} = F_{X_L}(s_0,\hat{\theta})$, $\hat{p}_{00} = 1 - \hat{p}_{01}$; $\hat{p}_{(n_D+2)(n_D+2)} = 1$. All the remaining probabilities are equal to 0.

In the same way the submatrix corresponding to MDS_CL2 can be defined. But really we are interested only to know the probability of the event $s^+ \le S_D$. So in the following let us use $\hat{P}$ instead of $\hat{P}_1$. Note ones again that the matrix $\hat{P}$ is the realization of the random matrix, since it is a function of the random sample $x_{L,1:n_C}$, which was used for estimation of the parameter $\hat{\theta}$ of CDF $F_{X_L}(x,\theta)$ and development of the corresponding MDS.

Let us designate the transition time from s_{i-1} to s_i by $\hat{T}_i$ in the MCh with the realization of the random transition-probability matrix $\hat{P}$. For the RV $\hat{T} = \hat{T}_1 + \hat{T}_2 + ... + \hat{T}_{n_D}$, i.e., the corresponding number of steps before the absorption (now this is the longevity of one link (one parallel system)), the estimate of the corresponding random CDF is as follows:

$$\hat{F}_T(t,\eta;(s,x_{L,1:n_C})^*) = \pi \hat{P}^t b,\ t = 1,2,3,...,$$

where vector $\pi = (\pi_0, \pi_1, ..., \pi_{n_D+1})$ is the vector of the prior probabilities in the possible initial states (in the simplest case, $\pi = (1,0,...,0)$, i.e., all n_C of the LE are active), the vector b is the vector-column in the form $(0,\ldots 0,1)'$, the vector η is the vector of parameters of the model (f_C, n_C,…), and s^+ is the initial nominal stress level. The average, $E(\hat{T}_i)$, and variance , $V(\hat{T}_i)$ of the RV realization $\hat{T}$ is easily calculated bearing in mind that the RVs $\hat{T}_i$, $i = 1,2,...,n_D + 1$ have a geometric distribution: $E(\hat{T}_i) = 1/p_i$, $V(\hat{T}_i) = (1-p_i)/p_i^2$.

Owing to the randomness of the sample $x_{L,1:n_C}$, it is necessary to average (e.g., using the Monte Carlo method) the obtained random CDF over the space of the samples included into $R_{(s,x_{L,1:n_C})^*}$ and then to calculate in the average value T and the quantiles we are interested in as functions of the initial cyclic stress s^+.

The example of supposed connection between the time of absorption $\hat{T}$ and corresponding number of cycles is shown in section 5.

4. THE CONNECTION BETWEEN THE RELIABILITY OF THE SEPARATE LINK AND THE SPS

Defected links can be present in the considered system even before its tests start or can appear in the process of the system's operation. Let us designate the number of defects in the *i*-th link at the time moment t (after t cycles) by $K_{Ci}(t)$, $0 \le K_{Ci}(t) \le n_C$. It is postulated that $K_{Ci}(t)$ does not decrease with time.

Two hypotheses for the connection between the longevity of the separate link and the SPS can be put forward if defected links are presented in the considered system before its tests start.

(a) The process of accumulation of damage of separate LEs occurs in all the links of the system, in which defects can appear; in addition, these processes are mutually independent and identically distributed.

(b) This process develops only in the link with the minimum residual strength (or maximum initial number of defected LEs).

In the first case (it can be called the RMinMax hypothesis), the failure moment of the ith link is the first moment when the event $R_{Di}(t) \le s^+$ takes place. So fatigue life of the whole SPS

$$T = \min_{1 \le i \le K_L} T_i = \min_{1 \le i \le K_L} \max_t (t : R_{Di}(t) > s^+)$$

where $R_{Di}(t)$ is the Daniels' residual strength in ith link, $i = 1, 2, ..., n_L$. In simplest case, when the strength of the defected LE is equal to zero, in Paramonov at all (2013a) the MinMax hypothesis was introduced with

$$T = \min_{1 \le i \le K_L} T_i = \min_{1 \le i \le K_L} \max_t (t : n_C - K_{Ci}(t) \ge 0) .$$

The corresponding CDF is as follows:

$$F_T(x) = \sum_{k=1}^{n_L} p_{K_L}(k)(1-(1-F_{T_1}(x))^k) ,$$

where $F_{T_1}(x)$ is the CDF of the random time to failure of some random link, T_1, and $p_{K_L}(k)$, $k = 1, 2, ...$ is the prior distribution of the RV K_L. If, e.g., $K_L = 1 + K$ and, when $n_L \to \infty$, K has Poisson's distribution with the parameter λ, then

$$F_T(x) = 1 - (1 - F_{T_1}(x)) \exp(-\lambda F_{T_1}(x)) .$$

If $P(K_L = n_L) = 1$ then $F_T(x) = 1 - (1 - F_{T_1}(x))^{n_L}$.

As applied to the second hypothesis, we designate the initial Daniels' residual strength in ith link before beginning of test by $R_{Di}(0)$ and the smallest one in a weakest link by $R_{Di^*}(0) = \min_{0 \le i \le K_L} R_{Di}(0)$. The assumption can be made that fatigue damage accumulation takes place only in the weakest link i^*. So the time to failure of SPS

$$T = \max_t (t : R_{Di^*}(t) > s^+) .$$

This hypothesis can be called RMaxMin. If the strength of defected LEs is equal to zero then

$$T = \max (t : M_{Ci^*}(0) - K_{Ci^*}(t) \ge 0)$$

where $M_{Ci^*}(0) = n_C$ - $K_{Ci^*}(0)$, $K_{Ci^*}(t)$ corresponds to the weakest link.

In this case, we obtain the CDF of RV T by averaging the CDF of RV $T_1(k)$, corresponding to the presence in one link random number intact LEs: $F_T(x) = \sum_{k=1}^{n_C - 1} F_{T_1(k)}(x) p_{M_{C^*}}(k)$

where

$$p_{M_{C^*}}(k) = P(M_{Ci^*}(0) = k) = F_{M_{Ci^*}}(k) - F_{M_{Ci^*}}(k-1) , \quad k = 1, 2, ..., n_C - 1;$$

$$F_{M_{C^*}}(k) = \sum_{k_L=1}^{n_L} p_{K_L}(k_L)(1-(1-F_{M_{C1}}(k))^{k_L}) .$$

Besides, if $K_L = 1 + K$, the RV K has Poisson's distribution with the parameter λ then,

$$F_{M_{C^*}}(k) = 1 - \left(1 - F_{M_{C1}}(k)\right) \exp\left(-\lambda F_{M_{C1}}(k)\right) .$$

Links with defects can also appear during cyclic loading. Assume that X_i, $i = 1, 2, ...,$ are the random intervals between the moments of the appearance of these links; X_1, $X_1 + X_2$, $X_1 + X_2 + X_3$,... are the time moments of their appearance; and T_i are the lives of the corresponding links. Then, if the failure of the system can occur only due to the failure of the links with defects, for $n_C = \infty$, we have $T = \min (T_1, T_2 + X_1, T_3 + X_1 + X_2, ...)$ or

$$T = \min (T_1, T^+ + X_1) , \tag{6}$$

where the RV T^+ has the same CDF as the RV *T*. It is appropriate to name the hypothesis for the model at which the CDF of the RV T meets equation (6) the MinTime hypothesis.

Equation (6) is true for arbitrary positive random variables. In the particular case of initiating faulty links in accordance with the Poisson process with the intensity μ for CDF of RV T, we obtain

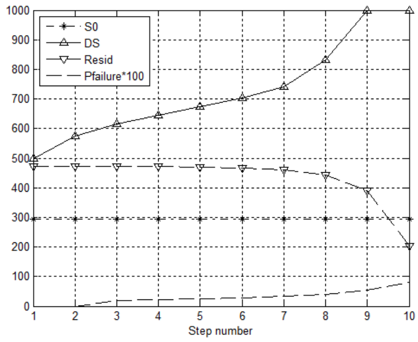
$$F_T(y) = 1-(1-F_{T_1}(y))\exp(-\mu\int_0^y F_{T_1}(t)dt)\,. \qquad (7)$$

The three considered hypotheses can find application in solving problems of investigating the longevity of composites with various physical properties (more or less fragile and more or less viscous). However, all these hypotheses are closed in the end for the problem of determining the CDF of the longevity of one link $F_{T_1}(t)$, the determination of which is the main problem considered in this paper. Later, to reduce the writing, we will write T instead of T_1.

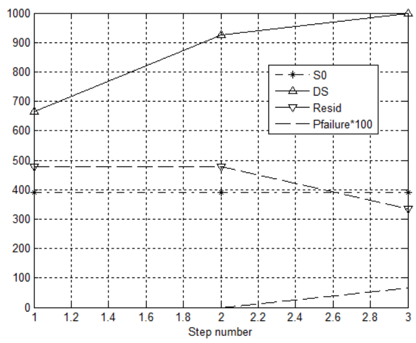
5. EXAMPLE OF PROCESSING THE DATA ON THE FATIGUE LIFE AND RESIDUAL STRENGTH

To verify the examined model, the fatigue tests of specimens made of a unidirectional composite (Udo UD ES 500/300 — SGL epo GmbH with LH 160 in "Composites HAVEL"; structure [0/45/0]; effective length, *l* = 60 mm) were carried out at the Riga Institute of Polymer Mechanics. The average static strength of specimen was 487.56 MPa. The fatigue tests was performed at stress ratio *R* = 0.1. The maximum stresses of the cycle of fatigue tests were 292.5, 341.3, and 390.5 MPa . It was assumed that the number of LEs in a "link" (it was assumed that n_L =1) n_C =1000, the number of defected ("weak") LEs is the RV with binomial distribution $b(p,n_C)$, $p=0.1$. The tensile strength of both LEs has the lognormal distribution with CDF of the type : $F(x)=\Phi((\log(x)-\theta_0)/\theta_1)$. The following parameters was found for fitting the test data: for undamaged LE θ_{0U} =6.757 (exp (θ_{0U})=860); for weak LE $\theta_{0W}=\theta_{0U}$ - log(k_F), k_F =3; for both cases θ_1 =0.3. It was assumed additionally that there is a stress concentration defined by the coefficient $k_C=1.7$.

In Fig.2 the nominal cycling stress level (-*- S0), examples of Daniels' sequences (-▲-DS), random residual Daniels' function (-▼- Resid) and a part (multiplied by 100) of already destroyed LEs (-- Pfailure*100) as function of DS step number are shown for cycling nominal stress levels 292.5and 390.5 MPa .



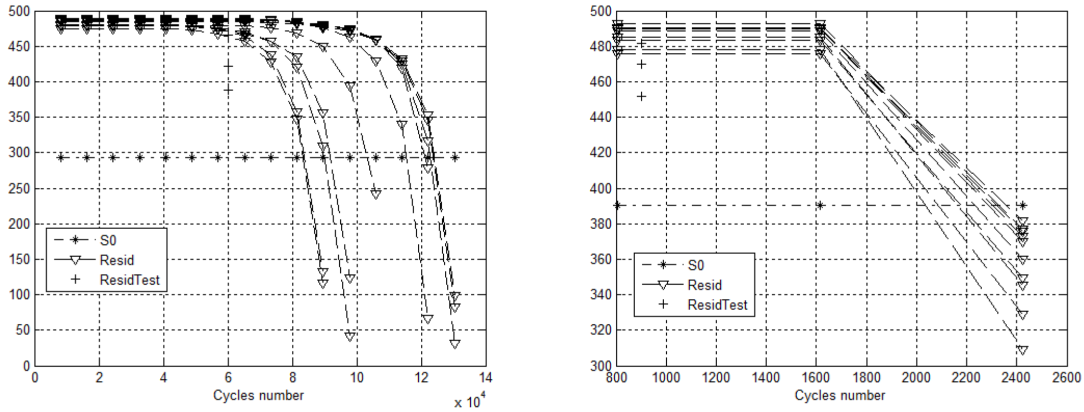
a



b

Fig.2. The nominal cycling stress level (-*- S0), the examples of Daniels' sequences (-▲-DS), random residual Daniels' function (-▼- Resid) and part (multiplied by 100) of already destroyed LEs (-- Pfailure*100) as the function of DS steps for cycling nominal stress levels 292.5 (a) and 390.5 MPa (b).

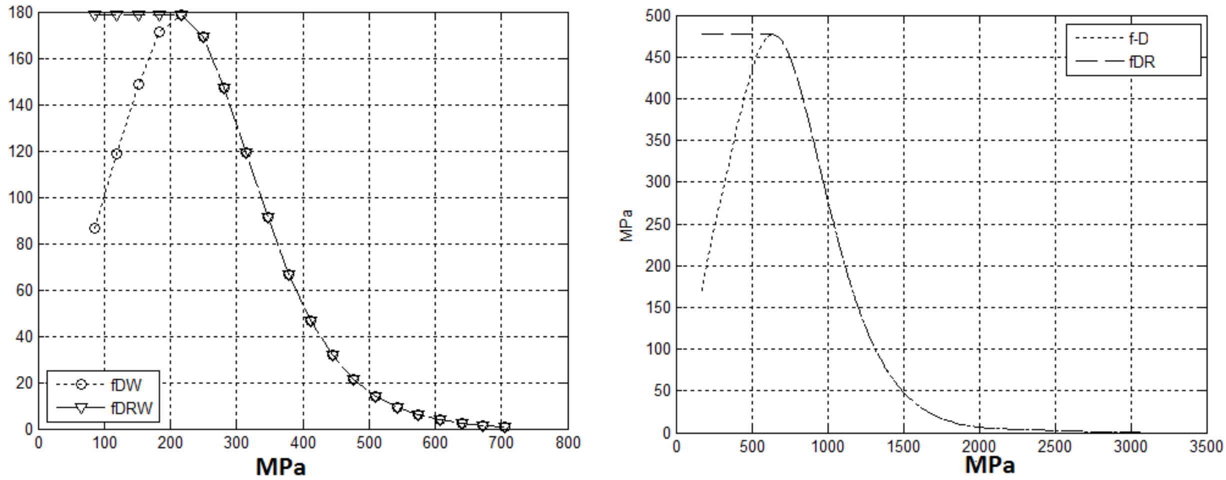
The nominal cycling stress level (-*- S0), the 10 examples of random Daniels' residual strength (-▼- Resid) and result of test of residual stress as function of cycles number for the same cycling nominal stress levels 292.5and 390.5 MPa are shown in Fig.3 .



a b

Fig.3 The nominal cycling stress level (-*- S0), the 10 examples of random residual Daniels' function (-▼- Resid) and result of test of residual stress (+ ResidTest) as function of cycles number.

In Fig.2 and 3 we see result of Monte Carlo calculation. The 'teoretical" Daniels' function and residual Daniels' function are shown in Fig.4 for damaged LEs (a) and for mixture of undamaged and damaged LEs (b).



a b

Fig.4. The residual (-▼- fDRW) and 'teoretical" (-o- fDW) Daniels' function for damaged LEs (a) and the same (-- fDR) and (… fD) for mixture of undamaged and damaged LEs (b).

The result of Monte Carlo calculation of fatigue curve (►is a smallest value,* is a mean value,◄ is a largest value) and the result of fatigue test (+) are shown in Fig. 5. For every Monte Carlo trial (n_W values of strength of weak (damaged) LEs and $n_U = n_C - n_W$ values of strength of undamaged LEs) the estimate of CDF $F_{X_L}(x,\hat{\theta})$, the realization of MDS, the items of matrix $\hat{P}$, the values of time to absorption $\hat{T}$ and the corresponding number of cycles N was calculated. The connection between $\hat{T}$ and N was approximated in following way : $N = k_m (R/s^+)^\gamma T$, where k_m, R, γ are parameter of nonlinear regression model. It is assumed that this approximation in some way takes into account that for failure of some set of LEs the accumulation of some energy is necessary (in first approximation, it is assumed that in every cycle this energy is proportional to $(s^+)^\gamma$, the corresponding number of cycles to failure is proportional to $1/(s^{+})^{\gamma}$). For fitting of considered in this paper data (Fig. 3 and 5) the estimates $k_m = 1.46$, $R = 860$, $\gamma = 8$ are used.

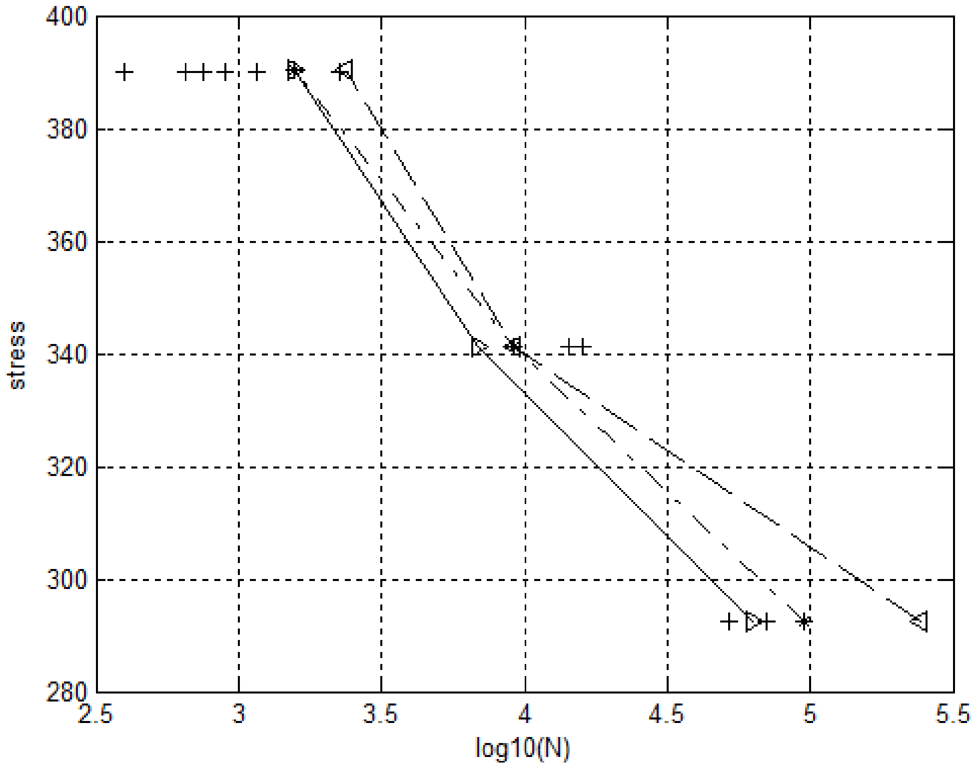


Fig. 5. The result of Monte Carlo calculation of fatigue curve (►is the smallest value,* is the mean value,◄ is the largest value) and the result of fatigue test (+) .

It can be sad that we have enough reasonable fitting of test data.

SUMMARY AND DEVELOPMENT LINES OF THE PROBLEM

1. The model of reliability of series-parallel system (SPS) under tension and cycling load can be considered as a model of reliability of UFC. MinMax, MaxMin, MinTime hypotheses explain the size effect and the connection of CDF of one link (weak micro volume) and the UFC itself.
2. The use of the MDS concept allows to build the model of reliability of a link as a part of SPS.
3. The numerical results of the use the MDS concept are very pure: the fatigue limit is too high, the fatigue life is very small.
4. The developed regression model based of MDS concept together with the use of Markov chains theory and Monte Carlo method allows to get simultaneously the reasonable fitting of the result of tests of fatigue life and residual strength:

(i) to get connection of CDF of tensile strength of component of UFC and the CDFs of the tensile strength, the fatigue life of composite itself and fatigue limit;

(ii) to explain the “strange” behavior of residual strength as function of the number of cycles of fatigue loading (weak dependence at the initial period of loading and sudden decreasing at the last one).

It is shown that it is of great importance to know the moment of drastic decreasing of residual strength. The special study of corresponding CDF is needed.

Parameters of considered regression model can be interpreted as parameters of CDF of the local strength of components of UFC

It is necessary to underline that the offered MDS concept is only the base for different type of regression models which can be based on it. The analysis of the numerical examples considered in this paper and previous publications show that this concept deserves to be studied more intensively. The main problem is the increasing of the number of steps of MDS. Probably it can be done using another approximation of distribution of local strength of LE. In framework of composite it has specific support

and it is not clear the "length" (or the number) of links in corresponding UFC. It (or n_L) can be considered as an additional parameter. Next important problem is to take into account the matrix of real composite material. Some possible approach was considered in Paramonov at all (2011) but it can not explain the existence of fatigue limit. Much more sophisticated hypothesis should be offered to take into account the accumulation of some energy necessary for fatigue failure.
The event " residual strength becomes lower than the maximum level of cycle stress" is only the necessary but not enough condition of the failure of UFC. Much more sophisticated hypothesis should be offered to take into account the accumulation of some energy necessary for fatigue failure. The theory of semi-Markov process with rewords can be used for solution of considered problem. Very important problem is to develop appropriate method of model parameter estimation in order to get better fitting of test result.

The search of the parameters of nonlinear regression for simultaneosly processing tensile, fatigue test and residual strength test result is a difficult task, but we think that, in due course, the structure of models suggested will be of interest not only for graduation theses of students, but also for engineering applications, in particular, for predicting variations in the parameters of strength and durability of UFCs upon changes in the parameters of their components.

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ESTIMATION AND OPTIMAL DESIGN OF CONSTANT STRESS PARTIALLY ACCELERATED LIFE TEST FOR GOMPERTZ DISTRIBUTION WITH TYPE I CENSORING

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ABSTRACT

This study deals with simple Constant Stress Partially Accelerated life test (CSPALT) using type-I censoring. The lifetime distribution of the test item is assumed to follow Gompertz distribution. The Maximum Likelihood (ML) Estimation is used to estimate the distribution parameters and acceleration factor. Asymptotic confidence interval estimates of the model parameters are also evaluated by using Fisher information matrix. Statistically optimal PALT plans are developed such that the Generalized Asymptotic Variance (GAV) of the Maximum Likelihood Estimators (MLEs) of the model parameters at design stress is minimized. In the last, to illustrate the statistical properties of the parameters, a simulation study is performed

KEYWORDS: Reliability; Partially Accelerated Life Tests; Acceleration factor; constant stress; maximum likelihood estimation; Fisher information matrix; generalized asymptotic variance; optimum test plans; time censoring

1 INTRODUCTION

The life test of the Products having high reliability under normal use conditions often requires a long period of time. In such problems, accelerated life tests (ALTs) are often used to quickly obtain information on the life time distribution of products by testing them at accelerated conditions than normal use conditions to induce early failures**.**

In ALT , the mathematical model relating to the lifetime of an item and stress is known or can be assumed.But,in some cases these relationships are not known and can not be assumed, i.e the data obtained from ALT can not be extrapolated to use condition.So, partially accelerated life test can be used in such cases in which the test items are run at both normal and higher than normal stress conditions. PALT can be carried out using constant-stress, step-stress, or Progressive-stress (linearly increasing stress). In constant stress PALT products are tested at either usual or higher than usual condition only until the test is terminated. The approach to accelerate failures is the step stress which increases the load applied to the products in a specified discrete sequence. A sample of test items is first run at use condition and, if it does not fail for a specified time, then it is run at accelerated condition until a pre specified numbers of failures are obtained or a pre specified time has reached.

There is an amount of literature on PALT which has been studied by many authors. Bai and Chung (1) discussed the optimall designing constant strss PALT or the test item having exponential distribution under Type I censoring.Bai et.al (2) discussed the PALT plan for lognormal distribution under time censored data.after that Bai et al (3) also considered the problemof failure-censored accelerated life-test sampling plans for lognormal and Weibull distributions. Abdel-Ghani (4) investigated some lifetime models under partially accelerated life tests. Ghaly et al. (5) discussed the PALT problem of parameter estimation for Pareto using Type I censoring and after that Ghaly et al. (6) considered the same problem under Type II censoring. Ismail (7) used the maximum likelihood method to estimate the acceleration factor and parameters of the Pareto distribution under PALT. Ismail (8) discuss the constant stress PALT for the Weibull failure distribution under failure censored case. Ismail (9) considered the problem of optimally designing a simple time-step-stress PALT which terminates after a pre-specified number of failures and developed optimum test plans for products having a two-parameter Gompertz lifetime distribution. Zarrin et al. (10) considered constant stress PALT with type-I censoring. Assuming Rayleigh distribution as the underlying lifetime distribution, the MLEs of the distribution parameter and acceleration factor were obtained. More recent Saxena et al (11) consider the PALT design for extreme value distribution using type I censoring and Kamal et al. (12) discuss the same problem for Inverted Weibull distribution.

This work was conducted for constant-stress PALT under type II censored sample. the problems of estimation in constant stress PALT are considered under Rayleigh distribution. Maximum likelihood estimates and confidence intervals for parameters and acceleration factor are obtained.

2 THE MODEL AND TEST METHOD

2.1 The Gompertz Distribution

The lifetimes of the test items are assumed to follow a Gompertz distribution. The probability density function (pdf) of the Gompertz distribution is given by

$$f(t) = \theta\, e^{\alpha t} \exp^{\left\{-\theta/_{\alpha}\left[e^{\alpha t}-1\right]\right\}} \qquad t > 0, \theta > 0, \alpha > 0. \tag{1}$$

where θ is the scale parameter and α is the shape parameter of the distribution.

And the cumulative distribution function is given by

$$F(t) = 1 - \exp^{\left\{-\theta/_{\alpha}\left[e^{\alpha t}-1\right]\right\}} \qquad t > 0, \theta > 0, \alpha > 0. \tag{2}$$

The reliability function of the Gompertz distribution is given by

$$R(t) = \exp^{\left\{-\theta/_{\alpha}\left[e^{\alpha t}-1\right]\right\}} \qquad t > 0, \theta > 0, \alpha > 0. \tag{3}$$

And the corresponding hazard rate is given by

$$h(t) = \theta\, e^{\alpha t}$$

When $\alpha \to 0$, the Gompertz distribution will tend to an exponential distribution, see Wu et al. (13).

The two-parameter Gompertz model is a commonly used survival time distribution in actuarial science, reliability and life testing. There are several forms for the Gompertz distribution given in the literature. Some of these are given in Johnson et al. (14, 15).The pdf formula given in Equation (1) is the commonly used form and it is unimodal. It has positive skewness and an increasing hazard rate function.

2.2 Constant stress PALT procedure

i. Total n items are divided randomly into two samples of sizes $n(1-s)$ and ns respectively where s is sample proportion. First sample is allocated to normal use condition and other is assigned to accelerated conditions.
ii. Each test item of every sample is run until the censoring time τ and the test condition is not changed.

2.3 Assumptions

i. The life time of the test product at use condition follows the Gompertz distribution given in (1).
ii. The life time of the test product at accelerated condition is obtained by using the relation $X = \beta^{-1}T$, where $\beta > 1$ is an acceleration factor. Therefore, the pdf at accelerated condition is given by equation (4) as follows

$$f(x) = \theta\beta\, e^{\alpha\beta x} \exp^{\left\{-\theta/\alpha \left[e^{\alpha\beta x}-1\right]\right\}} \qquad x > 0, \theta > 0, \alpha > 0. \tag{4}$$

iii. The lifetimes T_i , $i = 1,2,............n(1-s)$ of items allocated to normal use condition, are i.i.d. random variables.
iv. The lifetime X_j , $j = 1,2,............ns$ of items allocated to accelerated condition, are i.i.d random variables.
v. The lifetimes T_i and X_j are mutually statistically-independent.

3 MAXIMUM LIKELIHOOD ESTIMATION

The maximum likelihood parameter estimation is used to determine the estimates of the parameter that maximizes the likelihood of the sample data. Also the MLEs have the desirable properties of being consistent and asymptotically normal for large samples
Since, the type-I censoring test terminates after a pre specified time is reached, so, the observed lifetimes $t_{(1)} \le ....\le t_{(n_u)} \le \tau$ and $t_{(1)} \le ....\le t_{(n_a)} \le \tau$ are ordered failure times at normal use and accelerated conditions respectively, where τ is the pre specified time at which the test is terminated, n_u and n_a are the numbers of items failed at normal use and accelerated use conditions, respectively which are given by

$$n_u = \sum_{i=1}^{n(1-s)} \delta_{ui} \text{ and } n_a = \sum_{j=1}^{ns} \delta_{aj}$$

and $n_u + n_a = r$, r is the total number of the failed items.
Let δ_{ui} and δ_{aj} be the indicator functions such that

$$\delta_{ui} = \begin{cases} 1 & t_i \le \tau \qquad\qquad i = 1,2,..........n(1-s) \\ 0 & otherwise \end{cases}$$

And

$$\delta_{aj} = \begin{cases} 1 & x_j \le \tau \qquad\qquad j = 1,2,..........ns \\ 0 & otherwise \end{cases}$$

Then the likelihood function for (t_i, δ_{ui}), the likelihood function for (x_j, δ_{aj}) and the total likelihood function for $(t_1; \delta_{u1}, ..., t_{n(1-s)}; \delta_{un(1-s)}, x_1; \delta_{a1}, ..., x_{ns}; \delta_{ans})$ are respectively given by

$$L_{ui}(t_i, \delta_{ui} | \alpha, \theta) = \prod_{i=1}^{n(1-s)} \left[\theta e^{\alpha t_i} \exp\left\{ -\left(\frac{\theta}{\alpha}\right)\left[e^{\alpha t_i} - 1\right] \right\} \right]^{\delta_{ui}} \left[\exp\left\{ -\left(\frac{\theta}{\alpha}\right)\left[e^{\alpha \tau} - 1\right] \right\} \right]^{\bar{\delta}_{ui}} \tag{5}$$

$$L_{aj}(x_j, \delta_{aj} | \alpha, \theta) = \prod_{i=1}^{ns} \left[\beta\theta e^{\alpha\beta x_{j_i}} \exp\left\{ -\left(\frac{\theta}{\alpha}\right)\left[e^{\alpha\beta x_j} - 1\right] \right\} \right]^{\delta_{aj}} \left[\exp\left\{ -\left(\frac{\theta}{\alpha}\right)\left[e^{\alpha\beta\tau} - 1\right] \right\} \right]^{\bar{\delta}_{aj}} \tag{6}$$

$$\begin{aligned} L(t, x | \alpha, \beta, \theta) = & \prod_{i=1}^{n(1-s)} \left[\theta e^{\alpha t_i} \exp\left\{ -\left(\frac{\theta}{\alpha}\right)\left[e^{\alpha t_i} - 1\right] \right\} \right]^{\delta_{ui}} \left[\exp\left\{ -\left(\frac{\theta}{\alpha}\right)\left[e^{\alpha \tau} - 1\right] \right\} \right]^{\bar{\delta}_{ui}} \\ & \prod_{i=1}^{ns} \left[\beta\theta e^{\alpha\beta x_{j_i}} \exp\left\{ -\left(\frac{\theta}{\alpha}\right)\left[e^{\alpha\beta x_j} - 1\right] \right\} \right]^{\delta_{aj}} \left[\exp\left\{ -\left(\frac{\theta}{\alpha}\right)\left[e^{\alpha\beta\tau} - 1\right] \right\} \right]^{\bar{\delta}_{aj}} \end{aligned} \tag{7}$$

where $\bar{\delta}_{ui} = 1 - \delta_{ui}$ and $\bar{\delta}_{aj} = 1 - \delta_{aj}$

Taking log of above equation

$$\begin{aligned} l = \ln L = & \sum_{i=1}^{n(1-s)} \delta_{ui} \left[\ln\theta + \alpha t_i - \left(\frac{\theta}{\alpha}\right)\left[e^{\alpha t_i} - 1\right] \right] - \left(\frac{\theta}{\alpha}\right)\left[e^{\alpha\tau} - 1\right] \sum_{i=1}^{n(1-s)} (1 - \delta_{ui}) \\ & + \sum_{i=1}^{ns} \delta_{aj} \left[\ln\theta + \ln\beta + \alpha\beta x_j - \left(\frac{\theta}{\alpha}\right)\left[e^{\alpha\beta x_j} - 1\right] \right] - \left(\frac{\theta}{\alpha}\right)\left[e^{\alpha\beta\tau} - 1\right] \sum_{j=1}^{ns} (1 - \delta_{aj}) \end{aligned} \tag{8}$$

MLEs of α, β and θ are obtained by solving the equations $\frac{\partial l}{\partial \alpha} = 0, \frac{\partial l}{\partial \beta} = 0$ and $\frac{\partial l}{\partial \theta} = 0$.

$$\frac{\partial l}{\partial \theta} = \frac{r}{\theta} - \frac{1}{\alpha}\left[-n + \sum_{i=1}^{n(1-s)} \delta_{ui} e^{\alpha t_i} + \sum_{j=1}^{ns} \delta_{aj} e^{\alpha\beta x_j} + e^{\alpha\tau}\{n(1-s) - n_u\} + e^{\alpha\beta\tau}(ns - n_a) \right] \tag{9}$$

$$\begin{aligned} \frac{\partial l}{\partial \alpha} = & \sum_{i=1}^{n(1-s)} \delta_{ui} t_i + \beta \sum_{j=1}^{ns} \delta_{aj} x_j - \frac{\theta}{\alpha}\left[\sum_{i=1}^{n(1-s)} \delta_{ui} t_i e^{\alpha t_i} + \beta \sum_{j=1}^{ns} \delta_{aj} x_j e^{\alpha\beta x_j} + \tau e^{\alpha\tau}\{n(1-s) - n_u\} + \beta\tau e^{\alpha\beta\tau}(ns - n_a) \right] \\ & + \frac{\theta}{\alpha^2}\left[-n + \sum_{i=1}^{n(1-s)} \delta_{ui} e^{\alpha t_i} + \sum_{j=1}^{ns} \delta_{aj} e^{\alpha\beta x_j} + e^{\alpha\tau}\{n(1-s) - n_u\} + e^{\alpha\beta\tau}(ns - n_a) \right] \end{aligned} \tag{10}$$

$$\frac{\partial l}{\partial \beta} = \frac{n_a}{\beta} + \alpha \sum_{j=1}^{ns} \delta_{aj} x_j - \theta \sum_{j=1}^{ns} \delta_{aj} x_j e^{\alpha\beta x_j} - \theta\tau e^{\alpha\beta\tau}(ns - n_a) \tag{11}$$

It is difficult obtain a closed form solution to nonlinear equations to (9), (10) and (11). Newton-Raphson method is used to solve these equations simultaneously to obtain $\hat{\alpha}, \hat{\beta}$ and $\hat{\theta}$.

The asymptotic variance-covariance matrix of α, β and θ is obtained by numerically inverting the Fisher-information matrix composed of the negative second derivatives of the natural logarithm of the likelihood function evaluated at the ML estimates. The asymptotic Fisher-information matrix can be written as:

$$\frac{\partial^2 l}{\partial\theta^2} = -\frac{r}{\theta^2}$$

$$\frac{\partial^2 l}{\partial\alpha^2} = -\frac{\theta}{\alpha}\left[\sum_{i=1}^{n(1-s)} \delta_{ui} t_i^2 e^{\alpha t_i} + \beta^2 \sum_{j=1}^{ns} \delta_{aj} x_j^2 e^{\alpha\beta x_j} + \tau^2 e^{\alpha\tau}\{n(1-s)-n_u\} + \beta^2\tau^2 e^{\alpha\beta\tau}(ns-n_a)\right]$$

$$+\frac{2\theta}{\alpha^2}\left[\sum_{i=1}^{n(1-s)} \delta_{ui} t_i e^{\alpha t_i} + \beta \sum_{j=1}^{ns} \delta_{aj} x_j e^{\alpha\beta x_j} + \tau e^{\alpha\tau}\{n(1-s)-n_u\} + \beta\tau e^{\alpha\beta\tau}(ns-n_a)\right]$$

$$-\frac{2\theta}{\alpha^3}\left[\sum_{i=1}^{n(1-s)} \delta_{ui} e^{\alpha t_i} + \sum_{j=1}^{ns} \delta_{aj} e^{\alpha\beta x_j} + e^{\alpha\tau}\{n(1-s)-n_u\} + e^{\alpha\beta\tau}(ns-n_a)\right]$$

$$\frac{\partial^2 l}{\partial\beta^2} = -\frac{n_a}{\beta^2} - \alpha\theta\sum_{j=1}^{ns} \delta_{aj} x_j^2 e^{\alpha\beta x_j} - \alpha\theta\tau^2 e^{\alpha\beta\tau}(ns-n_a)$$

$$\frac{\partial^2 l}{\partial\theta\partial\alpha} = \frac{1}{\alpha^2}\left[-n + \sum_{i=1}^{n(1-s)} \delta_{ui} e^{\alpha t_i} + \sum_{j=1}^{ns} \delta_{aj} e^{\alpha\beta x_j} + e^{\alpha\tau}\{n(1-s)-n_u\} + e^{\alpha\beta\tau}(ns-n_a)\right]$$

$$-\frac{1}{\alpha}\left[\sum_{i=1}^{n(1-s)} \delta_{ui} t_i e^{\alpha t_i} + \beta \sum_{j=1}^{ns} \delta_{aj} x_j e^{\alpha\beta x_j} + \tau e^{\alpha\tau}\{n(1-s)-n_u\} + \beta\tau e^{\alpha\beta\tau}(ns-n_a)\right]$$

$$\frac{\partial^2 l}{\partial\theta\partial\beta} = -\sum_{j=1}^{ns} \delta_{aj} x_j e^{\alpha\beta x_j} + \tau e^{\alpha\beta\tau}(ns-n_a)$$

$$\frac{\partial^2 l}{\partial\alpha\partial\beta} = \sum_{j=1}^{ns} \delta_{aj} x_j - \theta\beta\sum_{j=1}^{ns} \delta_{aj} x_j^2 e^{\alpha\beta x_j} - \theta\beta\tau^2 e^{\alpha\beta\tau}(ns-n_a)$$

The variance covariance and covariance matrix of the parameter can be written as

$$\Sigma = \begin{bmatrix} -\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\beta} & -\frac{\partial^2 l}{\partial\alpha\partial\theta} \\ -\frac{\partial^2 l}{\partial\beta\partial\alpha} & -\frac{\partial^2 l}{\partial\beta^2} & -\frac{\partial^2 l}{\partial\beta\partial\theta} \\ -\frac{\partial^2 l}{\partial\theta\partial\alpha} & -\frac{\partial^2 l}{\partial\theta\partial\beta} & -\frac{\partial^2 l}{\partial\theta^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat\alpha) & ACov(\hat\alpha\hat\beta) & ACov(\hat\alpha\hat\theta) \\ ACov(\hat\beta\hat\alpha) & AVar(\hat\beta) & ACov(\hat\beta\hat\theta) \\ ACov(\hat\theta\hat\alpha) & ACov(\hat\theta\hat\beta) & AVar(\hat\theta) \end{bmatrix}$$

4 INTERVAL ESTIMATES FOR MODEL PARAMETER

To construct a confidence interval for a population parameter σ, assume that $L_\sigma = L_\sigma(y_1..........y_n)$ and $U_\sigma = U_\sigma(y_1.............y_n)$ are functions of the sample data $y_1................y_n$ then a confidence interval for a population parameter σ is given by

$$P[L_\sigma \le \sigma \le U_\sigma] = \xi \tag{12}$$

where the interval $[L_\sigma, U_\sigma]$ is called a two sided $\xi 100\%$ confidence interval for σ. L_σ and U_σ are the lower and upper confidence limits for σ, respectively.

For large sample size, the MLEs, under appropriate regularity conditions, are consistent and asymptotically normally distributed.

Therefore, the two sided approximate $\xi 100\%$ confidence limits for the MLE $\hat\sigma$ of a population parameter σ can be constructed, such that

$$P\left[-z \le \frac{\hat{\sigma}-\sigma}{\phi(\hat{\sigma})} \le z\right] = \xi \tag{13}$$

where z is the $\left[100\left(1-\xi/2\right)\right]^{th}$ standard normal percentile.

Therefore, the two sided approximate $\xi 100\%$ confidence limits for α, β and θ are given respectively as follows

$$L_{\alpha} = \hat{\alpha} - z\phi(\hat{\alpha}) \qquad U_{\alpha} = \hat{\alpha} + z\phi(\hat{\alpha})$$
$$L_{\beta} = \hat{\beta} - z\phi(\hat{\beta}) \qquad U_{\beta} = \hat{\beta} + z\phi(\hat{\beta})$$
$$L_{\theta} = \hat{\theta} - z\phi(\hat{\theta}) \qquad U_{\theta} = \hat{\theta} + z\phi(\hat{\theta})$$

5 OPTIMUM SIMPLEC ONSTANT-STRESS TEST PLAN

In this section the problem of optimally designing a simple constant stress PALT, which terminates after a pre specified time is discussed. Optimum test plan for the items having Gompertz distribution is developed.

Most of the test plans allocate the same number of test units at each stress i.e. they are equally spaced test stresses. Such test plans are usually inefficient for estimating the mean life at design stress, see Yang (16). To decide the optimal sample proportion allocated to each stress, statistically optimum test plans are developed. Therefore, to determine the optimal sample proportion s^* allocated to accelerated condition, s is chosen such that the GAV of the ML estimators of the model parameters is minimized. The GAV of the ML estimators of the model parameters as an optimality criterion is defined as the reciprocal of the determinant of the Fisher-Information matrix F (Bai, Kim and Chun [3]). That is

$$GAV\left(\hat{\theta}, \hat{\alpha}, \hat{\beta}\right) = \frac{1}{|F|}$$

The minimization of the GAV over s solves the following equation

$$\frac{\partial GAV}{\partial s} = 0$$

The solution to the above equation is not in the closed form, so the Newton- Raphson method is applied to determine s^* which minimize the GAV. Accordingly, the corresponding expected optimal numbers of items failed at normal use and accelerated use conditions can be respectively as follows

$n_u^* = n\left(1-s^*\right)P_u$ and $n_a^* = ns^* P_a$

where,

P_u = Probability that an item tested only use condition fails by τ.

P_a = Probability that an item tested only accelerated condition fails by τ

6 SIMULATION STUDY

In order to obtain MLEs of β, α and θ and to study the properties of these estimates through Mean squared errors (MSEs), variance of the estimators and confidence limits for 95% and 99% asymptotic confidence interval, a simulation study is performed. Furthermore, optimum test plans are developed.

For this purpose, several data sets generated from Gompertz distribution under type-I censored data are considered with sample sizes 100, 200, 300, 400 and 500 using 500 replications for each sample size. Under Type I censoring choose a proportion of sample units allocated to accelerated condition to be $s = 30\%$ and censoring time of a PALT to be $\tau = 55$ .The combinations (β, α, θ) of values of the parameters are chosen to be (1.4,0.4,3) and (1.2,0.6,5).Computer programs are prepared and the Newton-Raphson method is used for the practical application of the ML estimators of α, β and θ. Table (1) and Table (3) give the MSE, variance of the estimators and the two sided approximate confidence limits at 95% and 99% level of significance. Tables (2) and (4) represent the results of the test design in which, the optimal sample-proportion s^* allocated to accelerated use condition, the expected fraction failing at each stress, represented by n_u^* and n_a^* and the optimal GAV of the MLEs of the model parameters are obtained numerically for each sample size.

Table 1: Simulation result for the parameters (β, α, θ) set as (1.4, 0.4, 3) respectively, given as *s*=0.30 and $\tau = 55$ for different sized samples under type-I censoring in constant-stress PALT

n	Parameters $\begin{pmatrix}\hat\beta\\ \hat\alpha\\ \hat\theta\end{pmatrix}$	MSE	Variance	95%		99%	
				LCL	UCL	LCL	UCL
100	1.4921	0.0248	1.9972	1.1429	1.9733	1.1356	1.9334
	0.5193	0.0433	0.9341	0.1933	0.8153	0.1842	0.8615
	3.8653	0.0297	2.6411	2.4549	3.7177	2.4426	3.6961
200	1.4755	0.0212	1.6546	1.2684	1.9454	1.2569	1.9831
	0.5014	0.0398	0.7857	0.2408	0.6842	0.2371	0.6648
	3.6411	0.0172	2.1981	2.5113	3.6883	2.5049	3.6749
300	1.4592	0.0170	1.3427	1.3102	1.8931	1.2994	1.8912
	0.4822	0.1669	0.3942	0.3889	0.5978	0.3313	0.5410
	3.3983	0.0241	1.9428	2.6906	3.5917	2.6769	3.5236
400	1.4204	0.0043	1.2099	1.3863	1.7394	1.2312	1.7661
	0.4185	0.0136	0.2082	0.3862	0.4604	0.3042	0.4018
	3.2864	0.0109	1.1839	2.7694	3.3019	2.7526	3.3924
500	1.4112	0.0015	1.1478	1.3900	1.6876	1.3757	1.7019
	0.4023	0.0049	0.2143	0.3911	0.4395	0.3817	0.4962
	3.1482	0.0068	1.1017	2.8114	3.1198	2.8939	3.2991

Table 2: The results of optimal design of the life test for different sized samples under type-I censoring in constant-stress PALT

n	s^*	n_u^*	n_a^*	Optimal GAV
100	0.3521	38	42	1.0941
200	0.3832	45	115	0.0834
300	0.4645	56	184	0.0210
400	0.5137	73	247	0.0026
500	0.5592	88	312	0.0019

Table 3: Simulation result for the parameters (β, α, θ) set as (1.2, 0.6, 5) respectively, given as s=0.30 and $\tau = 55$ for different sized samples under type-I censoring in constant-stress PALT

n	Parameters $\begin{pmatrix}\hat{\beta}\\ \hat{\alpha}\\ \hat{\theta}\end{pmatrix}$	MSE	Variance	95% LCL	95% UCL	99% LCL	99% UCL
100	1.3162	0.0639	3.5956	0.7998	1.8944	0.5783	1.7603
	0.8673	0.0911	1.8960	0.2674	0.9032	0.2386	0.8172
	7.8018	0.0315	1.8555	3.7793	7.4439	3.4491	6.9927
200	1.3096	0.0389	2.8720	1.0260	1.7503	1.0168	1.6118
	0.7791	0.0617	0.8472	0.4055	0.8541	0.4302	0.8017
	6.5409	0.0276	1.3879	4.6841	6.8617	3.5169	6.8005
300	1.2873	0.0406	1.5612	1.0704	1.5633	1.1027	1.4997
	0.6371	0.0235	0.7395	0.4559	0.8878	0.4399	0.7548
	5.7365	0.0227	0.7258	4.7018	6.5609	4.5918	6.7541
400	1.2415	0.0209	1.3291	1.1712	1.3093	1.1626	1.2963
	0.6079	0.0195	0.2959	0.5874	0.7040	0.5032	0.6817
	5.6671	0.0185	0.3940	4.5084	6.6012	4.7203	6.6019
500	1.2055	0.0186	1.1089	1.1831	1.2994	1.1291	1.3022
	0.5909	0.0122	0.2468	0.5937	0.6991	0.5135	0.6530
	5.2841	0.0096	0.1874	4.9163	6.8278	4.8021	5.9879

Table 4: The results of optimal design of the life test for different sized samples under type-I censoring in constant-stress PALT

n	s^*	n_u^*	n_a^*	Optimal GAV
100	0.3988	36	44	3.0849
200	0.4503	62	88	1.8692
300	0.5844	86	144	0.0672
400	0.5994	98	262	0.0151
500	0.6348	104	316	0.0039

7 Conclusions

This study deals with the problem of estimation and optimally designing simple constant stress PALT for the Gompertz distribution under type-I censored data. From the table (1) and (2), it is observed that that the ML estimates approximate the true values of the parameters as the sample size n increases. Also, we find that, for a fixed α, β and θ the mean squared errors and asymptotic variances of the estimators are decreasing with the increasing value of *n*. It is also noticed that when the sample size increases, the interval of the estimators are decreases.

Tables (2) and (4) present the optimal GAV of the ML estimators of the model parameters which is obtained numerically with *s** in place of *s* for different sized samples. As expected, the optimal GAV decreases as the sample size *n* increases. The minimization of the GAV of the MLEs of model parameters was adopted as an optimality criterion. It may be concluded that the PALT model is an appropriate plan. In practice, the optimum test plans are important for improving the level of precision in parameter estimation and thus improving the quality of the inference. So, statistically, optimum plans are needed, and the experimenters are advised to use it for estimating the life distribution at design stress because it enables us to save time and money in a limited time without necessarily using a high stress to all test units.

As a result , it is right to say that the proposed model work well which helps to save time and money considerably without using a high stress to all test units.

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