
MATHEMATICAL MODELS OF ECOLOGICALLY HAZARDOUS RAIL TRAFFIC ACCIDENTS

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ABSTRACT

The paper discusses the queuing system mathematical models simulating the development of processes of rail traffic accidents with hazardous materials as well as elimination of such accidents ecologically dangerous consequences. A new theoretical approach is proposed that enables a more rational dislocation of emergency detachments on railroad network and more successful actions by such detachments aimed at minimization of environmental damage and cargo losses.

1 INTRODUCTION

Research of ecologically hazardous rail traffic accidents with dangerous goods indicates that the accidents are of complex nature, having as possible final results the emergence of serious consequences such as explosions, fires, destruction of rolling stock and facilities, injuries or deaths, or environmental pollution.

Consideration of typical hazardous emergencies shows the possible scenarios of development [1, 2]:

- Slow accumulation of negative factors of ecologically dangerous situation, but not to the level of critical values, which in turn does not cause an explosion or fire;
- Slow accumulation of negative factors of ecologically dangerous situation beyond their critical values, followed by an explosion or fire;
- The rapid accumulation of negative factors of ecologically dangerous situation beyond their critical limits that is associated with an explosion or fire.

The analysis necessary action on the localization of these effects showed that measures must be made, within a wide range - from manual labor to use of complicated mechanisms, relevant emergency teams of railway companies and other ministries and agencies involved in carrying out localization action.

Therefore, analysis of the situation that has developed as a result of the emergence of dangerous goods transportation event, making a timely and adequate decision by task force leader as an emergency response, especially in lack of time, due to the need of sooner of traffic restoration and increase over time of environmental and other damage is a complex process that requires the use of advanced information technologies, including decision support systems (DSS).

Problems of automation of control localization hazardous consequences of emergencies with dangerous goods are dealt with in a number of works that discuss various aspects of automation for relevant team leader control [3, 4].

An important aspect of this task force leader is to determine the required number of capabilities for complex operations, develop an action plan and evaluate its effectiveness.

One of the problems of developing an action plan is the problem of concentration of emergency detachments based on their places of permanent deployment, terrain, availability of water sources, the development of rail and road infrastructure, meteorological conditions, the nature of changes in the emergency hazardous parameters, as well as general and hazardous properties of dangerous goods and so on.

This problem is compounded if it is accompanied by emergency of dangerous goods fire or the hazardous effects localization process must be preceded by localization of fire.

Naturally, the success of localization work is gravely affected by the level of staff training and serviceability of means.

Processes of railway traffic accidents occurrence and their development are of complex probabilistic nature. To determine the nature of interdependencies between the flow of emergency hazardous factors that occurred as a result of traffic accident, and performance of capabilities needed for accident recovery, taking into account the period of time required for their focus, in our view, it is advisable to employ methods of queuing theory, which is one of the most developed branches of probability theory.

2. MODELING EMERGENCY UNITS ACTION

Consider an object where localization works are made by accident recovery teams as a queuing system "emergency object - recovery units."

The exponential flow of customers (being emergency hazardous factors) arrives to n -channel queuing system (where n is the number of localization units) with intensity λ . The service time is exponential with parameter μ . The process of service has a feature that before it starts servicing, the device should prepare for service. The server preparation time T_{cf} has exponential distribution with parameter ν . The customer that catches service device free comes to be served. The catches all servers occupied is waiting for service in the queue.

So random variable T_{ar} consists of two phases – preparation and service:

$$T_{ar} = T_{cf} + T_{ew}. \quad (1)$$

Thus, a random variable T_{cf} is distributed according to generalized Erlang law of order 2 with parameters ν and μ . The probability density distribution of the law is described by the formula [5]:

$$g(t) = \int_0^t \nu e^{-\nu t_1} \mu e^{-\mu(t-t_1)} dt_1 = \frac{\nu\mu(e^{-\nu t} - e^{-\mu t})}{\mu - \nu}, (t > 0), \quad (2)$$

where: $\nu = \frac{1}{M[T_{cf}]}$, $f_1(t) = \nu e^{-\nu t}$; $\mu = \frac{1}{M[T_{ew}]}$, $f_2(t) = \mu e^{-\mu t}$.

The arrival pattern in that QS is not Poissonian, that system is not Markovian, so it is not possible to find probabilities for QS states using method for Markovian processes with discrete states and continuous time.

We know that abuse of Poisson distribution of probabilities in any queuing system shifts it from Markovian system to non-Markovian one and, as emphasized above, the direct output and use of Kolmogorov equations is impossible. Therefore, to analyze the QS the most common two areas of analytical methods for non-Markovian systems are [5 - 8]:

- The area based on the use of conventional theory of Markov chains, but the system studied needs its phase space of possible states to be expanded (pseudo-states method) [5 - 8];
- The area that involves the use of more sophisticated mathematical tools, but without increasing the number of system states (semi-Markov process method) [6, 7, 8];

Both of these areas have much in common, but differ in their capabilities and degree of difficulty for calculation.

Using a semi-Markov process involves examining the behavior of the system only in the change of state (in moments of jumps of the process), resulting in the formation of a Markov chain. In this case, no aftereffect do not come any moment of time, as is the case in a Markov process, but only in jump moments. The effectiveness of this method depends on the ways of setting semi-Markov process. In any case, must be known finite set of possible states of system under study that is linked in Markov chain, as well as directions of possible transition from one state to another and system's original state.

Pseudo-states method is only used when an arrival flow and service intensity have the Poisson probability density distribution function, which is a composition of exponential distributions with the same parameter. It allows you to simplify analysis, from a mathematical point of view, using normal notation of Kolmogorov equations to analyze a queuing system both in sustainable and in transient modes of operation. But this method complicates the structure of the original graph states, leading to computational complexity [5, 6].

The artificial extension of the phase space of states of non-Markov (Erlang) system by introducing into it additional (false) states that shifts it to Markov system, allowing consider the original non-Markov process as nested inside another, more complex process possessing Markovian properties [5, 6].

Consider a queuing system functioning as an object within which localization works are made, provided that there are restrictions on the length of the queue and environmentally dangerous emergency factors have negative impact on recipients that are within limited danger zones. Graph of states for such a QS is presented in Figure 1.

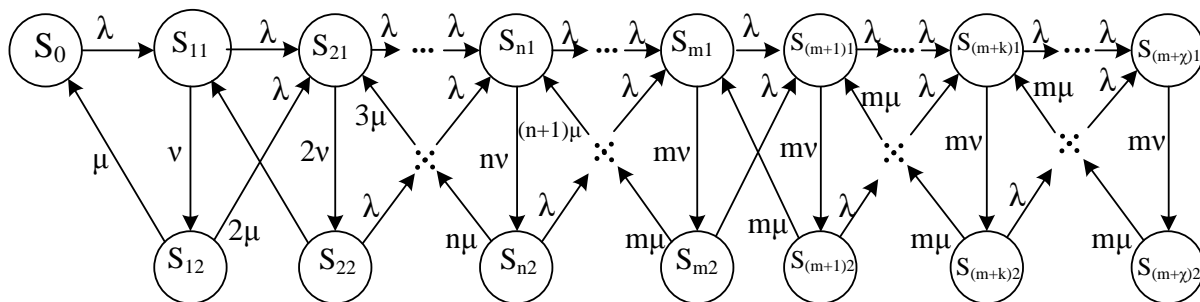


Figure 1 Graph of QS states with limited length of the queue

QS states:

S_0 – QS is free;

S_{11} – one customer in QS, service in the first phase;

S_{12} – one customer in QS, service in the second phase;

.....

S_{m1} – m customers in QS, service in the first phase, no queue;

S_{m2} – m customers in QS, service in the second phase, no queue;

.....

$S_{(m+\chi)1} - (m+\chi)$ customers in QS (m in service, χ in queue), service in the first phase;
 $S_{(m+\chi)2} - (m+\chi)$ customers in QS (m in service, χ in queue), service in the second phase.
 Algebraic equations for the final probabilities of system states are the following:

$$\begin{aligned} \lambda P_0 &= \mu P_{12}; \\ (\lambda + \nu)P_{11} &= \lambda P_0 + 2\mu P_{22}; \\ (\lambda + \mu)P_{12} &= \nu P_{11}; \\ (\lambda + 2\nu)P_{21} &= \lambda(P_{11} + P_{12}) + 3\mu P_{32}; \\ (\lambda + 2\mu)P_{22} &= \nu P_{21}; \\ &\dots\dots\dots \\ (\lambda + m\nu)P_{m1} &= \lambda(P_{(m-1)1} + P_{(m-1)2}) + m\mu P_{(m+1)2}; \\ (\lambda + m\mu)P_{m2} &= m\nu P_{m1}; \\ &\dots\dots\dots \\ (\lambda + m\nu)P_{k1} &= \lambda(P_{(k-1)1} + P_{(k-1)2}) + m\mu P_{(k+1)2}; \\ (\lambda + m\mu)P_{k2} &= m\nu P_{k1}; \\ &\dots\dots\dots \\ m\nu P_{(m+\chi)1} &= \lambda(P_{(m+\chi-1)1} + P_{(m+\chi-1)2}); \\ m\mu P_{(m+\chi)2} &= m\nu P_{(m+\chi)1}; \end{aligned} \tag{3}$$

Normalizing condition is:

$$P_0 + \sum_{i=1}^{m+\chi} \sum_{j=1}^2 P_{ij} = 1$$

The probabilities of states of Erlang QS are:

$$P_{1e} = P_{11} + P_{12}; P_{2e} = P_{21} + P_{22}; P_{ne} = P_{n1} + P_{n2}; \dots P_{ne} = P_{n1} + P_{n2}; \dots P_{(m+\chi)e} = P_{(m+\chi)1} + P_{(m+\chi)2}.$$

Define the characteristics of QS:

The probability of customer service:

$$P_{ar} = 1 - P_1 = 1 - P_f, \tag{4}$$

where $P_1 = P_{(m+\chi)e} - \frac{\sum_{i=2}^{m+\chi} P_{(i-1)2}}{2} = P_{(m+\chi)2} - \frac{P_{12} + P_{22} + \dots + P_{(m+\chi-1)2}}{2}$.

The average number of customers that are in queue:

$$\bar{r} = \frac{1}{2} (1 \cdot P_{(m+1)e} + 2P_{(m+2)e} + \dots + \chi P_{(m+\chi)e}). \tag{5}$$

The average number of customers that are in the system:

$$\bar{s} = \frac{1}{2} (1 \cdot P_{1e} + 2P_{2e} + \dots + (m+\chi)P_{(m+\chi)e}). \tag{6}$$

Average customer time spent in queue:

$$\bar{t}_q = \frac{\bar{r}}{\lambda}. \tag{7}$$

The average customer time spent in the system:

$$\bar{t}_s = \frac{\bar{s}}{\lambda}. \tag{8}$$

The average number of servers occupied:

$$\bar{k} = \frac{1}{2} (1 \cdot P_{1e} + 2P_{2e} + \dots + mP_{me} + \dots + mP_{(m+\chi)e}). \tag{9}$$

Explore functioning example of a M / E2 / 2/3 system, which is shown in the graph of Figure 2.

$$P_{ar} = 1 - P_1 = 1 - P_f;$$

$$P_1 = P_{5e} - \frac{P_{12} + P_{22} + P_{32} + P_{42}}{2}.$$

The average number of customers in queue:

$$\bar{r} = \frac{1}{2}(1 \cdot P_{3e} + 2P_{4e} + 3P_{5e}). \tag{11}$$

The average number of customers that are in the system:

$$\bar{s} = \frac{1}{2}(1 \cdot P_{1e} + 2P_{2e} + 3P_{3e} + 4P_{4e} + 5P_{5e}).$$

Average time spent in queue:

$$\bar{r}_q = \frac{\bar{r}}{\lambda}.$$

The average time spent in the system:

$$\bar{s}_q = \frac{\bar{s}}{\lambda}.$$

The average number of employees servicing devices:

$$\bar{k} = \frac{1}{2}(1 \cdot P_{1e} + 2P_{2e} + 2P_{3e} + 2P_{4e} + 2P_{5e}).$$

To study the two-channel queuing system shown in Fig. 3, for the accepted range of values $0.5 \leq P_{ar} \leq 0.75$ we define the QS parameters. However the problem is that the number of the QS parameter values (λ, ν, μ) is infinite. Therefore, when adopted λ , then incrementally changing ν and find the values of μ that makes P_{ar} equal to values taken from the range.

Fig. 3 shows a graph of function $\mu = f_1(P_{ar})$.

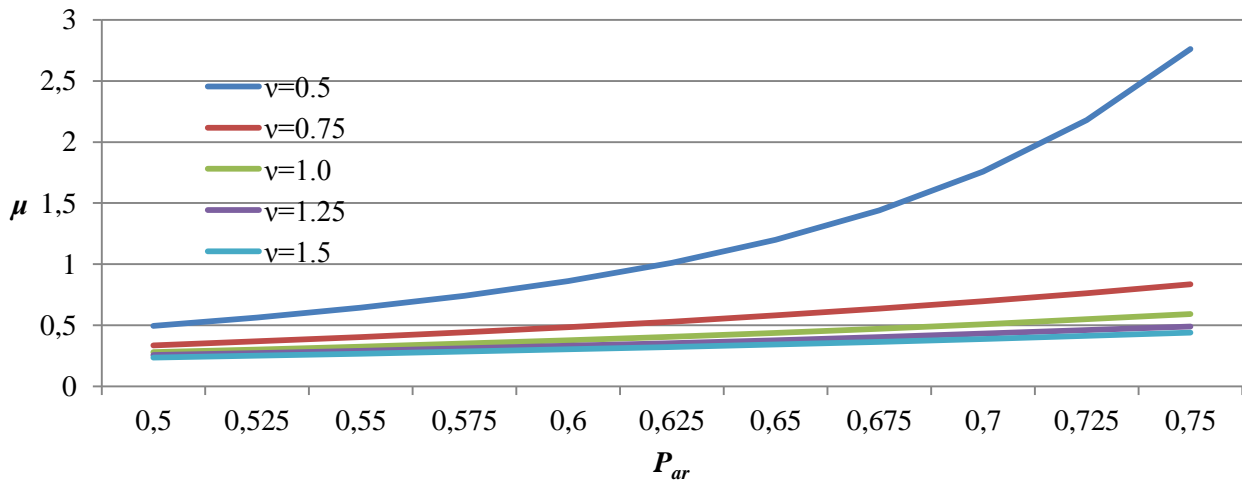


Fig. 3 Graph of function $\mu = f_1(P_{ar})$.

If an acceptable range of values for probability of successful elimination of an emergency environmentally hazardous consequences is $(0.5 \leq P_{ar} \leq 0.75)$, then reducing time of focusing localization units in emergency place (increase value ν) allows the use of less productive means of recovery operations. More details can be seen from the graph (Figure 4).

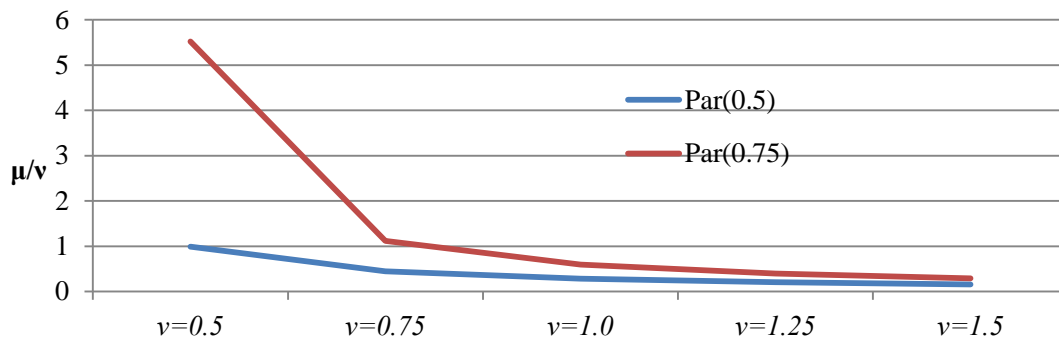


Fig. 4 Graph of function $\mu = f_2(\nu)$ for given values of P_{ar} .

In an acceptable range of $0,5 \leq P_{ar} \leq 0,75$, the ratio μ/ν is non-linear. Moreover, with decreasing ν (to $\nu = 0,5$), the need for fold increase of μ increases rapidly, that is the most characteristic for the upper band of P_{ar} .

More information is provided on a generalized graph of the probability of liquidation of consequences of environmentally hazardous emergency versus QS parameters (Fig. 5).

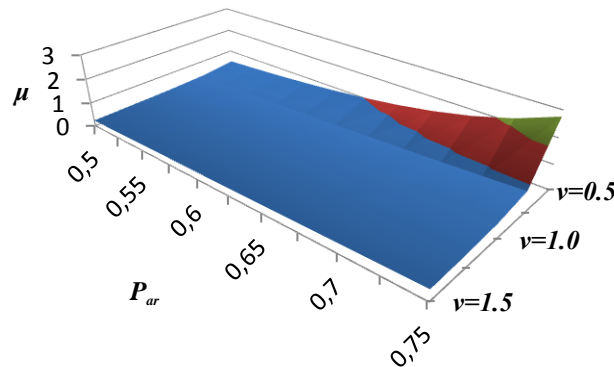


Fig. 5 A generalized graph of the probability of liquidation of consequences of environmentally hazardous emergency versus QS parameters.

Of particular note is the dependence of the probability of a negative impact on the environment (P_{ni}) on values of λ, ν, μ . Graph of $P_{ni} = f_3(\mu)$ is shown in Figure 6.

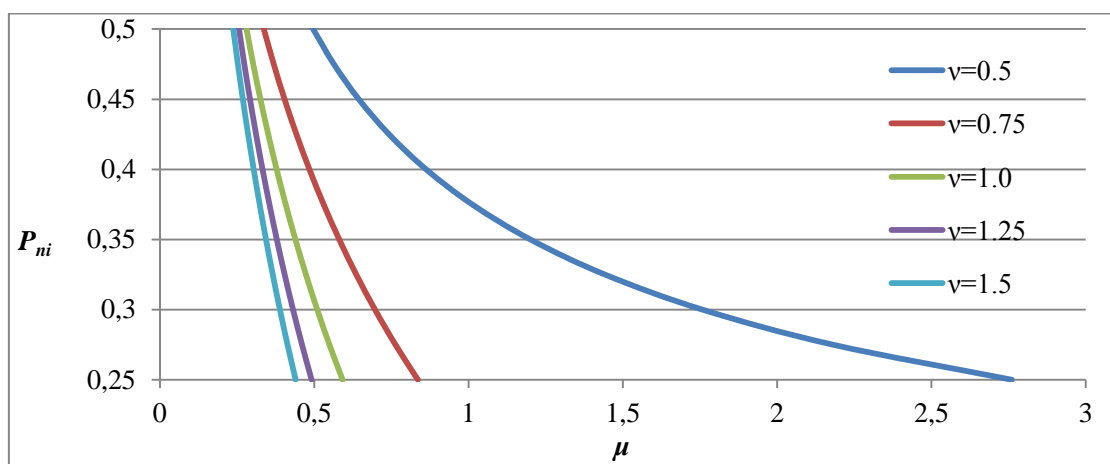


Fig. 6. Graph of $P_{ni} = f_3(\mu)$ for different values of ν .

It is known that the productivity of means elimination of environmentally hazardous consequences is defined in their technological specifications within $(\mu_{min_i} \leq \mu_i \leq \mu_{max_i})$. If $\mu_i \leq \mu_{min_i}$, even provided sooner concentration of emergency units at the site of an accident, the probability P_{ni} is rather high. The following values $\mu_i = \mu_{min_i}$ and $v = v_{max}$, where μ_i is constant, that is extremely inadequate performance. When $\mu_{min_i} \leq \mu_i \leq \mu_{max_i}$ with increasing v P_{ni} value decreases. When $\mu_i > \mu_{max_i}$, and with large values of emergency units concentrating time, P_{ni} is still relatively large, ie, the concentration is impractical.

In other words, if the means of the emergency response is not appropriate to the nature of accident and/or extremely unproductive, even with their short time of focusing on the accident site, they will not be effective. On the other hand, even if the means of eliminating are effective enough, but their focus on the scene was late, they also do not give proper effect.

The problem of negative impact on the environment at different duration of work by the liquidation units attracts special attention. Fig. 7 presents a graph of the duration of consequences recovery works versus performance of means, ie finding the QS customer full cycle of service.

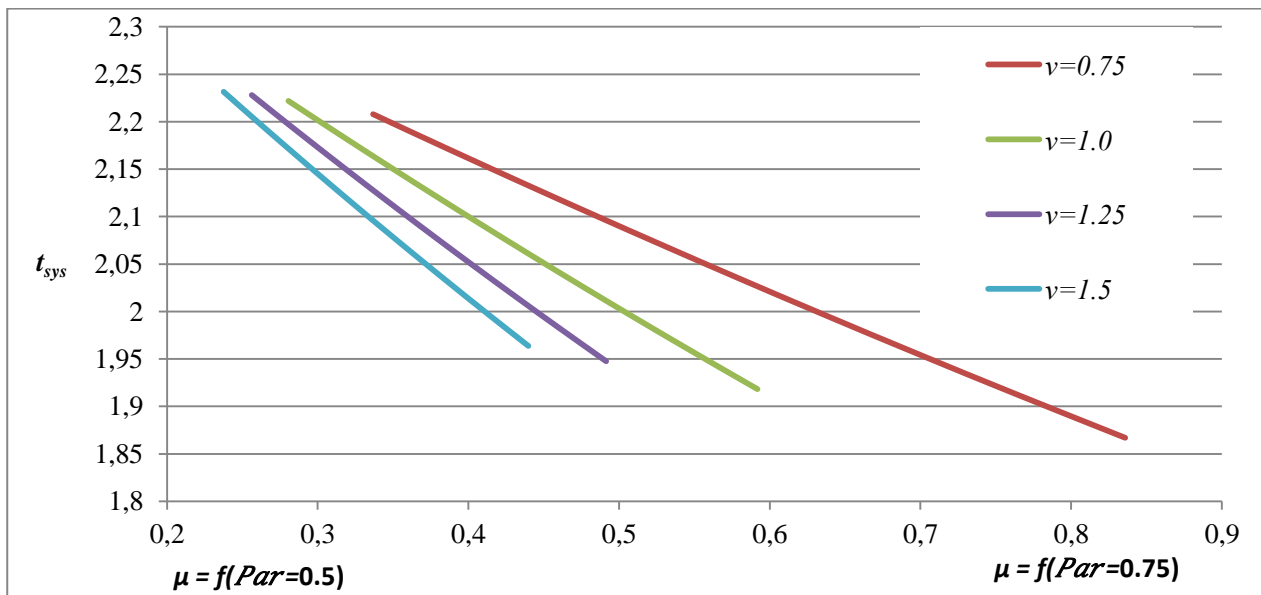


Fig. 7 Graph of average time spent in the system $t_{sys} = f_4(\mu)$ for different values of v in the acceptable range of values $0.5 \leq P_{ar} \leq 0.75$.

From Fig. 7 it is seen that a significant reduction of duration of works as well as the negative effects of environmentally hazardous emergency are possible with reducing the emergency units concentration time and using corresponding capabilities. Extending the concentration time needs to increase the productivity of these capabilities in times.

Consider a binary operation QS as operation of two emergency units (Figure 8).

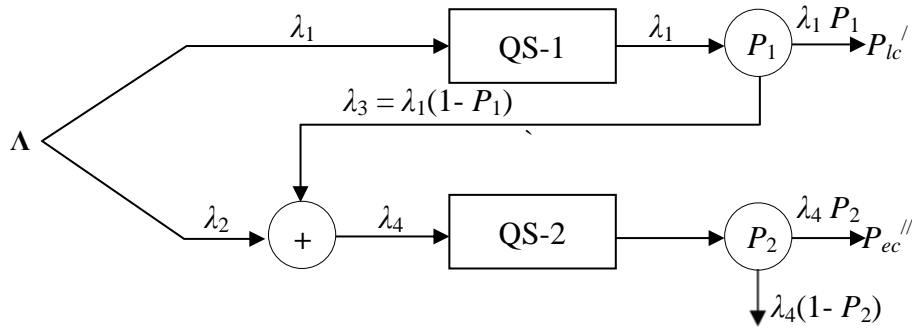


Fig. 8 Scheme of binary operation QS with consequent link of components

The arrival flow at the input of binary QS is exponential of Λ intensity, which is a superposition of two simple flows λ_1 and λ_2 , ie $\Lambda = \lambda_1 + \lambda_2$.

After service refusal in QS-1 failed customers flow with intensity $\lambda_3 = \lambda_1(1-P_1)$ immediately arrives to the input of the QS-2.

At the input of QS-2, either customers flow with intensity λ_1 is coming, or flow with intensity λ_3 . The probability of simultaneous arrival of two various customer flows is zero.

With the prescribed probability limits of environmentally hazardous emergency elimination ($0,8 \leq P_{qs} \leq 0,95$) determine the probability of the emergency elimination in QS-1 (P'_{lc}) and in QS-2 (P''_{lc}) using the relation:

$$(1-P_{qs})=(1-P'_{lc})(1-P''_{lc}),$$

where $P_{qs} = 1-(1-P'_{lc})(1-P''_{lc})$.

Set the values P'_{lc} and P''_{lc} and define P_1 and P_2 using relationships:

$$P'_{lc}=\lambda_1P; P''_{lc}=\lambda_4P_2,$$

$$\text{then: } P_1 = \frac{P'_{lc}}{\lambda_1}; P_2 = \frac{P''_{lc}}{\lambda_2+\lambda_1(1-P_1)}.$$

If accepted values $\lambda_1, \lambda_2, \mu_1$ and μ_2 define the value v1 and v2 by calculation using MathCad and MS Excel.

QS components are queuing systems of M / E2 / 1 / 3 type. Graph of QS-1 and QS-2 is given in Fig. 9.

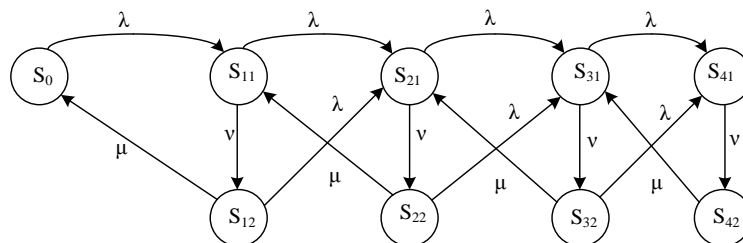


Fig. 9 Graph of QS components.

Matrix Λ_e of transitions intensities of such QS is as below:

$$\Lambda = \begin{pmatrix} -\lambda & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & -(\lambda + \nu) & 0 & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & \mu & -(\lambda + \mu) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & \lambda & -(\lambda + \nu) & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & \nu & -(\lambda + \mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & \lambda & -(\lambda + \nu) & 0 & 0 & \mu \\ 0 & 0 & 0 & 0 & 0 & \nu & -(\lambda + \mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & \lambda & -\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \nu & -\mu \end{pmatrix}$$

Algebraic equations for the final state probabilities of QS are:

$$\begin{aligned} \lambda P_0 &= \mu P_{12}; \\ (\lambda + \nu) P_{11} &= \lambda P_0 + 2\mu P_{22}; \\ (\lambda + \mu) P_{12} &= \nu P_{11}; \\ (\lambda + 2\nu) P_{21} &= \lambda P_{11} + 2\mu P_{32} + \lambda P_{12}; \\ (\lambda + 2\mu) P_{22} &= 2\nu P_{21}; \\ (\lambda + 2\nu) P_{31} &= \lambda P_{21} + 2\mu P_{42} + \lambda P_{22}; \\ (\lambda + 2\mu) P_{32} &= 2\nu P_{31}; \\ \nu P_{41} &= \lambda P_{31} + \lambda P_{32}; \\ \mu P_{42} &= \nu P_{41}. \end{aligned} \tag{12}$$

$$P_0 + P_{11} + P_{12} + P_{21} + P_{22} + P_{31} + P_{32} + P_{41} + P_{42} = 1$$

The probabilities of QS states are determined as:

$$P_{1e} = P_{11} + P_{12}; P_{2e} = P_{21} + P_{22}; P_{3e} = P_{31} + P_{32}; P_{4e} = P_{41} + P_{42};$$

QS features are:

$$\begin{aligned} P_{ar} &= 1 - P_l = 1 - P_f, \\ P_l &= P_{4e} - \frac{P_{12} + P_{22} + P_{32}}{2}. \end{aligned}$$

The average number of customers which are in queue:

$$\bar{r} = \frac{1}{2} (1 \cdot P_{2e} + 2P_{3e} + 3P_{4e}). \tag{13}$$

The average number of customers that are in the system:

$$\bar{s} = \frac{1}{2} (1 \cdot P_{1e} + 2P_{2e} + 3P_{3e} + 4P_{4e}).$$

Average time spent in queue:

$$\bar{r}_q = \frac{\bar{r}}{\lambda}.$$

The average time spent in the system:

$$\bar{s}_q = \frac{\bar{s}}{\lambda}.$$

The average number of servers occupied:

$$\bar{k} = \frac{1}{2} (1 \cdot P_{1e} + 1P_{2e} + 1P_{3e} + 1P_{4e}).$$

Graphs of functions $P_{qs} = f_5(\nu)$ with taking into account the range for P_{ar} are given in Figure 10.

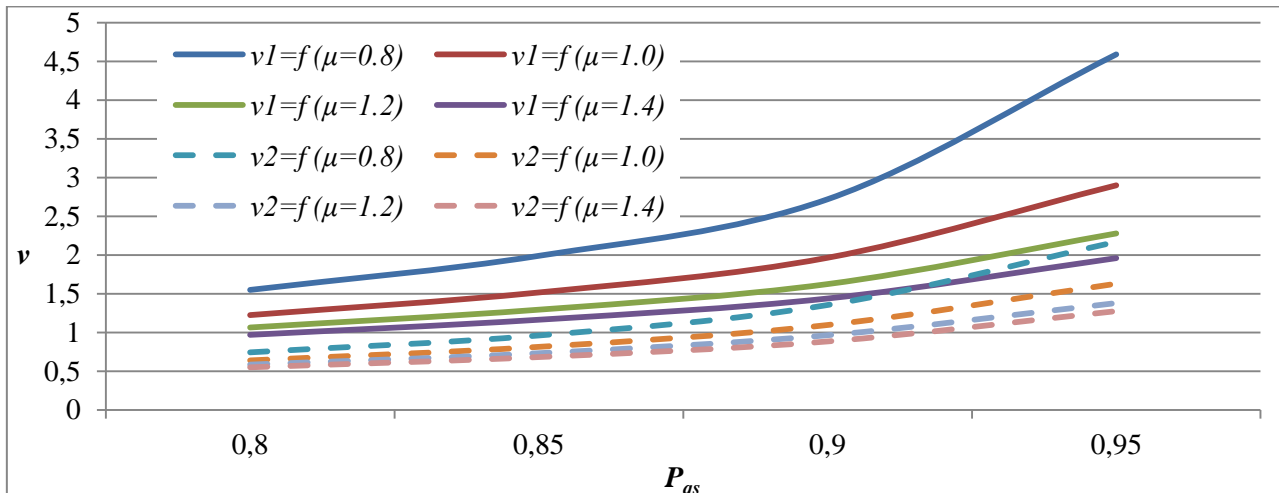


Fig. 10. Graphs of functions $P_{qs} = f_5(v)$ taking into account the range for P_{ar} .

The graphs (Figure 10) show the dependences of the QS-1 in continuous lines and the QS-2 in dotted lines.

As shown in the graphs (Figure 10), with an increase in the performance of QS-1 and QS-2, the needed intensity of concentration decreases. For a given value μ , and with increasing P_{qs} value, v increases non-linearly.

Providing certain probability P_{qs} s can be achieved by various parameters of QS-1 and QS-2, or by increasing the productivity of disposal capabilities and reducing the time of concentration. Moreover, the requirements for parameters QS-2 are significantly smaller than the parameters for QS-1.

3 CONCLUSIONS

The proposed approach to the study of the consequences of hazardous rail traffic accidents with dangerous goods:

- is the methodological basis for the creation and development of Decision Support System for Task Force leader in the aftermath of such accidents in a single automated control system of railway freight transportation;
- makes it possible to formulate reasonable requirements for the deployment of wreck, recovery and fire teams on the rail network, their equipment and the professional training for team leaders, managers and staff;
- enables to determine the probability and duration of the negative impact of environmentally hazardous transport accidents.

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