FATIGUE-PRONE AIRCRAFT FLEET RELIABILITY BASED ON THE USE OF A P-SET FUNCTION

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ABSTRACT

An inspection interval planning is considered in order to limit a probability of any fatigue failure (FFP) in a fleet of N aircraft (AC). A solution of this problem is based on a processing of the result of the acceptance fatigue test of a new type of an aircraft. During this test an estimate of the parameter of a fatigue crack growth trajectory can be obtained. If the result of this acceptance test is too bad then this new type of aircraft will not be used in service. A redesign of this project should be done. If the result of acceptance test is too good then the reliability of the aircraft fleet will be provided without inspections. For this strategy there is a maximum of FFP as a function of an unknown parameter θ. This maximum can be limited by the use of the offered here procedure of the choice of an inspection number.

1 INTRODUCTION

We suppose that in the interval (T_d, T_c), where T_d is a random time when the fatigue crack becomes detectable (the corresponding crack size $a(T_d)=a_d$ and T_c is a random time when the crack reaches its critical size (the corresponding crack size $a(T_c)=a_c$), the size of the crack can be approximated by the equation $a(t)=\alpha exp(Qt)$. Then we have:

$$T_c=(\log a_c-\log\alpha)/Q=C_c/Q,\ T_d=(\log a_d-\log\alpha)/Q=C_d/Q. \tag{1}$$

For the calculation of a probability of a fatigue crack detection during an inspection we need to know the probability of detection of fatigue crack as function of its size a and the distribution of the size for specific time of inspection. Usually for processing fatigue life the lognormal distribution is used, so for the considered numerical example here (see the 5-th section) we study the simplest case: a random variable (rv), $\log(Q)$, has a normal distribution with an parameter $\theta=(\theta_0,\theta_1)$, where θ_0 is an unknown mean but θ_1 is the known standard deviation. And we suppose that α, a_d, a_c are the known constants. The estimate $\hat{\theta}=(\hat{\theta}_0+\theta_1)$ of the parameter θ can be obtained by the regress analysis of the result of fatigue test of AC of the same type in laboratory (i.e. processing the observations of fatigue crack: pairs {(time, fatigue crack size)$_i$, i=1,…,m}, where m is a number of the fatigue crack observation.

2 CALCULATION OF A PROBABILITY OF A FATIGUE FAILURE OF ONE AIRCRAFT FOR THE KNOWN θ

For the known θ, there are two decisions: 1) the aircraft is good enough and the operation of this aircraft type can be allowed, 2) the operation of the new type of AC is not allowed and a redesign of AC should be made. In the case of the first decision, the vector $t=(t_1,\ldots,t_n)$, where t_i is the time moment of *i*-th inspection, should also be defined. If θ is known the different rules can be offered for the choice of structure of the vector t:1) every interval between the inspections is equal to the constant $d_t = t_{SL}/(n+1)$, where t_{SL} is the aircraft specified life (SL) (the retirement time), n is a number of inspections, 2) the conditional probabilities of a failure (under condition that the fatigue failure did not takes place in previous interval) in every interval is equal to the same value $P(T_C < t_{SL})/(n+1)$… In this paper we suppose the first type of the choice and the vector t is defined by the fixed t_{SL} and the choice of n.

For the substantiation of the choice of the inspection number we should know the probability of afatigue crtack detection as a functions of a crack size a. We suppose that this probability is defined by the equations

$$p_d(a) = w_0 w(a),\ w(a) = \begin{cases} 0, & if \quad \mathrm{a} \le \mathrm{a_{d0}}, \\ \dfrac{\mathrm{a}-\mathrm{a_{d0}}}{\mathrm{a_{d1}}-\mathrm{a_{d0}}}, & if \quad \mathrm{a_{d0}} < \mathrm{a} < \mathrm{a_{d1}}, \\ 1, & if \quad \mathrm{a} \ge \mathrm{a_{d1}}. \end{cases} \tag{2}$$

Where $\mathrm{a_{d0}}$, $\mathrm{a_{d1}}$ are some constants, see Fig.1; the constant w_0 can be considered as probability to carry out planned inspection (human factor).

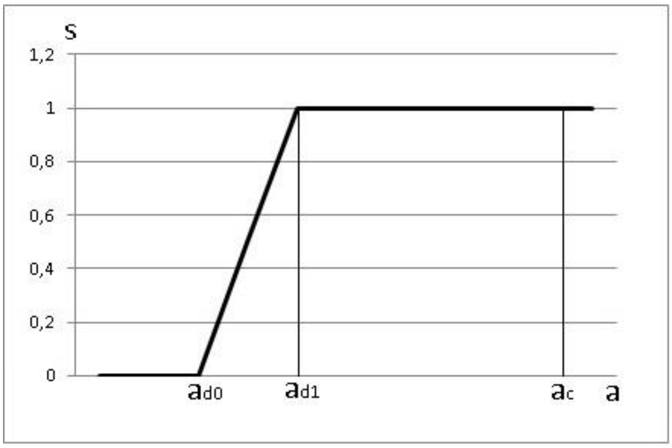

Figure 1. Crack detection probability function of crack size

In simplest case $w_0 = 1$, $\mathrm{a_{d1}} = \mathrm{a_{d0}}$ and we define

$$w(a) = \begin{cases} 0, & if \quad \mathrm{a} < \mathrm{a_{d0}}, \\ 1, & if \quad \mathrm{a} \ge \mathrm{a_{d0}}. \end{cases} \tag{3}$$

Then it is convenient the process of an operation of AC to consider as an absorbing Markov chain (MCh) with $(n+4)$ states. The states $E_1, E_2, \ldots, E_{n+1}$ correspond to an AC operation in the time intervals $[t_0,t_1),[t_1,t_2),\ldots,[t_n,t_{SL})$. States E_{n+2}, E_{n+3} and E_{n+4} are the absorbing states: AC is discarded from a service when the SL is reached or fatigue failure (FF), or fatigue crack detection (CD) take place.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	...	0	0	0	0	q_1	v_1
E_2	0	0	u_2	...	0	0	0	0	q_2	v_2
E_3	0	0	0	...	0	0	0	0	q_3	v_3
...	...	...	...	...	...	...	...	...	...	...
E_{n-1}	0	0	0	...	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	...	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	...	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	0	0	0	...	0	0	0	1	0	0
E_{n+3} (FF)	0	0	0	...	0	0	0	0	1	0
E_{n+4} (CD)	0	0	0	...	0	0	0	0	0	1

Figure 2. Probability matrix P_{AC}

In the corresponding transition probability matrix, P_{AC}, (see Fig.2.) let ν_i be the probability of a crack detection during the inspection number i, let q_i be the probability of the failure in service time interval $t \in (t_{i-1}, t_i)$, and let $u_i = 1 - \nu_i - q_i$ be the probability of the successful transition to the next state (next interval of aircraft service). In our model we also assume that an aircraft is discarded from a service at t_{SL} even if there are no any crack discovered by inspection at the time moment t_{SL}. This inspection at the end of $(n+1)$-th interval (in state E_{n+1}) does not change the reliability but it is made in order to know the state of the aircraft (whether there is a fatigue crack or there is no fatigue crack). It can be shown that $u_i = P(T_d > t_i | T_d > t_{i-1})$, $q_i = P(t_{i-1} < T_d < T_c < t_i | T_d > t_{i-1})$, $\nu_i = 1 - u_i - q_i$, $i = 1, \ldots, n+1$. In the three last lines of the matrix P_{AC} there are three units in the matrix diagonal because the states E_{n+2}, E_{n+3} and E_{n+4} are the absorbing states. All the other entries of this matrix are equal to zero (see Fig.2.). The structure of the considered matrix can be described in the following way: $P_{AC} = [QR;\ 0I]$,

Q	R
0	I

Figure 3. Structure of matrix P_{AC}

where in the second line of this structure the matrix 0 is the submatrix of zeroes, *I* is the submatrix of identity corresponding to the absorbing states of the matrix P_{AC}. Then the matrix of the probabilities of absorbing in the different absorbing states for the different initial transient states $B=(I-Q)^{-1}R$. The failure probability of a new AC is equal to $p_f = aBb$, where the vector row $a=(1,0,\ldots,0)$ means that all the aircraft begin an operation within the first interval (state E_1), the vector column $b=(0,1,0)'$. We need also to know the mean number of steps of MCh up to the absorption $E(T_{AC})=a(I-Q)^{-1}c$, where $c=(1,\ldots,1)'$ is the vector-column. The mean life of aircraft will be equal to $d_t E(T_{AC})$.

3 PROBABILITY OF ANY FATIGUE FAILURE IN THE FLEET OF AIRCRAFT FOR THE KNOWN θ

We consider the case when the operation of all N aircraft will be stopped if any fatigue crack will be detected. In order to limit the probability of fatigue failure in the fleet (FFPN) it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place.

3.1 Probability of detection is defined by (3)

For the case when the probability of fatigue crack detection is defined by equation (3) the corresponding probability is equal to the expected value of random variable $P_{fNW}=(1-w)^R$, where w is a human factor: a probability, that the planned inspection will be made, *R* is the total random number of inspections before the first failure in the whole fleet.

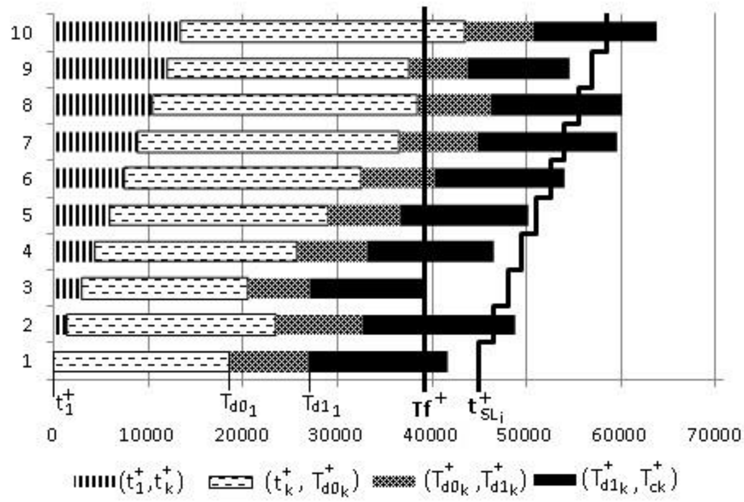

Figure 4. Operation of N aircraft

Let t_k^+, $t_{k-1}^+ < t_k^+$, $t_0^+ = 0$ to be "calendar" time moment when *k*-th aircraft begin the service, $T_{dk}^+ = t_k^+ + T_{dk}$, $T_{ck}^+ = t_k^+ + T_{ck}$, $k=1,2,...,N$ to be the random calendar time moments when fatigue crack can be discovered and fatigue failure of AC takes place correspondingly, see Fig.4. And let $K_{SL}=\{k : T_{ck} < t_{SL},\ k=1,2,...,N\}$ be a set of indexes of aircraft, the failure of which can take a place, if an inspection will not take the place, $T_f^+ = \min\{T_{fk}^+ : k \in K_{SL}\}$, $T_{fk}^+ = \min\{T_{ck}^+,\ T_f^+\},\ k \in K_{SL}$,

$R=\sum_{k\in K_{SL}} R_k$, where $R_k=\max(\{[(T^+_{fk}-t^+_k)/d_t]-[(T^+_{dk}-t^+_k)/d_t]\},0),\ k\in K_{SL}$, is the random inspection number of k-th aircraft from the set K_{SL} if inspection interval $d_t=t_{SL}/(n+1)$ (it is supposed a specific "calendar" schedule of the inspections for each aircraft: , $i=1,2,...,n+1$, $k\in K_{SL}$)

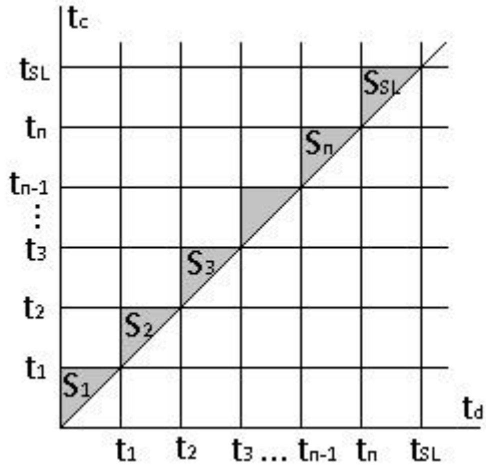

Figure 5a. Example of a "value" p-set function for one aircraft

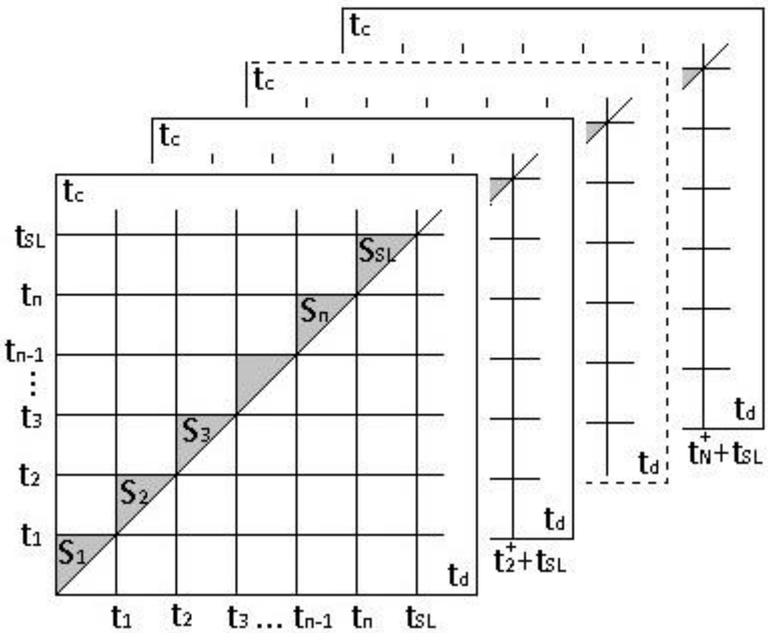

Figure 5b. Set of "values" of a p-set function for N aircraft with different beginning of operation.

Random variable Q is a speed of fatigue crack growth in logarithm scale. It has the specific realization for each aircraft and $Q_1,\ldots,Q_N$ are independent random variables. So mean value of random probability of failure in the fleet

$$E\left(P_{fNW}\right)=p_{fNW}\left(n,\theta\right)=\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty}\left(1-w\right)^{r(q)}dF_{Q_i}\left(q_1\right)\ldots dF_{Q_N}\left(q_N\right) \tag{4}$$

where $q=\left(q_1,\ldots,q_N\right)$, $r\left(q\right)$, is realization of rv R. For large number N the Monte Carlo method is appropriate for the calculation of p_{fNW}. If this function is known then the number of the inspections, $n(p,\theta)$, required to limit the FFPN by a value p is defined by the function $n(p,\theta)=\min(r: p_{fNW}(r,\theta)\le p \text{ for all } r>n(p,\theta),\ r=1,2,...)$.

3.1. General definition of probability of detection

For general definition of the function $p_d(a)$ it is necessary to make more detailed analysis. Let us denote by t^+_{kj} the calendar time of j-th inspection of k-th aircraft $k\in K_{SL}$, $j\in J_k$, $k\in K_f$, where $J_k=\left\{j: t^+_{kj}<T^+_{fk}, j=1,2,...\right\}$ is the set of indexes of inspections of k-th aircraft; $t^+_{kj}=t_k+jd_t, j=1,2,...,\ K_f=\left\{k: k\in K_{SL}, t^+_k<T^+_f\right\}$.

Let $q=(q_1,...,q_N)$ be realization of random vector $Q=(Q_1,...,Q_N)$ and let $a_{jk}=\alpha\exp(q_k d_t j)$ is a size of fatigue crack at j-th inspection of k-th aircraft for which the growth of fatigue crack is defined by specific value of q_k which is the realization of random variable Q_k. Corresponding probability of fatigue crack discovery is equal to $p_{dkj}=p_d(a_{jk})$. For specific value of vector q probability that not any fatigue crack will not be discovered is equal to

$$p_f(q)=\begin{cases}0, & if\ \ \mathrm{K}_f=\varnothing,\\ \prod_{k\in \mathrm{K}_f}\prod_{j\in J_k}(1-p_{dkj}).\end{cases} \tag{5}$$

By modeling random vector Q using Monte Carlo method, we can calculate mean value of this probability: $p(n,\theta)=E_\theta(p_f(q))$. Now we can choose number of the inspection $n(p,\theta)$ in such a way that the failure probability will be equal to p.

4. SOLUTION FOR UNKNOWN θ

First, we consider the problem of a limitation of FFP1 in an operation of one AC if the probability of detection is defined by equation (3), the human factor $w_0=1$. This means that if there is a detectable fatigue crack, then during the inspection after T_d we see it with probability 1 and the limitation of FFP1 of AC is provided by the choice of the specific p-set function, Paramonov *et al* (2011). Let us take into account that the operation of a new type of aircraft will not take place if the result of acceptance fatigue test in a laboratory is "too bad" (previously, the redesign of the new type of AC should be made). We say that in this case the event $\hat\theta\notin\Theta_0$, $\Theta_0\subset\Theta$, takes place (for example, $\hat\theta\notin\Theta_0$ if fatigue life T_C is lower than some limit; or $n(p,\hat\theta)$ is too large,…). Let us define some binary set function

$$S(\hat\theta,\Theta_0,n)=\begin{cases}\bigcup_{i=1}^{n+1}S_i(n)\ \ if\,\hat\theta\in\Theta_0\\ \varnothing\ if\,\hat\theta\notin\Theta_0\end{cases} \tag{6}$$

where $S_i=\{(t_d,t_c): t_{i-1}<t_d, t_c\le t_i\}$, $t_i=it_{SL}/(n+1), i=1,...,n+1$; $\varnothing$ is an empty set.
We call this function *binary p-set function* if

$$\sup_\theta\sum_{i=1}^{n+1}P(Z\in S_i(n)\bigcap\hat\theta\in\Theta_0)=p$$

Here we take into account that if $\hat\theta\notin\Theta_0$ operation of AC is not allowed and corresponding the failure probability is equal to 0. Examples of value of binary p-set functions are shown in Fig.5.

It can be shown that for very wide range of the definition the set Θ_0 and the requirements to limit FFP1 by the value p^*, where $(1-p^*)$ is a required reliability, there is a preliminary "designed" choice of allowed FFP1, p_{fD}, such that corresponding set function $S(\Theta_0, n(p_{fD}, \hat{\theta}))$ is p-set function of the level p^* for the vector $Z=(T_d, T_c)$. The value of p_{fD} is defined by equation

$$\sup_\theta \sum_{i=1}^{n+1} P(Z \in S_i(n(p_{fD,} \hat{\theta}) \bigcap \hat{\theta} \in \Theta_0) = p \tag{7}$$

For this p_{fD} the FFP1 will be limited by the value p^* for any unknown $\theta \in \Theta$.

Now we consider the reliability of the fleet of N AC when there is an information exchange and the operation of all aircraft will be stopped if fatigue crack will be found during an inspection of any AC and, as it was told already, in order to prevent the failure in the fleet, it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place. Let us define some multiple set function:

$$S^+\left(\hat{\theta}, \Theta_0, n\right) = \bigcup_{k \in K_{SL}} S_k^+\left(\hat{\theta}, \Theta_0, n\right) \tag{8a}$$

where

$$S_k^+\left(\hat{\theta}, \Theta_0, n\right) = \begin{cases} \bigcup_{i=1}^{n+1} S_{i,k}(n) if \hat{\theta} \in \Theta_0 \\ \varnothing, if \hat{\theta} \notin \Theta_0 \end{cases} \tag{8b}$$

$S_{ik(n)}^+ = \{(t_{dk}^+, t_{ck}^+) : t_{(i-1)k} < t_{dk}, t_{ck} \le t_{ik}\}$, $t_{ik}^+ = t_k^+ + t_i$, $t_i = it_{SL}/(n+1)$, $i = 1, ..., n+1$, $k = 1, 2, ..., N$. Again, it can be shown that for very wide range of the definition the set Θ_0 and the requirements to limit FFPN by the value p^*, there is a preliminary "designed" choice of allowed FFPN, p_{fD}, such that corresponding multiple set function $S^+(\Theta_0, n(p_{fD}, \hat{\theta}))$ is p-set function of the level p^* for the set of vectors $\{Z_k^+,\ k \in K_{SL}\}$, where $Z_k^+ = (T_{dk}^+, T_{fk}^+)$:

$$v(p_{fD}) = p*, \tag{9a}$$

$$\text{where } v(p) = \sup_\theta v(\theta, p) \tag{9b}$$

$$v(\theta, p) = E\left\{ \sum_{k \in K_{SL}} \sum_{i=1}^{n+1} P\left(Z_k^+ \in S_{ik}^+\left(n\left(p, \hat{\theta}\right)\right) \bigcap \hat{\theta} \in \Theta_0\right) \right\} \tag{9c}$$

That means that FFPN will be limited by the value p^* for any unknown θ.

5 NUMERICAL EXAMPLE

In this numerical example we assume that $t_{SL} = 45000$, $w_0 = 0.95$, processing the result of full scale fatigue test we get the estimate of fatigue crack parameters $\hat{\theta} = -8.5885$, $\alpha = 0.286$ mm, the standard deviation of $\log(Q)$ is equal to 0.346 (see Fig.2.32 in [1]), and let for considered inspection technology the detection probability p_d is defined by (2) with $a_{d0} = 10$ mm, $a_{d1} = 20$ mm, $a_c = 237$ mm. There are 10 aircraft in the fleet, the interval between the aircraft putting into operation $d_t = 1000$; allowed failure probability $p^* = 0.01$, the set Θ_0 is defined by the condition : if $\hat{n} = n(0.01, \theta) > 20$ then the redesign of AC should be made. Using the Monte Carlo calculation we get $\hat{n} = n(0.01, \theta) > 7$, see Fig.6.

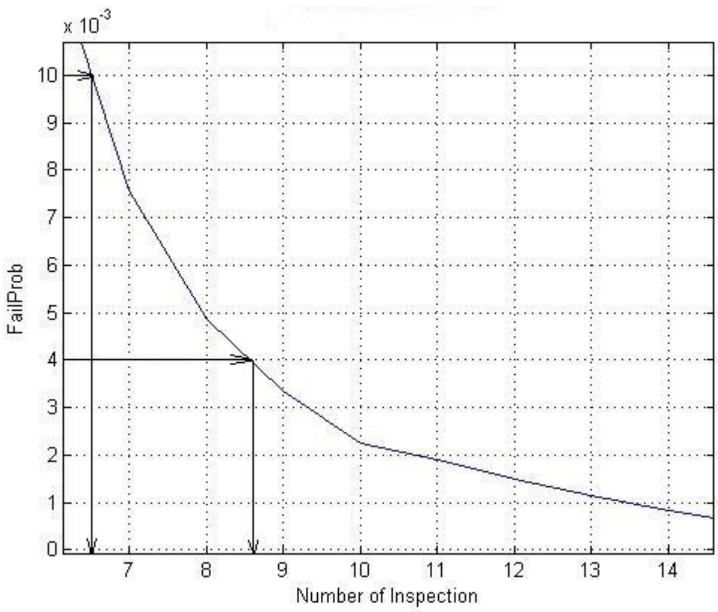

Figure 6. Example of function $p_{fNW}(n,\theta)$ (probability of crack detection) for θ_0=-8.5885

But this calculation is correct only if in the service the same value of $\theta_0 = -8.5885$ takes place. In reality we do not know the θ_0. If θ_0 value is changed, then selected number of inspections to provide required reliability level will be changed as well. This effect could be seen in Fig.7.

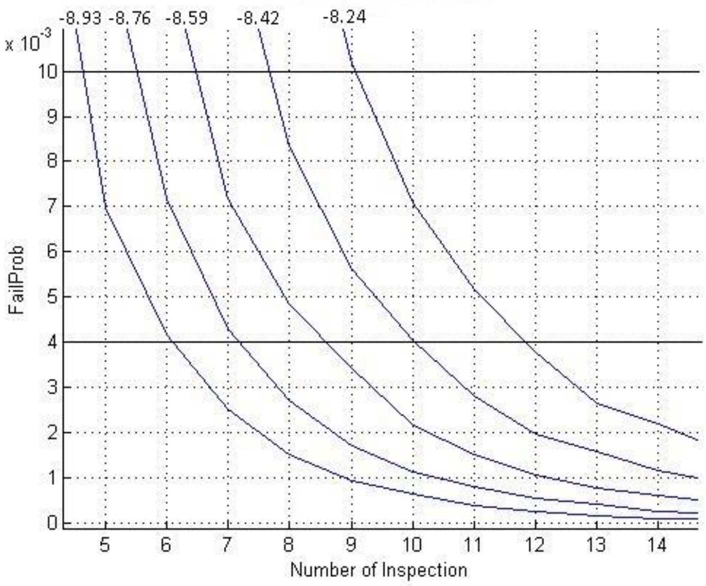

Figure 7. Example of function $p_{fNW}(n,\theta)$ (probability of crack detection) for different θ_0 values

This means that selected inspection program $\hat{n} = n(0.01,\theta) = 7$ is not appropiate for all possible θ_0 values. In service it is possible that some fatigue cracks require higher number of inspections. Number of inspections dependence on θ_0 value for different p^* could be seen on Fig.8.

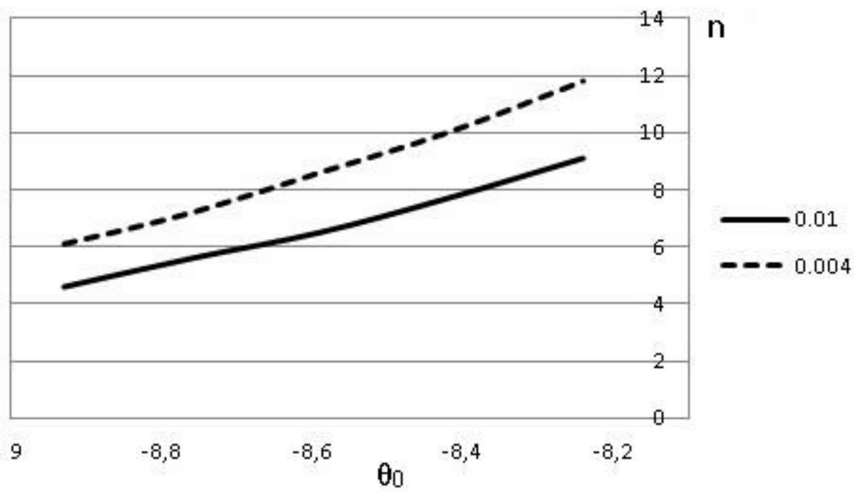

Figure 8. Example of function $n(p^{*},\theta)$ for p*=0.01 and p*=0.004

We should limit the maximal possible failure probability for any θ_0. It can be done by the choice of specific "designed" failure probability, p_{fD}. The family of the functions $v(\theta, p)$ for different p is shown in Fig.9, where the corresponding calculations for parallel axis are made for corresponding "Mean durability" $= C_C / Q$ for $C_C = \log a_c - \log\alpha$, $Q = \exp(\theta_0)$

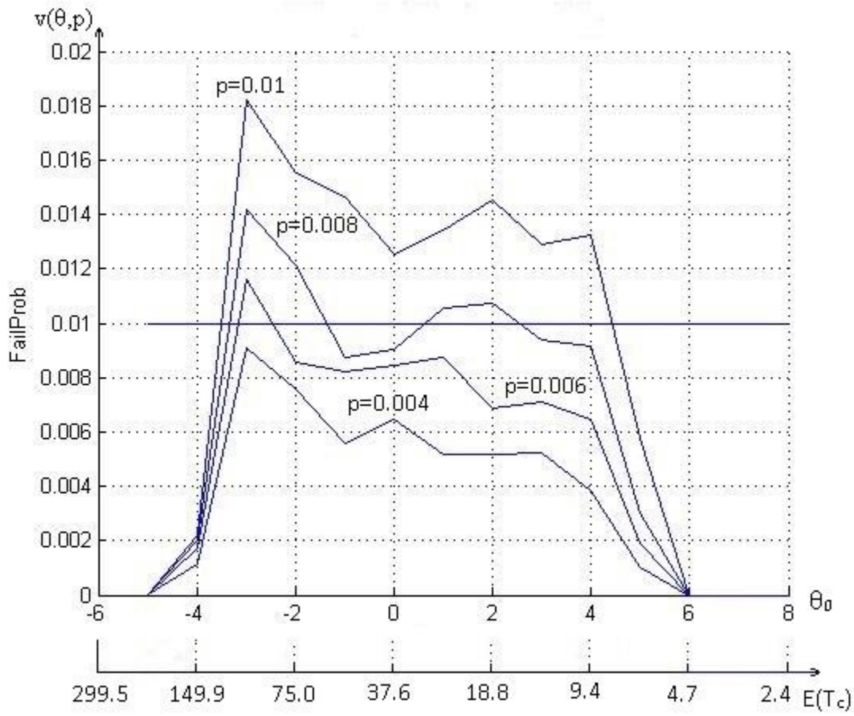

Figure 9. The function v(θ,p) for different p. In parallel axis the flight hours divided by 10^3 of the corresponding "Mean durability" $= C_C / Q$ are given for $C_C = \log a_c - \log\alpha$, $Q = \exp(\theta_0)$

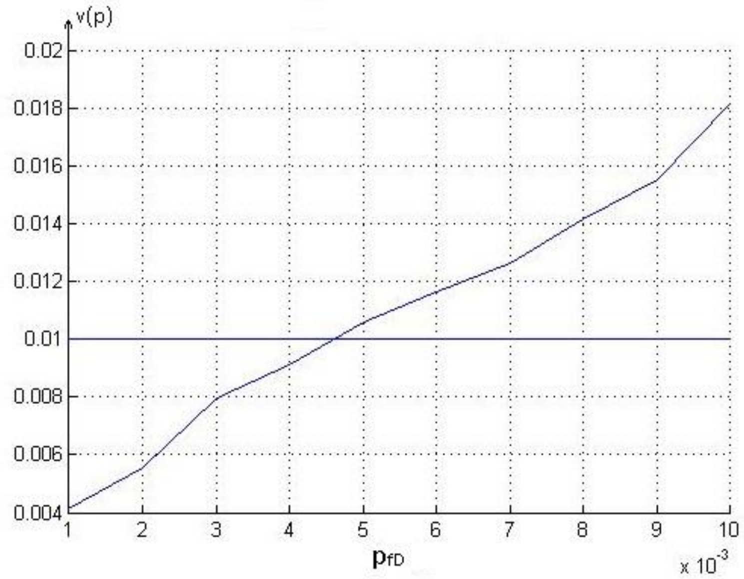

Figure 10. The function v(p)

In Fig.10 the function v(p) is shown for considered example data. We see that in order to limit FFPN by value $p^* = 0.01$ the value $p_{fD} = 0.004$ should be chosen. Now using the function $p_{fNW}(n,\theta)$ which is shown in Fig.7 for the test estimate of fatigue crack parameters $\hat{\theta}_0 = -8.5885$, the number of inspections should be chosen equal to $\hat{n} = n(0.004, \theta) = 9$.

6. CONCLUSIONS

It is found, how, using the estimate of the unknown parameter $\hat{\theta}$ (after the acceptance fatigue test), one of the two decisions should be chosen: 1) to do the redesign of a new type of AC if the result of the test is “too bad” or 2) to make a choice of the number of inspections $n = n(p_{fD}, \hat{\theta})$ as a function of $\hat{\theta}$ and specific p_{fD}.

7. REFERENCES

1. Paramonov Yu., Kuznetsov A., Kleinhofs M. (2011). *Reliabilty of fatigue-prone airframes and composite materials*. Riga: RTU.
2. Hauka M., Paramonov Yu. Inspection program development using minimax method. In: *Proceedings of the Conference ‘Relstat, 2011’* , Reliability and Statistics in Transportation and Communication 19-22 October 2011, pp. 105-114.
3. Paramonov Yu., Tretyakov S. Reliability of fleet of aircraft taking into account the information exchange about the fatigue crack discovery and human factor. *Aviation.* Vol. 16, No 4, 2012, pp. 103-108.