p-BIRNBAUM SAUNDERS DISTRIBUTION: APPLICATIONS TO RELIABILITY AND ELECTRONIC BANKING HABITS

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ABSTRACT

Birnbaum and Saunders (1969) introduced a two-parameter lifetime distribution which has been used quite successfully to model a wide variety of univariate positively skewed data. Diaz-Garcia and Leiva-Sanchez proposed a generalized Birnbaum Saunders distribution by using an elliptically symmetric distribution in place of the normal distribution. In this paper, we construct a new distribution, say p-Birnbaum-Saunders distribution, by introducing a new parameter 'p', which influences both Skewness and Kurtosis. The deviation from the behaviour of Birnbaum and Saunders distribution can be accommodated in the new p-Birnbaum Saunders (p-BS) distribution. Different properties of this distribution are obtained. Most of the data from Reliability and Banking sector is having skewness and their frequency curve is from among the class of p-BS distribution. A data set from internet banking sector is considered.

1 INTRODUCTION

There are many distributions for modeling lifetime data. Among the known parametric models, the most popular are the Birnbaum-Saunders, Gamma, lognormal and the Weibull distributions. Sometimes, these are not appropriate for a given data. The two-parameter Birnbaum-Saunders (BS) distribution was originally proposed by Birnbaum and Saunders [1969] as a failure time distribution for fatigue failure caused under cyclic loading. It considered only the material specimens which are subjected to fluctuating stresses by a periodic loading. Size of stress and crack are random.

Although the BS distribution was originally proposed as a failure time distribution for fatigue failure under the assumption that the failure is due to development and growth of a dominant crack, a more general derivation was provided by Desmond [1985] based on a biological model. Desmond [1985] also strengthened the physical justification for the use of this distribution by relaxing the assumptions made originally by Birnbaum and Saunders [1969]. Some recent work on the BS distribution can be found in Balakrishnan et al. [2007], Chang and Tang [1993,1994], Dupuis and Mills [1998], From and Li [2006], Lemonte et al. [2007], Rieck [1995,1999], Ng et al. [2003,2006], Owen [2006] and Xie and Wei [2007]. A review of different developments on the BS distribution until 1995 can be found in the book by Johnson et al. [1995].

The objective of this work is to study the behavior of Birnbaum Saunders distribution with one more **relevant** parameter. Not to argue its particular merits in applications over other distributions. It is a reasonable generalization. We study some behavior of the Birnbaum-Saunders distribution with one more parameter p, which influence both skewness and kurtosis strongly.

The paper is arranged as follows. Section 2 discussed basic definition of Birnbaum Saunders distribution. Section 3 introduced, a new distribution, p-Birnbaum Saunders distribution and studied some of its properties. Maximum likelihood estimation is given in section 4. Application to reliability analysis and Banking Habits are given at the section 5. Conclusion is given in last section.

2 BIRNBAUM SAUNDERS DISTRIBUTION

We considered the Birnbaum-Saunders data. It shows departure from the normality with s skewness and peaked frequency curve. Distribution function of Birnbaum-Saunder distribution is

$$\boldsymbol{F(t,\alpha,\beta) = N\left(\frac{1}{\alpha}\xi\left(\frac{t}{\beta}\right)\right), t > 0}$$

where N(.) is the Normal distribution evaluated at $\boldsymbol{\xi\left(\frac{t}{\beta}\right)} = [\boldsymbol{(t/\beta)^{1/2} - (t/\beta)^{-1/2}}]$, α>0 and β>0. The parameters α and β are the shape and scale parameters, respectively. Moreover, β is the median of the BS distribution. Consider the histogram of BS data and Normal curve drawn through it.

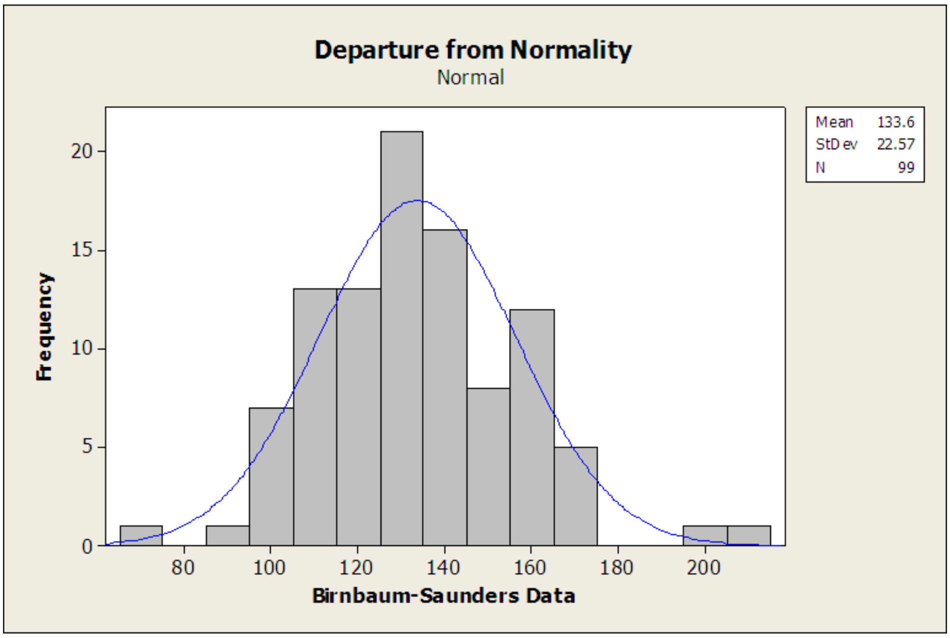

Table 1. Histogram of Birnbaum Saunders Data

The inference on this plot is this: It is a skewed data on positive axis. Mean of the distribution is placed left to the Normal mean. Curve is more peaked than Normal. Left and right tails have slight positive mass. Tail thickness is more than that of Normal tail. Existence of moments can be ensured. Flatness is very low.

From the above inference we can make some reasonable conclusions. The actual distribution will be positively skewed. Peakedness may vary with situation of varying stresses. But asymmetry still remains. Either Birnbaum Saunders, log-Normal, skew-Normal, Weibull etc distributions can be chosen as an approximate model.

But for the correctness in model selection, think about the smooth frequency curve drawn through the histogram. What will be the function, having desirable distributional properties, that suits our objective? Can we generalize Birnbaum Saunders distribution to approach more accuracy in probability calculation? Choosing a suitable frequency curve is an important area of research of distribution theory. A plausible model can be attained by aattaching one more parameter to the Birnbaum Saunders distribution. We observed the behavior with different values of parameters.

3 p-BIRNBAUM SAUNDERS DISTRIBUTION

Use $\xi_p\left(\frac{t}{\beta}\right) = [(t/\beta)^p - (t/\beta)^{-p}]$ instead of $\xi\left(\frac{t}{\beta}\right) = [(t/\beta)^{1/2} - (t/\beta)^{-1/2}]$ in BS distribution.

It gives us more suitable flexible model for the data. We define p-Birnbaum Saunders distribution as follows

$$F(t,\alpha,\beta,p) = N\left(\frac{1}{\alpha}\xi_p\left(\frac{t}{\beta}\right)\right), t > 0$$

Probability density function ***is***

$$f(t,\alpha,\beta,p) = \frac{1}{\alpha\beta p\sqrt{2\pi}}\left[\left(\frac{\beta}{t}\right)^{1-\frac{1}{p}} + \left(\frac{\beta}{t}\right)^{1+\frac{1}{p}}\right] e^{-1/2\alpha^2[\left(\frac{t}{\beta}\right)^{\frac{2}{p}}+\left(\frac{\beta}{t}\right)^{\frac{2}{p}}-2]}, t > 0$$

Here α is the shape parameter and β is the scale parameter. The parameter p governs both shape and scale. For all values of α the PDF is unimodal. Mode cannot be obtained in explicit form, it has to be obtained by solving a non-linear equation in α. Clearly, the median is at β, for all α. Table 2, table 3 and table 4 give the shape of probability density function for various values of parameters.

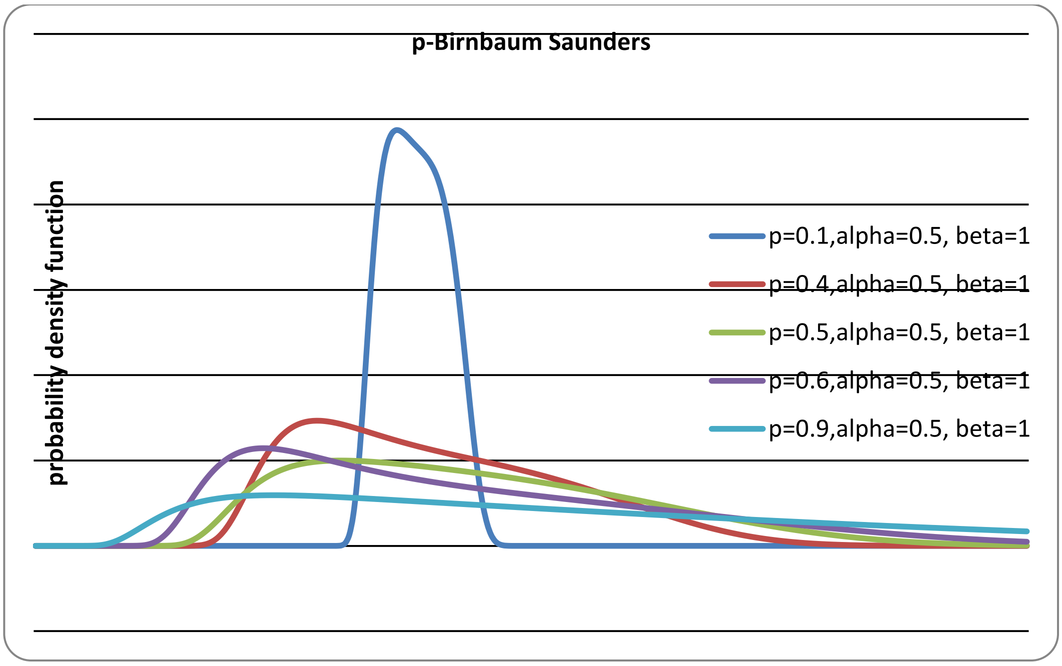

Table 2. Probability density function of p-Birnbaum Distribution, alpha=0.5 and beta=1

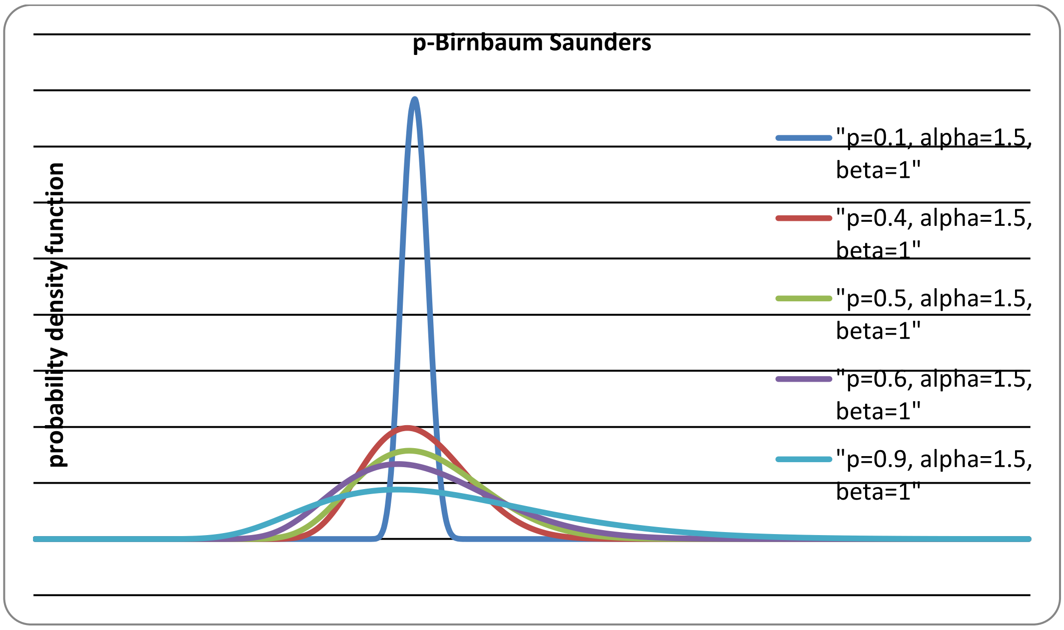

Table 3. Probability density function of p-Birnbaum Distribution, alpha=2 and beta=1

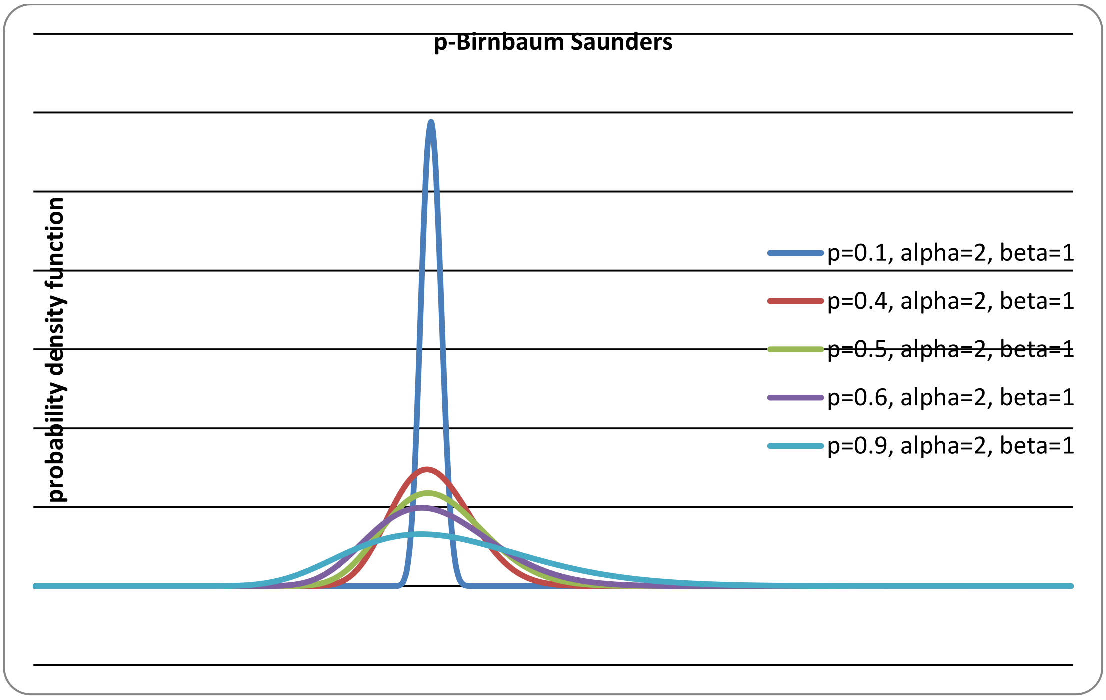

Table 4. Probability density function of p-Birnbaum Distribution, alpha=2 and beta=1.

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All these models have slight skewness and departure from symmetric Normal distribution. They are defined on positive axis. Peakedness changes with p. p-is a parameter which control both flatness and skewness (Shape).

Let $X = 1/2[(t/\beta)^{1/p}-(t/\beta)^{-1/p}]$ then

$$\boldsymbol{X \sim N(0, \frac{\alpha^2}{4})}$$

Using this transformation Expectation, variance, skewness and kurtosis can be obtained.

$$\boldsymbol{E(T) = \beta E\left(X + (X^2 - 1)^{\frac{1}{2}}\right)^p}$$

If p=2, we get
If p=2, we get

$$\boldsymbol{E(T) = \beta(1 + \alpha^2/2)}$$

$$\boldsymbol{V(T) = (\beta\alpha)^2(1 + 5\alpha^2/4)}$$

$$\boldsymbol{\beta_1(T) = (4\alpha)^2 \frac{(11\alpha^2 + 6)}{(5\alpha^2 + 4)^3}}$$

$$\boldsymbol{\beta_2(T) = 3 + 6(\alpha)^2 \frac{(93\alpha^2 + 41)}{(5\alpha^2 + 4)^2}}$$

3.1 properties of p-Birnbaum Saunders distribution

We obtained two important properties of p-Birnbaum Saunders distribution.

Theorem 1: If T has life distribution p-BS, $\boldsymbol{F(t, \alpha, \beta, p) = N\left(\frac{1}{\alpha}\xi_p\left(\frac{t}{\beta}\right)\right), t > 0}$ then its reciprocal 1/T has $\boldsymbol{F(t, \alpha, \beta^{-1}, p)}$ distribution.

Proof: Using transformation, we can obtain the required result.

Theorem 2: If T has life distribution p-BS, $\boldsymbol{F(t, \alpha, \beta, p) = N\left(\frac{1}{\alpha}\xi_p\left(\frac{t}{\beta}\right)\right), t > 0}$ then *kT* has $\boldsymbol{F\big(t, \alpha, k\beta \quad, p\big)}$ distribution.

Proof: Using transformation, we can obtain the required result.

4. MAXIMUM LIKELIHOOD ESTIMATION

The log likelihood function is

$$\boldsymbol{logL = -nlog\alpha - nlog\beta - nlogp + \sum_{i=1}^{n}\{-\frac{1}{2}log(2\pi) - \frac{1}{2}\alpha^{-2}\xi_p\left(\frac{t_i}{\beta}\right) + log\xi'_p(\frac{t_i}{\beta})\}}$$

Then,

$$-\frac{\alpha^3}{n}\frac{\partial logL}{\partial \alpha} = \alpha^2 - \frac{1}{n}\sum_{i=1}^{n} \xi_p^2\left(\frac{t_i}{\beta}\right)$$

$$\frac{\partial logL}{\partial \beta} = -\frac{n}{\beta} + (\beta\alpha)^{-2}\sum_{i=1}^{n} t_i \xi_p\left(\frac{t_i}{\beta}\right)\xi_p{}'\left(\frac{t_i}{\beta}\right) - \frac{1}{\beta^2}\sum_{i=1}^{n}\frac{t_i}{\xi_p(\frac{t_i}{\beta})}\xi_p''\left(\frac{t_i}{\beta}\right)$$

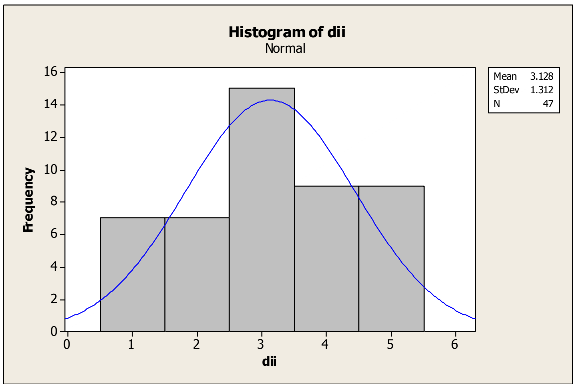
$$\frac{\partial logL}{\partial p} = -\frac{n}{p} + p\alpha^{-2}\sum_{i=1}^{n} \xi_p\left(\frac{t_i}{\beta}\right)\xi_{logt_i}\left(\frac{t_i}{\beta}\right) - \frac{1}{\beta^2}\sum_{i=1}^{n}\frac{1}{\xi(\frac{t_i}{\beta})}\xi_{logt_i}''\left(\frac{t_i}{\beta}\right)$$

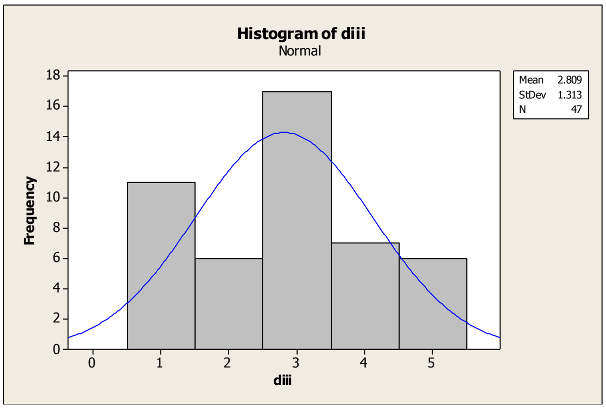
All these three equations are non-linear, we need to use numerical procedure to solve it.

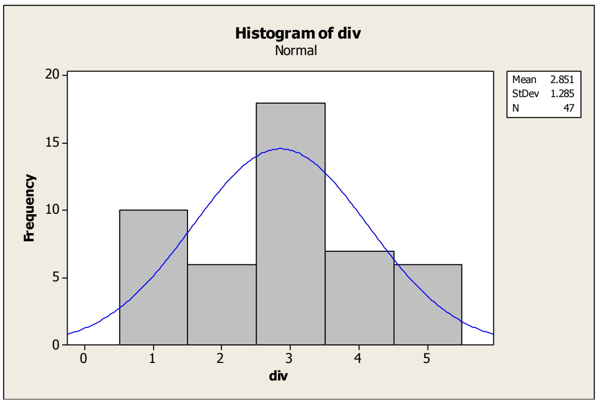
5. APPLICATIONS TO RELIABILITY AND ELECTRONIC BANKING HABITS

The motivation of this work is coming from the fatigue failure data given in Birnbaum and Saunder (1969). Also many real situations in banking sector shows the behaviour of the p-Birnbaum Saunders distribution.

Deepa Paul (2012) conducted a survey of Banking habits in the usage of internet banking, ATM, mobile banking and branch banking. Some variables have the property specified in Theorem 1 and Theorem 2. Variables in five point likert scale are dii, diii, and div, where dii=Technology enabled services are quick to use than visiting the bank branch personally, diii= ATM saves much time, div= ATMs are more accurate than human tellers.







All these histograms shows departure from normality and more suitable for p-Birnbaum Saunders distribution.

6. CONCLUSIONS

This three parameter distribution is more plausible model for the distribution of fatigue failure. Moreover the shape of density curve with various skewness and kurtosis provide a well defined class of life distributions useful in reliability and social sciences. Snedecor's *F* distribution has the property that reciprocal is also *F*, similar property holds for p-Birnbaum Saunders distribution.

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