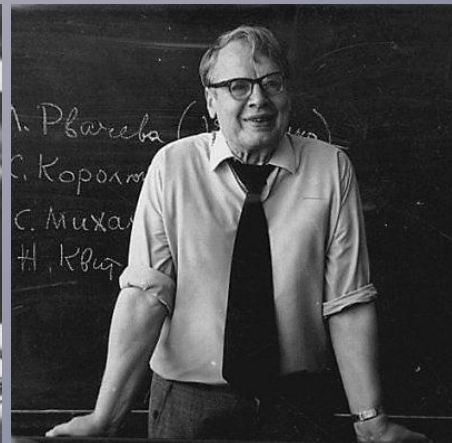


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San Diego

In Memory Of Igor Ushakov

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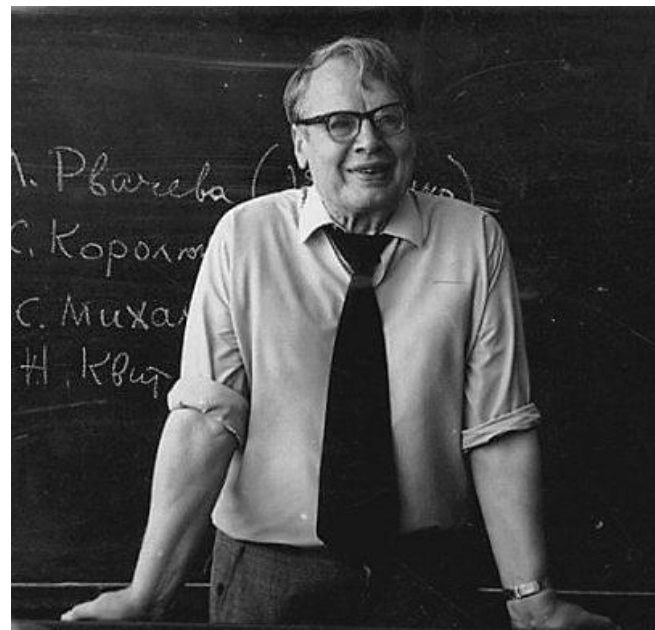
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In Memory Of Igor Ushakov

San Diego
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Theoretical papers have to contain new problems, finger practical applications and should not be overloaded with clumsy formal solutions.

Priority is given to descriptions of case studies.

General requirements for presented papers

1. Papers have to be presented in English in MSWord format. (Times New Roman, 12 pt, 1.5 intervals).
2. The total volume of the paper (with illustrations) can be up to 15 pages.
3. A presented paper has to be spell-checked.
4. For those whose language is not English, we kindly recommend to use professional linguistic proofs before sending a paper to the journal.

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An inspection interval planning is considered in order to limit a probability of any fatigue failure (FFP) in a fleet of N aircraft (AC). A solution of this problem is based on a processing of the result of the acceptance fatigue test of a new type of an aircraft. During this test an estimate of the parameter of a fatigue crack growth trajectory can be obtained. If the result of this acceptance test is too bad then this new type of aircraft will not be used in service. A redesign of this project should be done. If the result of acceptance test is too good then the reliability of the aircraft fleet will be provided without inspections. For this strategy there is a maximum of FFP as a function of an unknown parameter θ . This maximum can be limited by the use of the offered here procedure of the choice of an inspection number.

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Process and Nuclear industries use Independent Protection Layers (IPLs) to prevent initiating abnormal events from becoming accidents. They form layers of protection that acts to prevent an abnormal situation from escalating. IPLs can be hardware (Basic Control System-BPCS) or operator actions, active (Safety Instrumented System-SIS) or passive (Dike walls) or a combination of all these factors. Safety Instrumented System (SIS) is the protection layer that comes in to action in case of failure of BPCS and operator action. Therefore reliability and ability of the SIS to respond should be higher than that of the layer like the BPCS. Reliability of SIS is usually specified in terms of Safety Integrity Level (SIL). The required SIL is calculated by analyzing the Probability of Failure of Demand (PFD) of all the IPLs in the case of an Initiating Event (IE) and comparing the Mitigated Consequence Frequency with a pre-established Tolerable Frequency (TF). The calculations involve probability of failure of each of layers and are usually done through spreadsheet or proprietary software. Bayesian methods are suited to handle these calculations due the nature of conditional probabilities inherent in the system. Further Bayesian methods can analyze the influencing factors affecting the PFD of the IPLs. This paper will present analysis of IPLs and its influencing factors using Bayesian methods including application of Common Cause Failures (CCF) and NoisyAnd distribution to Conditional Probability Tables (CPTs).

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Single Electric Energy System of Russia (SEES) is one of the largest energy systems in the world. It includes 69 regional power systems that, in turn, form seven interconnected power systems (IPS), namely: IPS of the East, IPS of Siberia, IPS of the Urals, IPS of the Middle Volga, IPS of the South, IPS of the Centre, and IPS of the North-West. Electric energy complex of SEES of Russia includes more that 700 power plants whose capacity exceeds 5 MW. As of the end of 2013 the installed capacity of power plants within SEES of Russia totalled 226,470.18 MW. All the IPS are connected by inter-system high-voltage 220-500 kV (and higher) power lines that operate in parallel. Power system of the Crimea is currently referred to technologically independent power systems (PS) of Russia.

V.M.Chacko, Mariya Jeeja P V, Deepa Paul p-BIRNBAUM SAUNDERS DISTRIBUTION: APPLICATIONS TO RELIABILITY AND ELECTRONIC BANKING HABITS	70
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Birnbaum and Saunders (1969) introduced a two-parameter lifetime distribution which has been used quite successfully to model a wide variety of univariate positively skewed data. Diaz-Garcia and Leiva-Sanchez proposed a generalized Birnbaum Saunders distribution by using an elliptically symmetric distribution in place of the normal distribution. In this paper, we construct a new distribution, say p-Birnbaum-Saunders distribution, by introducing a new parameter 'p', which influences both Skewness and Kurtosis. The deviation from the behaviour of Birnbaum and Saunders distribution can be accommodated in the new p-Birnbaum Saunders (p-BS) distribution. Different properties of this distribution are obtained. Most of the data from Reliability and Banking sector is having skewness and their frequency curve is from among the class of p-BS distribution. A data set from internet banking sector is considered.

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Ranging of objects is widely applied at the decision of operational problems. However, it is spent mainly intuitively. There are developed method and algorithm of ranging of objects of a power supply system on independent parameters of reliability and profitability of work with the recommendation of the basic directions of improvement of these parameters.

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The amount of works in the reliability domain is really huge although this field of research has not yet evolved as a science. It is worth reminding that Gnedenko took the first step to create the reliability science. He adopted the deductive logic – typical of exact sciences such as mechanics, electronics and thermodynamics – and demonstrated the general form of the reliability function. However the hazard rate, which tunes up this function, has not been demonstrated so far. We believe that one should follow and complete the Gnedenko’s seminal work and should demonstrate the various trends of the hazard rate using the deductive approach. We have conducted a theoretical research using traditional mathematical methods and have even introduced a new tool named Boltzmann-like entropy. The present paper makes a summary of various contributions published in the past decade and means to show the deductive implications developed.

IN MEMORY OF PROFESSOR IGOR USHAKOV

Mikhail Yastrebenetsky
Gnedenko Forum Vice President

•
Alexander Bochkov
Gnedenko Forum So-founder

On February 28, 2015, our colleague and dear friend Professor Igor Ushakov passed away. Igor was a famous scientist, and one of the key people in the theory of reliability over the past 50 years. He was the Editor-in-Chief of the e-Journal “Reliability: Theory and Application,” as well as founder and president of “Gnedenko e-Forum” – an informal International Association of professionals in the field of reliability.

Igor was a marvelous and uncommon example of the Renaissance man - he was not just a professor, but also a writer, a poet, a painter, and an evangelist of science. It seems that he knew all and could make all in the world.

Igor was born on January 22, 1935 in St. Petersburg, Russia to the family of a military electrical engineer. The most part of Igor life to 1989 was connected with Moscow. Igor graduated from school with a gold medal in 1952 and then from the Moscow Aviation University (Electronics School, Dept of Self-controlled surface-to-air missiles) in 1958. Igor worked as an engineer in Aviation Construction Bureau and participated in military projects and tests of the anti-aircraft warfare missiles.

His first step into the field of reliability took place in 1959 when he began working in the first USSR reliability department. This is when he published his first paper on reliability. (The word “first” was typical throughout Igor’s life!). This was also the beginning of his contacts with group of mathematics at the Moscow State University that was headed by academic Boris Gnedenko.

Igor earned a Candidate of Science in Reliability Engineering (PhD) in 1963, and after 5 years he became a Doctor of Science (Dr.Sc.) of System Analysis. In both cases, B. Gnedenko was his opponent. Igor was one of the youngest Dr.Sc. in USSR. A handbook of reliability evaluation, which Igor prepared together with B. Kozlov, was one of the first in the world - it was issued in 1966. This textbook was translated into different languages and published many times. To this day, it is a desktop reference for reliability specialists.



From 1963 to 1975, Igor served as Chief of Operations Research Dept. of Research and Design Institute. His area of interest included reliability & availability analysis of control systems for anti-aircraft system and control system for intercontinental ballistic missiles. From 1975 to 1989, Igor served as a Chair of Dept of Operations Research at the Computer Center of USSR





Academy of Science.

He was one of the founders and Deputy Chair of Central Consulting Center on Reliability (Moscow). Throughout the same time from 1968, he had been heading departments at such prestigious Russian educational centers as Moscow Energy Institute (MEI) and Moscow Institute of Physics and Technology (MFTI, a Russian analogue to Massachusetts Institute of Technology). Igor created his own scientific schools in all institutions where he worked. Igor was the head of 61 PhD theses of post-graduated students.

Igor's life changed in 1989 when he received a letter from George Washington University (Washington DC, USA) with an invitation to work there as an honorary invited professor. Then, in US he worked at:

- **SOTAS, Inc., Rockville, Maryland as *Chief Scientist*.** Directed modeling of telecommunication networks. Supervised a group of programmers and statisticians.
- **QUALCOMM, Inc., San Diego, California as *Principal Engineer and Senior Consultant*** **Availability analysis of telecommunication systems. Planning of spare stocks for Globalstar base stations.**
- **University of California San Diego as *Full Professor*.** Taught Courses on applied probability and statistics.
- **Hughes Network Systems, Bethesda, Maryland *Senior Consultant*.** Logistics, Inventory control, Maintenance scheduling, Reliability analysis, Quality Assurance problems.
- **ManTech International, Bethesda, Maryland *Senior Consultant*.** Cost-effectiveness analysis of system maintenance, optimization of spare supply for Na.

Areas of Igor's expertise included:

- Operations Research
- Cost-Effectiveness Analysis
- Logistics, Inventory Control
- Survivability & Safety Analysis
- Probabilistic Modeling
- Applied Statistics

Main results of Igor in reliability related to:

- Reliability optimization problems including optimal redundancy;



- Evaluation of systems effectiveness including systems effectiveness computation at design state;
- Highly reliable systems design;
- Analysis of complex network structure;
- Telecommunication reliability;
- Reliability analysis of computer systems and networks;
- Reliability of mechanical equipment;
- Reliability of big power systems.



A little bit more about the last item in the enumeration listed above. Acad. Yu. Rudenko and Siberian Energetic Institute allowed him to work on different tasks related to power system reliability. One of them was problem of survivability of these systems- their ability to resist deliberate impacts. The importance of this problem can scarcely be overestimated. The book by Yu.Rudenko and I. Ushakov “Reliability of Power Systems” received award of Russian Academy of Science in 1994.

He was a member of Editorial Board of Russian journal “Reliability and Quality Control” (since 2000 “Methods of Quality Management”); Executive Secretary of the journal of the Soviet Academy of Sciences “Engineering Cybernetics” (in USA it is published as “Soviet Journal of Computer and System Sciences”); Kybernetes (Great Britain); European Journal on Operational Research (USA); the journal of the Soviet Academy of Sciences “Informatics and its Applications”; International Journal of Performability Engineering; “Reliability” (Russia).



As editor and reviewer, he was always unbiased and held the strongest principles. Regardless of who wrote a book or an article, a famous scientist or even his good friend – he would always judge the work for its quality and merit and show on lacks. He was especially strict with criticizing articles in mathematic juggling that would not have any actionable application.

He was the author of about 30 monographs and over 300 scientific papers published in the prestigious National and International scientific and engineering journals. If Igor had not pursued the field of technology, he could have been a brilliant journalist. In fact, at some point, he was actually debating which one of these fields to pursue. His autobiography: “Notes of an uninteresting person,” is popular among many groups of people regardless of their profession. For specialists in applied mathematics, reliability, operations research – this book is perceived as the history of our areas of knowledge. One thing you just can’t do is agree with the title of the book – “uninteresting person,” as Igor is everything but that.

In 1976, Igor had a great misfortune of being in the Byelorussia and getting exposed to the nuclear cloud that was the result of the incident in the Chernobyl Nuclear Power Plant. He got very sick as a result. There is no doubt that healthcare in US did a lot to extend his life. But...

In his last 11 years, he worked on creating and developing an international project – an online forum where he tried to connect reliability professionals across the globe.

Representatives of the scientific schools of the USSR in Moscow, Leningrad, Kiev, Riga , Irkutsk, Tashkent , Gorky, Minsk , Tbilisi, Yerevan , Vladivostok and other cities after the tragic collapse of the country and due to the rapid development of globalization trends (including in science) in the world, appeared in various parts of the world. Of course, many leading experts continued scientific and personal contacts, but these personal contacts were sporadic and unsystematic character. How did he envision this project to be?



The Gnedenko Forum (the Forum) is to serve as an umbrella organization for fulfilling the professional needs of all individuals who have an interest in the scientific and technical aspects of reliability, risk analysis and safety, theoretical and applied.

The Forum will be virtual and internet - based, unencumbered by obligations to commercial and profit-making entities. It is designed to encompass all technical specialists interested in the topic of reliability, safety and risk analysis irrespective of their physical and organizational co-ordinates.

The Forum will also serve as an unbiased and neutral entity that disseminates scientific information to the press and the public on matters pertaining to the risk and reliability of complex technological systems. It will publish newsletters, technical papers, technical reports, and expository essays for timely dissemination of knowledge and information, but with a strong emphasis on scientific

credibility. It will also serve as a vehicle for disseminating information on scholarships, fellowships, and academic and professional positions in reliability, safety and risk analysis all over the world.



An informal international group of experts on reliability (International Group on Reliability, IGOR) organized in 2005 a worldwide network - Internet Forum website Gnedenko (GNEDENKO e-FORUM, www.gnedenko-forum.org), named in honor of the outstanding Soviet mathematician, specialist probability theory, mathematical statistics, probability and statistical methods, Corresponding Member (1945) and academician (1948) Ukrainian Academy of Sciences Boris Vladimirovich Gnedenko (1912-1995).

Gnedenko Forum - voluntary informal association, the main purpose of which - business discussion platform and establishing professional and personal contacts in the international community of experts on the theory of probability and statistics and their various applications, such as the theory of reliability, quality control, queuing theory, the theory of inventory management, risk analysis, etc.

The basic form of the Forum - the exchange of professional information via the website of the Forum participants and personal contacts. The Forum is a non-profit organization.

Areas of Interest Forum activities include (but are not limited to) the following models and their application :

- system reliability analysis;
- analysis model queues ;
- models to counter terrorism;
- operating efficiency;
- safety;
- survivability (vulnerability) ;
- risk analysis ;
- cost-effectiveness analysis ;
- optimal distribution ;
- maintainability;

- optimum redundancy ;
- optimum inventory management ;
- reliability analysis;
- statistical process control ;
- quality assurance issues
- and other related topics .

The main objectives of Gnedenko Forum:

- establishing professional contacts among experts in the field of reliability theory in the world;
- sharing information on upcoming events in the activities of the Forum ;
- exchange of information on new publications in the field of activities of the Forum ;
- participation in international conferences and meetings;
- scientific publications in the electronic journal of the Forum;
- assistance in organizing the participation of members of the Forum for advice and grants.

The Forum can participate as personally (mostly) any expert in the theory of reliability and risk analysis, both individually and collectively (companies engaged in applied probability theory and statistics projects , the relevant departments of universities, etc.). There are no special requirements for membership except professional affiliation to the area of the Forum.

President of the Forum elected annually and has a casting vote in the decision-making process and the discussions at the Forum. The Vice President is elected annually and, accordingly, has a consultative voice. Under the rules of the Forum Vice President automatically becomes President for the second term of their activities.

Starting from January 2006 Gnedenko Forum began releasing its quarterly electronic magazine "Reliability: Theory and Practical Applications» («Reliability: Theory & Applications»). The journal is registered in the Library of Congress (ISSN 1932-2321). Over 8 years have passed since the publication of the first issue, in 31 issue of the journal published article 421. Articles undergo a compulsory stage of editing and are published in PDF format on the journal's website. The journal publishes articles, reviews, reviews, memories, information and bibliographies on theoretical and applied aspects of reliability and quality control, security, survivability, maintenance and methods of analysis and risk management. Preference is given to the editorial board materials, reflecting the practical application of these methods in the articles of a theoretical nature, must necessarily contain new problems designation practical application and should not



be excessive use of formal calculations.

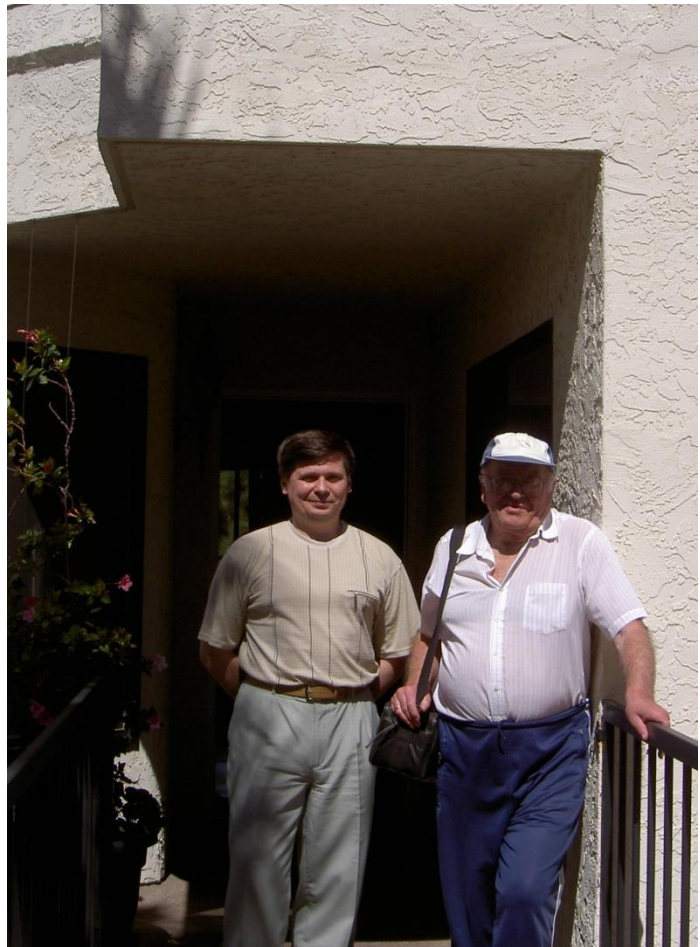
In the editorial board of the journal includes scientists and experts who are recognized experts in their fields of activity and well versed in the essence of the problems discussed in the magazine.

Publication in the magazine is equivalent to publication in scientific journals. Articles recommended by the members of the editorial board for review are routed. The editors reserve the right to change the title of the article, as well as spend editing. The author retains full right to use their materials after publication in the journal of your choice (to send them to other publications, to submit to conferences, etc.)

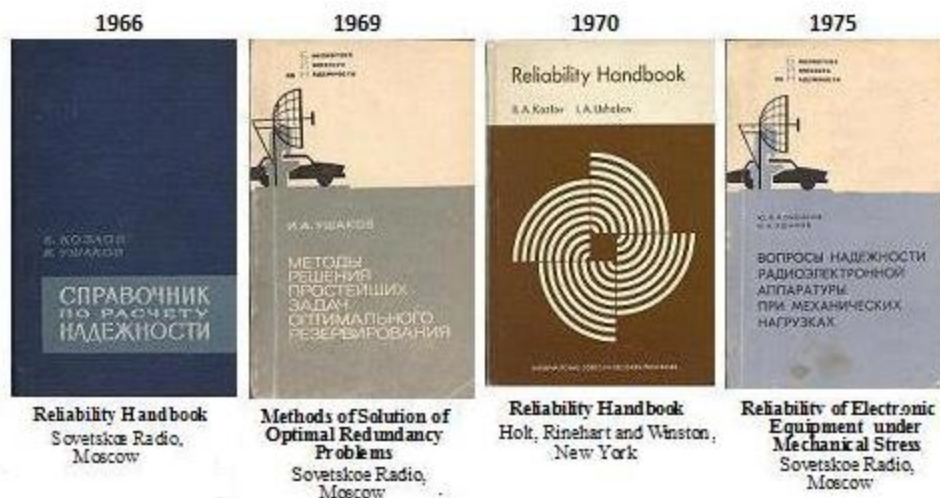
Forum regularly informs its members about upcoming conferences, seminars, when new monographs in the field of reliability theory and risk analysis.

Currently Forum brings together 347 participants from 47 countries. This is how this form exists today. I think that the best memory about Igor would be this forum.

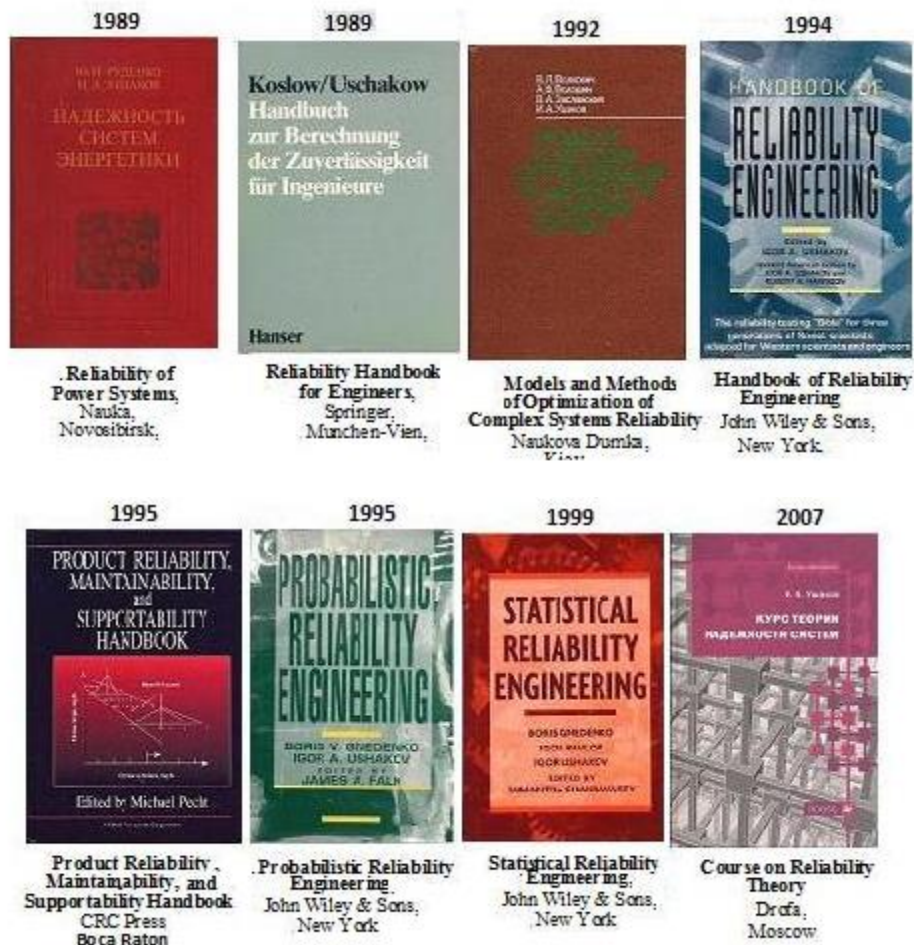
Igor was the author of about 30 monographs and over 300 scientific papers published in prestigious National and International scientific and engineering journals. He also published eight books of prose, lyrics and poems (in Russian).



Dr. Ushakov's List of Publications







Books Edited (in English)

1. **Theory of Probability** by *B.V. Gnedenko*. Gordon and Breach Science Publishers, **1997**.

Books Edited (in Russian)

18. **Reliability and Maintenance** by *F. Beihelt and P. Franken*. Radio i Svyaz, Moscow, **1988**
17. **Network Flow Programming** by *P. Jensen and J.W. Barnes*. Radio i Svyaz, Moscow, **1984**
16. **Statistical Methods of Reliability Evaluation of Complex Systems by Test Results** by *I. Pavlov*. Radio i Svyaz, Moscow, **1981**
15. **Problems of Optimal Failure Diagnosis in Electronic Equipment** by *G. Pashkovsky*. Radio i Svyaz, Moscow, **1981**
14. **Reliability in Engineering Design** by *K. Kapur and L. Lamberson*. Mir, Moscow, **1980**
13. **Fundamentals of Reliability Theory of Complex Systems** by *A. Raikin*. Sovetskoe Radio, Moscow, **1978**
12. **Reliability Evaluation for Systems with Time Redundancy** by *P. Kredentser*. Naukova Dumka, Kiev, **1978**
11. **Inventory control with random requests** by *G. Rubalsky*. Sovetskoe Radio, Moscow, **1977**

10. **Mathematical Models of Arms Control and Disarmament** by *T. Saaty*. Sovetskoe Radio, Moscow, **1977**
9. **Design of Reliable Electronic Circuits** by *P. Becker and F. Jensen*. Sovetskoe Radio, Moscow, **1977**
8. **Statistical Theory of Reliability and Life Testing** by *R.Barlow and F.Proshan*. Nauka, Moscow, **1975**
7. **English-Russian Dictionary on Reliability and Quality Assurance**. Moscow, **1975**
6. **Optimization in Integers and Related Extremal Problems** by *T. Saaty*. Mir, Moscow, **1973**
5. **Fundamentals in Operations Research** by *R. Ackoff and M. Sasieni*. Mir, Moscow, **1973**
4. **A concept of Corporate Planning** by *R. Ackoff*, Sovetskoe Radio, Moscow, **1972**
3. **On Purposeful Systems** by *R. Ackoff and F. Emery*. Sovetskoe Radio, Moscow, **1972**
2. **Mathematical Theory of Reliability** by *R.Barlow and F.Proshan*. Sovetskoe Radio, Moscow, **1969**
1. **Operations Research** by *P. Rivet and R.Ackoff*, Mir, Moscow, **1966**.

Brochures

15. **Methods of Research in Telecommunications Reliability** (An Overview of Research in the Former Soviet Union), RTA, Springfield, Virginia, **1994**
14. **Reliability Analysis of Computer Systems and Networks** (*Russian*). Mashinostroenie, Moscow, **1989**
13. **Reliability Evaluation of Repairable Systems**, with Ya.G. Genis (*Russian*). Znanie, Moscow, **1986**
12. **Analysis of Complex Network Structures**, with E.I. Litvak (*Russian*). Znanie, Moscow, **1985**
11. **Evaluation of System Effectiveness** (*Russian*). Znanie, Moscow, **1985**
10. **System Effectiveness Computation at Design Stage** (*Russian*). Znanie, Moscow, **1983**
9. **Problems of Reliability Prediction** (*Russian*). Znanie, Moscow, **1981**
8. **Optimal Redundancy Problems** (*Russian*). Znanie, Moscow, **1979**
7. **Reliability of Complex Systems**, with V.A. Gadasin (*Russian*). Znanie, Moscow, **1978**
6. **Analysis of System Performance** (*Russian*). Znanie, Moscow, **1976**
5. **Highly Reliable System Design** (*Russian*). Znanie, Moscow, **1976**
4. **Reliability of Mechanical Equipment**, with Yu.K. Konyonkov (*Russian*). Znanie, Moscow, **1974**
3. **Reliability Estimation by Test Data**, with F.I. Fishbein (*Russian*). Znanie, Moscow, **1973**
2. **Reliability Optimization Problems** (*Russian*). Znanie, Moscow, **1971**
1. **Engineering Methods of Reliability Analysis** (*Russian*). Znanie, Moscow, **1970**

Igor was very ill, but any illness can't force him to leave off work. This is the list of his articles for last 15 years:

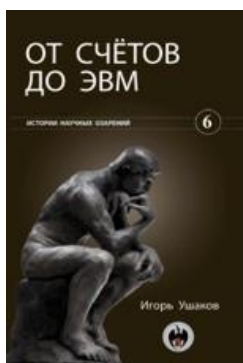
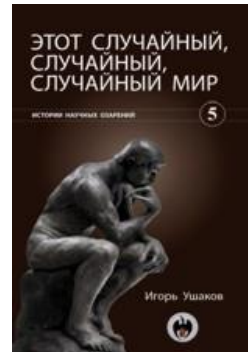
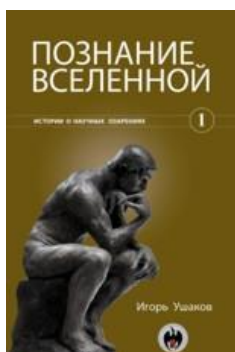
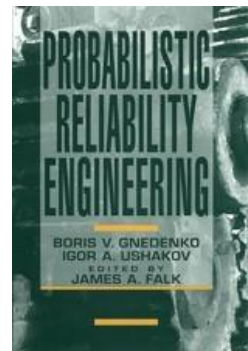
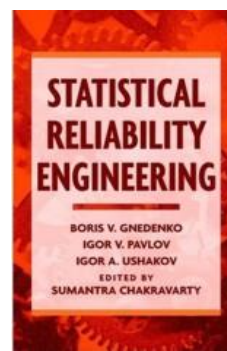
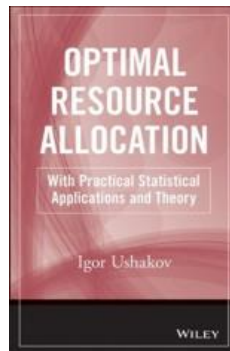
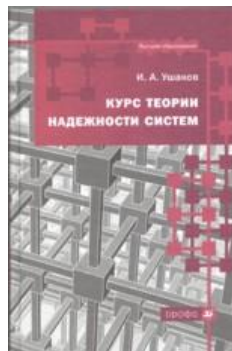
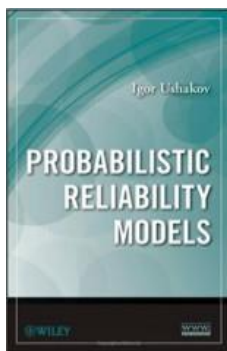
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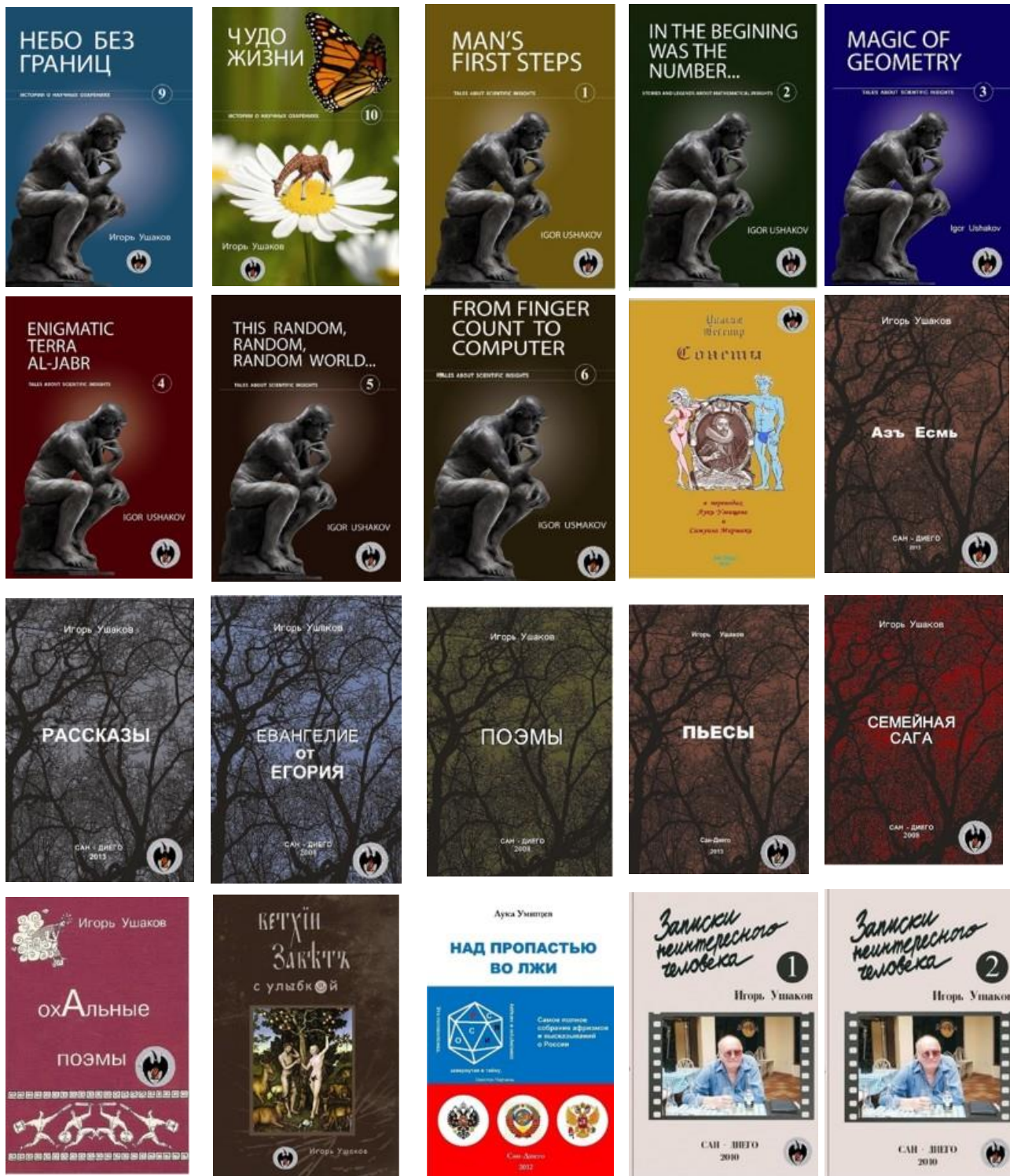
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A width of his views and ability to work were amazing. Besides scientific books, his tireless nature found expressions in fiction, poems, humor stories.

The set of books «History of scientific inspirations» is unique event in modern popular- science literature. History of genius inspiration and invention is described so interesting that these books became as fascinating textbook for young people and friend for adult readers. Rather it isn't a textbook. It is a collection of stories about mathematical, scientist and engineering inspiration and about creators of new ideas in different areas of human activity. Read these books, present one to you children and grandchildren!





We understand what a serious loss it is to us and our Forum, caused by the passing away of Igor Ushakov. He was a remarkable person and great scientist and will always be remembered by all friends and acquaintances.

That he built up such a Forum from scratch speaks of the great talents and business acumen he had. He was the father of our Forum and the fountain-head of all progressive ideas. He was a source of strength and inspiration to many other scientists and researchers. Some of his pioneering work will go a long way in benefiting the future generations.



The gap left behind by the deceased is difficult to be filled. Please accept our sincerest sympathies on this sudden shock. We are pray for peace for the soul. We are express our heart-felt condolences on this sad occasion. May the departed soul rest in peace and be a driving force to all of us...

HUMAN LIFESPAN DISTRIBUTION WITH MAXIMUM LIFESPAN PARAMETER

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ABSTRACT

The problem of existence of maximum human lifespan is discussed. Assuming that the maximum human lifespan exists, a new lifespan distribution is suggested. In opposite to the popular lifespan distributions (Gompertz, Weibull, Extended Weibull, etc.) supported on the semi-infinite interval $[0, \infty)$, the suggested lifespan distribution is supported on the finite interval $[0, a)$, in which a is the non-random maximum lifespan. The suggested lifespan distribution was applied to three death rate datasets (Australia, France, and Switzerland) from the Human Mortality Database, for which the parameters of the suggested lifespan distribution were estimated. The fitted death rates have the high proportion of variance explained by the models ($R^2 \geq 0.96$), and the estimated maximum lifespan is about 200 years. A more adequate lifespan distribution might be a distribution having two competing risks – the risk of death from diseases, and the risk of death from “pure” aging.

INTRODUCTION

The existence of maximum human lifespan is of a great interest and debate to scientists as well as lay audience. This question also belongs to the wider problem of human ageing, which is investigated using multidisciplinary approaches, e.g., integrating population genetics methods with the principles of epidemiological and demographic investigation (Tan et al. 2004; Hayflick 2000).

An important methodological tool for ageing studies is the mortality data analysis (Tan et al. 2004). Note that any data analysis is based on the respective probabilistic model, which, in the case considered, is the lifespan distribution. Various distributions were suggested as the human lifespan models such as Gompertz, Weibull and Extended Weibull distributions (Tan et al. 2004; Weon and Je 2009).

It should be noted that all these distributions are supported on the semi-infinite interval $[0, \infty)^1$, which means that the lifespan (age at death) can take on any value from interval $[0, \infty)$. Using the distributions with infinite or semi-infinite support obviously makes it difficult to suggest a simple and non-random definition of the maximum lifespan. The common definition, which states that the maximum lifespan is “the maximum observed lifespan of a species” (Tan et al. 2004; Hayflick 2000), just reduces it to a realization of an unknown random variable. At most, it can be considered as a common sense point estimate of the maximum lifespan.

According to the suggested below definition, the maximum human lifespan is a non-random distribution characteristics, like the mean, median, mode, etc.

¹ In the case of normal distribution also used in the ageing studies, the interval is infinite, i.e. $(-\infty, +\infty)$ [4]

In the framework of lifespan distribution models approach, the question we pose is -- does the human lifespan, as a random variable, have a non-random limit? If it does, the respective lifespan distribution should have a finite positive support.

Below, we introduce a new lifespan distribution supported on the finite interval $[0, a)$, $a > 0$, in which a is the non-random maximum lifespan, and it is also one of the distribution parameters. Then, we consider some case studies demonstrating how the suggested distribution fits the mortality data for three countries having high life expectancy.

LIFESPAN DISTRIBUTION

The suggested distribution is based on the assumption that the human lifespan is a random variable having a nonrandom upper limit. The distribution is introduced through its death rate (mortality rate, hazard rate, of failure rate), which is given by

$$h(t) = \frac{b}{(a-t)^\beta}, \quad (1)$$

where t is the lifespan ($0 \leq t < a$), parameter $a > 0$ is the maximum lifespan, and b and β are the other distribution parameters (one can call β the *shape parameter*).

Using the death rate (1), the main probabilistic functions related to our distributions can be obtained. The respective cumulative death rate is

$$\begin{aligned} H(t) &= b \int_0^t \frac{1}{(a-\tau)^\beta} d\tau \\ &= \frac{b}{(\beta-1)(a-t)^{\beta-1}} \end{aligned} \quad (2)$$

Using (2), the cumulative distribution function of lifespan can be written as

$$F(t) = 1 - \exp\left[-\frac{b}{(\beta-1)(a-t)^{\beta-1}}\right], \quad (3)$$

and the survival function as

$$S(t) = \exp\left[-\frac{b}{(\beta-1)(a-t)^{\beta-1}}\right] \quad (4)$$

The respective lifespan probability density function (pdf) is

$$f(t) = \frac{b}{(a-t)^\beta} \exp\left[-\frac{b}{(\beta-1)(a-t)^{\beta-1}}\right] \quad (5)$$

Using Equations (4) and (5), the quantile of level p , or the 100 p th percentile ($0 < p < 1$), can be written as

$$t_p = a - \left[\frac{1-\beta}{b} \ln(1-p)\right]^{\frac{1}{1-\beta}} \quad (6)$$

Using the pdf (5), one can find the respective mode as

$$t_{mode} = a - \left(\frac{\beta}{b}\right)^{\frac{1}{1-\beta}} \tag{7}$$

The mean, variance and other moments can't be found in closed forms. For example, the distribution mean can be written as

$$E(t) = \frac{a}{1-\beta} \left[\frac{b}{\beta-1} a^{\beta-1}\right]^{-\frac{1}{\beta-1}} \Gamma\left(\frac{1}{\beta-1}, \frac{\beta a^{\beta-1}}{\beta-1}\right) \tag{8}$$

or

$$E(t) = \frac{1}{1-\beta} \left[\frac{b}{\beta-1}\right]^{-\frac{1}{\beta-1}} \Gamma\left(\frac{1}{\beta-1}, \frac{\beta a^{\beta-1}}{\beta-1}\right)$$

where $\Gamma(a, b)$ is the incomplete gamma function, which is not easy to calculate. However, numerical evaluation of the distribution moments, based on their definitions is not difficult.

CASE STUDIES

The suggested lifespan distribution was applied to three datasets from the Human Mortality Database (HMD) that “was created to provide detailed mortality and population data to researchers, students, journalists, policy analysts, and others interested in the history of human longevity. The project began as an outgrowth of earlier projects in the Department of Demography at the University of California, Berkeley, USA, and at the Max Planck Institute for Demographic Research in Rostock, Germany “(Human Mortality Database 2014).

The data sets are the total (male and female) death rates. The three sets of data, which were analyzed, are Australia 2009, France 2012, and Switzerland 2011. All these countries have the high life expectancy (83.0, 82.3, and 82.8 respectively (Overall Life Expectancy 2012). The parameters of the death rate model (1) were estimated using the non-linear least squares (NLLSQ) regression procedure. The estimates of the parameters are displayed in Table 1, and the fitted death rates are shown in Figures 1, 2, and 3 below.

Table 1. Estimates of distribution (1) parameters

Country, Year	Estimates of distribution parameters			Proportion of variance of $h(t)$ explained by model, R^2
	a	$\ln(b)$	β	
Australia, 2009	186.3	47.44	10.86	0.98
France, 2012	228.8	69.77	14.58	0.96
Switzerland, 2011	204.8	62.01	13.51	0.96

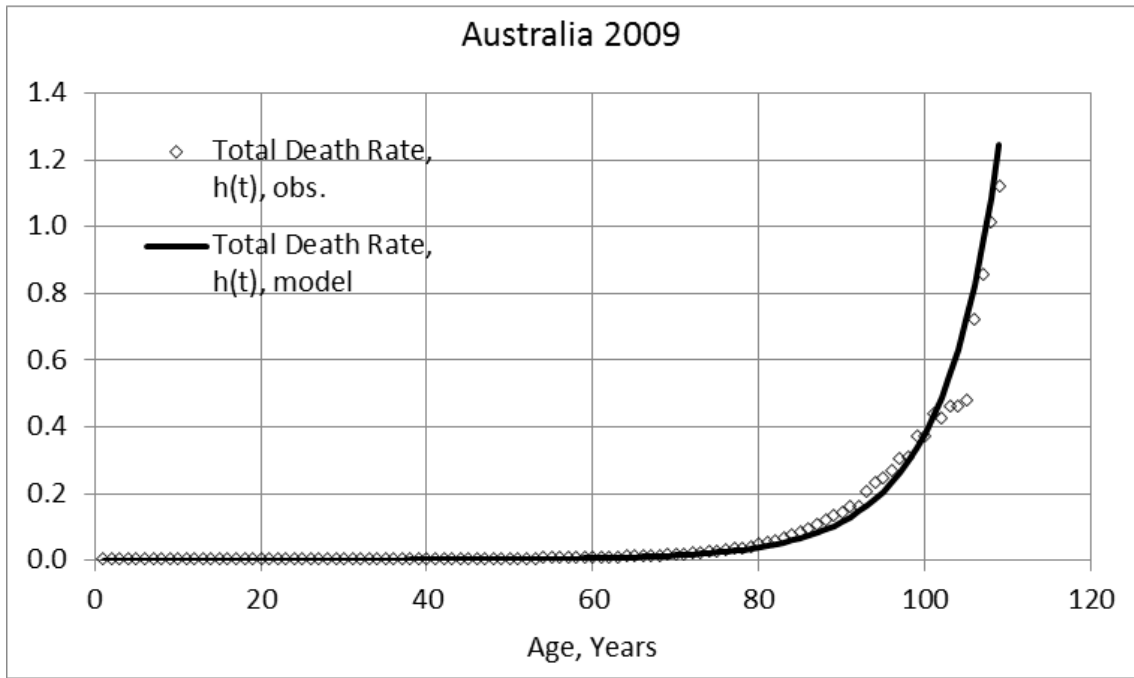


Figure 1. Total (male and female) death rates, Australia, 2009

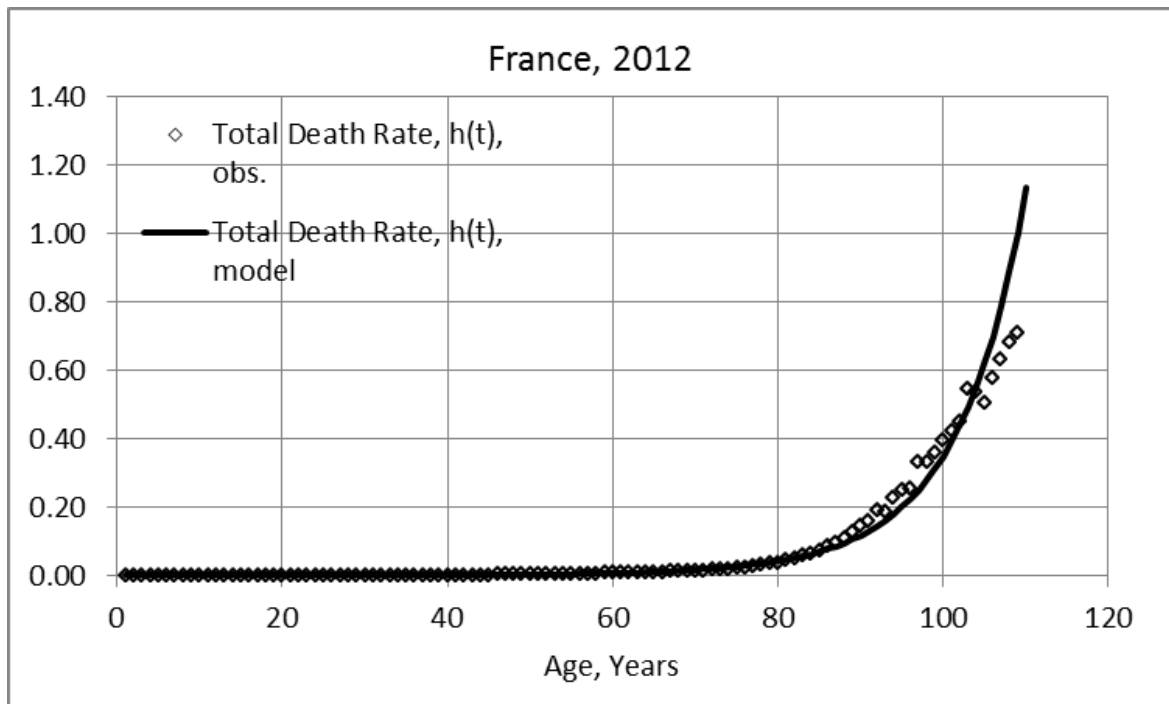


Figure 2. Total (male and female) death rates, France, 2012

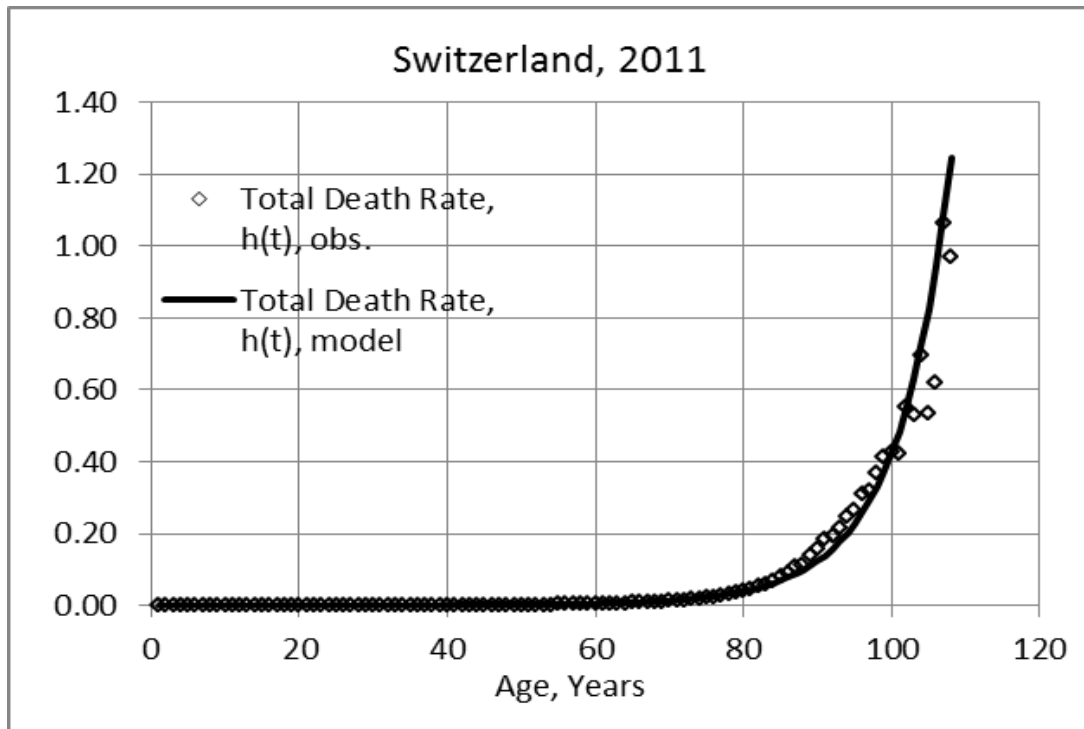


Figure 3. Total (male and female) death rates, Switzerland, 2011

Table 2. The two-sided 95% confidence limits on the model (1) parameters for the Australia 2009 data set

	Estimate	Lower Confidence Limit	Upper Confidence Limit
$\ln(b)$	47.4434	38.2577	56.6292
β	10.8602	9.2662	12.454
a	186.3213	169.073	203.57

Table 3. Australia 2009. Correlation matrix of estimates

	b	β	a
b	1.000000	0.999853	0.996621
β	0.999853	1.000000	0.995114
a	0.996621	0.995114	1.000000

RESULTS AND DISCUSSION

With all the high proportions of variance explained by the model (R^2) for the data sets considered, the model fit could not be called perfect. The confidence intervals on the lifespan distribution parameters are rather wide, as revealed by Table 2, and the estimates of the parameters are strongly correlated as illustrated by Table 3. This is, to an extent, a typical situation, when the NLLSQ method is applied. On the other hand, one can notice that for all three data sets, the data points are above the fitted curve in the age interval of 80 – 100 years, and they are under the fitted model for the ages older 100 years. This model misfit for the older ages might be indication that a better lifespan distribution should be a competing risk distribution (Kaminskiy 2012). According to L. Hayflick (2000), “More than 75% of all human deaths in developed countries now occur in those

over the age of 75. If the causes of these deaths are resolved we will not become immortal but we will have revealed how death occurs in the absence of disease.”

Based on this, we can assume that more adequate lifespan distribution will have two competing risks – the risk of death from diseases, and the risk of death from “pure” aging as “inexorable loss of physiological capacity in the cells of vital organs. . .” (Hayflick 2000).

ACKNOWLEDGMENT

The author is most grateful to the Human Mortality Database (<http://www.mortality.org>) allowing free access to the mortality data.

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MATHEMATICAL MODELS OF ECOLOGICALLY HAZARDOUS RAIL TRAFFIC ACCIDENTS

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ABSTRACT

The paper discusses the queuing system mathematical models simulating the development of processes of rail traffic accidents with hazardous materials as well as elimination of such accidents ecologically dangerous consequences. A new theoretical approach is proposed that enables a more rational dislocation of emergency detachments on railroad network and more successful actions by such detachments aimed at minimization of environmental damage and cargo losses.

1 INTRODUCTION

Research of ecologically hazardous rail traffic accidents with dangerous goods indicates that the accidents are of complex nature, having as possible final results the emergence of serious consequences such as explosions, fires, destruction of rolling stock and facilities, injuries or deaths, or environmental pollution.

Consideration of typical hazardous emergencies shows the possible scenarios of development [1, 2]:

- Slow accumulation of negative factors of ecologically dangerous situation, but not to the level of critical values, which in turn does not cause an explosion or fire;
- Slow accumulation of negative factors of ecologically dangerous situation beyond their critical values, followed by an explosion or fire;
- The rapid accumulation of negative factors of ecologically dangerous situation beyond their critical limits that is associated with an explosion or fire.

The analysis necessary action on the localization of these effects showed that measures must be made, within a wide range - from manual labor to use of complicated mechanisms, relevant emergency teams of railway companies and other ministries and agencies involved in carrying out localization action.

Therefore, analysis of the situation that has developed as a result of the emergence of dangerous goods transportation event, making a timely and adequate decision by task force leader as an emergency response, especially in lack of time, due to the need of sooner of traffic restoration and increase over time of environmental and other damage is a complex process that requires the use of advanced information technologies, including decision support systems (DSS).

Problems of automation of control localization hazardous consequences of emergencies with dangerous goods are dealt with in a number of works that discuss various aspects of automation for relevant team leader control [3, 4].

An important aspect of this task force leader is to determine the required number of capabilities for complex operations, develop an action plan and evaluate its effectiveness.

One of the problems of developing an action plan is the problem of concentration of emergency detachments based on their places of permanent deployment, terrain, availability of water sources, the development of rail and road infrastructure, meteorological conditions, the nature of changes in the emergency hazardous parameters, as well as general and hazardous properties of dangerous goods and so on.

This problem is compounded if it is accompanied by emergency of dangerous goods fire or the hazardous effects localization process must be preceded by localization of fire.

Naturally, the success of localization work is gravely affected by the level of staff training and serviceability of means.

Processes of railway traffic accidents occurrence and their development are of complex probabilistic nature. To determine the nature of interdependencies between the flow of emergency hazardous factors that occurred as a result of traffic accident, and performance of capabilities needed for accident recovery, taking into account the period of time required for their focus, in our view, it is advisable to employ methods of queuing theory, which is one of the most developed branches of probability theory.

2. MODELING EMERGENCY UNITS ACTION

Consider an object where localization works are made by accident recovery teams as a queuing system "emergency object - recovery units."

The exponential flow of customers (being emergency hazardous factors) arrives to n -channel queuing system (where n is the number of localization units) with intensity λ . The service time is exponential with parameter μ . The process of service has a feature that before it starts servicing, the device should prepare for service. The server preparation time T_{cf} has exponential distribution with parameter ν . The customer that catches service device free comes to be served. The catches all servers occupied is waiting for service in the queue.

So random variable T_{ar} consists of two phases – preparation and service:

$$T_{ar} = T_{cf} + T_{ew}. \quad (1)$$

Thus, a random variable T_{cf} is distributed according to generalized Erlang law of order 2 with parameters ν and μ . The probability density distribution of the law is described by the formula [5]:

$$g(t) = \int_0^t \nu e^{-\nu t_1} \mu e^{-\mu(t-t_1)} dt_1 = \frac{\nu \mu (e^{-\nu t} - e^{-\mu t})}{\mu - \nu}, (t > 0), \quad (2)$$

where: $\nu = \frac{1}{M[T_{cf}]}$, $f_1(t) = \nu e^{-\nu t}$; $\mu = \frac{1}{M[T_{ew}]}$, $f_2(t) = \mu e^{-\mu t}$.

The arrival pattern in that QS is not Poissonian, that system is not Markovian, so it is not possible to find probabilities for QS states using method for Markovian processes with discrete states and continuous time.

We know that abuse of Poisson distribution of probabilities in any queuing system shifts it from Markovian system to non-Markovian one and, as emphasized above, the direct output and use of Kolmogorov equations is impossible. Therefore, to analyze the QS the most common two areas of analytical methods for non-Markovian systems are [5 - 8]:

- The area based on the use of conventional theory of Markov chains, but the system studied needs its phase space of possible states to be expanded (pseudo-states method) [5 - 8];
- The area that involves the use of more sophisticated mathematical tools, but without increasing the number of system states (semi-Markov process method) [6, 7, 8];

Both of these areas have much in common, but differ in their capabilities and degree of difficulty for calculation.

Using a semi-Markov process involves examining the behavior of the system only in the change of state (in moments of jumps of the process), resulting in the formation of a Markov chain. In this case, no aftereffect do not come any moment of time, as is the case in a Markov process, but only in jump moments. The effectiveness of this method depends on the ways of setting semi-Markov process. In any case, must be known finite set of possible states of system under study that is linked in Markov chain, as well as directions of possible transition from one state to another and system's original state.

Pseudo-states method is only used when an arrival flow and service intensity have the Poisson probability density distribution function, which is a composition of exponential distributions with the same parameter. It allows you to simplify analysis, from a mathematical point of view, using normal notation of Kolmogorov equations to analyze a queuing system both in sustainable and in transient modes of operation. But this method complicates the structure of the original graph states, leading to computational complexity [5, 6].

The artificial extension of the phase space of states of non-Markov (Erlang) system by introducing into it additional (false) states that shifts it to Markov system, allowing consider the original non-Markov process as nested inside another, more complex process possessing Markovian properties [5, 6].

Consider a queuing system functioning as an object within which localization works are made, provided that there are restrictions on the length of the queue and environmentally dangerous emergency factors have negative impact on recipients that are within limited danger zones. Graph of states for such a QS is presented in Figure 1.

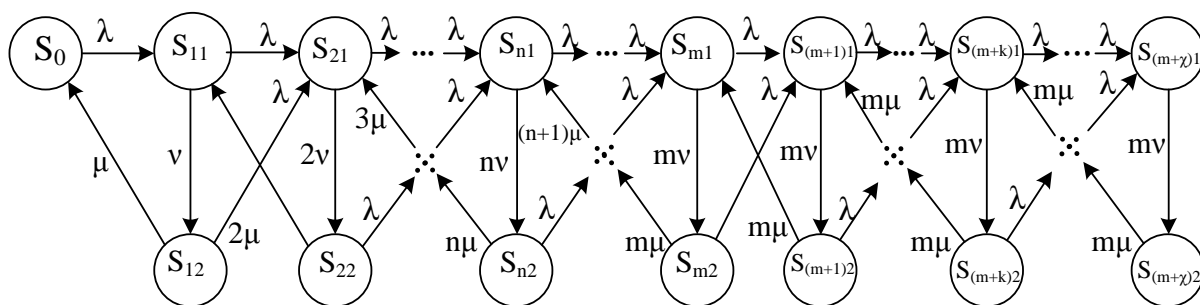


Figure 1 Graph of QS states with limited length of the queue

QS states:

S_0 – QS is free;

S_{11} – one customer in QS, service in the first phase;

S_{12} – one customer in QS, service in the second phase;

.....

S_{m1} – m customers in QS, service in the first phase, no queue;

S_{m2} – m customers in QS, service in the second phase, no queue;

.....

$S_{(m+\chi)1} - (m+\chi)$ customers in QS (m in service, χ in queue), service in the first phase;
 $S_{(m+\chi)2} - (m+\chi)$ customers in QS (m in service, χ in queue), service in the second phase.
 Algebraic equations for the final probabilities of system states are the following:

$$\begin{aligned} \lambda P_0 &= \mu P_{12}; \\ (\lambda + \nu)P_{11} &= \lambda P_0 + 2\mu R_{22}; \\ (\lambda + \mu)P_{12} &= \nu P_{11}; \\ (\lambda + 2\nu)P_{21} &= \lambda(P_{11} + P_{12}) + 3\mu P_{32}; \\ (\lambda + 2\mu)P_{22} &= \nu P_{21}; \\ &\dots\dots\dots \\ (\lambda + m\nu)P_{m1} &= \lambda(P_{(m-1)1} + P_{(m-1)2}) + m\mu P_{(m+1)2}; \\ (\lambda + m\mu)P_{m2} &= m\nu P_{m1}; \\ &\dots\dots\dots \\ (\lambda + m\nu)P_{k1} &= \lambda(P_{(k-1)1} + P_{(k-1)2}) + m\mu P_{(k+1)2}; \\ (\lambda + m\mu)P_{k2} &= m\nu P_{k1}; \\ &\dots\dots\dots \\ m\nu P_{(m+\chi)1} &= \lambda(P_{(m+\chi-1)1} + P_{(m+\chi-1)2}); \\ m\mu P_{(m+\chi)2} &= m\nu P_{(m+\chi)1}; \end{aligned} \tag{3}$$

Normalizing condition is:

$$P_0 + \sum_{i=1}^{m+\chi} \sum_{j=1}^2 P_{ij} = 1$$

The probabilities of states of Erlang QS are:

$$P_{1e} = P_{11} + P_{12}; P_{2e} = P_{21} + P_{22}; P_{ne} = P_{n1} + P_{n2}; \dots P_{ne} = P_{n1} + P_{n2}; \dots P_{(m+\chi)e} = P_{(m+\chi)1} + P_{(m+\chi)2}.$$

Define the characteristics of QS:

The probability of customer service:

$$P_{ar} = 1 - P_1 = 1 - P_f, \tag{4}$$

where $P_1 = P_{(m+\chi)e} - \frac{\sum_{i=2}^{m+\chi} P_{(i-1)2}}{2} = P_{(m+\chi)2} - \frac{P_{12} + P_{22} + \dots + P_{(m+\chi-1)2}}{2}$.

The average number of customers that are in queue:

$$\bar{r} = \frac{1}{2} (1 \cdot P_{(m+1)e} + 2P_{(m+2)e} + \dots + \chi P_{(m+\chi)e}). \tag{5}$$

The average number of customers that are in the system:

$$\bar{s} = \frac{1}{2} (1 \cdot P_{1e} + 2P_{2e} + \dots + (m+\chi)P_{(m+\chi)e}). \tag{6}$$

Average customer time spent in queue:

$$\bar{t}_q = \frac{\bar{r}}{\lambda}. \tag{7}$$

The average customer time spent in the system:

$$\bar{t}_s = \frac{\bar{s}}{\lambda}. \tag{8}$$

The average number of servers occupied:

$$\bar{k} = \frac{1}{2} (1 \cdot P_{1e} + 2P_{2e} + \dots + mP_{me} + \dots + mP_{(m+\chi)e}). \tag{9}$$

Explore functioning example of a M / E2 / 2/3 system, which is shown in the graph of Figure 2.

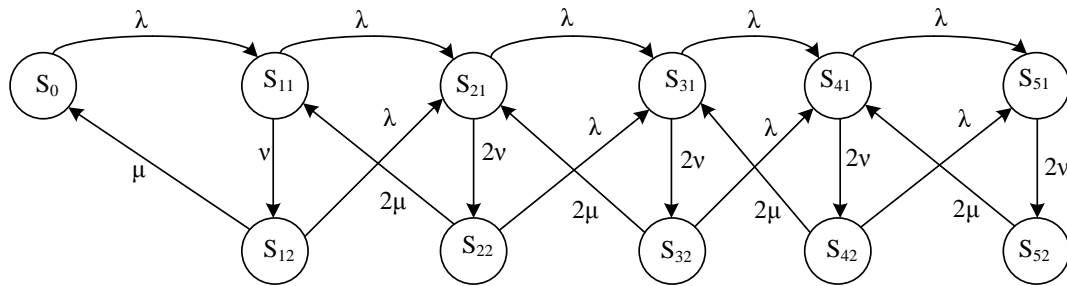


Fig. 2 Graph of two-channel queuing system with three places in the queue.

Matrix Λ_e of transition intensities of the graph is:

$$\Lambda = \begin{pmatrix}
 -\lambda & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \lambda & -(\lambda + \nu) & 0 & 0 & 2\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \nu & -(\lambda + \mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \lambda & \lambda & -(\lambda + 2\nu) & 0 & 0 & 2\mu & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 2\nu & -(\lambda + 2\mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & \lambda & \lambda & -(\lambda + 2\nu) & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 2\nu & -(\lambda + 2\mu) & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \lambda & \lambda & -(\lambda + 2\nu) & 0 & 0 & 0 & 2\mu \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\nu & -(\lambda + 2\mu) & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & \lambda & -2\nu & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\nu & -2\mu & 0
 \end{pmatrix}$$

Algebraic equations for the final probabilities of system states are:

$$\begin{aligned}
 \lambda P_0 &= \mu P_{12}; \\
 (\lambda + \nu) P_{11} &= \lambda P_0 + 2\mu P_{22}; \\
 (\lambda + \mu) P_{12} &= \nu P_{11}; \\
 (\lambda + 2\nu) P_{21} &= \lambda P_{11} + 2\mu P_{32} + \lambda P_{12}; \\
 (\lambda + 2\mu) P_{22} &= 2\nu P_{21}; \\
 (\lambda + 2\nu) P_{31} &= \lambda P_{21} + 2\mu P_{42} + \lambda P_{22}; \\
 (\lambda + 2\mu) P_{32} &= 2\nu P_{21}; \\
 (\lambda + 2\nu) P_{41} &= \lambda P_{31} + 2\mu P_{52} + \lambda P_{32}; \\
 (\lambda + 2\mu) P_{42} &= 2\nu P_{41}; \\
 2\nu P_{51} &= \lambda P_{41} + \lambda P_{42}; \\
 2\mu P_{52} &= 2\nu P_{51};
 \end{aligned} \tag{10}$$

$$P_0 + P_{11} + P_{12} + P_{21} + P_{22} + P_{31} + P_{32} + P_{41} + P_{42} + P_{51} + P_{52} = 1.$$

The probability of states of Erlang QS is determined as:

$$P_{1e} = P_{11} + P_{12}; P_{2e} = P_{21} + P_{22}; P_{3e} = P_{31} + P_{32}; P_{4e} = P_{41} + P_{42}; P_{5e} = P_{51} + P_{52}.$$

The QS features are the following:

$$P_{ar} = 1 - P_1 = 1 - P_f;$$

$$P_1 = P_{5e} - \frac{P_{12} + P_{22} + P_{32} + P_{42}}{2}.$$

The average number of customers in queue:

$$\bar{r} = \frac{1}{2}(1 \cdot P_{3e} + 2P_{4e} + 3P_{5e}). \tag{11}$$

The average number of customers that are in the system:

$$\bar{s} = \frac{1}{2}(1 \cdot P_{1e} + 2P_{2e} + 3P_{3e} + 4P_{4e} + 5P_{5e}).$$

Average time spent in queue:

$$\bar{r}_q = \frac{\bar{r}}{\lambda}.$$

The average time spent in the system:

$$\bar{s}_q = \frac{\bar{s}}{\lambda}.$$

The average number of employees servicing devices:

$$\bar{k} = \frac{1}{2}(1 \cdot P_{1e} + 2P_{2e} + 2P_{3e} + 2P_{4e} + 2P_{5e}).$$

To study the two-channel queuing system shown in Fig. 3, for the accepted range of values $0.5 \leq P_{ar} \leq 0.75$ we define the QS parameters. However the problem is that the number of the QS parameter values (λ, ν, μ) is infinite. Therefore, when adopted λ , then incrementally changing ν and find the values of μ that makes P_{ar} equal to values taken from the range.

Fig. 3 shows a graph of function $\mu = f_1(P_{ar})$.

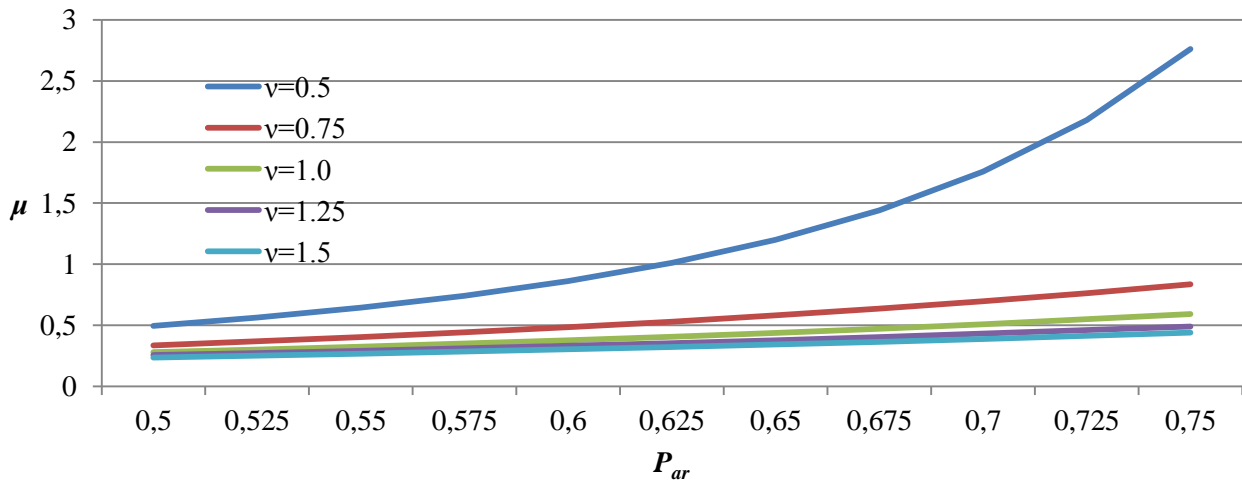


Fig. 3 Graph of function $\mu = f_1(P_{ar})$.

If an acceptable range of values for probability of successful elimination of an emergency environmentally hazardous consequences is $(0.5 \leq P_{ar} \leq 0.75)$, then reducing time of focusing localization units in emergency place (increase value ν) allows the use of less productive means of recovery operations. More details can be seen from the graph (Figure 4).

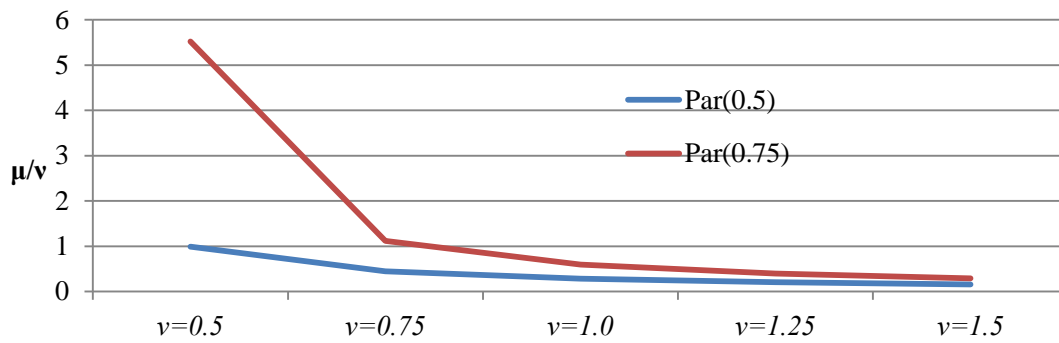


Fig. 4 Graph of function $\mu = f_2(\nu)$ for given values of P_{ar} .

In an acceptable range of $0,5 \leq P_{ar} \leq 0,75$, the ratio μ/ν is non-linear. Moreover, with decreasing ν (to $\nu = 0,5$), the need for fold increase of μ increases rapidly, that is the most characteristic for the upper band of P_{ar} .

More information is provided on a generalized graph of the probability of liquidation of consequences of environmentally hazardous emergency versus QS parameters (Fig. 5).

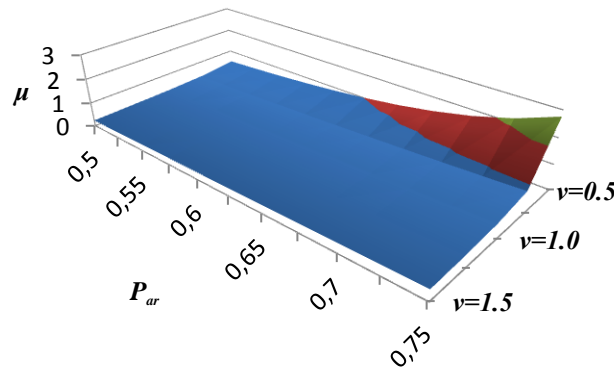


Fig. 5 A generalized graph of the probability of liquidation of consequences of environmentally hazardous emergency versus QS parameters.

Of particular note is the dependence of the probability of a negative impact on the environment (P_{ni}) on values of λ, ν, μ . Graph of $P_{ni} = f_3(\mu)$ is shown in Figure 6.

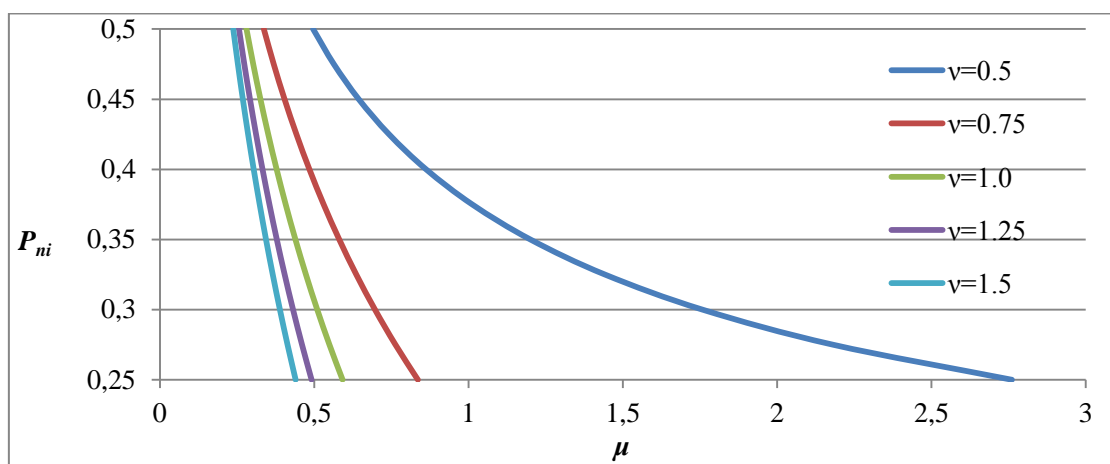


Fig. 6. Graph of $P_{ni} = f_3(\mu)$ for different values of ν .

It is known that the productivity of means elimination of environmentally hazardous consequences is defined in their technological specifications within $(\mu_{min_i} \leq \mu_i \leq \mu_{max_i})$. If $\mu_i \leq \mu_{min_i}$, even provided sooner concentration of emergency units at the site of an accident, the probability P_{ni} is rather high. The following values $\mu_i = \mu_{min_i}$ and $v = v_{max}$, where μ_i is constant, that is extremely inadequate performance. When $\mu_{min_i} \leq \mu_i \leq \mu_{max_i}$ with increasing v P_{ni} value decreases. When $\mu_i > \mu_{max_i}$, and with large values of emergency units concentrating time, P_{ni} is still relatively large, ie, the concentration is impractical.

In other words, if the means of the emergency response is not appropriate to the nature of accident and/or extremely unproductive, even with their short time of focusing on the accident site, they will not be effective. On the other hand, even if the means of eliminating are effective enough, but their focus on the scene was late, they also do not give proper effect.

The problem of negative impact on the environment at different duration of work by the liquidation units attracts special attention. Fig. 7 presents a graph of the duration of consequences recovery works versus performance of means, ie finding the QS customer full cycle of service.

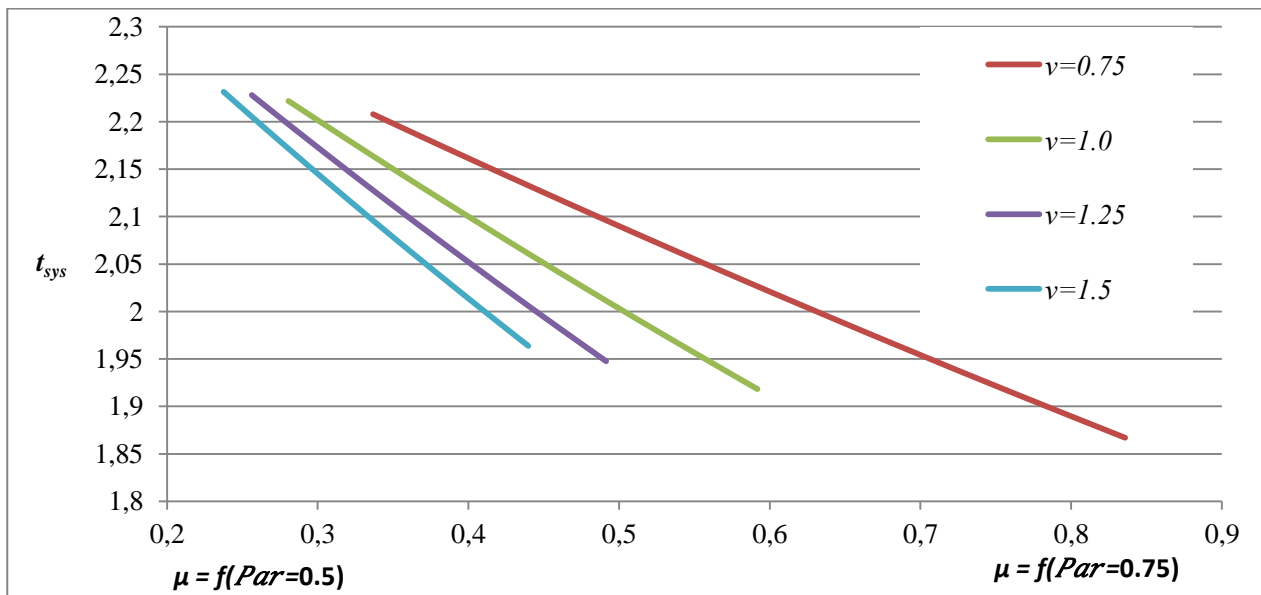


Fig. 7 Graph of average time spent in the system $t_{sys} = f_4(\mu)$ for different values of v in the acceptable range of values $0.5 \leq P_{ar} \leq 0.75$.

From Fig. 7 it is seen that a significant reduction of duration of works as well as the negative effects of environmentally hazardous emergency are possible with reducing the emergency units concentration time and using corresponding capabilities. Extending the concentration time needs to increase the productivity of these capabilities in times.

Consider a binary operation QS as operation of two emergency units (Figure 8).

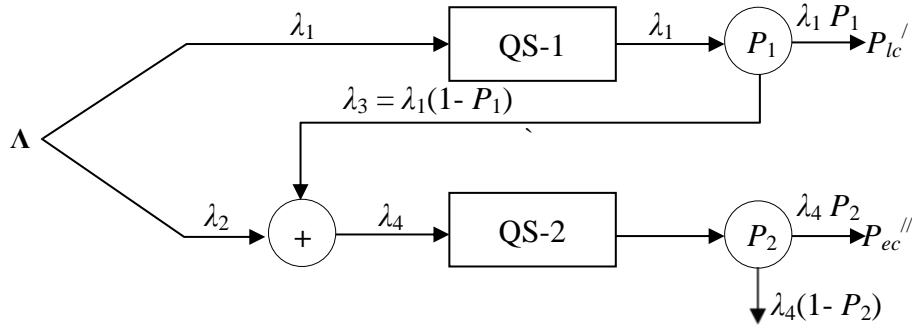


Fig. 8 Scheme of binary operation QS with consequent link of components

The arrival flow at the input of binary QS is exponential of Λ intensity, which is a superposition of two simple flows λ_1 and λ_2 , ie $\Lambda = \lambda_1 + \lambda_2$.

After service refusal in QS-1 failed customers flow with intensity $\lambda_3 = \lambda_1(1-P_1)$ immediately arrives to the input of the QS-2.

At the input of QS-2, either customers flow with intensity λ_1 is coming, or flow with intensity λ_3 . The probability of simultaneous arrival of two various customer flows is zero.

With the prescribed probability limits of environmentally hazardous emergency elimination ($0,8 \leq P_{qs} \leq 0,95$) determine the probability of the emergency elimination in QS-1 (P'_{lc}) and in QS-2 (P''_{lc}) using the relation:

$$(1-P_{qs})=(1-P'_{lc})(1-P''_{lc}),$$

where $P_{qs} = 1-(1-P'_{lc})(1-P''_{lc})$.

Set the values P'_{lc} and P''_{lc} and define P_1 and P_2 using relationships:

$$P'_{lc}=\lambda_1P; P''_{lc}=\lambda_4P_2,$$

$$\text{then: } P_1 = \frac{P'_{lc}}{\lambda_1}; P_2 = \frac{P''_{lc}}{\lambda_2+\lambda_1(1-P_1)}.$$

If accepted values $\lambda_1, \lambda_2, \mu_1$ and μ_2 define the value v1 and v2 by calculation using MathCad and MS Excel.

QS components are queuing systems of M / E2 / 1 / 3 type. Graph of QS-1 and QS-2 is given in Fig. 9.

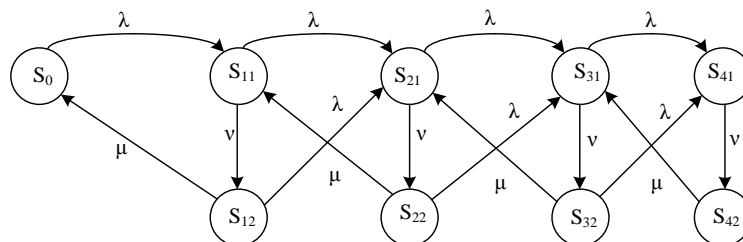


Fig. 9 Graph of QS components.

Matrix Λ_e of transitions intensities of such QS is as below:

$$\Lambda = \begin{pmatrix} -\lambda & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & -(\lambda + \nu) & 0 & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & \mu & -(\lambda + \mu) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & \lambda & -(\lambda + \nu) & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & \nu & -(\lambda + \mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & \lambda & -(\lambda + \nu) & 0 & 0 & \mu \\ 0 & 0 & 0 & 0 & 0 & \nu & -(\lambda + \mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & \lambda & -\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \nu & -\mu \end{pmatrix}$$

Algebraic equations for the final state probabilities of QS are:

$$\begin{aligned} \lambda P_0 &= \mu P_{12}; \\ (\lambda + \nu) P_{11} &= \lambda P_0 + 2\mu P_{22}; \\ (\lambda + \mu) P_{12} &= \nu P_{11}; \\ (\lambda + 2\nu) P_{21} &= \lambda P_{11} + 2\mu P_{32} + \lambda P_{12}; \\ (\lambda + 2\mu) P_{22} &= 2\nu P_{21}; \\ (\lambda + 2\nu) P_{31} &= \lambda P_{21} + 2\mu P_{42} + \lambda P_{22}; \\ (\lambda + 2\mu) P_{32} &= 2\nu P_{31}; \\ \nu P_{41} &= \lambda P_{31} + \lambda P_{32}; \\ \mu P_{42} &= \nu P_{41}. \end{aligned} \tag{12}$$

$$P_0 + P_{11} + P_{12} + P_{21} + P_{22} + P_{31} + P_{32} + P_{41} + P_{42} = 1$$

The probabilities of QS states are determined as:

$$P_{1e} = P_{11} + P_{12}; P_{2e} = P_{21} + P_{22}; P_{3e} = P_{31} + P_{32}; P_{4e} = P_{41} + P_{42};$$

QS features are:

$$\begin{aligned} P_{ar} &= 1 - P_l = 1 - P_f, \\ P_l &= P_{4e} - \frac{P_{12} + P_{22} + P_{32}}{2}. \end{aligned}$$

The average number of customers which are in queue:

$$\bar{r} = \frac{1}{2} (1 \cdot P_{2e} + 2P_{3e} + 3P_{4e}). \tag{13}$$

The average number of customers that are in the system:

$$\bar{s} = \frac{1}{2} (1 \cdot P_{1e} + 2P_{2e} + 3P_{3e} + 4P_{4e}).$$

Average time spent in queue:

$$\bar{r}_q = \frac{\bar{r}}{\lambda}.$$

The average time spent in the system:

$$\bar{s}_q = \frac{\bar{s}}{\lambda}.$$

The average number of servers occupied:

$$\bar{k} = \frac{1}{2} (1 \cdot P_{1e} + 1P_{2e} + 1P_{3e} + 1P_{4e}).$$

Graphs of functions $P_{qs} = f_5(\nu)$ with taking into account the range for P_{ar} are given in Figure 10.

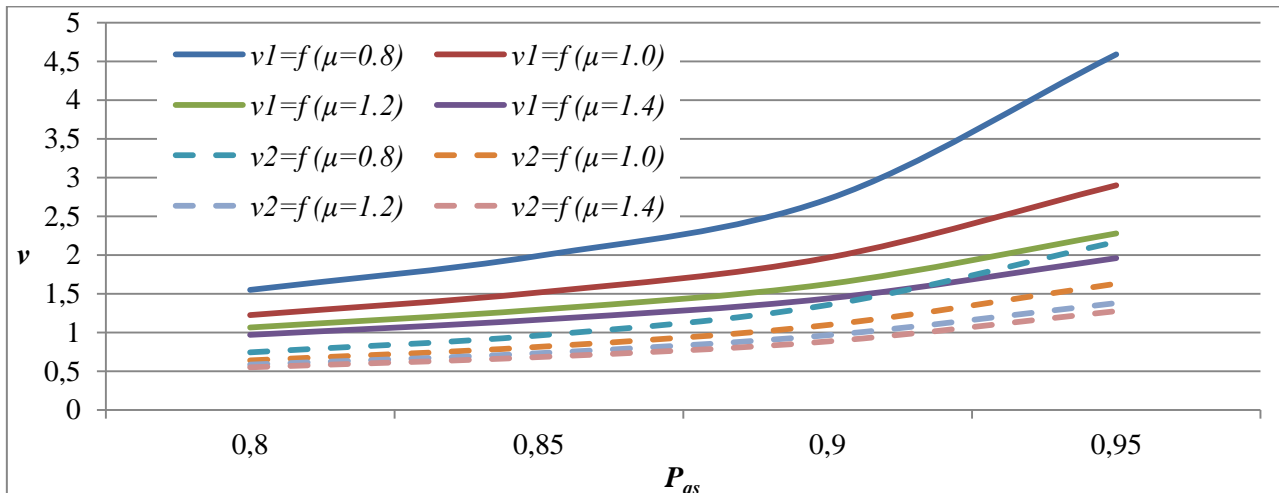


Fig. 10. Graphs of functions $P_{qs} = f_5(v)$ taking into account the range for P_{ar} .

The graphs (Figure 10) show the dependences of the QS-1 in continuous lines and the QS-2 in dotted lines.

As shown in the graphs (Figure 10), with an increase in the performance of QS-1 and QS-2, the needed intensity of concentration decreases. For a given value μ , and with increasing P_{qs} value, v increases non-linearly.

Providing certain probability P_{qs} s can be achieved by various parameters of QS-1 and QS-2, or by increasing the productivity of disposal capabilities and reducing the time of concentration. Moreover, the requirements for parameters QS-2 are significantly smaller than the parameters for QS-1.

3 CONCLUSIONS

The proposed approach to the study of the consequences of hazardous rail traffic accidents with dangerous goods:

- is the methodological basis for the creation and development of Decision Support System for Task Force leader in the aftermath of such accidents in a single automated control system of railway freight transportation;
- makes it possible to formulate reasonable requirements for the deployment of wreck, recovery and fire teams on the rail network, their equipment and the professional training for team leaders, managers and staff;
- enables to determine the probability and duration of the negative impact of environmentally hazardous transport accidents.

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FATIGUE-PRONE AIRCRAFT FLEET RELIABILITY BASED ON THE USE OF A P-SET FUNCTION

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ABSTRACT

An inspection interval planning is considered in order to limit a probability of any fatigue failure (FFP) in a fleet of N aircraft (AC). A solution of this problem is based on a processing of the result of the acceptance fatigue test of a new type of an aircraft. During this test an estimate of the parameter of a fatigue crack growth trajectory can be obtained. If the result of this acceptance test is too bad then this new type of aircraft will not be used in service. A redesign of this project should be done. If the result of acceptance test is too good then the reliability of the aircraft fleet will be provided without inspections. For this strategy there is a maximum of FFP as a function of an unknown parameter θ . This maximum can be limited by the use of the offered here procedure of the choice of an inspection number.

1 INTRODUCTION

We suppose that in the interval (T_d, T_c) , where T_d is a random time when the fatigue crack becomes detectable (the corresponding crack size $a(T_d) = a_d$ and T_c is a random time when the crack reaches its critical size (the corresponding crack size $a(T_c) = a_c$), the size of the crack can be approximated by the equation $a(t) = \alpha \exp(Qt)$. Then we have:

$$T_c = (\log a_c - \log \alpha) / Q = C_c / Q, \quad T_d = (\log a_d - \log \alpha) / Q = C_d / Q. \quad (1)$$

For the calculation of a probability of a fatigue crack detection during an inspection we need to know the probability of detection of fatigue crack as function of its size a and the distribution of the size for specific time of inspection. Usually for processing fatigue life the lognormal distribution is used, so for the considered numerical example here (see the 5-th section) we study the simplest case: a random variable (rv), $\log(Q)$, has a normal distribution with an parameter $\theta = (\theta_0, \theta_1)$, where θ_0 is an unknown mean but θ_1 is the known standard deviation. And we suppose that α , a_d , a_c are the known constants. The estimate $\hat{\theta} = (\hat{\theta}_0 + \theta_1)$ of the parameter θ can be obtained by the regress analysis of the result of fatigue test of AC of the same type in laboratory (i.e. processing the observations of fatigue crack: pairs $\{(time, fatigue\ crack\ size)_i, i=1, \dots, m\}$, where m is a number of the fatigue crack observation.

2 CALCULATION OF A PROBABILITY OF A FATIGUE FAILURE OF ONE AIRCRAFT FOR THE KNOWN θ

For the known θ , there are two decisions: 1) the aircraft is good enough and the operation of this aircraft type can be allowed, 2) the operation of the new type of AC is not allowed and a redesign of AC should be made. In the case of the first decision, the vector $t = (t_1, \dots, t_n)$, where t_i is the time moment of i -th inspection, should also be defined. If θ is known the different rules can be offered for the choice of structure of the vector t : 1) every interval between the inspections is equal to the constant $d_i = t_{SL} / (n+1)$, where t_{SL} is the aircraft specified life (SL) (the retirement time), n is a number of inspections, 2) the conditional probabilities of a failure (under condition that the fatigue failure did not takes place in previous interval) in every interval is equal to the same value $P(T_C < t_{SL}) / (n+1) \dots$ In this paper we suppose the first type of the choice and the vector t is defined by the fixed t_{SL} and the choice of n .

For the substantiation of the choice of the inspection number we should know the probability of a fatigue crack detection as a functions of a crack size a . We suppose that this probability is defined by the equations

$$p_d(a) = w_0 w(a), \quad w(a) = \begin{cases} 0, & \text{if } a \leq a_{d0}, \\ \frac{a - a_{d0}}{a_{d1} - a_{d0}}, & \text{if } a_{d0} < a < a_{d1}, \\ 1, & \text{if } a \geq a_{d1}. \end{cases} \quad (2)$$

Where a_{d0} , a_{d1} are some constants, see Fig.1; the constant w_0 can be considered as probability to carry out planned inspection (human factor).

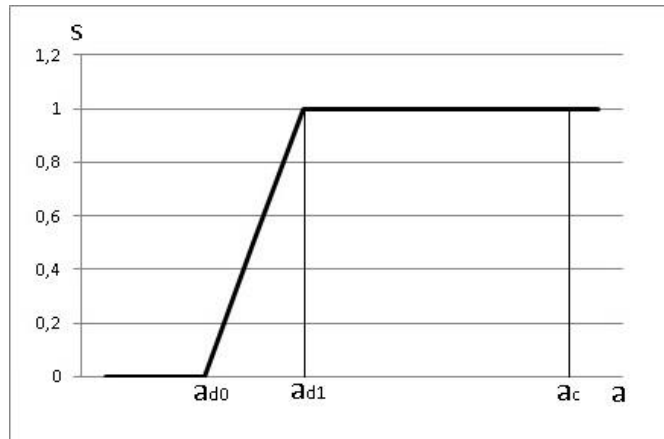


Figure 1. Crack detection probability function of crack size

In simplest case $w_0 = 1$, $a_{d1} = a_{d0}$ and we define

$$w(a) = \begin{cases} 0, & \text{if } a < a_{d0}, \\ 1, & \text{if } a \geq a_{d0}. \end{cases} \quad (3)$$

Then it is convenient the process of an operation of AC to consider as an absorbing Markov chain (MCh) with $(n+4)$ states. The states E_1, E_2, \dots, E_{n+1} correspond to an AC operation in the time

intervals $[t_0, t_1), [t_1, t_2), \dots, [t_n, t_{SL})$. States E_{n+2} , E_{n+3} and E_{n+4} are the absorbing states: AC is discarded from a service when the SL is reached or fatigue failure (FF), or fatigue crack detection (CD) take place.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	...	0	0	0	0	q_1	v_1
E_2	0	0	u_2	...	0	0	0	0	q_2	v_2
E_3	0	0	0	...	0	0	0	0	q_3	v_3
...
E_{n-1}	0	0	0	...	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	...	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	...	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	0	0	0	...	0	0	0	1	0	0
E_{n+3} (FF)	0	0	0	...	0	0	0	0	1	0
E_{n+4} (CD)	0	0	0	...	0	0	0	0	0	1

Figure 2. Probability matrix P_{AC}

In the corresponding transition probability matrix, P_{AC} , (see Fig.2.) let v_i be the probability of a crack detection during the inspection number i , let q_i be the probability of the failure in service time interval $t \in (t_{i-1}, t_i)$, and let $u_i = 1 - v_i - q_i$ be the probability of the successful transition to the next state (next interval of aircraft service). In our model we also assume that an aircraft is discarded from a service at t_{SL} even if there are no any crack discovered by inspection at the time moment t_{SL} . This inspection at the end of $(n+1)$ -th interval (in state E_{n+1}) does not change the reliability but it is made in order to know the state of the aircraft (whether there is a fatigue crack or there is no fatigue crack). It can be shown that $u_i = P(T_d > t_i | T_d > t_{i-1})$, $q_i = P(t_{i-1} < T_d < T_c < t_i | T_d > t_{i-1})$, $v_i = 1 - u_i - q_i$, $i = 1, \dots, n+1$. In the three last lines of the matrix P_{AC} there are three units in the matrix diagonal because the states E_{n+2} , E_{n+3} and E_{n+4} are the absorbing states. All the other entries of this matrix are equal to zero (see Fig.2.). The structure of the considered matrix can be described in the following way: $P_{AC} = [QR; 0I]$,

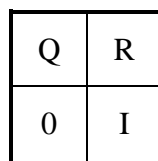


Figure 3. Structure of matrix P_{AC}

where in the second line of this structure the matrix 0 is the submatrix of zeroes, I is the submatrix of identity corresponding to the absorbing states of the matrix P_{AC} . Then the matrix of the probabilities of absorbing in the different absorbing states for the different initial transient states $B = (I - Q)^{-1}R$. The failure probability of a new AC is equal to $p_f = aBb$, where the vector row $a = (1, 0, \dots, 0)$ means that all the aircraft begin an operation within the first interval (state E_1), the vector column $b = (0, 1, 0)$. We need also to know the mean number of steps of MCh up to the absorption $E(T_{AC}) = a(I - Q)^{-1}c$, where $c = (1, \dots, 1)$ is the vector-column. The mean life of aircraft will be equal to $d_i E(T_{AC})$.

3 PROBABILITY OF ANY FATIGUE FAILURE IN THE FLEET OF AIRCRAFT FOR THE KNOWN θ

We consider the case when the operation of all N aircraft will be stopped if any fatigue crack will be detected. In order to limit the probability of fatigue failure in the fleet (FFPN) it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place.

3.1 Probability of detection is defined by (3)

For the case when the probability of fatigue crack detection is defined by equation (3) the corresponding probability is equal to the expected value of random variable $P_{fNW} = (1 - w)^R$, where w is a human factor: a probability, that the planned inspection will be made, R is the total random number of inspections before the first failure in the whole fleet.

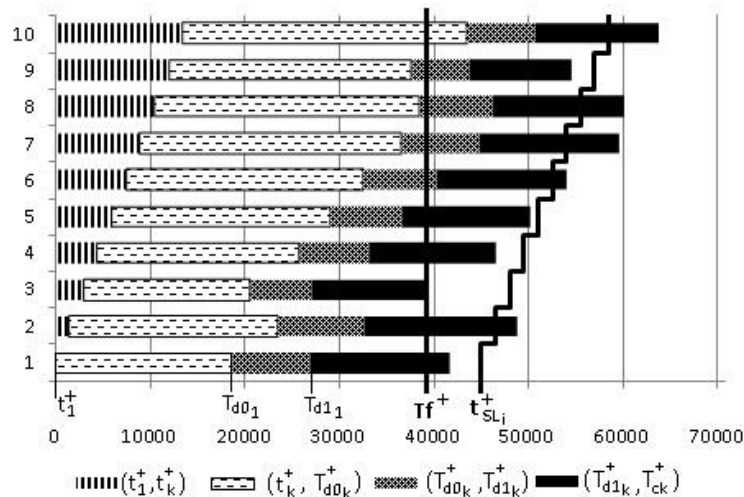


Figure 4. Operation of N aircraft

Let $t_k^+, t_{k-1}^+ < t_k^+, t_0^+ = 0$ to be “calendar” time moment when k -th aircraft begin the service, $T_{dk}^+ = t_k^+ + T_{dk}, T_{ck}^+ = t_k^+ + T_{ck}, k = 1, 2, \dots, N$ to be the random calendar time moments when fatigue crack can be discovered and fatigue failure of AC takes place correspondingly, see Fig.4. And let $K_{SL} = \{k : T_{ck} < t_{SL}\}, k = 1, 2, \dots, N\}$ be a set of indexes of aircraft, the failure of which can take a place, if an inspection will not take the place, $T_f^+ = \min\{T_{fk}^+ : k \in K_{SL}\}, T_{fk}^+ = \min\{T_{ck}^+, T_f^+\}, k \in K_{SL}$,

$R = \sum_{k \in K_{SL}} R_k$, where $R_k = \max(\{[(T_{fk}^+ - t_k^+) / d_t] - [(T_{dk}^+ - t_k^+) / d_t]\}, 0)$, $k \in K_{SL}$, is the random inspection number of k -th aircraft from the set K_{SL} if inspection interval $d_t = t_{SL} / (n + 1)$ (it is supposed a specific “calendar” schedule of the inspections for each aircraft: , $i = 1, 2, \dots, n + 1$, $k \in K_{SL}$)

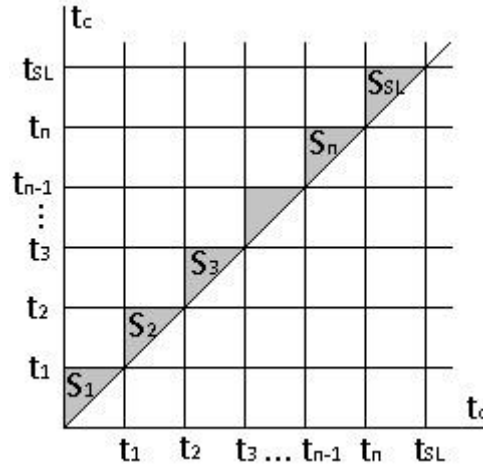


Figure 5a. Example of a “value” p-set function for one aircraft

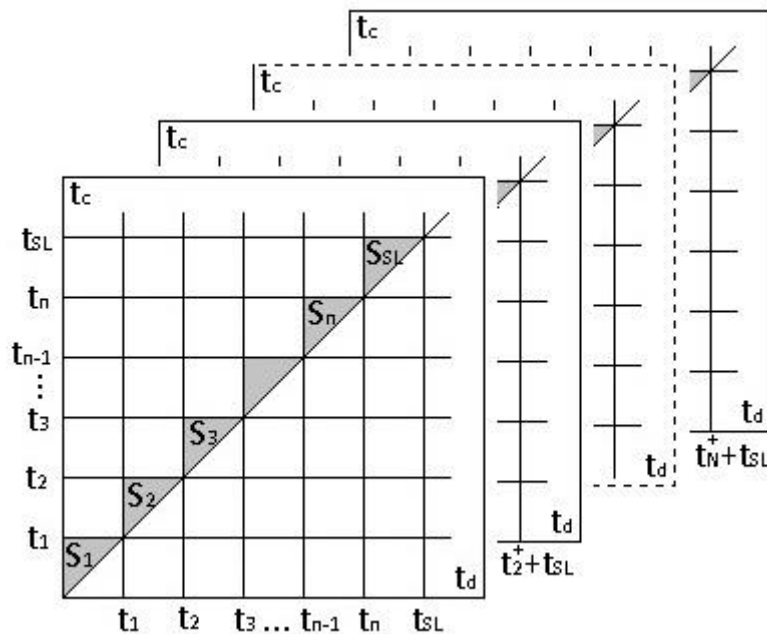


Figure 5b. Set of “values” of a p-set function for N aircraft with different beginning of operation.

Random variable Q is a speed of fatigue crack growth in logarithm scale. It has the specific realization for each aircraft and Q_1, \dots, Q_N are independent random variables. So mean value of random probability of failure in the fleet

$$E(P_{fNW}) = p_{fNW}(n, \theta) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} (1-w)^{r(q)} dF_{Q_1}(q_1) \dots dF_{Q_N}(q_N) \tag{4}$$

where $q = (q_1, \dots, q_N)$, $r(q)$, is realization of rv R . For large number N the Monte Carlo method is appropriate for the calculation of p_{fNW} . If this function is known then the number of the

inspections, $n(p, \theta)$, required to limit the FFPN by a value p is defined by the function $n(p, \theta) = \min(r: p_{fNW}(r, \theta) \leq p \text{ for all } r > n(p, \theta), r = 1, 2, \dots)$.

3.1. General definition of probability of detection

For general definition of the function $p_d(a)$ it is necessary to make more detailed analysis. Let us denote by t_{kj}^+ the calendar time of j -th inspection of k -th aircraft $k \in K_{SL}, j \in J_k, k \in K_f$, where $J_k = \{j: t_{kj}^+ < T_{fk}^+, j = 1, 2, \dots\}$ is the set of indexes of inspections of k -th aircraft; $t_{kj}^+ = t_k + jd_t, j = 1, 2, \dots, K_f = \{k: k \in K_{SL}, t_k^+ < T_f^+\}$.

Let $q = (q_1, \dots, q_N)$ be realization of random vector $Q = (Q_1, \dots, Q_N)$ and let $a_{jk} = \alpha \exp(q_k d_t j)$ is a size of fatigue crack at j -th inspection of k -th aircraft for which the growth of fatigue crack is defined by specific value of q_k which is the realization of random variable Q_k . Corresponding probability of fatigue crack discovery is equal to $p_{dkj} = p_d(a_{jk})$. For specific value of vector q probability that not any fatigue crack will not be discovered is equal to

$$p_f(q) = \begin{cases} 0, & \text{if } K_f = \emptyset, \\ \prod_{k \in K_f} \prod_{j \in J_k} (1 - p_{dkj}), & \end{cases} \quad (5)$$

By modeling random vector Q using Monte Carlo method, we can calculate mean value of this probability: $p(n, \theta) = E_\theta(p_f(q))$. Now we can choose number of the inspection $n(p, \theta)$ in such a way that the failure probability will be equal to p .

4. SOLUTION FOR UNKNOWN θ

First, we consider the problem of a limitation of FFP1 in an operation of one AC if the probability of detection is defined by equation (3), the human factor $w_0 = 1$. This means that if there is a detectable fatigue crack, then during the inspection after T_d we see it with probability 1 and the limitation of FFP1 of AC is provided by the choice of the specific p-set function, Paramonov *et al* (2011). Let us take into account that the operation of a new type of aircraft will not take place if the result of acceptance fatigue test in a laboratory is “too bad” (previously, the redesign of the new type of AC should be made). We say that in this case the event $\hat{\theta} \notin \Theta_0, \Theta_0 \subset \Theta$, takes place (for example, $\hat{\theta} \notin \Theta_0$ if fatigue life T_c is lower than some limit; or $n(p, \hat{\theta})$ is too large, ...). Let us define some binary set function

$$S(\hat{\theta}, \Theta_0, n) = \begin{cases} \bigcup_{i=1}^{n+1} S_i(n) & \text{if } \hat{\theta} \in \Theta_0 \\ \emptyset & \text{if } \hat{\theta} \notin \Theta_0 \end{cases} \quad (6)$$

where $S_i = \{(t_d, t_c): t_{i-1} < t_d, t_c \leq t_i\}, t_i = it_{SL} / (n+1), i = 1, \dots, n+1; \emptyset$ is an empty set.

We call this function *binary p-set function* if

$$\sup_\theta \sum_{i=1}^{n+1} P(Z \in S_i(n) \cap \hat{\theta} \in \Theta_0) = p$$

Here we take into account that if $\hat{\theta} \notin \Theta_0$ operation of AC is not allowed and corresponding the failure probability is equal to 0. Examples of value of binary p-set functions are shown in Fig.5.

It can be shown that for very wide range of the definition the set Θ_0 and the requirements to limit FFP1 by the value p^* , where $(1-p^*)$ is a required reliability, there is a preliminary “designed” choice of allowed FFP1, p_{fd} , such that corresponding set function $S(\Theta_0, n(p_{fd}, \hat{\theta}))$ is p -set function of the level p^* for the vector $Z = (T_d, T_c)$. The value of p_{fd} is defined by equation

$$\sup_{\theta} \sum_{i=1}^{n+1} P(Z \in S_i(n(p_{fd}, \hat{\theta})) \cap \hat{\theta} \in \Theta_0) = p \quad (7)$$

For this p_{fd} the FFP1 will be limited by the value p^* for any unknown $\theta \in \Theta$.

Now we consider the reliability of the fleet of N AC when there is an information exchange and the operation of all aircraft will be stopped if fatigue crack will be found during an inspection of any AC and, as it was told already, in order to prevent the failure in the fleet, it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place. Let us define some multiple set function:

$$S^+(\hat{\theta}, \Theta_0, n) = \bigcup_{k \in K_{SL}} S_k^+(\hat{\theta}, \Theta_0, n) \quad (8a)$$

where

$$S_k^+(\hat{\theta}, \Theta_0, n) = \begin{cases} \bigcup_{i=1}^{n+1} S_{i,k}(n) & \text{if } \hat{\theta} \in \Theta_0 \\ \emptyset, & \text{if } \hat{\theta} \notin \Theta_0 \end{cases} \quad (8b)$$

$S_{ik(n)}^+ = \{(t_{dk}^+, t_{ck}^+) : t_{(i-1)k} < t_{dk}, t_{ck} \leq t_{ik}\}$, $t_{ik}^+ = t_k^+ + t_i$, $t_i = it_{SL} / (n+1)$, $i = 1, \dots, n+1, k = 1, 2, \dots, N$. Again, it can be shown that for very wide range of the definition the set Θ_0 and the requirements to limit FFPN by the value p^* , there is a preliminary “designed” choice of allowed FFPN, p_{fd} , such that corresponding multiple set function $S^+(\Theta_0, n(p_{fd}, \hat{\theta}))$ is p -set function of the level p^* for the set of vectors $\{Z_k^+, k \in K_{SL}\}$, where $Z_k^+ = (T_{dk}^+, T_{fk}^+)$:

$$v(p_{fd}) = p^*, \quad (9a)$$

$$\text{where } v(p) = \sup_{\theta} v(\theta, p) \quad (9b)$$

$$v(\theta, p) = E \left\{ \sum_{k \in K_{SL}} \sum_{i=1}^{n+1} P(Z_k^+ \in S_{ik}^+(n(p, \hat{\theta})) \cap \hat{\theta} \in \Theta_0) \right\} \quad (9c)$$

That means that FFPN will be limited by the value p^* for any unknown θ .

5 NUMERICAL EXAMPLE

In this numerical example we assume that $t_{SL} = 45000$, $w_0 = 0.95$, processing the result of full scale fatigue test we get the estimate of fatigue crack parameters $\hat{\theta} = -8.5885$, $\alpha = 0.286$ mm, the standard deviation of $\log(Q)$ is equal to 0.346 (see Fig.2.32 in [1]), and let for considered inspection technology the detection probability p_d is defined by (2) with $a_{d0} = 10$ mm, $a_{d1} = 20$ mm, $a_c = 237$ mm. There are 10 aircraft in the fleet, the interval between the aircraft putting into operation $d_i = 1000$; allowed failure probability $p^* = 0.01$, the set Θ_0 is defined by the condition : if $\hat{n} = n(0.01, \theta) > 20$ then the redesign of AC should be made. Using the Monte Carlo calculation we get $\hat{n} = n(0.01, \theta) > 7$, see Fig.6.

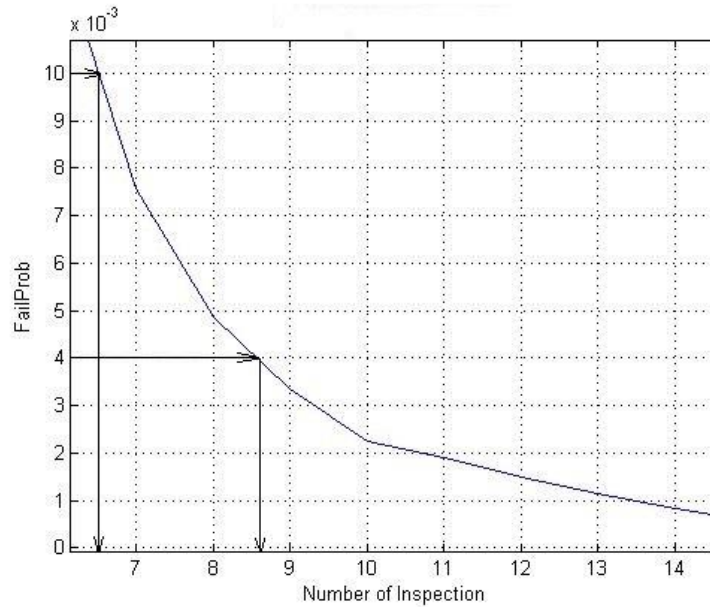


Figure 6. Example of function $p_{fNW}(n, \theta)$ (probability of crack detection) for $\theta_0 = -8.5885$

But this calculation is correct only if in the service the same value of $\theta_0 = -8.5885$ takes place. In reality we do not know the θ_0 . If θ_0 value is changed, then selected number of inspections to provide required reliability level will be changed as well. This effect could be seen in Fig.7.

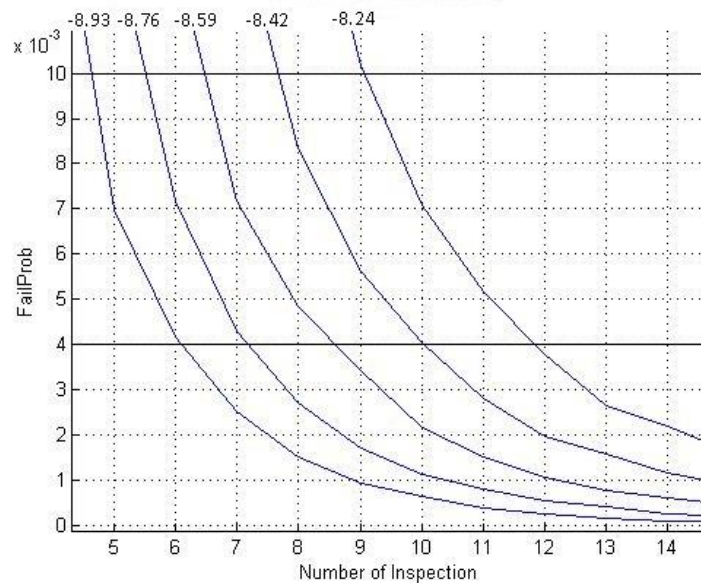


Figure 7. Example of function $p_{fNW}(n, \theta)$ (probability of crack detection) for different θ_0 values

This means that selected inspection program $\hat{n} = n(0.01, \theta) = 7$ is not appropriate for all possible θ_0 values. In service it is possible that some fatigue cracks require higher number of inspections. Number of inspections dependence on θ_0 value for different p^* could be seen on Fig.8.

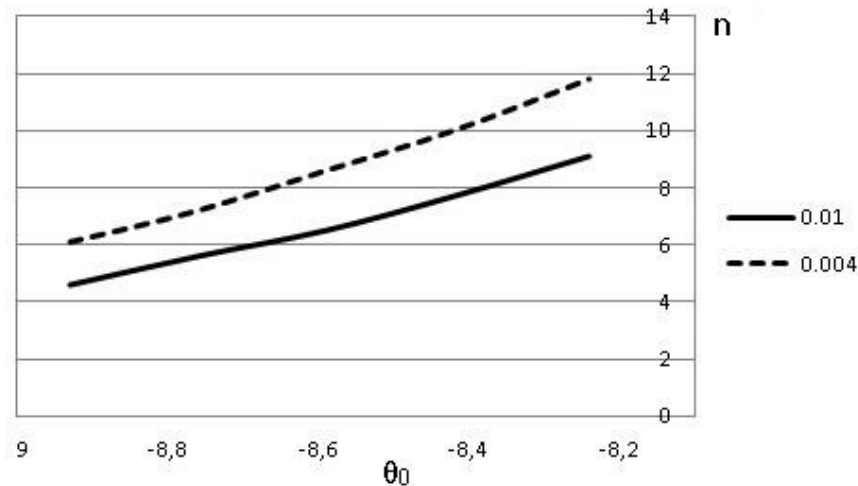


Figure 8. Example of function $n(p^*, \theta)$ for $p^*=0.01$ and $p^*=0.004$

We should limit the maximal possible failure probability for any θ_0 . It can be done by the choice of specific “designed” failure probability, p_{fd} . The family of the functions $v(\theta, p)$ for different p is shown in Fig.9, where the corresponding calculations for parallel axis are made for corresponding ”Mean durability“ = C_c / Q for $C_c = \log a_c - \log \alpha$, $Q = \exp(\theta_0)$

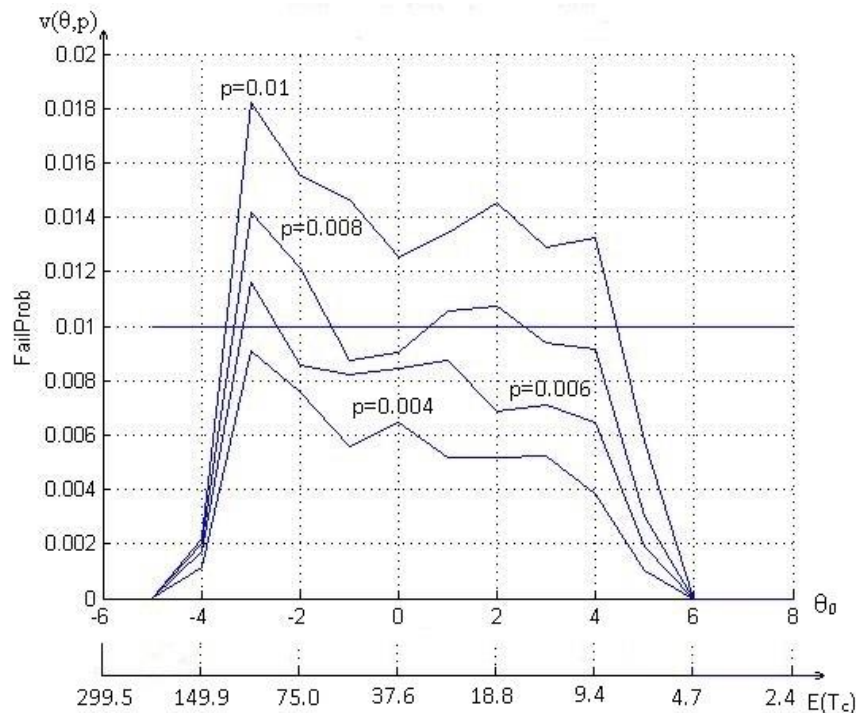


Figure 9. The function $v(\theta, p)$ for different p . In parallel axis the flight hours divided by 10^3 of the corresponding “Mean durability” = C_c / Q are given for $C_c = \log a_c - \log \alpha$, $Q = \exp(\theta_0)$

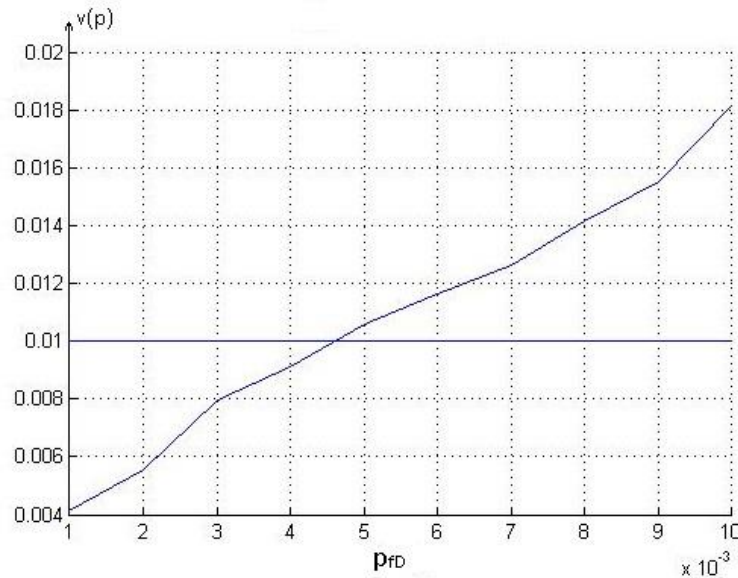


Figure 10. The function $v(p)$

In Fig.10 the function $v(p)$ is shown for considered example data. We see that in order to limit FFPN by value $p^* = 0.01$ the value $p_{FD} = 0.004$ should be chosen. Now using the function $p_{FNW}(n, \theta)$ which is shown in Fig.7 for the test estimate of fatigue crack parameters $\hat{\theta}_0 = -8.5885$, the number of inspections should be chosen equal to $\hat{n} = n(0.004, \theta) = 9$.

6. CONCLUSIONS

It is found, how, using the estimate of the unknown parameter $\hat{\theta}$ (after the acceptance fatigue test), one of the two decisions should be chosen: 1) to do the redesign of a new type of AC if the result of the test is “too bad” or 2) to make a choice of the number of inspections $n = n(p_{FD}, \hat{\theta})$ as a function of $\hat{\theta}$ and specific p_{FD} .

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ANALYSIS OF INDEPENDENT PROTECTION LAYERS AND SAFETY INSTRUMENTED SYSTEM FOR OIL GAS SEPARATOR USING BAYESIAN METHODS

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ABSTRACT

Process and Nuclear industries use Independent Protection Layers (IPLs) to prevent initiating abnormal events from becoming accidents. They form layers of protection that acts to prevent an abnormal situation from escalating. IPLs can be hardware (Basic Control System-BPCS) or operator actions, active (Safety Instrumented System-SIS) or passive (Dike walls) or a combination of all these factors. Safety Instrumented System (SIS) is the protection layer that comes in to action in case of failure of BPCS and operator action. Therefore reliability and ability of the SIS to respond should be higher than that of the layer like the BPCS. Reliability of SIS is usually specified in terms of Safety Integrity Level (SIL). The required SIL is calculated by analyzing the Probability of Failure of Demand (PFD) of all the IPLs in the case of an Initiating Event (IE) and comparing the Mitigated Consequence Frequency with a pre-established Tolerable Frequency (TF). The calculations involve probability of failure of each of layers and are usually done through spreadsheet or proprietary software. Bayesian methods are suited to handle these calculations due the nature of conditional probabilities inherent in the system. Further Bayesian methods can analyze the influencing factors affecting the PFD of the IPLs. This paper will present analysis of IPLs and its influencing factors using Bayesian methods including application of Common Cause Failures (CCF) and NoisyAnd distribution to Conditional Probability Tables (CPTs).

1. INTRODUCTION

Independent Protection Layers (IPLs) form the safety barriers that prevent Initiating Events (IE) from becoming hazardous consequences (accidents) and are used extensively in Nuclear and Process industries. The concept and methodologies of such layers are described in Center for Chemical Process Safety's (CCPS) book Layers Of Protection Analysis (2001) [1]. Layers Of Protection Analysis (LOPA) is a formalized procedure used to assign Safety Integrity Levels (SIL) to the Safety Instrumentation Systems (SIS) in accordance with the International Electro-technical Commission's standard (IEC) 61511 meant for process industries. SIL levels involve calculation of the Mitigated Consequence frequency using the Probability of Failure on Demand (PFDs) of each of the IPLs and comparing the value with Tolerable Frequency for the events. If the calculated Mitigated Consequence frequency is higher than the Tolerable Frequency, the reliability of the SIS layer has to be increased. The calculations are usually implemented through spreadsheet or proprietary software. Due it probabilistic nature such calculations can be easily done through Bayesian Networks (BN). BN models can offer easy inclusion of several influencing factors that affect the IPLs PFDs. Common Cause Failure (CCF) and other uncertainties or noise in the system can also be modelled with BN. This paper will describe the usage of BNs to model the IPLs including CCF and other uncertainties in the system through NoisyAnd distribution.

2. LAYERS OF PROTECTION.

Figure 1 illustrates the concept of successive layers that protect personnel, environment and assets from the harmful consequences of a loss of containment in a process system. The failures of protection layers are considered in series. Except for the Design & BPCS rest of the protection layers act only on demand. (Demand mode operation).

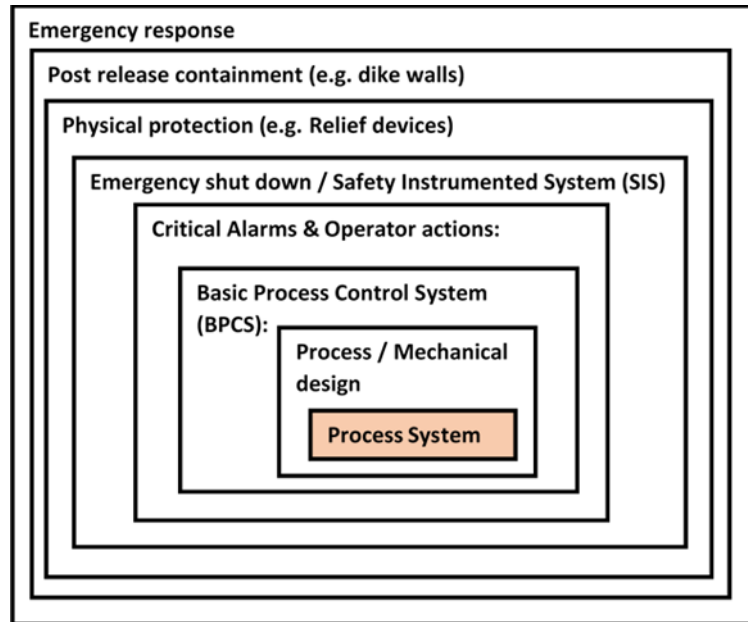


Figure 1: Layers Of Protection for a Process System

3. CALCULATIONS FOR DETERMINING SAFETY INTEGRITY LEVEL (SIL)

Calculations to determine SIL for the Safety Instrumented System (SIS) involve the following steps.

Step 1: Each of the layers of protection has a PFD associated with it. The sequential failure of the IPLs can be readily put in the equation 1 below:

$$PFD_{IPLs} = \prod_{n=1}^N PFD_n \quad (1)$$

N is the total number of IPLs.

In calculations for SIL, the PFD of SIS layer is set to 1; that is no credit is taken for the SIS already provided

Step 2: If the probability of Initiating Event is IE, then the probability of Mitigated Consequence is given by

$$\text{Mitigated Consequence} = IE * \prod_{n=1}^N PFD_n \quad (2)$$

Sometimes the probability of Initiating Events are modified by enabling conditions (EC) (for example presence of operators) and conditional modifiers (CM) (example: probability of gas cloud ignition) and the same can be included in the above equation 2 to give equation 3.. Center for Chemical Process Safety's (CCPS) Criteria for evaluating Enabling Conditions and Conditional Modifiers in Layers Of Protection Analysis (2014) [2] gives details on the above.

$$\text{Mitigated Consequence} = IE * EC * CM * \prod_{n=1}^N PFD_n \tag{3}$$

Step 3: This step is the comparison with established Tolerable Frequencies. (TF). Table 1 below shows the commonly used values for TFs. These values could vary depending upon the country and nature of loss. Some companies use more categories and tolerable frequencies. Lewis (2007) [3] summarizes the subject.

Category	Tolerable Frequency (TF)
Multiple Personnel fatality	$1 * 10^{-6}$
Environment	$1 * 10^{-4}$
Property (Assets)	$1 * 10^{-4}$

Table 1: Category of Consequences & Tolerable Frequencies

The required PFD of the SIS is obtained by dividing the TF by the total PFDs of IPLs (excluding SIS) and is given by the following equation.

$$PFD \text{ required for SIS} = \frac{TF}{\text{Mitigated Consequence}} \tag{4}$$

Equation 4 is repeated for each category of loss and corresponding Tolerable Frequency in Table 1 and the highest value of SIL obtained is taken for implementing the SIS.

Step 5: The PFD required for SIS is categorized as per IEC 61511 shown on Table 2.

Range of failures. Average Probability of Failure on Demand	Risk Reduction Factor	Category of Safety Integrity Level SIL
$\geq 1 * 10^{-5}$ to $< 1 * 10^{-4}$	$>10,000$ to $\leq 100,000$	SIL 4 ^{**}
$\geq 1 * 10^{-4}$ to $< 1 * 10^{-3}$	>1000 to $\leq 10,000$	SIL3
$\geq 1 * 10^{-3}$ to $< 1 * 10^{-2}$	>100 to ≤ 1000	SIL2
$\geq 1 * 10^{-2}$ to $< 1 * 10^{-1}$	>10 to ≤ 100	SIL1
$\geq 1 * 10^{-1}$ to $< 1 * 10^1$	>10	SILa [*]

^{**} SIL4 is not normally used in Process industries

^{*}SILa denotes that there is no need to assign a SIL level to the SIS under consideration.

Table 2: Range of Average Probability of Failures on Demand & Safety Integrity Levels.

4. APPLICATION TO IPLS OF OIL AND GAS SEPARATOR

The calculations described under 3 are illustrated for a typical industrial Oil and Gas Production separator shown in Figure 2.

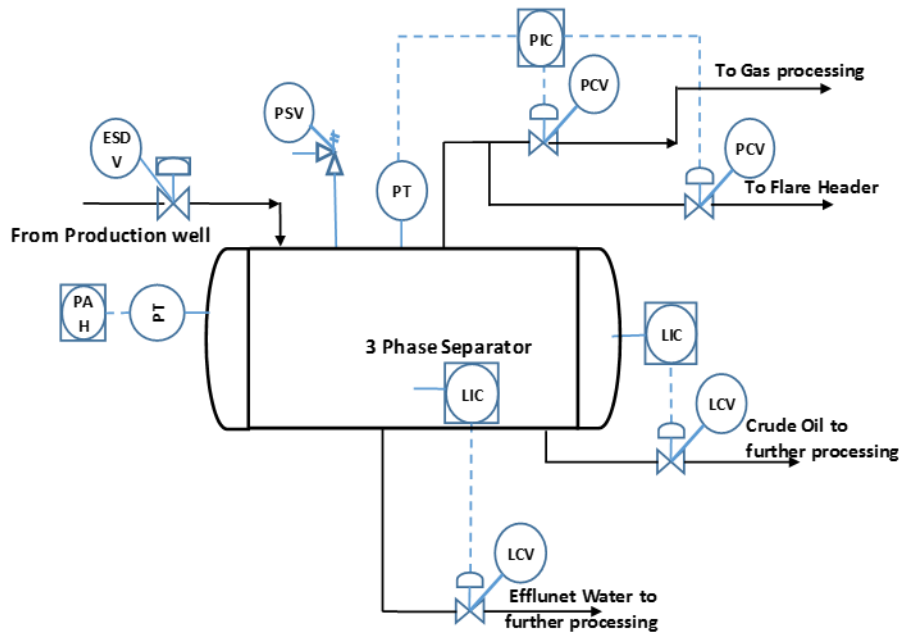


Figure 2: Typical Oil & Gas separator showing the Independent Protection Layers

The Initiating Events for an overpressure scenario in the separator are:

- a) Pressure surge from upstream well which suddenly raises the pressure inside the Separator vessel. Frequency 0.1 per year.
- b) Fail to open situation for the Pressure Safety Valve (PSV). PFD =0.000212 (Based on CCPS & HSE UK database)

The hazardous consequences are vessel failure, loss of containment, fire and explosion which are of highest severity.

The IPLs are:

IPL1: Adequate process and mechanical design of the separator vessel is the first layer of protection, which is not usually considered in SIL calculations (PFD=1.0)

IPL2: Basic Process Control Systems-here there are two, the Pressure Control Valve PCV for controlling the vessel pressure (BPCS1) and the other PCV for letting the gas out to the flare in case the pressure goes up beyond the set point (BPCS2). They are not independent and therefore PFD of both the control systems together are taken as 0.10.

IPL3: The SIS forms the next IPL; namely the Emergency Shutdown Valve (ESDV) that comes into action independently once the BPCS and Operator action has failed. PFD is taken as 0.0008 from CCPS & HSE UK database. SIL calculations are done without considering this. PFD is set to 1.

The (PAH) alarm coming from the control system is meant to initiate Operator action to control the sudden rise in pressure. However Operator action is not considered as an IPL in this paper. Depending on company's policies this IPL may be included in SIL calculations.

With Initiating Event frequency of 0.1 per year and Enabling Event probability of 0.1, calculation for the SIS is put in a spread sheet given below in Figure 3:

Tolerable Risk 1E-06

Initiating event description	Initiating Event Frequency / Year	Enabling Event Probability	Probability of Conditional modifiers	INDEPENDANT PROTECTION LAYERS						Mitigated Consequence Frequency without SIS
				IPL1 Process / Mechanical Design	IPL2 BPCS1	IPL3 Operator Response to Alarm-NotConsidered	IPL4 Independent SIF	IPL5 F & G Detection	IPL6 Others-None	
U/S OR D/S DISTURBANCE	1.00E-01	0.1	1.0	1.0	0.1	1.0	1.0	1.0	1.0	1.00E-03
Pressure Safety Valve	2.12E-04									2.12E-04
TOTAL CAUSE FREQUENCY	1.00E-01						F (event - without SIS)		TOTAL	1.21E-03

PFD Required For SIS	8.25E-04
Risk Reduction Factor	1212
Required SIL for SIS	3

Figure 3: Spreadsheet calculations for determining the Safety Integrity Level of the ESDV

In this case the Mitigated Consequence frequency is 1.00 E-03, whereas, based on the severity, the TF of consequences is placed at 1.00 E-06, (see Table 1) which is lower. Therefore the SIL level of the SIS is arrived by substituting the above values in to equation 4.

$$PFD \text{ required for SIS} = \frac{1 * E - 06}{1 * E - 03} = 1 * E - 03$$

The above value is in category of SIL3 (See Table 2) and thus the SIS has to designed with SIL3 reliability.

5. BAYESIAN NETWORKS

A detailed description of Bayesian Network (BN) is not attempted in the paper. Interested readers can go through any of the several books on the subject e.g. Pourret et al (2008) [4], Kjærulf et al (2005) [5], Neapolitan (2003) [6]. Briefly BN is a directed acyclic graph (DAG) in which the nodes represent the system variables and the arcs symbolize the dependencies or the cause–effect relationships among the variables. A BN is defined by a set of nodes and a set of directed arcs. Probabilities are associated with each state of the node. The probability is defined, a priori for a root (parent) node and computed in the BN by inference for the others (child nodes). Each child node has an associated probability table called Conditional Probability Table (CPT).

The computation of the net is based on the Bayes Theorem which states that if P (B) is probability of B happening, then P (A/B) is probability of A happening given that B has happened, given P (B) not equal to zero

Following gives the most common form of Bayes equation

$$P(A|B) = \frac{P(B|A)*P(A)}{P(B)} \tag{5}$$

$$\text{Where } P(B) = P(B|A) * P(A) + P(B|A') * P(A') \tag{6}$$

A' stands for A not happening

The equation 5, right hand side represents the prior situation –which when computed gives the left hand side –called posterior values. The value P (A) is the prior probability and P (B | A) is the likelihood function –which is data specific to the situation. P (B) is the probability of B- which is calculated from equation 6.

6. USING BAYESIAN NET (BN) FOR SIS CALCULATIONS

6.1 Kannan [7] used BN to calculate the PFD of the IPLs specifically for a separator using BN. He described the failure IPLs consisting of Level Control Valve and 2 Emergency Shutdown Valves (ESDVs) in series including the components; Level Transmitter (LT) and Programmable Logic Controller (PLC). Common cause failure due to cold weather of the LT was illustrated in the paper. SIL calculations were not given.

The spreadsheet calculations given in Figure 3 are mapped to BN using Netica software. See Figure 4.

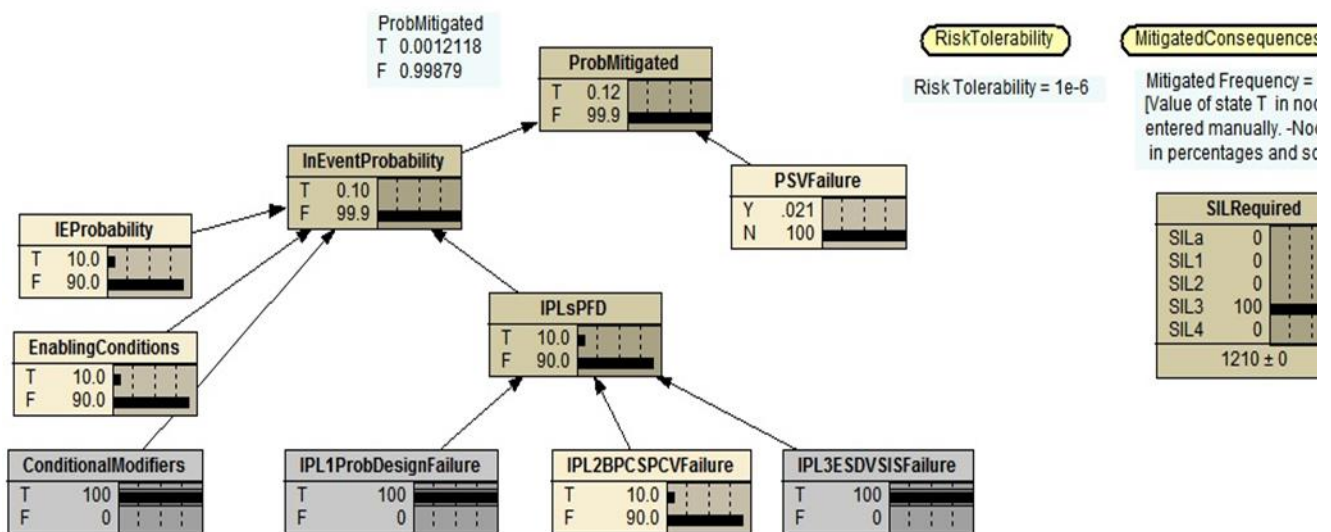


Figure 4: Bayesian Network for IPLs for Oil & Gas Separator

Note: Netica nodes do not display decimals beyond 4 digits. Higher decimals are obtained through the Report feature in the Menu and added in the Figure.

The BN in Figure 4 is equivalent to the spreadsheet including the SIL calculations.

Initiating Events are InEventProbability from upstream disturbance and PSV failure. InEventProbability is the child node of IEProbability, EnablingConditions and ConditionalModifiers nodes. IPLsPFD is the child node for the parents of IPL1 to IPL3 nodes. The nodes are parameterized with the probability values used in the spreadsheet. The PFD calculation in the node IPLsPFD is implemented using the AND Gate feature in Netica. No credit is taken for the SIS layer (PFD =1).

Once the probability for consequences is calculated by the BN (value of state T in node ProbMitigated), the value is input manually into the constant node named MitigatedConsequences. The SIL is calculated and presented in the node SILRequired. The values in Netica nodes are in percentage probability and so when entering the value manually the same has to be divided by 100 to match with the probability values in which the Risk Tolerability is expressed. This BN represents a general structure of network for any IPLs with a SIS.

The probability values of any of the nodes can be changed and the resultant change in the Mitigated Consequence node can be seen easily, including backward propagation of probabilities when a child node is changed.

6.2 Adding Influencing Factors

Unlike that in spreadsheet calculations, influencing factors that affect the BN can be easily added to the BN. For example testing affects the PFD of PSV. Adding of the influencing factor of PSV testing for the node PSVFailure is given in Figure 5. The states are OnSchedule and NotOnSchedule. The input probabilities for this node are the initial or prior states, which can be updated based on actual data. For BN in Figure 5, the PSV failure data given earlier (PFD = 0.00021) is assumed to be when the testing as per schedule. When testing is not as per schedule the PFD is taken as 0.0007. These values are entered in CPT for node PSVFailure.

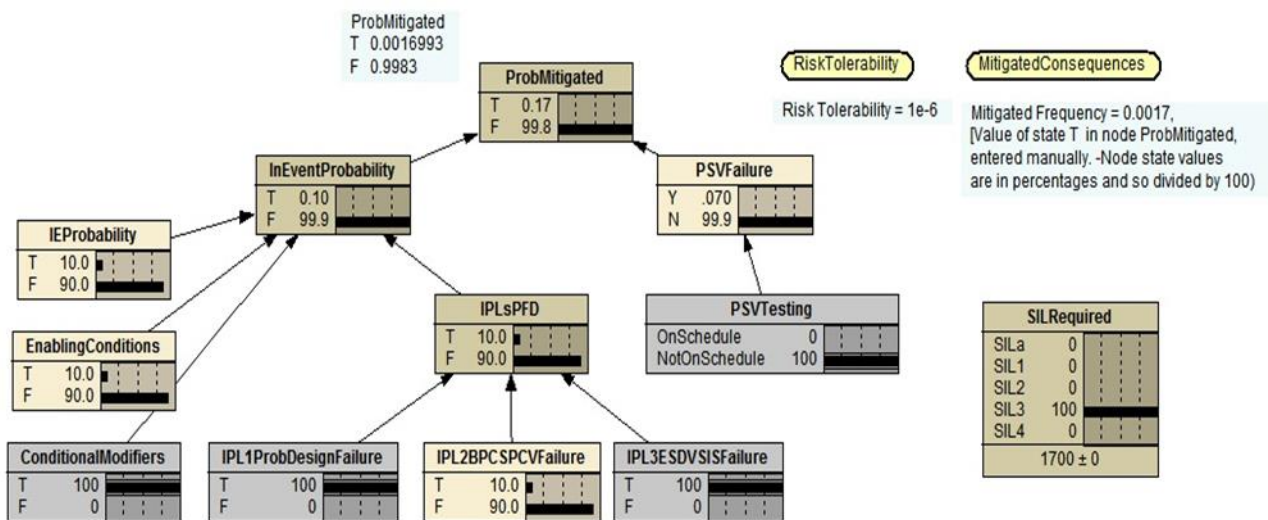


Figure 5: BN for IPLs with addition of influencing factor for PSV failure.

6.2.1 Changing influencing factors

The influencing factors can be changed based on actual situation. For example, if the PSV testing is not as per schedule the state in NotOnSchedule can be set 100, to see the effect of the same on other nodes. See Figure 5. The PFD for the PSV goes up to 0.0007 and the probability of Mitigated Consequence goes up to 0.0017 from 0.0010. Though the SIL rating is not affected in this case, visualizing such influencing factors gives better insight into the state of the IPLs.

6.3 Common Cause Failures (CCF)

CCFs can be implemented quite easily in BN. See the Figure 6, where a CCF for the Control Valve & ESDV failure has been added to the BN. The CCF is based on a scenario of fire. If there is fire, there is a probability that the instrument air piping and cables could be damaged rendering both the valves ineffective. In the node for Fire, probability of Fire is entered as 0.02 & in CCF node CPT, the probability of CCF being true when there is fire is entered as 0.80. It be seen that there is slight increase in the PFD for BPCS (goes up to 0.105 from 0.10) resulting in an increase of Mitigated Consequence to 0.0013 from 0.0012.

Though the change in the probability is very minor, it may impact the SIL level sometimes.

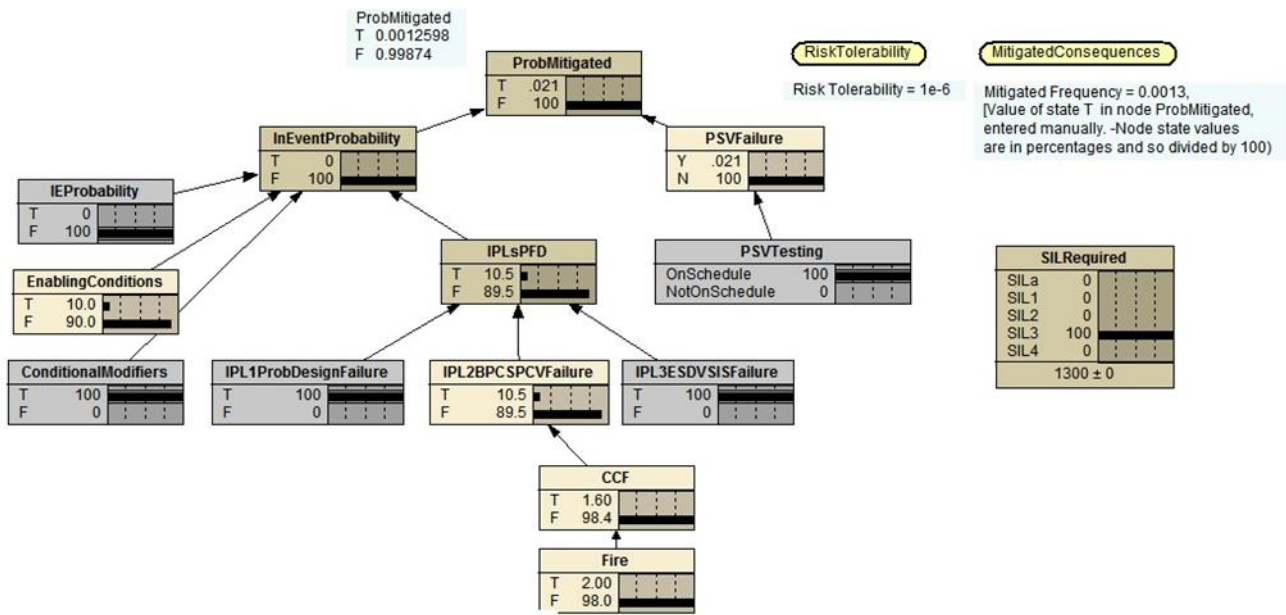


Figure 6: BN showing addition of Common Cause Failure of control valves due to fire

6.4 Noisy-And Distribution

Netica has a facility to use Noisy-OR or Noisy-And distribution. This can be used to model the PFD of failure of IPLs more realistically. Noisy-(logical) distribution essentially represents the noise in the system, which cannot be adequately modelled since in reality all causes to an event or consequence cannot be identified. Further if there are many causes (parent nodes), the entries in the CPT rises exponentially.

Noisy-OR distribution can be used when there are several possible causes for an event, any of which can cause the event by itself, but only with a certain probability. Also, the event can occur spontaneously (without any of the known causes being true), which can be modelled with probability 'leak'. (This can be zero if it cannot occur spontaneously).

Noisy-And distribution is used when there are several possible requirements for an event, and each of which has a probability that will actually be necessary. Each of the necessary requirements must pass for the event to occur. Noisy-And can also model a situation where the event may not occur even when all requirements are passed.

In the case of IPLs for Oil & Gas separator, Noisy-And distribution is appropriate since all the IPLs have to fail necessarily for event to occur.

The equation for NoisyAnd is written in Netica as

$$P(IPLsPFD | IPL1ProbDesignFailure, IPL2BPCSPCVFailure, IPL3ESDVSISFailure) = \text{NoisyAndDist}(IPLsPFD, 0.0, IPL1ProbDesignFailure, 0.6, IPL2BPCSPCVFailure, 0.6, IPL3ESDVSISFailure, 0.6)$$

(7)

The probability values in equation 7 are assumed values and are not based on actual data. No probability leak is assumed; that is, for combined IPLs given by node IPLsPFD, the condition T=100 is possible only when all IPLs have failed.

Netica Help file at Norsys [8] provides further details of the syntax for the above distribution.

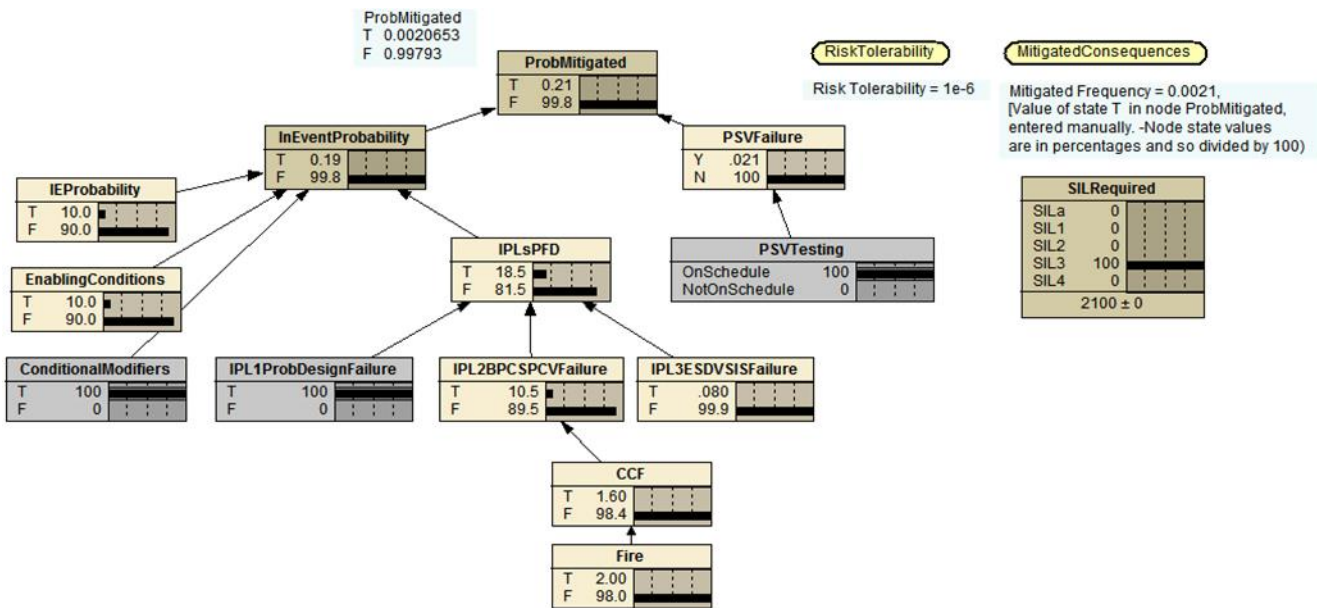


Figure 7: BN showing failure probabilities with NoisyAnd distribution

The equation populates CPT table for IPLsPFD with values based on the equation as given Table 3. The compiled BN given in Figure 7 shows that there is a small increase in the overall probability of Mitigated Consequence to 0.0021. Thus the NoisyAnd distribution offers a method to input probabilistic values to the IPLsPFD node instead of the AND feature that computes the CPT based on T & F only as used in BN shown in Figure 4.

T	F	IPL1ProbDesignFailure	IPL2BPCSPCVFailure	PL3ESDVSISFailure
1	0	T	T	T
0.4	0.6	T	T	F
0.4	0.6	T	F	T
0.16	0.84	T	F	F
0.4	0.6	F	T	T
0.16	0.84	F	T	F
0.16	0.84	F	F	T
0.064	0.936	F	F	F

Table 3: Probability values in Conditional Probability Table populated by NoisyAnd distribution

7. DISCUSSION

BN can model the IPLs, its influencing factors and failure rates in a visually easy and understandable way. SIL calculations can be implemented as a part of BN. Influencing factors and CCFs can be added to any node and its impact other nodes can be studied in detail. The ability to include probabilistic Noisy-And distribution to populate the CPTs increases the power of and applicability of the BN. Fine tuning the Noisy-And distribution based on site specific data is a challenge, and more work need to be done in this area. Application of Bayesian Methods to analysis of IPLs and SIL calculations will help improve the predictive & diagnostic power of the model.

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CLASSIFICATION OF POWER RESERVES OF ELECTRIC POWER SYSTEMS

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A.A. Rykova

INTRODUCTION

Single Electric Energy System of Russia (SEES) is one of the largest energy systems in the world. It includes 69 regional power systems that, in turn, form seven interconnected power systems (IPS), namely: [IPS of the East](#), [IPS of Siberia](#), [IPS of the Urals](#), [IPS of the Middle Volga](#), [IPS of the South](#), [IPS of the Centre](#), and [IPS of the North-West](#). Electric energy complex of SEES of Russia includes more than 700 power plants whose capacity exceeds 5 MW. As of the end of 2013 the installed capacity of power plants within SEES of Russia totalled 226,470.18 MW. All the IPS are connected by inter-system high-voltage 220-500 kV (and higher) power lines that operate in parallel [1]. Power system of the Crimea is currently referred to as technologically independent power systems (PS) of Russia.

The main specified functions of IPS are [2]:

- Functions related to their purpose;
- Functions related to the fact of their creation.

Ability of the object to perform the specified function stipulated by its purpose is referred to as power supply reliability [3-5]. The purpose of a power system is to supply power for consumers. Power supply reliability is PS ability to supply consumers with the power of required quality and following the specified power consumption schedule. Power supply reliability is ensured by: change in the power system configuration and structure, including control means; change in the reliability and in other performances of its components (the main and auxiliary equipment, control devices); creation of different redundancies (power reserve, transmission capacity reserve for power lines, fuel stock at thermal power plants (TPP), water stock at hydro power plants (HPP), etc.; perfection of PS operation [6].

CAPACITY RESERVE

Capacity reserve is one of the main ways of raising the power system reliability. Reserve (réserve in French) is a stock of something for the case of need; a source of new materials, forces [7]. Capacity reserve is a method of raising the object reliability by incorporating the redundancy. As applied to power systems we consider structural, functional, temporal and informational reserves [6].

Capacity reserve may be located in the generating and consuming parts of the system [8] (Fig. 1). Difference between the available PS capacity and its load at a given time moment is referred to as generating capacity reserve of the system [6]. *Reserve in the consuming part of the system is power the consumer is ready to forsake on the reimbursement base in case of the PS operation failure.* Maintaining the balance between power production and consumption that is ensured at the expense of consumers should be considered as a temporary measure and, as a rule, is used in the emergency and post-emergency PS operation provided that all the generating capacity

reserves of required mobility have been exhausted. If necessary, the generating capacity reserve of required mobility will be recovered by the reserve start-up in the consuming part of the system. At the same time the availability of generating capacity reserves (but of insufficiently high mobility) and its further start-up would allow connection of disconnected consumers, i.e., reserve recovery in the consuming part of the system. Thus, in this event the generating capacity reserve is replaced by the reserve of the consuming part of the system and vice versa.

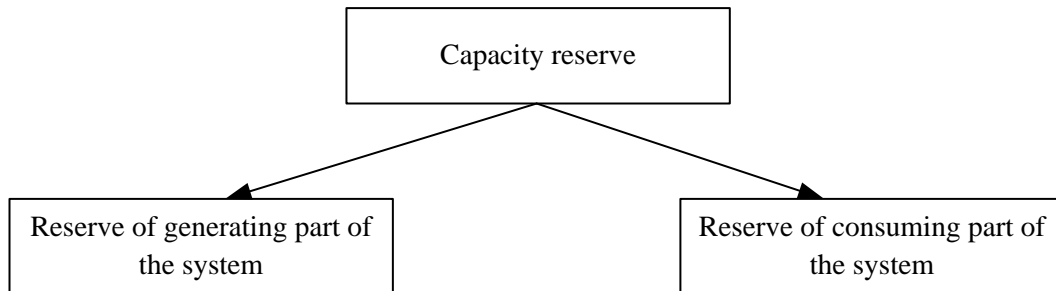


Fig. 1 Power reserve location in the system

GENERATING CAPACITY RESERVE

Functionally the PS generating capacity reserve is divided into two components: maintenance reserve and operating reserve.

Maintenance reserve is needed for balancing the reduction of PS working capacity when the main equipment of the system is under scheduled maintenance or modification [6]. Scheduled maintenances are deemed to be the main way of controlling the technical state of equipment and restoring its resource. Scheduled maintenances include regular maintenances of equipment and equipment overhauls.

Operating reserve is intended to balance the imbalance between power generation and consumption that is caused by equipment failure and sudden deviation of consumer load from the expected value. For this reason the operating reserve is divided into emergency reserve and load reserve (Fig. 2). Emergency reserve is needed for compensating the losses of power used for meeting the load at emergency and during non-scheduled shutdowns of the main equipment of power plants. Load reserve is intended to balance the power imbalance caused by load deviation (its growth or fall) from the expected one. Unfortunately, currently there is no sufficiently well grounded classification of the operating reserve of generating capacity except for its division into emergency and load reserves.

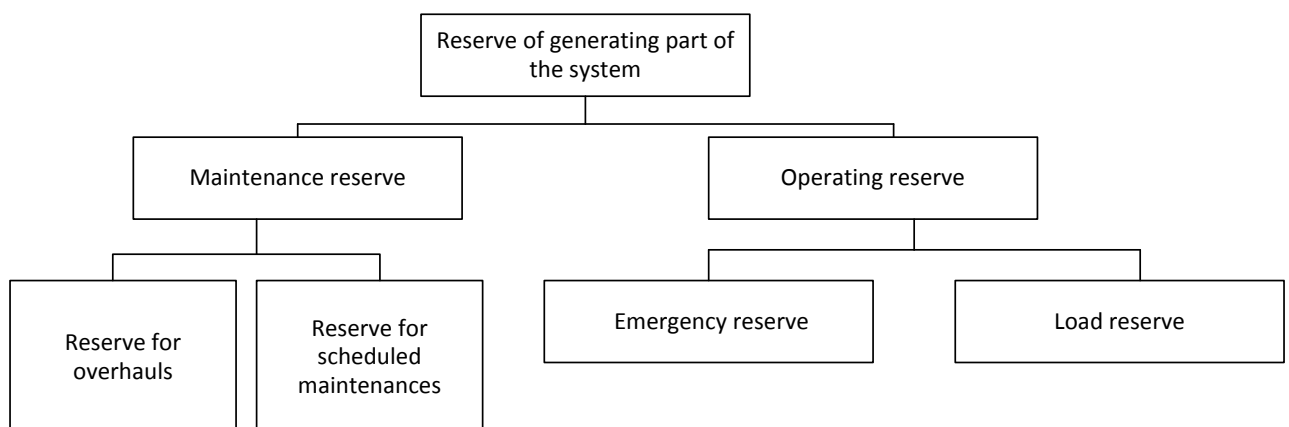


Fig. 2 Classification of generating capacity reserve

Terminology used in literature for engineering workers can mislead the reader. Some examples:

- «Operating ("hot") capacity reserve is a share of a capacity reserve intended for balancing the imbalance between production and consumption that is caused by equipment failures, emergency or accidental reduction in the PS operating capacity or by unexpected on-line increase of the consumer's load" [1]. According to this definition hot reserve is actually an operating reserve.

- "Hot (or spinning) reserve is created by boilers under steam pressure and by idle running turbine generators" [9]. According to this definition a hot reserve is a spinning reserve.

- "Spinning reserve of active capacity is capacity reserve located at operating units and units with the start-up time of up to 5 minutes" [1]. According to this definition the fast-start reserve is also a spinning reserve.

- "Cold reserve is ensured by special reserve units with small start-up and spin-up time" [9]. Period of the cold reserve start-up is deemed to be from 2 to 24 hours and more. Units with small start-up time usually have a power-on reserve.

- A hot reserve is sometimes referred to a power-on reserve though time of its start-up makes 1-2 hours, whereas start-up time of the power-on reserve is just minutes [6, 10]. A hot reserve should be referred to a standing reserve.

Further we give classification of the operating reserve that takes into account such characteristics as state of equipment on which the reserve is located, method of its start-up, mobility degree and functionality.

An operating reserve of generating capacity is located on the equipment that can be in different states. Depending on the equipment state it can be the power-on and standing reserve (Fig. 3).

Under the power-on reserve we mean reserve capacity of currently operating units that can be used immediately (started-up in minutes). A power-on reserve includes:

- A spinning reserve is an operating reserve of the system that is placed on the operating underloaded units of power plants;

- A fast start-up reserve is an operating reserve of the system that is placed on the fast start-up units whose full loading time does not exceed the time of spinning reserve start-up.

- A standing reserve is capacity of idle properly operating units of power plants within the energy system. A standing reserve equals to difference between operating and power-on capacity of the power system. Standing reserve includes:

- A hot reserve of the system is an operating reserve located at TPP units where a boiler is in hot reserve [10].

- A cold reserve of the system is an operating reserve located at TPP units where boilers are in cold reserve.

It should be mentioned that a hot reserve of the system also includes an operating reserve located at TPP with transverse relations where the boiler is maintained in the hot state, and the turbine generator is shutdown.

When the boiler is immobilized for a hot reserve, the specified steam pressure and temperature are maintained in it.

A power-on reserve is started-up into minutes, whereas activation of a standing reserve requires 1-2 hours and more.

Functionally (with account of mobility) we can single out the following types of reserves:

- First priority reserve for primary frequency control (started up into seconds);
- Second priority reserve for secondary control (power flows limitation; started-up into tens of seconds);
- Third priority reserve for the secondary control (frequency control and control of power flows over transmission lines; started-up into up to five (5) minutes);
- Fourth priority reserve for tertiary control (fast mode adjustment for the system transition into a more preferable (from the standpoint of reliability) state, including for restoring the control range of power plants connected to an automatic load-frequency control system; start-up time is up to 15-20 minutes);
- Fifth priority reserve for balancing the power imbalances and for complete optimization within the considered hour of the system operation;
- Sixth priority reserve for compensating the power imbalances that can be identified at a lead time exceeding the start-up time of a standing reserve (replacement of the fifth priority reserve and optimization of operating conditions within the considered day of the system operation).

Thus, we can conclude that *a power-on reserve is a reserve that ensures primary, secondary and tertiary control in PS.*

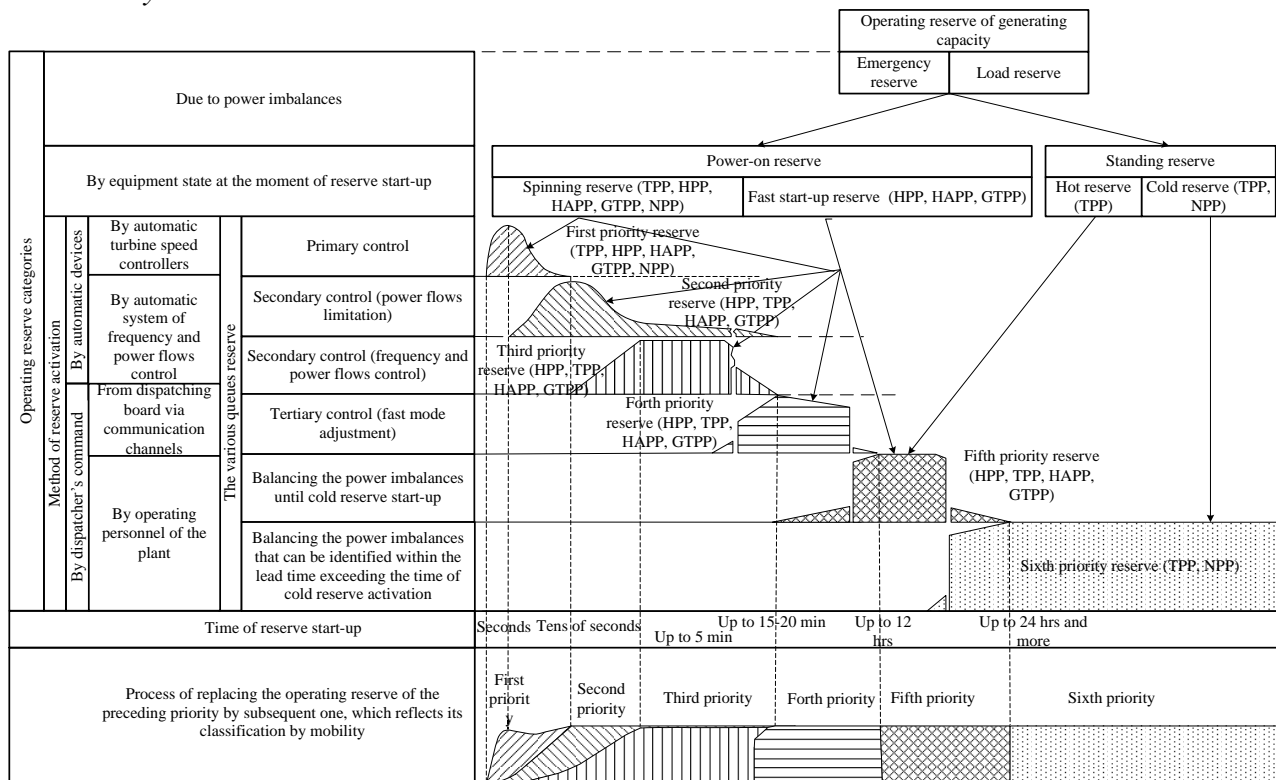


Fig. 3. Classification of the operating reserve of generating capacity of the system

(Note: HAPP – hydroaccumulating power plants; GTPP – gas-turbine power plants; NPP – nuclear power plants)

Within the first several minutes the power imbalance is covered by the first priority reserve that is started up by turbine speed control within several seconds (primary control). Primary control is characterized by certain statics and does not maintain the required frequency.

Secondary control ensures recovery of the given value of frequency and power flows in the cross-sections where they are controlled. In this case units of power plants that are not involved in the secondary control but participate in the primary control go back to initial operating mode.

For restoring the control ranges of power plants connected to an automatic load-frequency control system the reserve is started up by the operating personnel of power plants following dispatcher's command or directly from the dispatching board of the system (fast adjustment of operating conditions), i.e., tertiary control.

Subsequent (slow) economic adjustment of operating conditions is done observing all the limitations related to the fifth priority reserve start-up. A reserve ensuring the tertiary control in this case becomes available. Sixth priority reserve is started up and releases the fifth priority reserve for reducing the operating costs in the system. Such an alternate start-up and replacement of reserve of the preceding priority by the subsequent one ensures the required controllability of the system.

RESERVE OF THE CONSUMING PART OF THE SYSTEM

Similarly to a generating capacity reserve a reserve of the consuming part of the system participates in the power and frequency control. Functionally the reserve of the consuming part of the system is an operating and maintenance reserve (Fig. 4).

Operating reserve is deemed to be the capacity spared due to disconnection or limitation of consumers for balancing the power imbalances caused by emergency shutdown of generating and main equipment or in the event of load excess over the expected one.

Maintenance reserve is deemed to be the capacity spared due to disconnection or limitation of consumers for balancing the power imbalances caused by scheduled maintenances of generating equipment in the event of insufficient maintenance reserve of generating capacities in the power system. In the event of insufficient or lack of generating capacity reserve the reserve is formed at the expense of consumers since operation of electric power systems (including maintenances of generating equipment) without generating capacity reserve is not possible. Thus, generating capacity reserve in these events is formed owing to efficient use of reserves in the consuming part of the system. The difference between a generating capacity reserve and a reserve in the consuming part of the system lies in the fact that *the generating capacity reserve is, as a rule, a structural reserve, whereas reserve in the consuming part of the system can be considered as an example of a functional reserve.*

Further we consider classification of the operating reserve in the consuming part of the system.

Depending on the cause of power imbalances this reserve is divided into emergency and load ones (Fig. 4).

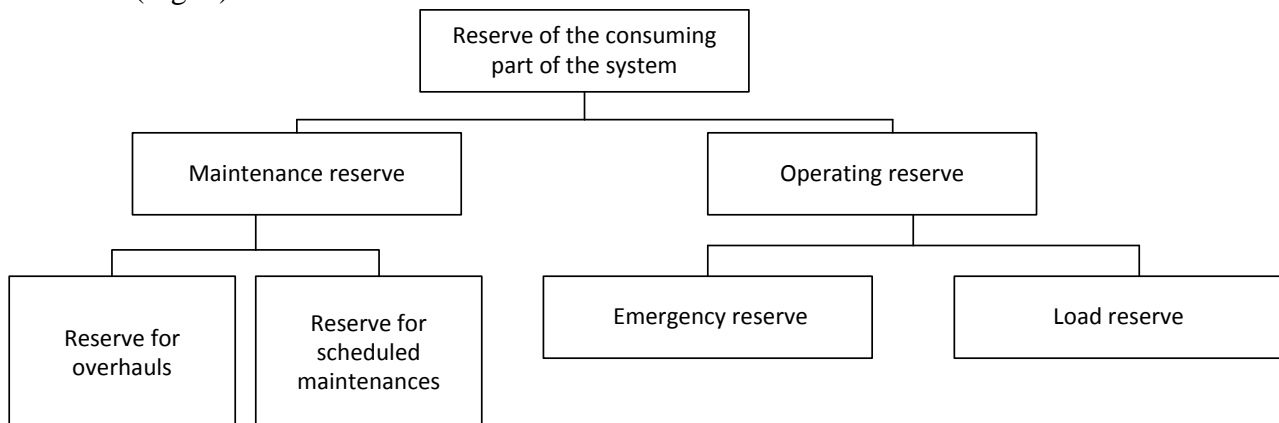


Fig. 4 Classification of the reserve of the consuming part of the system

Load reserve is deemed to be the capacity spared due to disconnection or limitation of consumers for balancing the power imbalances caused by deviation of the load from expected one. An emergency reserve in the consuming part of the system is capacity spared due to disconnection or limitation of consumers for balancing the power imbalances caused by emergency shutdown of the generating and main equipment of the system.

Reserve of the consuming part of the system includes consumers with controllable load and consumers-controllers.

A group of consumers with controllable load includes consumers that due to their operating conditions can rapidly reduce power consumption from the network. Such consumers provide (on the reimbursement base) services for emergency avoidance in the Single Electric Energy System of Russia. Certain groups of consumers are offered special (lower) tariff on condition that they can be disconnected at any moment should such an event occur in the network [1].

A load consumer-controller is a power or heat consumer whose operating mode allows power or heat consumption limitation both in emergencies and in peak hours for levelling the load schedule in the power system and for raising the load during periods of minimum consumption. Based on the difference between the basic and privileged tariffs one can assess the economic efficiency of HPP share in the manoeuvrable secondary frequency control after emergencies or after considerable imbalance between power production and consumption in PS [11].

Classification of the reserve of the consuming part of the system based on its functionality with account of mobility is given in Fig. 5.

With account of the consumers' state at the moment of the reserve start-up the operating reserve is divided into power-on and standing reserves:

- A power-on reserve: its start-up time exceeds several minutes (not more than 10-15 minutes);
- A standing reserve: its start-up time does not exceed 12 hours.

A power-on reserve includes:

- Consumers ensuring the load control effect;
- Consumers with controllable load that are connected to the Automatic Load Disconnection System (ALDS) and to the Automatic Line Overload Limitation System (ALOLS);
- Consumers connected to the system of automatic underfrequency load shedding systems I&II;
- Consumers that can be disconnected following the dispatcher's command or directly from the control board.

A standing reserve may include consumers that allow limitations.

A consuming part of the system participates in primary, secondary and tertiary frequency and power control and in balancing the power imbalances by starting up the standing reserve (by limitation of consumers).

Primary control is performed, first, due to load control effect (in the event of frequency raise the primary controlling power of interconnected consumers is positive, i.e., consumption grows; in the event of frequency fall it is negative, i.e., self-unloading occurs), second, by emergency control

devices (underfrequency load shedding systems I, ALDS, and ALOLS). Start-up time is from fractions of seconds to 0.3 second [12].

Secondary control is performed by consumers connected to underfrequency load shedding systems II. Time of the reserve start-up is up to 90 seconds.

Tertiary control is ensured by consumers disconnected by a dispatcher, and by consumers-controllers (ensuring fast adjustment of the operating mode) that are ready to reduce (raise) the power consumption following the dispatcher's command. Time of the reserve start-up is up to 15 minutes.

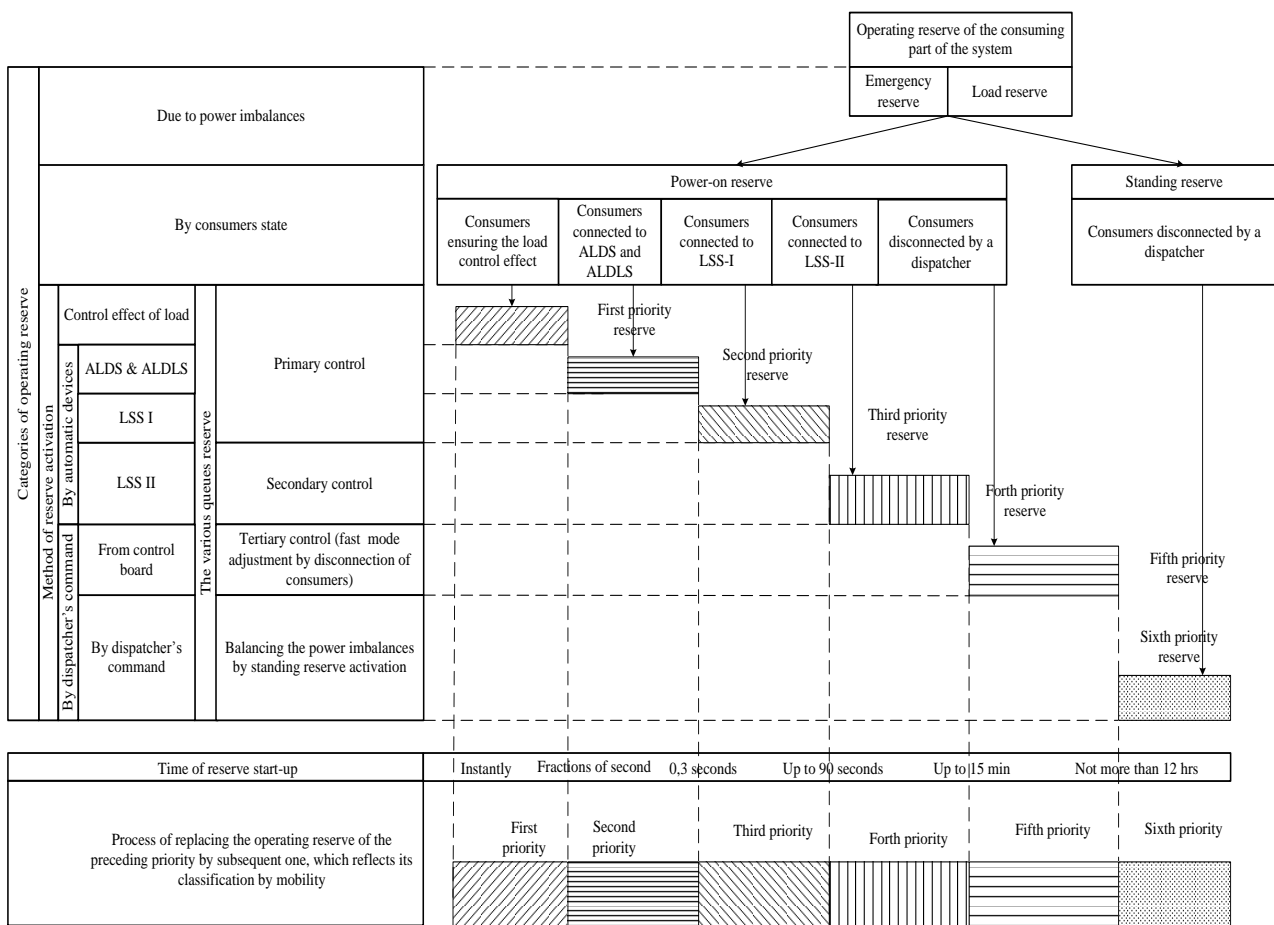


Fig. 5. Classification of the operating reserve in the consuming part of the system.

Now we may offer classification of the operating reserve of the consuming part of the system based on its functionality with account of mobility:

- First priority reserve for primary frequency control due to load control effect (started up practically instantly);
- Second priority reserve for primary frequency control and power flows limitation owing to using the reserve located at consumers with controllable load that are connected to ALDS and ALOLS systems (started-up into fractions of seconds but it is less than start-up time of the reserve connected to the underfrequency load shedding system I (LSS-I));

- Third priority reserve for primary frequency control due to using the reserve located at consumers connected to the underfrequency load shedding system I (started-up in not more than 0.3 seconds);
- Fourth priority reserve for secondary frequency control due to using the reserve located at consumers connected to the underfrequency load shedding system II (LSS-II); start-up time is not more than 90 seconds;
- Fifth priority reserve for tertiary control due to using the reserve located at consumers that are disconnected by a dispatcher from the control board; start-up time is not more than 15 minutes;
- Sixth priority reserve for balancing the power imbalances by starting up the standing reserve located at consumers that allow temporary power limitation; start-up time is not more than 12 hours.

Consumers are disconnected and limited in accordance with the schedules of emergency disconnection or limitation of consumers.

Schedules of limiting the electric capacity use mode and schedules of limiting the electric power consumption mode are schedules according to which the consumers are preliminarily notified about the need to limit the electric power (capacity) consumption and they themselves perform technical (technological) measures ensuring the consumption reduction in the volumes and in the periods of the day indicated in the notice. Such schedules can be implemented without disconnection of the energy receiving devices and/or power lines.

Grounds for using the schedules of emergency limitation of power (capacity) consumers include occurrence or threat of occurrence of emergency conditions, when parameters of electric conditions go beyond the permissible values, including due to [7]:

- Occurrence of non-permissible power shortage and capacity in the power system or in its separate parts that leads to the electric current frequency shedding below 49.8 Hz or to voltage reduction below minimum permissible levels;
- Insufficient power and electricity production in separate parts of the system that leads to inadmissible overload of power lines, transformers and other main equipment or originates the risk of such overload;
- Damage of the main equipment, including in the event of emergencies, natural and man-induced disasters;
- Damage of technological control systems, technological networks and emergency control devices.

Schedules of temporary disconnection of consumers are schedules following to which the power producing company disconnects power lines without preliminary notice to consumers. Disconnection of power units directly by consumer's staff is also possible.

Schedules of temporary disconnection of consumers are activated if it is impossible to apply the schedules of power consumption limitation mode in the time period needed for preventing the emergency electric conditions, in the event the consumers do not follow dispatcher's commands on activating the power consumption limitation mode, as well as in the event the causes necessitated the activation of power consumption limitation mode persist after introduction of the consumption limitation mode.

Power Supply Company shall notify the consumers about temporary power disconnection schedules immediately after their activation.

Schedules of power (electricity) consumption limitation mode are introduced starting from 0 hr 00 min of the following day. The power supply company shall notify the power consumers about that not later than at 2:00 pm of the current day [13].

Schedules of power (electricity) consumption limitation mode are introduced following the dispatcher's Order of the regional dispatching board. The Order shall include:

- Area (operating zone, power sector, and/or power facility) where emergency limitations are introduced;
- Grounds for limitations;
- Time of the limitations start and end.

Primary receivers of commands on emergency limitations distribute the limitation volumes specified by a dispatching centre and deliver information on introduction of the consumption limitation mode to secondary receivers of commands on emergency limitations and to appropriate power supply companies.

In the event some consumer does not follow the commands on introduction of power consumption limitation mode, the operating personnel of the main company has the right to disconnect that consumer from power supply centres, up to the emergency reservation [13].

SUMMARY

1. Definition of the operating reserve of generating capacity is given vs the state of equipment it is placed on. Based on that feature the spinning reserve, fast start-up reserve, hot reserve, and cold reserve are distinguished.

2. Consideration is given to such notions as power-on and standing reserves, their relation to the spinning reserve, fast start-up reserve, hot reserve, and cold reserve.

3. It has been shown that primary, secondary and tertiary control in the power system is ensured by the power-on reserve of generating capacity.

4. The given classification of the operating reserve of generating capacity is based on its functionality with account of mobility. Several priorities of the reserve have been singled out, which allows one to: make correlation between the method of the reserve start-up and its mobility; uniquely determine requirements to the operating reserve the observation of which would ensure the required controllability of PS owing to automatic devices and operating & dispatching personnel.

5. Definition of the reserve in consuming part of the system has been proposed (first).

6. Classification of the reserve in consuming part of the system has been given with account of its mobility and functionality.

7. Consideration was given to capabilities of the reserve of consuming part of the system under primary, secondary and tertiary control.

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p-BIRNBAUM SAUNDERS DISTRIBUTION: APPLICATIONS TO RELIABILITY AND ELECTRONIC BANKING HABITS

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ABSTRACT

Birnbaum and Saunders (1969) introduced a two-parameter lifetime distribution which has been used quite successfully to model a wide variety of univariate positively skewed data. Diaz-Garcia and Leiva-Sanchez proposed a generalized Birnbaum Saunders distribution by using an elliptically symmetric distribution in place of the normal distribution. In this paper, we construct a new distribution, say p-Birnbaum-Saunders distribution, by introducing a new parameter 'p', which influences both Skewness and Kurtosis. The deviation from the behaviour of Birnbaum and Saunders distribution can be accommodated in the new p-Birnbaum Saunders (p-BS) distribution. Different properties of this distribution are obtained. Most of the data from Reliability and Banking sector is having skewness and their frequency curve is from among the class of p-BS distribution. A data set from internet banking sector is considered.

1 INTRODUCTION

There are many distributions for modeling lifetime data. Among the known parametric models, the most popular are the Birnbaum-Saunders, Gamma, lognormal and the Weibull distributions. Sometimes, these are not appropriate for a given data. The two-parameter Birnbaum-Saunders (BS) distribution was originally proposed by Birnbaum and Saunders [1969] as a failure time distribution for fatigue failure caused under cyclic loading. It considered only the material specimens which are subjected to fluctuating stresses by a periodic loading. Size of stress and crack are random.

Although the BS distribution was originally proposed as a failure time distribution for fatigue failure under the assumption that the failure is due to development and growth of a dominant crack, a more general derivation was provided by Desmond [1985] based on a biological model. Desmond [1985] also strengthened the physical justification for the use of this distribution by relaxing the assumptions made originally by Birnbaum and Saunders [1969]. Some recent work on the BS distribution can be found in Balakrishnan et al. [2007], Chang and Tang [1993,1994], Dupuis and Mills [1998], From and Li [2006], Lemonte et al. [2007], Rieck [1995,1999], Ng et al. [2003,2006], Owen [2006] and Xie and Wei [2007]. A review of different developments on the BS distribution until 1995 can be found in the book by Johnson et al. [1995].

The objective of this work is to study the behavior of Birnbaum Saunders distribution with one more **relevant** parameter. Not to argue its particular merits in applications over other distributions. It is a reasonable generalization. We study some behavior of the Birnbaum-Saunders distribution with one more parameter p, which influence both skewness and kurtosis strongly.

The paper is arranged as follows. Section 2 discussed basic definition of Birnbaum Saunders distribution. Section 3 introduced, a new distribution, p-Birnbaum Saunders distribution and studied some of its properties. Maximum likelihood estimation is given in section 4. Application to reliability analysis and Banking Habits are given at the section 5. Conclusion is given in last section.

2 BIRNBAUM SAUNDERS DISTRIBUTION

We considered the Birnbaum-Saunders data. It shows departure from the normality with s skewness and peaked frequency curve. Distribution function of Birnbaum-Saunders distribution is

$$F(t, \alpha, \beta) = N\left(\frac{1}{\alpha} \xi \left(\frac{t}{\beta}\right)\right), t > 0$$

where $N(\cdot)$ is the Normal distribution evaluated at $\xi \left(\frac{t}{\beta}\right) = [(t/\beta)^{1/2} - (t/\beta)^{-1/2}]$, $\alpha > 0$ and $\beta > 0$. The parameters α and β are the shape and scale parameters, respectively. Moreover, β is the median of the BS distribution. Consider the histogram of BS data and Normal curve drawn through it.

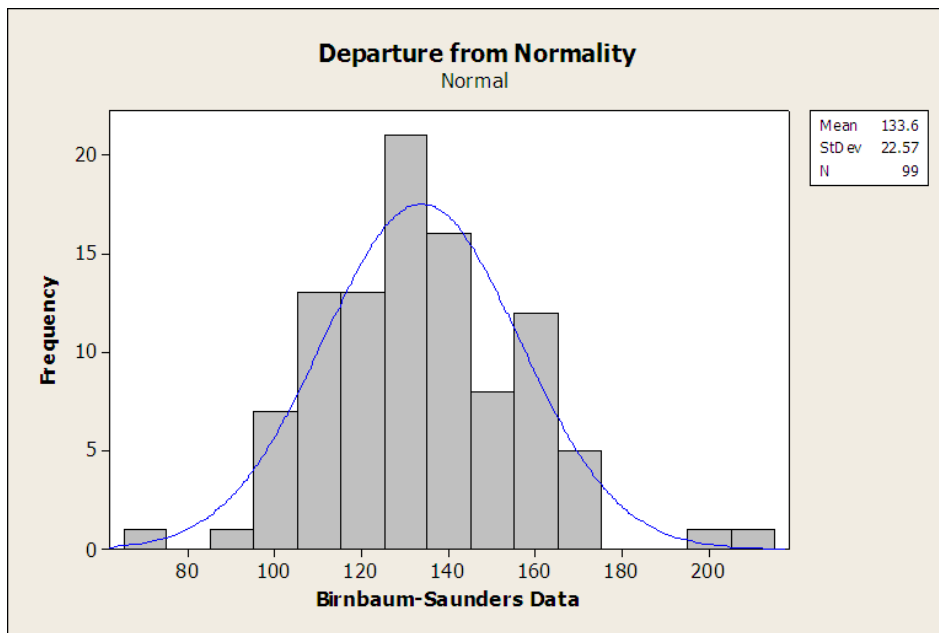


Table 1. Histogram of Birnbaum Saunders Data

The inference on this plot is this: It is a skewed data on positive axis. Mean of the distribution is placed left to the Normal mean. Curve is more peaked than Normal. Left and right tails have slight positive mass. Tail thickness is more than that of Normal tail. Existence of moments can be ensured. Flatness is very low.

From the above inference we can make some reasonable conclusions. The actual distribution will be positively skewed. Peakedness may vary with situation of varying stresses. But asymmetry still remains. Either Birnbaum Saunders, log-Normal, skew-Normal, Weibull etc distributions can be chosen as an approximate model.

But for the correctness in model selection, think about the smooth frequency curve drawn through the histogram. What will be the function, having desirable distributional properties, that suits our objective? Can we generalize Birnbaum Saunders distribution to approach more accuracy in probability calculation? Choosing a suitable frequency curve is an important area of research of distribution theory. A plausible model can be attained by attaching one more parameter to the Birnbaum Saunders distribution. We observed the behavior with different values of parameters.

3 p-BIRNBAUM SAUNDERS DISTRIBUTION

Use $\xi_p\left(\frac{t}{\beta}\right) = [(t/\beta)^p - (t/\beta)^{-p}]$ instead of $\xi\left(\frac{t}{\beta}\right) = [(t/\beta)^{1/2} - (t/\beta)^{-1/2}]$ in BS distribution.

It gives us more suitable flexible model for the data. We define p-Birnbaum Saunders distribution as follows

$$F(t, \alpha, \beta, p) = N\left(\frac{1}{\alpha}\xi_p\left(\frac{t}{\beta}\right)\right), t > 0$$

Probability density function is

$$f(t, \alpha, \beta, p) = \frac{1}{\alpha\beta p\sqrt{2\pi}} \left[\left(\frac{\beta}{t}\right)^{1-\frac{1}{p}} + \left(\frac{\beta}{t}\right)^{1+\frac{1}{p}} \right] e^{-1/2\alpha^2\left[\left(\frac{t}{\beta}\right)^{\frac{2}{p}} + \left(\frac{\beta}{t}\right)^{\frac{2}{p}} - 2\right]}, t > 0$$

Here α is the shape parameter and β is the scale parameter. The parameter p governs both shape and scale. For all values of α the PDF is unimodal. Mode cannot be obtained in explicit form, it has to be obtained by solving a non-linear equation in α . Clearly, the median is at β , for all α . Table 2, table 3 and table 4 give the shape of probability density function for various values of parameters.

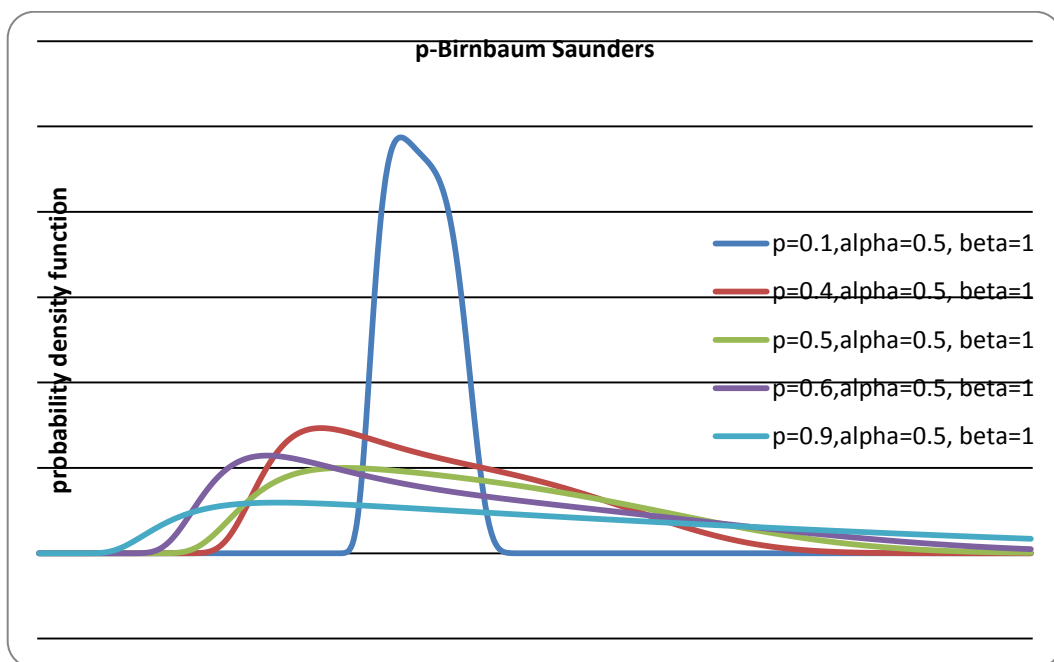


Table 2. Probability density function of p-Birnbaum Distribution, alpha=0.5 and beta=1

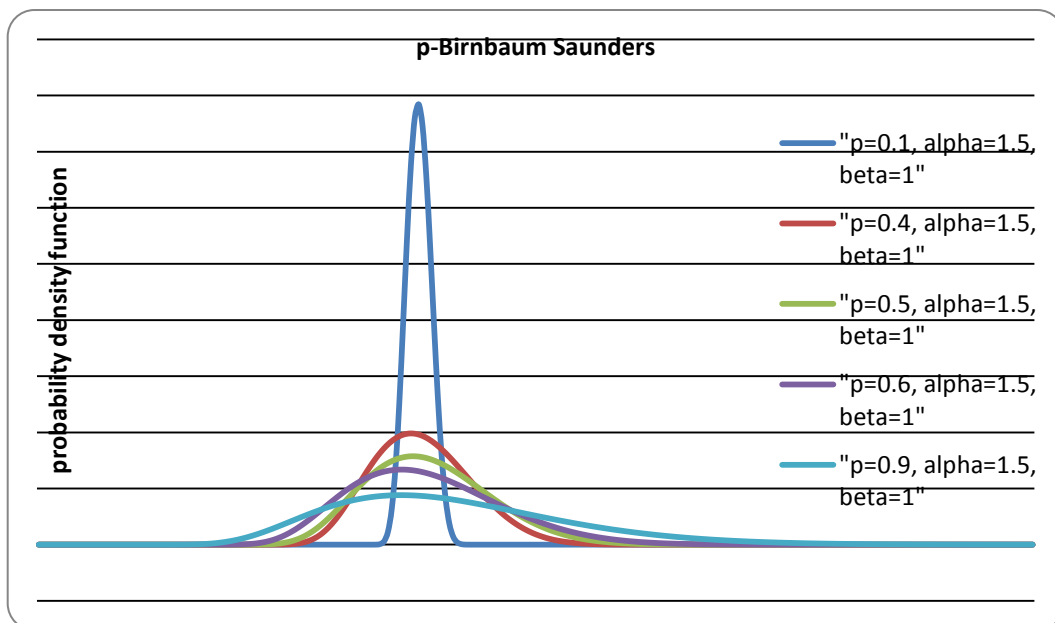


Table 3. Probability density function of p-Birnbaum Distribution, alpha=2 and beta=1

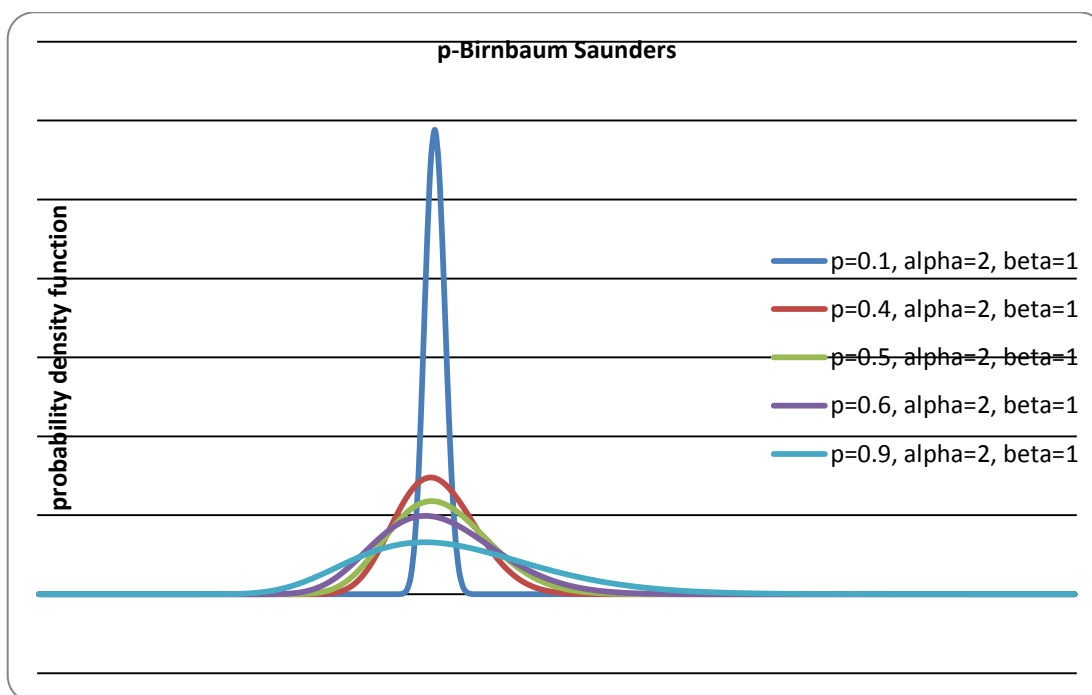


Table 4. Probability density function of p-Birnbaum Distribution, alpha=2 and beta=1.

All these models have slight skewness and departure from symmetric Normal distribution. They are defined on positive axis. Peakedness changes with p. p-is a parameter which control both flatness and skewness (Shape).

Let $X = 1/2[(t/\beta)^{1/p}-(t/\beta)^{-1/p}]$ then

$$X \sim N\left(0, \frac{\alpha^2}{4}\right)$$

Using this transformation Expectation, variance, skewness and kurtosis can be obtained.

$$E(T) = \beta E\left(X + (X^2 - 1)^{\frac{1}{2}}\right)^p$$

If p=2, we get

If p=2, we get

$$E(T) = \beta(1 + \alpha^2/2)$$

$$V(T) = (\beta\alpha)^2(1 + 5\alpha^2/4)$$

$$\beta_1(T) = (4\alpha)^2 \frac{(11\alpha^2 + 6)}{(5\alpha^2 + 4)^3}$$

$$\beta_2(T) = 3 + 6(\alpha)^2 \frac{(93\alpha^2 + 41)}{(5\alpha^2 + 4)^2}$$

3.1 properties of p-Birnbaum Saunders distribution

We obtained two important properties of p-Birnbaum Saunders distribution.

Theorem 1: If T has life distribution p-BS, $F(t, \alpha, \beta, p) = N\left(\frac{1}{\alpha} \xi_p\left(\frac{t}{\beta}\right)\right)$, $t > 0$ then its reciprocal 1/T has $F(t, \alpha, \beta^{-1}, p)$ distribution.

Proof: Using transformation, we can obtain the required result.

Theorem 2: If T has life distribution p-BS, $F(t, \alpha, \beta, p) = N\left(\frac{1}{\alpha} \xi_p\left(\frac{t}{\beta}\right)\right)$, $t > 0$ then kT has $F(t, \alpha, k\beta, p)$ distribution.

Proof: Using transformation, we can obtain the required result.

4. MAXIMUM LIKELIHOOD ESTIMATION

The log likelihood function is

$$\log L = -n \log \alpha - n \log \beta - n \log p + \sum_{i=1}^n \left\{ -\frac{1}{2} \log(2\pi) - \frac{1}{2} \alpha^{-2} \xi_p\left(\frac{t_i}{\beta}\right) + \log \xi'_p\left(\frac{t_i}{\beta}\right) \right\}$$

Then,

$$-\frac{\alpha^3}{n} \frac{\partial \log L}{\partial \alpha} = \alpha^2 - \frac{1}{n} \sum_{i=1}^n \xi_p^2 \left(\frac{t_i}{\beta} \right)$$

$$\frac{\partial \log L}{\partial \beta} = -\frac{n}{\beta} + (\beta\alpha)^{-2} \sum_{i=1}^n t_i \xi_p \left(\frac{t_i}{\beta} \right) \xi_p' \left(\frac{t_i}{\beta} \right) - \frac{1}{\beta^2} \sum_{i=1}^n \frac{t_i}{\xi_p \left(\frac{t_i}{\beta} \right)} \xi_p'' \left(\frac{t_i}{\beta} \right)$$

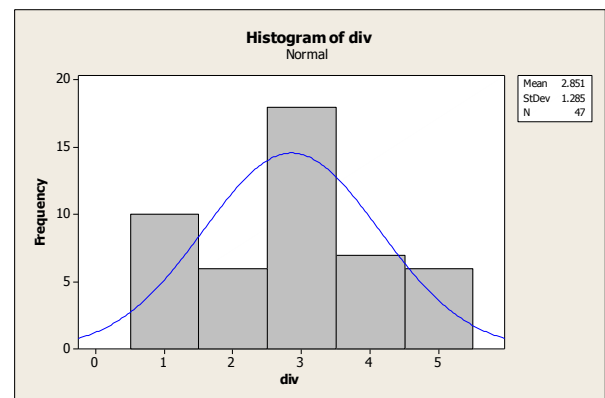
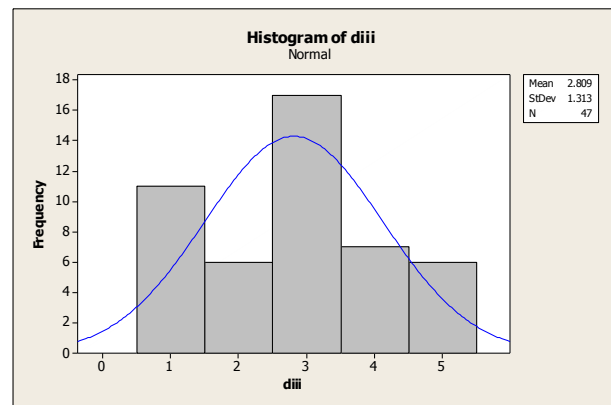
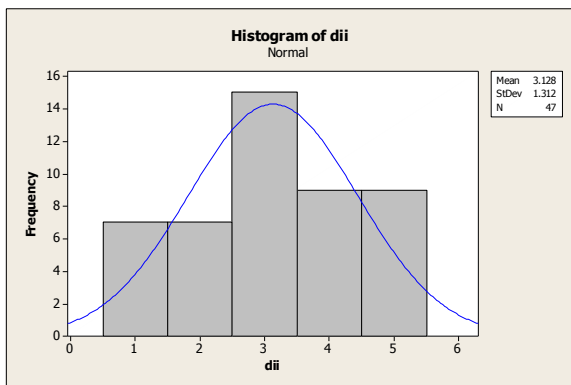
$$\frac{\partial \log L}{\partial p} = -\frac{n}{p} + p\alpha^{-2} \sum_{i=1}^n \xi_p \left(\frac{t_i}{\beta} \right) \xi_{\log t_i} \left(\frac{t_i}{\beta} \right) - \frac{1}{\beta^2} \sum_{i=1}^n \frac{1}{\xi \left(\frac{t_i}{\beta} \right)} \xi_{\log t_i}'' \left(\frac{t_i}{\beta} \right)$$

All these three equations are non-linear, we need to use numerical procedure to solve it.

5. APPLICATIONS TO RELIABILITY AND ELECTRONIC BANKING HABITS

The motivation of this work is coming from the fatigue failure data given in Birnbaum and Saunder (1969). Also many real situations in banking sector shows the behaviour of the p-Birnbaum Saunders distribution.

Deepa Paul (2012) conducted a survey of Banking habits in the usage of internet banking, ATM, mobile banking and branch banking. Some variables have the property specified in Theorem 1 and Theorem 2. Variables in five point likert scale are dii, diii, and div, where dii=Technology enabled services are quick to use than visiting the bank branch personally, diii= ATM saves much time, div= ATMs are more accurate than human tellers.



All these histograms shows departure from normality and more suitable for p-Birnbaum Saunders distribution.

6. CONCLUSIONS

This three parameter distribution is more plausible model for the distribution of fatigue failure. Moreover the shape of density curve with various skewness and kurtosis provide a well defined class of life distributions useful in reliability and social sciences. Snedecor's F distribution has the property that reciprocal is also F , similar property holds for p-Birnbaum Saunders distribution.

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METHOD AND ALGORITHM OF RANGING OF RELIABILITY OBJECTS OF THE POWER SUPPLY SYSTEM

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ABSTRACT

Ranging of objects is widely applied at the decision of operational problems. However, it is spent mainly intuitively. There are developed method and algorithm of ranging of objects of a power supply system on independent parameters of reliability and profitability of work with the recommendation of the basic directions of improvement of these parameters.

INTRODUCTION

Ranging of the equipment and devices (objects) of an electro power system on reliability and profitability of work is widely used at the decision of many operational problems, including at the organization of maintenance service and repair. Known, that reliability and profitability of work of objects characterized by a number of parameters (for example, factor of readiness, specific charge of conditional fuel, etc.). To group objects by way of increase of their reliability and profitability on each of these parameters does not represent difficulty. However, often the situation when these parameters contradict each other observed. For example, on size specific the charge of fuel the power unit can exceed average value on power station. At the same time, under the charge of the electric power in system of own needs - to be it is less, than average value. As an example in table 1 some monthly average parameters of eight power units 300 MWt are resulted.

Table 1. Data on work of power units of power station

N	Parameter	Index number of the power unit							
		1	2	3	4	5	6	7	8
1	Operating ratio of the established capacity, %	62,8	27,9	68,6	71,3	80,0	76,8	75,9	78,9
2	Average loading, MW _T	222	211	216	229	276	231	228	237
3	The charge el. energy on own needs, %	4,1	4,4	4,0	3,9	3,5	4,0	3,7	3,5
4	The specific charge of conditional fuel, q/(kW _{T.C})	374,6	371,0	368,4	369,7	336,7	373,9	363,0	374,2

We will use these data in the further for an illustration of methodology of ranging of objects. They concern to a class of discrete multivariate data with a nominal scale of measurement [1] as each of noted above parameters considered as an attribute with a quantitative scale of measurement of continuous sizes. It is necessary to note, that alongside with discrete multivariate data there are also multivariate data of continuous random variables. For example, initial information for calculation of parameters of individual reliability. Features of classification of these data are

considered in [2,3]. Practical realization of algorithm of ranging of objects is preceded with transformation of initial data

Transformation of initial data provides overcoming the difficulties connected with natural distinction of units of measure and a scale of quantitative estimations of parameters, distinction of their orientation of change, with elimination of interrelation of these parameters. For example, the charge of the electric power on own needs differs from the specific charge of conditional fuel both on units of measure, and on scale. The interconnected parameters at ranging initial data result not only in increase in labour input of calculations, but also to erroneous result. Therefore, classification of used parameters on independent groups makes one of the primary goals of transformation of data.

Overcoming of distinction of units and scales of measurement of parameters is reached by normalization (standardization). Normalization in practice spent on one of following formulas:

$$Z_1 = \frac{X}{\bar{X}^*}; Z_2 = \frac{X}{\sigma^*(X)}; Z_3 = \frac{X}{L^*(X)}; Z_4 = \frac{X - \bar{X}^*}{\bar{X}^*}; Z_5 = \frac{X - \bar{X}^*}{\sigma^*(X)}; Z_6 = \frac{X - \bar{X}^*}{L^*(X)}$$

where X and Z -quantitative estimations of parameters before transformation; $\bar{X}^* = m^{-1} \sum_{i=1}^m X_i$;

$$L^*(X) = (X_{\max} - X_{\min}); \sigma^*(X) = \sqrt{\frac{(X - \bar{X}^*)^2}{m-1}}; X_{\max} = \{X_1, X_2, \dots, X_m\}; X_{\min} = \{X_1, X_2, \dots, X_m\}; m-$$

number of objects.

Comparative estimation of expediency of these transformations has shown [4]:

1. Transition as a result of the certain transformations to sizes Z_1, Z_2 and Z_3 (unlike Z_4, Z_5 and Z_6) does not solve a problem of distinction of scale of measurement;
2. Sizes $\sigma^*(X)$ and $L^*(X)$ are correlated. The factor of correlation is significant, but the size of scope $L^*(X)$ demands less calculations, than an average quadratic deviation $\sigma^*(X)$. The size $\sigma^*(X)$ provides presence of general population of random variables. Real statistical data concern to statistical data of multivariate type and are small. Data on distribution of realizations of attributes are absent. The information on attributes is concentrated in statistical function of distribution (s.f.d.) realizations of attributes $F_{\Sigma}^*(\Pi_1)$. Advantages of scope $L^*(X)$ cause expediency of its application;
3. Comparison of transformations $Z_5 = \frac{X - \bar{X}^*}{\sigma^*(X)}$ also $Z_6 = \frac{X - \bar{X}^*}{L^*(X)}$ shows, that factor of correlation

between $(X - \bar{X}^*)$ and \bar{X}^* it is essential below, than between $(X - \bar{X}^*)$ and $L^*(X)$;

$$\text{Thus, the most effective should consider transformation } Z_6 = \frac{X - \bar{X}^*}{L^*(X)}.$$

As follows from table 1, the vector of parameters has a various orientation. If operating ratio of the established capacity (K_E) and average loading of one power unit (P_A) similar parameters for other power unit with the minimal risk of the erroneous decision it is possible to conclude, that exceed reliability and profitability of work of the first power unit above. The conclusion will be erroneous for the charge electric power on own needs (E_{ON}) and the specific charge of conditional fuel (S_F). Heuristic character of discussion of this question demands formalization of the decision. For what take advantage of concepts and methods of the correlation analysis.

Results of calculations of factors of correlation (r) between K_E, P_A, E_{ON} and S_F are resulted in table 2. Calculations spent under the formula [5]

$$r = m^{-1} \sum_{\substack{j=1 \\ k=1 \\ j \neq k}}^m [\Pi_{i,j} - M^*(\Pi_i)][\Pi_{k,j} - M^*(\Pi_k)] / [\sigma^*(\Pi_i) \cdot \sigma^*(\Pi_k)],$$

Which, in particular, testifies to independence of factor of correlation of an orientation of change of a parameter

Table 2. Estimations of factors of correlation of parameters

Parameters	K_E	P_A	E_{ON}	S_F
K_E	-	0,59	-0,83	-0,30
P_A	0,59	-	-0,77	-0,84
E_{ON}	-0,83	-0,77	-	0,52
S_F	-0,30	-0,84	0,52	-

Analysis of data of table 2 confirms the distinction of an orientation of vectors of attributes noted above. Orientation K_E and P_A differs from orientation E_{ON} and S_F . Factors of correlation on size are significant and allow assuming interrelation of considered parameters.

Casual character of realizations of parameters causes casual character of observable interrelation. To consider this feature, critical values of factors of correlation $[\bar{r}; \bar{r}]$ pay off with the set significance value α . This problem solved as follows:

1. Two samples of random variables are modeled ξ with uniform distribution in an interval $[0,1]$. Number of elements of the first and the second samples we shall designate through m_v ;
2. Calculate factor of correlation r between realizations samples;
3. Items 1 and 2 repeat N time;
4. On realizations of factor of correlation is under construction s.f.d. $F^*(r)$ critical values \bar{r}_α and $\bar{r}_{(1-\alpha)}$ for of some significance values also are defined α ;
5. Under standard programs are established in view of symmetry $F^*(r)$ dependences $\bar{r} = f(m_v)$. These dependences with the big assurance look like $\bar{r}_{(1-\alpha)} = A m_v^{-B}$. Some results of calculation of factors R^2 , A , B , m_v , X and \bar{r} are resulted in table 3

Table 3. Results of calculations of factors of the equation $\bar{r} = f(m_v)$

(1-2 α)	R^2	A	B	Estimations \bar{r} at m_v equal						
				3	5	8	10	20	30	50
0.8	0.997	1.65	-0.56	0,990	0,690	0,507	0,442	0,397	0,240	0,168
0.9	0.999	1.88	-0.52	0,992	0,811	0,624	0,549	0,379	0,310	0,240
0.95	0.95	2.00	-0.5	0,995	0,880	0,712	0,629	0,444	0,360	0,281
0.975	0.975	2.02	-0.46	0,999	0,927	0,774	0,700	0,499	0,410	0,320

As follows from table 3 at $n \leq 5$ to establish dependence between two parameters it is practically impossible, since even at $\alpha=0,05$ absolute values of factors of correlation independent samples random variables (\bar{r}) \bar{r} not less than 0,81. At $m_v=8$ and $\alpha=0,05$ according to table 2 it is possible to approve presence of dependence between K_E and E_{ON} , P_A and E_{ON} , P_A and S_F (table.2). At the same time dependence between K_E and P_A , K_E and S_F , and also E_{ON} and S_F can be casual. A graphic illustration of dependence $\bar{r} = f(m_v)$ at $\alpha=0,025$ it is resulted in figure 1.

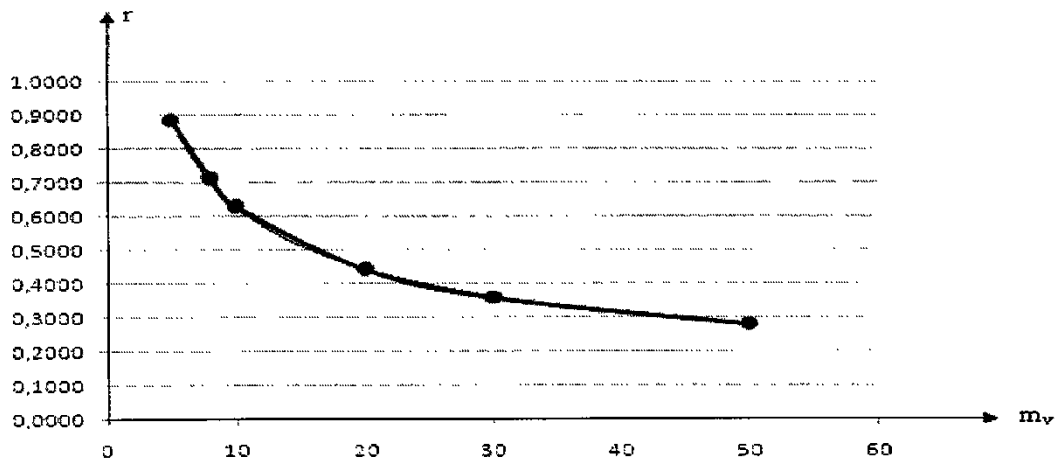


Fig.1. A graphic illustration of change of critical values of factor of correlation independent samples random variables

These curves show, what even at $m_v=50$ absolute size of critical values \underline{r} and \bar{r} with a significance value $\alpha=0,05$ not less than 0,2.

To eliminate distinction in an orientation of vectors of parameters we shall enter into consideration an opposite parameter on sense «factor of underexploitation of the established capacity», calculated as $K_U=1-K_E$, and instead of P_A we shall enter size $\Delta P_A = P_{NOM} - P_A$.

At small number of objects, probably essential influence of casual character of factor of correlation on result of the analysis of interrelation of attributes. Validity of the analysis is provided by comparison of an estimation r with bottom \underline{r}_α and top $\bar{r}_{(1-\alpha)}$ critical values. Absence of interrelation of parameters with probability α takes place either at $r < \underline{r}_\alpha$ or at $r > \bar{r}_{(1-\alpha)}$.

Algorithm ranking objects. Ranking of objects of a power supply system spent in following sequence:

1. Realizations of each of the parameters describing reliability and profitability of object, we shall consider as population of random variables $\{\Pi_i\}_{m_\Sigma}$;
2. Let's calculate a number of their statistical parameters. Namely, average arithmetic value $M_\Sigma^*(\Pi_i)$, the minimal $\Pi_{i,min}$ and maximal values $\Pi_{i,max}$, scope $L_\Sigma^*(\Pi_i)$ under formulas:

$$M_\Sigma^*(\Pi_i) = m_\Sigma^{-1} \sum_{j=1}^{m_\Sigma-1} \Pi_{i,j}$$

$$\Pi_{i,min} = \min\{\Pi_i\}_{m_\Sigma}$$

$$\Pi_{i,max} = \max\{\Pi_i\}_{m_\Sigma}$$

$$L_\Sigma^*(\Pi_i) = [\Pi_{i,max} - \Pi_{i,min}]$$

where $i=1, n_\Sigma$; n_Σ - number of parameters

List of parameters is caused by necessity of representation of each population two samples as versions of i -th attribute (parameter) with $i=1, n_\Sigma$;

3. Realizations for which $\Pi_i > M_\Sigma^*(\Pi_i)$, we carry to the first sample (to the first version i -th an attribute). Realizations, for which $\Pi_i < M_\Sigma^*(\Pi_i)$ -(to the second the second version i -th an attribute). Such classification is widely used in practice, physically proved;

4. For both samples (v) each data population average arithmetic values $M_{v,1}^*(\Pi_i)$ and $M_{v,2}^*(\Pi_i)$ with $i=1, n_\Sigma$ are calculated. Thus, the minimal value of realizations i- th a parameter of the first sample– $\Pi_{i,1,\min}$, and the maximal value in the second sample– $\Pi_{i,2,\max}$. Notice, that essential distinction $M_{v,1}^*(\Pi_i)$ and $M_{v,2}^*(\Pi_i)$ are caused by distinction of number of realizations samples $\{m_v, 1 \neq m_v, 2\}$;

5. Under formulas

$$\delta M_{v,1}^*(\Pi_i) = [M_\Sigma^*(\Pi_i) - M_{v,1}^*(\Pi_i)] / L_\Sigma^*(\Pi_i)$$

$$\delta M_{v,2}^*(\Pi_i) = [M_\Sigma^*(\Pi_i) - M_{v,2}^*(\Pi_i)] / L_\Sigma^*(\Pi_i)$$

are calculated normalization values of absolute size of average value of a relative deviation;

6. The greatest absolute size of average value of a relative deviation under the formula is defined

$$\delta M_{v,\max}^*(\Pi_i) = \max\{\delta M_{v,j}^*(\Pi_i)\}_{j=1,2, i=1, n_\Sigma}$$

That defines sample, which to the greatest degree differs from corresponding set. It is necessary to note, that as $\Pi_{i,2,\max} < M_\Sigma^*(\Pi_i) < \Pi_{i,1,\min}$ both considered samples are unpreventable (not representative). In other words, the group of objects that versions to the greatest degree differ from other objects on j-oh i- th an attribute allocated;

7. Further from this group of objects, the subgroup for which distinction on i- th to an attribute from the value average on set is even more allocated. This subgroup can be allocated under condition of a finding of the second significant attribute. Recognition of a subgroup is spent as follows:

7.1. For the allocated group of objects the matrix of realizations j-oh versions κ - th an attribute, where $k=1, n_{v,j,i}$ and $k \neq i; n_{v,j,i}$ – number of realizations of sample on j-oh versions i- th an attribute;

7.2. Each realization κ - th an attribute with $k=1, n_{v,j,i}$ and $k \neq i$ in a matrix it is replaced with realization corresponding everyone object i- th an attribute;

7.3. According to the transformed matrix average arithmetic values of realizations κ -to an attribute with $k=1, n_{v,j,i}$ and $k \neq i$ are calculated;

7.4. The greatest size among these average values defined;

7.5. This greatest value is normalized and compared with $\delta M_{v,\max}$. If at $j=1$ it is more, and at $j=2$ it is less, than $\delta M_{v,\max}$ classification of data on i- th to an attribute is expedient. Otherwise – it is inexpedient;

7.6. If the lead classification has appeared inexpedient:

7.6.1. In a basis data on previous stage of classification undertake;

7.6.2. From the general list of objects the objects having essential features (by results of expedient classification) withdrawn;

7.6.3. We pass to classification of the remained list of objects with constant sequence of the analysis.

Control criterion representativity of samples. Above at the analysis representative samples we started with unconditional position conformity with which sample it is considered not representative if at $M_\Sigma^*(\Pi_i) < M_v^*(\Pi_i)$ size $\Pi_{i,\min}$ as would exceed $M_\Sigma^*(\Pi_i)$, and at $M_\Sigma^*(\Pi_i) > M_v^*(\Pi_i)$ size $\Pi_{i,\max}$ would be less $M_\Sigma^*(\Pi_i)$ with $i=1, n_\Sigma$. It has made meaning to not distract from algorithm of classification of data. Actually the criterion of the control representative samples is more strict, since a place of conditions $\Pi_{i,\min} > M_\Sigma^*(\Pi_i)$ and $\Pi_{i,\max} < M_\Sigma^*(\Pi_i)$ parities $\overline{M_{v,\beta}^{**}(\Pi_i)} > M_\Sigma^*(\Pi_i)$ and $\overline{M_{v,(1-\beta)}^{**}(\Pi_i)} < M_\Sigma^*(\Pi_i)$, where $\overline{M_{v,\beta}^{**}(\Pi_i)} > \Pi_{i,\min}$ are checked, and $\overline{M_{v,(1-\beta)}^{**}(\Pi_i)} < \Pi_{i,\max}$;

$\overline{M_{v,\beta}^{**}}(\Pi_i)$ and $\overline{M_{v,(1-\beta)}^{**}}(\Pi_i)$ - cvantil distributions $F^*\{M_v^{**}(\Pi_i)\}$ for $F^*\{M_v^{**}(\Pi_i)\} = \beta = 0,05$ and $F^*\{M_v^{**}(\Pi_i)\} = 1 - \beta = 0,95$; $M_v^{**}(\Pi_i)$ - modeled on $F_v^*(\Pi_i)$ estimations of average arithmetic values n_v realizations Π_i ; n_v – number of realizations of sample.

CONCLUSIONS

1. The method and algorithm of ranking of objects by way of increase of reliability and profitability of their work is developed;
2. In real conditions when the number of the factors influencing reliability and profitability of work of objects is great, classification of objects at an intuitive level leads to essential risk of the erroneous decision;
3. The automated ranking of objects allows:

3.1. To classify objects on two groups. Provided that with increase in quantitative value of parameters of reliability and profitability of work of objects their reliability and profitability increases

- The first group includes "bad" objects for which the quantitative estimation of the most significant parameters exceeds their average value on all objects;
- The second group includes "good" objects, for which quantitative estimation of the most significant parameters less their average value on all objects;

3.2. To define the basic ways of increase of reliability and profitability of work

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CAN THE RELIABILITY THEORY BECOME A SCIENCE?

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ABSTRACT

The amount of works in the reliability domain is really huge although this field of research has not yet evolved as a science. It is worth reminding that Gnedenko took the first step to create the reliability science. He adopted the *deductive logic* – typical of exact sciences such as mechanics, electronics and thermodynamics – and demonstrated the general form of the reliability function. However the hazard rate, which tunes up this function, has not been demonstrated so far. We believe that one should follow and complete the Gnedenko’s seminal work and should demonstrate the various trends of the hazard rate using the deductive approach.

We have conducted a theoretical research using traditional mathematical methods and have even introduced a new tool named *Boltzmann-like entropy*. The present paper makes a summary of various contributions published in the past decade and means to show the deductive implications developed.

1 INTRODUCTION

Reliability theory is an abstract approach aimed to gain theoretical insights into engineering and biology. The vast majority of researchers make conclusions about populations based on information extracted from random samples. Authors usually adopt statistical methods which furnish various algorithms and likelihood criteria for qualifying the finish of an inductive reasoning process. The final statement never turns out to be absolutely true or false. By definition, the premises of *inductive logic* provide some degree of support for the final statement of an argument.

The inductive approach is ruling over reliability theory that has not yet evolved into a science. In fact exact sciences – such as mechanics, electronics, and thermodynamics – comply with *deductive logic*, that is to say theorists derive the results from principles and axioms using theorems. Each theoretical construction starts with precise assumptions and, using the rules of logical inference, draws conclusions from them. A sequel of logic implications lies at the base of the deductive approach

$$\mathfrak{A} \Rightarrow \mathfrak{B} \quad (1)$$

Each close \mathfrak{B} is certain provided that the argument’s assumption \mathfrak{A} is true. The sentence \mathfrak{B} is undisputable no matter it expresses a deterministic or indeterministic statement. Also probability functions, which authors usually adopt to describe reliable systems, can be established through deductive mathematical demonstrations as Gnedenko taught us. A deductive reasoning regardless it is *deterministic* or *indeterministic* lies far apart from statistical modeling since the truth of the premises provides a guarantee of the truth of the conclusion.

Ancient philosophers popularize the term ‘deduction’ as *reasoning from the general to the specific*; and specified ‘induction’ as *reasoning from the specific to the general*. These definitions, which modern thinkers deem not perfectly accurate, have nonetheless the virtue of making more

evident the peculiarity of the deductive mode which proceeds from the general principle \mathfrak{A} to the specific conclusion \mathfrak{B} .

Gnedenko was the first to adopt the deductive mode and laid the first stone for the construction of the reliability science [Gnedenko et al. 1965]. He assumes that the system S is a Markov chain and from this assumption concludes that the probability of good functioning without failure is *the general exponential function*:

$$P(t) = e^{-\int_0^t \lambda(t) dt} \quad (2)$$

Where $\lambda(t)$ determines the reliability of the system in each instant:

$$\lambda(t) = -P'(t)/P(t) \quad (3)$$

Gnedenko demonstrates that function (2) comes from the conditional probability typical of Markov chains. Equation (2) originates from the operations that a system executes one after the other, and the following formal statement summarizes Gnedenko's inference:

$$\text{Chained Units} \Rightarrow \text{General Exponential Function} \quad (4)$$

The *hazard* (or *mortality*) *function* $\lambda(t)$ is the key element in that $\lambda(t)$ tunes up the general exponential function (2), although there is dispute about the hazard function. Some authors believe that curve begins with a decreasing trend typical of the time period during which factory defective items are flushed out. The population thereafter reaches the useful life period where the failure rate is minimized. The curve is completed by the third phase where many essential parts of systems fail in an increasingly larger number. However significant evidence contradicts the tripartite form of $\lambda(t)$ usually called '*bathtub*' curve. For instance, researchers show the irregular degeneracy of electronic circuits [Jensen 1996]. The hazard rate presents humps so evident that Wong [1989] labels this: "roller coaster distribution". In biology $\lambda(t)$ has very differing trends. The recent article published in Nature [Owen et al. 2014] examines the mortality of 11 mammals, 12 other vertebrates, 10 invertebrates, 12 vascular plants and a green alga. The authors of this ponderous experimental research show the evident discrepancy between empirical data and the bathtub model. Ascher [1968] claims that "the bathtub curve is merely a statement of apparent plausibility which has never been validated". More recently Zairi [1991] and Klutke with others [2003] share skeptical judgments.

We observe that the hazard rate has not yet determined in a deductive and general manner. As second, it would be desirable that the reliability domain evolves as an exact science. In our opinion we should proceed with the method inaugurated by Gnedenko and should complete inference (4). We started a project that pursues these objectives and we published partial results during the last decade. The present paper brings together various upshots in order to provide the comprehensive view of this research and highlight its deductive pathway.

2. A LESSON FROM THERMODYNAMICS

The project has used some mathematical tools shared in literature and in addition has prepared a new mathematical tool that we briefly present in this section.

The second law of thermodynamics claims that the entropy of an isolated system will increase as the system goes forward in time. This entails – in a way – that physical objects have an inherent tendency towards disorder, and a general predisposition towards decay. Such a wide-spreading process of annihilation hints an intriguing parallel with the decadence of biological and artificial systems to us. The issues of reliability theory are not far away from some issues inquired by thermodynamics and this closeness suggests us to introduce the entropy function for the study of reliable/repairable systems.

As first we detail the Markovian model introduced by Gnedenko and assume that the continuous stochastic system S has m states which are mutually exclusive:

$$S = (A_1 \text{ OR } A_2 \text{ OR } \dots \text{ OR } A_m), \quad m > 0. \quad (5)$$

Each state is equipped with a set of *sub-states* or *components* or *parts* which work together toward the same purpose. Formally, the generic state A_i ($i = 1, 2, \dots, m$) has this form:

$$A_i = (A_{i1} \text{ AND } A_{i2} \text{ AND } \dots \text{ AND } A_{in}), \quad n > 0. \quad (6)$$

We consider that the states of the stochastic system S can be more or less reversible, and mean to calculate the reversibility property using the *Boltzmann-like entropy* H_i where P_i is the probability of the generic state A_i :

$$H_i = H(A_i) = \ln(P_i) \quad (7)$$

The proof is in [Rocchi 2000]. We confine our attention to:

- The *functioning state* A_f and the *reliability entropy* H_f ;
- The *recovery state* A_r and the *recovery entropy* H_r .

The meanings of H_f and H_r can be described as follows. When the functioning state is irreversible, the system S works steadily. In particular, the more A_f is irreversible, the more H_f is high and S is capable of working and reliable. On the other hand, when H_f is low, S often abandons A_f in the physical reality. The system switches to recovery state since S fails and is unreliable.

The recovery entropy calculates the irreversibility of the recovery state, this implies that the more H_r is high, the more A_r is stable. In practice S is hard to be repaired and/or cured in the world. In summary, *the reliability entropy H_r expresses the aptitude of S to work or to live without failures; the entropy H_r illustrates the disposition of S toward reparation or restoration to health.*

As an application, suppose a and b are two devices in series with probability of good functioning: $P_f(a) = 10^{-200}$, $P_f(b) = 10^{-150}$. We can calculate the probability of the overall system and later capability of good working of S with the entropy:

$$P_f(S) = [P_f(a) \cdot P_f(b)] = 10^{-350} \quad (8)$$

$$H_f(S) = \log[P_f(S)] = \log(10^{-350}) = -805.9 \quad (9)$$

The Boltzmann-like entropy is additive and one can follow this way with the same result:

$$\begin{aligned} H_f(S) &= [H_f(a) + H_f(b)] = \\ &= \log[P_f(a)] + \log[P_f(b)] = \\ &= \log[P_f(a) \cdot P_f(b)] = \\ &= \log[10^{-200} \cdot 10^{-150}] = \\ &= \log(10^{-350}) = -805.9 \end{aligned} \quad (10)$$

As second case, suppose a device degrades during the interval (t_1, t_2) ; and the probability of good functioning are the following: $P_f(t_1) = 10^{-10}$, $P_f(t_2) = 10^{-200}$. The entropies $H_f(t_1) = \log(10^{-10}) = -23.0$; and $H_f(t_2) = -460.5$ qualify the irreversibility of the device and one obtains how much the capability of good functioning has sloped down:

$$\Delta H_f = H_f(t_2) - H_f(t_1) = -460.5 - (-23.0) = -437.5 \quad (11)$$

3. BASIC ASSUMPTION ON INTRINSIC FACTORS

Real events are multi-fold in the sense that mechanical, electrical, thermal, chemical and other *intrinsic effects* interfere with S . The generic component A_{fg} ($g = 1, 2, \dots, n$) involves a series of collateral physical mechanisms that run in parallel with A_{fg} and change it with time. Parallel interferences damage A_{fg} by time passing and at last impede the correct functioning to A_{fg} . Thus we can establish a general property for the system components:

$$\text{The sub-state } A_{fg} \text{ degenerates as time goes by.} \quad (12)$$

For example, Carnot defines a model for the heat engine that includes two bodies at temperature T_1 and T_2 ($T_1 \neq T_2$), and the gas A_{fg} that does the mechanical work via cycles of contractions and expansions. The mounting disorder of the molecules – qualified by the thermodynamic entropy – results in the decreasing performances of A_{fg} . Several unwanted side effects – e.g. the attrition amongst the gears and the heat dispersion – impact on the components and progressively harm the effectiveness of the heat engine.

4. SIMPLE DEGENERATION OF SYSTEMS DUE TO INTRINSIC FACTORS

Hypothesis (12) is written in words, hence it is necessary to translate (12) into mathematical terms. We establish the regular degeneration of A_{fg} using the reliability entropy H_{fg} that decreases linearly as time goes by:

$$H_{fg}(t) = -c t, \quad c > 0. \quad (13)$$

From hypothesis (13) one can prove that the probability of good functioning P_f – that is to say the probability that A_f runs until the first failure – follows the *exponential law* with constant hazard rate:

$$P_f = P_f(t) = e^{-ct} = e^{-\lambda t}. \quad (14)$$

Where the *hazard function is constant*:

$$\lambda = c. \quad (15)$$

The proof may be found in [Rocchi 2005].

5. EXTREME SHOCKS

Beside the side effects that are extrinsic factors of failure, there are *extreme shocks* coming from the external that are able to stop S . Intrinsic effects work in such a manner as could not be otherwise; instead extreme shocks are not certain to happen. Several long-living systems do not run

into such a kind of external attacks. In consequence of this irregular conduct, modern authors assume that extreme shocks have random magnitude and occur without any order [Gut & Hüsler 1999], in particular they divide the system lifetime into regular intervals with the constraints:

- a) The shocks can occur only one per time interval,
 - b) The shock occurs with the same probability in each time interval,
 - c) The occurrence of the shocks takes place independently from each other in the time intervals.
- (16)

6. SIMPLE DEGENERATION OF SYSTEMS DUE TO EXTRINSIC FACTORS

Coeval authors demonstrate that under assumptions (16) extreme shocks can be modeled by the Poisson distribution. In particular, supposing $N(t)$ independent random shock loads which occur in the time interval $(0, t)$, they obtain that the probability of k shocks occurring in the interval $(0, t)$ is:

$$P[N(t) = k, t] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \quad k = 0, 1, 2, \dots; \lambda > 0. \quad (17)$$

Where λ is the parameter typical of a certain distribution. From (17) one can derive the probability of good functioning until the first failure that is *exponential of time*:

$$P_f = P_f(t) = e^{-\lambda t}. \quad (18)$$

Where *the hazard function λ is constant*. The proof may be found in [O'Connor & Kleyner 2011].

Eqn (18) – well known in current literature – perfectly match with (14) and (15). Two results which deriving from distinct and far different assumptions lead to the same rule. Physical phenomena that have intrinsic and extrinsic origins and present differing behaviors, can be treated by identical mathematical tools.

7. COMPLEX DEGENERATION OF SYSTEMS

When assumption (13) comes true over a certain period of time, the components $A_{j1}, A_{j2}, \dots, A_{jn}$ worsen to the extent that they set up a *cascade effect* [Rocchi 2006]. The cascade effect consists of the generic part A_{ig} that spoils one or more close components while the system proceeds to run. A cascade effect can be linear or otherwise compound. In the first stage we assume the component A_{ig} harms the close part A_{ik} and in turn this damages the subsequent part and so on:

The cascade effect is linear. (19)

The linear cascade effect occurs while principle (13) is still true of necessity and one can prove that the probability of good functioning is the *exponential-power function*:

$$P_f = P_f(t) = b e^{-a t^n}, \quad a, b > 1. \quad (20)$$

The hazard function is a *power of time*:

$$\lambda(t) = at^{n-1}. \quad (21)$$

In the second stage we suppose that the component A_{ig} damages the components all around:

$$\text{The cascade effect is compound.} \quad (22)$$

This hypothesis – alternative to linear waterfall effect – yields that the probability of functioning is *the exponential-exponential function*:

$$P_f = P_f(t) = g e^{-de^t}, \quad g, d > 1. \quad (23)$$

And the hazard rate is *exponential of time*:

$$\lambda(t) = de^t. \quad (24)$$

The proofs of equations (16) and (19) may be found in [Rocchi 2005, 2006].

8. CONCLUSIVE REMARKS

The closing comments are subdivided into four sections.

(A) Gnedenko derives the general exponential function (1) under the assumption that S is a Markovian chain [Gnedenko et al. 1965]. We specify the shape and behavior of this model using various narrow hypotheses such as eqns. (5) and (6) that depict special Markovian chains. The present paper develops the ensuing logical inferences that determine various trends of the hazard rate:

$$\begin{aligned} \text{Regular degeneration of system's components} &\Rightarrow \text{Exponential Function} \\ \text{Random extreme shocks} &\Rightarrow \text{Exponential Function} \end{aligned}$$

$$\text{Regular degeneration} + \text{linear cascade effect} \Rightarrow \text{Exponential-Power Function}$$

$$\text{Regular degeneration} + \text{composite cascade effect} \Rightarrow \text{Exponential-Exponential Function} \quad (25)$$

It is worth underlying that the mathematical results (14), (18), (20) and (23) are special cases of (1). Gnedenko's seminal work and the present study are consistent despite the different mathematical techniques in use.

(B) The present approach adopts the deductive logic which relates eqns. (15), (21) and (24) to precise causes and not to precise periods of system lifetime. In other words, the function $\lambda(t)$ can be a constant, it can follow the power or exponential distributions in any interval of the system life. Each result in (25) derives from a precise hypothesis that may come true during the system juvenile period, the maturity and the senescence alike.

(C) Authors recognize that sometimes the organs of appliances and biological beings degenerate at constant rate – in accord to (15) – during the middle age. Several machines have

linear structures and the probability of good functioning follows the Weibull distribution during ageing that corresponds to equation (20). The body of animals and humans appear rather intricate and during ageing equation (23) comes true in agreement with the Gompertz distribution. In conclusion, on one hand the present frame does not hold that the bathtub curve is the standard form of $\lambda(t)$ in accordance with empirical evidence. On the other hand the theoretical results obtained here do not exclude that a special system can take after the bathtub curve. The bathtub curve is a concept that may be used for describing a very special system.

(D) The Boltzmann entropy plays a fundamental role on the theoretical plane as it clarifies why systems follow the second law of thermodynamics; instead it is not so common in engineering calculations. The Boltzmann-like entropy has the same virtues and limits of the Boltzmann entropy. It helps us to pass from studying “how” a system declines, to studying “why” a system declines, though the use of the Boltzmann-like entropy in applications is not so manageable. It is our opinion, that the Boltzmann-like entropy sustains a promising approach for developing a deductive construction integrating mathematical methods with engineering notions and specific biological knowledge.

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