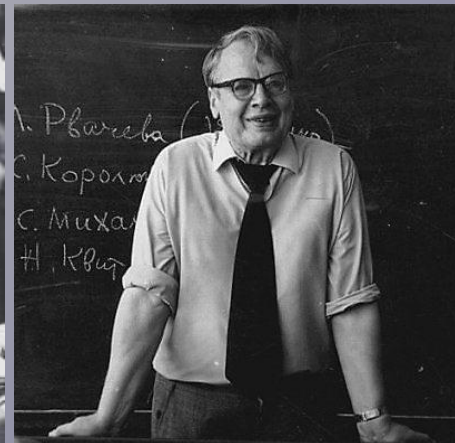


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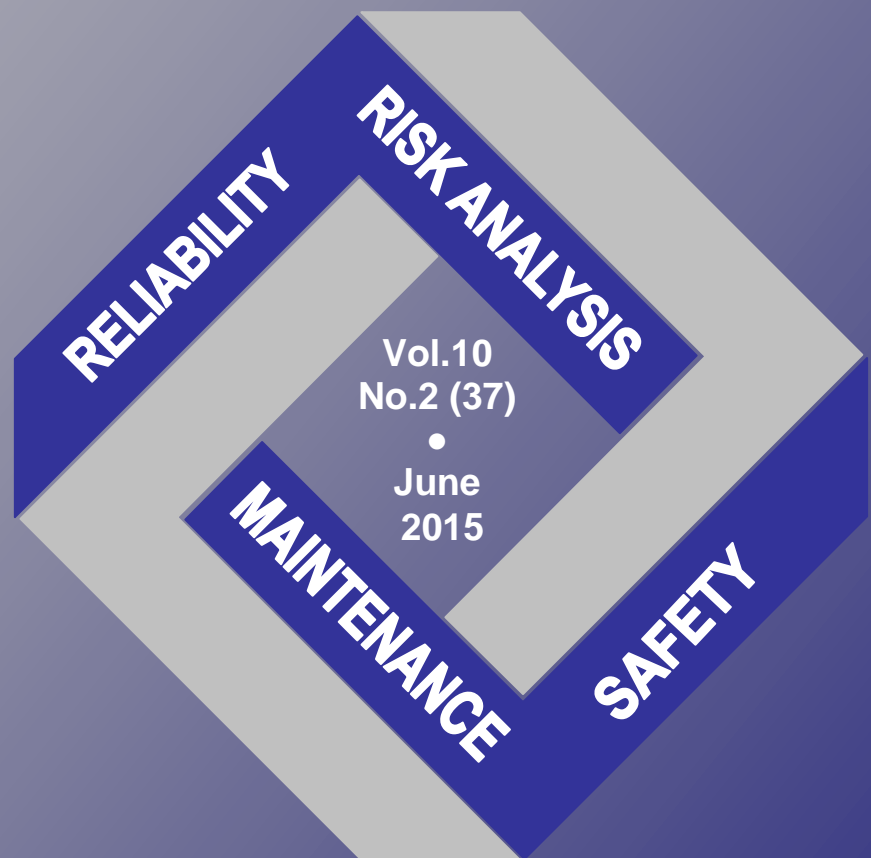


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San Diego

*In Memory Of Igor Ushakov*

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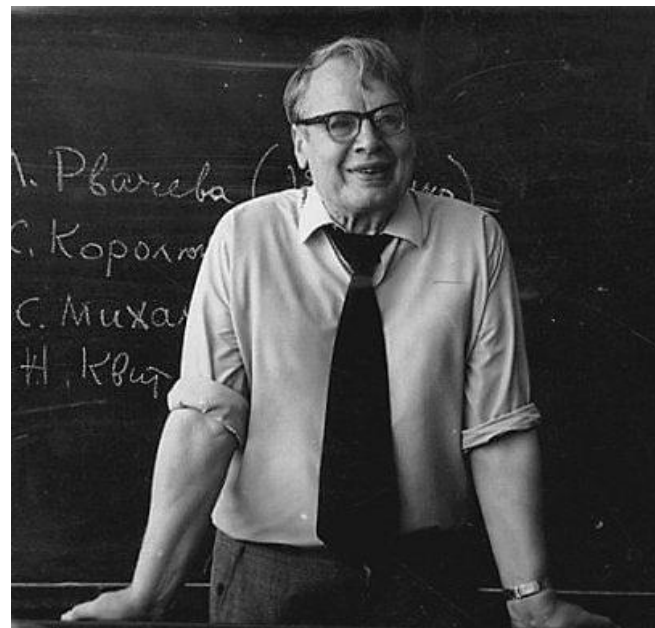
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# RELIABILITY: THEORY & APPLICATIONS

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*In Memory Of Igor Ushakov*

San Diego  
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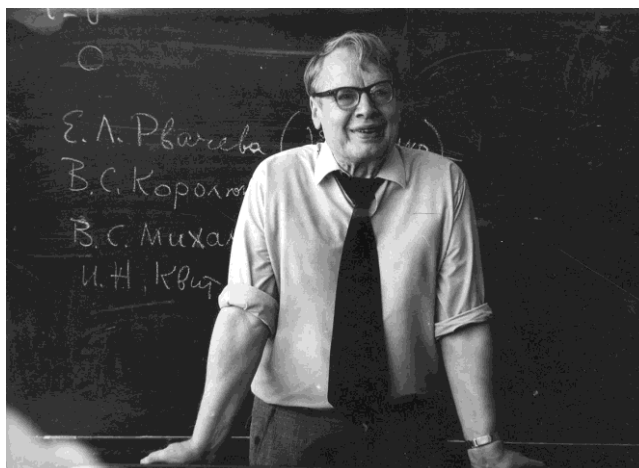
The focus of this paper is to assess ways to improve plant availability of an organisation in order to reduce its reactive maintenance from its current level of 30%. The pharmaceutical products the organisation (name withheld) produces are in short supply. It is vital the equipment that produces these products are reliable and produce quality products all the time. Loss of one batch due to equipment malfunction not only equates to loss of revenue but also the loss of vital medications which could jeopardise patient safety. This paper solely focuses and reports on the SIRFrt method to assess plant reliability in order to achieve maintenance excellence. There are seven elements that can impact maintenance excellence, however the focus of this paper is on reliability improvement since the audit scores were low in comparison to the other elements considered under the SIRFrt method. As a result of using feedback gained through the SIRFrt method assessment, it is anticipated that reactive maintenance will reduce below 10%.

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**BORIS VLADIMIROVICH GNEDENKO**  
**(01.01.1912 – 27.12.1995)****Yu.K. Belyaev**•  
Professor of Umea University, Sweden  
e-mail: [Yuri.Belyaev@math.umu.se](mailto:Yuri.Belyaev@math.umu.se)**ABSTRACT**

This article was published on journal «Markov Processes and Related Fields» (2014, Volume 20, Number 3). This article contain papers based on the talks given at the international conference “Probability Theory and Its Applications” (Moscow, June 26–30, 2012) in commemoration of the centennial of Boris Vladimirovich Gnedenko (01.I.1912 – 27.XII.1995)

One hundred years have passed since the birth of B.V. Gnedenko, the out- standing mathematician. B.V. Gnedenko’s contributions to the general theory of asymptotic properties of distributions of sums and the maxima of sequences of independent random variables are known worldwide. His contributions to mathematical methods in the Queuing theory are essential to reliability re- search, especially in the area of the evaluation of reliability characteristics of technical units and complex technical systems. B.V. was the principal organizer of research in these areas of applied mathematics in our country.



Boris Vladimirovich Gnedenko

The interests and activities of B.V. delved far wider than these areas of his scientific research where he is well known due to important results. For example, besides numerous discussions of the principal problems in Reliability and Queuing, there were numerous aspirants (Ph.D. candidates) presenting their dissertation research at the special seminars held in the lecture room at the Laboratory of Probability at the Department of Probability Theory. B.V. Gnedenko together with Yu.K. Belyaev and A.D. Soloviev were the organizers of these seminars. The seminars hosted speakers from different universities and research institutes. Seminar discussions were interesting



and useful, with the large number of participants always filling the notably large lecture room designed for around 50 people.

Through his many papers, seminars and meetings he organized, as well as his presentations at major scientific conferences, B.V. has outlined the necessity of higher education and its role as a critical priority in conducting significant and efficient scientific investigations and engineering applications. He emphasized that in the process of enabling valuable higher education, the most talented students will be discovered and will form the next generation of significant researchers and teachers. Gnedenko's lectures and presentations at research seminars were amazingly transparent in presenting basic ideas for the audience.

B.V. inherited the ability to provide an elegant and accessible presentation of basic ideas from his teacher A.Ya. Hinchin. B.V. considered questions of mathematical education in elementary schools, high schools and universities in broad aspects that included the influence of this education on the formation of pupils and students with a scientific world-view. He also maintained that the elements of mathematical education naturally promote and facilitate students to develop their own critical thinking skills. B.V. believed that the mathematical education should be illustrated by real applications containing real empirical data related to the future profession of students. For example, at seminars for students of bio-medical professions, mathematical exercises should be illustrated by the application of mathematical methods to analysis of empirical bio-medical data.

The last years of his life have coincided with the disintegration of the Soviet Union and with deterioration of its educational system. B.V. expressed his concern over the hazards associated with the loss of the prestige of the teaching profession, and following loss of experience and educational traditions specific to the older generations of professors, teachers and researches at universities and research institutions. He pointed out that these hazards would lead to a long-standing trend of low modernization and slow development of the Russian economy because education and progress in sciences represent a key factor in successful development of a nation.

B.V. was interested in history of mathematics. In his work he often highlighted history of scientific publications in the world and especially in Russia. In the book "Essay history of the Probability theory" he presented the history of the Probability theory as an evolution of the basic concepts: probabilities of events, random variables and their mathematical expectations. He showed that this approach to the historical analysis of the Probability theory is preferable because it allows to trace the evolution of all these basic concepts and, therefore, leads to their better understanding.

B.V. understood the importance of applying the results of mathematical theory to development of mathematics itself, as well as the growing importance of mathematics in other branches of science. The influence of this understanding is reflected in numerous contributions of the Department of Probability Theory in the Faculty of Mechanics and Mathematics at the Lomonosov Moscow State University while B.V. was the department head. Publications in prestigious academic journals, participation in research grants, as well as various remarkable mandatory and elective courses in probability and its applications offered in the department, show the high scientific potential of the members of the Department of Probability Theory at that time. In a photo taken on the occasion of the 50th anniversary of creation of the Department of Probability Theory (1985), we can see nearly all members of this Department.





From left to right there are: in the first row, D.B. Gnedenko (the Department of Numbers Theory), Yu.N. Tyurin, N.G. Himchenko, Boris Vladimirovich Gnedenko, E.V. Bulinskaya, O.P. Vinogradov; in the second row, M.V. Menshikov, B.A. Sevastyanov, Ya.G. Sinai, A.D. Soloviev, Yu.K. Belyaev, V.N. Tutubalin, E.V. Chepurin, T.N. Dugina, N.I. Cherevichkina, V.A. Lebedev; in the third row, A.D. Ventsel, V.V. Kozlov, S.A. Molchanov, V.I. Piterbarg, N.P. Korovina, L.G. Afanasieva, F.I. Falin, V.A. Belyaeva, A.G. Diyachkov, E. Balasanova, G.V. Martynov, A.P. Makarov. It should be also added to the above list absent in the photo V.A. Malyshev, V.P. Nosko, N.V. Chistyakova and G.K. Nosko.

Benevolent, hospitable B.V. was ready to discuss scientific research with colleagues, from different parts of Russia and foreign universities, who were visiting the Moscow Lomonosov State University. He had been often invited and warmly accepted by the colleagues across the globe. During his conference trips, B.V. always used the opportunity to learn about different cultural and historical heritage, observing towns and nature in different parts of Russia and abroad. The list of such invited trips is impressive. B.V. liked such travels.

B.V. made a major contribution to organizing lecture series for engineers at the Moscow Polytechnic Museum. The main aim was to increase the qualification and capabilities of engineers to produce products of high quality and reliability. This activity was launched in 1961 when B.V. and engineer Ya.M. Sorin organized a Moscow seminar on the problems of quality and reliability. The authorities of the Moscow Polytechnic Museum offered rooms for consulting and lectures, and the so-called “Reliability Chamber” had been organized there. The series of books and booklets devoted to quality and reliability was issued by the publishing houses “Znanie” and “Soviet Radio”.

These publications were intended for a wide audience of engineers. Successful experience of “Reliability Chamber” attracted the attention of many foreign scientists. Often scientists from different countries received get-to-know visits to “Reliability Chamber”. Season tickets for a series of lectures devoted to methods of the Reliability and the Quality Control theories were distributed with the help of the “Reliability Chamber”. Some of these lectures attracted 400 – 500 participants. Visiting lectures on these topics were also organized in different towns of the former USSR.

In February 1987 B.V.’s wife Natalia Konstantinovna Gnedenko died after a grave illness. She was his faithful friend and the love of his life. This doleful event proved to be fateful for his health. In 1988 he became seriously ill and had a complex surgical operation. The anxiety associated with the country- wide economic difficulties in higher education and scientific research during the transition to the market economy contributed to his health problems. In spite of all these problems B.V. continued to make notable contributions developing brand-new courses in his department and trying to accomplish publishing his numerous research ideas. In particular, he organized a new course in the Department of Probability Theory that detailed the study of insurance methods against different risks, which was an innovative step for Russia at that time. In the final years of his life, B.V. managed to prepare (together with I.A. Ushakov and I.V. Pavlov) two books devoted to applications of mathematical methods of evaluation reliability characteristics. These books were published in 1995 and 1999 by John Wiley publishing house. B.V. also prepared several other books for publishing and, with the help of his son Dmitry Borisovich Gnedenko, finished the memoirs. Dmitry managed to edit and compile the resulting 620-page book “My life in mathematics and mathematics in my life” which was published by the publishing house URSS in 2012 – the year of the 100th anniversary of B.V.’s birth. In his apartment, B.V. had collected a rich home library containing thousands of books reflecting different directions of research in Probability, Statistics and their applications. A part of this collection is gifted to the Library of Moscow State University. In this short essay I have only reflected B.V.’s activity and contributions related to the Moscow period of his life from 1960. There was also an interesting and extremely fruitful period in Moscow and in Ukraine before B.V. and his family relocated in 1960 to Russia at the Lomonosov Moscow State University, where he was invited by A.N. Kolmogorov.

The rich inheritance, which B.V. left to us in his publications and in his students’ works, will be useful to new generations of mathematicians. As an acknowledgement of B.V.’s scientific contributions, the International Conference “Probability Theory and Its Applications” was organized at Moscow Lomonosov State University in 2012. This conference was held in Commemoration of the Centennial of Boris Vladimirovich Gnedenko’s birth. Hundreds of probability specialists, B.V.’s followers and former students have come to get together to remember the Teacher and to continue his good work.

---

## B.V. GNEDENKO AND SOVIET PROBABILITY

In the period 1950–1980 any probability school in the USSR was somehow related to the name, works and personality of A.N. Kolmogorov. However, besides pure science, there always exists scientific politics initiated by some personalities. Kolmogorov, like any great scientist, did not participate much in this politics, being however an arbiter in some situations. Most political issues developed independently of his will. There were 4 main probability centers in the USSR of that time: Mathematical (Steklov) Institute in Moscow, its Leningrad and Novosibirsk branches, and the Department of Probability Theory in the Faculty of Mechanics and Mathematics at the Moscow State University (mechmath for short). I do not mention the groups in the Soviet republics like Kiev, Vilnius groups etc., most of which were under strong influence of the above mentioned groups. Steklov Institute controlled the central journal in the field — “Probability theory and its applications”, and because of this its content concerned mostly several classical topics in probability, for example, sums of independent random variables. In mechmath the situation was quite different. This was a multipolar world with different and bright personalities, whose interests varied from pure and fundamental research to very applied, and who coexisted without big problems, in comfort and safety. This situation was very much due to Boris Vladimirovich who could patiently listen and accept different views on the science. Despite the occurring conflicts the atmosphere of good will prevailed in the Department of Probability Theory. B.V. Gnedenko was a wise leader and it was thanks to him that all appearing intrigues, quarrels and contradictions were quickly extinguished.

V.A. Malyshev

# OPTIMAL DESIGN OF STEP STRESS PARTIALLY ACCELERATED LIFE TEST UNDER PROGRESSIVE TYPE-II CENSORED DATA WITH RANDOM REMOVAL FOR FRECHET DISTRIBUTION

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## ABSTRACT

In this article, progressive censoring and step stress partially accelerated life test are combined to develop a step-stress PALT with Progressively type-II Censored Data with the random removal. The removals from the test are assumed to have binomial distribution and uniform distribution and the life time of the testing products are considered to follow Frechet distribution. The parameters are estimated by using the maximum likelihood method and asymptotic confidence interval estimates of the model parameters are also evaluated by using Fisher information matrix. Statistically optimal PALT plans are developed such that the Generalized Asymptotic Variance (GAV) of the Maximum Likelihood Estimators (MLEs) of the model parameters at design stress is minimized. At the end, simulation study is performed to illustrate the statistical properties of the parameters.

**KEYWORDS:** Partially Accelerated Life Tests; Binomial Removal; Uniform Removal; Progressive Censoring; Maximum Likelihood Estimator; Generalized Asymptotic Variance

## 1 INTRODUCTION

When the product of high reliability is tested, the result of the some commonly used life test gives no or very few failures by the end of the test. In these types of the testing, the accelerated life testing (ALT) is used to obtain failures quickly. In such cases the testing is done at higher than usual use conditions. Three types of testing such as constant-stress, step-stress and progressive-stress are commonly used. In ALT, the mathematical model relating the lifetime of the unit and the stress is known or can be assumed. For detailed study of ALT see Nelson [1]. So as to, ALT data cannot be extrapolated to normal use condition. So, in such cases, partially accelerated life testing (PALT) is a more appropriate test to be used to estimate the statistical model parameters. Ismail et al. [2] introduced the Optimum Simple Time-Step Stress Plans for Partially Accelerated Life Testing with Censoring.

In many life tests, the experiment does not observe the failure times of all components. In such cases, the censored sampling arises. The most common censoring schemes are type-I censoring and type-II censoring. These two censoring schemes do not allow for units to be removed from the test at the points other than the final termination point. Moreover, there are some cases in which components are lost or removed from the test before failure. This would lead to progressive censoring. For progressive censoring see Balakrishnan and Aggarwala [3] and Balakrishnan [4]. Under the progressive type II censoring scheme, the experimenter puts  $n$  components on test at time zero. The first failure is observed at  $Y_1$  and then  $R_1$  of surviving components is randomly selected and removed. When the second failure occurs at time  $Y_2$ ,  $R_2$  of surviving components is randomly

selected and removed and when  $(m-1)^{th}$  failure is observed at the time  $Y_{m-1}, R_{m-1}$  of the surviving units are randomly selected and removed from the experiment, the experiment terminates when the  $m^{th}$  failure component is observed at  $X_m$  and  $R_m = n - m - \sum_{i=1}^{m-1} R_i$  all removed. In this censoring scheme  $R_1, R_2, \dots, R_m$  are all prefixed. However, in some practical experiments, these numbers cannot be pre-fixed and they occur at random. Inference based on progressively Type II censored data is discussed by many authors. Yuen and Tse [5] considered the estimation problem for Weibull distribution under progressive Censoring with random removals. Yang et al. [6] statistically analyzed the Weibull Distributed Lifetime Data under Type-II Progressive Censoring with Binomial Removals. Wu [7] used progressively Type-II censored data with uniform removals to estimate the parameters of Pareto distribution. Ismail et al. [8] introduced the Optimal Design of Step-Stress Life Test with Progressively type-II Censored Exponential Data with binomial removals. Bander [9] estimated the maximum likelihood for Generalized Pareto Distribution under Progressive Censoring with Binomial Removals. Chang et al. [10] studied the progressive censoring with Random Removals for the Burr Type XII Distribution.

## 2 THE MODEL AND TEST METHOD

### 2.1 The Frechet Distribution

The Frechet distribution is a special case of the generalized extreme value distribution. The generalized extreme value (GEV) distribution is a family of continuous probability distributions developed within extreme value theory to combine the Gumbel, Fréchet and Weibull families also known as type I, II and III extreme value distributions. The lifetimes of the test items are assumed to follow a Frechet distribution. The probability density function (pdf) of the Gompertz distribution is given by

$$f(t) = \alpha \theta^\alpha t^{-\alpha-1} \exp\left(-\left(\frac{t}{\theta}\right)^{-\alpha}\right) \quad (1)$$

And the cumulative distribution function is given by

$$F(t) = \exp\left(-\left(\frac{t}{\theta}\right)^{-\alpha}\right) \quad (2)$$

The survival function of the Frechet distribution is given by

$$\bar{F}(t) = 1 - \exp\left(-\left(\frac{t}{\theta}\right)^{-\alpha}\right)$$

### 2.2 Assumptions

- $n$  identical and independent units are put on the life used condition and the lifetime of each testing unit follows Frechet distribution.
- The test is terminated at the  $m^{th}$  failure, where  $m$  is prefixed ( $m \leq n$ ).
- Each of the  $n$  units is first run under normal use condition. If it does not fail or remove from the test by a pre-specified time  $\tau$ , it is put under accelerated condition.

- At the  $i^{\text{th}}$  failure a random number of the surviving units,  $R_i, i = 1, 2, \dots, m-1$ , are randomly selected and removed from the test. Finally, at the  $m^{\text{th}}$  failure the remaining surviving units  $R_m = n - m - \sum_{i=1}^{m-1} R_i$  are all removed from the test and the test is terminated.
- The lifetime, say  $Y$ , of a unit under SS-PALT can be written as

$$Y = \begin{cases} T & \text{if } T \geq \tau \\ \tau + (T - \tau)/\beta & \text{if } T < \tau \end{cases} \quad (3)$$

where  $T$  is the lifetime of the unit under normal use condition,  $\tau$  is the stress change time and  $\beta$  is the acceleration factor;  $\beta > 1$ . Therefore, the pdf of  $Y$  can be written as in the following form

Therefore probability density function (pdf) of  $Y$  can be written as

$$f(y) = \begin{cases} 0 & y \leq 0 \\ f_1(y) & 0 < y \leq \tau \\ f_2(y) & y > \tau \end{cases}$$

$$f(y) = \begin{cases} 0 & y \leq 0 \\ \alpha \theta^\alpha y^{-\alpha-1} \exp\left(-\left(\frac{y}{\theta}\right)^{-\alpha}\right) & 0 < y \leq \tau \\ \alpha \theta^\alpha \beta (\tau + \beta(y - \tau))^{-\alpha-1} \exp\left(-\left(\frac{\tau + \beta(y - \tau)}{\theta}\right)^{-\alpha}\right) & y > \tau \end{cases} \quad (4)$$

$$F(y) = \begin{cases} 0 & y \leq 0 \\ \exp\left(-\left(\frac{y}{\theta}\right)^{-\alpha}\right) & 0 < y \leq \tau \\ \exp\left(-\left(\frac{\tau + \beta(y - \tau)}{\theta}\right)^{-\alpha}\right) & y > \tau \end{cases} \quad (5)$$



### 3 MAXIMUM LIKELIHOOD ESTIMATION

#### 3.1 Parameter Estimation with the Binomial Removals

The number of units removed from the test at each failure time follows a binomial distribution and any individual unit being removed is independent of the others but with the same probability  $p$ . That is,  $R_1 \sim \text{bino}(n - m, p)$  and for  $i = 2, 3, \dots, m - 1$ ,  $R_i \sim \text{bino}\left(n - m - \sum_{j=1}^{i-1} r_j, p\right)$

and  $r_m = n - m - r_1 - r_2 - \dots - r_{m-1}$ .

Let  $(y_i, r_i, \delta_{1i}, \delta_{2i}), i = 1, 2, \dots, m$  denote the observation obtained from a progressively type-II censored sample with random removals in a step-stress PALT. Here  $y_{(1)} \leq y_{(2)} \leq \dots \leq y_{(m)}$ .

Thus for the progressive censoring with the pre determined number of the removals  $R = (R_1 = r_1, \dots, R_{m-1} = r_{m-1})$ , the conditional likelihood function of the observations  $y = \{(y_i, r_i, \delta_{1i}, \delta_{2i}), i = 1, 2, \dots, m\}$  can be defined as follow

$$L(y_i; \alpha, \beta, \theta, \delta_{1i}, \delta_{2i} | R = r) = \prod_{i=1}^m \left\{ [f_1(y_i)(\bar{F}_1(y_i))^{r_i}]^{\delta_{1i}} [f_2(y_i)(\bar{F}_2(y_i))^{r_i}]^{\delta_{2i}} \right\} \tag{6}$$

$$L(y; \theta, \alpha, \beta, \delta_{1i}, \delta_{2i} / R = r) = \prod_{i=1}^m \left\{ \left[ \alpha \theta^\alpha y_i^{-\alpha-1} \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right) \left(1 - \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right)\right)^{r_i} \right]^{\delta_{1i}} \right. \\ \left. \left[ \alpha \theta^\alpha \beta (\tau + \beta(y_i - \tau))^{-\alpha-1} \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right) \left(\exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right)^{r_i} \right]^{\delta_{2i}} \right\} \tag{7}$$

The number of units removed at each failure time follows a binomial distribution such that

$$P(R_1 = r_1) = \binom{n - m}{r_1} P^{r_1} (1 - P)^{n - m - r_1}$$

And for  $i = 2, 3, \dots, m - 1$

$$P(R_i = r_i | R_{i-1} = r_{i-1}, \dots, R_1 = r_1) = \binom{n - m - \sum_{j=1}^{i-1} r_j}{r_i} P^{r_i} (1 - P)^{n - m - \sum_{j=1}^i r_j}$$

where  $0 \leq r_i \leq n - m - (r_1 + r_2 + \dots + r_{i-1})$ . Furthermore, suppose that  $R_i$  is independent of  $Y_i$  for all  $i$ . Then the joint likelihood function can be found as

$$L(y_i; \alpha, \beta, \theta, p, \delta_{1i}, \delta_{2i}) = L_1(y_i; \alpha, \beta, \theta, p, \delta_{1i}, \delta_{2i} | R = r) P(R, p) \tag{8}$$

where  $P(R, p)$  is the joint probability distribution of  $R = (r_1, r_1, r_1, \dots, r_m)$  and is given by



$$\begin{aligned}
P(R, p) &= P(R_{m-1} = r_{m-1}, R_{m-2} = r_{m-2}, \dots, R_1 = r_1) \\
&= P(R_{m-1} = r_{m-1} / R_{m-2} = r_{m-2}, \dots, R_1 = r_1) \times P(R_{m-2} = r_{m-2} / R_{m-3} = r_{m-3}, \dots, R_1 = r_1) \\
&\quad \times P(R_{m-3} = r_{m-3} / R_{m-4} = r_{m-4}, \dots, R_1 = r_1) \times \dots \times P(R_2 = r_2 / R_1 = r_1) P(R_1 = r_1) \\
&= \frac{(n-m)!}{\binom{n-m-\sum_{i=1}^{m-1} r_i}{\sum_{i=1}^{m-1} r_i} \prod_{i=1}^{m-1} r_i!} p^{\sum_{i=1}^{m-1} r_i} (1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1} (m-i)r_i} \quad (9)
\end{aligned}$$

Now by substituting  $L_1(y_i; \alpha, \beta, \theta, p, \delta_{1i}, \delta_{2i} | R = r)$  and  $P(R, p)$  from the equation (7) and (9) in (8) we get the likelihood function

$$\begin{aligned}
L(y, \theta, \beta, p, \delta_{1i}, \delta_{2i}) &= \prod_{i=1}^m \left\{ \left[ \alpha \theta^\alpha y_i^{-\alpha-1} \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right) \left(1 - \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right)\right)^{r_i} \right]^{\delta_{1i}} \right. \\
&\quad \left. \left[ \alpha \theta^\alpha \beta (\tau + \beta(y_i - \tau))^{-\alpha-1} \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right) \left(\exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right)^{r_i} \right]^{\delta_{2i}} \right\} \quad (10) \\
&\quad \times \frac{(n-m)!}{\binom{n-m-\sum_{i=1}^{m-1} r_i}{\sum_{i=1}^{m-1} r_i} \prod_{i=1}^{m-1} r_i!} p^{\sum_{i=1}^{m-1} r_i} (1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1} (m-i)r_i}
\end{aligned}$$

The log-likelihood of the above equation is given by

$$\begin{aligned}
\log L &= \left[ m \log \alpha + m \alpha \log \theta - (\alpha + 1) \sum_{i=1}^{m_u} \log y_i - \sum_{i=1}^{m_u} (y_i / \theta)^{-\alpha} + r_i \sum_{i=1}^{m_u} \log \left(1 - \exp\left(-\left(y_i / \theta\right)^{-\alpha}\right)\right) \right] \\
&\quad \left[ m_a \log \beta - (\alpha + 1) \sum_{i=1}^{m_a} \log \left(\tau + \beta(y_i - \tau)\right) - \sum_{i=1}^{m_a} \left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha} \right. \\
&\quad \left. + r_i \sum_{i=1}^{m_a} \log \left(1 - \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right) \right] + \log c_1 + \sum_{i=1}^{m_i} r_i \log p \\
&\quad + \left[ (m-1)(n-m) - \sum_{i=1}^{m-1} (m-i)r_i \right] \log(1-p)
\end{aligned}$$

The maximum likelihood estimators of  $\beta$  and  $\theta$  can be derived directly by maximizing the equation (7) instead of (10) because  $P(R, p)$  does not involve the parameter  $\beta$  and  $\theta$ . Similarly the binomial parameter  $p$  does not depend on  $L_1(y_i; \alpha, \beta, \theta, p, \delta_{1i}, \delta_{2i} | R = r)$ , hence the MLE of  $p$  can be

found by maximizing  $P(R; p)$  directly. Thus, the maximum likelihood estimates (MLEs), of  $\beta$  and  $\theta$  can be found by solving the following equations:

$$\frac{\partial \log L}{\partial \theta} = \frac{m\alpha}{\beta} - \alpha\theta^{\alpha-1} \sum_{i=1}^{m_u} y_i^{-\alpha} + \alpha\theta^{\alpha-1} \sum_{i=1}^{m_u} \frac{r_i y_i^{-\alpha} \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right)}{1 - \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right)} - \alpha\theta^{\alpha-1} \sum_{i=1}^{m_a} (\tau + \beta(y_i - \tau))^{-\alpha} + \alpha\theta^{\alpha-1} \sum_{i=1}^{m_a} \frac{r_i (\tau + \beta(y_i - \tau))^{-\alpha} \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)}{1 - \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)} \tag{11}$$

$$\frac{\partial \log L}{\partial \beta} = \frac{m_a}{\beta} - (\alpha + 1) \sum_{i=1}^{m_a} \frac{y_i - \tau}{\tau + \beta(y_i - \tau)} + \alpha\theta^\alpha \sum_{i=1}^{m_a} \frac{y_i - \tau}{(\tau + \beta(y_i - \tau))^{\alpha+1}} - \alpha\theta^\alpha \sum_{i=1}^{m_a} \frac{r_i (y_i - \tau) (\tau + \beta(y_i - \tau))^{-\alpha-1} \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)}{\left[1 - \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right]} \tag{12}$$

Independently, the MLE of the binomial parameter  $p$  can be obtained by solving the following equation:

$$\frac{\partial \log L}{\partial p} = \sum_{i=1}^{m-1} \frac{r_i}{p} - \frac{\left[ (m-1)(n-m) - \sum_{i=1}^{m-1} (m-i)r_i \right]}{1-p} \tag{13}$$

Therefore we get  $\hat{p}$  from equation (13)

$$\hat{p} = \frac{\sum_{i=1}^{m-1} r_i}{\sum_{i=1}^{m-1} r_i + (m-1)(n-m) - \sum_{i=1}^{m-1} (m-i)r_i}$$

### 3.2 Estimation with the Uniform Removal

The number of units removed from the test at each failure time follows a uniform discrete distribution. That is,  $R_1 \sim Unif(0, n - m)$  and for  $i = 2, 3, \dots, m - 1$ ,  $R_i \sim Unif\left(0, n - m - \sum_{j=1}^{i-1} r_j\right)$  and  $r_m = n - m - r_1 - r_2 - \dots - r_{m-1}$ . Such that,

$$P(R_1 = r_1) = \frac{1}{n - m + 1}$$

And for  $i=2, 3 \dots m-1$ .

$$P(R_i = r_i | R_{i-1} = r_{i-1}, \dots, R_1 = r_1) = \frac{1}{n - m - \sum_{j=1}^{i-1} r_j + 1}$$

where  $P(R)$ , the joint probability distribution of  $R = (r_1, r_1, r_1, \dots, r_m)$  and is given by

$$P(R = r) = \frac{1}{n - m - \sum_{i=1}^{m-1} r_i + 1} \tag{14}$$

where  $0 \leq r_i \leq n - m - (r_1 + r_2 + \dots + r_{i-1}), i = 1, 2, \dots, m - 1$ .

It is clear that  $P(R=r)$  does not depend on the parameters  $\beta$  and  $\theta$  and, hence the maximum likelihood estimators can be derived directly by maximizing the equations (7) and then solving the equations (11)

#### 4 FISHER INFORMATION MATRIX & ASYMPTOTIC CONFIDENCE INTERVAL

The asymptotic variance-covariance matrix of the ML estimators of the parameters can be approximated by numerically inverting the Fisher-information matrix F and The Fisher information matrix is obtained by taking the negative second partial derivatives of the log-likelihood function and for the binomial removal it can be written

$$F = \begin{bmatrix} -\frac{\partial^2 l}{\partial \theta^2} & -\frac{\partial^2 l}{\partial \theta \partial \beta} & -\frac{\partial^2 l}{\partial \theta \partial p} \\ -\frac{\partial^2 l}{\partial \beta \partial \theta} & -\frac{\partial^2 l}{\partial \beta^2} & -\frac{\partial^2 l}{\partial \beta \partial p} \\ -\frac{\partial^2 l}{\partial p \partial \theta} & -\frac{\partial^2 l}{\partial p \partial \beta} & -\frac{\partial^2 l}{\partial p^2} \end{bmatrix}$$

And for the uniform removal, fisher information matrix can be written as

$$F = \begin{bmatrix} -\frac{\partial^2 l}{\partial \theta^2} & -\frac{\partial^2 l}{\partial \theta \partial \beta} \\ -\frac{\partial^2 l}{\partial \beta \partial \theta} & -\frac{\partial^2 l}{\partial \beta^2} \end{bmatrix}$$

Elements of Fisher Information matrix are

$$\begin{aligned} \frac{\partial^2 \log L}{\partial \theta^2} &= -\frac{m\alpha}{\theta^2} - \alpha(\alpha-1) \sum_{i=1}^{m_u} y_i^{-\alpha} \theta^{\alpha-2} + \alpha(\alpha-1) \theta^{\alpha-2} \sum_{i=1}^{m_u} (\tau + \beta(y_i - \tau))^{-\alpha} \\ &\quad + \alpha \sum_{i=1}^{m_u} \frac{r_i y_i^{-\alpha} \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right) \left[(\alpha-1)\theta^{\alpha-2} - y_i^{-\alpha}\right]}{\left[1 - \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right)\right]} - \alpha^2 \theta^{2\alpha-2} \sum_{i=1}^{m_u} \frac{r_i y_i^{-2\alpha} \exp\left(-2\left(\frac{y_i}{\theta}\right)^{-\alpha}\right)}{\left[1 - \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right)\right]^2} \\ &\quad + \alpha \sum_{i=1}^{m_a} \frac{r_i (\tau + \beta(y_i - \tau))^{-\alpha} \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right) \left[(\alpha-1)\theta^{\alpha-2} + \alpha\theta^{2\alpha-2} (\tau + \beta(y_i - \tau))^{-\alpha}\right]}{\left[1 - \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right]} \\ &\quad - \alpha^2 \theta^{2\alpha-2} \sum_{i=1}^{m_a} \frac{r_i (\tau + \beta(y_i - \tau))^{-2\alpha-1} \exp\left(-2\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)}{\left[1 - \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right]^2} \\ \frac{\partial^2 \log L}{\partial \beta^2} &= -\frac{m_a}{\beta^2} + (\alpha+1) \sum_{i=1}^{m_a} \frac{(y_i - \tau)^2}{[\tau + \beta(y_i - \tau)]^2} - \alpha(\alpha+1) \theta^\alpha \sum_{i=1}^{m_a} \frac{(y_i - \tau)^2}{(\tau + \beta(y_i - \tau))^{\alpha+2}} \\ &\quad - \alpha \theta^\alpha \sum_{i=1}^{m_a} \frac{r_i (y_i - \tau)^2 (\tau + \beta(y_i - \tau))^{-\alpha-2} \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right) \left[-(\alpha+1) + \alpha\theta^\alpha \left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right]}{1 - \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)} \\ &\quad - \alpha \sum_{i=1}^{m_a} \frac{r_i (y_i - \tau)^2 (\tau + \beta(y_i - \tau))^{-2\alpha-2} \exp\left(-2\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)}{\left[1 - \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right]^2} \\ \frac{\partial^2 \log L}{\partial p^2} &= -\sum_{i=1}^{m_i} \frac{r_i}{p^2} - \frac{\left[(m-1)(n-m) - \sum_{i=1}^{m-1} (m-i)r_i\right]}{(p-1)^2} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \log L}{\partial \theta \partial \beta} &= \frac{\partial^2 \log L}{\partial \beta \partial \theta} = \alpha^2 \theta^{\alpha-1} \sum_{i=1}^{m_a} (\tau + \beta(y_i - \tau))^{-\alpha-1} (y_i - \tau) \\ &- \alpha^2 \sum_{i=1}^{m_a} \frac{r_i (y_i - \tau) (\tau + \beta(y_i - \tau))^{-\alpha-1} \theta^{\alpha-1} \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right) \left[1 + \theta^\alpha (\tau + \beta(y_i - \tau))^{-\alpha}\right]}{\left[1 - \exp\left(-\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)\right]} \\ &+ \alpha^2 \sum_{i=1}^{m_a} \frac{r_i \theta^{2\alpha-1} (y_i - \tau) (\tau + \beta(y_i - \tau))^{-2\alpha-1} \exp\left(-2\left(\frac{\tau + \beta(y_i - \tau)}{\theta}\right)^{-\alpha}\right)}{\left[1 - \exp\left(-\left(\frac{y_i}{\theta}\right)^{-\alpha}\right)\right]^2} \end{aligned}$$

$$\frac{\partial^2 \ln L}{\partial p \partial \theta} = \frac{\partial^2 \ln L}{\partial \theta \partial p} = \frac{\partial^2 \ln L}{\partial p \partial \beta} = \frac{\partial^2 \ln L}{\partial \beta \partial p} = 0$$

The variance covariance and covariance matrix of the parameter for the binomial removal can be written

$$\Sigma = \begin{bmatrix} -\frac{\partial^2 l}{\partial \beta^2} & -\frac{\partial^2 l}{\partial \beta \partial \theta} & -\frac{\partial^2 l}{\partial \beta \partial p} \\ -\frac{\partial^2 l}{\partial \theta \partial \beta} & -\frac{\partial^2 l}{\partial \theta^2} & -\frac{\partial^2 l}{\partial \theta \partial p} \\ -\frac{\partial^2 l}{\partial p \partial \beta} & -\frac{\partial^2 l}{\partial p \partial \theta} & -\frac{\partial l}{\partial p^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\beta}) & ACov(\hat{\beta}\hat{\theta}) & ACov(\hat{\beta}\hat{p}) \\ ACov(\hat{\theta}\hat{\beta}) & AVar(\hat{\theta}) & ACov(\hat{\theta}\hat{p}) \\ ACov(\hat{p}\hat{\beta}) & ACov(\hat{p}\hat{\theta}) & AVar(\hat{p}) \end{bmatrix}$$

And for the uniform removal case it can be written as

$$\Sigma = \begin{bmatrix} -\frac{\partial^2 l}{\partial \beta^2} & -\frac{\partial^2 l}{\partial \beta \partial \theta} \\ -\frac{\partial^2 l}{\partial \theta \partial \beta} & -\frac{\partial^2 l}{\partial \theta^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\beta}) & ACov(\hat{\beta}\hat{\theta}) \\ ACov(\hat{\theta}\hat{\beta}) & AVar(\hat{\theta}) \end{bmatrix}$$

The  $100(1-\xi)\%$  asymptotic confidence interval for  $\theta, \beta$  and  $p$  can be written as

$$\left[ \hat{\theta} \pm Z_{1-\frac{\xi}{2}} \sqrt{AVar(\hat{\theta})} \right], \left[ \hat{\beta} \pm Z_{1-\frac{\xi}{2}} \sqrt{AVar(\hat{\beta})} \right] \text{ and } \left[ \hat{p} \pm Z_{1-\frac{\xi}{2}} \sqrt{AVar(\hat{p})} \right]$$

## 5 OPTIMUM TEST PLAN

The present criterion by which one can choose the optimal value of  $\tau$  is based on the determinant of the Fisher's information matrix. Maximization of that that determinant is equivalent to minimization of the generalized asymptotic variance (GAV) of the MLE of the model parameters. The GAV is the reciprocal of the determinant of the Fisher's information matrix  $F$  that is

$$GAV = \frac{1}{|F|}$$

So, the optimal value of  $\tau$  is chosen in such a way that the determinant of the Fisher's information matrix  $F$  is maximized and then the GAV is minimized. This is called the D-optimality criterion.

## 6 SIMULATION STUDY

In order obtain MLEs of  $\beta, \theta$  and  $p$  and to study the properties of these estimates through Mean squared errors (MSEs,) and the coverage rate of asymptotic confidence intervals for different sample sizes, a simulation study is performed. Moreover, we will determine the optimal stress change time which minimizes the generalized asymptotic variance of the MLE of parameters. To perform the simulation study, we used the following steps

- a) First specify the value of  $n$  and  $m$ .
- b) The value of the parameters are chosen to be  $\alpha = 2.87, \theta = 3.02, \beta = 2.62, p = 0.67, \tau = 3.5$ .
- c) Generate a random sample with size  $n$  and censoring size  $m$  with random removals,  $r_i, i = 1, 2, \dots, m-1$  from the random variable  $Y$  given by (4).
- d) Generate a group value  $R_i \sim \text{bino}\left(n - m - \sum_{j=1}^{i-1} r_j, p\right)$  and also  $R_i \sim \text{Unif}\left(0, n - m - \sum_{j=1}^{i-1} r_j\right)$   
where,  $r_m = n - m - r_1 - r_2 - \dots - r_{m-1}$ .
- e) For different sample sizes  $n = 20, 60, 80, 100$  and  $120$ , compute the ML estimates.
- f) The mean squared error (MSE), the coverage rate of the 95% confidence interval of parameters and Bias are obtained associated with the MLE of the parameters, optimal value of  $\tau$  and also the Optimal GAV of the MLEs of the model parameters are obtained numerically for each sample size.

**Table 1(i);** Simulation study results with Binomial Removals for  $\alpha = 2.87, \theta = 3.02, \beta = 2.62, p = 0.67, \tau = 3.5$ .

n	m	Binomial case			95% Confidence interval coverage			$\tau$	$ F^{-1} $
		$\hat{\theta}$	$\hat{\beta}$	$\hat{p}$	$CP_{\hat{\theta}}$	$CP_{\hat{\beta}}$	$CP_{\hat{p}}$		
20	9	2.983612	4.889763	0.897212	0.92039	0.90121	0.90313	3.8746	1.336
	19	2.977351	4.886342	0.895871	0.92141	0.90341	0.90583	3.8786	1.465
60	9	2.953811	4.867830	0.847492	0.92156	0.90547	0.90876	3.7424	1.493
	19	2.907351	4.858361	0.808313	0.92183	0.90645	0.90963	3.7413	1.502
	29	2.895634	4.888907	0.804721	0.92190	0.90673	0.90991	3.7409	1.573
	39	2.893631	4.683670	0.800838	0.92199	0.90843	0.91234	3.7289	1.638
	49	2.890731	4.642846	0.799743	0.92213	0.90863	0.91425	3.7263	1.693
	59	2.865341	4.619843	0.795982	0.92254	0.91633	0.91473	3.7084	1.699
80	9	2.862563	4.983741	0.769371	0.92261	0.91740	0.91533	3.7052	1.712
	19	2.860726	4.738421	0.766932	0.92275	0.91834	0.91642	3.5566	1.734
	29	2.846535	4.597361	0.759826	0.93280	0.91876	0.91735	3.5503	1.782
	39	2.818732	4.55836	0.685821	0.93289	0.91899	0.91841	3.4371	1.791
	49	2.815721	4.387461	0.588763	0.93385	0.92934	0.91893	3.5778	1.832
	59	2.687631	4.334524	0.559831	0.93481	0.92997	0.91934	3.0766	1.854
	69	2.665434	4.284712	0.530841	0.93541	0.93013	0.91953	3.0355	1.871
	79	2.646213	4.097361	0.508349	0.94753	0.93084	0.91979	2.7009	1.889
100	9	2.619736	3.869763	0.487354	0.95130	0.93099	0.91991	2.7987	1.920
	19	2.605531	3.898731	0.379421	0.95353	0.93194	0.92421	2.7354	1.943
	29	2.576435	3.757365	0.339741	0.95365	0.93245	0.92632	2.7354	2.132
	39	2.557261	3.728371	0.336821	0.95475	0.93385	0.92713	2.7047	2.223
	49	2.397251	3.686510	0.309431	0.95573	0.93642	0.92795	2.7028	2.264
	59	2.307360	3.428761	0.304814	0.95752	0.93752	0.92846	2.7011	2.349
	69	2.152841	3.087361	0.233193	0.95883	0.93840	0.92896	2.7006	2.382
	79	2.119423	2.787361	0.230341	0.95992	0.94671	0.92888	2.6937	2.467
	89	2.094381	2.629834	0.178287	0.96862	0.94689	0.92913	2.6795	2.484
	99	2.007378	2.198347	0.145931	0.97432	0.94778	0.92999	2.6654	2.961
120	9	2.003841	2.007973	0.089831	0.97652	0.95032	0.93252	2.6473	2.999
	19	1.997763	1.775983	0.089720	0.97743	0.95075	0.93419	2.5531	3.012
	29	1.947345	1.999631	0.084566	0.97832	0.95174	0.93555	2.5139	3.058
	39	1.917371	1.929831	0.059741	0.97865	0.95195	0.93860	2.4961	3.184
	49	1.886351	1.909832	0.057631	0.97921	0.95348	0.94642	2.4741	3.452
	59	1.807363	1.905987	0.005574	0.98134	0.95534	0.94875	2.3961	3.872
	69	1.743251	1.889874	0.005174	0.98353	0.95613	0.95875	2.3756	3.891
	79	1.797356	1.899642	0.003752	0.98463	0.95732	0.95999	2.3367	3.928
	89	1.586352	1.858943	0.001734	0.98561	0.95822	0.96641	2.3205	3.963
	99	1.559736	2.81874	0.001538	0.98673	0.95913	0.96831	2.1858	3.971
	109	1.5372651	1.77321	0.009634	0.98751	0.95989	0.97654	2.0751	3.984
	119	1.386345	1.79731	0.002752	0.98462	0.96143	0.97943	2.0356	3.991



**Table 1(ii);** Simulation study results with Binomial Removals for  $\alpha = 2.87, \theta = 3.02, \beta = 2.62, p = 0.67, \tau = 3.5.$

n	m	Bias $\hat{\theta}$	Bias $\hat{\beta}$	Bias $\hat{p}$
20	9	0.006691	0.009879	0.089431
	19	0.006687	0.009773	0.067909
60	9	0.005982	0.005928	0.063989
	19	0.005791	0.005721	0.047298
	29	0.005194	0.004823	0.045901
	39	0.003791	0.004594	0.028432
	49	0.003913	0.003909	0.018931
	59	0.003492	0.003791	0.015986
80	9	0.003389	0.003588	0.014982
	19	0.002780	0.002791	0.011955
	29	0.002678	0.002279	0.011577
	39	0.002569	0.002254	0.008793
	49	0.002378	0.002093	0.003985
	59	0.002354	0.002056	0.003416
	69	0.002334	0.001973	0.001567
	79	0.001682	0.001671	0.001391
100	9	0.001494	0.001498	0.001198
	19	0.001475	0.001289	0.001203
	29	0.000971	0.001182	0.001982
	39	0.000849	0.000678	0.000689
	49	0.000692	0.000451	0.000486
	59	0.000578	0.000381	0.000198
	69	0.000387	0.000078	0.000139
	79	0.000234	0.000029	0.000116
	89	0.000209	$7.35 \times 10^{-5}$	0.000104
	99	0.000209	$3.74 \times 10^{-5}$	0.000101
120	9	$9.87 \times 10^{-5}$	$9.59 \times 10^{-6}$	0.000094
	19	$9.31 \times 10^{-5}$	$9.38 \times 10^{-6}$	0.000047
	29	$7.52 \times 10^{-5}$	$9.27 \times 10^{-7}$	0.000029
	39	$4.39 \times 10^{-5}$	$5.62 \times 10^{-7}$	0.000016
	49	$2.58 \times 10^{-5}$	$5.39 \times 10^{-7}$	$8.79 \times 10^{-5}$
	59	$5.81 \times 10^{-7}$	$2.76 \times 10^{-7}$	$5.91 \times 10^{-5}$
	69	$4.79 \times 10^{-7}$	$5.73 \times 10^{-8}$	$4.13 \times 10^{-5}$
	79	$3.35 \times 10^{-7}$	$4.79 \times 10^{-8}$	$9.95 \times 10^{-6}$
	89	$2.79 \times 10^{-7}$	$3.79 \times 10^{-8}$	$8.88 \times 10^{-6}$
	99	$2.12 \times 10^{-7}$	$1.87 \times 10^{-8}$	$6.83 \times 10^{-6}$
	109	$2.07 \times 10^{-7}$	$8.67 \times 10^{-9}$	$4.67 \times 10^{-6}$
	119	$6.87 \times 10^{-8}$	$5.19 \times 10^{-9}$	$1.39 \times 10^{-6}$

**Table 2;** Simulation study results with uniform removals for  $\alpha = 2.87, \theta = 3.02, \beta = 2.62, p = 0.67$  and  $\tau = 3.5$

n	m	MLE		95% Confidence Interval coverage		$Bias_{\hat{\theta}}$	$Bias_{\hat{\beta}}$	$\tau^*$	$ F^{-1} $
		$\hat{\theta}$	$\hat{\beta}$	$CP_{\hat{\theta}}$	$CP_{\hat{\beta}}$				
20	9	3.94710	4.98997	0.91018	0.87329	0.097631	0.009959	5.9829	1.009
	19	3.92741	4.98975	0.91317	0.87440	0.093859	0.009936	5.9693	1.119
60	9	3.91931	4.99742	0.91712	0.87489	0.073185	0.009368	5.8746	1.239
	19	3.91673	4.95836	0.91738	0.87511	0.068166	0.008489	5.7837	1.265
	29	3.91391	4.98890	0.91740	0.87546	0.057217	0.008299	5.4728	1.363
	39	3.91832	4.99888	0.91765	0.87599	0.043608	0.006943	5.4643	1.371
	49	3.67721	4.99817	0.91785	0.87632	0.041735	0.006509	5.4489	1.398
	59	3.65360	4.98736	0.91889	0.87790	0.030360	0.006297	5.4098	1.403
80	9	3.48721	4.95742	0.91940	0.87793	0.018429	0.006098	5.4071	1.412
	19	3.46831	4.92646	0.91985	0.87888	0.009588	0.006024	5.3064	1.425
	29	3.47974	4.90896	0.91990	0.89999	0.005981	0.005949	5.1984	1.451
	39	3.29346	4.90693	0.91998	0.91354	0.005945	0.005439	5.1697	1.463
	49	3.25312	4.78931	0.92011	0.91616	0.005674	0.005190	5.0983	1.470
	59	3.21038	4.75726	0.92042	0.91659	0.005395	0.004987	5.0582	1.623
	69	3.09531	4.73842	0.92086	0.91923	0.005194	0.004956	5.0193	1.674
79	3.09836	4.73571	0.92090	0.91987	0.004004	0.004547	4.9875	1.680	
100	9	3.05647	4.59831	0.92119	0.92156	0.003598	0.003757	4.8572	1.731
	19	3.05563	4.56828	0.92187	0.92181	0.003283	0.002899	4.6365	2.643
	29	3.03844	4.29784	0.92319	0.92615	0.003093	0.002875	4.5324	2.684
	39	3.03573	4.27641	0.92355	0.90842	0.003062	0.002598	4.5084	2.299
	49	3.01963	4.25989	0.92488	0.92476	0.000999	0.002429	4.3948	2.384
	59	2.98450	4.23791	0.92556	0.92589	0.000739	0.002125	4.2874	2.715
	69	2.95741	4.09912	0.92580	0.92757	0.000721	0.001356	4.2683	2.764
	79	2.93474	4.06983	0.92666	0.92783	0.000699	0.000896	4.2543	2.754
	89	2.91093	4.06728	0.92691	0.92791	0.000570	0.000597	4.1974	2.794
	99	2.90983	4.01734	0.92921	0.92798	0.000398	0.000496	4.0746	2.790
120	9	2.90657	3.93837	0.92957	0.92799	0.000096	0.000063	3.9973	2.917
	19	2.90633	3.90973	0.93421	0.93523	0.000068	0.000039	3.8374	2.932
	29	2.90435	3.68347	0.93511	0.92645	0.000036	0.000019	3.6467	2.938
	39	2.90313	3.65531	0.92585	0.92685	0.000019	0.000013	3.5621	2.972
	49	2.86414	3.48327	0.92881	0.92985	$9.96 \times 10^{-5}$	$9.02 \times 10^{-5}$	3.4788	2.977
	59	2.84531	3.45177	0.93183	0.93145	$9.28 \times 10^{-7}$	$8.84 \times 10^{-5}$	3.1845	2.999
	69	2.84313	3.42641	0.93371	0.93351	$9.09 \times 10^{-7}$	$8.36 \times 10^{-5}$	3.1477	3.031
	79	2.75443	3.28421	0.93612	0.93831	$8.45 \times 10^{-7}$	$7.79 \times 10^{-5}$	3.1248	3.074
	89	2.73249	3.26947	0.93722	0.935145	$8.33 \times 10^{-7}$	$7.34 \times 10^{-5}$	3.0983	3.187
	99	2.71734	3.24874	0.94513	0.93690	$7.84 \times 10^{-7}$	$6.78 \times 10^{-5}$	3.0387	3.191
	109	2.59931	3.19834	0.94721	0.93800	$6.69 \times 10^{-7}$	$6.34 \times 10^{-5}$	3.0276	3.284
	119	2.51677	3.15893	0.94882	0.93641	$4.05 \times 10^{-7}$	$5.99 \times 10^{-6}$	2.9975	3.291

## 7 CONCLUSION

This paper considers the SS-PALT under type-II progressive censoring with Binomial and uniform removals assuming frechet distribution. Comparison between both removal are shown. The Newton-Raphson method is applied to obtain the optimal stress-change time  $\tau^*$  which minimizes the GAV.

The numerical study for obtaining the optimum plan for binomial removal is tabulated in table 1 for different sample size and table 2 describes uniform removal for possible values of scale and shape parameters. From the above results it is easy to find that for the fixed values of the parameters, the error and optimal time decrease with increasing sample size  $n$ .

Performance of testing plans and model assumptions are usually evaluated by the properties of the maximum likelihood estimates of model parameters. Hence from the numerical result we can conclude that estimates of binomial are more stable with relatively small error with increasing sample size. Therefore, the test design obtained here is robust design and work well for binomial removal.

As a future work, this study can be extended to explore the situation under type-I progressive censoring

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# ASYMPTOTIC ANALYSIS OF FEW NODES FAILURE IN ORIENTED RANDOM GRAPH

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## ABSTRACT

In this paper an oriented graph with high reliable nodes is considered. A node stops its work if there is failed node which has a way to this node. Asymptotic formulas for a calculation of a probability that some nodes in the graph stop their work are obtained for different conditions on a graph structure and on a number of the nodes. These formulas allow to obtain conditions on reserve in considered graphs if nodes have different failure probabilities.

**Keywords:** an oriented graph, a high reliable node, a failure probability.

## INTRODUCTION

In the reliability theory there is a problem of an investigation of graphs with unreliable edges and absolutely reliable nodes [1], [2], graphs with unreliable nodes and absolutely reliable edges [3], [4], [5]. Graphs analyzed in cited articles are not oriented.

In this paper a model of a random graph in which a node stops its work if there is failed node which has a way to this node. This model is considered in the monograph [6] devoted to failures trees in technological systems. Such model also is connected with an idea of random network controllability which is used in medical - biological applications [7].

In this paper asymptotic formulas are obtained for a calculation of a stop probability of few nodes in a condition that graph nodes are high reliable but have different failure probabilities. First results in this direction have been obtained in a case of identical failure probabilities of nodes [8]. The formulas allow to analyze a problem of a reservation in considered graphs. One of approaches to produce such formulas is a construction of incompatible events sequence [9]. But in our case this approach leads to calculations with geometrical complexity by a number of graph nodes. Obtained result is based on an asymptotic expansion in the Poincare inclusion and exclusion formula. Specifics of the obtained result is an inclusion of nodes failures weights into asymptotic formulas.

## MAIN RESULTS

Consider the oriented graph  $G$  with the finite set of nodes  $I$ . On the set  $I$  define a relation of a partial order:  $i' \succcurlyeq i''$ , if in the graph  $G$  there is a way from the node  $i'$  to the node  $i''$ . For each node  $i \in I$  contrast the set of nodes  $F_i = \{i' : i' \succcurlyeq i\}$ . Assume that the graph  $G$  nodes work independently with the probabilities  $p_i, i \in I$  and if in  $F_i$  there is a failed node then the node  $i$  stops its work.

Consider the subset of nodes  $R = \{1, \dots, r\} \subseteq I$  and calculate the probability  $Q(R)$  that all nodes in  $R$  stop their work. Denote  $c(i_1, \dots, i_k)$  the sum of all  $c_i$  such that  $i \in T(i_1, \dots, i_k) = \bigcap_{j=1}^k F_{i_j}$ ,  $1 \leq i_1 < \dots < i_k \leq r$ ,  $1 \leq k \leq r$ .

**Theorem 1.** If  $p_i = \exp(-c_i h)$ ,  $c_i > 0$ ,  $i \in I$ , then for any natural  $r$  we have the relation  $Q(R) = h c(1, \dots, r) + O(h^2)$ ,  $h \rightarrow 0$ .

**Proof.** Denote  $D(k)$  the random event that there are not fails in the set  $F_k$ . Then from the Poincare formula of inclusions and exclusions we have

$$Q(R) = 1 - P\left(\bigcup_{k=1}^r D(k)\right) = 1 - \sum_{k=1}^r (-1)^{k-1} \sum_{1 \leq i_1 < \dots < i_k \leq r} P\left(\bigcap_{j=1}^k D(i_j)\right), \quad (1)$$

$$P\left(\bigcap_{j=1}^k D(i_j)\right) = \exp(-h\bar{N}), \quad \bar{N} = \sum_{s=1}^k (-1)^{s-1} \sum_{1 \leq j(1) < \dots < j(s) \leq k} c(i_{j(1)}, \dots, i_{j(s)}).$$

Substituting the relation (2) into Formula (1) and using the Taylor expansion of the exponent we obtain for  $h \rightarrow 0$ :

$$Q(R) = 1 - \sum_{k=1}^r (-1)^{k-1} \sum_{1 \leq i_1 < \dots < i_k \leq r} (1 - h\bar{N}) + O(h^2) =$$

$$= l_0 + h \sum_{k=1}^r \sum_{1 \leq i_1 < \dots < i_k \leq r} c(i_1, \dots, i_k) l(i_1, \dots, i_k) + O(h^2). \quad (2)$$

Here  $l(i_1, \dots, i_k)$  are some integer coefficients. As each transposition of indexes  $i_1, \dots, i_k$ ,  $1 \leq i_1 < \dots < i_k \leq r$ ,  $1 \leq k \leq r$ , in Formula (2) does not change  $Q(R)$ , so we have the equalities

$$l(i_1, \dots, i_k) = l(1, \dots, k), \quad l_0 = 1 - \sum_{k=1}^r (-1)^{k-1} C_r^k = \sum_{k=0}^r (-1)^k C_r^k = 0,$$

$$l(1, \dots, s) = \sum_{k=s}^r (-1)^{k-s} C_r^{k-s} = 0, \quad 1 \leq s \leq r-1, \quad l(1, \dots, r) = 1.$$

Theorem 1 is proved.

For  $c(1, \dots, r) = 0$  from Theorem 1 we obtain that  $Q(R) = O(h^2)$ . But a problem is to formulate sufficient conditions of the relation  $Q(R) = O(h^{t+1})$ ,  $t \geq 2$ . Such conditions may be obtained from the following statement.

**Theorem 2.** If  $p_i = \exp(-c_i h)$ ,  $c_i > 0$ ,  $i \in I$ , and for any subset of different nodes  $i, j, k \in R$  we have the relation  $F_i \cap F_j \cap F_k = \emptyset$  then for  $2 < 2t < r$  the equality  $Q(R) = O(h^{t+1})$  is true.

**Proof.** Denote  $\bar{C}(i_1, \dots, i_k)$  the sum of all  $c_i$  such that  $i \in \bigcup_{j=1}^k F_{i_j}$  and put

$$S_l = \sum_{k=1}^r (-1)^{k-1} \sum_{1 \leq i_1 < \dots < i_k \leq r} \bar{C}^l(i_1, \dots, i_k), \quad (3)$$

$$\bar{C}(i_1) = c(i_1), \quad \bar{C}(i_1, \dots, i_k) = \sum_{j=1}^k c(i_j) - \sum_{1 \leq j < l \leq k} c(i_j, i_l), \quad 2 \leq k \leq r.$$

From Formula (1) and Theorem 2 conditions we have that  $Q(R) = -\sum_{l=2}^k (-1)^l \frac{h^l}{l!} S_l + O(h^{t+1})$ .

Prove the equalities  $S_l = 0$ ,  $2 \leq l \leq r$ , for a simplicity bounding ourselves by the case  $l = 2$ . Denote  $b(i_1, \dots, i_4)$  the number of different nodes in the set  $\{i_1, \dots, i_4\}$ ,  $i_1, \dots, i_4 \in R$ . From Formula (3) we have that for some coefficients  $a(i_1, \dots, i_4)$

$$S_2 = \sum_{k=1}^r (-1)^{k-1} \sum_{1 \leq i_1 < \dots < i_k \leq r} \sum_{1 \leq j(1), \dots, j(4) \leq k} a(i_{j(1)}, \dots, i_{j(4)}) c(i_{j(1)}, i_{j(2)}) c(i_{j(3)}, i_{j(4)})$$

with  $a(i_{j(1)}, \dots, i_{j(4)}) = \sum_{k=b(i_{j(1)}, \dots, i_{j(4)})}^r (-1)^{k-b(i_{j(1)}, \dots, i_{j(4)})} C_{r-b(i_{j(1)}, \dots, i_{j(4)})}^{k-b(i_{j(1)}, \dots, i_{j(4)})} = 0$ .

From Formula (3) the equality  $S_2 = 0$  takes place. Theorem 2 is proved.

**Corollary 1.** If  $p_i = \exp(-c_i h)$ ,  $c_i > 0$ ,  $i \in I$ , and for any set of different nodes  $i_1, \dots, i_{m+1} \in R$  the relation  $\bigcap_{s=1}^{m+1} F_{i_s} = \emptyset$  is true then for  $mt < r$  the equality  $Q(R) = O(h^{t+1})$  takes place.

**Remark 1.** Theorem 2 gives conditions when it is possible to form a reserve in the network with high reliable nodes.

**Remark 2.** All calculations are made without any restrictions on an oriented graph structure. But for a convenience of the graph representation it is possible to factorize oriented graph nodes by a relation of a cyclic equivalence and to introduce an edge between two factors if there is edge between some nodes of these factors. Then a graph structure becomes more simple (it is acyclic) but a factorization procedure is sufficiently complicated.

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# SYSTEM OF ADAPTIVE MANAGEMENT OF RAILWAY TRANSPORT INFRASTRUCTURE TECHNICAL MAINTENANCE (URRAN PROJECT)

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## ABSTRACT

The paper considers issues related to well-balanced management of resources for JSC RZD infrastructure maintenance under the conditions of scarce finances. The authors have made an analysis of the RAMS methodology and its further transformation into a complex of Russian standards and normative and methodology documentation base applied for management of life cycle processes of railway transport systems (URRAN) in JSC RZD. The paper studies the prerequisites and key aspects for development of the innovative technology of management decision making support for increase of dependability and functional safety of transport in Russia at all life-cycle stages.

**Keywords:** Dependability, safety, availability, longevity, risks, life cycle, information support, decision making support system.

## INTRODUCTION

The technical maintenance of the network of Russian railways requires tremendous expenditures related to maintaining the dependability of infrastructure facilities and ensuring the safety of transportation process. Under the conditions of scarce resources, an inadequate decision may cause mistakes in planning repair works for infrastructure sections which according to the existing rules call for repair but at the same time have a sufficiently high level of dependability [1]. On the contrary, infrastructure sections displeasing in terms of dependability are still being operated without refurbishment, capital repairs, or at least on-going remedial works. This in turn entails risks of traffic accidents.

At least two of the below prerequisites are required for good governance of scarce resources:

1. Real-time acquisition of actual information on the status of dependability and functional safety of all infrastructure assets of rail transport.
2. Setting-up of a system for decision making support of maintenance of infrastructure assets of railway transport at the unit, regional and network layers.

The first prerequisite implies establishment and deployment of an automated system for acquisition, analysis and processing of data about failures of infrastructure facilities and traffic accidents over the network of Russian railway. The second prerequisite means that since it is fundamentally impossible to set up and further operate a facility with absolute dependability and safety, then for solving this problem it is expedient to invest as many funds as really available and justifiable in terms of dependability reduction and safety ensuring. In other words, the residual risk of traffic accidents should have an accepted level.

Under these conditions, there arises a contradiction between the commercial interests of a railway infrastructure owner aimed at intense exploitation of infrastructure, on the one hand, and the need for routine breaks in operation for the purpose of maintaining the required levels of dependability and safety, on the other hand. This contradiction may be eliminated through the

development of a system for integrated management of maintenance works on railway transport infrastructure. This system should ensure automation of processes dealing with real-time acquisition and processing of data on failures of facilities and traffic accidents, processes of identification of track sections which are most displeasing in terms of dependability, as well as automation of decision-making support for distribution of scarce economic resources for maintenance of infrastructure provided that accepted levels of safety and required levels of dependability of its component facilities are achieved.

## **1. PRINCIPLES, PURPOSE AND GOALS OF INTEGRATED MANAGEMENT OF DEPENDABILITY, RISKS, AND LIFE-CYCLE COST ON RAILWAY TRANSPORT**

### **1.1. RAMS methodology**

In 2010 JSC Russian Railways started to develop and introduce a set of standards, methods and guidelines used for management of life-cycle processes of railway systems (URRAN). To that end, Russian railways initiated the process of harmonization of the Russian infrastructure management regulatory framework with the RAMS standards widely used by the EU and US railway companies.

RAMS is a methodology for ensuring Reliability, Availability, Maintainability, and Safety on railway transport. This is a corporate effort of the European Union formalized by the standards EN 50126 and IEC 62278. The RAMS methodology is based on the ALARP principle (as low level of residual risk as it is reasonably possible) in ensuring safety and dependability at all life-cycle stages of a railway transport facility.

RAMS in relation to the conventional standards in the sphere of dependability of engineering systems features the following:

1. Integrated management of dependability and safety of a facility with its life-cycle stages taken into account;
2. Decision making as to management of dependability and safety of assets based on risk assessment;
3. Management of dependability and safety of an asset in terms of quantitative indicators as well as based on recommendations proven by the international community as listed, for instance, in the railway-related application standards EN50126/IEC62278, EN 50128/IEC62279, EN50129/IEC62425Ed, EN50159 (the first and second parts);
4. In the RAMS methodology, there are four safety integrity levels. Each level is characterized by numeric values as well as a set of requirements specified for the technology of product development and pertaining to the implementation of these levels.

### **1.2. URRAN project purpose and goals**

RAMS targeting at manufacturers of technical equipment did not satisfy the goals of JSC RZD which are focused around operational activity. During the practical application of the RAMS methodology on Russian railways, several critical drawbacks thereof were revealed. Key ones are:

- Integrated analysis does not take longevity of facilities into account. This circumstance does not let us relate longevity and safety of facilities, assess the risks of transition from a set service life to their limiting state during operation and even correctly estimate the limit state of a facility;
- Life-cycle cost of a facility is estimated in isolation from its dependability and safety, i.e. not included into the RAMS methodology. This circumstance hinders reasonable distribution of investments into a facility at various stages of its life cycle;

– RAMS methodology is well developed for the stages of designing and manufacturing facilities and is practically not developed for the stages of their maintenance, modernization, decommissioning and disposal. For railway transport, of key importance is the management of dependability and safety of compound facilities at the stages of their maintenance and modernization;

– Issues of risk management are interpreted in the RAMS standards at the conceptual level and call for all-round development;

– Issues of system resilience (robustness) under unfavorable impacts are not covered;

– RAMS methodology does not cover the issues related to safety of technological processes, impact upon the environment.

All the above has required to transform the RAMS methodology into a complex of Russian standards, methods, guidelines applied for the management of life-cycle processes of railway transport systems (URRAN) in JSC RZD [2,3]. The conceptual framework of the URRAN system is represented in Fig. 1.

A system life cycle is meant as a sequence of stages, each containing specific tasks. This sequence fully grasps the entire system service life – from the primary concept till decommissioning and disposal. The life cycle provides the framework for planning, management and control of all the system indicators for the purpose of manufacturing quality products at reasonable price and within agreed time limits. The life-cycle of railway transport including 15 stages and represented in the form of a V-shaped model is shown in Fig. 2.

The downward (left-hand part) of the V-shaped model is normally referred to as the system designing or development and is a process of system evolution that ends down in the manufacture of system components. The upward (right-hand) part is referred to the assembly, installation, acceptance and follow-up maintenance of a system.

The V-shaped representation has gained widespread application in the industry. It implies that the acceptance procedure is closely linked to the system designing and development, since the system under development should in the end be verified for compliance with the requirements. The approval and acceptance of a system are based on the requirements specification and are planned at earlier stages of the life cycle – during the designing or development. Such representation of a life cycle is effective for the tasks of system check-up and approval during the life cycle. The purpose of check-up is to confirm that for certain source data the output data at each stage fully meets the requirements of the given stage. The purpose of the approval is to ascertain that the system being reviewed fully meets the imposed requirements at each stage of development and after installation.

Within the URRAN project, an object-element model of railway-related application has been developed which is based on the specially introduced concept of the reference element; also, dependability performance and operational safety indicators of railway transport have been developed and linked to the amounts of operational work performed. Table 1 represents the measurement units for the scopes of operational work performed for the divisions of JSC RZD which are used as arguments in dependability and safety performance indicators. In several cases, indicators which are functions of a facility's operation time are used. This applies only to the assets whose dependability and safety does not depend of the volume of operational work performed in transportation process.

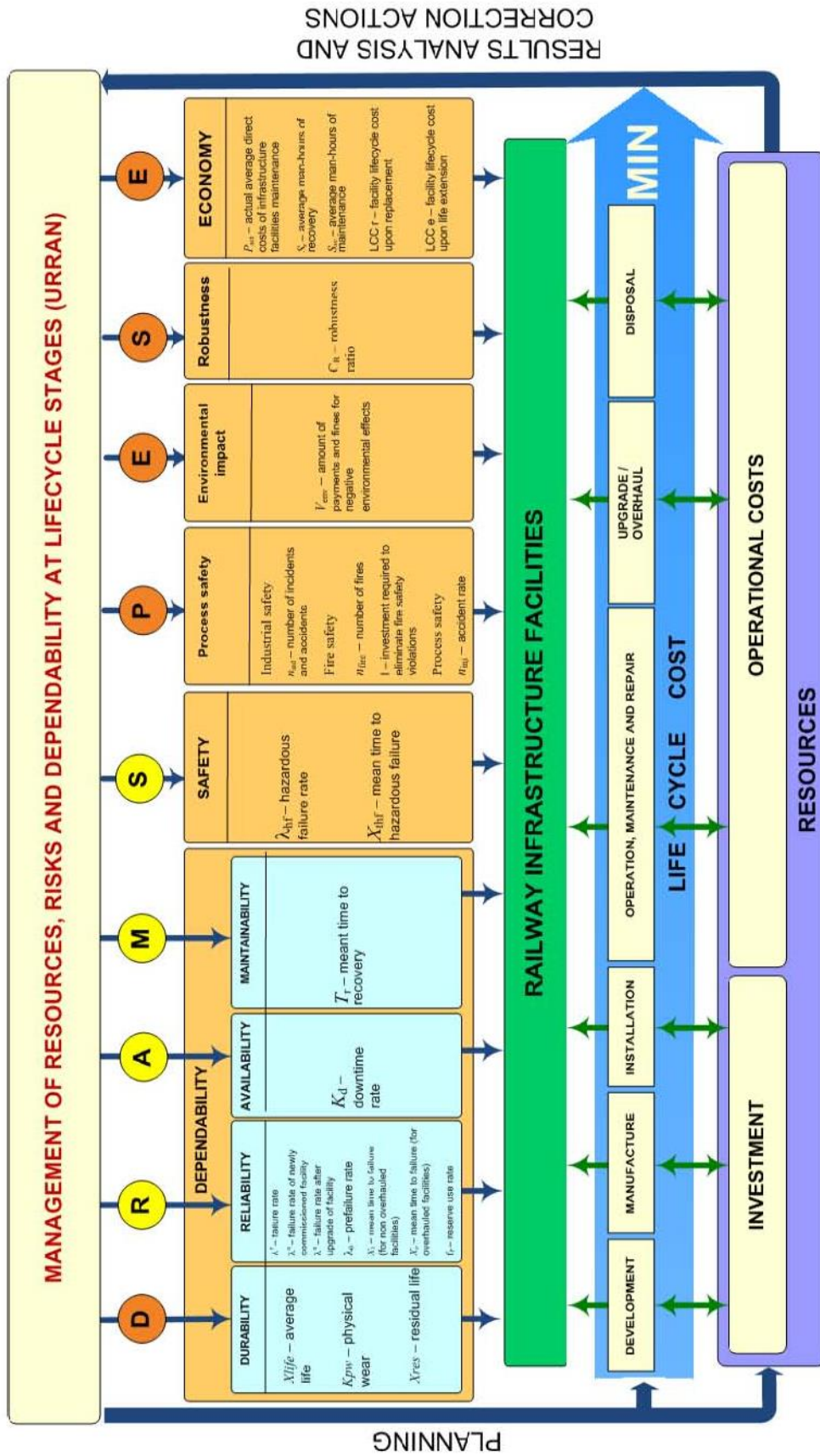


Fig. 1. Concept of the URRAN system



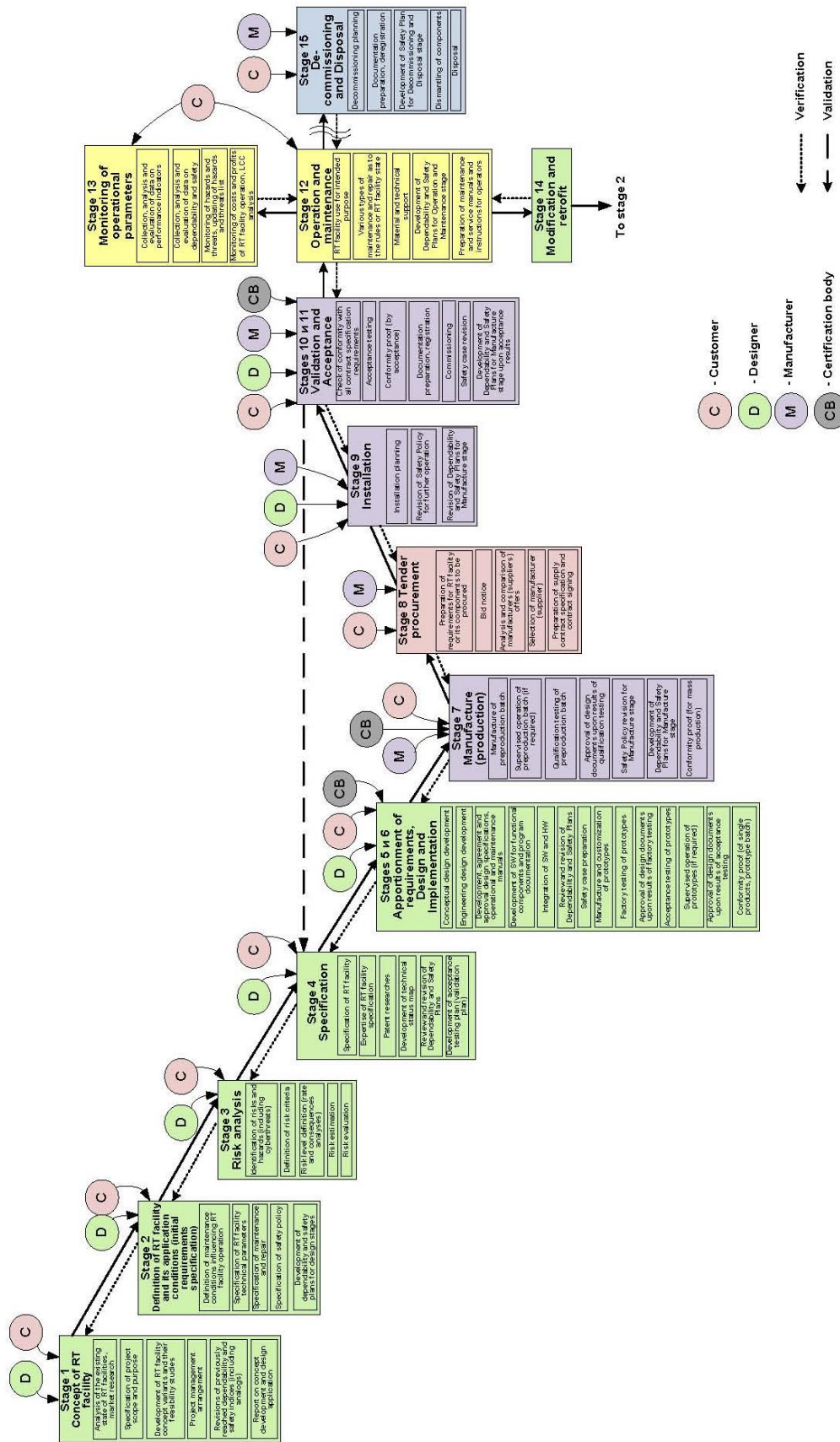


Fig. 2. Life cycle of railway transport facility including the process of manufacturing

Table 1. Quantities to measure the scopes of operational work performed in divisions of JSC RZD

No.	Division	Scope of field operation performed		
		Legend	Measurement unit	Dimensions
1	Division of tracks and structures	<i>Tr</i>	bln t*km of ton-kilometer operation	1
2	Division of signalling and remote control	<i>S</i>	mln train*km	1
3	Division of telecommunication	<i>T</i>	mln train*km	1
4	Division of electrification and power supply	<i>E</i>	mln kW*hr of transformed electrical energy	100
5	Locomotive division	<i>L</i>	mln locomotive*km of total mileage	1
6	Wagon division	<i>W</i>	mln wagon*km of total mileage	100
7	Passenger division	<i>P</i>	mln wagon*km of total mileage (passenger wagons)	10

The purpose of URRAN introduction is to increase the efficiency of railway transport operation based on the adaptive management under the conditions of resource scarcity. The object of URRAN application is the aggregate of technical facilities, systems, and technological processes of railway transport.

By adaptive management we imply the form and methods for control over business entities that assume the possibility and capability of the control system to change the parameters and structure of the regulator and the control subsystem in general depending on the change of internal parameters of the managed asset or the external environment (disturbances), as well as the changes in strategic goals.

The URRAN project solves the problems of optimization of resource management on the basis of dependability and safety performance criteria with risk assessment taken into account. At the same time, much attention is paid to the consideration of effect of the human factor in technological processes of the company's operation. In the conditions of shortage of funds, URRAN allows for increasing a set service life of railway transport assets to the limiting state on the basis of risk assessment and redistributing investments for the maintenance of dependability and safety of the most displeasing facilities.

## 2. URRAN NORMATIVE AND METHODOLOGICAL FRAMEWORK

Currently, all the works related to infrastructure maintenance are arranged on the basis of a facility's standard service life, with disregard for its current state. Such national standards of dependability management as GOST 27.002-89, GOST R 53480-2009, GOST R 51.901.2002, GOST R 51.901.12-2007 etc. do not cover the issues of management of running maintenance investments and costs. For the practical implementation of the URRAN system, it was necessary to

develop a set of regulatory and methodology documents which represent the principles and essential aspects of the system.

Due to the formation of the Customs Union, cooperation of organizations in the member countries has become more active in the sphere of railway transport, including cooperation in the sphere of ensuring safety and dependability of railway engineering. It is also of note that not only the JSC RZD is the largest railway organization in the countries of the former USSR, but it is also the most active participant of works aimed at standardization at the inter-state level since it is a secretariat of the International Committee for standardization 524 “Railway transport”. Since the matters related to the dependability and safety of railway transport are important for all the organizations and users of railway transport services, a decision was made on the necessity of upgrading the status of the main URRAN standards to the inter-state and national level. That also facilitated the improvement of interaction between JSC RZD and organizations that are involved in railway sector but are not part of RZD’s holding.

Within the URRAN project, GOST 32192-2013 “Dependability in railway equipment. Basic concepts. Terms and definitions” has been developed, which defines all the required terms and concepts of the subject matter with regard for the specifics of railway e and the best practices elaborated in the URRAN project [4,5,6] and helps the correct application of concepts and better mutual understanding among all the parties.

To establish the general rules for risk management on railway transport related to the traffic safety and operation, the following national standards have been developed:

- GOST R 54504-2011 “Functional safety. Safety policy and safety plan. Safety case of railway transport facilities”;
- GOST R 54505-2011 “Functional safety. Risk management on railway transport”.

The technical regulations of the Customs Union introduce the concept “risk” that is fundamental for safety. But the inter-state standards in risk management are missing. JSC RZD strives to extend its best practices [7,8] over the entire 1520 gauge space, so the company will continue activities in that direction jointly with the Rosstandard.

By now, within the URRAN project, 1 inter-state standard, 5 national standards of the Russian Federation and 17 corporate standards of JSC RZD have been developed. These standards set the requirements in the sphere of risk assessment, functional safety, dependability and life cycle cost of railway transport facilities. These standards are just the tool which may help solve the major issues in the development and introduction of innovations and enhance the factors that favor the success of the URRAN project. The standards provide the framework for elaboration of the common language of communications among all the interested parties that participate in the development and introduction of the new system.

The national and corporate standards developed within the URRAN project set the procedures for introduction and use of the URRAN methodology on railway transport and contain the requirements for management of resources, risks, and dependability as applied to railway transport.

For the practical implementation of the standards, 8 methodology recommendations and 55 methodology instructions have been developed. They allow for assessing technical condition of infrastructural facilities, providing data for decision making as to the necessity of conducting technical maintenance and repairs, planning investments for facility maintenance, as well as evaluating damages and generating risk matrices for each type of infrastructure facilities as the framework for the management of technological processes on railway transport.

### 3. THE UNIFIED CORPORATE PLATFORM OF URRAN (UCP URRAN)

#### 3.1. Development directions of information systems in JSC RZD and prerequisites for the development of the KASANT system

Currently, JSC RZD uses a lot of information systems which cover almost all the aspects of the company's production activities, which have been developed and introduced into commercial operation over a long time.

The general direction of development for information systems (IS) and automatic control systems (ACS) meets the worldwide trends:

- from accounting systems (transactional systems, OLTP systems) which allow to get the precise answer to the questions “what?”, “where?”, “when?”, “how much?” that are related to the past — to analytical systems (of OLAP class) which not only let us answer the question “why?”, but also let us move to forecasts of development;
- from systems which in many ways do not meet the scopes of activities of JSC RZD and its divisions — to systems that are based on large world-renowned industrial platforms (SAS Intelligence Storage, SAP AG, Oracle, IBM Maximo) which proved well at the world's largest corporations;
- from systems that are based on the “file” and “client-server” technologies — to systems based on Internet technologies (http, SOAP protocols, mark-up languages html, xml, Java programming language, thin client);
- from isolated railway information systems, from systems that ensure information support for specific directions of activities of JSC RZD and its divisions (for example, financial or personnel management) — to integrated centralized systems which cover the entire life cycle of transportation process.

*The unique character of dependability management on railway transport is in the following:*

- *Many facilities of railway transport are distributed in space and have a hierarchical structure;*
- *To manage dependability of infrastructure facilities and railway transport in general, it is needed to combine in real time the above two directions: acquisition and processing of statistical data on facility failures, modeling of the dependability of facilities;*
- *A vast variety of facilities, on the one hand, and essential differences in operational modes and conditions for even same type facilities, on the other hand;*
- *The extent of impact of facility dependability on transportation process depends on the properties of fail-safety as well as maintainability of facilities and on the properties of transportation process itself (traffic intensity, train speed, duration of delays of trains because of repairs of infrastructure and/or rolling stock).*
- *The mentioned circumstances have caused the need to develop specialized automated control systems (ACS) on railway transport to manage the processes of gathering data about the schedule of realized train traffic, on the current state of elements of infrastructure divisions (GID-URAL, ASOUP-2, ASU-P, ASU-Sh-2, ASU-E) [9]. These systems provide information about failures in the elements of divisions. However, this information has a disparate character and not systematized by categories and impacts upon train delays. The abundance of automated control systems that have no common interface, insufficiency of information on dependability and safety of facilities represented by them, uncertainty of the information itself, no possibility in these systems to analyze failures, etc. have become the prerequisites for development of a common automated control system for management of acquisition, analysis and pre-processing of data on failures and recoveries of compound elements of railway transport facilities.*



The basis of the information technology that ensures integrated management of transportation process dependability is the “Integrated automated system for accounting, control of fault elimination in facilities and analysis of their dependability” (KASANT). To acquire trustworthy information on the state of facilities and implement the principle of maximum usage of human-independent forms of data acquisition, the KASANT system has been integrated with the existing industrial automated control systems as regards exchange of data on failures. To arrange effective interaction with the KASANT system, the mentioned automated systems has been adapted in terms of storage and access to data for track-recording wagons, results of decoding speed gauges, KLUB-U legal registration units, and other devices for registration of train movement parameters. Due to these works, objective sources form the basis of primary information on the status of technical facilities in the URRAN project.

Based on the real data of the KASANT system, it is possible to acquire summary information on faulty facilities regarding the company in general. The total amount of users of the system by the end of 2014 was 35 thousand employees of JSC RZD [10]. Therefore, in the company now there is already a fully automated process of analyzing failures of technical facilities that impact the stability of traffic.

Based on the achieved positive results, in line with the general methodology of URRAN, Russian experts have started the work related to the analysis of violations of technological processes in the company’s divisions. In the long run, this will let us provide economical feasibility of the infrastructure component value in cargo and passenger transportation tariffs through a life cycle cost and ensuring an acceptable risk level, as well as take into account the impact of errors in the planning of traffic, including staff errors.

### **3.2. Development of the URRAN unified corporate platform**

Currently, JSC RZD uses a lot of information systems that cover almost all the sides of the company's production activities and have been developed and introduced into commercial operation for a long time. Under the conditions of structural differentiation by the types of activities, the role of computerization as the technical means of coordination and centralization of control increases. It is needed to perform transition from isolated railway division information systems, from systems that ensure information support for specific directions of activities of JSC RZD and its divisions [11] to the integrated centralized platform that provisions the entire life cycle of transportation process.

The object of computerization for the unified corporate platform of URRAN is business processes of technical maintenance work of JSC RZD’s infrastructure.

The enterprises of JSC RZD, subsidiary companies participating in maintenance of the company's infrastructure assets will be able to use data in the unified IT environment, standardized systems of data processing and transmission. It will also allow combining the benefits of the scale and coordination with the benefits of distributed production.

The key groups (complexes) of business processes aimed at the management of the infrastructure maintenance are:

- Strategic management of maintenance of infrastructure facilities;
- Management of monitoring and diagnostics of infrastructure facilities;
- Management of running maintenance and scheduled preventive repairs;
- Management of maintenance of self-propelled vehicles, instruments, machinery and equipment of enterprises, ensuring availability and good condition of engineering means.

The basis for support of the above described management principles and processes are technologies implemented in the UCP URRAN. The structure of the UCP URRAN system is represented in Fig. 3. The UCP URRAN is a system for decision making support whose goal is to aid the top managers making decisions in severe conditions to perform an all-round and impartial analysis of substantive work.

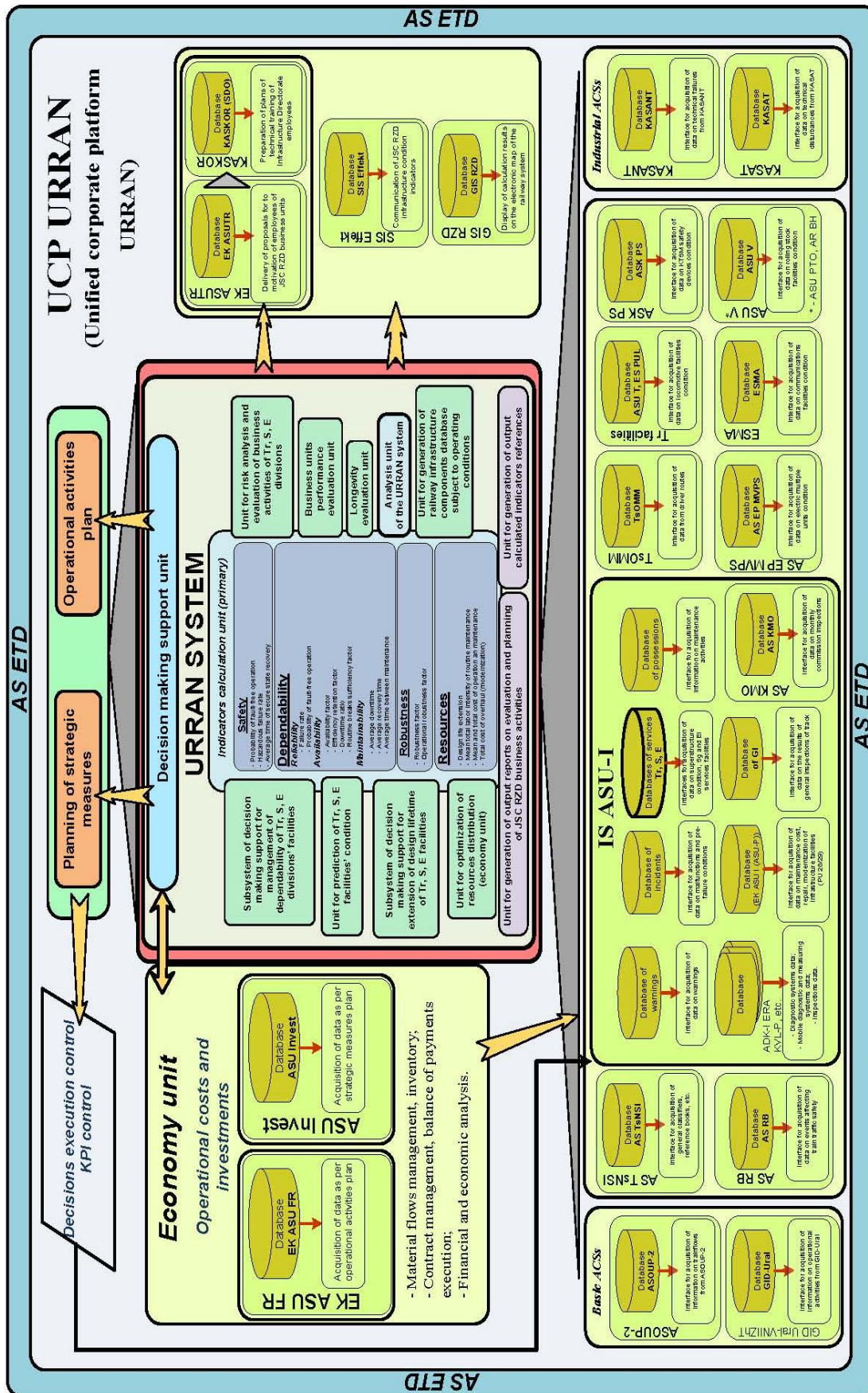


Fig. 3. Structure of the UCP URRAN system

With the help of decision-making support for integrated management of dependability, risks, and cost of a life cycle on railway transport, resources' provision of the company's activities is formed: when budgeting, the company's strategic goals and purposes are related to the volumes of funds it currently possesses or will get soon. When planning, the system assists in viewing data from previous years, reveal trends and regularities and use them in preparing a budget, thus making it more well-targeted and accurate.

The UCP URRAN is meant to solve the following problems:

- automation of primary processing of statistic data on failures in technical facilities of infrastructure facilities and rolling stock of railway transport;
- quantification of indicators for operational dependability and safety of infrastructure facilities;
- quantitative evaluation of production activities of infrastructure divisions and rolling stock with regard for failures and maintenance as well as operation of infrastructure facilities;
- monitoring, correlation, and motivation for activities of structural subdivisions within divisions based on the indicators of operational dependability and safety;
- evaluation of compliance of achieved indicators of operational dependability and safety with set norms;
- preparation of estimated data to draw up recommendations for decreasing risk levels;
- identification of vulnerable facilities based on risk assessment;
- preparation of draft plans for maintenance of infrastructure and rolling stock;
- preparation of projects for distribution of investments into the most displeasing facilities of railway transport.

## CONCLUSION

The RZD holding is Russia's largest transport business system that offers vast potentials for increasing efficiency in meeting the growing needs of the national economy and population in transport services and delivers quality transport services both at the Russian and international markets.

To achieve this goal is not possible without ensuring a high level of safety and dependability of transportation process, which is a guarantee for safekeeping stable competitive edges of the RZD holding at transport markets.

The URRAN project implies development of an innovative technology for supporting executive decision-making aimed at increasing the dependability and functional safety of Russia's railway transport at all the stages of a life cycle.

Russian experts have developed the key provisions of the URRAN methodology that as an improvement to the European RAMS methodology provides for integrated management of dependability and safety as well as the cost of a life cycle of railway transport facilities with regard for assessment of risks and longevity. The developed system of operational indicators provides estimates of the properties of dependability, safety, life-cycle cost achieved for the railway transport infrastructure with regard for the volume of field operation performed by facilities. Under the scarcity of financial means, the URRAN allows for assigning repairs to the most displeasing sections and ensuring reliable functioning of the infrastructure and traffic safety, managing the maintenance of railway transport facilities based on their current status of dependability and safety, which, in particular, ensures prolongation of a set service life of facilities provided that acceptable levels of safety are maintained. Assessment of risks of hazardous situations on railway transport allows for forecasting occurrence of traffic accidents at sections of railway lines in poor state.

The basis for supporting the URRAN methodology is technologies implemented in the UCP URRAN which is a system of decision-making support in severe conditions, including the conditions of uncertainty.

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# TO THE QUESTION ON DISTRIBUTION OF LOADING BETWEEN POWER UNITS TES

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## ABSTRACT

Practical realization of existing programs of calculation of distribution of loading between power units TES being in working order demands authentic power characteristics. In conditions of increasing ageing the core and auxiliaries of power units, maintenance of reliability of power characteristics in itself represents serious problems. The method of distribution of loadings between power units TES in view of their technical condition, based on registered technical and economic parameters is developed. Practical approbation of this method testifies to essential advantage of a recommended method, as in comparison with uniform distribution of loading between power units, and in comparison with the intuitive approach to distribution of loading.

## INTRODUCTION

A problem of distribution of loading between the same power units (EB) TES well-known [1]. Corresponding algorithms and programs of calculation are developed. Practical realization demands, first, authentic power characteristics, that in conditions of increasing ageing the core and auxiliaries EB in it represents a serious problem [2]. In this connection heuristic approach when because of an operational experience, loading EB appointed seldom applied.

In these conditions, the great value got with methodical support to management TES in the form of recommendations on expedient distribution of loading between EB depending on reliability and profitability of their work.

These recommendations received by estimations of integrated parameters (B) reliability and the profitability of work EB calculated on actual values of technical and economic parameters EB TES [3]. We shall notice that desire simultaneously to raise reliability and profitability of work in some cases is amazing, since for maintenance of reliability of work additional expenses are required. In addition, it is true. However, in considered statement it is a question only of operational expenses, which at greater reliability, naturally, is less.

**Method of calculation of distribution of loadings.** As initial data for calculations serve:

$n$  - the general number same EB

$n_b$  – number being in working order EB,  $n_b \leq n$ ;

$P_{\min, s}$  – minimally safe load EB;

$P_{\text{nom}}$  – rated power EB;

In – an integrated parameter of reliability and profitability of work EB;  
 $P_{av} = P \sum / n_b$  – average loading on one EB, where P - loading TES;

Calculation of distribution of loading between  $n_b$  EB spent in following sequence:

1. The relative size of factors of the importance of the technical condition (TC) i-th EB under the formula defined

$$b_i = \frac{B_i}{B_\Sigma} \quad (1)$$

where  $i=1, n_b$ ;  $B_\Sigma = \sum_{i=1}^{n_b^+} B_i^+ = \left| \sum_{j=1}^{n_b^-} B_j^- \right|$ ;  $B^+$  и  $B^-$  -accordingly, positive (+) and negative (-) values  $B_i$ ;

$n_b^+$  and  $n_b^-$  - accordingly number being in working order EB with  $B^+$  и  $B^-$ ;

2. Are defined minimal ( $b_{min}$ ) and maximal ( $b_{max}$ ) value of realizations of an integrated parameter  $b_i$  under formulas:

$$b_{min} = \min (b_1, b_2, \dots \dots b_{nb}) \quad (2)$$

$$b_{max} = \max (b_1, b_2, \dots \dots b_{nb}) \quad (3)$$

It is obvious, that  $b_{min} < 0$ , and  $b_{max} > 0$ ;

3. Intervals of possible decrease are defined ( $\Delta P^-$ ) and increases ( $\Delta P^+$ ) average loading EB under formulas

$$\Delta P^- = P_{cp} - P_{min, s} \quad (4)$$

$$\Delta P^+ = P_{nom} - P_a \quad (5)$$

4. If  $\Delta P^- \leq \Delta P^+$  calculation of distribution of loadings between  $n_b$  EB in view of their reliability and profitability is spent under the formula:

$$P_i = P_{av} + \Delta P^- \cdot b_i \quad (6)$$

5 If  $\Delta P^- > \Delta P^+$ , under the formula:

$$P_i = P_{av} + \Delta P^+ \cdot b_i \quad (7)$$

## EXAMPLES OF CALCULATION OF DISTRIBUTION OF LOADING BETWEEN EB TES IN VIEW OF THEIR TC

1. On fig.1 the fragment of monthly result of the analysis of technical and economic parameters and recommendations in the basic directions of increase of overall performance EB is resulted.

**The basic directions of increase of an overall performance of power units in \_\_ month 20\_\_ year**

Results of calculations have allowed to establish and recommend:

1. Technical and economic parameters of power units (EB), not meeting shown requirements for \_\_\_\_\_ month

EB	Technical and economic parameters			
	The name	Relative deviation	Actual value	Recommended value
2	The maximal electric loading	<b>-1,439</b>	220,00	270,00
	Share el. energy for own needs	<b>-1,333</b>	5,50	4,10
	Average loading	<b>-1,330</b>	160,00	179,29
	Operating ratio established capacities	<b>-1,310</b>	11,10	44,84
	The specific charge of conditional fuel	<b>-1,186</b>	344,00	329,61
3	The specific charge of conditional fuel	<b>-1,072</b>	342,80	329,61
	Operating ratio established capacities	<b>-0,996</b>	19,20	44,84
	Share el. energy for own needs	<b>-0,857</b>	5,00	4,10
	The maximal electric loading	<b>-0,571</b>	250,00	270,00
6	Average loading	<b>-0,296</b>	175,00	179,29
7	Average loading	<b>-0,365</b>	174,00	179,29

2. Factors of the importance TC EB are equal

EB	1	2	3	4	5	6	7	8
Factor of the importance of the TC	0,486	- <b>1,318</b>	- <b>0,634</b>	0,482		0,126	0,255	0,604

3. To group "bad" concern 2 and 3 EB. Decrease in their loading in inverse proportion relative sizes of factors of the importance of the TC is recommended.

4. Least effective of operating EB should consider 2 EB. This EB is recommended to switching-off on scheduled repair, and preliminary - in a reserve or to the greatest possible decrease in loading

5. To group "good" concern 8, 1 4, 7, and 6 EB. Increase of their productivity to proportionally relative sizes of factors of the importance of the TC supposed.

6. To the most effective is 8 EB. Its work with as much as possible admissible productivity is expedient.

Fig.1. Fragment of monthly result of the analysis of technical and economic parameters and recommendations in the basic directions of increase of overall performance EB.

2. Results of calculations of relative values of factors  $b_i$  under the formula (1) describing TC EB are resulted in table 1

Table 1. Realizations of relative sizes of factors of the importance of TC EB

N (i)	Serial number of power units							
	1	2	3	4	5	6	7	8
$b_i$	0.249	-0.675	-0.325	0.242	-	0.064	0.13	0.309

Results of calculations distribution of loading between EB TES for of some values  $P_{av}$  are resulted in table 2.

Table 2. Results of calculations distribution of loading between EB TES for of some values  $P_{av}$ 

Loadings $P_{av}$ , MWt	Conditional numbers of power units							
	1	2	3	4	5	6	7	8
110	115	96,5	103,5	114,9	-	111,3	112,6	116,2
130	140	103	117	139,9	-	132,6	135,2	142,4
150	164,9	109,5	130,5	164,8	-	153,9	157,8	168,6
170	189,9	116	144	189,7	-	175,2	180,4	194,8
190	214,9	122,5	157,5	214,7	-	196,4	203	220,9
210	232,4	149,2	180,8	232,2	-	215,8	221,7	237,9
230	247,4	182,7	207,3	247,3	-	234,5	239,1	251,7
250	262,4	216,2	233,8	262,3	-	253,2	256,5	265,5

Experience of calculations of distribution of loadings between EB shows, that application of formulas (6) and (7), despite of their faultlessness, insufficiently full use adjusting intervals EB ( $\Delta P^+$  and  $\Delta P^-$ ). Essentially greater effect turns out, if instead of formulas (6) and (7) to use formulas (8) and (9), looking likes:

$$P_i = P_{av} - \Delta P^- \frac{b_i}{b_{\min}} = P_{av} - (P_{av} - P_{\text{nim},s}) \frac{b_i}{b_{\min}} \quad (8)$$

$$P_i = P_{av} + \Delta P^+ \frac{b_i}{b_{\max}} = P_{av} + (P_{\text{nom}} - P_{av}) \frac{b_i}{b_{\max}} \quad (9)$$

where  $i=1, n_b$

Thus the formula 8 is used, if  $\Delta P^- / b_{\min} \leq \Delta P^+ / b_{\max}$ . If  $\Delta P^- / b_{\min} > \Delta P^+ / b_{\max}$ , the formula (9) is used. Results of calculations of distribution of loadings between EB TES under formulas (8) and (9) are resulted in table 3.

Table 3. Recommended distribution of loading of power station between EB for of some values  $P_{av}$ .

Loadings $P_{cp}$ , MWt	Conditional numbers of power units							
	1	2	3	4	5	6	7	8
110	117,4	90	100,4	117,3	-	111,9	113,9	119,2
130	144,8	90	110,7	144,6	-	133,8	137,7	148,3
150	172,1	90	121,1	171,9	-	155,7	161,6	177,5
170	199,5	90	131,5	199,2	-	177,6	185,5	206,7
190	226,9	90	141,9	226,5	-	199,5	209,3	235,8
210	254,3	90	152,2	253,9	-	221,5	233,2	265
230	281,6	90	162,6	281,2	-	243,4	257	294,2
250	290,2	140,9	197,5	289,9	-	260,4	271,1	300

We shall define an interval of change of loading EB at the first and second ways of calculation of distribution of loadings. We shall assume, that  $\Delta P^- \leq \Delta P^+$ . At calculation by the first way:

- The bottom boundary value of loading ( $\underline{P}$ ) according to the formula (6) will be equal:

$$\underline{P}^{(1)} = P_{av} + \Delta P^- \cdot b_{\min}$$

- The top boundary value of loading ( $\bar{P}$ ) according to the formula (7) will be equal:

$$\bar{P}^{(1)} = P_{av} + \Delta P^- \cdot b_{\max}$$



- The size of an interval of change of loading calculated under the formula:

$$\Delta_1 = \bar{P}^{(1)} - \underline{P}^{(1)} = \Delta P^- (b_{\max} - b_{\min}) \quad (10)$$

At work as the second (2) ways size of an interval of change of loading ( $\Delta_2$ ) it calculated under the formula:

$$\Delta_2 = \bar{P}^{(2)} - \underline{P}^{(2)} = \Delta P^- \left( \frac{b_{\max} - b_{\min}}{b_{\min}} \right) \quad (11)$$

Degree of change of an interval of loading EB we shall define from parity  $\Delta_2$  and  $\Delta_1$

$$\frac{\Delta_2}{\Delta_1} = b_{\min}^{-1} \quad (12)$$

Thus, the interval of change of loading increases in  $|b_{\min}^{-1}| = 1/0.675 = 1.48$ .

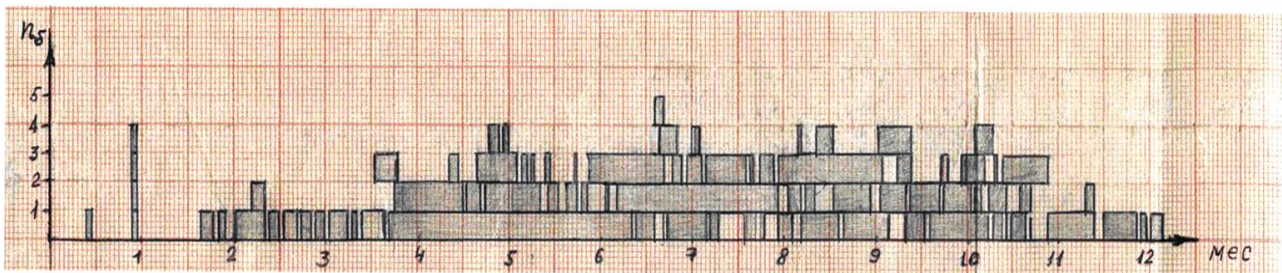
If  $\Delta P^- > \Delta P^+$ , similar calculations allow to establish, that

$$\frac{\Delta_2}{\Delta_1} = b_{\max}^{-1} = 1/0.309 = 3.23 \quad (13)$$

Essential excess  $\Delta_2$  above  $\Delta_1$  testifies to doubtless economic advantages of the second way.

## ACCOUNT OF CHARACTER OF CHANGE OF NUMBER OF THE POWER UNITS THAT ARE BEING IN WORKING ORDER IN TIME.

As an example on fig.2 dynamics of change  $n_b$  within a year is resulted.



As change  $n_b$  the casual character caused by automatic switching-off EB, their switching-off under the emergency application, in a reserve or on scheduled repair follows from fig.2 monthly have in many respects. Inclusion and switching-off EB demands revision of distribution of loading between EB. The basic stage thus is the estimation of relative values of integrated parameters (B), describing reliability and profitability of everyone EB. As the initial information for calculation  $B_i$  with  $i=1, n_{\Sigma}$  last results of measurement and calculation of technical and economic parameters EB. The special program preparing this information is developed.

It is natural, that at calculations that part of these data which reflects the TC being in working order  $n_b$  EB is used only. It is necessary to have in view of, that planned loading PES

specified and known not less than for 15 minutes before execution. This feature makes the basic difficulty of operative management of operating modes EB. Fluctuation of loading can be operatively considered at constant sizes  $B_i$   $c i=1, n_{\Sigma}$ . The block diagram of algorithm of definition of loading EB in view of their reliability and profitability is resulted on fig.3.

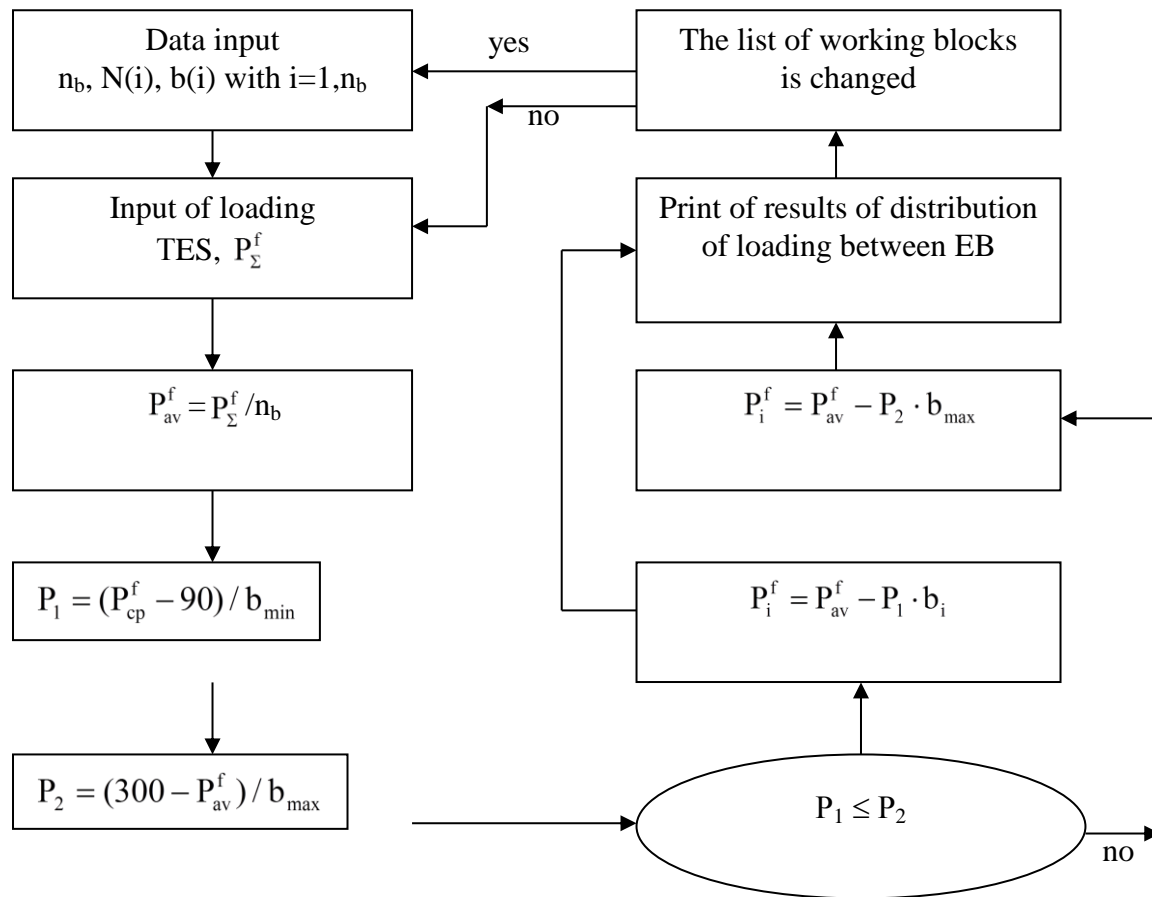


Fig.3 The Block diagram of algorithm of calculation of distribution of loading between EB TES

### ESTIMATION OF ECONOMIC EFFICIENCY

The basic purpose of the account of reliability and profitability of work EB at distribution of loading YES, is decrease in operational expenses and first of all, the charge of fuel. The sparing mode of operation insufficiently reliable and economic EB by restriction of their loading allows not only to lower the charge of fuel on TES, but also to reduce speed of deterioration and an idle time in emergency repair. Most operatively and precisely efficiency of the account of reliability and profitability EB can be estimated by calculation of the charge of conditional fuel. We shall consider some features of these calculations.

1. Economic profit ( $\delta U_{\Sigma}$ ) in the form of relative reduction of size of the charge of fuel can be calculated by comparison of the charge of conditional fuel at identical actual (f) loadings EB equal  $P_{av}^f$  (designate it through  $U_{\Sigma,1}$ ) and recommended (r) in view of reliability and profitability of work of loadings EB  $P_{av}^r$  (we shall designate it through  $U_{\Sigma,2}$ ).

$$\delta U_{\Sigma} = 100 \frac{U_{\Sigma,1} - U_{\Sigma,2}}{U_{\Sigma,1}} \% \quad (14)$$

where  $U_{\Sigma,1}$  and  $U_{\Sigma,2}$  can be calculated:

- For each hour (h) daily production schedule TES under the formula:

$$\left. \begin{aligned} u_{\Sigma,1,i}^{(h)} &= P_{av,i}^{(f)} \cdot \Delta t_i \sum_{j=1}^{n_b} U_{av,j}^{(m)} \\ U_{\Sigma,2,i}^{(h)} &= \sum_{j=1}^{n_b} P_{i,j}^{(f)} \cdot U_{av,j}^{(m)} \cdot \Delta t_j \end{aligned} \right\} \quad (15)$$

where  $i=1,2,4$ ;  $U_j$  – average value of the specific charge of conditional fuel of j-th EB for preceded month;  $\Delta t \leq 1$ ;

- For each day (d) month:

$$\left. \begin{aligned} U_{\Sigma,1}^{(d)} &= \sum_{i=1}^{24} U_{\Sigma,1,i} \\ U_{\Sigma,2}^{(d)} &= \sum_{i=1}^{24} U_{\Sigma,2,i} \end{aligned} \right\} \quad (16)$$

- For each month (m) year:

$$\left. \begin{aligned} U_{\Sigma,1}^{(m)} &= \sum_{j=1}^{n_b} P_{av,j}^{(f)} \cdot \Delta T_j \cdot U_{av,j}^{(m)} \cdot \Delta t_j \\ U_{\Sigma,2}^{(m)} &= \sum_{j=1}^{n_b} P_{av,j}^{(f)} \cdot \Delta T_j \cdot U_{av}^{(m)} \cdot \Delta t_j \end{aligned} \right\} \quad (17)$$

If to consider, that cost 1 t. conditional fuel on 01.01.2014г. was equal  $S_0=229\$$ . That economic benefit in cost expression calculated under the formula:

$$S_{\Sigma} = (U_{\Sigma,1} - U_{\Sigma,2}) S_0 \quad (18)$$

In table 4 settlement, values are resulted  $\delta U_{\Sigma}$  and  $S_{\Sigma}$  for of some  $P_{av}$  for an interval of time 1 hour.

Table 4. Parameters of efficiency of uniform distribution of loadings EB for of some  $P_{av}$  and  $\Delta t=1$ hour.

$P_{av}$	110	130	150	170	190	210	230	250	270
$\delta U_{\Sigma}, \%$	0,23	0,42	0,54	0,62	0,7	0,75	0,8	0,53	0,3
$S_{\Sigma}, \%$	137	296,4	433,2	570	707	821	980,4	752	456

As one would expect with increase  $P_{av}$   $\delta U_{\Sigma}$  and  $S_{\Sigma}$  increase, reach the greatest value equal:

$$P_{av} = \frac{P_{nom} \cdot |b_{min}| + P_{min} \cdot b_{max}}{b_{max} + |b_{min}|} \quad (19)$$

and further decrease.

According to table.1

$$P_{av,max} = \frac{300 \cdot 0.675 + 90 \cdot 0.309}{0.309 + 0.675} = 234 \text{ MWt}$$

Thus, the most economic mode TES takes place, when  $P_{av}/n_b = P_{av} = P_{av,max}$ . This dependence of change  $\delta U_{\Sigma} = f(P_{av})$  and  $S_{\Sigma} = f(P_{av})$  it is typical and does not depend on technical and economic parameters. Their change conducts only to change  $P_{av,max}$ .

## CONCLUSION

1. The new method of distribution of loadings between power units TES, considering reliability and profitability of each power unit is developed;
2. At formation of structure of working power units, distribution of loading is practically instantly calculated for each step of daily production schedule TES;
3. Change of structure of working power units demands recalculation of factors of the importance of a technical condition;
4. Economic benefit of distribution of loading a recommended method makes approximately 0.45% from total expenses for fuel.

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# SYSTEM OF AUTOMATIC REGULATION OF REACTIVE POWER BY MEANS OF FUZZY LOGIC

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## ABSTRACT

For increasing efficiency of distribution of electric energy the problem of indemnification of reactive-power by means of practical and theoretical researches is considered. On the basis of theoretical researches the algorithm of control of power of condenser by low sensitive to parametric indignations with the use of fuzzy theory is worked out.

**Keywords:** Distributive electric networks, reactive capacity, voltage, regulation of power, fuzzy logic.

## 1. INSTRUCTION

One of basic problems decided on the stage of design and exploitation of the system of power supply (SPS) of industrial enterprise is indemnifications of reactive power (RP) including some calculation of the compensated power, choice of method of indemnification, regulation and placing in plant.

For creation of economic work rejime of system of power supply in an enterprise and increase of efficiency distribution of electric power with minimum expenses on exploitation is expedient to carry out indemnification of reactive power with the use of static condenser units with the automatically adjusted power [3]. Application of condenser units with automatic adjusted power allows to decrease the losses of electric power in the elements of nets of the system of power supply, and also is one of the means of regulation of tension and upgrading of electric power.

On founding of data of research of SPS of the brickwork manufacture with the purpose of indemnification of RP it was defined that like the difficult system it functions at non determination conditions with different rejimes of operations of receivers of electric energy. Taking into account individuality of change of loading every consumer of electric power of the brickwork manufacture it is necessary to apply the centralized method of indemnification of RP, with placing of condenser unit in decreasing losts of the substation of the plant with connection of them on tires 0,4 kV. Power of the condenser unit is defined on the basis of calculation of loading the plant, and power of condenser battery is defined taking into account symmetric distribution them on phases and minimum level of indemnification of RP. At present the applied systems of automatic regulation of RP possess the large zone of insensitivity owing to preliminary tuning of them on the some value of the controlled parameter. In this connection, the error of the system of automatic regulation increases at regulation of power of the condenser unit [3], and possibility of creation of the economic rejime of operations of SPS and effective distribution of electric power goes down.

The aim of this article is development of the system and control of algorithm by power of the condenser unit of not sensitive to self-reactance indignations with the use of the theory of fuzzy sets [4-7].

## II. STRUCTIVE SCHEME AND ALGORITHM OF THE SYSTEM OF AUTOMATIC REGULATION OF REACTIVE POWER

Structure of technical realization of the system of automatic regulation which provides rational control of RP in SPS of the investigated plant is given on the figure 1.

System automatic regulation consists: 1 - the loading unit; 2 - sensor of calculable block of reactive power; 3 - "Dynamics" (dynamics of change of reactive power); 4 - sensor of tension on the tires of loading; 5 - measure equipment of amount of switching accomplished for a day; 6 - fuzzyficater that is connected by means of 5, intended for transformation of clear signals to fuzzy sets; 7 - a table of linguistic rules (TLR), i.e. totality of fuzzy rules describing an unclear relation between the entry and output parameters of controller; 8 – defuzzyficater, where the got fuzzy value after defuzzyficater as clear controlling influence acts into the entrance of block of commutation (BC); 9 - BC.

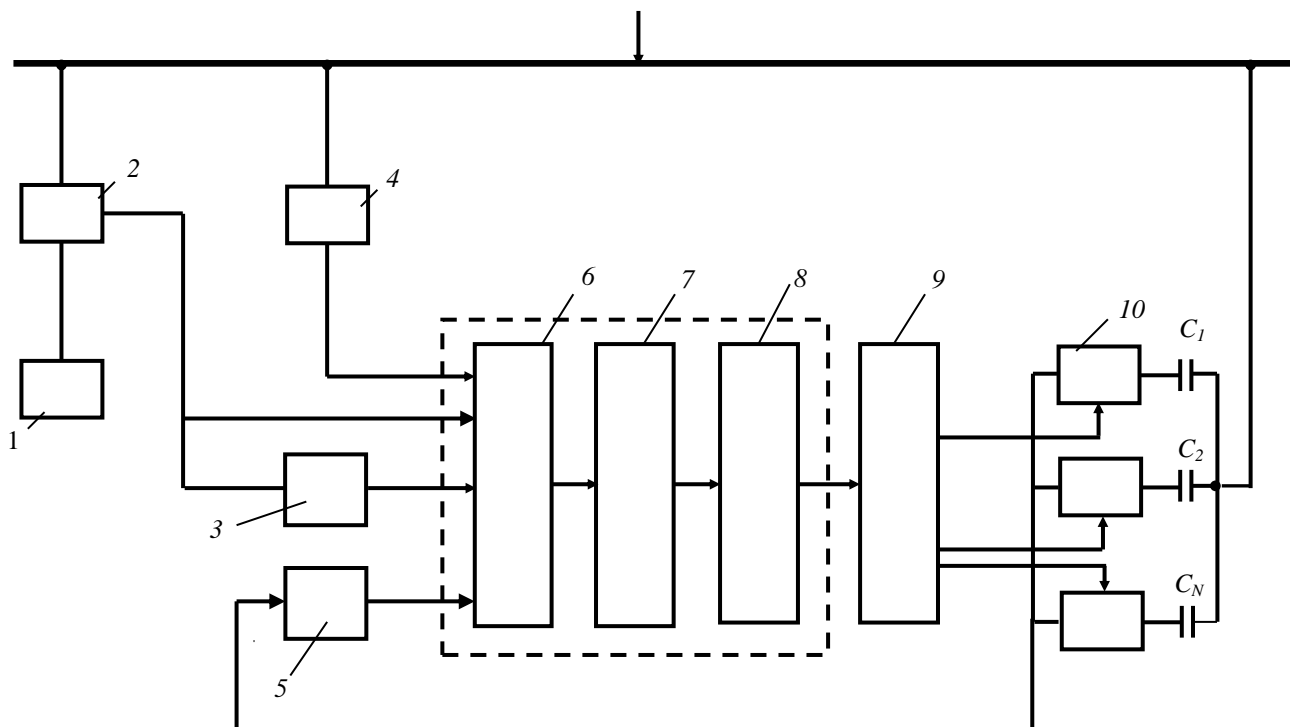


Fig. 1. Structure scheme of the system of automatic regulation (SAR) of reactive power

For SAR as fuzzy algorithm the algorithm of Mamdani was chosen. This algorithm is most often applied in practice, as it showed itself very well in many problems of control at regime of real-time. Mathematically it can be described by the following of sequence of steps [2]:

- Fuzzy: there are degrees of truth for initial conditions of every rule

$$A_1(x_0), A_2(x_0), B_1(y_0), B_2(y_0)$$

Fuzzy conclusion: there are levels of "chopping" for initial conditions of each of rules:

$$\alpha_1 = A_1(x_0) \wedge B_1(y_0)$$

$$\alpha_2 = A_2(x_0) \wedge B_2(y_0)$$

where " $\wedge$ " is an operation of logical minimum (min) in the truncated membership function:

$$C'_1 = (\alpha_1 \wedge C_1(z))$$

$$C'_2 = (\alpha_2 \wedge C_2(z))$$

- 3. Composition: with the use of operation of max (designated as " $\vee$ ") association of the found truncated functions is produced. The date operation supports getting results of final fuzzy subset for the variable of exit with the membership function

$$\mu_{\Sigma}(z) = C(z) = C'_1(z) \vee C'_2(z) = (\alpha_1 \wedge C_1(z)) \vee (\alpha_2 \wedge C_2(z))$$

- Finally, a coercion to the clearness (for finding  $z_0$ ) is executed for example by means of the Centerior method:
- 

$$z_0 = \frac{\int_{\Omega} z \mu_{\Sigma}(z) dz}{\int_{\Omega} \mu_{\Sigma}(z) dz}$$

### III. MEMBERCHIP FUNCTIONS AND FUZZY TERMS

Membership functions used in the date article were mainly four kinds: triangle, trapezoidal, S and Z -figures. They are described by follows:

**S – form**. They are represented with two parameters – a, b:

$$\mu(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a < x < b \\ 1, & b < x \end{cases}$$

**Z – form**. They are represented with two parameters – b, c:

$$\mu(x) = \begin{cases} 1, & x < b \\ \frac{c-x}{c-b}, & b < x < c \\ 0, & c < x \end{cases}$$

**Triangle.** They are represented with three parameters – a, b, c:

$$\mu(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a < x < b \\ \frac{c-x}{c-b}, & b < x < c \\ 0, & c < x \end{cases}$$

**Trapezoidal.** They are represented with four parameters -a, b<sub>1</sub>, b<sub>2</sub>, c:

$$\mu(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b_1-a}, & a < x < b_1 \\ \frac{c-x}{c-b_2}, & b_2 < x < c \\ 0, & c < x \end{cases}$$

During work of controller on the algorithm of Mamdani in the distributive nets (maintenance of rejection of reactive-power in the rationed limits) [1]: on the entrance of fuzzy controller the calculated value of reactive power, dynamics of change of reactive power, calculated value of tension and most of switching for a day are given. From the exit of controller the values of linguistic variables of "Direct", "Delay" were taken off.

All entrance and output sizes are real values, as they are taken off from the real devices of the automatics. Further, in a controller, these parameters already will be transformed to the fuzzy values [5]. After the wearing-out of fuzzy rules, the got output variables again will be transformed to real form.

For working the fuzzy controller the following linguistic variables are used:

1. In the input of the controller (input variables):

- **Reactive power.** Fuzzy parameters of these linguistic variables are the following (fig. 2):  
**Very small** – Z-form membership function with parameters (0 0,05);  
**Small** – trapezoidal membership function with parameters (0 0,05 0,15 0,2);  
**Middle** - trapezoidal membership function with parameters (0,15 0,2 0,4 0,45);  
**Large** - trapezoidal membership function with parameters (0,4 0,45 0,85 0,95);  
**Very large** – S-form membership function with parameters (0,8 0,91 1).



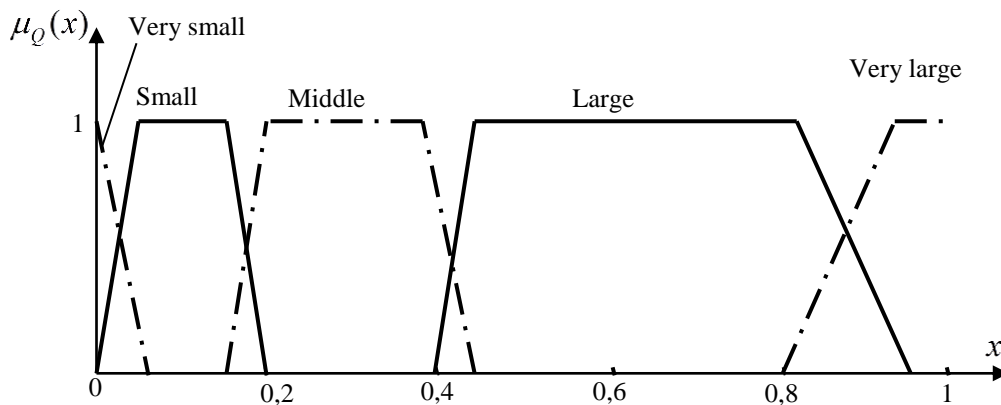


Fig. 2. Fuzzy values of variable of “Reactive power”

- Dynamic.** Dynamic of changing reactive power (output of reactive power). The data of that variable (fig. 3).  
**Negative** – Z-form of membership function with parameters (-0,5 0);  
**Zero's** – membership function with parameters (-0,8 0 0,8);  
**Positive** – S – form of membership function with parameters (0 0,5).

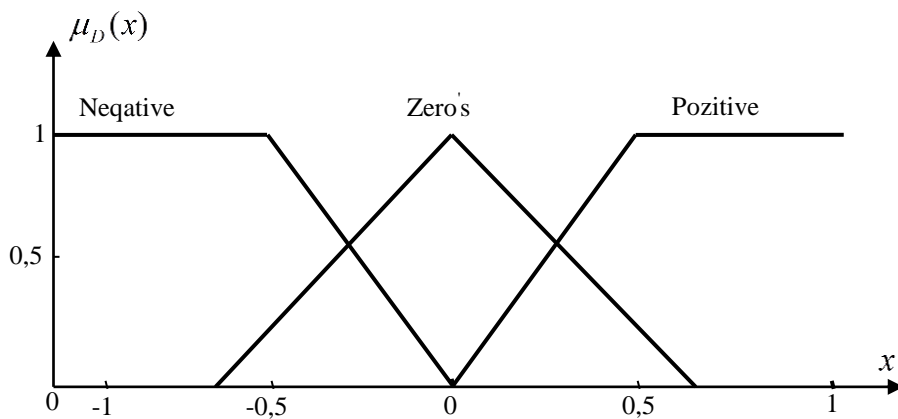


Fig. 3. Fuzzy value of the variable of “Dynamic”

**Voltage.** Voltage in the bus of loading. Fuzzy parameters of that linguistic variable are the following (fig. 4):

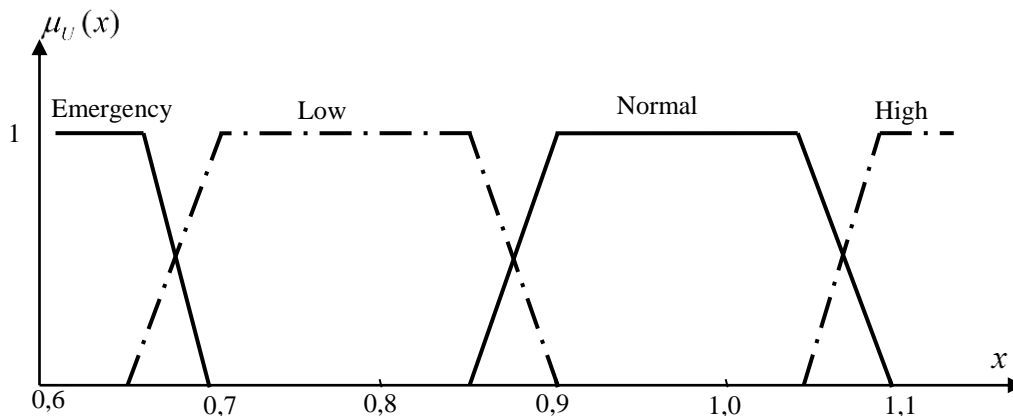


Fig. 4. Fuzzy value of the variable of “Voltage”

- **Emergency** – trapezoidal membership function with parameters (0,69 0,749 0,89 0,95);
- **“Normal”**- trapezoidal membership function with parameters (0,89 0,94 1,05 1,1);
- **“High”**- S-form membership function with parameters (1,05 1,1);
- **“Low”** – Z-form membership function with parameters (0,69 0,74).
- **Amount.** Amount of switching executed by the compensation equipment for a day. Value of that variable (fig. 5):

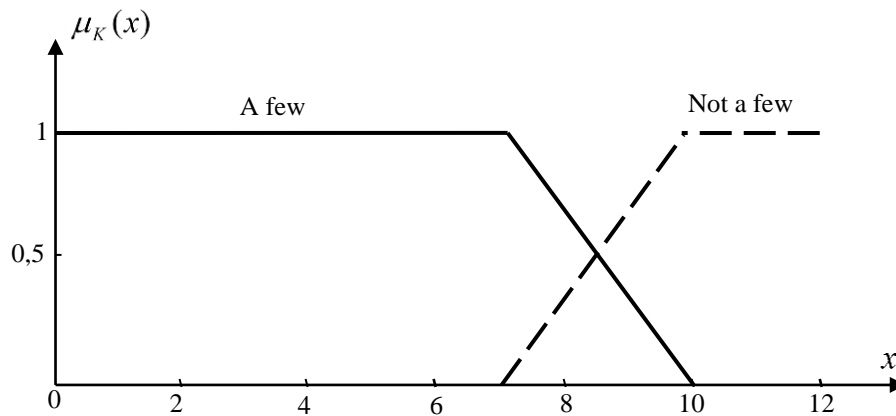


Fig. 5. Fuzzy value of the variable “Amount”

**A few** – Z-form membership function with parameters (7 10);

**Not a few** – S-form membership function with parameters (7 10).

2. From the exit of controller the values of the following linguistic variables (output variables) were taken off;

- **Direct.** Direction of the following switch. Value of the variable (fig. 6):
- **Down** - trapezoidal membership function with parameters (-1.5 -1.25 -0.75 -0.5);
- **Up** - trapezoidal membership function with parameters (0.5 0.75 1.25 1.5);
- **Stay** - trapezoidal membership function with parameters (-0.5 -0.25 0.25 0.5).

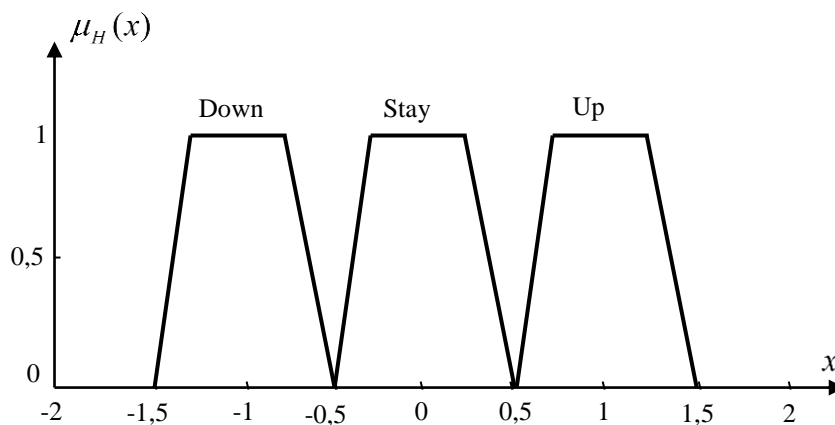


Fig. 6. Fuzzy value of the variable “Direct”

- **Delay.** Delay of time before switching. Fuzzy values of a variable (fig. 5b):
- **Very Small** – Z-form membership function with parameters (0 0.05);
- **Short** - trapezoidal membership function with parameters (0.05 0.15 0.2);
- **Mean** - trapezoidal membership function with parameters (0.15 0.2 0.4 0.45);
- **Long** - trapezoidal membership function with parameters (0.4 0.45 0.85 0.95);

**Very Long** - S-form membership function with parameters (0.8 0.9 1.1).

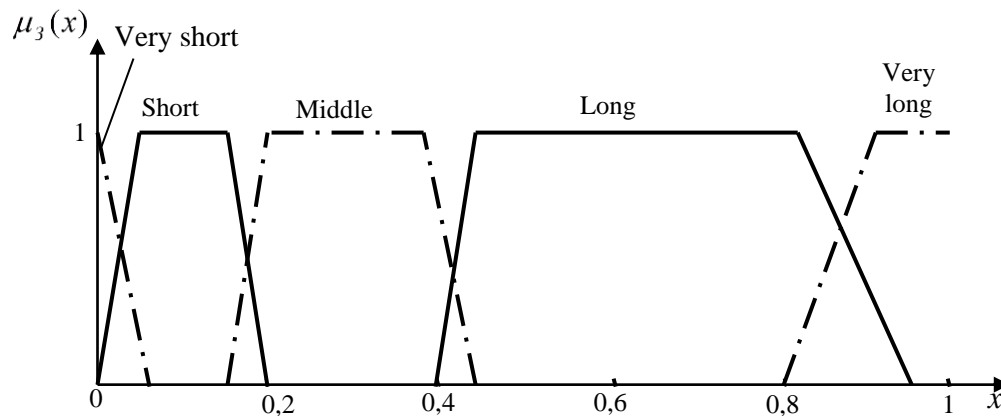


Fig. 7. Fuzzy value of the variable "Delay"

#### IV. SIMULATION RESULTS

By the given variables there were created 32 rules of fuzzy outputs for the date system. The got rules are given as:

1. If (Reactive power is Very small) (Voltage is normal) and (Amount is a few) and (Dynamic is Negative) then (Delay is very short) (Direction is To up);
2. If (Reactive power is Small) (Voltage is normal) and (Amount is a few) and (Dynamic is Negative) then (Delay is very short) (Direction is To up);
3. If (Reactive power is Mean) (Voltage is Emergency) and (Amount is a few) and (Dynamic is Negative) then (Delay is very short) (Direction is To up);
4. If (Reactive power is Large) (Voltage is Emergency) and (Amount is a few) and (Dynamic is Negative) then (Delay is short) (Direction is To up);
5. If (Reactive power is Very Great) (Voltage is Emergency) and (Amount is a few) and (Dynamic is Negative) then (Delay is short) (Direction is To up);
6. If (Reactive power is Mean) (Voltage is Emergency) and (Amount is a few) and (Dynamic is Positive) then (Delay is very short) (Direction is down).

During work of the researched model there were graphics of changing reactive power by time (fig. 8). First graphic, showed in the graphic, is represented as changing reactive power. The second graphic is represented as changing reactive loading. Third graphics are represented as spleens of working the condensation equipment.

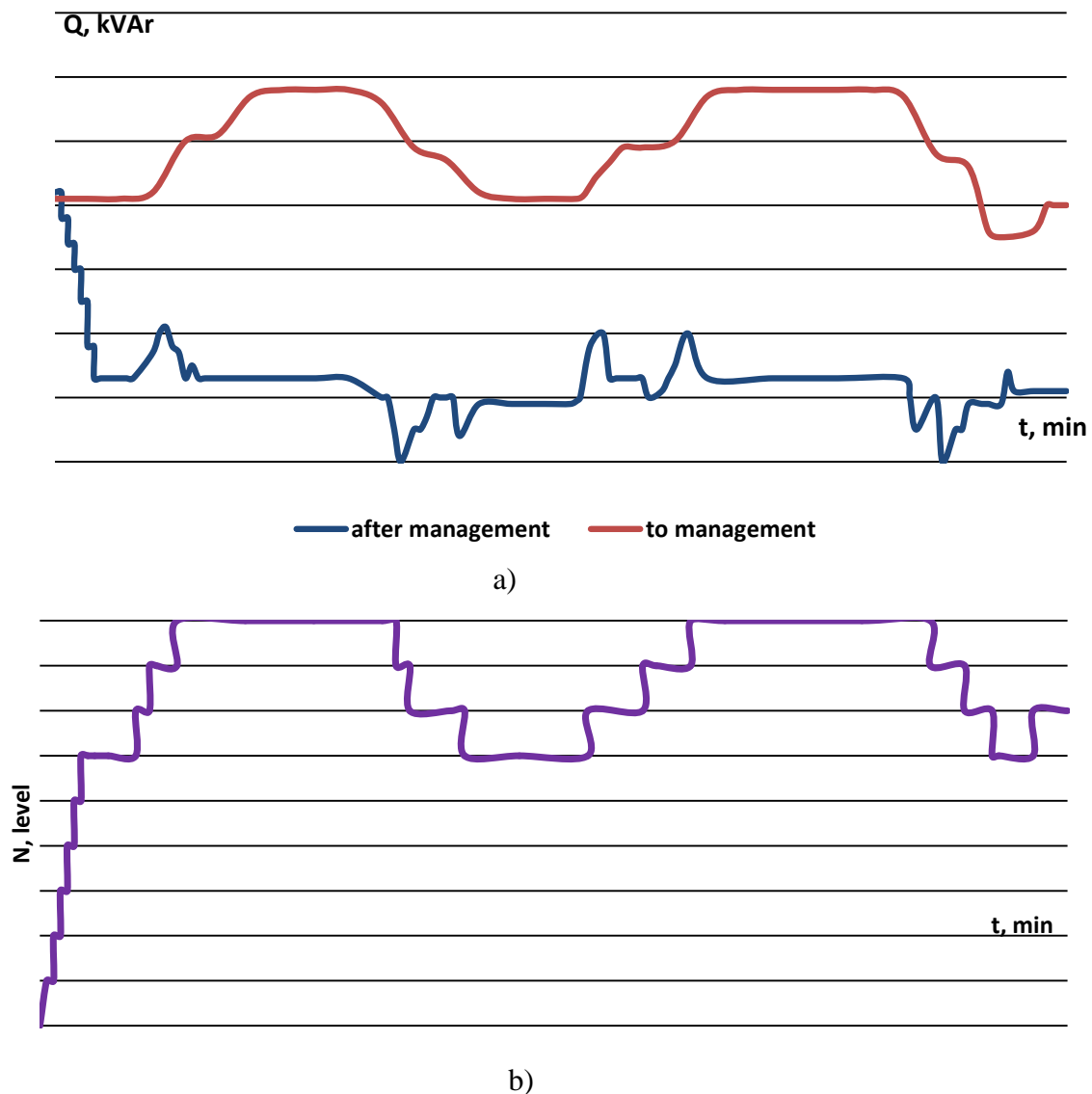


Fig. 8. The graphics of reactive loading, changing reactive loading (a) and spleens of working the condenser equipment (b)

## V. CONCLUSIONS

It was considered fuzzy algorithm of regulation of reactive power by means of the algorithm of Mamdani. Regulation of reactive power was realized in the distributive nets for supporting reactive power from a consumer in the rationed limits. At researching work of fuzzy controller which functioned on the basis of the algorithm of Mamdani (at regulation of reactive power in the distribution nets of the factory) it was got that fuzzy algorithm produces less amount of switching as compared to traditional systems. That process provides rising efficiency of working equipments of switching steps of the condenser battery.

Device on the basis of fuzzy logic possess another important property - comparatively by simple expansibility. Addition of new properties, new functional possibilities for such devices are easily and undifficult. In case of occurring of necessarily to additional functions it is possibility to include them to work of fuzzy device.

It is possible to assert that devices on the basis of fuzzy logic are more preferable for regulation of reactive-power on the bus of consumers, than devices on the basis of ordinary logic. In this case regulation gets more quality, the amount of switching goes down, a condenser battery serves longer and probability of its refuse falls down.

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## SIRFrt METHOD: ASSESSING AND IMPROVING MAINTENANCE RELIABILITY

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### ABSTRACT

The focus of this paper is to assess ways to improve plant availability of an organisation in order to reduce its reactive maintenance from its current level of 30%. The pharmaceutical products the organisation (name withheld) produces are in short supply. It is vital the equipment that produces these products are reliable and produce quality products all the time. Loss of one batch due to equipment malfunction not only equates to loss of revenue but also the loss of vital medications which could jeopardise patient safety. This paper solely focuses and reports on the SIRFrt method to assess plant reliability in order to achieve maintenance excellence. There are seven elements that can impact maintenance excellence, however the focus of this paper is on reliability improvement since the audit scores were low in comparison to the other elements considered under the SIRFrt method. As a result of using feedback gained through the SIRFrt method assessment, it is anticipated that reactive maintenance will reduce below 10%.

**Keywords** - SIRFrt method, maintenance excellence, reliability-centered maintenance, asset management, risk assessment, maintenance management.

## 1 INTRODUCTION

The approach to maintenance management has changed over the past few decades. Effective maintenance management requires a multi-disciplinary approach where maintenance is viewed strategically from a business centred perspective. These include the integration of technical and commercial issues and a quantitative approach involving mathematical models, the use of all relevant information and continuous improvement in maintenance management (Kobbacy & Murthy, 2008).

Maintenance excellence focusses on maintenance efficiency and effectiveness of processes. It aims to ensure that practices are consistent, sustainable and replicable. Maintenance excellence is not about fixing things when it breaks, it is about delivering value for money and improving plant and equipment performance (Afefy, 2010). It is also about aligning engineering goals with corporate goals (Plösch, Pomberger & Stallinger, 2011).

The challenge that confronts most organisations is to be leading maintenance organisations and develop and follow performance standards as a part of continuous improvement program. In this pursuit and in order to meet production targets, organisations often neglect scheduled maintenance and this contributes to poor maintenance performance (Tomlinsong, 2006). The motivation for implementing maintenance or reliability excellence within an organisation is to optimise manufacturing assets' health and processes, maximise production at the lowest unit cost, improve product quality and improve safety (Dhillon, 2006; Bakhshi et al., 2011; Moubray, 2012). Maintenance of equipment is a significant fraction of the total operating cost in many industry sectors.

If maintenance and manufacturing functions are not clear on the definition of reliability, it is possible the desired outcome of plant reliability will not be achieved. Hence it is vital that definitions of reliability are correctly understood by the maintenance team. Kleine (2009) has provided three paradigms where the definition of reliability takes on a whole new meaning.

Paradigm one changes the mindset from reliability being fewer breakdowns to less interventions. This argument is based on having increased condition monitoring tasks, which allow Engineering Department (ED) to intervene and carry out repairs outside manufacturing hours. Although this is good from a manufacturing perspective with regard to fewer breakdowns, it is still not the best outcome for maintenance due to the high cost of labour and material.

Paradigm two is about shifting the focus that reliability is used to determine equipment performance, to determining the performance of all activities. This involves taking the holistic view that there are many other issues that can impact the business other than equipment reliability. This involves environmental, health and safety, planning & scheduling and training. There is very little value to the business if the focus is only on the equipment while the majority of the management resources are spent on occupational health and safety and environmental issues.

Paradigm three challenges the notion that reliability belongs to the shop floor compared to the board room. There have been many examples when improvement programs have never really taken off due to the lack of commitment from senior managers. There is a good chance of an improvement activity succeeding if it has the commitment of senior management. ED's strategic plan along with its objectives has to align with the organisational strategic plan.

Hence, it is necessary to focus on reliability problems and identify the major contributors to loss and equipment malfunctions. In order to provide successful maintenance operations, it is crucial to implement a systematic approach for assessment and improvement. One such systematic approach is promulgated by SIRFrt. SIRFrt organises the Australasian Maintenance Excellence Awards (AMEA) annually (SIRFrt, 2013). As part of the AMEA audit, organisations are recognised for their commitment to maintenance excellence.

The SIRFrt Australasian Maintenance Excellence Awards (AMEA) audit focuses on the following seven elements (SIRF Roundtables, 2013) in its journey towards maintenance excellence:

1. Leadership: This element focusses on assessing the role of corporate executive leadership in creating and promoting a culture that supports continuous improvement in its maintenance program. The executive leadership is examined for its dedication to a planned and reliability culture.
2. People: This element focusses on assessing the extent to which people at different levels in the organisation are committed and appropriately trained to safely and effectively achieve the maintenance goals and objectives of the organisation. This element also aims to assess

the management and evaluation of the contribution of its staff in achieving organisational objectives.

3. **Planning and Scheduling:** This element evaluates how the organisation builds, controls and improves planning and scheduling of its maintenance work. This category encompasses how the organisation develops its overall maintenance plans. It also looks at how well the organisation incorporates its maintenance plan with the production plan and finally with its business plan. This is the key for improving wrench time.
4. **Maintenance Process and Practices:** This element evaluates the way the organisation selects the best maintenance strategy for its equipment. This element focusses upon methods and processes carried out to implement, monitor and improve maintenance processes and practices. Data and methods to predict appropriate maintenance goals form an important part of this assessment.
5. **Reliability Improvement:** This element focusses on assessing how an organisation achieves reliability at the lowest costs. The key focus is to assess reliability improvement processes and identify whether adequate preventive and predictive maintenance techniques are being used.
6. **Resource Management:** This element focusses on assessing the processes used by the organisation to efficiently allocate and utilise its materials and resources for carrying out maintenance activities.
7. **Business Performance:** This element aims to assess the extent to which the maintenance operation demonstrates sustained improvement and its integration and contribution to organisational goals.

This paper solely focuses on the SIRFrt method to assess and improve maintenance excellence. The remainder of the paper is organised as follows. The next section outlines the research methodology adopted for this study. Section three outlines the problem faced by an organisation in their maintenance excellence journey. Section four presents the discussion and key findings with key emphasis on the elements used to evaluate maintenance excellence. Finally in the conclusion, the key premises of the paper have been summarised and the limitations of this study are explicitly stated with an outlook for possible future research.

## 2 METHODOLOGY

This study is based on the current maintenance practices of an Australian pharmaceutical organisation that specialises in the manufacture of specialised pharmaceutical products. The name of the organisation explored as part of this study has been withheld for confidentiality reasons and the pseudonym XLean has been used instead. The analysis applies to only one plant of XLean as it operates in multiple locations. The engineering group at the site consists of the ED and the Projects department.

An exploratory case study method helps to investigate a ‘contemporary phenomenon within its real-life context; when the boundaries between phenomenon and context are not clearly evident; and in which multiple sources of evidence are used’ (Yin, 1984, p.23). Therefore case study research method was chosen as it provides clarity and in-depth understanding of complex matters and adds strength to what is already established through previous studies.



With all the progress XLean has made in the past two decades in the areas of research and development, new products, acquisitions and innovations it has a long way to go before being the best in the world when it comes to carrying out maintenance of its plant and equipment. The next section outlines some of the problems that confront XLean.

### 3 THE ORGANISATIONAL PROBLEM

Due to increased production demands, the manufacturing cycle of different products in XLean has reduced from 7 days to 5.6 days since July 2010 and then down to a 5 day cycle since January 2011. This reduced cycle time meant that the available time to carry out maintenance in between the cycles is less and hence overall maintenance time is reduced. Plant reliability cannot be maintained with less ‘windows of opportunity’ to carry out maintenance. This has resulted in an increased level of reactive maintenance which is currently at 30% of all maintenance work. The way forward is to revisit and challenge the current maintenance practices.

The organisational problem challenging XLean is the reduced manufacturing cycle times cannot be sustained with poor equipment reliability. Reduced cycle time also means the maintenance resource structure should be able to respond to ‘windows of opportunity’ at a short notice. This problem cannot be resolved at the current rate of reactive maintenance being undertaken. Reactive maintenance includes breakdowns and non-scheduled work that breaks into the weekly schedule. Plant availability is measured as part of the current key performance indicators.

XLean, similar to other companies, has difficulty in seeing asset reliability initiatives in the same light as safety improvements. Most companies are proud to publish that safety comes first but not many companies proclaim that asset reliability is top priority. During difficult times the first process to be cut will be continuous improvement activities which would result in reduced asset reliability. Once continuous improvements’ process breaks then asset reliability starts to suffer.

Following are the main elements that need to be resolved by the ED, if XLean is to meet the reduced cycle times and achieve higher plant availability:

- Reactive maintenance to be reduced from the current level of 30% to less than 10%;
- Improve the way planning and scheduling activities are undertaken;
- Align the manufacturing plan to the maintenance plan;
- Carry out asset criticality as well as PM task criticality;
- Complete a high level Reliability Centered Maintenance (RCM) activity on all assets that have a PM schedule; and
- Undertake a Preventive Maintenance Optimisation (PMO) exercise on the current PM’s to ensure only value adding activities are conducted.

The consequence of not achieving the five day production cycle will result in not meeting the contractual obligations between XLean and its key customers. In order to meet the five day batch cycle XLeans’ maintenance strategy needs to be reviewed and reactive maintenance work needs to be brought down to 10% from the current level of 30%. A maintenance resource structure needs to

be developed to meet the reduced cycle time. Ability to plan and schedule the work in advance will help achieve the targets as well as improve wrench time.

The ‘maintenance hero’ mentality is alive at XLean but there is a lack of strategy to combat the current issues. XLean maintenance is good at getting the horse back into the barn after it has escaped. One of the main challenges XLean faces is how to change the mindset that keeping the horse in the barn is what maintenance is about. XLean is no different to most companies who think that maintenance is a necessary evil. The purpose of maintenance at XLean is seen as repairing failed assets, thus adding little value to the organisation. It is only recently that XLean has started to believe that maintenance is about delivering value to the business and improving plant reliability. It is about making the paradigm shift that maintenance is a reliability function and not a repair function.

XLean has come to the conclusion that it cannot be a world leader in the production of pharmaceutical products if it does not have a reliable and robust maintenance process working together with manufacturing. It has been recognised within the organisation that equipment reliability is essential, if progressing down the path of excellence. Senior management is of the strong belief that the organisation cannot continue to develop and seek new opportunities if the existing foundation (plant reliability) is not stable.

#### **4 DISCUSSION AND FINDINGS**

The philosophy of XLean’s maintenance strategy is to be proactive and not reactive. XLean plans to achieve this by striving for maintenance excellence. One of the key strategies that will be used to achieve maintenance excellence in XLean will be the SIRFrt method in exploring and resolving its maintenance problems.

A self-assessment audit was undertaken on each of the seven elements of the SIRFrt AMEA audit to determine XLean’s maintenance excellence journey. Each of the seven elements is applied to the Intent, Approach, Deployment, Results, and Improve (IADRI) five-point inquiry model, which forms the basis for self-assessment and scoring (SIRFrt, 2010). The IADRI model sets the context for a challenging critical review of the completeness and effectiveness (NSW Trade & Investment, 2012) of the current maintenance processes at XLean.

The advantage of scoring using the IADRI model is that points are allocated based on many facets and not on the individuals’ perception. The categories under the IADRI model are:

**Intent:** highlights the purpose and expected outcomes aimed by the organisation.

**Approach:** aims at identifying the planning carried out.

**Deployment:** aims at outlining how and where the activities/processes were carried out.

**Result:** aims at identifying whether the anticipated outcomes have been achieved.

**Improvement:** targets at identifying the way past improvements were carried out and assessing their effectiveness.

The IADRI model sets the context for a complete self-audit on all the 7 elements of the SIRFrt method as listed below along with the maximum achievable score.

1. Leadership (140 points)
2. People (180)
3. Planning and scheduling (110)
4. Maintenance process and practices (160)
5. Reliability improvement (180)
6. Resource management (110)
7. Business performance (120)

The above scores were taken from the SIRFrt maintenance excellence awards criteria and these evaluation categories total up to 1000 points. The audit was conducted by the local branch of the SIRFrt and included XLean staff from Manufacturing, Engineering and the Quality Assurance departments.

It was considered to be beyond the scope of this paper to list all the individual assessment sub-elements under each main element. Hence the average scores that were the outcome of the audit across each of the 7 elements are presented in Figure 1.

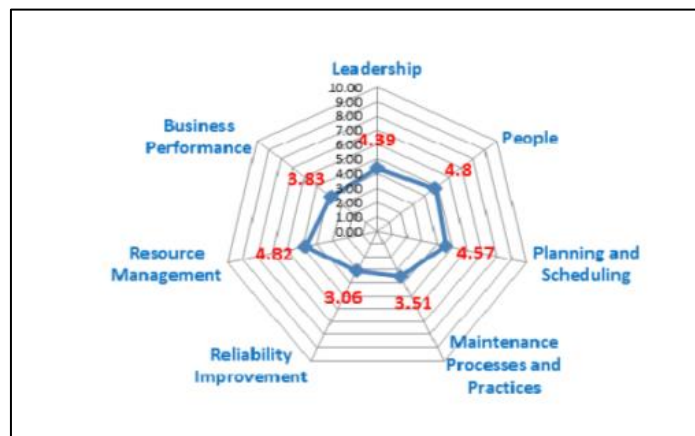


Figure1: Audit summary - Average Scores

When this self-assessment was conducted at XLean, the reliability improvement element scored the lowest and hence required further analysis.

Reliability improvements in the audit were assessed against the following five sub-elements. Figure 2 shows the scores for the five sub-elements for reliability improvement.

1. Reliability improvement identification (processes used to determine when reliability improvement is required) - Currently there is no documented procedure when or how to capture certain repeated failures and pass this information on to the reliability group.

2. Reliability improvement process (processes used to deliver reliability improvements) - The audit identified that there was not a process in place to work on reliability issues. There were various RCA methodologies being used on site.
3. Achieved reliability improvement (Improvements to asset performances that have been achieved as a result of improved reliability) - There were no measures in place to see if the reliability program was having an impact in improving the uptime. There was no system in place to capture all the reliability improvements that were carried out over the past year. The current KPIs need revisiting.
4. Data for overall improvement (Data being collected and its usage to improve the overall performance of the business) - The maintenance management system did not have a category for reliability improvement. It was difficult to differentiate between reliability work and other PM work. The information gathered was not being used for new plant design or for improving the current maintenance strategy. Past trends and data were not being used for failure analysis.
5. Learnings from reliability improvements being used for new projects (Maintenance learning incorporated into the design and purchase of new equipment) - It was found that there was insufficient communication and procedural exchange between Maintenance and the Projects department. This resulted in the experience of the maintenance engineers not being captured in new projects. It was found in the audit that this resulted in the same deficiencies being reintroduced in new installations. The audit also found that the project engineers were not aware of the requirements of the maintenance department when it came to handing over projects. This included minimum training, spares and no PM's in place for the new equipment.

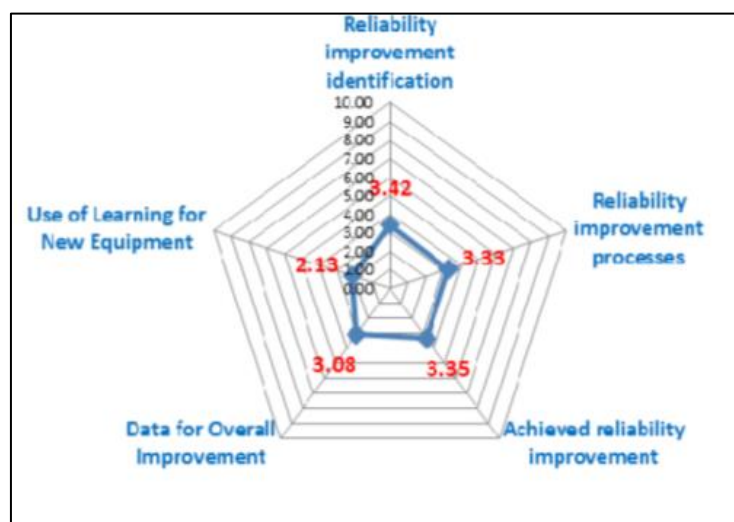


Figure 2: Reliability Improvement

To counteract these problems, findings from the maintenance self-assessment audit revealed that focus groups should be set up to deal with the following high priority issues which would free up

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time and resources for the organisation to concentrate on all the elements required to achieve maintenance excellence.

1. Time wasted due to production personnel not giving enough information on the work order.
2. No proper system to ensure that specialised tools are returned to the store after use.
3. Better planning and scheduling.
4. Spares parts removed from the store need to be accounted for.
5. Need to have a reliability culture and not a breakdown culture.
6. Work closely with production to maximise plant availability.
7. Set up a Reliability group with a clear mandate.
8. Leadership and team building.

It is expected that the findings will help in improving plant reliability at XLean. Improvements made should be validated by conducting a repeat audit once the improvements have been implemented. It is important that a culture of proactive rather than reactive maintenance is developed to support the operations of the organisation and reduce cycle times and the culture is in sync with its philosophy. An overall maintenance strategy that is designed to ensure reliability and minimisation of resource consumption should be looked into by XLean and finally it should be ensured that the continuous improvement cycle of Plan-Do-Check-Act should be carried out uninterrupted.

## 5 CONCLUSION

The aim of this paper was to assess ways to improve plant availability of an organisation by reducing reactive maintenance from its current level of 30%. It is understood that the reduction in cycle times will allow more batches to be produced thereby reducing the shortage of life saving products produced at this plant. However reduced cycle time also reduces the opportunity to carry out critical maintenance activities. This increases reactive maintenance due to breakdowns which consequently impacts the cycle time.

SIRFRt method was found suitable to review XLean's journey towards maintenance excellence. As evident from the SIRFRt audit, reliability improvement was an area that needs attention. It is vital to optimise the current maintenance practices under the current regime of reduced production cycle and high reactive maintenance. It is highly recommended that organisations facing similar challenges assess their maintenance strategies and conduct regular audits to improve their maintenance systems.

Although this paper only reviewed reliability elements, it is important to have a more holistic view of maintenance rather than just investigating one element. It is recommended that the other elements be addressed as part of future studies. Generalisability of the findings is not advised due to the nature of case study methodology.

This research can be further extended to the remaining six elements identified to achieve maintenance excellence. An important part of maintenance is managing people and hence leadership needs to be considered as one of the key areas of interest for cultural change. Benchmarking against similar companies could be beneficial in improving maintenance management.

XLean along with many other pharmaceutical companies has realised the importance of maintenance, which in turn has increased its awareness of the maintenance processes. The audit indicated that it is difficult to achieve maintenance excellence by improving all seven elements simultaneously. It has therefore identified that roll out of the reliability program in the first phase of implementation of the maintenance excellence plan is needed.

It is vital that maintenance patterns at XLean changes from being reactive to proactive where senior management need to play an important role in ushering this change and foster a culture that drives right staff attitudes and adequate resource allocation.

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