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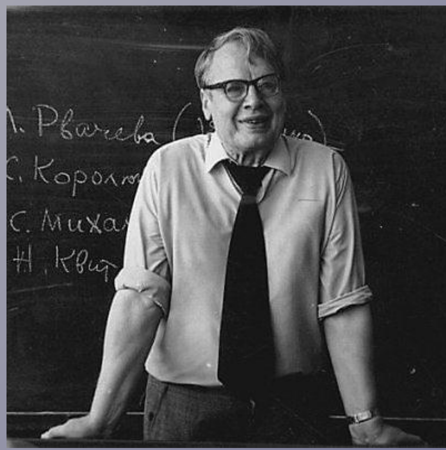

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ISSN 1932-2321

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VOL.10 NO.3 (38)
SEPTEMBER, 2015

RELIABILITY:
THEORY&APPLICATIONS

RELIABILITY
RISK ANALYSIS
MAINTENANCE
SAFETY

Vol.10
No.3 (38)
•
September
2015

In Memory Of Igor Ushakov

San Diego

ISSN 1932-2321

http://www.gnedenko-forum.org/Journal/index.htm

Journal of International Group on Reliability

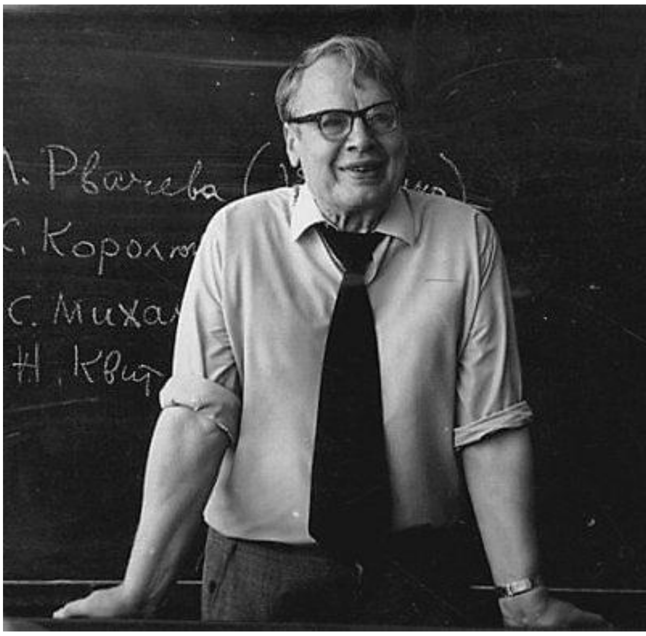

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Table of Contents

There are considered questions of a choice and placing of devices of the battery of static condensers in distributive electric networks of power supply systems for indemnification of reactive capacity, on the basis of technical and economic indicators and investment possibilities of a power supply system. Given results of settlement experiments of realization of an offered methods for the typical distributive network, characterizing technical and economic efficiency of the made decision are resulted.

In this paper a problem of hierarchical classification of some objects by similarity matrix solution is solved. This approach gives single solution of classification problem. Each hierarchical level is defined by some critical value of a similarity. Using critical value the similarity matrix is transformed into contiguity matrix of some no oriented graph in which connectivity components are constructed. Increasing successfully critical values it is possible to define hierarchical classification of initial objects. This approach is closely connected with reliability theory and mathematical statistics in which a reaching of critical value is one of important problems.

Many electronic devices operate in a cyclic mode. This should be considered when forecasting reliability indicators at the design stage. The accuracy of the prediction and the planning for the event to ensure reliability depends on correctness of valuation and accounting greatest possible number of factors. That in turn will affect the overall progress of the design and, in the end, result in the quality and competitiveness of products.
This study (research grant № 15-05-0029) was supported by the National Research University Higher School of Economics' Academic Fund Program in 2015.

The tampered Brownian motion process (BMP) arises in the context of partial step-stress accelerated life testing when the underlying system fatigue accumulated over time is modeled by two constituent BMPs, one governing up to the predetermined time point at which the stress level is elevated and the other afterwards. A conditioning argument obtains the probability distribution function (pdf) of the corresponding time-to-failure random variable. This result has been reported and studied in the literature, but its derivation has not been published.

A definition of binary lambda - set function is introduced. It is used for the inspection interval planning in order to limit a probability of fatigue failure rate (FFR) of an airline (AL). A solution of this problem is based on a processing of the result of the acceptance full - scale fatigue test of a new type of an aircraft. Numerical example is given.

METHODS INDEMNIFICATION OF REACTIVE CAPACITY IN THE ELECTRIC NETWORK FOR INCREASE OF REGIME RELIABILITY OF THE POWER SUPPLY SYSTEM

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ABSRACT

There are considered questions of a choice and placing of devices of the battery of static condensers in distributive electric networks of power supply systems for indemnification of reactive capacity, on the basis of technical and economic indicators and investment possibilities of a power supply system. Given results of settlement experiments of realization of an offered methods for the typical distributive network, characterizing technical and economic efficiency of the made decision are resulted.

Keywords: power supply systems, a distributive electric network, batteries of static condensers, indemnification of reactive capacity.

I. INTRODUCTION

Problems of rise of efficiency and regime reliability of electric power systems at the expense of decrease of losses of the electric power in electrical networks by means of a reactive power compensation (RPC) is the important provision on an energy conservation. Thus the all-round energy conservation can be observed as alternative in relation to the large-scale escalating of power powers on large power stations.

Methodically the problem of sampling and disposing of devices RPC dares as follows. The supply authority should install meaning of a reactive power which should be transmitted from an electric power system in a consumer web (an economic reactive power – $Q_{эк}$), and a deficiency of a reactive power of consumer webs should become covered at the expense of its generation in webs at the expense of batteries of statistical capacitors (BSC) [1-4] or other modes, for example, by application of a thrystors radiant of a reactive power [5].

In the conditions of market economy electrical networks of electric power systems belong to various departments and consequently solving problems BSC it is represented as a complex technical and economic problem. Thus determination $Q_{эк}$ should dare on the basis of technical-and-economic indexes and investment possibilities of an electric power system, and consumer webs are obliged to realize local BSC ensuring set $Q_{эк}\,(tg\varphi_{эк})$, or to them sanctions in the form of the allowance to the tariff for use by electric energy should be applied.

Practically the technique of sampling and disposing of operated compensating devices (CD) in electrical networks of electric power systems consists in definition of summarized power CD, and then their optimum disposing, with determination of sequence of their installation, in electric power system knots.

The sampling and disposing problem operated BSC in distributive electrical networks of electric power systems is in-process observed.

II. CRITERION FUNCTIONS AND ACCEPTED RESTRICTION AT CRC

On the basis of experience of the solution of the given problem and the developed economic mutual relations at the optimum complex solution of problem CRC the optimization equations in a following aspect [6] are used:

1. Criterion function

$$F = \sum \Delta\Delta W_{ann} = \Sigma\left(\Delta W_{ann}^{b.c} - \Delta W_{ann}^{a.c}\right) \to \max,$$

$$\Sigma E_{ann} = \left(\sum \Delta\Delta W_{ann} \cdot \beta - \frac{\sum C_{CD}}{n_c}\right) \to \max$$

$$T_{pb} = \frac{\sum C_{CD}}{\sum \Delta\Delta W_{ann} \cdot \beta} \to min$$

where: $\Delta\Delta W_{ann}$ - annual economy of the electric power owing to compensation;

$\Delta W_{ann}^{b.c}$, $\Delta W_{ann}^{a.c}$ - annual losses of energy before and after compensation;

$\beta -$ the cost price of manufacture of the electric power in power supply system;

$\sum C_{CD}$ - total expenses for CD installation;

n_c - serviceability of the capacitor battery.

2. The restriction equations:

$$\sum Q_c^{add} \le \left(P_{c,\max} \cdot a - Q_{k,exs}\right)$$

$Q_{ci} \le Q_{H,i,min} -$ non - adjustable CD

$Q_{ci} \le Q_{H,i,max} -$ adjustable CD

$$\sum C_{CD} = C_c \sum Q_c^{add} \le C_{inv.}$$

$$U_{\kappa \min} < U_k < U_{k\max}$$

where

$\sum Q_c^{add}$ - in addition necessary CD power;

$P_{c,max}$ - the maximal power supply system;

a - the equipment factor of the CD power supply system $(a \approx 0{,}2)$;

$Q_{c,exs} -$ capacity of existing CD;

$Q_{ci,}$ $Q_{H,i,min,,}$ $Q_{H,i,max}$ - necessary CD power, minimal and maximal reactive power in i node;

C_c - specific cost of CD$\left(\frac{\$}{MBAp}\right)$;

C_{inv} - amount of the allocated investment on CD installation in a power supply system (in region);

$U_{\kappa\min}, U_{k\max}$ - the minimal and maximal values of the voltage in k node.

3. Constraint equation.

The program of calculation of the established mode of the electric network (CEMEN) - for definition $\sum \Delta W = \sum \Delta P \cdot \tau$

$m_i = \frac{\partial \sum \Delta\Delta W}{\partial Q_c}$ - for definition of the sequence of CD installation

where $\Delta P, \tau$ - ctive power losses in a mode of the maximal loading, number of hours of use of the maximal losses *(τ= 4000-5000 hours)*.

m_i - sensitivity of alteration of losses at changing of the CD power in *i* node.

Economic benefit from CRP is achieved owing to:

- reduction of energy losses in air and cable lines and in transformers.

$$\Delta\Delta W = \Delta\Delta P \cdot \tau\ ; \quad C_{\Delta P} = \Delta\Delta W \cdot \beta$$

- increase in throughput of lines and transformers, which is taken into account by corresponding shares of their cost, i.e.:

for lines with admissible current $\dot{I}_d$

$$\Delta K_l = K_l (I_1^2 - I_2^2) / I_d$$

for transformers

$$\Delta K_T = K_T (S_1 - S_2) / S_1$$

Here K_l and K_T - cost of lines and transformers.

Annual economic benefit

$$E_{ann} = C_{\Delta P} - C_{CD} / n$$

Pay-back period

$$T_{pb} = (C_{CD} - \Delta K_T - \Delta K_l) / C_{\Delta P}$$

III. METHOD OF SAMPLING CD IN DISTRIBUTIVE ELECTRICAL NETWORKS OF ELECTRIC POWER SYSTEMS

The principle of sampling CD in distributive electrical networks is based on security of an economic reactive power - $Q_{эк}$ or $tg\varphi_{эк}$. Thus CD, ensuring $Q_{эк}$ in a point of association of a distribution net (DN) to an electric power system, it can be installed in various points [7-9].

At installation CD on buses of a high voltage of the feed source, i.e. in a point 4 power CD will be

$$Q_{к4} = Q_4 - Q_{ek} = Q_4 - P_4 \cdot tg\varphi \tag{1}$$

At installation CD on a low leg of substation on supply net buses, in a point 3 $Q_{к3}$ it is had:

$$Q_4^{ac} = Q_3 - Q_{k3} + \Delta Q_{T1}^{ac} = Q_{ek} = P_4^{ac} \cdot tg\varphi$$

Where $P_4^{ac}, Q_4^{ac}, \Delta Q_{T1}^{ac}$ – active and reactive powers on buses 4 and reactive power losses in transformer T1 after compensation.

Here

$$Q_{k3} = Q_4 - Q_{4ek} = Q_3 + \Delta Q_{T1} - P_4^{ac} \cdot tg\varphi = Q_3 + \Delta Q_{T1}{}^{ac} - (P_3 + \Delta P_{T1}{}^{ac})tg\varphi =$$
$$= Q_3 - P_3 tg\varphi + \frac{P_3^2 + (Q_3 - Q_{k3})^2}{U_3^2} x_{T1} - \frac{P_3^2 + (Q_3 - Q_{k3})^2}{U_3^2} r_{T1} tg\varphi =$$
$$= Q_3 - P_3 tg\varphi + \frac{P_3^2 + Q_3^2}{U_3^2} x_{T1} - \frac{P_3^2 + Q_3^2}{U_3^2} r_{T1} tg\varphi - 2Q_3 Q_{k3} (\frac{x_{T1} - r_{T1} tg\varphi}{U_3^2}) + \qquad (2)$$
$$+ Q_{k3}^2 (\frac{x_{T1} - r_{T1} tg\varphi}{U_3^2}) = Q_{k4} - [2Q_3 Q_{k3} - Q_{k3}^2] \frac{x_{T1} - r_{T1} tg\varphi}{U_3^2}$$

In expression (2) $2Q_3 Q_{k3} - Q_{k3}{}^2 > 0$, since $Q_3 > Q_{k3}$, hence, $Q_{k3} < Q_{k4}$.

Meaning of power CD Q_{k3} we will discover from the following quadratic equation gained after transformation (2)

$$\frac{x_{T1} - r_{T1} tg\varphi}{U_3^2} Q_{k3}^2 - \frac{x_{T1} - r_{T1} tg\varphi}{U_3^2} 2Q_3 Q_{k3} - Q_{k3} + (P_3^2 + Q_3^2) \frac{x_{T1} + r_{T1} tg\varphi}{U_3^2} + Q_3 - P_3 tg\varphi = 0 \qquad (3)$$

In observed case Q_{k3} it is defined according to $U_3, P_3, Q_3, tg\varphi, r_{T1}, x_{T1}$.

From two roots of an equation (3) it is accepted $0 > Q_{k3} < Q_3$ which will ensure demanded tgφ on a substation high-tension side.

At desire of definition Q_{k3} it is direct according to P_4, Q_4 in the formula (3) meanings P_3, Q_3 are substituted from expressions:

$$\mathrm{P}_3 = P_4^{bc} - \frac{(P_4^{bc})^2 + (Q_4^{bc})^2}{U_4^2} r_{T1}$$

$$\mathrm{Q}_3 = Q_4^{\mathrm{bc}} - \frac{(P_4^{\mathrm{bc}})^2 + (Q_4^{bc})^2}{U_4^2} x_{T1}$$

Where, P_4^{bc} Q_4^{bc} - meaning active and a reactive power before compensation.

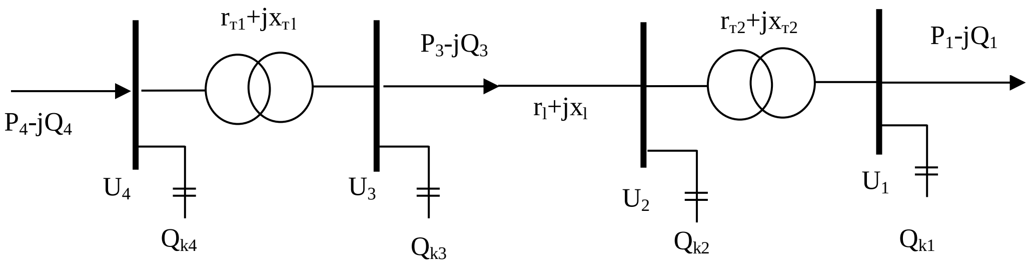

Fig. 1. The typical circuit design of a supply net

Let's accept:

$$A = \frac{x_{T1} - r_{T1} tg\varphi}{U_3^2}$$

$$B = \left(\frac{x_{T1} - r_{T1} tg\varphi}{U_3^2} \cdot 2Q_3 - 1\right)$$

$$C = (P_3^2 + Q_3^2)\,\frac{x_{T1} + r_{T1} tg\varphi}{U_3^2} + Q_3 - P_3 tg\varphi$$

Then

$$AQ_{k3}^2 - BQ_{k3} + C = 0$$

Apparently from (3) meaning of reactive power CD it is defined affiliated active both a reactive power of a knot and active and jet by resistance between a point of association of a supply net to an electric power system. Therefore at installation CD in a knot 2 to resistance r_{T1} and x_{T1} it is necessary to add r_l and x_l lines 2-3, and at installation CD in a knot 1 still r_{T2} and x_{T2}, voltage led to one step, for example to voltage U_4.

In case the supply net is represented several departing from buses 3 (10 kV) lines it is expedient to define at first the general power which is necessary for installing in this supply net, and then it to distribute between knots of loadings to proportionally their reactive powers.

IV. RECOMMENDATIONS ABOUT DISPOSING BSC IN ELECTRIC POWER SYSTEM NETS

In existing webs for definition of a place of disposing new CD it is necessary to have the information on reactive loadings on substation. The most authentic data are the winter and summer indications spent in an electric power system. On the basis of the indications spent in real electric power system in winter phase, the analysis of a relationship jet and an active-power ($tg\varphi$) on all central substations *110 kV* is made φand knots with the greatest meanings of these relationships are revealed. For 7 substations where $tg\varphi > 0{,}4$ powers CD are defined, adoption in limits (0,2-0,3) $P_{\max}$ and their effectiveness's are defined, at various meanings τ, C_{CD} resulted in table 1. In association about volume of investments summarized power CD, and on it annual economic benefit E_a and pay-back period the Current is chosen.

For reaching of the greatest efficiency it is necessary to install also sequence of installation CD in the electrical network of electric power systems. The analysis on the basis of factor design of experiments can be for this purpose used. Thus in the capacity of factors powers CD in various, most probable load buses with the greatest factors of a reactive power can be used.

The regression equation will have the following appearance:

$$\bar{Y} = f(\vec{X}) = b_0 + \sum_{i=1}^{k} b_i X_i \tag{4}$$

Where, $\bar{Y}$ – average meanings of the sized up parameter; X_i – current meanings of input parameters; b_0, b_i – estimations of factors of the equation of a regression.

Table 1

Outcomes of accounts of sampling CD and performances of their efficiency in PC electric power systems

$Q_{K\Sigma}$	Q_{Ki}, MVAr							ΔP	ΔΔP	$\Delta\Delta W_{year}$, kVt.h		$C_{\Delta\Delta W}$, a dale	
	$Q_{k\ I}$	$Q_{K\ II}$	$Q_{k\ III}$	$Q_{k\ IV}$	$Q_{K\ V}$	Q_{kVI}	$Q_{k\ VII}$	MVt	MVt	τ=4500	τ=5000	1	2
0	0	0	0	0	0	0	0	127,38	-	-	-	-	-
142	18	18	18	25	19	20	24	123,78	3,60	16200000	18000000	810000	900000
158	20	16	16	30	16	30	30	123,53	3,85	17325000	19250000	866250	962500
180	20	20	20	30	20	40	30	123,16	4,22	18990000	21100000	949500	1055000
194	20	20	24	40	20	40	30	123,01	4,37	19665000	21850000	983250	1092500
235	25	25	25	45	25	45	45	122,09	5,29	23805000	26450000	1190250	1322500
255	35	30	30	50	25	50	35	122,05	5,33	23985000	26650000	1199250	1332500

Table 1 prolongation

C_{CD}, in USD		$E_y=C_{\Delta\Delta W}-(C_{cd}/n)$, in USD				$T_{pb}=C_{CD}/C_{\Delta\Delta W}$, year			
15000	20000	1	2	3	4	1	2	3	4
-	-	-	-	-	-	-	-	-	-
2130000	2840000	668000	758000	620667	710667	2,6	2,4	3,5	3,2
2370000	3160000	708250	804500	655583	751833	2,7	2,5	3,6	3,3
2700000	3600000	769500	875000	709500	815000	2,8	2,6	3,8	3,4
2910000	3880000	789250	898500	724583	833833	3,0	2,7	3,9	3,6
3525000	4700000	955250	1087500	876917	1009167	3,0	2,7	3,9	3,6
3825000	5100000	944250	1077500	859250	992500	3,2	2,9	4,3	3,8

The greatest and least meanings of factors can make (0,15-0,35) P_{max}. Annual economic efficiency and pay-back period application CD will be responses (outcomes). The sequence will be defined on positive greatest regression coefficient for annual economic efficiency and on negative greatest factor for pay-back period. On these in factors it is possible to choose as sequence, and most an effective value of power CD.

For sampling of sequence of installation CD in the real electrical network *110 kV* electric power systems have been carried out also researches on disposing CD in 12 knots resulted in table 2.

Table 2

Outcomes of indications on active and a reactive power for separate substations

№	SS	P_{max} MVt	Q_{max} MVAr	tg φ	P_{min} MVt	Q_{min} MVAr	tg φ
1	SS № 17	121,15	76,86	0,63	104,86	47,01	0,45
2	SS № 162	111,72	57,25	0,51	32,38	9,01	0,28
3	SS № 9	252,86	85,75	0,34	38,03	87,57	2,30
4	SS № 195	159,34	72,48	0,45	49,16	17,32	0,35
5	SS № 298	87,44	44,17	0,51	67,88	37,12	0,55
6	SS № 290	194,31	81,89	0,42	43,97	16,82	0,38
7	SS № 99	163,63	52,33	0,32	29,37	13,94	0,47
8	SS № 26	194,41	64,72	0,33	28,06	46,51	1,66
9	SS № 23	93,19	36,78	0,39	28,5	31,13	1,09
10	SS № 112	73,92	32,14	0,43	12,24	5,05	0,41
11	SS № 31	64	32	0,50	25	14,07	0,56
12	SS № 21	106,3	81,97	0,77	139,6	40	0,29
THE SUM:		1378,05	572,23	0,42	599,05	365,55	0,61

The plot of a fractional factorial experiment of type $2^{12-8}=16$ is made. Following adequate equations of a regression are gained:

$$\Delta\Delta P = 7{,}75 + 0{,}514X_7 + 0{,}180X_5 + 0{,}149X_2 + 0{,}146X_9 + 0{,}139X_3 +$$
$$+ 0{,}131X_8 + 0{,}110X_{11} + 0{,}105X_{10} + 0{,}104X_6 + 0{,}101X_1 + 0{,}080X_{12} + 0{,}066X_4 \quad (MVt)$$
$$\Delta\Delta W = 38{,}753 + 2{,}571X_7 + 0{,}903X_5 + 0{,}746X_2 + 0{,}734X_9 + 0{,}696X_3 +$$
$$+ 0{,}659X_8 + 0{,}553X_{11} + 0{,}528X_{10} + 0{,}521X_6 + 0{,}509X_1 + 0{,}403X_{12} + 0{,}334X_4 \quad (\text{mln.kVt.h})$$
$$E_{ann} = 1{,}47 + 0{,}108X_7 + 0{,}035X_5 + 0{,}024X_2 + 0{,}023X_9 + 0{,}021X_3 +$$
$$+ 0{,}019X_8 + 0{,}019X_{10} + 0{,}014X_{11} + 0{,}012X_1 + 0{,}006X_6 + 0{,}006X_{12} + 0{,}003X_4 \quad (\text{мln.dol.})$$
$$T_{pb} = 3{,}587 + 0{,}100X_6 + 0{,}062X_{12} + 0{,}050X_1 + 0{,}050X_4 + 0{,}050X_{11} +$$
$$+ 0{,}037X_2 + 0{,}037X_3 + 0{,}037X_8 + 0{,}025X_9 + 0 \cdot X_{10} - 0{,}012X_5 - 0{,}087X_7 \quad (\text{year})$$

From the analysis of equations E_a and *the Current* follows the following sequence of installation CD in knots: 7-5-2-9-3-10-8-11-1-12-4-6.

V. ANALYSIS CRC IN A TYPICAL SUPPLY NET

The typical electric radial supply net *110/10/0,4 kV* is observed (fig. 2), in which buses *110 kV* are a feeding knot of an electric-power supply, and transformer T*1 (110/10 kV)*, a line *10 kV* and transformer *T2 (110/10 kV)* belong to a supply net. Summarized loading of users makes

25+j11,5MVA, distributed between knots *1 (0,4 kV)*, *2* and *3 (10 kV)*. Equivalent parameters of lines and transformers are resulted in table 3.

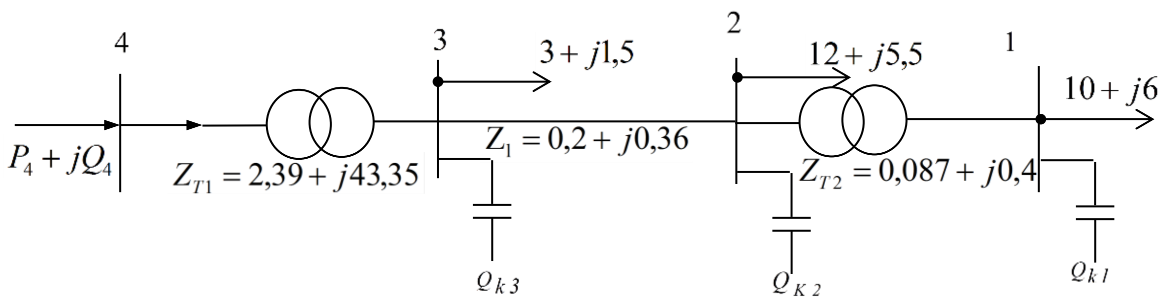

Fig. 2. The circuit design observed typical SS

Table 3

Equivalent parameters of lines and transformers

Net element	Type, brand	S, MVA	I, A	R_{ekv}, Om	X_{ekv}, Om
Tr-r T1	2xTDN 16/110	2x16=32		2,19	43,35
Tr-r T2	8xTM 1,6/10	8x1,6=12,8		0,0875	0,41
Line	4xAC-120		4x390=1560	0,21	0,358
Loading					
S_1		10 - j6			
S_2		12 - j5,5			
S_3		3 - j1,5			

Outcomes of account before and after CRC in the distributive electrical network of electric power systems are resulted in table 4.

Table 4

Outcomes of account before and after CRC in the distributive electrical network

Indexes	Before compensation	After compensation				Efficiency of compensation			
		1	*2*	*3*	*4*	*1*	*2*	*3*	*4*
ΔP_{T2}	0,119	0,119	0,119	0,119	0,088	0	0	0	0,031
ΔQ_{T2}	0,556	0,556	0,556	0,556	0,413	0	0	0	0,143
ΔP_{T1}	0,157	0,121	0,118	0,119	0,117	0,039	0,038	0,038	0,04
ΔQ_{T1}	3,115	2,388	2,33	2,358	2,32	0,727	0,785	0,757	0,795
ΔP_l	1,33	1,33	1,036	1,132	1,032	0	0,294	0,198	0,298
ΔQ_l	2,28	2,28	1,77	1,934	1,763	0	0,51	0,346	0,517
$\sum I_{add}$	1560	1560	1560	1560	1560				
$\sum I_l$	1456	1456	1284	1342	1281	0	172	114	175
U_1	356	366	390	382	398	10	34	26	42
S_{T1}	32,66	27,68	27,7	27,51	27,28	4,98	4,96	5,11	5,38

S_{T_2}	12,05	12,05	12,05	12,05	10,2	0	0	0	1,75
$\sum \Delta\Delta P$ MVt						0,039	0,332	0,236	0,369
$\sum_{\Delta\Delta W} = \sum_{\Delta\Delta} \mathrm{P} \cdot \tau \cdot 10^3$ кVt.h. $(\tau = 4500)$						177,5	1494	1062	1660
$C_{\Delta\Delta W} = \sum_{\Delta\Delta} W \cdot \beta \cdot 10^3\ dol.$ $\left(\beta = 0{,}07\ \frac{\mathrm{dol}}{\text{кVt.h}}\right)$						12,42	104,58	74,34	116,23
$C_{CD} = C_{ко} \cdot Q_k \cdot 10^3\ dol.$ $\left(C_{ко} = 20 \cdot 10^3\ \frac{\mathrm{dol.}}{\mathrm{MVAr}}\right)$						200	200	200	200
$Э_y = \left(C_{\Delta\Delta W} - \frac{\mathrm{C_{CD}}}{\mathrm{n_l}}\right) \cdot 10^3 dol.$ $(n_l = 10\ year)$						-7,58	84,6	54,34	96,2
$T_{pb} = \frac{C_{CD}}{C_{\Delta\Delta W}}$ $year$							1,93	2,6	1,72

It is observed following alternatives of disposing CD in knots:

1) $Q_{К3} = 10{,}5\,MVAr$

2) $Q_{К2} = 10\,MVAr$

3) $Q_{К3} = 5MVAr \quad Q_{К2} = 5\,MVAr$

4) $Q_{К2} = 5\,MVAr \quad Q_{К1} = 5\,MVAr$

From the analysis of outcomes of account follows, that in a typical supply net *10-0,4 kV* on everyone installed *1MVAr* BSC the economy on losses (100-160 thousand *kVt.h* is gained. The electric power in a year, with pay-back period BSC 2-2,6 years. Thus, loading of transformers decreases more than on 15 %, and lines on 12 %. Voltage levels in the most remote knot raises on 7-11 %. The Most effective is installation CD more close to users, especially on voltage 0,4 sq. Installation CD in the accepted level will ensure system of an electric-power supply of a supply net with necessary economic power in a knot of association of a supply net, with *tgφ=0,21* which before compensation made *tgφ=0,6*. Installed in supply net CD makes, $\frac{10MVAr}{25MVt} = 0{,}4\,\frac{MVAr}{MVt}$ that matches to an average value of equipment CD for supply nets.

VI. CONCLUSIONS

1. On the basis of technical-and-economic indexes and investment possibilities of an electric power system it is given a technique of sampling and disposing of devices a reactive power compensation in the distributive electrical network, raising effectiveness and regime reliability of electric power systems at the expense of decrease of losses of the electric power.

2. For reaching of the greatest efficiency it is installed sequence of installation CD in the electrical network of electric power systems with use of the analysis on the basis of factor design of experiments and the adequate equations of a regression for parameters of efficiency CRP are accordingly gained. From the analysis of the matching equations are defined sequence of installation CD in 12 knots of electric power systems.

3. Outcomes of implementation of the offered technique for a typical supply net has shown, that on everyone installed *1MVAr* BSC the economy on 100-160 thousand losses *kVt.h* is gained. The electric power in a year, with pay-back period BSC 2-2,6 years. Installed in supply net CD makes, $0{,}4\,MVAr/MVt$ that matches to an average value of equipment CD for supply nets.

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CONSTRUCTION OF HIERARCHICAL CLASSIFICATION BY SIMILARITY MATRIX

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ABSTRACT

In this paper a problem of hierarchical classification of some objects by similarity matrix solution is solved. This approach gives single solution of classification problem. Each hierarchical level is defined by some critical value of a similarity. Using critical value the similarity matrix is transformed into contiguity matrix of some no oriented graph in which connectivity components are constructed. Increasing successfully critical values it is possible to define hierarchical classification of initial objects. This approach is closely connected with reliability theory and mathematical statistics in which a reaching of critical value is one of important problems.

1 INSTRUCTIONS

A problem of a classification by a matrix of pair similarities (differences) between objects is widely used in data processing (Aivazian, Enukov, Meshalkin, 1983). These problems usually are considered as a solution of some minimum – maximum problem in which a similarity between objects in a class is maximized (difference between objects in a class are minimized) and a similarity between classes is minimized (difference between classes is maximized). One of disadvantages of such a statement of a question is no uniqueness of this problem solution and difficulties with an enumeration of all possible solutions.

But last years in different applications a tendency to solve classification problem in a connection with a construction of hierarchical classification intensifies. Such statement of a question increases demands to a uniqueness of classification problem. In this paper the uniqueness is reached by a transformation of similarity matrix into zero – one matrix via a comparison of similarity matrix elements with some critical value. This zero – one matrix becomes contiguity matrix of some no oriented graph. Then connectivity components of this graph are constructed. These components are identified with some objects classes. To define hierarchical classification of the objects critical values are increased. As a result these classes are divided into subclasses and so on. In such a way hierarchical classification is constructed by matrix of pair similarities (of pair differences).

Another applied problem of the classification is a definition of upper boundary "supremum" for similarity matrix (of lower boundary "infinum" for difference matrix) of critical values for which classification procedure gives single solution. This problem may be solved using the method of dichotomous division. For this aim on initial step we take a pair of critical values: zero and a maximum of pair similarities (a minimum and a maximum of pair differences).

Consequently hierarchical classification transforms into a definition of connectivity components in some no oriented graph. Known algorithms (Kormen, Leizerson, Rivest, 2004), (Graham, Hell, 1985) of connectivity components construction are based on a search into a depth and into a width. A disadvantage of these algorithms is in repeated calls to earlier considered graph edges in "search tree" and a definition of for the nodes all nodes contiguity with them. In this paper we suggest algorithm which has not these disadvantages. If considered graph is connected this algorithm is similar with algorithm of spanning tree construction (Eppstein, 1999).

2 ALGORITHM OF HIERARCHICAL CLASSIFICATION

Consider n objects and denote μ_{ij} their similarity measure i, j, $i \neq j$. Then similarity matrix between these objects is $M = \| \mu_{ij} \|_{i,j=1}^{n}$. It consists of nonnegative numbers with $\mu_{ii} = \overline{m}$, where $\overline{m} = \max_{1 \le i \neq j \le n} \mu_{ij} + 1$. To each integer number $k,\, 0 \le k \le \overline{m}$, contrast matrix $M^{(k)} = \| m_{ij}^{(k)} \|_{i,j=1}^{n}$, where $m_{ij}^{(k)} = 1$, if $\mu_{ij} \ge k$, else $m_{ij}^{(k)} = 0$. The matrix $M^{(k)}$ consists of zeros and units and may be considered as contiguity matrix of some no oriented graph $G^{(k)}$ with n nodes which designate initial objects.

In the graph $G^{(k)}$ construct connectivity components $J_1^{(k)}, J_2^{(k)}, \ldots, J_{n(k)}^{(k)}$, so that for any two nodes $i, j \in J_t^{(k)}$ in the graph $G^{(k)}$ there is a way which connect them. If $i \in J_t^{(k)},\, j \in J_l^{(k)},\, t \neq l$ then there are not ways which connect the nodes i, j in the graph $G^{(k)}$. Remark that when we transit from k to $k+1$ each set $J_i^{(k+1)}$ completely contains to some set $J_j^{(k)}$ or does not intersect with it.

Consequently the subsets $J_1^{(k)}, J_2^{(k)}, \ldots, J_{n(k)}^{(k)}$ create a decomposition of the set $\{1, \ldots, n\}$ into classes by the levels $k,\, 0 \le k \le \overline{m}$,

$$\{J_1^{(0)}, J_2^{(0)}, \ldots, J_{n(0)}^{(0)}\},$$
$$\{J_1^{(1)}, J_2^{(1)}, \ldots, J_{n(1)}^{(1)}\}, \ldots,$$
$$\{J_1^{(\overline{m})}, J_2^{(\overline{m})}, \ldots, J_{n(\overline{m})}^{(\overline{m})}\},$$

in which for any class $J_t^{(k+1)}$ of the level $k+1$ there is the class $J_l^{(k)}$, satisfying the inclusion $J_t^{(k+1)} \subseteq J_l^{(k)}$. Further construct the tree D with the hight $\overline{m}$, its root is the node $J_1^{(0)} = \{1, \ldots, n\}$. On the level $k = 1$ consider the nodes $J_1^{(1)}, J_2^{(1)}, \ldots, J_{n(1)}^{(1)}$ and connect them by edges with the node $J_1^{(0)}$. On the level $k = 2$ consider nodes $J_1^{(2)}, J_2^{(2)}, \ldots, J_{n(2)}^{(2)}$ and connet the node $J_t^{(2)}$ by the edge with the node $J_l^{(1)}$ of the level 1, if there is the inclusion $J_t^{(2)} \subseteq J_l^{(1)}$. This procedure continues to the level $\overline{m}$, on which the set $\{1, \ldots, n\}$ is divided into n one node subsets. To simplify the description of the tree it is possible to replace the inclusion $J_t^{(k+1)} \subseteq J_l^{(k)}$ by the inclusion $J_t^{(k+1)} \subset J_l^{(k)}$. If $J_t^{(k+1)} = J_l^{(k)}$ then the nodes $J_t^{(k+1)}, J_l^{(k)}$ of the tree D are glued.

To construct connectivity components in no oriented graph g with n nodes we use the following algorithm. On the step 1 take the node 1 and construct the connectivity component $K_1^{(1)} = \{1\}$. Assume that on the step $t-1$ the set of nodes $\{1, \ldots, t-1\}$ is divided into connectivity components $K_i^{(t-1)},\, i \in L_{t-1}$:

$$K_i^{(t-1)} \bigcap K_j^{(t-1)} = \varnothing,\ i \neq j,\ \ i, j \in J,$$
$$\bigcup_{i \in L_{t-1}} K_i^{(t-1)} = \{1,\ldots,t-1\}.$$

On the step t consider the next node t and calculate $c_i = \max\limits_{j \in K_i^{(t-1)}} m_{tj}^{(k)},\ i \in L_{t-1}$ and put

$$I = \{i \in L_{t-1} : c_i = 1\},\ K_i^{(t)} := K_i^{(t-1)},\ i \in L_{t-1} / I,$$
$$K_t^{(t)} := \{t\} \bigcup \left[\bigcup_{i \in I} K_i^{(t)} \right],\ L_t := (L_{t-1} / I) \bigcup \{t\}.$$

This means that the classes $K_i^{(t-1)},\ i \in L_{t-1},$ with which the node t is connected by some edges, are aggregated with t into new class $K_t^{(t)}$.

3 NUMERICAL EXAMPLE

Assume that similarity matrix of 15 objects has the form ($\overline{m} = 8$):

$$\begin{pmatrix}
8 & 3 & 2 & 0 & 1 & 1 & 2 & 0 & 2 & 0 & 2 & 1 & 0 & 0 & 1 \\
3 & 8 & 5 & 1 & 1 & 3 & 2 & 0 & 2 & 0 & 3 & 1 & 0 & 0 & 0 \\
2 & 5 & 8 & 2 & 1 & 7 & 7 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 2 & 8 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 8 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 3 & 7 & 0 & 1 & 8 & 7 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\
2 & 2 & 7 & 2 & 2 & 7 & 8 & 1 & 0 & 0 & 0 & 1 & 0 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 8 & 1 & 0 & 5 & 3 & 0 & 0 & 3 \\
2 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 8 & 1 & 5 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 8 & 0 & 0 & 0 & 0 & 0 \\
2 & 3 & 1 & 0 & 0 & 0 & 0 & 5 & 5 & 0 & 8 & 3 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 2 & 1 & 3 & 2 & 0 & 3 & 8 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 8
\end{pmatrix}$$

For any critical level $k = 0, 1, \ldots, 8$ the graph $G^{(k)}$ has the following connectivity components (connectivity components with single element do not repeat on successive levels):
the level $k = 0$: $J_1^{(0)} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$
the level $k = 1$: $J_1^{(1)} = \{1,2,3,4,5,6,7,8,9,10,11,12,14,15\}$, $J_2^{(1)} = \{13\}$
the level $k = 2$: $J_1^{(2)} = \{1,2,3,4,5,6,7,8,9,11,12,14,15\}$, $J_2^{(2)} = \{10\}$
the level $k = 3$: $J_1^{(3)} = \{1,2,3,6,7,8,9,11,12,15\}$, $J_2^{(3)} = \{4\}$, $J_3^{(3)} = \{5\}$, $J_4^{(3)} = \{14\}$
the level $k = 4$: $J_1^{(4)} = \{1\}$, $J_2^{(4)} = \{2,3,6,7\}$, $J_3^{(4)} = \{8,9,11\}$, $J_4^{(4)} = \{12\}$, $J_5^{(4)} = \{15\}$
the level $k = 5$: $J_1^{(5)} = \{2,3,6,7\}$, $J_2^{(5)} = \{8,9,11\}$
the level $k = 6$: $J_1^{(6)} = \{2\}$, $J_2^{(6)} = \{3,6,7\}$, $J_3^{(6)} = \{8\}$, $J_4^{(6)} = \{9\}$, $J_5^{(6)} = \{11\}$
the level $k = 7$: $J_1^{(7)} = \{3,6,7\}$
the level $k = 8$: $J_1^{(8)} = \{3\}$, $J_2^{(8)} = \{6\}$, $J_3^{(8)} = \{7\}$
Then we construct the tree D with the hight 7 with glued nodes: $J_2^{(6)}$ with $J_1^{(7)}$, $J_2^{(4)}$ with $J_1^{(5)}$, $J_3^{(4)}$ with $J_2^{(5)}$ (Fig. 1).

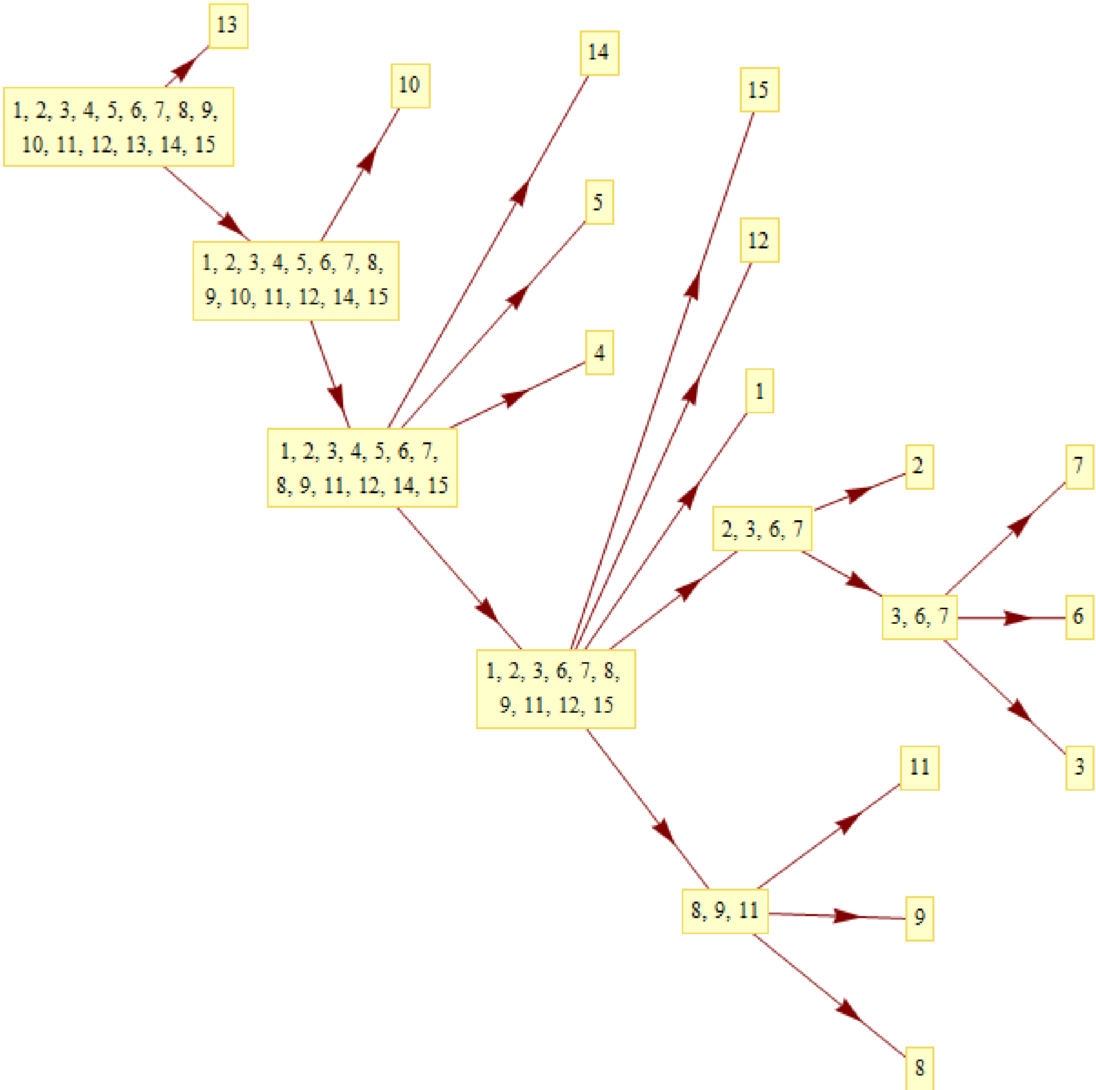

Figure 1. The tree D with the hight 7.

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CURRENT APPROACHES TO ANALYSIS OF THE PROJECT RELIABILITY OF ELECTRONIC DEVICES OF CYCLIC USE

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ABSTRACT

Many electronic devices operate in a cyclic mode. This should be considered when forecasting reliability indicators at the design stage. The accuracy of the prediction and the planning for the event to ensure reliability depends on correctness of valuation and accounting greatest possible number of factors. That in turn will affect the overall progress of the design and, in the end, result in the quality and competitiveness of products.
This study (research grant № 15-05-0029) was supported by the National Research University Higher School of Economics' Academic Fund Program in 2015.

1. Introduction

As stated in [1] for the calculation of reliability, it is necessary to make an accurate identification of the object, its operating conditions and other factors determining the reliability and reproducibility of the most important results of the real model of functioning calculations. There are many methods to estimate the parameters of reliability of electronic devices, among which the main groups are analytical, numerical and statistics.

Standard [2] recommends using of several methods of reliability prediction, which are shown in Table 1.

Table 1. Comparison of reliability calculating methods

№	Method	Distribution reliability requirements /goals	Qualitative analysis	Quantitative analysis	Recommen dations
1	Predicting failure rate	Applicable for sequential systems without redundancy	Can be used to analyze the maintenance strategy	The calculation of the failure rate and MTTF* for electronic components and equipment	Support
2	Fault tree analysis	Applicable if the behavior of the system depends on the time and sequence of events	An analysis of the combination of faults	Calculation of indicators of dependability and efficiency and of the relative contribution of the subsystems in the system	Applicable
3	Event Tree Analysis	Possible	Sequence analysis of failures	The calculation of failure rates	Applicable
4	Analysis of structural reliability schemes	Applicable for systems which can distinguish independent blocks	Analysis of ways of working capacity	Calculation of performance and dependability of systems complex reliability indices	Applicable

№	Method	Distribution reliability requirements /goals	Qualitative analysis	Quantitative analysis	Recommen dations
5	A Markov analysis	Applicable	Analysis of failures Sequence	Calculation of reliability indices and of complex systems dependability indices	Applicable
6	Analysis of Petri nets	Applicable	Analysis of failures Sequence	Preparing the description of the system for a Markov analysis	Applicable
7	Mode and Effect Analysis (critical) of failure FME (C) A	Applicable for systems which have dominated the single failure	Analysis of the impact of failures	The calculation of failure rates (and their criticality) for the system	Applicable
8	The truth table (the analysis of the functional structure)	Not applicable	Possible	Calculation of reliability indices and of complex systems dependability indices	Support
9	Statistical methods for reliability	Possible	Analysis of the impact of faults	Determination of quantitative estimates of reliability indicators with the uncertainty	Support

Note: Word-notation of the table: "Applicable"- the method is recommended for the solution of the problem; "Possible" - the method may be used for solving the problem, given that it has some disadvantages compared with other methods; "Support" -method is applicable to apart of the problem and can be used to solve the whole problem only in combination with other methods; "Not applicable" - the method must not be used to solve the problem.

Table 1 shows that, for calculating the reliability of electronic devices received widespread methods № 1-5, 8 and 9. At the same time models, built on them, allow the assessment of the reliability for the components in continuous use with a constant failure rate. But in the electronic devices operating in "session" mode there is a lot more difficulties in calculating. One of them is the use of models to predict reliability for redundant group, that is, how to take into account the group redundant periodic cycles of work/storage.

2. The basic principles of the calculation model of reliability for electronic devices

Consider this issue on an example of calculation of reliability of the electronic module, which has a series connection of elements working in the "session" mode.

In this case, the element failure rate function, $\lambda(t)$, is periodic (see. Fig.1).

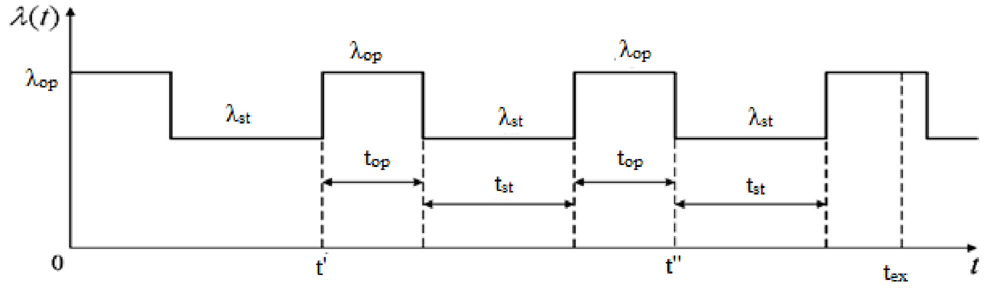

Figure1. Cyclogram of component operation.

In accordance with the general equation for determining the probability of failure-free operation of the component:

$$P(t) = e^{-\int_0^t \lambda(\tau) d\tau}\,, \tag{1}$$

where:

$$\lambda(\tau) = \begin{cases} \lambda_{op}, for\ t' < t < t' + t_{op} \\ \lambda_{st}, for\ t'' < t < t'' + t_{st} \end{cases}.$$

t=t_{ex} = the set time of existence.

If the calculation is performed on integer work and storage plots, the ratio of the estimated probability of failure-free operation of the component is:

$$P(t) = e^{-(\lambda_{op} \cdot T_{op} + \lambda_{st} \cdot T_{st})}, \tag{2}$$

where: $T_{op} = m \cdot t_{op}$ = cumulative operating time (for the time t_{ex}) for all m working areas;
$T_{st} = m' \cdot t_{st}$ = cumulative storage time (for the time t_{ex}) for all m' working areas;

$$m' = \begin{cases} m \\ m-1 \\ m+1 \end{cases}$$

- number of areas depending on whether the area (work or storage) starts and ends the time interval t_{ex}.

The reliability graph constructed by the equation (1) is continuous, but have breaks at the points of the t' and t'' in which the failure rate function has a jump.

In cases where calculation is to be made on a predetermined number m of work areas and storage areas obtain:

$$P\left[m\left(t_{op} + t_{st}\right)\right] = e^{-m(\lambda_{op} \cdot t_{op} + \lambda_{st} \cdot t_{st})}, \tag{3}$$

where: $t_{ex} = m\ (t_{op} + t_{st})$.

The calculations then resorted to the determination of the average (per period of work and storage) component failure rate, which is obtained from the following equation:

$$\lambda_{av} = \frac{\lambda_{op} \cdot t_{op} + \lambda_{st} \cdot t_{st}}{t_{op} + t_{st}}\,, \tag{4}$$

where:
$\lambda_{op} \cdot t_{op}$ = the proportion of the influence of the failure rate in operation mode for the period;
$\lambda_{st} \cdot t_{st}$ = the proportion of the influence of the failure rate in the storage mode for the period;
$t_{per} = t_{op} + t_{st}$ = the period.

Probability of failure-free operating is calculated by the model 2.

As an example, consider the assembly of electronic device, which has a single unloaded reserve (see Fig.2) having the following inputs:

1. Assembly is working in sessions on the amount of in work mode for 32,000 hours and in the storage mode for 55600 hours. The two components are the same and have the following parameters:
   - The failure rate during work mode (λ_{op}) $1.232992 \cdot 0^{-6}$1/h.;
   - The failure rate in the storage mode (λ_{st}) $2{,}194 \cdot 10^{-8}$ 1/h.

2. The failure criterion is the following:

- In the work mode the main element 1 is working, a reserve element 2 is switched off; in case of failure the second element switches on;

- In storage mode, both elements are disconnected from the power supply and it is necessary to consider whether the first element will work after turning on or not. If not, a second element will turn on (both elements together are in loaded reserve with failure rate parameter for storage mode).

Consider a few ways to calculate:

1. Using a model for calculating the probability of failure-free operation for redundant groups from [2], [3] or the scientific literature, and the parameter of the failure rate is determined by the model (4), periods of work and storage are known. According to [2] - a method №4;

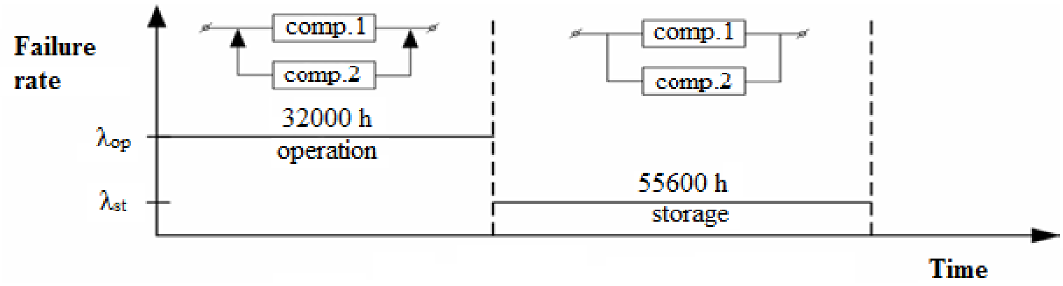

Figure 2. The block diagram of the reliability of the product

2. Construction of a model for calculating the probability of failure-free operation by a method of search of hypotheses based on the total probability formula [4] in accordance with the time schedule specified in [1], where the main parameters will be the failure rate in session mode and storage mode and time intervals. According to [2] - a method №5;

3. The method of decomposition of the time is used – a partition model for calculating the probability of failure-free operation in accordance with a predetermined schedule of operation for the work mode and storage mode. For each of the segments is selected model to calculate the probability of failure-free operation from a standard set or other sources provided that the probability of failure-free operation of a storage mode is determined basing on failure criterion at the time of switching. According to [2] - a methods №1 and 4;

4. Building a model for calculating the probability of failure-free operation using the Monte Carlo method, where the main parameters are the failure rate in the "session" mode, in work mode and in the mode of storage, storage and work periods. According to [2] – a method №9.

Now consider each method in more detail.

When using the first method, choose model for calculating the probability of failure-free operation, based on the criteria of failure for the entire service life of 87600 hours:

$$P_1(t) = \frac{\prod_{j=0}^{m}(n+j\alpha)}{\alpha^m m!} \sum_{i=0}^{m}(-1)^i \frac{C_m^i}{n+i\alpha} e^{-(n+i\alpha)\lambda_{av}t}\,, \qquad (5)$$

where: n = the number of basic components (in this case 1) and m = redundant components (in this case 1);
$\alpha = \frac{\lambda_{st}}{\lambda_{op}}$ - coefficient of proportionality;
λ_{op}, λ_{st} = the failure rate of components in the work and storage modes.

The result of the calculation is following value of the probability of failure-free operation over the lifetime (87600 hours): 0.999180711554146.

The second method involves the output probabilities of all scenarios that lead to the operation at the end of the period of exploitation on the basis of the above criteria of a failure of the electronic assembly, of temporary work schedule (see. Fig. 2), the list of incompatible successful hypothesis [1]. As a result, get the following model:

$$P_2(t) = e^{-\lambda_{av}t_\gamma} + \int_0^{t_\gamma} \lambda_{av} \cdot e^{-\lambda_{av}\cdot t} \cdot e^{-\lambda_{st}\cdot t} \cdot e^{-\lambda_{av}(t_\gamma - t)} dt \qquad (6)$$

As seen from the mathematical model (6), it takes into account all aspects of the assembly, including the time schedule, failure criteria and a transition of component 2 from storage mode to work mode dependent on status of the first component. Thus, this model will be considered as "Pareto standard" for evaluating the error.

As a result of the calculation get the following value of the probability of failure-free operation over the lifetime: 0.999157335573541.

The third method is analytical calculation by using method of temporal decomposition, it means to estimate separately the probability of failure-free operation of the electronic device for the time of work and time of storage, considering the event of failure at any stage of exploitation independent. In this case, the calculation is divided into three stages:

- The first phase is calculated probability of failure-free operation for the structural scheme of electronic device in work mode for a single period of work;
- The second phase describes the structure of the electronic device in storage mode and similarly the probability of failure-free operation on a single period of storage is estimated;
- At the final stage estimates the probability of failure-free operation for electronic device for the entire exploitation period, taking into account the amount of work and storage periods.

In general, the design equation for determining the probability of failure-free operation over the exploitation period is as follows:

$$P_{gen} = \prod_{i=1}^{m} P_{st}(\tau_{sti}) \cdot \prod_{j=1}^{n} P_{op}(\tau_{opj}) \,, \tag{7}$$

where: P_{gen} = general probability of failure-free operation for system; P_{st} = function of the probability of failure-free operation of the system in storage mode; τ_{sti} = the *i*-th storage interval, h.; P_{op} = function of the probability of failure-free operation of the system in work mode; τ_{opj} = *j*-th interval of work, h.

Using equation (7) in practice quite inconvenient, especially when there is redundancy in the system, which implies a fairly complex function defining the probability of failure of the system in either storage mode or in work mode. In addition, the design phase is usually not know the exact schedule of the electronic device (in general, it may be the case), it is known only to the expected ratio of time of work and time of storage, in this case suggest that the duration of sessions is constant. Based on this and on the use of mathematical models with time-constant failure rates equation (7) is transformed to the following form:

$$P_{gen} = (P_{st}(\tau_{st}))^{n} \cdot (P_{op}(\tau_{op}))^{m} \tag{8}$$

Also, based on the use of the exponential model for failure of electronic devices, intervals of work and storage can be combined:

$$P_{gen} = (P_{st}(\sum \tau_{st})) \cdot (P_{op}(\sum \tau_{op})) = P_{st}(t_{st}) \cdot P_{op}(t_{op}) \tag{9}$$

where: t_{st} = cumulative time of storage, h.; t_{op} = cumulative work time, h.

Equation (9) is valid for a strictly exponential mathematical models of failures of investigated electronic devices. However, in practice (9) is used to estimate the probability of failure-free operation of redundant systems, which failure model no longer corresponds to an exponential form.

Consider the application of (9) at the example of the investigated electronic assembly. It is possible to determine that during storage the failure rates of the main and the reserve component groups are same as both chains are identical and are stored under identical conditions. Thus, during storage, the electronic assembly is a loaded reserve, which probability of failure-free operation is described by the following expression [2]:

$$P_{st} = 1 - (1 - e^{-\lambda_{st} t_{st}})^{2} \tag{10}$$

In mode of work the components of the redundant group are located in different conditions, main chain performs its functions and is under load, while the backup continues to be stored unloaded. In case of failure in the main chain, the backup chain will be loaded. In this case, the system works on the scheme of facilitated reserve, probability of failure-free operation of which is determined by the equation [2]:

$$P_{op} = e^{-\lambda_{op} t_{op}}\left(1 + (1 - e^{-\lambda_{st} t_{st}}) \cdot \frac{\lambda_{op}}{\lambda_{st}}\right) \tag{11}$$

Thus, substituting (10) and (11) into (9) we can determine the overall probability of failure on the entire period of operation. The result of calculation is the following value: 0.99922699998305.

One of the sources of error in this method is that the failures in the storage mode and operation mode considered to be independent, that is only true to a simple linear structural reliability schemes where a failure of any component in any mode is the failure of all electronic devices. In (9, 10, 11) are considered two groups of two parallel elements as independent, that is, order to system is considered not failed enough that to the end of the period of work to keep working capacity of any one component of the two examined groups. This statement is incorrect, since the event of failures of system components in the first and second group are dependent.

Consider the specific of inaccuracy of the assumption of independence of groups of components on an example: the mathematical model (9) finds a workable system in which the first group declined 2 component, the second - 1, while it is physically the same components, and such a situation will lead to failure. That is, in such a calculation deliberately introduced an error, leading to an overestimation of the result, which is unacceptable in assessing of the reliability.

3. The simulation method

An alternative for analytical method is a simulation method, in theory it allows to take into account any correlation between failures. Using the simulation method for calculating the reliability parameters involves the construction of a model describing the operation of the device and the process of its failure over time. To build such a model can be used any programming or simulation language, but the most convenient to use specialized simulation languages already containing the blank for simulation of the systems studied. With regard to this problem is advantageous to use specialized software ASONIKA-K-RES containing a standard model to describe the electronic devices with a complex structure [5].

The tool implements a simulation method for the problem of determining parameters of reliability of electronic equipment with a complex structure and the presence of reconfiguration during operation. To perform simulation formal model that describes all the components of the electronic devices of distribution, failure criteria and possible events in the operation is build. Proceedings of the construction of a simulation model, consider on an example of the investigated system. It represents only 2 components, and has a fairly simple the algorithm performance for descriptive tools of the source language.

Primarily for model announces the laws of the distribution of component failure, in this case, can be declared two laws, each of which is characterized by its failure rate - work and storage. Fig. 3 shows an example of syntax classified exponential distribution laws.

```
distribution  Dis_Save (2,194e-8);
distribution  Dis_Work (1,232992e-6);
```

Figure 3. Announcement of the laws of distribution of failures.

After the announcement of the distribution laws can describe the working components, their formal model builds on the basis of possible states of the components and the laws of distribution of time spent in each state. Moreover component transfer from storage mode to work mode is not considered, as it will be implemented separately. Description of the first component shown in Fig. 4, it has two states corresponding to the storage and work and one operation mode.

```
knot K1
{
state: Fail, Work;
mode: Save,Normal;
startState: Work;
startMode: Normal;

cntrlMode: unDistribution;

tableDistribution:
       | Save       |     Normal    |
Work   |  Dis_Save |     Dis_Work  ;

tableStateChange:
          Normal   | Save
Work     |Fail    |Fail ;

};
```

Figure 4. A formal model of component part 1.

Description of the second component part is somewhat more complicated: it has two modes, one corresponds with the proper operation of the K1 (or component part 1), the second with the failure. Accordingly, in a serviceable K1 during the operation of the K2 (or component part 2) continue to be stored under the same crashed into operation instead, which is reflected in the table of distribution laws (see Fig. 5). In more detail the principles of construction of formal models of components are covered in [6].

```
knot K2
{
state: Fail, Work, Rezerv;
mode: Save,Normal;
startState: Rezerv;
startMode: Normal;

cntrlMode: unDistribution;

tableDistribution:
       | Save            | Normal    |
Work   |  Dis_Save       | Dis_Work |
Rezerv |  Dis_Save       | Dis_Save ;

tableStateChange:
            Normal   | Save
Work       |Fail    |Fail    |
Rezerv     |Fail    |Fail    ;
};
```

Figure 5. A formal model of K2.

Further, into the model introduces a conditional component simulating assembly of electronic tools in general and the failure criterion for it, which is the simultaneous failure of K1 and K2, in the framework of a formal model of the operators can be described as a very simple logical expression (see Fig. 6). This expression represents that the assembly is running if any of the components K1 and K2 are serviceable.

```
function FunctREA
{
return K1|K2;
};
```

Figure 6. The criterion of failure of the electronic equipment.

Now to the model of assembly is necessary to add an event associated with the failure of K1, which should lead to the switching the component K2 to the work mode, if it is serviceable at this time. For these tasks, it provides a formal model of a specialized tool «switch_event», which consists of conditions and reconfiguration actions. In this case, the condition is simple: failure of K1, and the action is only one - to change the mode of operation of K2. In the case of an earlier failure of a component K2 change mode operator just will not have any effect. A formal record of this action is shown in Fig. 7.

```
switch_Event EL_1_fail
( ->K1:Fail & K2:Rezerv )
{

set_state ( K2:Work );

} ;
```

Figure 7. A formal description of the switching of K2.

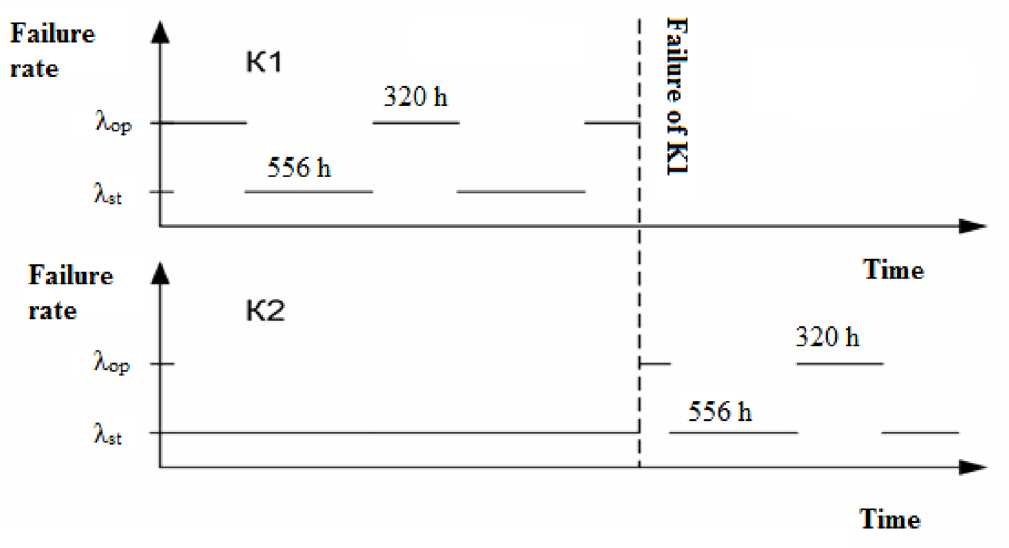

Figure 8. The timing diagram for changes of failure rates of components in the simulation.

After direct describing the structure of electronic assembly it is necessary to start modeling session mode, that is carried out through the introduction into a model additional conditional component of the periodic switching of the components of the state stored in the state of work. Since it is not known the exact real time distribution of work and storage of assembly, but only their ratio, the period of turning on and off are selected on the base that they are much smaller (by at least two orders of magnitude) than the operation period and maintaining overall ratio and time of storage and work. Based on these conditions, we assume that the duration of the work period is 320 hours and 556 hours of storage period. Thus we obtain a uniformly distributed the storage and work areas over the period of operation (see. Fig. 8).

Call the switch component SK, and announce its corresponding distributions (see Fig. 9). In this formal model it turns out that SK is fixated between the two states, while in each of which a constant. This technique is a standard for modeling periodic events by means of software ASONIKA-K-RES and it allows you to simulate not only the hard-coded time of work/storage, but unlikely individual impact on the analyzed assembly, the main requirement is the availability of information on the distribution law, corresponding to the impact.

```
distribution  Dis_per_Work (const 320);
distribution  Dis_per_Save (const 556);

knot Switcher
{
state: Fail, StW, StS;
mode: Normal;
startState: StW;
startMode: Normal;

cntrlMode: unDistribution;

tableDistribution:
       | Normal       |
StW    | Dis_per_Work |
StS    | Dis_per_Save ;

tableStateChange:
       | Normal      |
StW    | StS         |
StS    | StW         ;

};
```

Figure 9. Model optional assembly SK.

Declaring assembly SK, which is a timer of transition from one state to another, it can be associated with it changes in the functioning of the electronic equipment. This is done as well as regime change of component K2, through operator switch_event, formal description of these events is shown in Fig. 10.

```
switch_Event SwitchOn
( ->Switcher:StW )
{

set_mode ( KR2:Normal );
set_mode ( K1:Normal );
} ;

switch_Event SwitchOn
( ->Switcher:StS )
{

set_mode ( K2:Save );
set_mode ( K1:Save );
} ;
```

Figure 10. Simulation session mode of electric assembly.

The resulting formal model introduced in the software tool ASONIKA-K-RES is compiled and checked for compliance with the functioning of the algorithm. Since the model consists of only a few components, the verification process is not difficult and can easily be formed directly by the developer of formal model. Then one can start modeling. Programming model can be subjected to several types of tests - a test on the probability of failure-free operation (for a specific period of operation), and the MTBF. And in the second version obtained statistical data can be used to plot the probability of failure-free operation and of the failure rate of the investigated electronic devices.

The results of calculation of the fourth procedure get the following value is the probability of failure-free operation over the lifetime: 0.9991667.

4. Conclusions

Fig. 11 is a summary histogram of the probability of failure-free operation over the lifetime calculated by 4 ways, which shows that the smallest values provide methods 2 and 4, i.e., they may be used as a lower limit estimation of probability of failure-free operation that is acceptable in terms of calculations [3].

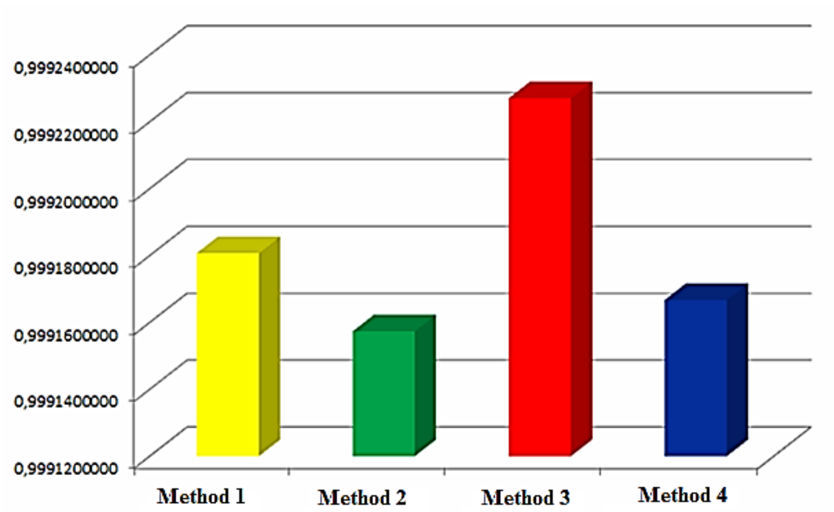

Figure 11. Summary histogram of probabilities of failure-free operation over the lifetime (87600 hours).

As a result, obtain the following values of the probability of failure-free operation over the lifetime (see Table 2). A second method is taken as a reference to calculate the relative error (method of constructing a model based on the equation for the total probability) for reasons which have been described above.

Table 2. Results of calculating the probability of failure-free operation and the relative error

№	Name of the calculation method	A value of probability of failure-free operation for the lifetime for scheme №1: one primary and one reserve	A value of probability of failure-free operation for uptime for the lifetime for the schema №2: 1 main and 2 reserve	The value of the relative error of the probability of failure-free operation		The value of the relative error of probability of failure	
				The scheme №1	The scheme №2	The scheme №1	The scheme №2
1	Method 1 (method of standard structural reliability schemes)	0,999180711554146	0,999988538873878	$2{,}339569532504992\cdot10^{-5}$	$8{,}709636600984844\cdot10^{-5}$	0,027741	0,883701
2	Method 2 (by sorting hypotheses)	**0,999157335573541**	**0,99990145109112**	**0**	**0**	0	0
3	Method 3 (temporal decomposition method)	0,99922699998305	0,999990059235791	$6{,}972316273800213\cdot10^{-5}$	$8{,}861687776787736\cdot10^{-5}$	0,082672	0,899129
4	Method 4 (simulation method)	0,9991667	0,9999876	9,72e-6	8,7057e-5	0,011113	0,883307

As can be seen from Table 2 coincidence between the results obtained by simulation (method 4) and by exact analytical calculation (Method 2) with a very small error (relative accuracy of probability of failure from 0.011113 to 0.883307). But the probability of failure when building a simulation model is minimized (as well as for a simple model) due to special means of verification model, while the probability of failure in the construction of a mathematical model (9) (Method 3) starts to grow because of its complexity and difficulty of verification. Thus, it becomes evident that the simulation can be regarded as an alternative to accurate analytical methods (method of sorting hypotheses) [2] in predicting the reliability of complex electronic devices in terms of types of redundancy and get a more accurate estimate of reliability parameters.

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DERIVATION OF THE DISTRIBUTION FUNCTION FOR THE TAMPERED BROWNIAN MOTION PROCESS MODEL

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ABSTRACT

The tampered Brownian motion process (BMP) arises in the context of partial step-stress accelerated life testing when the underlying system fatigue accumulated over time is modeled by two constituent BMPs, one governing up to the predetermined time point at which the stress level is elevated and the other afterwards. A conditioning argument obtains the probability distribution function (pdf) of the corresponding time-to-failure random variable. This result has been reported and studied in the literature, but its derivation has not been published.

1 INTRODUCTION

The tampered BMP (Bhattacharyya 1987, Lu & Storer 2001) arises in the context of partial step-stress accelerated life testing when the underlying system fatigue accumulated over time t, $B(t)$, is modeled by two separate BMPs, one applicable before the stress level is elevated at a predetermined fixed time point τ and the other afterwards (assuming that an item under test has not failed by time τ). Specifically, let

$$B(t) = \begin{cases} B_1(t), & t \leq \tau \\ B_1(\tau) + B_2(t - \tau), & t > \tau, \end{cases} \tag{1}$$

where $B_i(t) = B_i(t;\eta_i,\delta)$, i = 1, 2, are independent BMPs with positive drifts η_i and a common diffusion parameter δ^2, and the system fails when $B(t)$ first attains a critical threshold value ξ. The ordering $\eta_2 > \eta_1$ ensures that fatigue accrues relatively faster at the higher stress value.

A primary impetus for prescribing the representation (1) is its plausible physical basis. Additionally, the corresponding single stress setting problem is known to yield the prominent inverse Gaussian (IG) distribution for the first passage time of the BMP with respect to a critical boundary (Shrödinger 1915, Smoluchowski 1915, Tweedie 1945). The IG pdf accommodates a spectrum of shapes, adheres to the structure of an exponential family, and supports well-developed statistical inference procedures (Folks & Chhikara 1978). It has been applied extensively in the modeling of reliability, fatigue life, and long-tailed phenomena (Chhikara & Folks 1977, Bhattacharyya & Fries 1982b, Seshadri 1999). The IG pdf and cdf take the forms:

$$g(t) = g(t;\mu,\lambda) = \sqrt{\frac{\lambda}{2\pi t^3}}\, exp\left[\frac{-\lambda(t-\mu)^2}{2\mu^2 t}\right], \tag{2}$$

$$G(t) = G(t;\mu,\lambda) = \Phi\left(\sqrt{\frac{\lambda}{t}}\left(\frac{t}{\mu} - 1\right)\right) + exp\left(\frac{2\lambda}{\mu}\right)\Phi\left(-\sqrt{\frac{\lambda}{t}}\left(\frac{t}{\mu} + 1\right)\right),$$

denoting the mean as μ, the shape parameter as λ, and the standard N(0,1) cdf as $\Phi(\cdot)$.

Doksum & Hóyland (1992) examine variable accelerated life testing experiments for which the time-to-failure distribution is expressed in terms of linear time-transformed IG distribution functions, a construct that is incompatible with (1). Both Lu & Storer (2001) and Doksum & Hóyland (1992) employ a common characterization of the failure time: $T = \inf\{t: B(t) > \xi\}$. Different representations of $B(t)$, however, lead to distinct pdfs. Based on (1), Lu & Storer (2001) report the pdf for their tampered BMP model to be:

$$f(t) = \begin{cases} \sqrt{\frac{\lambda}{2\pi t^3}}\, exp\left\{-\frac{\lambda t c^2(\mu_1,t)}{2}\right\} & t \leq \tau, \\ \sqrt{\frac{\lambda}{2\pi t^3}}\, exp\left\{-\frac{\lambda t}{2}\left[c^2(\mu_1,t) + \frac{t}{\mu_2^2}c(\tau,t)\right]\right\} s(t), & t > \tau, \end{cases} \tag{3}$$

specifying $\lambda = \xi^2/\delta^2$, $\mu_i = \xi/\eta_i$, for i = 1, 2, $c(a,b) = (1/a - 1/b)$ for a, $b \neq 0$, $\Delta = \tau \cdot c(\mu_2,\mu_1)$, $s(t) = q(t,\Delta+1,\lambda) - q(t,\Delta-1,\lambda)$, and $q(t,a,\lambda) = a\cdot exp(½a^2\lambda c(\tau,t))\cdot\Phi(a(\lambda c(\tau,t))^{½})$. On the interval $(0,\tau]$, $f(t)$ matches $g(t)$, the IG pdf given in (2), with parameters μ_1 and λ. For larger values of t, $f(t)$ incorporates μ_2 from $B_2(t)$ and takes on an unwieldy form. Lu & Storer (2001) establish numerous properties of (3): $f(t)$ is continuous and may be either unimodal or bimodal; all positive integer moments exist; and maximum likelihood estimators are unique with probability tending to 1, are strongly consistent, and are asymptotically normally distributed.

Lu and Storer (2001) state that (3) was obtained *after* Bhattacharyya (1987) and attribute the derivation to Bhattacharyya – contradicting Bhattacharyya (1987, p. 156): "The distribution … *has been derived* by using a conditioning approach *which led to* a closed form expression for the pdf" *[emphasis added]*. I derived the tampered BMP pdf (Fries 1982) while awaiting my PhD defense. Gouri Bhattachatyya, my advisor, posed the problem to me (Bhattacharyya 1982) and shortly thereafter crafted a skeleton of a draft manuscript (Bhattacharyya & Fries, 1982a) streamlining portions of my exposition and introducing the exact parameterization (3). Section 2 below details the approach taken in the derivation.

2 PDF DERIVATION

Two lemmas support the development of (3). Both were obtained from first principles in 1982, but at present it suffices to cite published sources. Lemma 1 establishes the probability that a BMP in the future will attain a particular value, given that it earlier had reached a specified point at some prescribed instance in time – a fundamental probability arising naturally in the context of conditioning arguments. Lemma 2 simplifies certain integral expressions involving exponential functions.

Lemma 1 (Wang & Pötzelberger 1997, Eq. (2)). Let $B^*(t) = B^*(t;\eta,\delta)$ be a BMP with positive drift η and variance δ^2, and let τ, a, and b be positive constants. Then, independent of η,

$$P\left[\sup_{s\,\epsilon\,[0,\tau]} B^*(s) \geq a \mid B^*(\tau) = b\right] = \begin{cases} exp\left(\dfrac{-2a(a-b)}{\delta^2\tau}\right) & if\ b < a, \\ 1 & if\ b \geq a. \end{cases}$$

Lemma 2 (Gradshteyn & Ryzhik 2007, pp. 365 & 1030). Let $f(\cdot) = \Phi'(\cdot)$ denote the standard normal pdf. For $\alpha > 0$,

$$I(\alpha,\beta) \equiv \int_0^\infty \vartheta \cdot exp\left(-(\alpha\vartheta^2 + \beta\vartheta)\right) \mathrm{d}\vartheta = \frac{1}{2\alpha}\left(1 - \left(\frac{\beta}{\sqrt{2\alpha}}\right)\frac{\Phi\left(-\frac{\beta}{\sqrt{2\alpha}}\right)}{\phi\left(-\frac{\beta}{\sqrt{2\alpha}}\right)}\right).$$

Derivation of (3). On the time interval $(0,\tau]$, it is straightforward to determine the associated component of the cdf $F(t) = \mathrm{P}[T \le t]$. For a fixed $t \le \tau$, $B(t) = B_1(t)$ and

$$P[T \le t] = P\Big[\sup_{s\,\epsilon\,[0,t]} B(s) > \xi\Big] = P\Big[\sup_{s\,\epsilon\,[0,t]} B_1(s) > \xi\Big] = G(t;\mu_1,\lambda).$$

For the non-trivial case, $t > \tau$, the derivation proceeds by conditioning on $B_1(\tau)$ and invoking the independence of $B_1(\cdot)$ and $B_2(\cdot)$:

$$P[T \le t] = P[T \le \tau] + P[\tau < T \le t]$$

$$= P[T \le \tau] + P\left[\Big[\sup_{s\,\epsilon\,[0,\tau]} B(s) < \xi\Big] \cap \Big[\sup_{s\,\epsilon\,(\tau,t]} B(s) \ge \xi\Big]\right]$$

$$= P[T \le \tau] + \int_{-\infty}^{\infty} P\left[\left(\Big[\sup_{s\,\epsilon\,[0,\tau]} B(s) < \xi\Big] \cap \Big[\sup_{s\,\epsilon\,(\tau,t]} B(s) \ge \xi\Big]\right) \mid B_1(\tau) = b\right] f_{B_1(\tau)}(b)\mathrm{d}b \qquad (4)$$

$$= P[T \le \tau] + \int_{-\infty}^{\infty} P\left[\left(\Big[\sup_{s\,\epsilon\,[0,\tau]} B_1(s) < \xi\Big] \cap \Big[\sup_{s\,\epsilon\,(0,t-\tau]} B_2(s) \ge \xi - b\Big]\right) \mid B_1(\tau) = b\right] f_{B_1(\tau)}(b)\mathrm{d}b$$

$$= P[T \le \tau] + \int_{-\infty}^{\infty} P\Big[\sup_{s\,\epsilon\,[0,\tau]} B_1(s) < \xi \mid B_1(\tau) = b\Big] \cdot P\Big[\sup_{s\,\epsilon\,(0,t-\tau]} B_2(s) \ge (\xi - b)\Big] f_{B_1(\tau)}(b)\mathrm{d}b.$$

Lemma 1 enables the first term appearing in the final integrand to be evaluated directly, and effectively restricts the upper limit of the integral to be ξ. The second element in the integrand is recognized to be an IG cdf, $G(t-\tau;(\xi-b)/\eta_2,(\xi-b)^2/\delta^2)$. Note that this is the only factor in (4) that involves t. The last component of the integrand can be written as a normal pdf since $B_1(\tau)$ has the distribution $\mathrm{N}(\eta_1\tau,\delta^2\tau)$. Substituting back into (4), rearranging terms, and reparameterizing via the transformation $\upsilon = \xi - b$ yields:

$$F(t) = P[T \le \tau] + \int_0^\infty G\left(t-\tau;\frac{\upsilon}{\eta_2},\frac{\upsilon^2}{\delta^2}\right)\left(1 - exp\left(-\frac{2\xi\upsilon}{\delta^2\tau}\right)\right)\frac{1}{\delta\sqrt{\tau}}\phi\left(\frac{\upsilon-\xi+\eta_1\tau}{\delta\sqrt{\tau}}\right)\mathrm{d}\upsilon,$$

$$f(t) = F'(t) = \int_0^\infty g\left(t-\tau;\frac{\upsilon}{\eta_2},\frac{\upsilon^2}{\delta^2}\right)\left(1 - exp\left(-\frac{2\xi\upsilon}{\delta^2\tau}\right)\right)\frac{1}{\delta\sqrt{\tau}}\phi\left(\frac{\upsilon-\xi+\eta_1\tau}{\delta\sqrt{\tau}}\right)\mathrm{d}\upsilon.$$

Incorporating (2) and expanding the exponential function terms gives:

$$\begin{aligned} f(t) = {} & \frac{1}{2\pi\delta^2}\frac{1}{\sqrt{\tau(t-\tau)^3}} \cdot exp\left(-\frac{1}{2\delta^2}\left[\frac{(\eta_1\tau-\xi)^2}{\tau} + \eta_2^2(t-\tau)\right]\right) \\ & \cdot \int_0^\infty \upsilon \cdot exp\left(-\frac{1}{2\delta^2}\left[\upsilon^2\left(\frac{1}{t-\tau}+\frac{1}{\tau}\right) - 2\upsilon\left(\eta_2-\eta_1+\frac{\xi}{\tau}\right)\right]\right)\left(1 - exp\left(-\frac{2\xi\upsilon}{\delta^2\tau}\right)\right)\mathrm{d}\upsilon \\ = {} & \frac{1}{2\pi\delta^2}\frac{1}{\sqrt{\tau(t-\tau)^3}} \cdot exp\left(-\frac{1}{2\delta^2}\left[\frac{(\eta_1\tau-\xi)^2}{\tau} + \eta_2^2(t-\tau)\right]\right) \\ & \cdot \left\{ I\left(\left[\frac{\frac{1}{t-\tau}+\frac{1}{\tau}}{2\delta^2}\right], -\left[\frac{\eta_2-\eta_1+\frac{\xi}{\tau}}{\delta^2}\right]\right) - I\left(\left[\frac{\frac{1}{t-\tau}+\frac{1}{\tau}}{2\delta^2}\right], -\left[\frac{\eta_2-\eta_1-\frac{\xi}{\tau}}{\delta^2}\right]\right)\right\}. \end{aligned}$$

The precise form of (3) follows by application of Lemma 2 (observing that the first additive term in that result cancels out due to the difference being taken between the two I terms), assimilating the parameter definitions accompanying the initial statement of (3), and routine algebra.

3 DISCUSSION

The original derivation of the pdf for the tampered BMP, over three decades old but hitherto unpublished, has been presented. Extensions to encompass experiments with three or more stress levels conceptually could be developed following analogous conditioning arguments, but cumbersome analytical expressions are encountered, e.g.,

$$\int_0^\infty \Phi(\alpha_1 + \beta_1 x)\, \phi(\alpha_2 + \beta_2 x) dx.$$

This integral does not seem to be representable in a closed form or even a single series expansion; Fayed & Atiya (2014) establish that a related integral can be written as an infinite series of the normalized incomplete Gamma function and the Hermite polynomial. The identical analytical complexity arises when attempting to integrate the *F(t)* expression under (4) to directly obtain the tampered BMP cdf.

Upon reading an early draft of this paper, Nozer Singpurwalla noted that realizations of an underlying BMP with positive drift are not necessarily monotonically increasing. While such a construct plausibly may model many physical phenomena (e.g., when fatigue or degradation can be partially mitigated by regenerative or restorative processes), it would not realistically portray circumstances for which accumulated levels cannot decrease over time. For these situations, he endorsed modeling based on an underlying Wiener Maximum Process (introduced in Singpurwalla 2006), i.e., the customary $B(t;\eta,\delta)$, a BMP with drift $\eta > 0$ and variance $\delta^2 > 0$, would be replaced by $M(t;\eta,\delta) \equiv sup_{0<s\leq t} B(s;\eta,\delta)$. Since the distribution of the first hitting time of a threshold barrier is derived from considerations of the maximum attained value, one obtains the standard IG pdf (2) regardless of whether the phenomenon of interest is modeled by the standard BMP or by its maximum. The derivation of the tampered BMP (3) presented in this paper only considers standard BMPs. It does not account for the prospect that $B_1(t;\eta,\delta)$ and $M_1(t;\eta,\delta)$ are not identical.

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BINARY LAMBDA-SET FUNCTION AND RELIABILITY OF AIRLINE

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ABSTRACT

A definition of binary λ - set function is introduced. It is used for the inspection interval planning in order to limit a probability of fatigue failure rate (FFR) of an airline (AL). A solution of this problem is based on a processing of the result of the acceptance full - scale fatigue test of a new type of an aircraft. Numerical example is given.

1 INTRODUCTION

This paper is really the addition to the previous author paper [1] devoted to the reliability of an aircraft (AC) and presentation [2]. Here we consider the reliability of the process of operation of an airline when after the specified life is reached (retirement time), fatigue failure discovery or fatigue failure takes place a new aircraft is acquired and the operation of airline is continued up to infinity. We consider again the case when for solution of the problem of limitation of fatigue failure rate (FFR) of airline we know the type of distribution function of fatigue life of AC but do know the parameter of this function, for estimation of which we have the result of the acceptance full - scale fatigue test of a new type of an aircraft.

Despite of all the simplicity, the equation $a(t)=\alpha\exp(Qt)$ gives us rather comprehensible description of fatigue crack growth in the interval (t_d,t_c), where (we recall) t_d is the time when a fatigue crack becomes detectable ($a(t_d)=a_d$) and t_c is the time when the crack reaches its critical size ($a(t_c)=a_c$) and fatigue failure takes place. It can be assumed that corresponding random variables $T_d=(\log a_d-\log\alpha)/Q=C_d/Q$ and $T_c=(\log a_c-\log\alpha)/Q=C_c/Q$ have the lognormal distribution because, as it is assumed usually, normal distribution of $\log T_c$ can take place only if either both $\log C_c$ and $\log Q$ are normally distributed or if one of these components is normally distributed while another one is constant. We suppose also, that vector $(X,Y)=(\log(Q),\log(C_c))$ has two dimensional normal distribution with vector-parameter $\theta=(\mu_X,\mu_Y,\sigma_X,\sigma_Y,r)$. It is worth to note, that for the case when a_c and a_d are constants then cdf of C_d is completely defined by the distribution of C_c because $C_d=C_c-\delta$, where $\delta=\log(a_c/a_d)$.

First, we consider solution of the problem for the known distribution parameter and then for the unknown one. Numerical example will be given.

2 PROBABILITY MODEL FOR THE KNOWN θ

Just as in paper [1] for the known θ, there are two decisions: 1) the aircraft is good enough and the operation of this aircraft type can be allowed, 2) the operation of the new type of AC is not allowed. A redesign of AC should be made. In the case of the first decision, the vector $t_{1:n} = (t_1, \ldots, t_n)$, where t_i, $i = 1,...,n$, is the time moment of *i*-th inspection, should be defined also. If θ is known the different rules can be offered for the choice of structure of the vector $t_{1:n}$: 1) every interval between the inspections is equal to the constant $t_{SL}/(n+1)$, where t_{SL} is the aircraft specified life (SL) (the retirement time), 2) the probabilities of a failure in every interval are equal to the same value … In this paper we consider the first approach, but really, our considerations can be applied in more general case when the vector $t_{1:n}$ is defined by two parameters, the fixed t_{SL} and the the number of inspections, n, in such a way that probability of failure tends to zero when n tends to infinity.

For the substantiation of the choice of inspection number we should know FFR and the gain of AL as a functions of vector $t_{1:n}$. For this purpose the process of an operation of AL can be considered as an Markov chain (MCh) with $(n+4)$ states. The states $E_1, E_2, \ldots, E_{n+1}$ correspond to an AC operation in the time intervals $[t_0, t_1), [t_1, t_2), \ldots, [t_n, t_{n+1})$, $t_0 = 0$, $t_{n+1} = t_{SL}$. States E_{n+2}, E_{n+3} and E_{n+4} correspond to the following events : the specified life SL is reached , fatigue failure (FF) or fatigue crack detection (CD) take place. In all these three cases the acquisition of new AC takes place.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	...	0	0	0	0	q_1	v_1
E_2	0	0	u_2	...	0	0	0	0	q_2	v_2
E_3	0	0	0	...	0	0	0	0	q_3	v_3
...	...	...	...	...	...	...	...	...	...	...
E_{n-1}	0	0	0	...	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	...	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	...	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	1	0	0	...	0	0	0	0	0	0
E_{n+3} (FF)	1	0	0	...	0	0	0	0	0	0
E_{n+4} (CD)	1	0	0	...	0	0	0	0	0	0

Figure 1. Matrix of transition probabilities P_{AL}.

In the corresponding transition probability matrix, P_{AL}, let ν_i be the probability of a crack detection during the inspection number i, let q_i be the probability of the failure in service time interval $\left(t_{i-1},t_i\right]$, and let $u_i=1-\nu_i-q_i$ be the probability of successful transition to the next state. In our model we also assume that an aircraft is discarded from a service at t_{SL} even if there are no any crack discovered by inspection at the time moment t_{SL}. This inspection at the end of $(n+1)$-th interval (in state E_{n+1}) does not change the reliability but it is made in order to know the state of an aircraft (whether there is a fatigue crack or there is no fatigue crack). Here it is supposed that fatigue crack is discovered with probability equal to unit if inspection is made in interval (T_d,T_c). It can be shown that

$$u_i=P(T_d>t_i\,|\,T_d>t_{i-1}),\; q_i=P(t_{i-1}<T_d<T_c<t_i\,|\,T_d>t_{i-1}),\; \nu_i=1-u_i-q_i,\; i=1,\ldots,n+1. \quad (1)$$

In the three last lines of the matrix P_{AL} there are three units in the first column, corresponding to renewal of an operation of the airline (the AL operation returns to the first interval). All the other entries of this matrix are equal to zero, see Fig.1.

Using the theory of semi-Markov process with rewards and definition of P_{AL} we can get the vector of stationary probabilities, $\pi=(\pi_1,...,\pi_{n+4})$ which is defined by the equation system

$$\pi P_{AL}=\pi\,,\sum_{i=1}^{n+4}\pi_i=1 \quad (2)$$

and the airline gain

$$g(n)=\sum_{i=1}^{n+4}\pi_i g_i(n)\;, \quad (3)$$

where

$$g_i(n)=\begin{cases}a_iu_i+b_iq_i+c_i\nu_i, i=1,\ldots,n+1,\\ d_i, i=n+2,\ldots,n+4,\end{cases}. \quad (4)$$

a_i is the reward defined by the successful transition from one operation interval to the following one and the cost of one inspection; b_i, c_i and d_i correspond to transition to the states E_{n+3} (FF), E_{n+4} (CD) and then to the state E_1 (the "cost" of FF of AC, fatigue crack detection, acquisition of new AC). Let us note that if $a_i=t_i-t_{i-1}$, $b=c=d=0$ then

$$g(n)=\sum_{i=1}^{n+4}\pi_i g_i(n)=g_t(n)=\sum_{i=1}^{n+1}\pi_i(t_i-t_{i-1}) \quad (5)$$

and $L_j=g_t(n,\theta)/\pi_j$ defines the mean step number of MCh to return to the same state E_j, $\lambda_F(n,\theta)=1/L_{n+3}\Delta_t$, where $\Delta_t=t_{SL}/(n+1)$, is the FFR.

If θ is known we calculate the gain as a function of n, $g(n,\theta)$, and choose the number n_g corresponding to the maximum of the gain :

$$n_g(\theta)=\arg\max_n g(n,\theta). \quad (6)$$

Then we calculate FFR as function of n, $\lambda_F(n,\theta)$, and choose n_λ in such a way that for any $n\ge n_\lambda$ the function $\lambda_F(n,\theta)$ will be equal or less than some value λ:

$$n_\lambda(\lambda,\theta)=\min\left\{n:\lambda_F(n,\theta)\le\lambda,\;\text{ for all } n\ge n_\lambda(\lambda,\theta)\right\}. \quad (7)$$

And finally

$$n=n_{g\lambda}(\lambda,\theta)=\max(n_g,n_\lambda)\,. \quad (8)$$

3. SOLUTION FOR AN UNKNOWN θ

In [1] the problem of a limitation of fatigue failure probability in an operation of one AC (FFP1) was considered using the definition of binary p-set function

3.1. Binary p-set function

Now let us take into account that we consider the case when the for the estimate of unknown parameter θ , $\hat{\theta}=\hat{\theta}(x_1,...,x_n)$, the result of acceptance test is used and the operation of a new type of aircraft will not take place if the result of the fatigue test in a laboratory is "too bad" (previously, the redesign of the new type of AC should be made). We say that in this case the event $\hat{\theta}\notin\Theta_0$, $\Theta_0\subset\Theta$ takes place (for example, $\hat{\theta}\notin\Theta_0$ if the test fatigue life T_C is lower than some limit; or $n(p,\hat{\theta})$ is too large,…).
Let us define some binary set function

$$S(\hat{\theta},\Theta_0,n)=\begin{cases}\bigcup_{i=1}^{n+1}S_i(n) & if \quad \hat{\theta}\in\Theta_0,\\ \varnothing, & if \quad \hat{\theta}\notin\Theta_0\end{cases}, \tag{9}$$

where $S_i(n)=\{(t_d,t_c):t_{i-1}<t_d,t_c\le t_i\}$, $t_i=it_{SL}/(n+1), i=1,...,n+1$; $\varnothing$ is an empty set.
It can be shown that for very wide range of the definition of the set Θ_0 and the requirements to limit FFP1 by the value p^*, where $(1-p^*)$ is a required reliability, there is a preliminary "designed allowed FFP1" , p_{FD}, such that corresponding set function $S(\hat{\theta},\Theta_0,n(p_{FD},\hat{\theta}))$ is *binary p-set function of the level p^* for the vector $Z=(T_d,T_c)$ on the base of the estimate $\hat{\theta}$:*

$$\sup_\theta\sum_{i=1}^{n+1}P(Z\in S_i(n(p_{FD},\hat{\theta}))\bigcap\hat{\theta}\in\Theta_0)=p^* . \tag{10}$$

This means that FFP1 will be limited by the value p^* for any unknown $\theta\in\Theta$.

3.2. Binary λ-set function

In similar way, it can be shown that for very wide range of the definition of the set Θ_0 and the requirements to limit FFR of AL by the value λ^*, where λ^* is a required fatigue failure intensity, there is a preliminary "designed allowed FFR" , λ_{FD}, such that corresponding set function $S(\hat{\theta},\Theta_0,n_\lambda(\lambda_{FD},\hat{\theta}))$ is *a binary λ-set function of the level λ^* for the vector $Z=(T_d,T_c)$ on the base of the estimate $\hat{\theta}$:*

$$\sup_\theta E((\lambda(n_\lambda(\lambda_{FD},\hat{\theta}))\,|\,\hat{\theta}\in\Theta_0)*P(\hat{\theta}\in\Theta_0))=\lambda^* . \tag{11}$$

This means that FFR will be limited by the value λ^* for any unknown $\theta\in\Theta$.

Let us note, that instead of the words a *binary λ-set function* we should use the words *binary λ_g-set function* if instead of $n_\lambda(\lambda_{FD},\hat{\theta})$ we use $n_{g\lambda}(\lambda_{FD},\hat{\theta})$.
For the requirement of a high reliability the choice of an inspection number will be defined by the limitation of FFR. For very high "cost" of FF of AC it will be defined by the maximum of the gain.

4 NUMERICAL EXAMPLE

The example of the solution of the reliability problem of aircraft fleet is considered in [1, 2]. Here we consider only the problem of reliability of AL.

We use the following definitions of the components of an AL income: for all $i = 1,...,n+1$ $a_i = a_0(n) + d_{insp}t_{SL}$, where $a_0(n) = a_{01}t_{SL} / (n+1)$, - is the reward related to successful transition from one operation interval to the following one, a_{01} defines the reward of operation in one time unit (one hour or one flight); $d_{insp}t_{SL}$ is the cost of one inspection (negative value) which is supposed to be proportional to t_{SL} ; $b_i = b_{01}t_{SL}$ is related to FF (negative value), $c_i = c_{01}a_0(n)$ is the reward related to transitions from any state $E_1,...,E_{n+1}$ to the state E_{n+4} (it is supposed to be proportional to a_0 because it is a part of a_0); $d_i = d_{01}t_{SL}$ is negative reward, the absolute value of which is the cost of new aircraft acquisition after events SL, FF or CD and transition to E_1 takes place. In numerical example we have used the following values: $t_{SL} = 40000$, b_{01}=-0.3; d_{insp}= - 0.05; a_{01}=1; c_{01}=0.1; d_{01}=- 0.3.

Suppose we have the following estimate of parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$: $\hat{\theta}$=(-8.58688044, 1.9424608, 0.155, 0.0778895, 0.796437) (see Fig 2.2 and Table 2.1 in [3]). It was assumed that the set Θ_0 corresponds to the decision to make redesign if the estimate of critical time to failure $t_C = \exp(\hat{\mu}_Y - \hat{\mu}_X)$ is too small: $t_C < 0.3t_{SL}$.

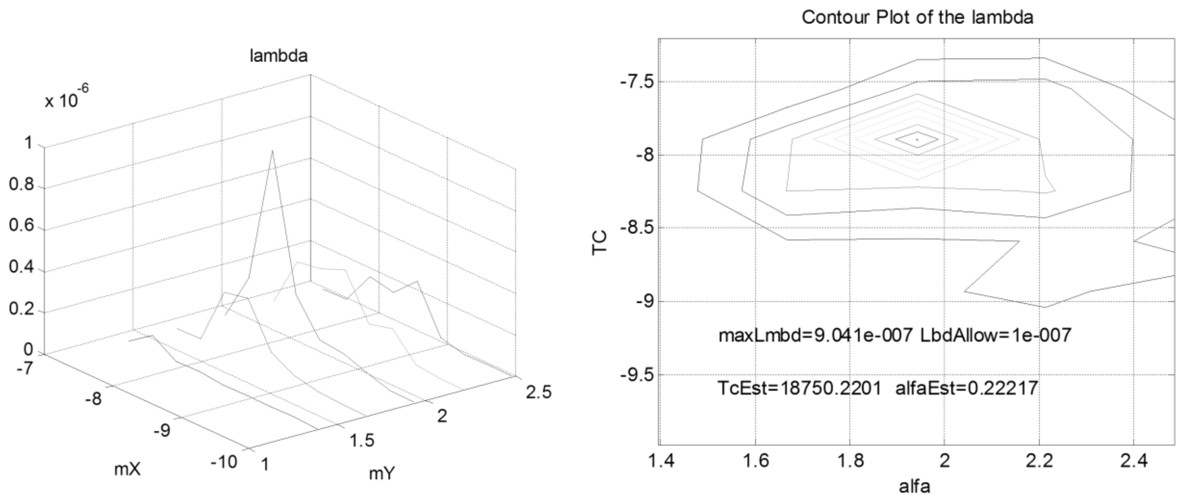

Figure 2. The value of $w(\theta, \lambda_{FD}, \Theta_0)$ for five value of μ_Y $(1.2415 \le \mu_Y \le 2.6435)$ in vicinity of maximum value of $w(\theta, \lambda_{FD}, \Theta_0)$ which is equal to 0.9041*10-6 for (σ_X, σ_Y, r) =(0.155128668, 0.0778895, 0.796437) .

Calculation of $w(\theta, \lambda_{FD}, \Theta_0) = E\{\lambda_F(\hat{\theta}, \lambda_{FD}, \Theta_0)\}$ was made for $(7.2029 \le \mu_X \le 9.9709)$, $(1.3972 \le \mu_Y \le 2.4877)$ assuming that the vector (σ_X, σ_Y, r) is the same for all different vectors (μ_X, μ_Y). It was found that for λ_{FD}=0.1*10-6 the maximum value of $w(\theta, \lambda_{FD}, \Theta_0)$ is equal to 0.9041*10-6 .

Suppose that the value 0.9041*10-6 is required reliability. Then for the known estimate of the parameter the calculation of $n_\lambda(\lambda_{FD}, \hat{\theta})$ for λ_{FD}=0.1*10-6 gives us the required number of inspection. It is equal to 6. For the considered estimate of θ t_C (realization of T_C is equal to

37.4574e+003 so the redesign is not needed. After the necessary calculation of $g(n,\theta)$ it is found $n_g = 4$. So finally, the required number of inspections $n = \max(n_g, n_\lambda)$ is equal to 6.

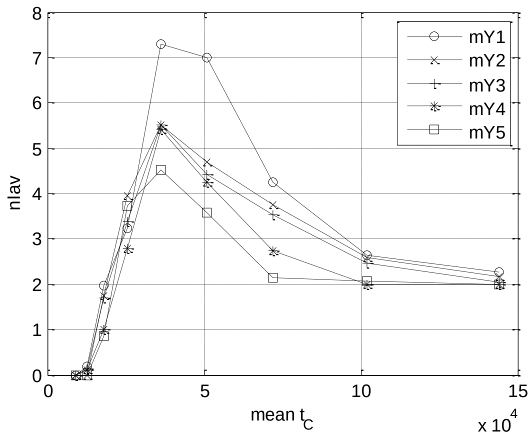

Figure 3. The value of $n_\lambda(\lambda_{FD}, \theta)$ for five value of μ_Y $(1.2415 \le \mu_Y \le 2.6435)$ for λ_{FD} =0.1*10-6 as function of equivalent mean value of T_C which was calculated as $\exp(\mu_Y - \mu_X)$

CONCLUSIONS

The problem of inspection planning on the bases of the result of acceptance full-scale fatigue test of AC structure is the choice of the sequence $\{t_1, t_2, ..., t_n, t_{SL}\}$ providing the limitation of FFR of AL if some requirements to the result of acceptance full-scale fatigue test are met. If these requirements are not met the redesign of the new type of an aircraft should be made.

The definition of binary λ-set function is introduced for description of corresponding mathematical procedure, based on the observation of some fatigue crack during the acceptance full-scale fatigue test of aircraft structure. In general case the the desire to increase the gain of airline service can be taken into account but under condition that required reliability is already provided. The limitation of FFR is provided for **any unknown** parameter of the fatigue crack model. The method of necessary calculation is provided.

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ISSN 1932-2321