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RELIABILITY:
THEORY&APPLICATIONS

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RISK ANALYSIS
MAINTENANCE
SAFETY

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In Memory Of Igor Ushakov

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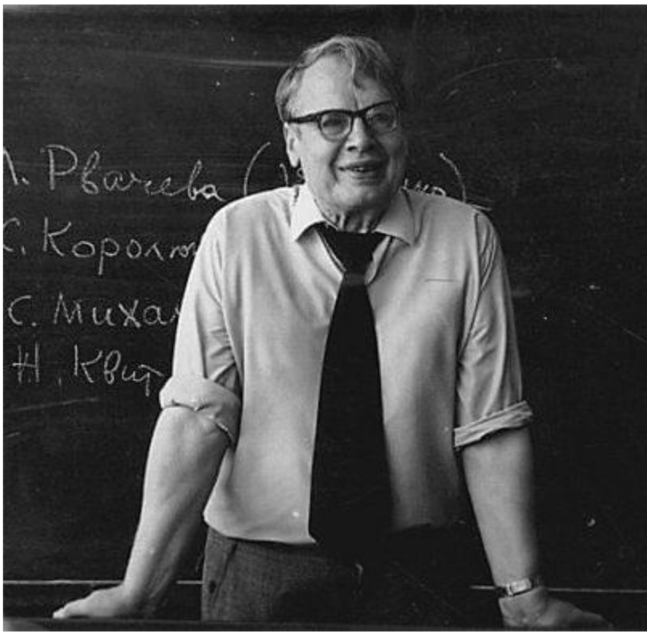

RELIABILITY:

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In many cases, reliability can be improved by using redundant components. This is an approach that is applied especially in information networks. In this paper we study redundant systems with imperfect switches. We show that there exists a limit as the number of redundant components tends to infinity. This limit is computed for components with exponential life time distributions, which is the typical distribution for digital equipment used in information systems. For components with distributions belonging to the NBUE or HNBUE classes, bound are derived.

A. Fries (Panel Chair), W.P. Cherry, R.G. Easterling, E.A. Elsayed, A.V. Huzurbazar, S.V. Wiel, P.A. Jacobs, W.Q. Meeker, Jr., N. Nagappan, M. Pecht, A. Sen, C. Citro, M.L. Cohen, M.J. Siri

This paper, extracted from National Research Council (2015), summarizes the findings and recommendations from a recent report from the Panel on Reliability Growth Methods for Defense Systems, operating under the auspices of the Committee on National Statistics (CNSTAT) within the National Research Council (NRC). The report offers recommendations to improve defense system reliability throughout the sequence of stages that comprise U.S. Department of Defense (DoD) acquisition processes – beginning with the articulation of requirements for new systems and ending with feedback mechanisms that document the reliability experience of deployed systems. A number of these recommendations are partially or fully embraced by current DoD directives and practice, particularly with the advent of recent DoD initiatives that elevate the importance of design for reliability techniques, reliability growth testing, and formal reliability growth modeling. The report supports the many recent steps taken by DoD, building on these while addressing associated engineering and statistical issues. The report provides a self-contained rendition of reliability enhancement proposals, recognizing that current DoD guides and directives have not been fully absorbed or consistently applied and are subject to change.

A. Breznická, A. Chovanec

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M. A. El-Damcese, N. H. El-Sodany

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BOUNDS FOR THE RELIABILITY OF DIFFERENT REDUNDANT SYSTEMS

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ABSTRACT

In many cases, reliability can be improved by using redundant components. This is an approach that is applied especially in information networks. In this paper we study redundant systems with imperfect switches. We show that there exists a limit as the number of redundant components tends to infinity. This limit is computed for components with exponential life time distributions, which is the typical distribution for digital equipment used in information systems. For components with distributions belonging to the NBUE or HNBUE classes, bound are derived.

1. INTRODUCTION

In order to improve the reliability of a system there are mainly two possibilities. The first one is to improve the reliability of the components, the second is to implement redundancy. Mainly this is done by using more than one component to fulfill the same function, see e.g. Barlow & Proschan (1976). Redundancy means that in a technical system there are more possibilities present to ensure a function, than the necessary minimum. If one discards influences as costs and needed space, one might come to the conclusion that using redundant items, one could improve system reliability up to an arbitrarily high level. In this paper we will discuss the problem whether it is possible to improve reliability up to an arbitrary high level. Using redundant components is an approach used mainly in networks, especially in telecommunication networks. If a certain link or node fails, traffic is rerouted to other nodes and links.

In this paper we will show that, under several assumptions, reliability cannot be improved further than to a certain limit.

In section 2 we will describe the main assumptions of our model. In the next two chapters we consider two extremal modes of standby, hot standby and cold standby. Hot standby means that the load on the standby component is the same as on the main component and that no load sharing between the redundant components occurs. Cold standby describes a situation, where the redundant devices do not age at all during their standby phase, i.e. when the main component provides the service. All other modes of standby will describe modes of ageing that are between these two situations of load on the redundant components.

In the third section we describe the situation of hot standby, the worst case regarding ageing.

In the fourth section, we discuss the situation of cold standby, no ageing of the standby components.

Section five provides an example and in section six we give a summary and conclusion.

2. MAIN ASSUMPTIONS

For the model the following assumptions shall hold

a) Detection and switching to another component is not perfect but fails. Here the probability of failure of switching from the failed component to the redundant one includes the failure of the switch itself in case of detection of the failure, the failure of the detection mechanisms when the switch is working as well as failure of both switch and detection mechanisms. This resulting probability is denoted by γ

b) The lifetime of the components is random and follows the lifetime distribution F(x) with F(0) = 0 and

$$\lim_{x\to\infty} F(x) = 1$$

c) The failure times of all redundant components are completely statistically independent from each other.

d) The number of redundant components is not limited.

e) All redundant components have the same lifetime distribution.

f) The lifetime distribution of the components is continuous, differentiable and has a finite mean.

The model has been described in more detail in Shubinsky (2012).

Parallel systems with imperfect switching to redundant components will be called imperfect systems in this paper.

The following figure shows an example of a system with redundant components. Each of the m, possibly different, components has n redundant replications. We will study this type of systems for $n \to \infty$

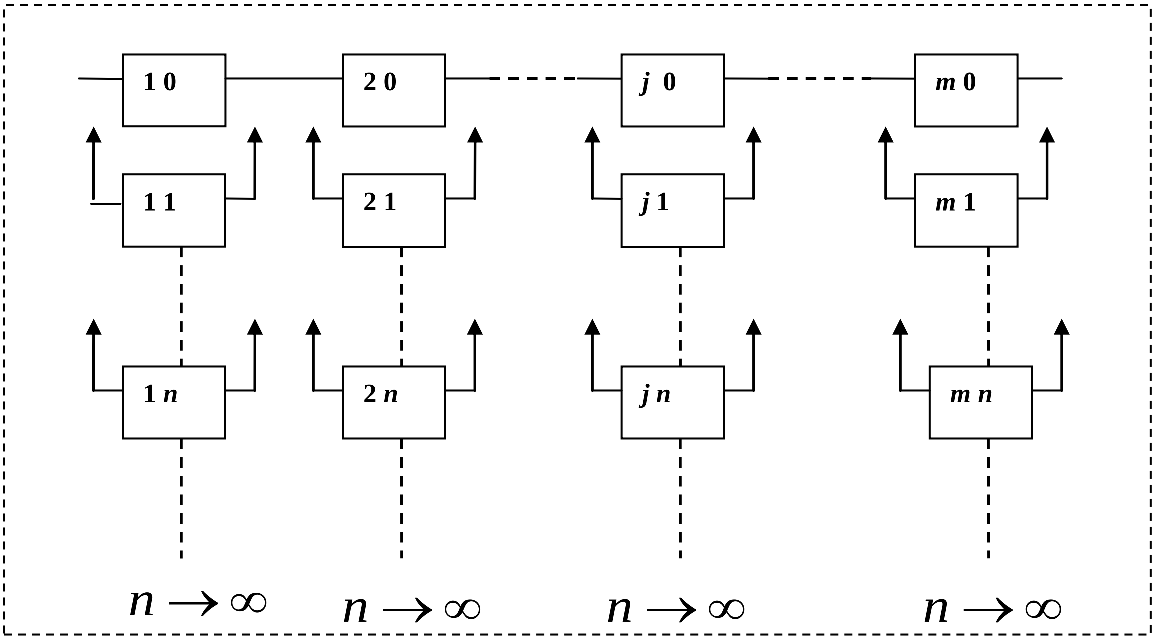

Figure 1. System with redundant components

In the following subsections we will simplify the system in figure 1 by considering only one component with its redundant replications.

3. HOT STANDBY

For hot standby, all components are under full load from the beginning. So this is in fact a situation of a simple parallel system. Assume that a component with lifetime distribution F(x) is connected in parallel with all its replications. The following figure 2 shows the reliability block diagram of the system. Assume that n components are connected in parallel.

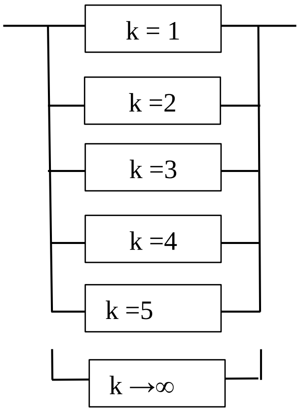

Figure 2. System with parallel structure of components

The lifetime distribution of the parallel system with hot standby can now be computed as follows.

In order to have achieve a redundancy of level k, i.e. that k are components functioning, k-1 successful switchovers are necessary with a failure on the k-th switch-over.

The probability of this event is $(1-\gamma)^{k-1}\gamma$. The distribution function of k identical units with lifetime distribution F(x) and connected in parallel is

$$1-(1-F(x))^k. \qquad (1)$$

Combining both expressions and summing up we arrive at

$$\sum_{i=1}^{k}\gamma(1-\gamma)^{i-1}(1-(1-F(x))^i) \qquad (2)$$

If now k tends to infinity, this gives

$$\sum_{i=1}^{\infty}\gamma(1-\gamma)^{i-1}F(x)^i = \frac{\gamma F(x)}{1-(1-\gamma)F(x)} = G(x), \qquad (3)$$

where G(x) denotes the distribution function of the lifetime of the redundant system.

Note that, the lifetime distribution of the parallel system is given by an analytic expression. Moreover, one can observe that

$$G(x) = \frac{\gamma F(x)}{1-(1-\gamma)F(x)} \le F(x) \qquad (4)$$

which follows easily from

$$\gamma F(x) \le F(x) - (1-\gamma)F(x)^2 \text{ and}$$
$$(1-\gamma)F(x) \ge (1-\gamma)F(x)^2.$$

The latter is obvious since $F(x) \ge F(x)^2$.

Considering (4) one can see that (4) is smaller than the distribution of a single component, but even in the limiting case, the failure probability does not vanish. This is only possible for

perfect switching, i.e. $\gamma=0$. For all positive values of γ which means imperfect switching, G(x) will form a lower bound for all systems with a large but finite number of redundant elements.
Now we can compute the mean lifetime by

$$m_G = \int_0^\infty (1-G(x))dx = \int_0^\infty \frac{1-F(x)}{1-(1-\gamma)F(x)} dx \tag{5}$$

For an exponential distribution, one computes

$$m_G = \int_0^\infty \frac{\exp(-\lambda x)}{1-(1-\gamma)(1-\exp(-\lambda x))} dx = \int_0^\infty \frac{\exp(-\lambda x)}{\gamma+(1-\gamma)\exp(-\lambda x)}) dx = -(1/\lambda) \ln(\gamma) / (1-\gamma). \tag{6}$$

For $\gamma=1$ this gives $1/\lambda$, which is the result for the exponential distribution without redundancy. Again, for imperfect switching, m_G always stays bounded and its value is determined by m_F and γ.

Now, for a function that belongs to the NBUE (new better than used in expectation) or NWUE (new worse than used in expectation) family we can show that an expression as (1) is an upper (lower) bound on the mean value of the distribution function G.

A lifetime distribution function belongs to the class NBUE (NWUE) if it satisfies

$$\int_x^\infty (1-F(t))dt \leq(\geq)\ m_F(1-F(x)) ,$$

where m_F is the mean of F(x), see e.g. Barlow and Proschan (1976)

If now F(x) belongs to the class NBUE (or NWUE) the following inequality holds

$$m_G \leq (\geq) -m_F \ln(\gamma)/(1-\gamma). \tag{7}$$

This result can be proven as follows.

We rewrite (6) in the following form:

$$m_G = \int_0^\infty \frac{1-F(x)}{1-(1-\gamma)F(x)} dx = - \int_0^\infty \frac{d \int_x^\infty (1-F(t))dt}{1-(1-\gamma)F(x)} \tag{8}$$

Integrating this expression by parts, we arrive at

$$m_G = m_F/(1-(1-\gamma)) + \int_0^\infty \int_x^\infty (1-F(t))dt \; d\, \frac{1}{1-(1-\gamma)F(x)} \tag{9}$$

Using the NBUE (NWUE) property this can be rewritten as

$$m_G \leq (\geq)\ m_F/(1-(1-\gamma)) - \int_0^\infty m_F\, (1-F(x))\; d\, \frac{1}{1-(1-\gamma)F(x)} \tag{10}$$

and integrating by parts again

$$m_G \le (\ge)\ m_F \int_0^{\infty} \frac{(1-F(x))dx}{1-(1-\gamma)F(x)}) = -m_F \ln(\gamma)/(1-\gamma) \tag{11}$$

This proves (7).

Using the expression (3), we can derive an inequality for the residual life function T_{RL}. The latter is defined by

$$T_{RL} = \int_x^{\infty}(1-G(t))dt\ .$$

Using (3) we arrive at

$$T_{RL} = \int_x^{\infty}\left(\frac{1-F(t)}{1-(1-\gamma)F(t)}\right)dt = -\int_x^{\infty}\left(\frac{1}{1-(1-\gamma)F(t)}\right)d\int_t^{\infty}(1-F(s))ds\ .$$

Integrating by parts, we get

$$T_{RL} = \frac{1}{1-(1-\gamma)F(x)}\int_x^{\infty}(1-F(t))dt + \int_x^{\infty}\int_t^{\infty}(1-F(s))ds\ d\left(\frac{1}{1-(1-\gamma)F(t)}\right).$$

For a NBUE (NWUE) distribution this leads to

$$-\frac{m_F(1-F(x))}{1-(1-\gamma)F(x)} - \int_x^{\infty} m_F(1-F(t))\ d\left(\frac{1}{1-(1-\gamma)F(t)}\right).$$

Integrating by parts again, this expression equals

$$T_{RL} \le (\ge)\ -m_F\int_x^{\infty}\frac{d(1-F(t))}{1-(1-\gamma)F(t)} = m_F\int_x^{\infty}\frac{dF(t)}{1-(1-\gamma)F(t)} = (m_F/\gamma)\ln\left(\frac{\gamma}{1-(1-\gamma)F(x)}\right)$$

Putting everything together, we arrive at

$$T_{RL} \le (\ge)\ (m_F/\gamma)\ln\left(\frac{\gamma}{1-(1-\gamma)F(x)}\right)$$

For the exponential distribution, the equality holds.

4. COLD STANDBY

The case of cold standby is the other extremal case. Here, the lifetime distribution of a parallel system is computed by

$$G(x) = \sum_{i=1}^{\infty}\gamma(1-\gamma)^{i-1}F^{(i)}(x), \tag{12}$$

where $F^{(i)}(x)$ denotes the i-fold convolution of the distribution function F(x). The convolution is defined by

$F^{(1)}(x) = F(x)$
for the first order convolution, all higher orders are defined iteratively by

$$F^{(k+1)}(x) = \int_0^x F^{(k)}(x-t)dF(t)\ . \qquad (13)$$

Formula (12) is derived from the probability $(1-\gamma)^{i-1}\gamma$ for a failure of the system when the switching to the i-th redundant component and the lifetime distribution $F^{(i)}(x)$ of i successively used components .

For the type of distributions given by (12), a general analytical solution does not exist. However, the following results can easily be obtained.

For an exponential distribution with density $f(x) = \lambda\ \exp(-\lambda x)$ one obtains (see /Shubinski/)

$$G(x) = 1-\exp(-\lambda\gamma x). \qquad (14)$$

If $\gamma=1$ (switching fails always), we arrive at the usual exponential distribution of a single component. The result (9) can be easily derived by using

$$f^{(k)}(x) = \lambda^{k}-1\exp(-\lambda x) /(k-1)! \qquad (15)$$

and computing the density g(x).

Using results of Schäbe (1986), we can also derive other analytical results for special Gamma distributions that have the following form

$$F(x) = \lambda^{\alpha}x^{\alpha-1}\exp(-\lambda x)/\Gamma(\alpha) \qquad (16)$$

The results are given in the following table.

Table 1. density functions g(x) for special types of gamma densities for f(x).

Parameters	**density g(x) of the parallel system**
$\alpha=1/2$	$\gamma\sqrt{\frac{\lambda}{\pi x}}\ \exp(-\lambda x)+\lambda\gamma(1-\gamma)\exp(-\lambda(1-\gamma)^{2}/2)\mathrm{erfc}(-\lambda(1-\gamma)\sqrt{x})$
$\alpha = 1$	$\lambda\gamma\ \exp(-\lambda\gamma x)$
$\alpha = 2$	$\frac{\gamma\lambda}{2\sqrt{1-\gamma}}\ (\exp(-(1-\sqrt{1-\gamma})\lambda x) - \exp(-(1+\sqrt{1-\gamma})\lambda x)$
$\alpha = 3$	$\frac{\lambda\gamma}{(1-\gamma)^{2/3}}\ \Big(\frac{1}{3}\exp(\lambda x(1-\gamma)^{1/3})-\frac{2}{3}\ \exp(-\lambda x(1-\gamma)^{1/3})\cos(\frac{3}{2}\lambda x(1-\gamma)^{1/3}-\pi/3)\Big)$
$\alpha = 4$	$\frac{\lambda\gamma}{2(1-\gamma)^{3/4}}\ \exp(-\lambda x)(\sinh(\lambda(1-\gamma)^{1/4}x- \sin(\lambda(1-\gamma)^{1/4}x)$

Also, it has been shown in Schäbe (1986), that

$$m_G = m_F/\gamma. \qquad (17)$$

Therefore, no approximation for m_G needs to be given.

One may note, that the mean is limited, even if the number of redundant devices becomes infinite. The distribution function G(x) has no closed form expression in the general case. So, it is worthwhile to have a bound on it. In Schäbe (1986) it has been shown in theorem 3.2 that if F belongs to the class NBUE (NWUE), the same holds for G. An analogous result has been proven for the class HNBUE (harmonic new better than used in expectation) and HNWUE (harmonic worse than used in expectation) in theorem 3.4. The latter result can be used to give a bound on G. If F is HNBUE (HNWUE), we have for the distribution G the following inequality for the residual life function, see Klefsö (1982)

$$\int_x^{\infty}(1-G(t))dt \le (\ge)\ m_G \exp(-x/m_G) = (m_F/\gamma) \exp(-\gamma x/m_F) \qquad (18).$$

Also this expression shows, that an infinite number of redundant devices is not able to improve the residual life function further than to a certain value. For HNBUE distributions, we derived an upper bound on an infinitely increasing number of redundant devices.

5. EXAMPLE

In this section we will show how the mean lifetime depends on the number of components used for redundancy and how it depends on the probability γ of failure of switching for a cold standby system.
From (5) we have.

$$G(x) = 1-\exp(-\lambda\gamma x).$$

For a system as in figure 1 consisting of m components connected in series each having k redundant replications this gets

$$G(x) = 1-\exp(-\lambda\gamma k x).$$

This distribution has mean $1/(\lambda\gamma k)$. Now the relative mean of the system with redundancy over a system consisting of one element with failure rate λ is

$$R = 1/(\gamma k).$$

Let us now denote by $\alpha=1-\gamma$ the probability that detection of a fault and switching to the redundant component is successful.

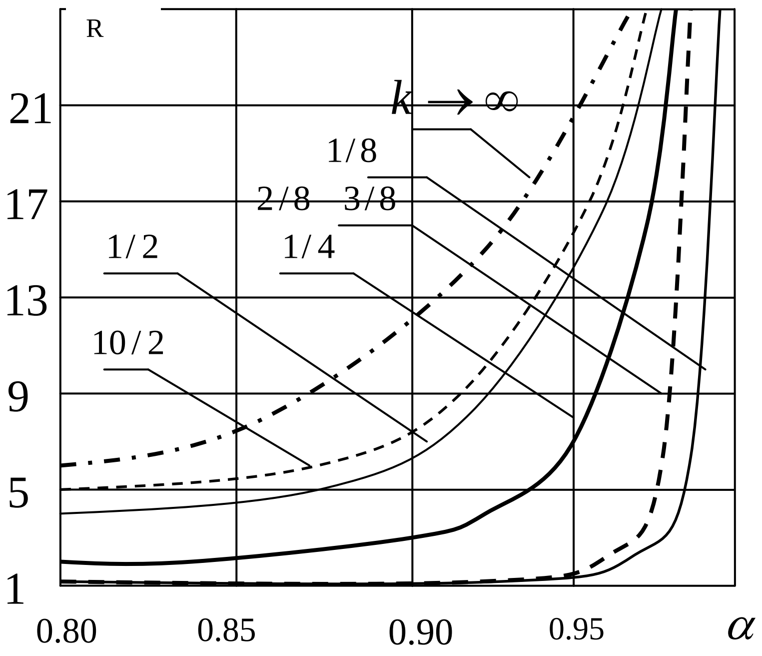

Figure 3. Relation of means 1/(γk) depending on α.

For k=1 the mean life time is plotted by a simple line. One can observe that with increasing degree of redundancy (k) the mean lifetime grows. Also, with increasing α, i.e. with increasing quality of switching, the mean lifetime also increases.

6. DISCUSSION AND CONCLUSIONS

Now we can provide the following limits for the different types of systems.

Table 2. Overview of the limit values for parallel systems with an independent number of components.

Characteristics	**Limit for hot standby**	**Limit for cold standby**
G(x)	$\frac{\gamma F(x)}{1-(1-\gamma)F(x)}$	$G(x) = \sum_{i=1}^{\infty} \gamma(1-\gamma)^{i-1} F^{(i)}(x)$
m_G	≤ (≥) $-m_F \ln(\gamma)/(1-\gamma)$ For F being NBUE (NWUE), equality for the exponential distribution	$m_G = m_F/\gamma$
Residual life $\int_x^{\infty}(1-G(t))dt$	≤ (≥) $(m_F/\gamma) \ln\left(\frac{\gamma}{1-(1-\gamma)F(x)}\right)$ For F being NBUE (NWUE), equality holds for the exponential distribution	≤ (≥) $(m_F/\gamma) \exp(-\gamma x/m_F)$ For F being HNBUE (HNWUE), equality holds for the exponential distribution

Note that, the limit itself is an upper bound for systems with a finite number of redundant components. So the upper bounds for real systems with a finite number of components is given by the NBUE / HNBUE limits. This is given in table 3

Table 3 upper bounds for imperfect parallel systems.

Characteristics	**Limit for hot standby**	**Limit for cold standby**
$G(x)$	$\frac{\gamma F(x)}{1-(1-\gamma)F(x)}$	$G(x) = \sum_{i=1}^{\infty}\gamma(1-\gamma)^{i-1}F^{(i)}(x)$
m_G	$-m_F \ln(\gamma)/(1-\gamma)$ For F being NBUE	$m_G = m_F/\gamma$
Residual life $\int_x^{\infty}(1-G(t))dt$	$(m_F/\gamma)\ln\left(\frac{\gamma}{1-(1-\gamma)F(x)}\right)$ For F being NBUE	$(m_F/\gamma)\exp(-\gamma x/m_F)$ For F being HNBUE

An imperfect system cannot achieve better values than given in the table above for components that satisfy the NBUE or HNBUE property.

In this paper we have obtained distribution functions for parallel systems in the case that switching to redundant devices is not perfect. It has turned out that there exists a limit and reliability cannot be improved up to 1. This can only be reached if switching is perfect.

This implies that at a certain stage of system development it is worthwhile to improve the reliability of the switching algorithm that to implement further additional redundant devices.

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SUMMARY OF THE NATIONAL RESEARCH COUNCIL REPORT ON "RELIABILITY GROWTH: ENHANCING DEFENSE SYSTEM RELIABILITY"

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A.V. Huzurbazar, S.V. Wiel, P.A. Jacobs, W.Q. Meeker, Jr.
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ABSTRACT

This paper, extracted from National Research Council (2015), summarizes the findings and recommendations from a recent report from the Panel on Reliability Growth Methods for Defense Systems, operating under the auspices of the Committee on National Statistics (CNSTAT) within the National Research Council (NRC). The report offers recommendations to improve defense system reliability throughout the sequence of stages that comprise U.S. Department of Defense (DoD) acquisition processes – beginning with the articulation of requirements for new systems and ending with feedback mechanisms that document the reliability experience of deployed systems. A number of these recommendations are partially or fully embraced by current DoD directives and practice, particularly with the advent of recent DoD initiatives that elevate the importance of design for reliability techniques, reliability growth testing, and formal reliability growth modeling. The report supports the many recent steps taken by DoD, building on these while addressing associated engineering and statistical issues. The report provides a self-contained rendition of reliability enhancement proposals, recognizing that current DoD guides and directives have not been fully absorbed or consistently applied and are subject to change.

1 INTRODUCTION

Reliability – the innate capability of a system to perform its intended functions – is one of the key performance attributes that is tracked during U.S. Department of Defense (DoD) Acquisition processes. Although every system is supposed to achieve a specified reliability requirement before being approved for acquisition, the perceived urgency to operationally deploy new technologies and military capabilities often leads to defense systems being fielded without having demonstrated adequate reliability. Between 2006 and 2011, one-half of the 52 major defense systems reported on by the DoD Office of the Director, Operational Test and Evaluation (DOT&E) to Congress failed to meet their prescribed reliability thresholds, yet all of the systems proceeded to full-rate production status.

Defense systems that fail to meet their reliability requirements are not only less likely to successfully carry out their intended missions, but also may endanger the lives of the Armed Service personnel who are depending on them. Such deficient systems are also much more likely than reliable systems to require extra scheduled and unscheduled maintenance and to demand more spare and replacement parts over their life cycles. In addition, the consequences of not finding fundamental flaws in a system’s design until after it is deployed can include costly and strategic delays until expensive redesigns are formulated and implemented and imposition of operational limits that constrain tactical employment profiles.

Recognizing these costs, the Office of the Secretary of Defense (OSD) – through DOT&E and the Office of the Under Secretary of Defense for Acquisition, Technology, and Logistics (USD AT&L) – in 2008 initiated a concerted effort to elevate the importance of reliability through greater use of design-for-reliability techniques, reliability growth testing, and formal reliability growth modeling. To this end, handbooks, guidance, and formal memoranda were revised or newly issued to provide policy to lead to the reduction of the frequency of reliability deficiencies. To evaluate the efficacy of that effort and, more generally, to assess how current DoD principles and practices could be strengthened to increase the likelihood of defense systems satisfying their reliability requirements, DOT&E and USD AT&L requested that the National Research Council conduct a study through its CNSTAT. The Panel on Reliability Growth Methods for Defense Systems was created to carry out that study.

2 SCOPE AND CONTEXT

The panel examined four broad topics: (1) the processes governing the generation of reliability requirements for envisioned systems, the issuance of requests for proposals (RFPs) for new defense acquisitions, and the contents of and evaluation of proposals in response; (2) modern design for reliability and how it should be utilized by contractors; (3) contemporary reliability test and evaluation practices and how they should be incorporated into contractor and government planning and testing; and (4) the current state of formal reliability growth modeling, what functions is it useful for, and what constitutes suitable use.

The current environment for defense system acquisition differs from the conditions that prevailed in DoD in the 1990s and also differs from the circumstances faced by commercial companies. Compared to the past, today’s DoD systems typically entail: greater design complexities (e.g., comprising dozens of subsystems with associated integration and interoperability issues); more dependence on software components; increased reliance on integrated circuit technologies; and more intricate dependencies on convoluted nonmilitary supply chains.

In commercial system development, all elements of program control are generally concentrated in a single project manager driven by a clear profit motive. In contrast, DoD acquisition processes are spearheaded by numerous independent “agents” – a system developer, one or more contractors and subcontractors, a DoD program manager, DoD testers, OSD oversight offices, and the military users – all of whom view acquisition from different perspectives and incentive structures. In addition, in the commercial sector the risk of delivering a poor reliability system is borne primarily by the manufacturer (in terms of reduced current and future sales, warranty costs, etc.), but for defense systems, the government and the military users generally assume most of the risk because the government is committed to extensive purchase quantities prior to the point where reliability deficiencies are evident.

Over the past few decades, commercial industries have developed two basic approaches to producing highly reliable system designs: techniques germane to the initial design, referred to as design-for-reliability methods; and testing in development phases aimed at finding failure modes and implementing appropriate design improvements to increase system reliability. In contrast, DoD has generally relied on extensive system-level testing, which is both time and cost intensive, to raise initial reliabilities ultimately to the vicinity of prescribed final reliability requirements. To monitor this growth in reliability, reliability targets are established at various intermediate stages of system developmental testing (DT). Upon the completion of DT, operational testing (OT) is conducted to examine reliability performance under realistic conditions with typical military users and maintainers. The recent experience with this DoD system development strategy is that operational reliability has frequently been deficient, and that deficiency can generally be traced back to reliability shortfalls in the earliest stages of DT.

Central to current DoD approaches to reliability are reliability growth models, which are mathematical abstractions that explicitly link expected gains in system reliability to total accrued testing time. They facilitate the design of defensible reliability growth testing programs and they support the tracking of the current system reliability. As is true for modeling in general, applications of reliability growth models entail implicit conceptual assumptions whose validity needs to be independently corroborated.

DoD reliability testing, unless appropriately modulated, does not always align with the theoretical underpinnings of reliability growth formulations, such as that system operating circumstances (i.e., physical environments, stresses that test articles are subjected to, and potential failure modes) do not vary during reliability growth periods.

The common interpretation of the term “reliability” has broad ramifications throughout DoD acquisition, from the statement of performance requirements to the demonstration of reliability in operational testing and evaluation. Because requirements are prescribed well in advance of testing, straightforward articulations, such as mean-time-between failures (MTBF) and probability of success, are reasonable. Very often, the same standard MTBF and success probability metrics will be appropriate for describing established levels of system reliability for the data from limited duration testing. But there may be instances – depending on sample sizes, testing conditions, and test prototypes – for which more elaborate analysis and reporting methods would be appropriate. More broadly, system reliabilities, both actual and estimated, reflect the particulars of testing circumstances, and these circumstances may not match intended operational usage profiles.

3 PANEL OBSERVATIONS AND RECOMMENDATIONS

The Panel on Reliability Growth Methods for Defense Systems offered 25 recommendations for improving the reliability of U.S. defense systems. These are listed in entirety in Section 4 below. Here we first summarize the panel’s primary observations that underlie the resultant recommendations. Then we highlight the content and substance of the individual recommendations. The panel’s conclusions cover the entire spectrum of DoD acquisition activities:

- DoD has taken a number of essential steps toward developing systems that satisfy prescribed operational reliability requirements and perform dependably once deployed.
- Fundamental elements of reliability improvement should continue to be emphasized, covering:
  - operationally meaningful and attainable requirements;
  - requests for proposal and contracting procedures that give prominence to reliability concerns;
  - design-for-reliability activities that elevate the level of initial system reliability;
  - focused test and evaluation events that grow system reliability and provide comprehensive examinations of operational reliability;

  - appropriate applications of reliability growth methodologies (i.e., compatible with underlying assumptions) for determining the extent of system-level reliability testing and the validity of assessment results;
  - empowered hardware and software reliability management teams that direct contractor design and test activities;
  - feedback mechanisms, spanning reliability design, testing, enhancement initiatives, and post-deployment performance, that inform current and future developmental programs; and
  - DoD review and oversight processes.
- Sustained funding is needed throughout system definition, design, and development, to:
  - incentivize contractor reliability initiatives;
  - accommodate planned reliability design and testing activities, including any revisions that may arise; and
  - provide sufficient state-of-the-art expertise to support DoD review and oversight.

Support for the panel's recommendations that are put forward throughout the panel's report. Here we present the content of the recommendations in terms of four aspects of the acquisition process: (1) system requirements, RFPs, and proposals; (2) design for reliability; (3) reliability testing and evaluation; and (4) reliability growth models.

The recommendations include a few "repeats" – endorsements of earlier CNSTAT and DoD studies, as well as reformulations of existing DoD acquisition procedures and regulations. These are presented to provide a complete self-contained rendition of reliability enhancement proposals, and because current DoD guidance and governance have not been fully absorbed, are inconsistently applied, and are subject to change.

3.1 System Requirements, RFPs, and Proposals

Prior to the initiation of a defense acquisition program, the performance requirements of the planned system, including reliability, have to be formally established. The reliability requirement should be grounded in terms of operational relevance (e.g., mission success) and be linked explicitly (within the fidelity available at this early stage) to the costs of acquisition and sustainment over the lifetime of the system. This operational reliability requirement also has to be technically feasible (i.e., verified to be within the state-of-the-art of current or anticipated near-term scientific, engineering, and manufacturing capabilities). Finally, the operational reliability requirement needs to be measureable and testable. The process for developing the system reliability requirement should draw on pertinent previous program histories and use the resources in OSD and the services (including user and testing communities). Steps should be reviewed and supplemented, as needed, by external subject-matter experts with reliability engineering and other technical proficiencies relevant to the subject system. [Recommendations 1, 2, 24, and 25]

The reliability requirement should be designated as a key performance parameter, making compliance contractually mandatory. This designation would emphasize the importance of reliability in the acquisition process and enhance the prospects of achieving suitable system reliability. During developmental testing, opportunities to relax the reliability requirement should be limited: it should be permitted only after high-level review and approval (at the level of a component acquisition authority or higher), and only after studying the potential effects on mission accomplishment and life-cycle costs. [Recommendations 3 and 5]

The government's RFP should contain sufficient detail for contractors to specify how they would design, test, develop, and qualify the envisioned system and at what cost levels. The RFP needs to elaborate on reliability requirements and justifications, hardware and software considerations, operational performance profiles and circumstances, anticipated environmental load conditions, and definitions of "system failure." The preliminary versions of the government's concept for a phased developmental testing program (i.e., timing, size, and characteristics of individual testing events) should also be provided. The government's evaluations of contractor proposals should consider the totality of the proffered reliability design, testing, and management processes, including specific failure definitions and scoring criteria to be used for contractual verification at various intermediate system development points. [Recommendations 1, 2, 4, 7, and 16]

3.2 Design for Reliability

High reliability early in system design is better than extensive and expensive system-level developmental testing to correct low initial reliability levels. The former has been the common successful strategy in non-DoD commercial acquisition; the latter has been the predominantly unsuccessful strategy in DoD acquisition.

Modern design-for-reliability techniques include but are not limited to: (1) failure modes and effects analysis, (2) robust parameter design, (3) block diagrams and fault tree analyses, (4) physics-of-failure methods, (5) simulation methods, and (6) root-cause analysis. The appropriate mix of methods will vary across systems. At the preliminary stages of design, contractors should be able to build on the details offered in RFPs, subsequent government interactions, and past experience with similar types of systems. [Recommendation 6]

The design process itself should rest on appropriately tailored applications of sound reliability engineering practices. It needs not only to encompass the intrinsic hardware and software characteristics of system performance, but also to address broader reliability aspects anticipated for manufacturing, assembly, shipping and handling, life-cycle profiles, operation, wear-out and aging, and maintenance and repair. Most importantly, it has to be supported by a formal reliability management structure and adequate funding (possibly including incentives) that provides for the attainment and demonstration of high reliability levels early in a system's design and development phases. If a system (or one or more of its subsystems) is software intensive, then the contractor should be required to provide a rationale for its selection of a software architecture and management plan, and that plan should be reviewed by independent subject-matter experts appointed by DoD. Any major changes made after the initial system design should be assessed for their potential impact on subsequent design and testing activities, and the associated funding needs should be provided to DoD. [Recommendations 6, 7, 15, and 18]

Three specific aspects of design for reliability warrant emphasis. First, more accurate predictions of reliabilities for electronic components are needed. The use of Military Handbook (MIL-HDBK) 217 and its progeny have been discredited as being invalid and inaccurate: they should be replaced with physics-of-failure methods and with estimates based on validated models. Second, software-intensive systems and subsystems merit special scrutiny, beginning in the early conceptual stages of system design. A contractor's development of the software architecture, specifications, and oversight management plan need to be reviewed independently by DoD and external subject-matter experts in software reliability engineering. Third, holistic design methods should be pursued to address hardware, soft- ware, and human factors elements of system reliability – not as compartmentalized concerns, but via integrated approaches that comprehensively address potential interaction failure modes. [Recommendations 6, 8, and 9]

3.3 Reliability Testing and Evaluation

Increasing reliability after the initial system design is finalized involves interrelated steps in planning for acquiring system performance information through testing, conducting various testing events, evaluating test results, and iteration. There are no universally applicable algorithms that precisely prescribe the composition and sequencing of individual activities for software and hardware developmental testing and evaluation at the component, subsystem, and system levels.

General principles and strategies, of which we are broadly supportive, have been espoused in a number of recent documents introduced to and utilized by various segments of DoD acquisition communities. While the reliability design and testing topics addressed in these documents are extensive, the presented expositions are not in-depth and applications to specific acquisition programs have to draw upon seasoned expertise in a number of reliability domains – reliability engineering, software reliability engineering, reliability modeling, accelerated testing, and the reliability of electronic components. In each of these domains, DoD needs to add appropriate proficiencies through combinations of in-house hiring, consulting or contractual agreements, and training of current personnel.

DoD also needs to develop additional expertise in advances in the state-of-the-art of reliability practices to respond to challenges posed by technological complexities and by endemic schedule and budget constraints. Innovations should be pursued in several domains: the foundations of design for reliability; early developmental testing and evaluation (especially for new technologies and for linkages to physical failure mechanisms); planning for efficient testing and evaluation and comprehensive data assimilation (for different classes of defense systems); and techniques for assessing aspects of near- and long-term reliability that are not well-addressed in dedicated testing.

Finally, to promote learning, DoD should encourage the establishment of information-sharing repositories that document individual reliability program histories (e.g., specific design and testing and evaluation initiatives) and demonstrated reliability results from developmental and operational testing and evaluation and post deployment operation. Also needed are descriptions of system operating conditions, as well as manufacturing methods and quality controls, component suppliers, material and design changes, and other relevant information. This database should be used to inform additional acquisitions of the same system and for planning and conducting future acquisition programs of related systems. In developing and using this database, DoD needs to ensure that the data are fully protected against the disclosure of proprietary and classified information. [Recommendations 22, 23, 24, and 25]

Planning for and conducting a robust testing program that increases system reliability, both hardware and software, requires that sufficient funds be allocated for testing and oversight of contractor and subcontractor activities. Such funding needs to be dedicated exclusively to testing so that it cannot be later redirected for other purposes. The amount of such funding needs to be a consideration in making decisions about proposals, in awarding contracts, and in setting incentives for contractors. The execution of a developer's reliability testing program should be overseen and governed by a formal reliability management structure that is empowered to make reliability an acquisition priority (beginning with system design options), retains flexibility to respond to emerging insights and observations, and comprehensively archives hardware and software reliability testing, data, and assessments. Complete documentation should be budgeted for and made available to all relevant program and DoD entities. [Recommendations 6, 7, 9, 12, 15, 16, 17, and 18]

The government and contractor should collaborate to further develop the initial developmental testing and evaluation program for reliability outlined in the RFP and described in the contractor's proposal. Reliability test plans, both hardware and software, should be regularly reviewed (by DoD and the developer) and updated as needed (e.g., at major design reviews) – considering what has been demonstrated to date about the attainment of reliability goals, contractual requirements, and intermediate thresholds and what remains uncertain about component, subsystem, and system reliability. Interpretations should be cognizant of testing conditions and how they might differ from operationally realistic circumstances. [Recommendations 4, 7, and 11]

The objectives for early reliability developmental testing and evaluation, focused at the component and subsystem levels, should be to surface failure mechanisms, inform design enhancement initiatives, and support reliability assessments. The scope for these activities, for both hardware and software systems, should provide timely assurance that system reliability is on track with expectations. The goal should be to identify and address substantive reliability deficiencies at this stage of development, when they are least costly, before designs are finalized and system-level production is initiated.

For hardware components and subsystems, there are numerous "accelerated" testing approaches available to identify, characterize, and assess failure mechanisms and reliability within the limited time afforded in early developmental testing and evaluation. They include exposing test articles to controlled nonstandard overstress environments and invoking physically plausible models to translate observed results to nominal use conditions. To manage software development in this early phase, contractors should be required to test the full spectrum of usage profiles, implement meaningful performance metrics to track software completeness and maturity, and chronicle results. For software-intensive systems and subsystems, contractors should be required to develop automated software testing tools and supporting documentation and to provide these for review by an outside panel of subject-matter experts appointed by DoD. [Recommendations 7, 9, 12, and 14]

When system prototypes (or actual systems) are produced, system-level reliability testing can begin, but that should not occur until the contractor offers a statistically supportable estimate of the current system reliability that is compatible with the starting system reliability requirement prescribed in the program's reliability demonstration plan. System-level reliability testing typically proceeds, and should proceed, in discrete phases, interspersed by corrective action periods in which observed failure modes are assessed, potential design enhancements are postulated, and specific design improvements are implemented. Individual test phases should be used to explore system performance capabilities under different combinations of environmental and operational factors and to demonstrate levels of achieved reliability specific to the conditions of that test phase (which may or may not coincide precisely with operationally realistic scenarios). Exhibited reliabilities, derived from prescribed definitions of system hardware and software failures, should be monitored and tracked against target reliabilities to gauge progress toward achieving the formal operational reliability requirement. Of critical importance is the scored reliability at the beginning of system-level developmental testing, which is a direct reflection of the quality of the system design and production processes. A common characteristic of recent reliability deficient DoD programs has been early evidence of demonstratively excessive observed failure counts, especially within the first phase of reliability testing. [Recommendations 7 and 19]

Inadequate system-level developmental testing and evaluation results in imprecise or misleading direct assessments of system reliability. If model based estimates (e.g., based on accelerated testing of major subsystems) become integral to demonstrating achieved system reliability and supporting major acquisition decisions, then the modeling should be subject to review by an independent panel of appointed subject-matter experts. To enhance the prospects of growing operational reliability, developmental system-level testing should incorporate elements of operational realism to the extent feasible. At a minimum, a single full-system, operationally relevant developmental test event should be scheduled near the end of developmental testing and evaluation – with advancement to operational testing and evaluation contingent on satisfaction of the system operational reliability requirement or other justification (e.g., combination of proximate reliability estimate, well-understood failure modes, and tenable design improvements). [Recommendations 13 and 20]

In operational testing, each event ideally would be of a sufficiently long duration to provide a stand-alone statistically defensible assessment of the system's operational reliability for distinct operational scenarios and usage conditions. When operational testing and evaluation is constrained (e.g., test hours or sample sizes are limited) or there are questions of interpretation (e.g., performance heterogeneity across test articles or operational factors is detected), nonstandard sophisticated analyses may be required to properly characterize the system's operational reliability for a single test event or synthesizing data from multiple developmental and operational test events. Follow-on operational testing and evaluation may be required to settle unresolved issues, and DoD should ensure that it is done. If the attainment of an adequate level of system operational reliability has not been demonstrated with satisfactory confidence, then DoD should not approve the system for full-rate production and fielding without a formal review of the likely effects that the deficient reliability will have on the probability of mission success and system life-cycle costs. [Recommendation 21]

The glimpses of operational reliability offered by operational testing are not well suited for identifying problems that relate to longer use, such as material fatigue, environmental effects, and aging. These considerations should be addressed in the design phase and in developmental testing and evaluation (using accelerated testing), and their manifestations should be recorded in the post deployment reliability history database established for the system. [Recommendation 22]

3.4 Reliability Growth Models

DoD applications of reliability growth models, focused on test program planning and reliability data assessments, generally invoke a small number of common analytically tractable constructs. The literature, however, is replete with other viable formulations – for time-to-failure data and discrete success/failure and both hardware and software systems (code). No particular reliability growth model is universally dominant for all potential applications, and some data complexities demand that common modeling approaches be modified in nonstandard and novel ways. [Recommendations 10, 11, and 19]

Within current formal DoD test planning documentation, each developmental system is required to establish an initial reliability growth curve (i.e., graphical depiction of how system reliability is planned to increase over the allotted developmental period) and to revise the curve as needed when program milestones are achieved or in response to unanticipated testing outcomes. The curve can be constructed from applying a reliability growth model, incorporating historical precedence from previous developmental programs, or customizing hybrid approaches. It should be fully integrated with overall system developmental test and evaluation strategies (e.g., accommodating other nonreliability performance issues) and retain adequate flexibility to respond to emerging testing results – while recognizing potential sensitivities to underlying analytical assumptions. The strategy of building the reliability growth curve to bring the system operational reliability at the end of developmental test and evaluation to a reasonable point supporting the execution of a stand-alone operational test and evaluation, with acceptable statistical performance characteristics, is eminently reasonable. Some judgment will always be needed in determining the number, size, and composition of individual developmental testing events, accounting for the commonly experienced DT/OT reliability gap, and in balancing developmental and operational testing and evaluation needs with schedule and funding constraints. [Recommendations 10 and 11]

Reliability growth models can be used, when supporting assumptions hold, as plausible “curve fitting” mechanisms for matching observed test results to prescribed model formulations – for tracking the development and maturity of software in early developmental testing, and for tracking the progression of system reliability during system-level testing. When overall sample sizes (i.e., numbers of recorded failures across multiple tests) are large, modeling can enhance the statistical precision associated with the last test event and support program oversight judgments. No elaborate modeling is needed, however, when the initial developmental testing experiences far more failures than anticipated by the planned reliability growth trajectory – indicative of severe reliability design deficiencies. [Recommendations 9, 10, and 19]

Standard applications of common reliability growth methods can yield misleading results when some test events are more stressful than others, when system operating profiles vary across individual tests, or when system functionality is added incrementally over the course of developmental testing. Under such nonhomogeneous circumstances, tenable modeling may need to require the development and validation of separate reliability growth models for distinct components of system reliability, flexible regression-based formulations, or other sophisticated analytical approaches. Without adequate data, however, more complex models can be difficult to validate: in this circumstance, too, reliability growth modeling needs to recognize the limitations of trying to apply sophisticated statistical techniques to the data. The utility and robustness of alternative specifications of reliability growth models and accompanying statistical methodologies can be explored via simulation studies. The general caution against model-based extrapolations outside of the range of the supporting test data applies to projections of observed patterns of system reliability growth to future points in time. One important exception, from a program oversight perspective, is assessing the reliability growth potential when a system clearly is experiencing reliability shortfalls during developmental testing – far below initial target values or persistently less than a series of goals. Reliability growth methods, incorporating data on specific exhibited failure modes and the particulars of testing circumstances, can demonstrate that there is little chance for the program to succeed unless major system redesigns and institutional reliability management improvements are implemented (i.e., essentially constituting a new reliability growth program). [Recommendations 10 and 19]

4 LIST OF RECOMMENDATIONS

RECOMMENDATION 1 The Under Secretary of Defense for Acquisition, Technology, and Logistics should ensure that all analyses of alternatives include an assessment of the relationships between system reliability and mission success and between system reliability and life-cycle costs.

RECOMMENDATION 2 Prior to issuing a request for proposal (RFP), the Office of the Under Secretary of Defense for Acquisition, Technology, and Logistics should issue a technical report on the reliability requirements and their associated justification. This report should include the estimated relationship between system reliability and total acquisition and life-cycle costs and the technical justification that the reliability requirements for the proposed new system are feasible, measurable, and testable. Prior to being issued, this document should be reviewed by a panel with expertise in reliability engineering, with members from the user community, from the testing community, and from outside of the service assigned to the acquisition. We recognize that before any development has taken place these assessments are somewhat guesswork and it is the expectation that as more about the system is determined, the assessments can be improved. Reliability engineers of the services involved in each particular acquisition should have full access to the technical report and should be consulted prior to the finalization of the RFP.

RECOMMENDATION 3 Any proposed changes to reliability requirements by a program should be approved at levels no lower than that of the service component acquisition authority. Such approval should consider the impact of any reliability changes on the probability of successful mission completion as well as on life-cycle costs.

RECOMMENDATION 4 Prior to issuing a request for proposal (RFP), the Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate the preparation of an outline reliability demonstration plan that covers how the department will test a system to support and evaluate system reliability growth. The description of these tests should include the technical basis that will be used to determine the number of replications and associated test conditions and how failures are defined. The outline reliability demonstration plan should also provide the technical basis for how test and evaluation will track in a statistically defendable way the current reliability of a system in development given the likely number of government test events as part of developmental and operational testing. Prior to being included in the request for proposal for an acquisition program, the outline reliability demonstration plan should be reviewed by an expert external panel. Reliability engineers of the services involved in the acquisition in question should also have full access to the reliability demonstration plan and should be consulted prior to its finalization.

RECOMMENDATION 5 The Under Secretary of Defense for Acquisition, Technology, and Logistics should ensure that reliability is a key performance parameter: that is, it should be a mandatory contractual requirement in defense acquisition programs.

RECOMMENDATION 6 The Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate that all proposals specify the design-for-reliability techniques that the contractor will use during the design of the system for both hardware and software. The proposal budget should have a line item for the cost of design-for- reliability techniques, the associated application of reliability engineering methods, and schedule adherence.

RECOMMENDATION 7 The Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate that all proposals include an initial plan for system reliability and qualification (including failure definitions and scoring criteria that will be used for contractual verification), as well as a description of their reliability organization and reporting structure. Once a contract is awarded, the plan should be regularly updated, presumably at major design reviews, establishing a living document that contains an up-to-date assessment of what is known by the contractor about hardware and software reliability at the component, subsystem, and system levels. The U.S. Department of Defense should have access to this plan, its updates, and all the associated data and analyses integral to their development.

RECOMMENDATION 8 Military system developers should use modern design-for-reliability (DFR) techniques, particularly physics-of- failure (PoF)-based methods, to support system design and reliability estimation. MIL-HDBK-217 and its progeny have grave deficiencies; rather, the U.S. Department of Defense should emphasize DFR and PoF implementations when reviewing proposals and reliability program documentation.

RECOMMENDATION 9 For the acquisition of systems and subsystems that are software intensive, the Under Secretary of Defense for Acquisition, Technology, and Logistics should ensure that all proposals specify a management plan for software development and also mandate that, starting early in development and continuing throughout development, the contractor provide the U.S. Department of Defense with full access to the software architecture, the software metrics being tracked, and an archived log of the management of system development, including all failure reports, time of their incidence, and time of their resolution.

RECOMMENDATION 10 The validity of the assumptions underlying the application of reliability growth models should be carefully assessed. In cases where such validity remains in question: (1) important decisions should consider the sensitivity of results to alternative model formulations and (2) reliability growth models should not be used to forecast substantially into the future. An exception to this is early in system development, when reliability growth models, incorporating relevant historical data, can be invoked to help scope the size and design of the developmental testing programs.

RECOMMENDATION 11 The Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate that all proposals obligate the contractor to specify an initial reliability growth plan and the outline of a testing program to support it, while recognizing that both of these constructs are preliminary and will be modified through development. The required plan will include, at a minimum, information on whether each test is a test of components, of subsystems, or of the full system; the scheduled dates; the test design; the test scenario conditions; and the number of replications in each scenario. If a test is an accelerated test, then the acceleration factors need to be described. The contractor's budget and master schedules should be required to contain line items for the cost and time of the specified testing program.

RECOMMENDATION 12 The Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate that contractors archive and deliver to the U.S. Department of Defense (DoD), including to the relevant operational test agencies, all data from reliability testing and other analyses relevant to reliability (e.g., modeling and simulation) that are conducted. This should be comprehensive and include data from all relevant assessments, including the frequency under which components fail quality tests at any point in the production process, the frequency of defects from screenings, the frequency of defects from functional testing, and failures in which a root-cause analysis was unsuccessful (e.g., the frequency of instances of failure to duplicate, no fault found, retest OK). It should also include all failure reports, times of failure occurrence, and times of failure resolution. The budget for acquisition contracts should include a line item to provide DoD with full access to such data and other analyses.

RECOMMENDATION 13 The Office of the Secretary of Defense for Acquisition, Technology, and Logistics, or, when appropriate, the relevant service program executive office, should enlist independent external, expert panels to review (1) proposed designs of developmental test plans critically reliant on accelerated life testing or accelerated degradation testing and (2) the results and interpretations of such testing. Such reviews should be undertaken when accelerated testing inference is of more than peripheral importance – for example, if applied at the major subsystem or system level, there is inadequate corroboration provided by limited system testing, and the results are central to decision making on system promotion.

RECOMMENDATION 14 For all software systems and subsystems, the Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate that the contractor provide the U.S. Department of Defense (DoD) with access to automated software testing capabilities to enable DoD to conduct its own automated testing of software systems and subsystems.

RECOMMENDATION 15 The Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate the assessment of the impact of any major changes to system design on the existing plans for design-for-reliability activities and plans for reliability testing. Any related proposed changes in fund allocation for such activities should also be provided to the U.S. Department of Defense.

RECOMMENDATION 16 The Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate that contractors specify to their subcontractors the range of anticipated environmental load conditions that components need to withstand.

RECOMMENDATION 17 The Under Secretary of Defense for Acquisition, Technology, and Logistics should ensure that there is a line item in all acquisition budgets for oversight of subcontractors' compliance with reliability requirements and that such oversight plans are included in all proposals.

RECOMMENDATION 18 The Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate that proposals for acquisition contracts include appropriate funding for design-for- reliability activities and for contractor testing in support of reliability growth. It should be made clear that the awarding of contracts will include consideration of such fund allocations. Any changes to such allocations after a contract award should consider the impact on probability of mission success and on life-cycle costs, and at the minimum, require approval at the level of the service component acquisition authority.

RECOMMENDATION 19 The Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate that prior to delivery of prototypes to the U.S. Department of Defense for developmental testing, the contractor must provide test data supporting a statistically valid estimate of system reliability that is consistent with the operational reliability requirement. The necessity for this should be included in all requests for proposals.

RECOMMENDATION 20 Near the end of developmental testing, the Under Secretary of Defense for Acquisition, Technology, and Logistics should mandate the use of a full-system, operationally relevant developmental test during which the reliability performance of the system will equal or exceed the required levels. If such performance is not achieved, then justification should be required to support promotion of the system to operational testing.

RECOMMENDATION 21 The U.S. Department of Defense should not pass a system that has deficient reliability to the field without a formal review of the resulting impacts the deficient reliability will have on the probability of mission success and system life-cycle costs.

RECOMMENDATION 22 The Under Secretary of Defense for Acquisition, Technology, and Logistics should emplace acquisition policies and programs that direct the services to provide for the collection and analysis of post-deployment reliability data for all fielded systems, and to make that data available to support contractor closed-loop failure mitigation processes. The collection and analysis of such data should be required to include defined, specific feedback about reliability problems surfaced in the field in relation to manufacturing quality controls and indicate measures taken to respond to such reliability problems. In addition, the contractor should be required to implement a comprehensive failure reporting, analysis and corrective action system that encompasses all failures (regardless whether failed items are restored/repaired/replaced by a different party, e.g., subcontractor or original equipment manufacturer).

RECOMMENDATION 23 After a system is in production, changes in component suppliers or any substantial changes in manufacturing and assembly, storage, shipping and handling, operation, maintenance, and repair should not be undertaken without appropriate review and approval. Reviews should be conducted by external expert panels and should focus on impact on system reliability. Approval authority should reside with the program executive office or the program manager, as determined by the U.S. Department of Defense. Approval for any proposed change should be contingent upon certification that the change will not have a substantial negative impact on system reliability or a formal waiver explicitly documenting justification for such a change.

RECOMMENDATION 24 The Under Secretary of Defense for Acquisition, Technology, and Logistics should create a database that includes three elements obtained from the program manager prior to government testing and from the operational test agencies when formal developmental and operational tests are conducted: (1) outputs, defined as the reliability levels attained at various stages of development; (2) inputs, defined as the variables that describe the system and the testing conditions; and (3) the system development processes used, that is, the reliability design and reliability testing specifics. The collection of these data should be carried out separately for major subsystems, especially software subsystems.

RECOMMENDATION 25 To help provide technical oversight regarding the reliability of defense systems in development, specifically, to help develop reliability requirements, to review acquisition proposals and contracts regarding system reliability, and to monitor acquisition programs through development, involving the use of design-for-reliability methods and reliability testing, the U.S. Department of Defense should acquire, through in-house hiring, through consulting or contractual agreements, or by providing additional training to existing personnel, greater access to expertise in these five areas: (1) reliability engineering, (2) software reliability engineering, (3) reliability modeling, (4) accelerated testing, and (5) the reliability of electronic components.

5 ACKNOWLEDGMENT

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MATHEMATIC AND SIMULATION MODELING FOR ANALYSIS PREDICTION OF RISK

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ABSTRACT

Specification of requirements for reliability of a transport means is first of all an issue of looking for an acceptable compromise between a requested level of reliability and a level of costs, which will be needed for its achievement. Provision of reliability in a stage of application is however dependent on allocated sources for a provision of maintenance.

1 GENERAL INSTRUCTIONS

The results of a simulation modeling provide for an intuitive perception on an implementation of small numerous events and on an approach to risks. It is obvious if we research them and implement in a large amount of simulation runs and so a long period of operation of mobile assets will approximate to statistic results. The above mentioned outputs and data processing from the performed experiments result in the following conclusions. Statistical characteristics of a failure-free operation of vehicles, particular groups and statistic characteristics of costs are more suitable for an application of risk theory and solution of tasks related with maintenance, logistic problems than quantitative assessment or semi-quantitative methods of risk assessment. Mathematical modeling and simulation is for an analysis, modeling and prediction of random events in operation, maintenance, logistics, and risk assessment very favorable, first of all for a possible visualization and monitoring through graphical outputs providing better perception and display of stochastic processes. There is a certain rate of uncertainty connected with each function of transport means, that it will be carried out in a different way than requested and that possible deviations from an expected function will have an unwanted consequence on a result of the function of the object as a whole. Therefore there is a certain risk, understood as a combination of probability, that a certain event occurs (a failure) and consequences (costs), which would occur, if an event would happen. From a course of costs distribution functions we can conclude a range in which the costs would occur.

1.1 Type area

However in technical areas by scientific approaches we can obtain assessments of probability of a rise of an unwanted event and data on its consequences that can be statistically processed and evaluated with rule of distribution of a random variable of a reviewed of an assumed event. In the same way we can express statistically other factors as well, e.g. time exposition, i.e. time period when the conditions last to generate a negative event or a value of an opportunity for application of protective measurements in stage of a threat.

That is an approach of a stochastic expression of a risk that can be used. Stochastic optimization issues utilize results of analyses in the final stage of solution. The input data of risk and influencing factors are real ones, their quantification through distribution of random variables complies with reality and the risk assessment and measures are quantified. The assessment algorithms are based on statistic results. In addition to a basic definition of a risk from a cause and consequence, we can statistically express additional factors, e.g. time exposition, i.e. time period of lasting conditions for a rise of a negative event and eventually a value of a possibility to apply protective measures in stage of a threat .

Then a risk function will look as follows:

$$R (t) = f /P (t), D (t), E (t), O (t), \ldots\ldots. Z (t) \qquad (1)$$

where:
R (t) ... risk,
P (t) ...probability of a rise of an unwanted event,
D (t) ... probability of a consequence,
E(t) ... time exposition /time period of lasting conditions for a rise/,
O (t) ... application of protective measures in stage of a threat [5].

The risk of meeting a mission supposing a fulfillment of transport tasks depends on:

- An anticipated drawing of operating units /overrun in kilometers, operating hours, time of operation,..../ vehicles,
- Failure-less operation of vehicles,
- Funds assigned,
- Maintenance provision.

Provision of readiness supposes an adequate volume, amount of funds.
The sources are allocated in the categories:

- Material costs
- Labor costs

Total costs are an aggregate of previous sources.

In practice there exist many probability models of distribution of random values being used in description of particular practical problems. For continuous values there are e.g. exponential, normal (Gaussian), regular, Student´s, Fisher – Snedocor´s Weibull and other distributions. For discrete random values there are e.g. alternative, binomial, Poisson, hypergeometric distributions. We statistically evaluated data on failure-less operation and on costs. From the results we defined hypotheses of research of a probable distribution of probabilities. There were used hypotheses of a normal distribution, exponential distribution and Weibull distribution of probability. We used the distribution parameters obtained for simulation of the same numbers of values as it is for number of data we had processed and assessed. Through comparing we can see that results from simulation of an exponential distribution and Weibull distribution of probability match with hypotheses. Of course, they do not significantly differ with regard to the parameter of a shape of the Weibull distribution being close to 1 value.

The costs for material, assessing a mean of probability 0.5, define an increasing order for costs in cost groups as electric installation, steering, body, and a frame, braking system, gear system, engine with systems.

2 THE ELEMENTS OF THE MATRIX

Standard expression of the risk matrix is formed on a principle of two participating distribution functions and their values in intervals < 0,1 >. Thereby, we reach, of course, results that in the probability matrix in the left corner we get small values of risks through a product of small values of a probability of causes and consequences. The elements of the matrix show the areas of acceptation or non-acceptation of the risk. Of course a non-acceptable area is on the right side up. A disadvantage is that a risk area is not defined by parameters of a cause and consequence. In case of a simulation modeling the fact of the phenomena appearance is defined by an appearance of values featured by probabilities of rise of phenomena participating in a risk, but assessed in unit formulation of parameters of participating phenomena.

2.1 Simulation capabilities risk matrix

We will use distribution of probability of a failure to generate a rise of a negative phenomenon – a failure and a distribution of probability of some kind of costs to generate amount of costs as a consequence of an unwanted event. Simulated values will be used for graphic display of an intersection of these phenomena in a point, the amount of costs on an y axis and amount of operational units course on x axis. It provides us with data and a perception of a rise of a risk situation. Burst of appearance and their quantification enables comparing of risks and costs for maintenance of objects being assessed.

Statistical processing of results of a simulation modeling enables displaying of a frequency, probability and assessment form a point of accepted hypotheses of a distribution kind participating on a risk and parameters of functions. Risk area is defined by a burst of points appearance within the range of the highest probabilities participating in probability density.

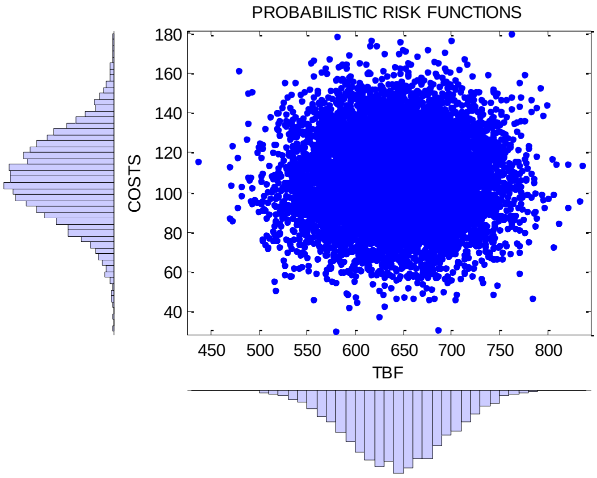

Figure 1. Display of an intersection of phenomena and frequency diagrams for 10000 simulations

2.2 The possibility of risk analysis

Risk analysis is a technique used to identify and assess factors that may jeopardize the success of a project or achieving a goal. This technique also helps to define preventive measures to reduce the probability of these factors from occurring and identify countermeasures to successfully deal with these constraints when they develop to avert possible negative effects.

- Categorize each hazard, threat, or peril according to how severe it is, how frequently it occurs, and how vulnerable you are.
- Develop strategies to deal with the most significant hazards, threats, or perils.
- Develop strategies to prevent hazards, threats, or perils that impact or might impact your organization and its people, operations, property, and environment.
- Develop strategies to mitigate hazards, threats, or perils that impact or might impact your organization and its people, operations, property, and environment.
- Develop strategies to prepare for hazards, threats, or perils that impact or might impact your organization and its people, operations, property, and environment.
- Develop strategies to respond to hazards, threats, or perils that impact or might impact your organization and its people, operations, property, and environment.
- Develop strategies to recover from hazards, threats, or perils that impact or might impact your organization and its people, operations, property, and environment.

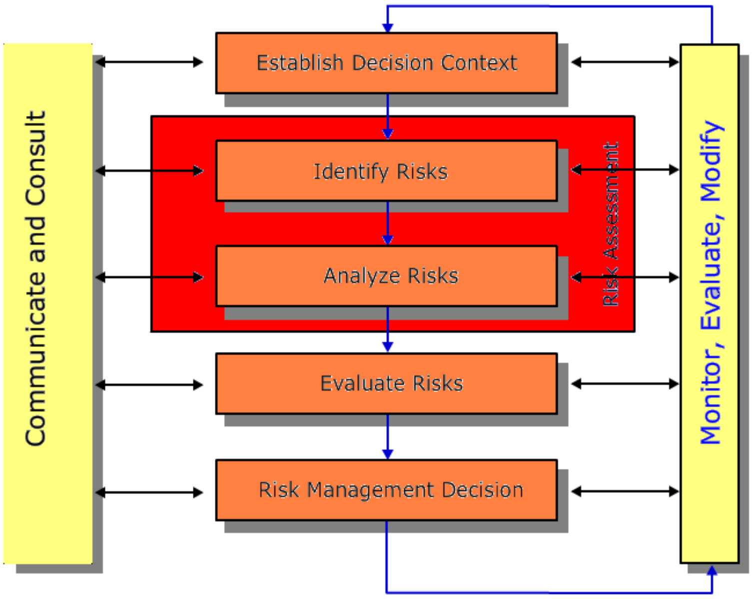

Figure 2. The risk assessment process

In case of a two-dimensional area described by vectors of a simulated probability of probability density we can change a scope of an acceptable risk of both participating functions through defining the quantiles. To define a rate of risk only intersections of generated events starting from a minimum value up to the defined values of quantiles are counted in.

The relation for a computation is: $$\boldsymbol{P} = \frac{\boldsymbol{n}}{\boldsymbol{N}} \tag{2}$$

where
- *n is* a number of executions being included into an area defined by quantiles,
- *N* number of simulated values.

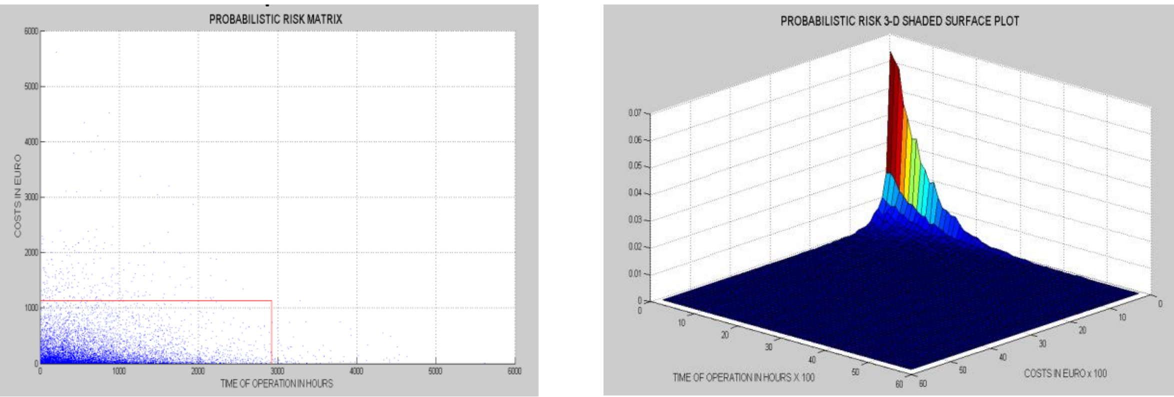

Figure 3. For example calculated values for 99 percent quantiles for 10000 simulations

Time period between failures in operational hours defined by a 99 percent quantile is 2922 hours. The costs per an operational hours defined by a 99 percent quantile are 113,5 Euros. Probability of a risk in this limited area is expressed through a value of 0.9657.

We use a function of density of a failure probability as a rise of a negative event – a failure and an amount of total costs as a result of an unfavourable event.

Visual expression of an intersection of these events gives us a notion about a rate of rise of critical situation. We can quantify this fact and to express it by probability of risk matrix.

We will use a distribution of a failure probability to generate a rise of a negative event – a failure and a distribution of a probability of costs to generate the amount of costs resulted from an unwanted event.

Graphic expression of an intersection of these events in a point of costs matrix and operation in hours provides us with a perception relating with quantification if a risk situation rises. Aggregations of their occurrence and their quantification on the legs enable comparing the risks from costs for maintenance of objects being assessed.

We can quantify this probability and to define it with a probability of elements, lines or columns of the risk matrix.

With an increased number of simulated events, representing a longer distance of kilometres driven, the ranges of affected risks increase as well.

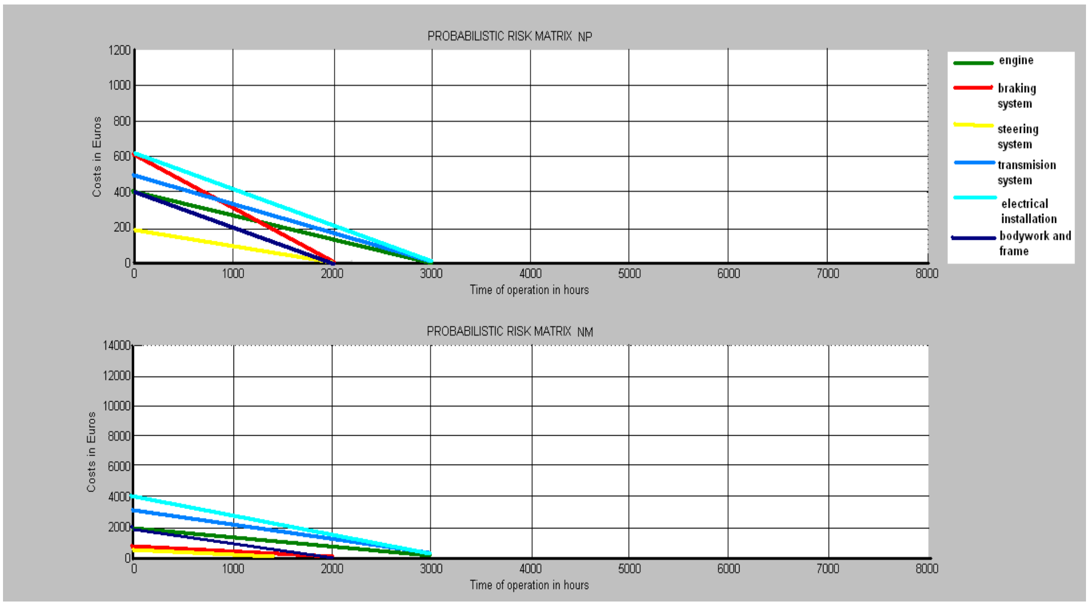

Figure 4. Probablistic risk matrix

Risk matrices are widely used in risk management. They are a regular feature in various risk management standards and guidelines and are also used as formal corporate risk acceptance criteria. It is only recently, however, that scientific publications have appeared that discuss the weaknesses of the risk matrix.

A sense of a mathematic expression of an availability factor has been supported, that relationship between reliability and maintainability expresses possibilities of an increase of availability of designed and operated devices that interfere with technological limits of periods when the activities are performed. Availability can be increased practically only through shortening of intervals of components of the device maintainability that interferes with technological limits of the action being performed. Asymptotic availability of a terrain vehicle is lower than the availability of groups, it becomes stabilized on 0.958- 0.966 level.

The statistic characteristics of a failure-free operation of vehicles and particular groups and statistic characteristics of costs are used for application of theory of risks and solution of tasks related to issues of maintenance and logistics issues.

3 CONCLUSIONS

Mathematic and simulation modelling is for an analysis, modelling and prediction of stochastic phenomena in the operation, maintenance, logistics, risk assessment very favourable, first of all for a possibility of monitoring through graphic outputs, which give more visual perception about stochastic processes.

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THE EVALUATION OF RISK OF THE DEVELOPMENT OF SOCIO-PRODUCTIVE STRUCTURE WITH USE OF THE PROGRAM COMPLEX OF ASM 2001

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ABSTRACT

How to manage socio-productive structure in modern economic conditions? On the basis of systematic analysis is formulated the concept "dangerous state of socio-productive structures". Described are three possible scenarios of failure of development of socio-productive structure. Proposed is LP-model of development of socio-productive structure. The calculations are made with using the software complex ASM 2001. The results obtained are help in making strategic solution on basing the assessment of the risks of unsuccessful development of socio-productive structure.

In conditions of unstable economy in the Russian Federation manifests the imperfection of the local law. Strategy for socio-economic development of the country was based on the formation of urban agglomerations. How to manage? In the thesis generally investigates the economic aspect of management or only businesses, or only territory.

Work (Gritskikh, 2009) this is the generalization of scientific works devoted to the social dimension of development in various Russian regions and single-industry towns. Two trends were highlighted: the formation of urban agglomerations and the growth of social tension in single-industry towns, but there is no model of effective management of social and industrial structure. An interesting work was written on the of flows management (Polenin, Gladkova, 2015), in which was investigated the transmission of electricity as a stream. In the National standard (GOST R 15704-2008) set out recommendations on the use of GERAM to improve the efficiency of the company.

And how to evaluate the success of the development of socio-productive structure as a system that combines production and industrial infrastructure, in which there are material, financial and information flows?

In general, the operation of the system can be represented as formal model of streams (see Fig. 1; formula 1).

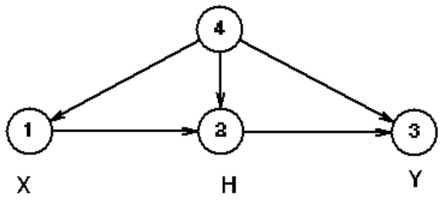

Figure 1. The formalized model of the flows

$$y(t) = F_S(x(t), h(t), z(t)) \tag{1}$$

Where: S – the flows of the socio-productive structure;

$x_i \in X, i = 1, 2, ..., n_x$ – the set of input streams;
$h_l \in H, l = 1, 2, ..., n_H$ – the set of internal influences;
$z_k \in Z, k = 1, 2, ..., n_Z$ – the set of impacts of the external environment;
$y_j \in Y, j = 1, 2, ..., n_Y$ – the set of output streams.

Optimization of movement flows of socio-productive structure cannot do without the methods of mathematical modeling with the use of a systematic approach.

First of all, we did the decomposition of socio-productive structure as system (S), allocating two subsystems: enterprise (S_1) and the infrastructure (S_2).For system development, is necessary successful development of all subsystems. The development of enterprises effects the development infrastructure. Development infrastructure supports the development of the enterprise.

For technical systems the dangerous condition this is a condition that can lead to the destruction of the object, damage and so on (Ryabinin, 2008, p.116).

What should be understood under the term "the dangerous condition of the socio-productive structure"? How to evaluate the probability of successful functioning of the socio-productive structure?

In the economic systems should be considered the not probability of success but probability of failure on the basis of logical-probabilistic modeling of risk and the effectiveness of the system (Solozhentsev, 2009, p.226).

The peculiarity of the socio-productive structure is the danger of full-scale experiments. The transition to a market economy in 1990-ies was accompanied by the destruction of the USSR and decline in living standards of the population. That is, the mathematical experiments for of study in this case more preferred.

As criteria for assessing the development of the socio-productive structure be choose: demography; damage; investments; tax revenues; subsidies, etc.

Let us formulate the hypothesis of the study. The dangerous condition for socio-productive structure is condition that may result:

A) Mass protests of the population for the purpose of destroying an existing management system (strikes, revolutions, etc.);

B) The Exodus of the population due to the impossibility of living in a particular area (technogenic accidents, natural disasters, etc.);

C) Termination of the production due to the loss of markets for manufactured products (the economic crisis; the decline in the quality of goods or services).

Logical and probabilistic theory this is of knowledge according to the calculations of the risk of accidents and catastrophes in the complex-structured systems (Ryabinin, 2008, p.125). The socio-productive structure represents the complex-structured systems. For assess the risk not successful development of socio-productive structure was an attempt of application of the software complex ASM 2001[1] (authors A. S. Mozhaev and I. A. Gladkova).

To build scenarios for the development of the socio-productive structure were selected three criteria for the evaluation of not successful development of socio-productive structure (see Fig. 2): environmental pollution, water and atmosphere (Y_1); a social explosion (Y_2); reduction of the amount of the taxes (Y_3).

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[1] SC ASM 2001 – software for automated structural and logical simulation of complex systems

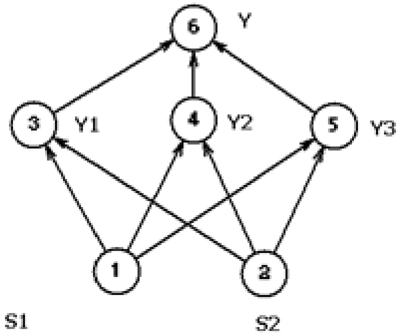

Figure 2. The three criteria for the evaluation of not successful development

In socio-productive structure always has a conflict of interests of different parties as a solution to social or environmental problems involves the reduction of the profit of the owner of fixed assets

Not all owners of the enterprises want observe laws that regulate the activities in the environmental field. Pollution of territories of subjects of RF the various substances occurs continuously.

For an assessment of risk of probability of unsuccessful development of socio-productive structure it is necessary to consider not less than three scenarios.

For an example we will make the scenario "Ecological Pollution" for social and production structure on one of material streams: resources → production → waste (see Fig. 3).

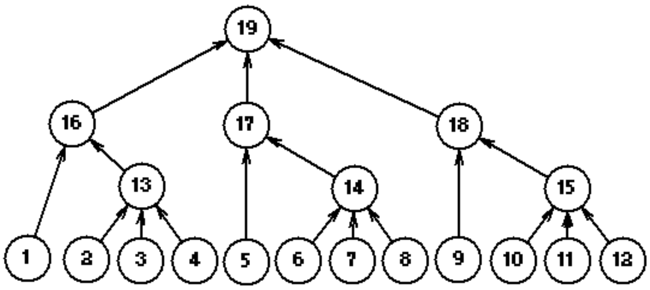

Figure 3. The scenario "The Ecological Pollution"

The designations:

x_1 – lack of filters of cleaning of gaseous waste;

x_2 – destruction of filters of cleaning of gaseous waste;

x_3 – filters of cleaning of gaseous waste aren't put into operation;

x_4 – the owner doesn't finance processes of cleaning of gaseous waste;

x_5 – lack of treatment facilities for liquid waste;

x_6 – destruction of treatment facilities for liquid waste;

x_7 – treatment facilities for liquid waste aren't put into operation;

x_8 – the owner doesn't finance processes of cleaning of liquid waste;

x_9 – absence of landfills;

x_{10} – destruction of landfills;

x_{11} – landfills aren't put into operation;

x_{12} – the owner doesn't finance waste disposal processes;

13 (x_{13}) – non-use of filters of cleaning of gaseous waste;
14 (x_{14}) – non-use of treatment facilities for liquid waste;
15 (x_{15}) – non-use of landfills;
16 (x_{16}) – emissions of gaseous waste in the atmosphere;
17 (x_{17}) – dumpings of liquid waste in reservoirs;
18 (x_{18}) – waste disposal out of landfills;
19 (Y_1) – risk of ecological pollution.

How to choose probabilities for the listed events?

If probability of an event is accept equal 0.05 (the event is improbable), then Ps = 0.1855 (the ecological pollution is improbable).

If probability of an event is accept equal 0.5 (an event equally possibly), then Ps = 0.9375 (the ecological pollution is possible).

If probability of an event is accept equal 0.95 (the event will practically be carried out), then Ps = 0.99999375 (the ecological pollution will take).

The increasing quantity of places (out of landfills) where solid waste is dumped on the earth, gives the grounds to accept value of probability of an event x_{12} not less than 0.3. One may to assume that the unwillingness of owners of enterprises to reduce the profit in the presence of financing of processes of utilization (burial) of waste is the reason of such actions.

The media occasionally publishes articles about discharges into water and emissions of various substances in the atmosphere, so we can assume that the probability of events x_4 and x_8 can be 0.3.

In the twenty-first century much public attention to the ecological state of the territories is initiating capital investments in the variety of the systems treatment of industrial and non-industrial waste, so the probability of events x_1, x_5, x_9 can be taken equal to 0.05 (unlikely).

The probability of events x_2, x_6, x_{10} depends on the regularity and quality of the preventive works; the natural disasters; the technological accidents of a large scale; terrorist acts, etc. Therefore, the probability of events x_2, x_6, x_{10} can be taken equal to 0.1.

The probability of events x_3, x_7, x_{11} depends on the timeliness of registration of project documentation; the quality of manufacturing of elements of systems for treatment of industrial and non-industrial waste, etc. Therefore, the probability of events x_3, x_7, x_{11} can be taken equal to 0.1. With this approach, we get Ps = 0.46135 (environmental contamination possible).

For the other two criteria assess the risk of probability of unsuccessful development of the socio-productive structure scenarios are created similarly.

Conclusions

It is necessary to create such economic conditions, to owner of the enterprises was not profitable evade the costs of improving waste treatment systems of various types.

Necessary to use unmanned aircraft for monitoring territories to identify:

A) vehicles, dumping of solid waste along roads (outside of landfills; outside waste recycling plants);

B) the objects that pollute the water bodies.

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TRANSITION FROM QUALITATIVE TO THE QUANTITATIVE APPROACH OF FORMATION OF DECISIONS ON INCREASE OF RELIABILITY OF OBJECTS OF ELECTRO POWER SYSTEMS

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ABSTRACT

The decision of problems of maintenance service arising at the organization and repair of the equipment and devices of electro power systems is resulted. To them concern: an estimation of the importance of a version of the attributes describing reliability and profitability of work; an estimation of parameters of individual reliability; an estimation of parameters of reliability of homogeneous groups (clusters). Methods, algorithms and programs of calculation of these estimations are developed. As the initial information statistical data of operation serve. These data are represented not as sample, of which general population, and as final population of multivariate data. The expediency of classification of these data on the set versions of attributes has or under condition of not casual character of a divergence of statistical functions of distribution of the initial data $F_{\Sigma}^{*}(X)$ constructed on all population and sample $F_{v}^{*}(X)$. As criterion of estimation it is accepted non-exceedance to an estimation of a parameter of reliability calculated on experimental data of sample $\Pi_{э}^{*}(X)$, and critical value of this parameter $\Pi_{к}$ for the set significance value (Errors I type). It is shown, that: decrease in number of versions of attributes has basic value for decrease in time of calculations; it is necessary to analyze not only character of a divergence of the average parameter of reliability Π_{Σ}^{*} and Π_{v}^{*}, but also Π_{v}^{*} various combinations of versions of an attribute; it would be erroneous to represent, that estimations of parameters of individual reliability are considered on statistical data of operation of the concrete equipment. It is simply not enough of them or not. Individuality is set by significant versions of attributes for this reason exist more than one unit of equipment and devices, parameters of which individual reliability are equal. These groups form cluster; Parameters of reliability clusters differ from parameters of individual reliability forming cluster the equipment and devices. Distinction is caused by that parameters of individual reliability are calculated on the set versions of attributes. At calculation parameters of reliability clusters are considered only versions of attributes for which $\Pi_{э}^{*}$ not casually differ not only from Π_{Σ} but also from all others Π_{v}^{*}.

INTRODUCTION

Increase of efficiency of the decision of operational problems in electro power systems (EPS) demands the objective account of reliability of the equipment and devices (objects). Traditionally this account is spent, basically, at a qualitative level (an operational experience of objects + intuition + high qualification of the personnel). Eventually:

- The share of objects, which service life exceeds settlement, became not less than 50 % and increases. Their technical condition worsens, opportunities decrease, demand special attention;
- Occurrence of new objects with other designs and principles of work, the control systems, an increasing variety of volume and norms of test and repair of objects, reduces the importance of the saved up operational experience and demands improvement of quality of preparation of experts, regular retraining of the personnel, improvement of professional skill;
- The automated control systems of operating modes of objects EPS, which service life, exceed settlement, consider change of power characteristics owing to ageing objects insufficiently and demand perfection. And the systems intended for the continuous control of a technical condition of objects, giving the unique information, unfortunately, not always form the decision on increase of reliability.

Thus, methods of the traditional account of reliability of objects demand perfection. One of the most significant directions in it is the increase in making information support of the personnel in the automated information systems of the analysis of a technical condition of objects [1]. Recommendations include:

- Ranking of objects on reliability and profitability (efficiency) of work;
- Instructions on « weak parts » the objects, the based reasons causing deterioration of a technical condition;
- Estimation of quality:
    - Managements of operating modes of objects;
    - Restoration of deterioration during scheduled repair;
    - Preservation during the compelled idle time and a number of others

In present clause, methods and the integrated algorithms of the decision of three interconnected problems providing information support noted above of the personnel are resulted.

1. Method and algorithm of an estimation of the importance of a version of an attribute

It is known, that at the analysis of refusals of objects EPS the big number of information attributes is considered and, first of all, because it is difficult to approve with confidence what of them will appear the most important and useful. Each of attributes is characterized by several versions (VA). On the basis of this information parameters and characteristics of reliability (PR) also pay off. However, at all this, the average estimations calculated, as a rule, and the variety of attributes and their versions at calculations PR practically not considered. These average quantitative estimations of reliability of work are used, first of all, for an illustration of application of methods of calculation PR, the decision of separate design problems. The choice of schemes of switching centers, an estimation of a reserve of capacity concern to such problems, etc.

Parameters, as a rule, are necessary for the decision of operational problems PR compared objects, i.e. and characteristics of individual reliability (PIR). However, it would be erroneous to think, that estimations PIR spend on statistical data about refusals and restorations the concrete generator, the transformer or the switch. To experts well known, that such information simply is not present. And when we speak about PIR is available in view of PR which pays off for significant VA objects. Traditionally, classification of statistical data on the some VA is spent and does not represent any difficulty. For example, PR pay off for objects of a various class of a voltage, either a various design, or various service life. Occasionally PR pays off for two VA. For example, estimates PR linear switches with rated voltage of 110 kV. Thus, questions of expediency of classification of statistical data on these VA are not considered. Let's notice, that the concept "expediency" is indissolubly connected with concept "importance": classification of statistical data is inexpedient for insignificant VA.

Let's consider an essence of a solved problem. Let a result of gathering and processing of data on refusals and restoration of objects EPS we have some population of statistically data, formalized in the form of the empirical table. As this population depends on a lot of casual and not casual factors, it concerns to a class multivariate and final population of multivariate data (FPMD) is called. For FPMD about refusals of objects EPS absence of general population and, as consequence, inexpediency of application is characteristic at the analysis developed for sample of general population of well-known statistical methods. So, on FPMD it is required to estimate PR on some group VA.

VA are set or corresponding classifiers or are appointed. Thus, as a rule, number VA gets out subjectively (greatest possible) in conformity with aspiration to specify character of change PR. For quantitative scales of change of attributes as a first approximation, it is possible to start with optimum number VA that calculated under formula Starges:

$$K = 1+1{,}44\cdot\ln M \tag{1}$$

where M – number of realizations τ, and K – the number of intervals, which length is defined under the formula:

$$h = {(\tau_{max} - \tau_{min})}/{K} \qquad (2)$$

τ_{max} and τ_{min} – accordingly, the greatest and least value of realizations τ.

In the illustrative purposes in table 1 recommended value for of some values of M.

Table 1. An illustration of dependence M=f (K)

Intervals of change of number of realizations of M of random variables τ	Number	
	VA	Combinations VA
11-23	5	30
24-46	6	62
74-91	7	126
92-183	8	254

We notice, that in conditions of a solved problem K optimum on number of random variables τ (Under condition of conformity of distribution F(τ) to the normal law of distribution), but, as a rule, essentially exceeds number significant VA.

In turn, laconic record of separate conditions of objects, for example, in dispatching schedules, often limits possible number of attributes and their versions. Having specified VA, having collected and having systematized in empirical table FPMD, we shall pass to an estimation of importance VA. The recommended method based on imitating modeling of casual character of estimations PR and application of substantive provisions of the theory of check of statistical hypotheses. As the account of casual character of estimations PR demands hundred, and more often thousand realizations, calculations carried out on the developed computer technology. This technology consists of following operations:

1. It is defined average PR on all FPMD. We shall designate it as Π^*_Σ

The note. It is obvious, that estimations PR calculated on significant VA should differ not casually as from Π^*_Σ, and among themselves. Hence, generally, it is necessary to speak not about significant attributes and their versions, and about significant combinations VA. The general number of combinations VA we shall designate it as K_S. It can be calculated under the formula:

$$K_S = \sum_{i=1}^{K-1} C_K^i = \sum_{i=1}^{K} \frac{K!}{i!(K-i)!} \qquad (3)$$

For example, if K=3, then K_S =6 and possible versions of combinations will be 1; 2; 3; 1 and 2; 2 and 3; 1 and 3.

2. For each of K_S combinations VA sample of continuous random variables is defined τ;
3. Estimations PR for each of K_S samples pay off. We shall designate them as $\Pi^*_{V,i}$ with $i=1, K_S$;
4. Check of the assumption (hypothesis) H_1 about casual character of distinction $\Pi^*_{V,i}$ from Π^*_Σ for $i=1, K_S$ is spent. The technique of such check is resulted in [2];
5. Combinations VA, PR are allocated, which not casually disperse with Π^*_Σ;
6. Groups VA with not casually differing PR are defined;
7. Ranking PR of these groups by way of increase in an Errors II types, i.e. reduction of capacity of criterion is spent. That, establishes significant combinations VA, classification FPMD on specified VA is spent.

Practical realization of this method has shown that the big number of possible combinations VA brings bulkiness in carrying out of calculations and demands is inadmissible big time of the count. So, at the automated analysis of regularity of change of average duration of idle time in emergency repair of power units 300 MVt on gas-and-oil fuel on months of year when number VA K=12, number of combinations VA K_S=4094, and speed of the analysis of expediency of classification it is equal 10 combinations in minute, time of calculations, even at reduction of number of realizations N in 25 times, it has appeared unacceptable. However these calculations have allowed establishing:

1. Combinations VA including insignificant VA are insignificant. So, according to table 1 if from eight VA only three are *insignificant*, size K_S decreases in 8,5 times;

2. The number of significant combinations VA does not exceed number of insignificant combinations VA. Hence the estimation of expediency of classification FPMD on set VA is necessary for spending by search of significant combinations VA;

3. With increase in absolute size of relative deviation PR $\Pi^*_{V,i}$ *from* Π^*_Σ *with i=1,K_S an Errors I type (α) result of comparison* $\Pi^*_{V,i}$ *also* Π^*_Σ *decreases, and an Errors II type (β) – increases. This conclusion defines a way of ranging PR samples which are supposed to be compared with* Π^*_Σ ;

4. Settlement it is necessary to consider such combinations VA for which relative deviations PR of VA having an identical sign. So if for the some j1th significant VA, PR is equal $\Pi^*_{V,j1}$, and for j2th significant VA, PR is equal $\Pi^*_{V,j2}$, association j1 and j2 is possible, if $\delta\Pi_{V,j2}=(\Pi^*_\Sigma-\Pi^*_{V,j2})\big/\Pi^*_\Sigma$ $\delta\Pi_{V,j1}=(\Pi^*_\Sigma-\Pi^*_{V,j1})\big/\Pi^*_\Sigma$ coincide;

5. The algorithm considered above allows dividing VA into three groups. First group VA has PR equals Π^*_Σ *, the second group VA has PR equals* $\Pi^*_V>\Pi^*_\Sigma$ *, and the third group VA has PR* $\Pi^*_V<\Pi^*_\Sigma$.

In view of these results, following transformations of a method of an estimation of importance VA recommended:

1. For each of set K of VA is defined sample of continuous random variables τ (See prg.2 algorithm);

2. Estimations PR for each of K sample (see prg.3 algorithm) pay off. Relative changes of each of i=1,K estimations PR under the formula $\delta\Pi^*_{V,i}=\left|(\Pi^*_\Sigma-\Pi^*_{V,i})\right|\big/\Pi^*_\Sigma$ are defined. Ranking of absolute values $\delta\Pi^*_{V,i}$ with i=1,K by way of their decrease is spent. The greatest (first) value of absolute sizes $\delta\Pi^*_{V,\max}$ is allocated;

3. Check of the assumption (hypothesis H_1) about casual character of distinction $\Pi^*_{V,\max}$ from Π^*_Σ (see prg.4 algorithm) is spent. The method of comparison Π^*_Σ also Π^*_V depends on type PR. If, for example, П* there is a model of distribution of a random variable of duration of idle time in emergency repair statistical functions of distribution (s.f.d.) FPMD $F^*_\Sigma(\tau_{ав})$ and s.f.d. samples $F^*_V(\tau_{ав})$ according to [6] are compared. If the observable divergence Π^*_Σ and Π^*_V is casual, a divergence with Π^*_Σ PR, calculated for the others VA also will be casual. In other words, classification FPMD on considered VA is inexpedient. Otherwise, when $\Pi^*_{V,\max}$ not casually differs from Π^*_Σ , we pass to PR the following in variation number VA and we check character of its

divergence with Π^*_Σ. This process proceeds until a divergence Π^*_Σ and Π^*_V it will not appear casual.

4. Three group' samples from FPMD are formed. Into the first group enter VA, estimations PR, which casually differs from Π^*_Σ. We shall designate number of VA of first group as K1. These K1 VA withdrawn from full list VA, as insignificant VA. Into second group enter VA, estimations PR, which it is not casual more Π^*_Σ. If number VA of the second group K2 more than one that, as simplification, PR this group are calculated as an average arithmetic estimations PR of VA of the second group. With this estimation PR it is compared integrated VA. For example, if the second group included objects with rated voltage 110 and 220 kV, then integrated VA will be (110 – 220) kV. For VA the third K3 groups PR, $\Pi^*_{V,III}$ it is calculated as an average arithmetic samples random variables τ_{em} for which, $\Pi^*_\Sigma > \Pi^*_{V,i}$, instead of Π^*_V casually differs from Π^*_Σ. PR of $\Pi^*_{V,III}$ also it is compared integrated VA.

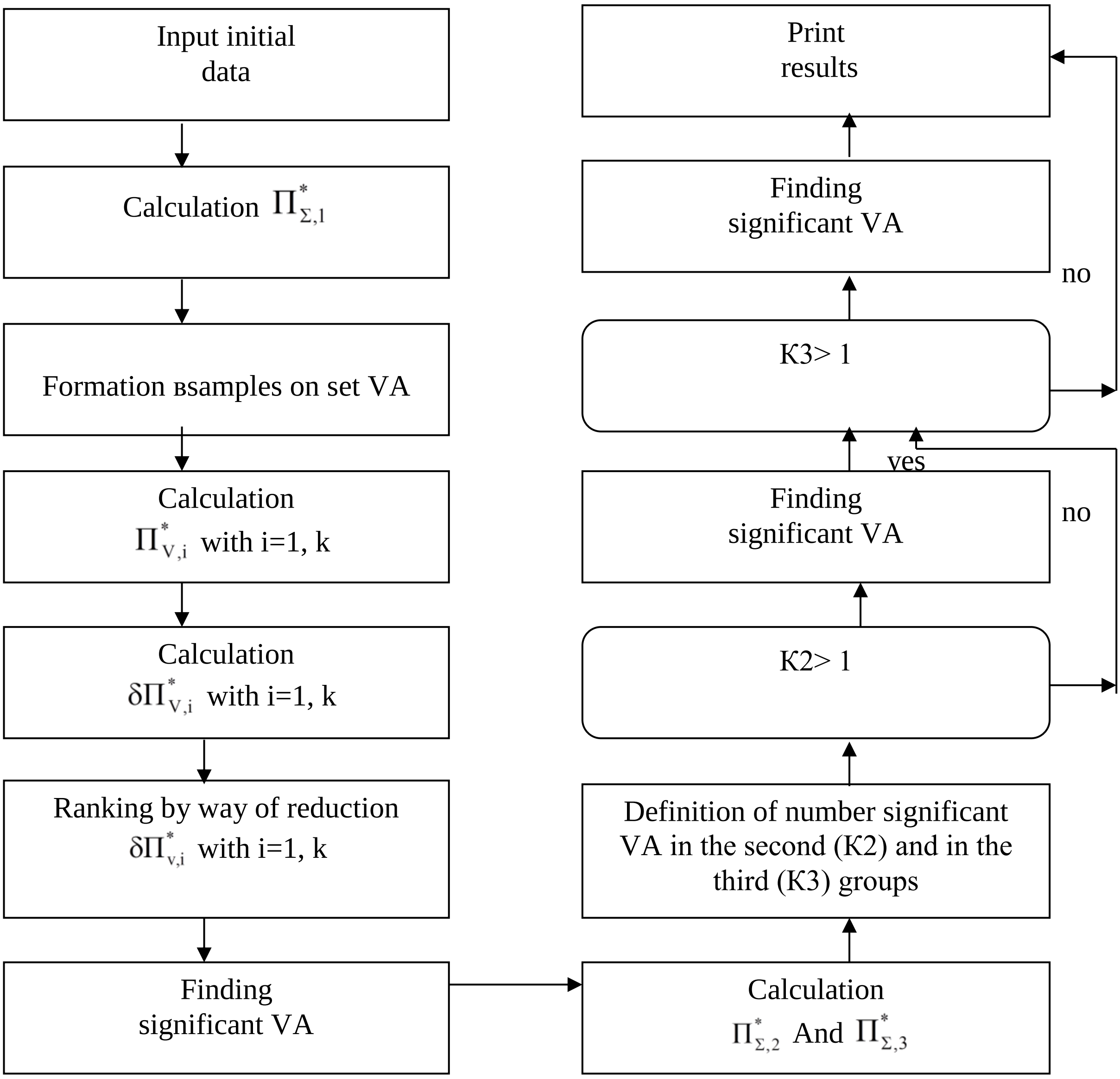

Fig.1. The integrated block scheme of algorithm of an estimation of importance VA.

On it construction of three-level dependence of change of estimations PR from VA comes to an end. In some cases (for example when it is required to establish group most or the least reliable objects) the additional information on object can be received, having increased number of levels of classification FPMD. For what, from samples of the second group (provided that their number К2>1) is formed the second FPMD, and from samples of the third group (at К3> 1) – is formed the third FPMD. Further, according to the sequence stated above the estimation of the importance of everyone VA and specification of their quantitative estimations PR is spent.

In figure 1 the integrated block diagram of algorithm of the decision of a problem about importance VA is resulted

Example 1. In table 2 statistical data about duration of emergency idle time are cited τ_{em} eight power units on gas-and-oil fuel capacity 300 MVt in the same interval of time.

Table 2. Data on emergency duration idle time of power units hs.

i	Serial numbers of power units							
	1	2	3	4	5	6	7	8
1	64,42	46,12	78,59	61,36	63,5	49,15	66,29	36,05
2	15,31	46,27	3,36	236,3	38,07	91,17	47,02	6,23
3	53,5	298,58	3,48	123,59		99,51	93,13	15,35
4	94,55	134,12	42,05	358,15		39,11	54,03	
5	69,37	35,51	45,15			133,24	79,21	
6	5,48		62,36				57,2	
7	185,0		18,15				66,1	
8			29,42				1,3	
9			7,43					
10			25,5					
$\tau_{em,i}$, hs	48,71	560,5	320,0	780,0	102,0	412,0	464,0	57,6

It is required to define estimations of average duration of idle time of power units in an emergency condition $M^*_{V,i}(\tau_{\text{ав}})$ with i=1,8. It is necessary to note, that analogue of a serial number of the power unit is service life. The preference to an attribute "serial number" is caused by an invariance of its versions while service life of power units annually changes.

Results of calculations of number of realizations n_v, average arithmetic value of realizations $M^*_{V,i}(\tau_{em})$, relative change $\delta M^*_{V,i}(\tau_{em})$, an Errors I type $\alpha_{v,i}$, i.e. probabilities of a errors of the conclusion (acceptance of hypothesis H_2) about not casual divergence of estimations $M^*_{V,i}(\tau_{\text{ав}})$ and $M^*_{\Sigma}(\tau_{\text{ав}})$, critical values for $\delta M^*_{V,i}(\tau_{em})$ at a significance value $\alpha_{c=0,05}$, the conclusion about character of a divergence $M^*_{V,i}(\tau_{em})$ with i=1,8 and $M^*_{\Sigma}(\tau_{em})$=72,4 hs. and recommended values $M^*_{V}(\tau_{\text{ав}})$ are resulted in table 3

Table 3. Results of an estimation of character of a divergence $M^*_{V,i}(\tau_{em})$ and $M^*_{\Sigma}(\tau_{em})$

Parameters	Serial numbers of power units							
	1	2	3	4	5	6	7	8
nv, i	7	5	10	4	2	5	8	3
$M^*_{V,i}(\tau_{em})$, Hour	69,7	112,1	32	195	51	82,4	58	19,2
$\delta M^*_{V,i}(\tau_{em})$, %	3,7	54,8	55,8	169,3	29,6	13,8	19,9	3,48
$\alpha_{v,\,i}$	0,96	0,03	<0,01	<0,01	0,37	0,60	0,35	0,02
$\delta_{0.05} M^*_{V,i}(\tau_{em})$, %	41,8	50,4	35,2	56,5	80,1	50,4	39,9	55,3
H	H1	H2	H2	H2	H1	H1	H1	H2

$M_V^*(\tau_{em})$, Hour	72,4	149	34,5	149	72,4	72,4	72,4	34,5

Estimations $M_{V,i}^*(\tau_{ав})$ are necessary, in particular, at calculations of duration of simultaneous idle time of some power units. In [3] it shown, that calculation of these estimations on the average parameters $M_\Sigma^*(\tau_{em})$ can lead to inadmissible inaccuracy. However, the inadmissible inaccuracy can be and owing to direct application in calculations of estimations $M_V^*(\tau_{em})$. As it has noted been earlier, application of estimations PR calculated on representative samples, is inexpedient. In other words, check of character of a divergence $M_\Sigma^*(\tau_{em})$ and $M_{V,i}^*(\tau_{em})$ with i=1,8 and is necessary for each of estimations $M_V^*(\tau_{ав})$ with other estimations. According to the algorithm stated above relative changes of estimations PR in percentage under the formula are allocated

$$\delta[M_{V,i}^*(\tau_{em})] = 100\frac{\left|[M_\Sigma^*(\tau_{em}) - M_{v,i}^*(\tau_{em})]\right|}{M_\Sigma^*(\tau_{em})},$$

On distribution of the possible realizations $\delta M_{V,i}^*(\tau_{em})$ modeled (**) according to [4], values of an Errors I type corresponding empirical values $\delta M_{V,i}^*(\tau_{em})$ are calculated $\alpha_{v,\ i}$. Further $\alpha_{v,\ i}$ are compared to critical value α_c, accepted equal 0,05. If $\alpha_{v,\ i} > \alpha_c$, then $H \Rightarrow H_2$ (an index ⇒ Designates "corresponds"), if $\alpha_{v,\ i} < \alpha_c$, $H \Rightarrow H_2$. Here results of calculation of critical values quantile of distributions $F^*\{\delta[M_{V,i}^*(\tau_{em})]\}$ under the formula [3] $\delta M_{V,i}^*(\tau_{em}) = 1.13/\sqrt{n_v}$ that confirms essential simplification of procedure of an estimation of expediency of classification of population of realizations are resulted τ_{em} on VA. As follows from table 2 of an estimation $M_V^*(\tau_{em})$ 1, 5, 6 and 7 power units casually differ from $M_\Sigma^*(\tau_{em})$ and equal 72,4 hours should be accepted, 3 and 8 power units concern to the second group (with the least values $M_V^*(\tau_{em})$), and 2 and 4 power units concern to the third group (with the greatest values $M_V^*(\tau_{em})$).

Average value of duration of emergency idle time of power units of the second group equally $M_{\Sigma,II}^*(\tau_{em}) = 34{,}5 hours.$, and the third group- $M_{\Sigma,III}^*(\tau_{em}) = 149 hours.$. In the illustrative purposes we shall estimate character of a divergence $M_\Sigma^*(\tau_{em}) = 72.4$ and $M_{\Sigma,II}^*(\tau_{em}) = 34{,}5$. For what we shall define:

1. $\delta M_{\Sigma,II}^*(\tau_{em}) = 100 \cdot \left|M_\Sigma^*(\tau_{em}) - M_{\Xi,II}^*(\tau_{em})\right| / M_\Sigma^*(\tau_{em}) = 52{,}3\%$
2. $\delta_{0,01} M_{\Sigma,II}^*(\tau_{em}) = 1.42 / \sqrt{n_{v,II}} = 39.5\%$

As $\delta M_{\Sigma,II}^*(\tau_{em}) >$ $\delta_{0.01} M_{\Sigma,II}^*(\tau_{em})$ the divergence $M_\Sigma^*(\tau_{em})$ also $M_{\Sigma,II}^*(\tau_{em})$ can be accepted not casual with a significance value not less α_c=0,01.

We shall assume now, that it is necessary for us to define the power unit with the least value $M_V^*(\tau_{em})$. For definition $M_{V,\min}^*(\tau_{em})$ it is spent following calculations:

1. As $M_{V,3}^*(\tau_{em}) = 32 > M_{V,2}^*(\tau_{em}) = 19.2$, size
   $\delta M_{v,8}^*(\tau_{em}) = 100 \cdot \left|M_\Sigma^*(\tau_{em}) - M_{v,8}^*(\tau_{em})\right| / M_{\Sigma,II}^*(\tau_{em}) = 44.3\%$
2. $\delta_{0,01} M_{v,8}^*(\tau_{em}) = 1.42 / \sqrt{3} = 82.1\%$

As $\delta M^*_{v,8}(\tau_{em}) < \delta_{0.01}M^*_{v,8}(\tau_{em})$, the assumption of not casual divergence $M^*_{\Sigma,II}(\tau_{em})$ and $M^*_{v,8}(\tau_{em})$ is erroneous.

We shall estimate character of a divergence $M^*_{\Sigma,II}(\tau_{em})$ and $M^*_{v,3}(\tau_{em})$, we calculate:

1. $\delta M^*_{v,3}(\tau_{em}) = {100\cdot\left|M^*_{\Sigma}(\tau_{em}) - M^*_{v,3}(\tau_{em})\right|}\Big/{M^*_{\Sigma,II}(\tau_{em})} = 7.2\%$
2. $\delta_{0,01}M^*_{V,3}(\tau_{em}) = 1.42\big/\sqrt{10} = 45.6\%$

Erroneous there was also an assumption of not casual divergence $M^*_{\Sigma,II}(\tau_{em})$ and $M^*_{v,3}(\tau_{em})$. Hence, $M^*_{v,3}(\tau_{em}) =$ $M^*_{v,8}(\tau_{em}) =$ 34,5hours.

Calculations for an estimation of character of a divergence $M^*_{\Sigma,III}(\tau_{em})$ are similarly lead and $M^*_{v,2}(\tau_{em})$, together with $M^*_{\Sigma,III}(\tau_{em})$ $M^*_{v,4}(\tau_{em})$. It is established, that $\delta M^*_{v,2}(\tau_{em}) = 24.8\%$ less than critical value $\delta_{0,05}M^*_{v,2}(\tau_{em}) = 50.4\%$, and $\delta M^*_{v,4}(\tau_{em}) = 30.9\% < \delta_{0,05}M^*_{v,4}(\tau_{em}) = 56.5\%$. Hence, $M^*_{V,2}(\tau_{em})$ and $M^*_{V,4}(\tau_{em})$, casually differ from $M^*_{\Sigma,III}(\tau_{em})$, and classification is inexpedient.

2. Method and algorithm of an estimation of parameters of individual reliability of objects

Despite of essential distinction of names of the first and second problem, algorithm of the decision of the second problem is easier. The estimation of expediency of classification FPMD on the set versions of one of attributes (the first problem) provides both an estimation significant VA, and an estimation of expediency of representation of an attribute the set list significant VA. The matter is that PR for some versions of one attribute, despite of the importance of these VA, can differ casually. This distinction can be caused by small number of realizations of random variables samples. I.e. the aspiration to so detailed representation of an attribute appears unjustified.

At estimation PIR analyze only importance VA, subjectively setting individuality of object [3]. The block scheme of algorithm of estimation PIR is resulted on fig. 2. If in algorithm of an estimation of importance VA (see fig. 1) is estimated the importance of each sample from FPMD in algorithm of estimation PIR consecutive classification originally FPMD is spent, further classification of the sample corresponding $\Pi^*_{V,\max}(X)$, further classification of the sample corresponding of two most significant VA, etc.

Calculations come to the end at the first casual divergence $\Pi^*_{\Sigma}(X)$ and $\Pi^*_{V}(X)$

Example 2. In the present example, we shall consider sequence of estimation PIR. For decrease in bulkiness of calculations, we shall consider only two attribute and their version – a serial number of the power unit and the basic devices of the power unit. Allocated: steam turbine and a boiler installation, system of own needs, a turbo generator, block transformers.

Data on duration of emergency idle time of the power unit, owing to refusal of one of these devices, are resulted in table 4.

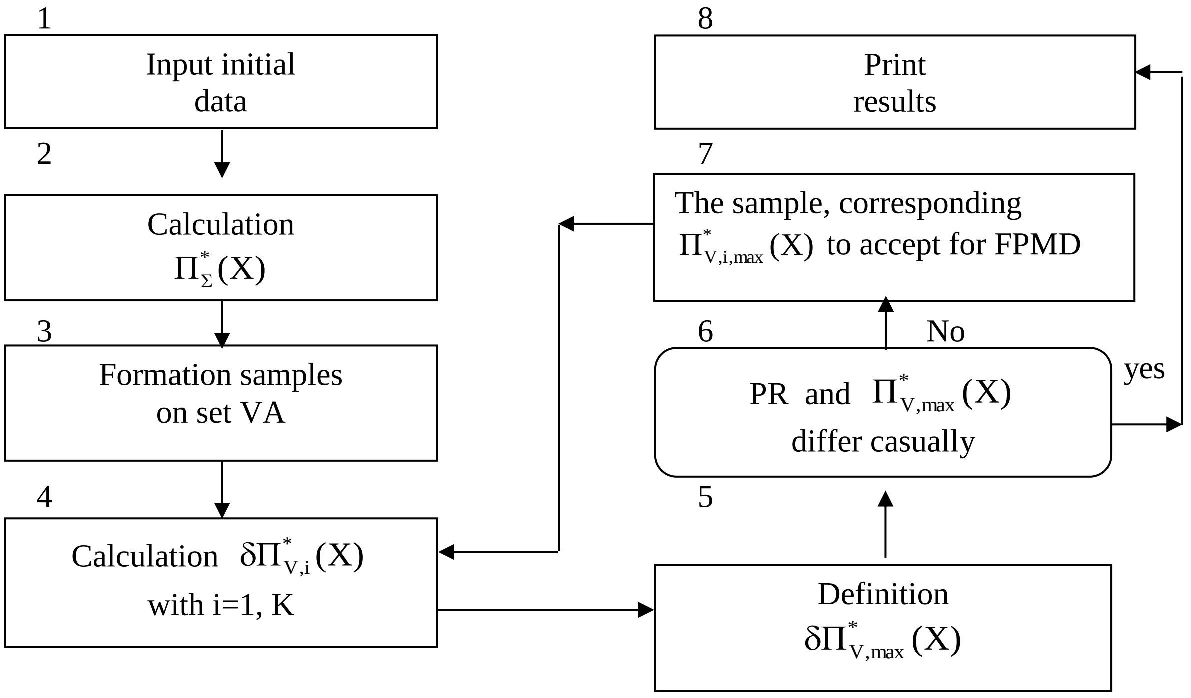

Fig. 2. The integrated block scheme of algorithm estimation PIR.

Table 4. Data on duration of emergency idle time at refusal of devices of power units, hs.

i	Devices									
	Steam turbine installations		Boiler installation		Own needs		Generators		Transformers	
	N PU	Hour	N PU	Hour	N PU	Hour	N PU	Hour	N PU	Hour
1	1	64,4	1	15,3	2	298,5	8	6,2	8	15,3
2	1	94,5	1	53,5	2	134,1	3	7,4	2	35,5
3	1	185,0	1	69,3	3	29,4	1	5,4		
4	2	46,2	2	46,1	3	25,5				
5	3	3,3	3	78,6	4	358,1				
6	3	3,4	3	42,0	6	133,2				
7	4	61,3	3	45,1						
8	4	236,3	3	62,3						
9	4	123,5	3	18,1						
10	6	49,1	5	63,5						
11	6	91,1	5	38,0						
12	7	66,2	6	99,5						
13	7	47,0	6	39,1						
14	7	93,1	7	54,0						
15	7	78,2								
16	7	57,2								
17	7	66,1								
18	7	1,3								
19	8	36,0								
$\tau_{em,i}$, hs	1404		725,2		979,2		19,2		50,9	
n_i	19		14		6		3		2	
$\Pi^*_{V,i}(X)$, hs	73,9		51,8		163,2		6,4		25,4	

Let's assume that it is necessary to estimate average duration of emergency idle time of the third power unit owing to refusal of boiler installation. As $M^*_{\Sigma}(\tau_{em}) = 72.4\,\text{hour}$, and $M^*_{V,3}(\tau_{ав}) = 32\text{hour}$ and $M^*_{V,ку}(\tau_{em}) = 51{,}8\,\text{hour}$, $\delta M^*_{\Sigma,\max}(\tau_{em}) = \delta M^*_{v,\max}(\tau_{em})$. But according to an example 1 the third power unit concerns to significant VA. We shall execute sample of realizations τ_{em} at refusals of boiler installation from a data population about refusals of the third power unit. It (see table.3): 78,59; 42,05; 45,15; 62,36; 18,15hour. It is necessary to establish expediency of such classification. According to algorithm fig.2 the relative deviation of average duration of emergency idle time of the power unit at refusals of its boiler installation $M^*_{V,3,bi}(\tau_{em}) = 49{,}3\,\text{hour}$ from $M^*_{V,3}(\tau_{em})$ will be equal

$$\delta M^*_{v,3,bi}(\tau_{em}) = \frac{100\cdot\left|M^*_{v,3,bi}(\tau_{em}) - M^*_{v,3,bi}(\tau_{em})\right|}{M^*_{v,3}(\tau_{em})} = 100\cdot|30-49.3|/32 = 59{,}3\%$$

Critical value $\delta_{0.05}M^*_{v,3,bi}(\tau_{em}) = 50{,}4\%$. Hence, with a significance value not less α_c=0.05 It is possible to approve, that $M^*_{V,3,bi}(\tau_{em})$ not casually differs from $M^*_{V,3}(\tau_{em})$ and it is equal 49,3hour.

3. Method and algorithm of an estimation of parameters and characteristics of reliability cluster' objects EPS

Having calculated PIR for population of the same objects it is easy to notice, that for significant VA there is not one, and the whole group of objects with equal PIR. For example, three-phase (the first VA), two winding (the second VA) transformers (the third VA), a voltage with 110 kV (the fourth VA), established on substations of distributive networks (the fifth VA) make about 20 % from the general number of transformers EPS.

In this connection, the opportunity of classification of analyzed objects on groups (clusters), their ranking by criterion of reliability of work and preparation of recommendations on perfection of system of maintenance service, the control of a technical condition and quality of repair of each group is of interest. Classification on three groups is as a first approximation sufficient: group high, group of average and group of low reliability.

If for calculation of parameters and characteristics of individual reliability initial data are one version of each attribute of object for calculation of parameters and characteristics of reliability clusters objects initial data are all versions of attributes. It would seem, enough to calculate PIR for of some the same objects and it is possible on VA to find clusters. However, this opinion as well as, equality PIR and PR clusters, wrongly. And first of all fixed VA at calculation PIR can appear significant, but classification on them – inexpedient. Also there is it because at calculations PIR character of a divergence between significant versions of same attribute is not considered. That is why significant it is necessary to consider VA, PR which differ not casually not only from PR, calculated on FPMD, but also between significant versions of same attribute. The essence of a method of calculation PR clusters FPMD reduced to following sequence of calculations:

1. For each of attributes of considered objects from the general number allocated VA significant combinations VA (see algorithm of an estimation of importance VA) are established. We shall designate number of significant combinations of versions i^{th} an attribute through r_i with i=1,m, where m - number of attributes of object;
2. For each attribute are calculated significant VA and are defined VA with the greatest estimation PR $\Pi^*_{V,i,\max}$, where $\Pi^*_{V,i,\max} = \max[\Pi^*_{V,1,1};\Pi^*_{V,2,2};\ldots\Pi^*_{C,n,r_i}]$; i=1, n; n – number of attributes; r_i – number of significant versions i^{th} an attribute;

3. Among $\Pi^*_{V,i,max}$ with i=1,n the greatest value PR $\Pi^*_{V,i,max} = \max[\Pi^*_{V,1,max}; \Pi^*_{V,2,max}; .... \Pi^*_{C,n,max}]$ is defined;
4. Sample of realizations of conformity $\Pi^*_{V,i,max}$ is represented as FPMD and for all significant combinations of each attribute, except for corresponding $\Pi^*_{V,i,max}$, the greatest values among versions of each attribute and the greatest average of all (m-1) attributes are calculated. Classification of this sample proceeds until estimation PR on FPMD and an estimation on sample with $\Pi^*_{V,i,max}$ will not disperse casually.

At achievement of this event, current FPMD it is withdrawn from initial FPMD. It is analyzed new FPMD. Process of classification FPMD proceeds until distinction of estimations PR calculated on FPMD and sample with $\Pi^*_{V,max}$ will not appear casual.

Example 3. To lower bulkiness of calculations illustration of estimation PR clusters we shall lead on statistical data tab. 2 and 4, i.e. classification we shall lead only to two attributes "serial number" of the power unit and "device" of the power unit. In an example 1 the sequence of calculations PR for an attribute – a serial number of the power unit has been resulted. It established that the greatest value $M^*_V(\tau_{ав})$ takes place for group of the second and fourth power units and 149 hour is equal. Results of the calculations, allowing estimating the importance of versions of an attribute of the device of power unit Thermal Power Stations (TPS), are resulted in table 5

Table 5. An estimation of character of a divergence $M^*_\Sigma(\tau_{ав})$ and $M^*_{V,i}(\tau_{ав})$ with i=1,5

Parameter	Devices of power unit TPS				
	Steam turbine installation	Boiler installation	Own needs	Turbo generator	The block transformer
$\delta M^*_{v,i}(\tau_{em})$, %	2,1	28,5	125	91.2	64,9
$\delta_{0,05}M^*_{v,i}(\tau_{em})$, %	25,9	30,2	46,1	65,3	80,1
H	H_1	H_1	H_2	H_2	H_1
$M^*_i(\tau_{em})$	72,4	72,4	163,2	6,4	72,4

As excess of a relative deviation $\delta M^*_{v,i}(\tau_{em})$ of critical value $\delta_к M^*_{v,i}(\tau_{em})$ follows from table 5 is observed at refusals in system of own needs and refusals of the generator. In other words, these two VA appear significant. Having established significant versions of each attribute, we shall define the most significant VA by comparison of relative deviations $\delta M^*_v(\tau_{em})$. These are realizations τ_{em} at refusals in system of own needs. We shall lead classification of six realizations τ_{em} at refusals in system of own needs (see table.4) on serial numbers of power units. We shall notice, that results of calculation of character of a divergence of estimations of average duration of emergency idle time of power units $M^*_{v,i}(\tau_{em})$ where i=1,8 with $M^*_\Sigma(\tau_{em})$ can and not coincide with an estimation of character of distribution of average duration of emergency idle time of power units owing to refusals in system of own needs. According to table 3 and the stipulated condition of classification $n_v>1$, $M^*_{v,i,ON}(\tau_{em})$ can be calculated and compared with $M^*_{v,ON}(\tau_{em}) = 163{,}2$ hour only for the second $M^*_{v,2,ON}(\tau_{em}) = 216{,}4$ hour and third $M^*_{v,3,ON}(\tau_{em}) = 27{,}5$ hour power units. However, considering, that $M^*_{v,3,ON}(\tau_{em}) < M^*_{v,ON}(\tau_{em}) < M^*_{v,2,ON}(\tau_{em})$, we shall be limited only to calculations for the second power unit. At nv=2 $\delta M^{*}_{v,2,ON}(\tau_{em}) = 100 \cdot \left| {(216.4-163.2)}/{163.2} \right| = 32.6\%$; $\delta_{0.05} M^{*}_{v,2,ON}(\tau_{em}) = 80.1\%$. Hence, sample τ_{em} for the second power unit at refusals in system of own needs the divergence between $M^{*}_{v,ON}(\tau_{em})$ and $M^{*}_{v,2,ON}(\tau_{em})$ with a high probability casually cannot be considered as unpresentable, i.e. For transition to the second stage of calculations from FPMD it is withdrawn six realizations τ_{em}, connected with refusals in system of own needs. For new FPMD are calculated:

- Average arithmetic value FPMD $M^{*}_{\Sigma,2}(\tau_{em}) = 58.6\,hs$;
- Average arithmetic value $M^{*}_{v}(\tau_{em})$ for everyone VA except for τ_{em} at refusals in system of own needs;
- Absolute value of a relative deviation $\delta M^{*}_{v}(\tau_{em})$ for everyone VA;
- Critical values $\delta_{\alpha_c} M^{*}_{v}(\tau_{em})$ at α_c=0,05;
- Are allocated significant VA;
- The most significant is defined VA.

Results of calculations are resulted in table 6 and 7

Table 6. Results of an estimation of the importance of versions of an attribute « number of the power unit »

Number of the power unit (i)	Parameters			
	$M^{*}_{v,i}(\tau_{em})$	$n_{v,i}$	$\delta M^{*}_{v,i}(\tau_{em})$	$\delta_{0.05} M^{*}_{v,i}(\tau_{em})$
1	69.7	7	18.9	42.6
2	42.6	3	27.3	65.3
3	33.1	8	43.5	39.9
4	140.6	3	140	65.3
5	51.0	2	12.9	79
6	69.7	4	18.9	56.5
7	58.0	8	1.0	43.5
8	19.2	3	67.2	65.3

Table 7. Results of an estimation of the importance of versions of an attribute of "device"

Devices	Parameters			
	$M^{*}_{v,i}(\tau_{em})$	$n_{v,i}$	$\delta M^{*}_{v,i}(\tau_{em})$	$\delta_{0.05} M^{*}_{v,i}(\tau_{em})$
Steam turbine installation	73,9	19	26,1	25,9
Boiler installation	51,8	14	11,6	30,2
System of own needs	-	-	-	-
Turbo generator	6,4	3	89	65,3
The block transformer	25,4	2	56,5	79

Analysis of given these tables show that to significant it is necessary to carry following VA: the third, fourth and second power units, steam turbine installation and a turbo generator. However, to compare follows only given the fourth power unit and idle times in emergency repair at refusal of a turbo generator. Excess of size $\delta M^{*}_{v,i}(\tau_{em})$ the fourth power unit above other power units, obviously. Classification of data of the fourth power unit is impossible, since all idle times in emergency repair passed at refusals steam turbine installations (see table 4).

To pass to the third stage, we shall exclude FPMD the second stage data about τ_{em} the fourth power unit. We shall receive $M^{*}_{\Sigma,3}(\tau_{em}) = 51\,\text{hour}$.

Having executed the calculations similar in detail presented for second stage, we shall receive:

1. For an attribute «number of the power unit » significant versions are absent;
2. For an attribute of "device», one significant version is revealed only: data about τ_{em} turbo generators. As average arithmetic value of realizations of this РП $M^*_{v,i}(\tau_{em}) < M^*_{v,3}(\tau_{em})$. Classification on VA with $M^*_{v,i}(\tau_{em})$ exceeding $M^*_{\Sigma}(\tau_{em})$ it is possible to consider that completed;
3. Classification of the sample corresponding significant VA is not spent, since number of realizations for each of three power units n_v=1

Calculations of the fourth stage of calculations testify to full absence significant VA, uniformity FPMD (for two attributes).

Thus, 4 groups of data are allocated. The first, most representative group, covers 73 % of data, has $M^*_v(\tau_{em}) = 55\,hour$. The second group reflects τ_{em} power units at refusals in system of own needs, it $M^*_v(\tau_{em}) = 163\,hour$. The third group characterizes τ_{em} the fourth power unit, it $M^*_v(\tau_{em}) = 140\,hour$. The fourth group allocates τ_{em} because of refusals of turbo generators observable in the considered period, it $M^*_v(\tau_{em}) = 6\,hour$. So small duration of idle time does not cause surprise if to consider, that this device includes not only actually a turbo generator, but also its system of cooling, system of excitation, system of relay protection, automatics and management, duration of which restoration of refusal are essentially various.

Average durations of idle time in emergency repair of separate groups allow to pass from s.f.d. realizations of duration of emergency idle time $F^*(\tau_{em})$ to integrated s.f.d. duration of emergency conditions T_{em} of power units $F^*(T_{em,i})$

CONCLUSION

1. Methods, algorithms and programs are developed:
    - Estimations of the importance of versions of attributes;
    - Estimations of parameters of individual reliability of objects;
    - Estimations of parameters of reliability clusters objects
2. Essential advantage of these methods is the opportunity to raise objectivity of the decision of many operational problems on had statistical data;
3. Results of researches allow pass from the traditional analysis of statistical data of operation of the equipment and devices of electro power systems as representative sample of general population to methods of the analysis, these data considering multivariate character

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RELIABILITY AND SENSITIVITY ANALYSIS OF THE K-OUT-OF-N:G WARM STANDBY PARALLEL REPAIRABLE SYSTEM WITH REPLACEMENT AT COMMON-CAUSE FAILURE USING MARKOV MODEL

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ABSTRACT

Standby redundancy is a technique that has been widely applied for improving system reliability and availability in system design. In this paper, probabilistic model for a redundant system with replacement at each common-cause failure has been developed to analyze the reliability measures using Markov models. We investigate the reliability and sensitivity analysis of k-out-of-n:G warm standby parallel repairable system. All failure and repair times of the system are exponentially distributed and when one of the operating primary units fails then it is instantaneously replaced by a warm standby unit if one is available. Comparative analysis of reliability measures between two dissimilar configurations has been developed. Configuration I is a 2-out-of-4:G warm standby parallel repairable system, while Configuration II is a 2-out-of-5:G warm standby parallel repairable system. We get a closed-form solution of the reliability measures of the system for the two configurations. Comparisons are performed for specific values of system parameter. Sensitivity analysis is also carried out to depict the effect of various parameters on the reliability function and mean time to failure of the system. Numerical example is given to illustrate the results obtained.

Keywords: k-out-of-n:G warm standby, reliability, parallel, common-cause failure, replacement, Markov model, sensitivity analysis.

1. Introduction

Standby redundancy is a technique used to improve system reliability and availability. Standby redundancy represents a situation with one unit operating and a number of units on standby. Gnedenko et al. (1969) classified standby redundancy according to failure characteristics; hot standby, cold standby, and warm standby. In hot standby redundancy, each unit has the same failure rate regardless of whether it is in standby or in operation. In cold standby redundancy, only one component will be working at any given time, the others being standbys and not working. One of the standby components starts working only when the currently working component fails. In warm standby redundancy, the standbys may fail in standby state but with a failure rate smaller than that of the primary component but is greater than zero. Operative and warm standby units can be considered to be repairable.

Warm standby repairable systems have received attention by several authors. Guo et al. (2012) analyzed the dynamic behavior of a two unit parallel system with warm standby and common-cause failure. The system is composed of three identical units; two units are operating and one unit is in warm standby state. Yun and Cha (2006) proposed a general method for modeling a warm standby system with three units and derived the system performance measures (system reliability and mean life); one unit is operating in an active state and two units wait in warm standby state. Hajeeh (2011) studied reliability and availability for four series configurations with both warm and cold standby and common-cause failure. Dhillon and Yang (1992) and Dhillon (1993) analyzed the reliability and availability of warm standby systems with common-cause failures and human errors. Labib (1991) proposed the stochastic analysis of a two-unit warm standby system with two switching devices. Singh (1989) considered a warm standby redundant system with (M+N) identical units, r repair facilities. The system is under common-cause failure and repair times are arbitrary distributed. Srinivasan and Subramanian (2006) developed reliability and availability functions of a three unit warm standby system with identical components. In this model, one unit is working at the beginning and the other two are in standby.

The k-out-of-n:G repairable system is one of the most popular and widely used systems in practice. The k-out-of-n:G systems have been studied in certain situations where redundancy is of importance. Redundancy is required not only to extend the functioning of the system but also to achieve a certain reliability of the system. The k-out-of-n:G systems can be classified into:

- **Active redundant systems (k-out-of-n:G system):** in which all the n units are active even though only k units are required for the proper functioning of the system;
- **Cold standby systems**: in which the n-k cold standby units will not be active and upon failure of one of the k active units, cold standby unit will instantaneously replace the failed unit;
- **Warm standby systems:** in which the n-k warm standby units will have a smaller failure rate compared to the k active ones;
- **Hot standby systems:** in which the n-k hot standby units and the k active ones will have the same failure rate.

Due to their importance in industries and design, the k-out-of-n:G systems have received attention from different researchers. El-Damcese and El-Sodany (2014) analyzed the reliability and availability of a k-out-of-n:G system with three failures using Markov model. El-Damcese (2009) presented the reliability and availability analysis of a k-out-of-(M+S):G warm standby system with time varying failure and repair rates in presence of common-cause failure. El-Damcese (2010) presented continuous-time homogeneous Markov process to evaluate availability, reliability and MTTF for circular consecutive k-out-of-n:G system with repairman. El-Damcese (2009) analyzed the k-out-of-n:G system model with critical human errors, common-cause failures and time dependent system repair-rate. Zhang et al. (2006) analyzed the k-out-of-(M+N):G warm standby system. In the system, not all components in standby can be treated as identical as they have different failure and repair rates. Kumar and Bajaj (2014) analyzed the vague reliability of k-out-of-n:G system (particularly, series and parallel system) with independent and non-identically distributed components, where the reliability of the components are unknown. The reliability of each component has been estimated using statistical confidence interval approach. Then converted these statistical confidence intervals into triangular fuzzy numbers. Based on these triangular fuzzy numbers, the reliability of the k-out-of-n:G system has been calculated. Moustafa (2008) presented a continuous time Markov chain (CTMC) model to obtain closed form expressions of the mean time between system failures (MTBF) for k-out-of-n:G systems subject to M exponential failure modes and repairs. She and Pecht (1992) made a brief review on standby redundancy techniques. In their research, a general closed form equation was developed for system reliability of a k-out-of-n warm standby system. Chryssaphinou et al. (1997) considered a 1-out-of-(m+1) warm standby system with non-identical units. Goel et al. (1989) analyzed a 1-out-of-3 warm standby system with two types of spare units a warm and a cold standby unit, and inspection. A general closed form equation was developed for system reliability of a k-out-of-n warm standby system where components in k-out-of-n:G standby systems were assumed to be statistically identical. An analysis on 1-out-of-2:G warm standby system has been presented by Henley and Kumamoto (1992). Yusuf and Bala (2013) analyzed the mean time to system failure (MTSF) of a repairable 2-out-of-4 warm standby system. Yusuf and Gimba (2013) analyzed the MTSF of 2-out-of-5 warm standby repairable system with replacement at the occurrence of each common-cause failure using Kolmogorov's forward equations method.

In recent years, it has been realized that in order to predict realistic reliability and availability of standby systems, the occurrence of common-cause failures must be considered. Common-cause failures can only occur in the system with more than one good unit. A common-cause failure is defined as any instance where multiple units or components fail due to a single cause. The concept of common-cause failure and its impact on reliability measures of system effectiveness has been introduced by several authors. Dhillon and Anude (1993) studied common-cause failure analysis of a non-identical unit parallel repairable system with arbitrary distributed repair times. Haggag (2009) studied cost analysis of a system involving common-cause failures and preventive maintenance. Vashisth (2011) have analyzed the reliability of redundant system with common-cause failure. Maintaining a system with common-cause failure is often an essential requisite.

Series system is a configuration in which all components are in series and all components have to work for the system to work. If any one of the system components fails, the system fails. Whereas the parallel system fails only when all the system components fails.

Parallel configuration is used to increase the reliability of a system without any change in the reliability of the individual components that form the system. The problem of evaluating the availability and reliability of the parallel system has been subject of many studies throughout the literature (Kolowrocki (1994), Pan and Nonaka (1995), Ebeling (2000) and Kwiatuszewska (2001)).

In this paper, we analyze the reliability measures of the k-out-of-n:G warm standby parallel repairable system with replacement at each common-cause failure with constant failure and repair rates of the operating primary units and warm standby units using Markov model.

The following reliability measures of the system are obtained using Markov model for two configurations. Configuration I is a 2-out-of-4:G warm standby parallel repairable system, while Configuration II is a 2-out-of-5:G warm standby parallel repairable system.

i. Availability and steady state availability of the system.
ii. Reliability and mean time to system failure.

We also perform sensitivity analysis for changes in the reliability characteristics along with changes in specific values of the system parameters.

This paper is organized as follows: Section 2 is devoted to the description and basic assumptions of the system. Section 3 is devoted to the reliability and availability analysis of the k-out-of-n:G warm standby parallel repairable system. In Section 4 we make a comparative analysis of the reliability measures of two dissimilar configurations of the k-out-of-n:G warm standby parallel repairable system. Sensitivity analysis is carried out to depict the effect of various parameters on the reliability function and mean time to failure of the system. In Section 5, a numerical example is given. In Section 6, some concluding remarks are given.

2. Model Description and Assumptions

The following assumptions are associated with the system:

1. The system under consideration is a k-out-of-n:G warm standby parallel repairable system. At least k units of the system are required for the system to work.
2. The system consists of k primary units and $n-k$ warm standby units and all the units are identical.
3. The system is subject to failure of a single unit and common-cause failure of more than one unit.
4. All primary units and warm standby units are considered to be repairable.
5. Each of the primary units fails independent of the state of the others, according to an exponential failure time distribution with parameter λ, and the available warm standby units can also fail according to an exponential failure time distribution with parameter $\lambda_s, \left(0 < \lambda_s < \lambda\right)$.
6. When one of the primary units fails, it is instantaneously replaced by a warm standby unit if one is available. Switching from warm standby to operative unit is perfect and instantaneous.
7. When a standby unit switches into the operating primary unit successfully, its failure characteristics will be the same as that of the operating primary units.
8. Whenever one of the operating units or warm standby units fail, it is immediately sent to a repair. After repairing, the failed unit works like a new one.
9. There is a single repairman who attends to the failed units.
10. The repairmen can repair only one failed unit at a time.
11. The failed system repair times are exponentially distributed. The units are repaired according to an exponential repair time distribution with parameter μ.
12. Common-cause failure and failure of a single unit are statistically independent.
13. The common-cause failure affects only the units in operation and the affected units are replaced instantaneously.
14. The system at any working state can completely fail due to common-cause failure with constant common-cause failure rate.
15. When the system fails, no failure will occur for other working components.

Notations:

S_i : state of the system, $i = 0,1,2,....,n-k,n$

λ : failure rate of a single primary unit

λ_s : failure rate of a single warm standby unit

μ : repair rate of a single unit

λ_{cj} / β_j : common-cause failure rate/replacement rate of j units, $j = 2,3,4,....,n$

$P_i(t)$: probability that the system is in state i at time t , $i=0,1,2,....,n-k,n$

$P_i^*(s)$: Laplace transformation of $P_i(t)$, $i=0,1,2,....,n-k,n$

$A_1(t)/A_2(t)$: availability of Configuration I / Configuration II

A_1/A_2 : steady state availability of the Configuration I / Configuration II

$R_1(t)/R_2(t)$: reliability of Configuration I / Configuration II

$R_1^*(s)/R_2^*(s)$: Laplace transformation of the reliability function of Configuration I / Configuration II

$MTTF_1/MTTF_2$: mean time to failure of Configuration I / Configuration II

3. Reliability and Availability Analysis of the System

With the help of the above notations and possible states of the system; the state transition diagram of the k-out-of-n:G warm standby parallel repairable system with replacement at each common-cause failure is shown in Figure 1.

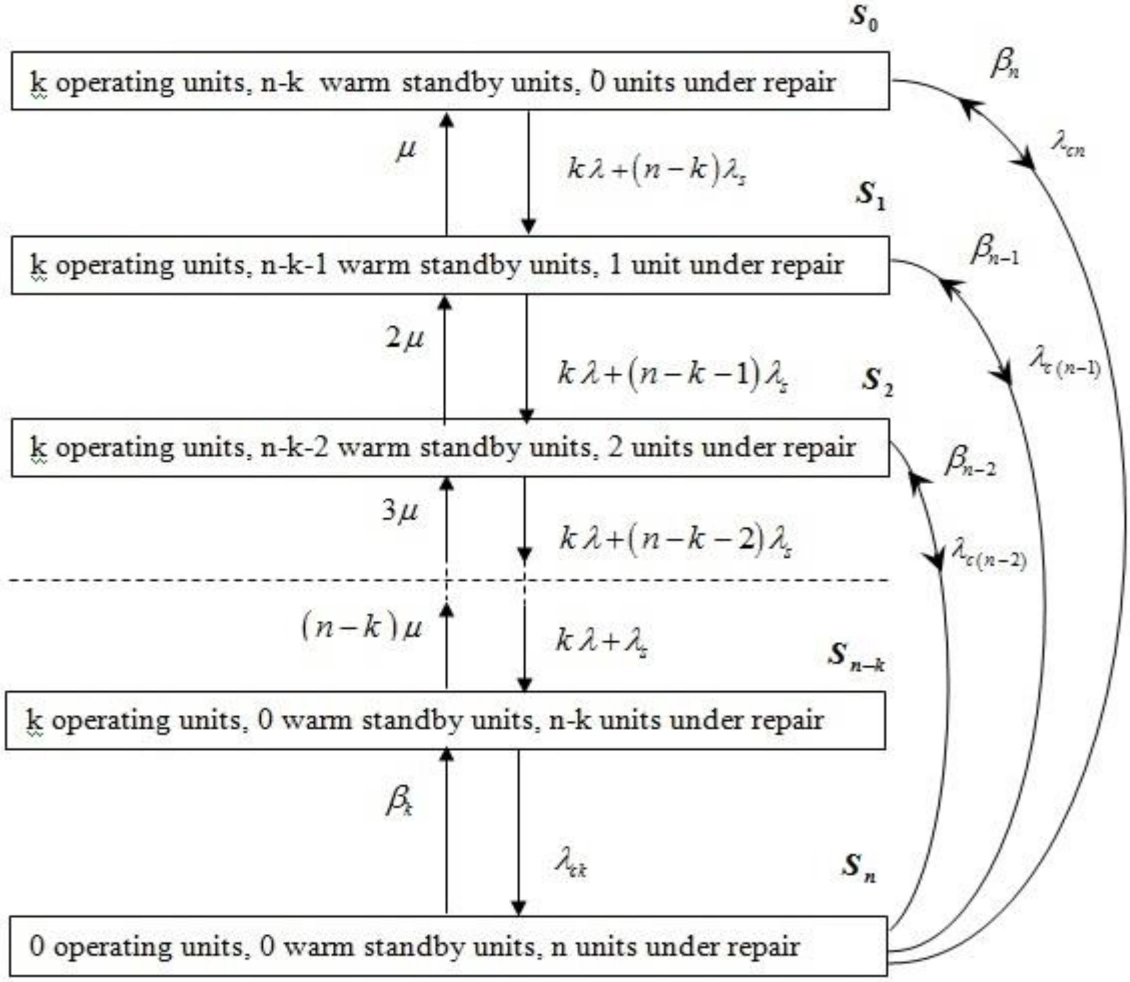

Figure 1: State transition diagram of the k-out-of-n:G warm standby parallel repairable system

Probability considerations gives the following set of differential difference equations associated with the state transition diagram of the k-out-of-n:G warm standby parallel repairable system:

$$\frac{d}{dt}P_0(t) = -\left(k\lambda + (n-k)\lambda_s + \lambda_{cn}\right)P_0(t) + \mu P_1(t) + \beta_n P_n(t) \tag{1}$$

$$\begin{aligned}\frac{d}{dt}P_i(t) = &-\left(k\lambda + (n-k-i)\lambda_s + \lambda_{c(n-i)} + i\mu\right)P_i(t) + \left(k\lambda + (n-k-i+1)\lambda_s\right)P_{i-1}(t)\\ &+(i+1)\mu P_{i+1}(t) + \beta_{n-i}P_n(t) \qquad ,0<i<n-k\end{aligned} \tag{2}$$

$$\frac{d}{dt}P_{n-k}(t) = -\left(\lambda_{ck} + (n-k)\mu\right)P_{n-k}(t) + \left(k\lambda + \lambda_s\right)P_{n-k-1}(t) + \beta_k P_n(t) \qquad ,n-k\geq 2 \tag{3}$$

$$\frac{d}{dt}P_n(t) = -\sum_{i=0}^{n-k}\beta_{n-i}P_n(t) + \sum_{i=0}^{n-k}\lambda_{c(n-i)}P_i(t) \qquad ,n-k\geq 2 \tag{4}$$

The system availability is given by:

$$A(t) = \sum_{i=0}^{n-k}P_i(t) \tag{5}$$

The initial conditions of the system are

$$\begin{aligned}&P_0(0) = 1\\ &P_i(0) = 0, \qquad i = 1,2...,n-k,n\end{aligned} \tag{6}$$

To obtain the reliability function of the system, we assume that the failed states are absorbing states and set all transition rates from these states equal to zero. Now let $P_i(t) \to \tilde{P}_i(t) \quad ,i = 0,1,....,n-k,n$ in Eqs.(1-4).

The system reliability is given by:

$$R(t) = \sum_{i=0}^{n-k}\tilde{P}_i(t) \tag{7}$$

4. Comparative Analysis of Reliability Measures

On the basis of the above description and assumptions, we investigate the reliability measures of two dissimilar configurations. Configuration I is a 2-out-of-4:G warm standby parallel repairable system, while Configuration II is a 2-out-of-5:G warm standby parallel repairable system.

4.1 Configuration I

Configuration I consists of 2 operating primary units and 2 warm standby units and all the units are identical and at least 2 units are required for the system to work.

i. Availability Analysis of the System

The state transition diagram of the 2-out-of-4:G warm standby parallel repairable system is shown in Figure 2.

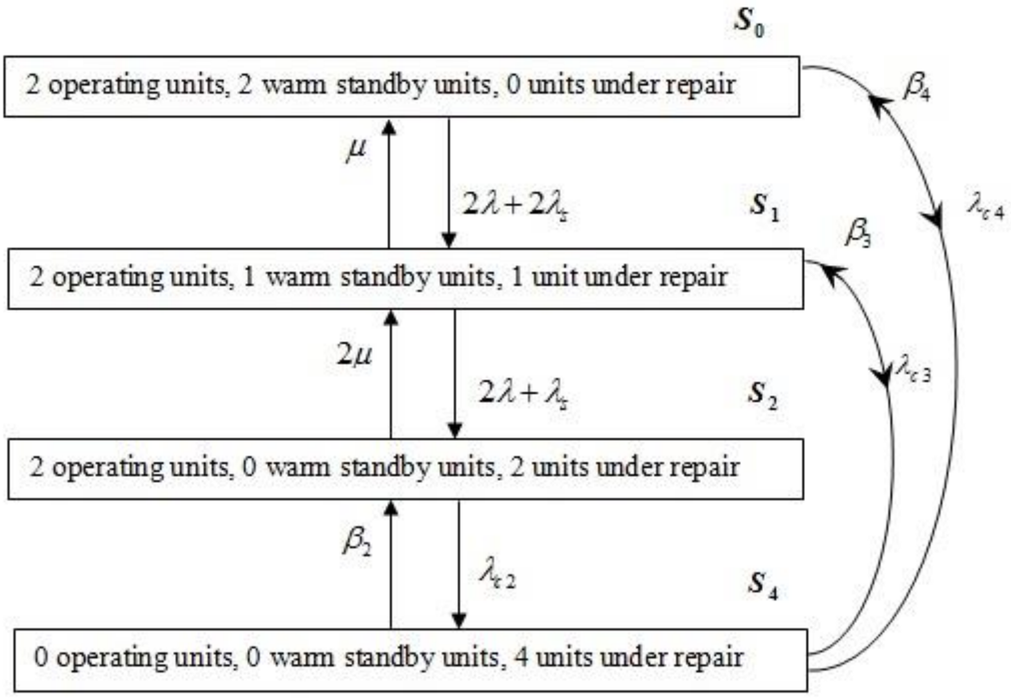

Figure 2: state transition diagram of 2-out-of-4:G warm standby parallel repairable system

The system of differential difference equations associated with the state transition diagram of the system are given by:

$$\frac{d}{dt}P_0(t)=-(2\lambda+2\lambda_s+\lambda_{c4})P_0(t)+\mu P_1(t)+\beta_4 P_4(t) \tag{8}$$

$$\frac{d}{dt}P_1(t)=-(2\lambda+\lambda_s+\lambda_{c3}+\mu)P_1(t)+(2\lambda+2\lambda_s)P_0(t)+2\mu P_2(t)+\beta_3 P_4(t) \tag{9}$$

$$\frac{d}{dt}P_2(t)=-(\lambda_{c2}+2\mu)P_2(t)+(2\lambda+\lambda_s)P_1(t)+\beta_2 P_4(t) \tag{10}$$

$$\frac{d}{dt}P_4(t)=-(\beta_2+\beta_3+\beta_4)P_4(t)+\lambda_{c4}P_0(t)+\lambda_{c3}P_1(t)+\lambda_{c2}P_2(t) \tag{11}$$

The system availability is given by

$$A_1(t)=P_0(t)+P_1(t)+P_2(t) \tag{12}$$

The initial conditions of the system are given by

$$\begin{aligned} &P_0(0)=1 \\ &P_1(0)=P_2(0)=P_4(0)=0 \end{aligned} \tag{13}$$

The steady state equations of the system are then

$$0=-\left(2\lambda+2\lambda_s+\lambda_{c4}\right)P_0+\mu P_1+\beta_4 P_4 \tag{14}$$

$$0=-\left(2\lambda+\lambda_s+\lambda_{c3}+\mu\right)P_1+\left(2\lambda+2\lambda_s\right)P_0+2\mu P_2+\beta_3 P_4 \tag{15}$$

$$0=-\left(\lambda_{c2}+2\mu\right)P_2+\left(2\lambda+\lambda_s\right)P_1+\beta_2 P_4 \tag{16}$$

$$0=-\left(\beta_2+\beta_3+\beta_4\right)P_4+\lambda_{c4}P_0+\lambda_{c3}P_1+\lambda_{c2}P_2 \tag{17}$$

Solving the system of linear Eqs.(14-17) using Maple program, we get the state probabilities determining the steady state availability of the system:

The steady state availability of the system is given by

$$A_1=P_0+P_1+P_2 \tag{18}$$

ii. System Reliability and Mean Time to Failure

We assume that the failed states are absorbing states and set all transition rates from these states equal to zero. Now let $P_i(t)\rightarrow\tilde{P}_i(t)\quad ,i=0,1,2,4$ in Eqs.(8-11).

The set of differential equations associated with the system are given by:

$$\frac{d}{dt}\tilde{P}_0(t)=-\left(2\lambda+2\lambda_s+\lambda_{c4}\right)\tilde{P}_0(t)+\mu\tilde{P}_1(t)$$
(19)

$$\frac{d}{dt}\tilde{P}_1(t)=-\left(2\lambda+\lambda_s+\lambda_{c3}+\mu\right)\tilde{P}_1(t)+\left(2\lambda+2\lambda_s\right)\tilde{P}_0(t)+2\mu\tilde{P}_2(t) \tag{20}$$

$$\frac{d}{dt}\tilde{P}_2(t)=-\left(\lambda_{c2}+2\mu\right)\tilde{P}_2(t)+\left(2\lambda+\lambda_s\right)\tilde{P}_1(t) \tag{21}$$

$$\frac{d}{dt}\tilde{P}_4(t)=\lambda_{c4}\tilde{P}_0(t)+\lambda_{c3}\tilde{P}_1(t)+\lambda_{c2}\tilde{P}_2(t) \tag{22}$$

The system reliability is given by

$$R_1(t)=\tilde{P}_0(t)+\tilde{P}_1(t)+\tilde{P}_2(t) \tag{23}$$

Taking Laplace transformation of Eqs.(19-22) using the initial conditions Eq.(13), we obtain:

$$\left(s+2\lambda+2\lambda_s+\lambda_{c4}\right)\tilde{P}_0^*(s)-\mu\tilde{P}_1^*(s)=1 \tag{24}$$

$$\left(s+2\lambda+\lambda_s+\lambda_{c3}+\mu\right)\tilde{P}_1^*(s)-\left(2\lambda+2\lambda_s\right)\tilde{P}_0^*(s)-2\mu\tilde{P}_2^*(s)=0 \tag{25}$$

$$\left(s+\lambda_{c2}+2\mu\right)\tilde{P}_2^*(s)-\left(2\lambda+\lambda_s\right)\tilde{P}_1^*(s)=0 \tag{26}$$

$$s\tilde{P}_4^*(s)-\lambda_{c4}\tilde{P}_0^*(s)-\lambda_{c3}\tilde{P}_1^*(s)-\lambda_{c2}\tilde{P}_2^*(s)=0 \tag{27}$$

On solving Eqs.(24-27) , we obtain the Laplace transformations $P_i^*(s),\ i=0,1,2,4$

The Laplace transformation of the reliability function of the system is given by

$$R_1^*(s)=\tilde{P}_0^*(s)+\tilde{P}_1^*(s)+\tilde{P}_2^*(s) \tag{28}$$

The mean time to system failure ($MTTF_1$) is obtained using:

$$MTTF_1=\int_0^\infty R_1(t)dt=\lim_{s\to 0}R_1^*(s)=R_1^*(0) \tag{29}$$

iii. Sensitivity Analysis of the Reliability and Mean Time to Failure of the System

The objective of reliability sensitivity analysis is to determine input variables that mostly contribute to the variability of the failure probability.

The results which can be obtained from any model are sensitive to many factors. In this paper, we concentrate our attention on parametric sensitivity analysis**.** Parametric sensitivity analysis helps in identifying the model parameters that could produce significant modeling errors.

One approach to parametric sensitivity analysis is to use upper and lower bounds on each parameter in the model to compute optimistic and conservative bounds on system reliability (Smotherman et al. (1986)). Our approach is to compute the derivative of the measures of interest with respect to the model parameters (Goyal et al. (1987) and Smotherman (1984)).

We first perform sensitivity analysis for changes in the system reliability $R_1(t)$ resulting from changes in parameters $\lambda,\lambda_{c2},\lambda_{c3},\lambda_{c4}$ and μ. We obtain the derivative of Eq.(23) with respect to the parameters $\lambda,\lambda_{c2},\lambda_{c3},\lambda_{c4}$ and μ.

Now we perform sensitivity analysis for changes in the mean time to failure $MTTF_1$ of the system resulting from changes in parameters $\lambda,\lambda_s,\lambda_{c2},\lambda_{c3},\lambda_{c4}$ and μ. We obtain the derivative of Eq.(29) with respect to the parameters $\lambda,\lambda_s,\lambda_{c2},\lambda_{c3},\lambda_{c4}$ and μ.

4.2 Configuration II

Configuration II consists of 2 operating primary units and 3 warm standby units and all the units are identical and at least 2 units are required for the system to work.

i. Availability Analysis of the System

The state transition diagram of the 2-out-of-5:G warm standby parallel repairable system is shown in Figure 3.

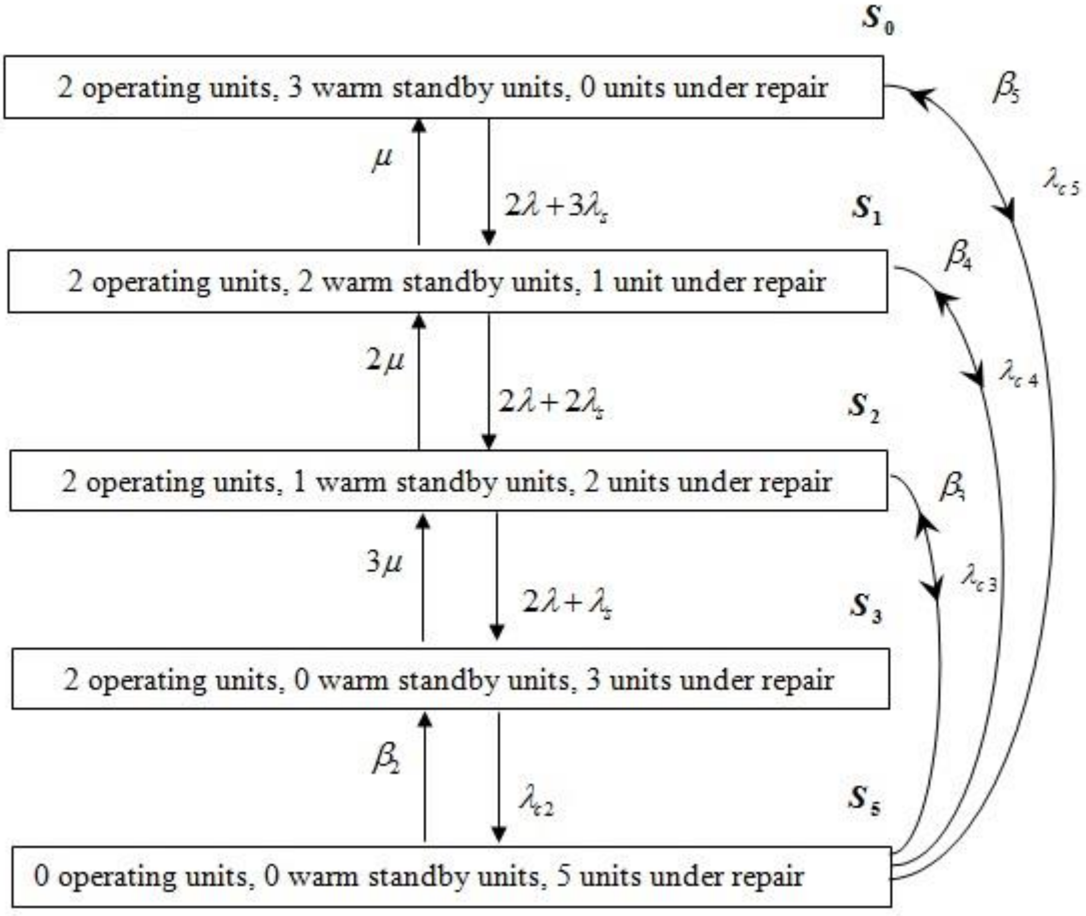

Figure 3: state transition diagram of 2-out-of-5:G warm standby parallel repairable system

The system of differential difference equations associated with the state transition diagram of the system are given by:

$$\frac{d}{dt}P_0(t)=-(2\lambda+3\lambda_s+\lambda_{c5})P_0(t)+\mu P_1(t)+\beta_5P_5(t) \tag{30}$$

$$\frac{d}{dt}P_1(t)=-(2\lambda+2\lambda_s+\lambda_{c4}+\mu)P_1(t)+(2\lambda+3\lambda_s)P_0(t)+2\mu P_2(t)+\beta_4P_5(t) \tag{31}$$

$$\frac{d}{dt}P_2(t)=-(2\lambda+\lambda_s+\lambda_{c3}+2\mu)P_2(t)+(2\lambda+2\lambda_s)P_1(t)+3\mu P_3(t)+\beta_3P_5(t) \tag{32}$$

$$\frac{d}{dt}P_3(t)=-(\lambda_{c2}+3\mu)P_3(t)+(2\lambda+\lambda_s)P_2(t)+\beta_2P_5(t) \tag{33}$$

$$\frac{d}{dt}P_5(t)=-(\beta_2+\beta_3+\beta_4+\beta_5)P_5(t)+\lambda_{c5}P_0(t)+\lambda_{c4}P_1(t)+\lambda_{c3}P_2(t)+\lambda_{c2}P_3(t) \tag{34}$$

The system availability is given by

$$A_2(t)=P_0(t)+P_1(t)+P_2(t)+P_3(t) \tag{35}$$

The initial conditions of the system are given by

$$\begin{aligned} &P_0(0)=1 \\ &P_1(0)=P_2(0)=P_3(t)=P_5(t)=0 \end{aligned} \tag{36}$$

Usually we are mainly concerned with systems running for a long time. The steady state availability of the system is the availability function as time approaches infinity. This can be obtained mathematically by taking $\frac{d}{dt}\to 0$ as $t\to\infty$ in the system of Eqs.(30-34) therefore, the system of Eqs.(30-34) reduces to the following system of linear equations:

$$0=-(2\lambda+3\lambda_s+\lambda_{c5})P_0+\mu P_1+\beta_5P_5 \tag{37}$$

$$0=-(2\lambda+2\lambda_s+\lambda_{c4}+\mu)P_1+(2\lambda+3\lambda_s)P_0+2\mu P_2+\beta_4P_5 \tag{38}$$

$$0=-(2\lambda+\lambda_s+\lambda_{c3}+2\mu)P_2+(2\lambda+2\lambda_s)P_1+3\mu P_3+\beta_3P_5 \tag{39}$$

$$0=-(\lambda_{c2}+3\mu)P_3+(2\lambda+\lambda_s)P_2+\beta_2P_5 \tag{40}$$

$$0=-(\beta_2+\beta_3+\beta_4+\beta_5)P_5+\lambda_{c5}P_0+\lambda_{c4}P_1+\lambda_{c3}P_2+\lambda_{c2}P_3 \tag{41}$$

Solving the system of linear Eqs.(37-41) using Maple program, we get the state probabilities determining the steady state availability of the system:

The steady state availability of the system is given by

$$A_2=P_0+P_1+P_2+P_3 \tag{42}$$

ii. System Reliability and Mean Time to Failure

To obtain the reliability function of the system, we assume that the set of failed states are absorbing states and set all transition rates from these states equal to zero. Now let $P_i(t)\to\tilde{P}_i(t)\quad ,i=0,1,2,3,5$ in Eqs.(37-41).

The set of differential equations associated with the system are given by:

$$\frac{d}{dt}\tilde{P}_0(t)=-(2\lambda+3\lambda_s+\lambda_{c5})\tilde{P}_0(t)+\mu\tilde{P}_1(t) \tag{43}$$

$$\frac{d}{dt}\tilde{P}_1(t)=-(2\lambda+2\lambda_s+\lambda_{c4}+\mu)\tilde{P}_1(t)+(2\lambda+3\lambda_s)\tilde{P}_0(t)+2\mu\tilde{P}_2(t) \tag{44}$$

$$\frac{d}{dt}\tilde{P}_2(t)=-(2\lambda+\lambda_s+\lambda_{c3}+2\mu)\tilde{P}_2(t)+(2\lambda+2\lambda_s)\tilde{P}_1(t)+3\mu\tilde{P}_3(t) \tag{45}$$

$$\frac{d}{dt}\tilde{P}_3(t)=-(\lambda_{c2}+3\mu)\tilde{P}_3(t)+(2\lambda+\lambda_s)\tilde{P}_2(t) \tag{46}$$

$$\frac{d}{dt}\tilde{P}_5(t)=\lambda_{c5}\tilde{P}_0(t)+\lambda_{c4}\tilde{P}_1(t)+\lambda_{c3}\tilde{P}_2(t)+\lambda_{c2}\tilde{P}_3(t) \tag{47}$$

The system reliability is given by

$$R_2(t)=\tilde{P}_0(t)+\tilde{P}_1(t)+\tilde{P}_2(t)+\tilde{P}_3(t) \tag{48}$$

Taking Laplace transformation of Eqs.(43-47) using the initial conditions Eq.(36), we obtain:

$$\left(s+2\lambda+3\lambda_s+\lambda_{c5}\right)\tilde{P}_0^*(s)-\mu\tilde{P}_1^*(s)=1 \tag{49}$$

$$\left(s+2\lambda+2\lambda_s+\lambda_{c4}+\mu\right)\tilde{P}_1^*(s)-\left(2\lambda+3\lambda_s\right)\tilde{P}_0^*(s)-2\mu\tilde{P}_2^*(s)=0 \tag{50}$$

$$\left(s+2\lambda+\lambda_s+\lambda_{c3}+2\mu\right)\tilde{P}_2^*(s)-\left(2\lambda+2\lambda_s\right)\tilde{P}_1^*(s)-3\mu\tilde{P}_3^*(s)=0 \tag{51}$$

$$\left(s+\lambda_{c2}+3\mu\right)\tilde{P}_3^*(s)-\left(2\lambda+\lambda_s\right)\tilde{P}_2^*(s)=0 \tag{52}$$

$$s\tilde{P}_5^*(s)-\lambda_{c5}\tilde{P}_0^*(s)-\lambda_{c4}\tilde{P}_1^*(s)-\lambda_{c3}\tilde{P}_2^*(s)-\lambda_{c2}\tilde{P}_3^*(s)=0 \tag{53}$$

On solving Eqs.(49-53), we obtain the Laplace transformations $P_i^*(s),\ i=0,1,2,3,5$.

The Laplace transformation of the reliability function of the system is given by

$$R_2^*(s)=\tilde{P}_0^*(s)+\tilde{P}_1^*(s)+\tilde{P}_2^*(s)+\tilde{P}_3^*(s) \tag{54}$$

The mean time to system failure ($MTTF_2$) is obtained using:

$$MTTF_2=\int_0^\infty R_2(t)dt=\lim_{s\to 0}R_2^*(s)=R_2^*(0) \tag{55}$$

iii. Sensitivity Analysis of the Reliability and Mean Time to Failure of the System

We first perform sensitivity analysis for changes in the reliability of the system $R_2(t)$ resulting from changes in parameters $\lambda,\lambda_{c2},\lambda_{c3},\lambda_{c4},\lambda_{c5}$ and μ. We obtain the derivative of Eq.(48) with respect to the parameters $\lambda,\lambda_{c2},\lambda_{c3},\lambda_{c4},\lambda_{c5}$ and μ.

Now we perform sensitivity analysis for changes in the mean time to failure $MTTF_2$ of the system resulting from changes in parameters $\lambda,\lambda_s,\lambda_{c2},\lambda_{c3},\lambda_{c4},\lambda_{c5}$ and μ. We obtain the derivative of Eq.(55) with respect to the parameters $\lambda,\lambda_s,\lambda_{c2},\lambda_{c3},\lambda_{c4},\lambda_{c5}$ and μ.

5. Numerical Example:

For comparative analysis of reliability measures between configuration I and configuration II; the failure, repair, common-cause failure and replacement rates are given by:

$\lambda = 0.2, \quad \lambda_s = 0.1, \quad \mu = 0.4, \quad \lambda_{c5} = 0.05, \quad \lambda_{c4} = 0.1, \quad \lambda_{c3} = 0.15,$
$\lambda_{c2} = 0.2, \quad \beta_5 = 0.25, \quad \beta_4 = 0.5, \quad \beta_3 = 0.6, \quad \beta_2 = 0.7$

Figures 4–5 show the availability and reliability for configuration I and II versus time. We conclude that the availability and reliability of configuration II is greater than the availability and reliability of configuration I. Figures 6–7 show the steady state availability of configuration I and II versus failure and repair rates. It can be observed that the steady state availability configuration II is greater than that of configuration I and the steady state availability of the two configurations decreases with the increase in the failure rate λ and increases with the increase in the repair rate μ. Sensitivity analysis for changes in the reliability functions $R_1(t)$ and $R_2(t)$ resulting from changes in system parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ are shown in Figures 8–9. We can easily observe that the system parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ has big impact on the reliability functions $R_1(t)$ and $R_2(t)$ of configuration I and II at the same time. The numerical results of the sensitivity analysis of the mean time to failure of configuration I and II resulting from changes in system parameters $\lambda, \lambda_s, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ are shown in Tables 1–2. It can be seen from Table 1 that the order of impacts of the system parameters on $MTTF_1$ are: $\lambda_{c4} > \lambda_{c3} > \lambda_{c2} > \lambda > \lambda_s > \mu$. From Table 2 the order of impacts of the system parameters on $MTTF_2$ are: $\lambda_{c4} > \lambda_{c5} > \lambda_{c3} > \lambda_s > \lambda > \mu > \lambda_{c2}$ and the mean time to failure of the two configurations are not sensitive to the replacement rates. It should be noted that these conclusions are only valid for the given values of system parameters. We may reach other conclusions for other values of the system parameters.

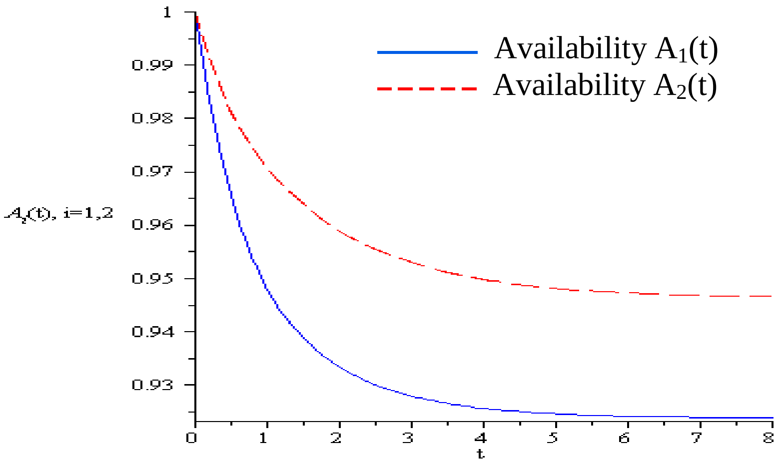

Figure 4: Availability $A_i(t)$ versus time, i=1,2

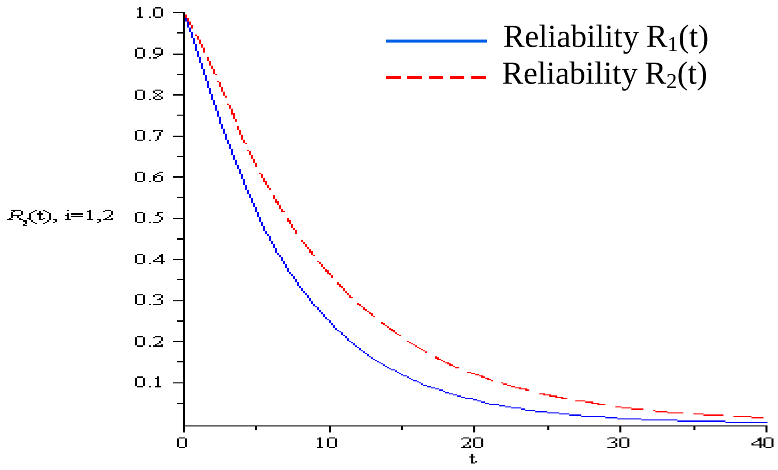

Figure 5: Reliability $R_i(t)$ versus time, i=1,2

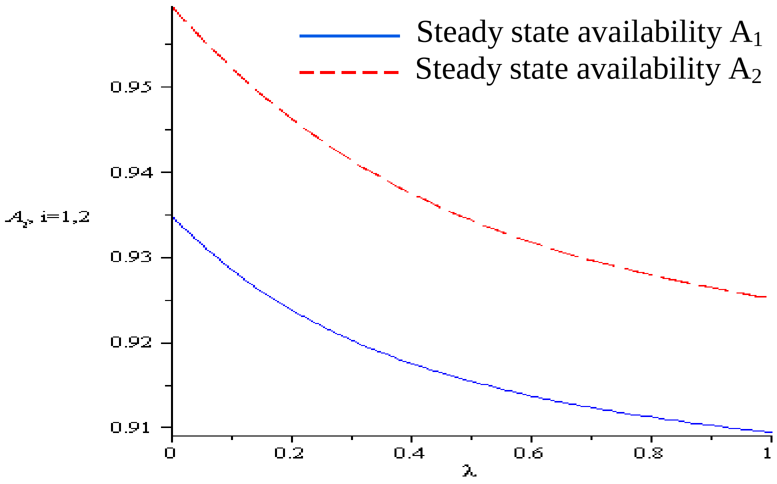

Figure 6: steady state availability A_i versus failure rate λ, i=1,2

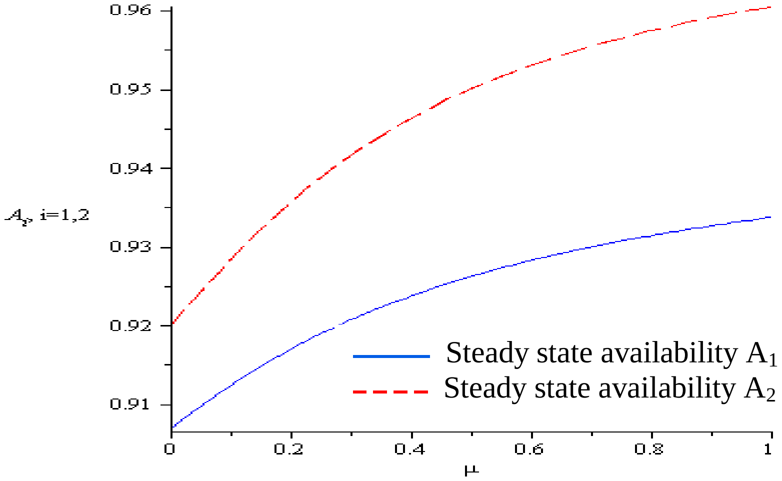

Figure 7: steady state availability A_i versus repair rate μ, i=1,2

Now we perform sensitivity analysis for changes in the reliability functions $R_i(t)$ $, i = 1, 2$ along with changes in specific values of the system parameters $\lambda, \lambda_{c2}, \lambda_{c3}, \lambda_{c4}, \lambda_{c5}$ and μ .

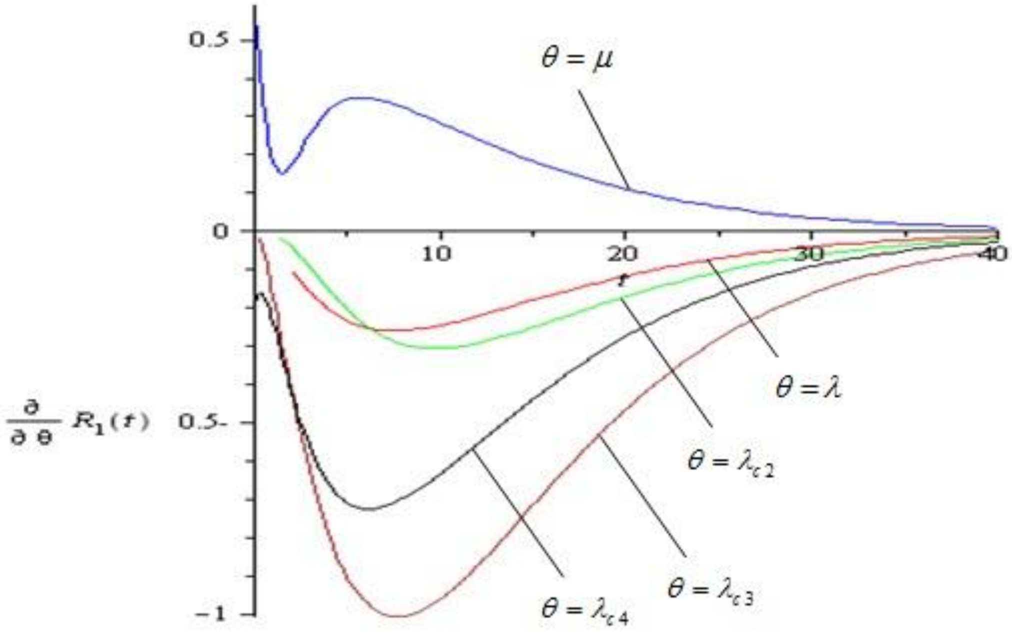

Figure 8: Sensitivity of the reliability of configuration I
with respect to system parameters

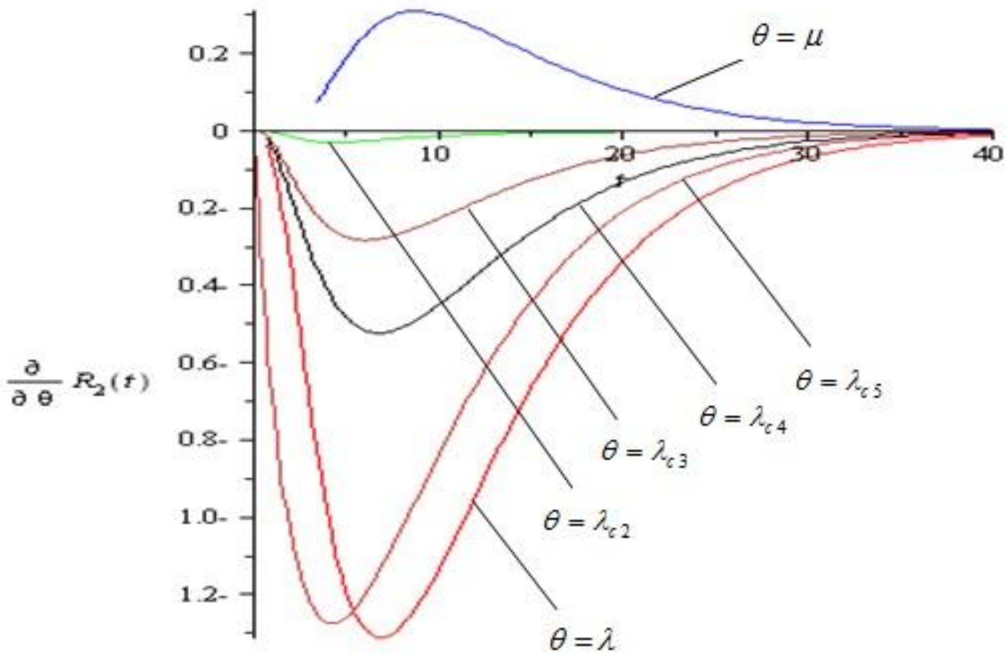

Figure 9: Sensitivity of the reliability of configuration II
with respect to system parameters

Finally we perform sensitivity analysis for changes in the mean time to failure $MTTF_i \ ,i=1,2$ along with changes in specific values of the system parameters $\lambda,\lambda_s,\lambda_{c2},\lambda_{c3},\lambda_{c4},\lambda_{c5}$ and μ.

Table 1: Sensitivity analysis for $MTTF_1$

	$\theta=\lambda$	$\theta=\lambda_s$	$\theta=\mu$	$\theta=\lambda_{c2}$	$\theta=\lambda_{c3}$	$\theta=\lambda_{c4}$
$\frac{\partial MTTF_1}{\partial\theta}$	-4.76	-3.78	2.27	-8.92	-18.82	-21.8

Table 2: Sensitivity analysis for $MTTF_2$

	$\theta=\lambda$	$\theta=\lambda_s$	$\theta=\mu$	$\theta=\lambda_{c2}$	$\theta=\lambda_{c3}$	$\theta=\lambda_{c4}$	$\theta=\lambda_{c5}$
$\frac{\partial MTTF_2}{\partial\theta}$	-11.51	-12.84	6.5	-6.06	-18	-33	-32.08

6. Conclusion

In this paper we have utilized the Markov model to develop the reliability measures of the k-out-of-n:G warm standby parallel repairable system**.** All failure and repair rates of the system are constant. Comparative analysis of reliability measures between two dissimilar configurations has been developed. Configuration I is a 2-out-of-4:G warm standby parallel repairable system, while Configuration II is a 2-out-of-5:G warm standby parallel repairable system. The system of differential equations with the initial conditions has been solved numerically using Laplace transformation by the aid of Maple program. Graphical representation of the reliability and availability of the two configurations versus time are made. Sensitivity analysis is also carried out to depict the effect of various parameters on the reliability function and mean time to failure of the system. Numerical example is given to illustrate the results obtained, and the results were shown graphically by the aid of Maple program. Results indicate that the reliability and availability of the system increase by increasing of the number of warm standby units. And the reliability and mean time to failure of the two configurations are sensitive to the failure and repair rates of the system and are not sensitive to the replacement rates.

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ON SENSITIVITY OF RELIABILITY SYSTEMS OPERATING IN RANDOM ENVIRONMENT TO SHAPE OF THEIR INPUT DISTRIBUTIONS

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ABSTRACT

Removable double redundant reliability system operating in Markov environment is considered. The problem of the system steady state probabilities insensitivity to shape of its elements life and repair time distributions is studied.

1 Introduction

Stability of systems behavior to small changing some of their parameters, initial states or exterior factors are central problem for all natural sciences. The stability of complex stochastic systems often can be represented in terms of sensitivity of their output characteristics to the shape of some input distributions. One of the earliest result concerning insensitivity of systems' characteristics to the shape of service time distribution has been done by B. Sevast'yanov [13], who proved insensitivity of the Erlang's formulas to the shape of service time distribution with fixed mean value for loss queueing systems with Poisson input.

I. Kovalenko [8] found the necessary and sufficient condition for insensitivity of stationary characteristics of reliability systems with exponential their elements life time to the shape of their elements general repair time distribution. It consists in possibility immediately begin to repair any failed element. The sufficiency of this condition for general life and repair time distributions has been found by V. Rykov in [9] with the help of multi-dimensional alternative processes theory. From another side as it is very known, in the case of limited possibilities for restoration these results are not true (see, for example in [7]). However as it was shown in the papers of B. Gnedenko and A. Solov'ev [2, 3, 14, 4] and others under quick restoration the reliability function of the system tends to exponential independently of their elements life and repair time distributions.

There are many papers, where reliability systems are studied in stable environment (see, for example, Cui & Hawkes [1]). However, the very important for applications problem of reliability characteristics insensitivity or character of their sensitivity to the shapes of some input distributions and the influence of environment randomness to their behavior are not enough studied yet. There are some papers devoted to operating of queueing systems in random environment. Some review of the previous and modern investigations on this topic one can find in [6]. Some previous results about sensitivity of reliability characteristics of the system operating in stable environment to the shape of their elements life and repair time distributions have been done in [10, 11]. In the paper these results are generalized to the system operating in random environment.

The paper is organized as follows. After the problem set and notations in the next section the steady state reliability characteristics of a double redundant systems with one repair unit, which is operate in Markov environment will be considered in two next sections. The numerical investigation of the model will be represented in the section 5. The paper ends with conclusion and some problems description.

2 The problem set and notations

Consider a simple cold double redundant repairable system that operates in a random environment. Throughout the paper we will use a generalization of Kendall's notation [5] for queueing systems. In this notation a closed queueing system, which operates in a Random Environment will be denoted by $< GI_n|GI|m(RE) >$, where the symbols "$< >$" stand for closed systems, or systems with finite population, where the flow of customers is generated by a finite number n of sources that is shown by index at the first position; GI means "General Independent" and at the first position of this notation specifies the general distributions of units' independent life times and at the second one – the general distributions of their independent repair times, at least the number "m" means the number of repair servers, and symbol "(RE)" means "Random Environment". These symbols can be changed by M for exponential distributions in a Markov case.

In the paper for simplicity and due to some calculation reasons we limited ourselves by studying of the sensitivity problem for simple cold double redundant models, operating in a two-state Markov Random Environment, where only one of distributions (life or repair time) distinguish from exponential, namely $< M_2|GI|1(ME) >$ and $< GI_2|M|1(ME) >$.

The units' random life time when the system operates in i-th environment is denoted by A_i and its random repair time is denoted by B_i. Their cumulative distribution functions (c.d.f.) are denoted respectively by $A_i(x)$ and $B_i(x)$. It is supposed the existence of the corresponding probability density functions (p.d.f.), which are denoted by $a_i(x)$ and $b_i(x)$. The mean life and repair times, the relative repair rate as well as the failure and repair hazard rate functions are denoted by

$$\begin{aligned}\bar{a}_i &= \int\limits_0^\infty (1 - A_i(x))\,dx, \quad \bar{b}_i = \int\limits_0^\infty (1 - B_i(x))\,dx \quad \text{and} \quad \rho_i = \frac{\bar{a}_i}{\bar{b}_i};\\ \alpha_i(x) &= \frac{a_i(x)}{1 - A_i(x)} \quad \text{and} \quad \beta_i(x) = \frac{b_i(x)}{1 - B_i(x)}.\end{aligned}$$

Define also moment generation functions (m.g.f.) of life A_i and repair B_i times, the Laplace-Stiltjes transforms (LST) of their distribution by following expressions,

$$\tilde{a}_i(s) = \int\limits_0^\infty e^{-sx} a_i(x)\,dx \quad \text{and} \quad \tilde{b}_i(s) = \int\limits_0^\infty e^{-sx} b_i(x)\,dx, \quad Re[s] \geq 0.$$

3 Cold redundancy $< M_2|GI|1(ME) >$ system, operating in Markov environment

Consider a two units cold redundant system $< M_2|GI|1(ME) >$ with one repair server, operating in two-state Markov environment. The elements have an exponential life time distributions with parameter α_i and general repair time distribution $B_i(t)$, when environment is in state i, and the environment change their states with intensities λ_i $(i = 1, 2)$. Denote by

$$\{Z(t)\}_{t\geq 0} = \{I(t),\, N(t),\, X(t)\}_{t\geq 0}$$

a three-dimensional stochastic process, where the first component $I(t)$ stands the state of environment, the second one $N(t)$ denote the number of failed units at time t and the third one $X(t)$ stands for the elapsed repair time of the unit at time t. The process $\{Z(t)\}_{t\geq 0}$ is obviously Markovian one with state space $E = \{(i,0),\, (i,n,x) : i,n \in \{1,2\},\, x \in \mathbb{R}_+\}$. Define the following state probabilities:

(1) $\pi_{(i,0)}(t) = \mathbf{P}\{I(t) = i,\ N(t) = 0\}$ – the probability of a “good” state of both of units, being the system in i-th state of environment at time t;

(2) $\pi_{(i,n)}(t;x)\,dx = \mathbf{P}\{I(t) = i,\ N(t) = n;\ x < X(t) \leq x + dx\}$ – the joint probability that at time t the system is in the i-th state of environment $(i = 1, 2)$, there are n $(n = 1, 2)$ failed units and the repairing unit has elapsed repair time between x and $x + dx$.

By usual approach with the help of Large Number Law for these probabilities the system of forward Kolmogorov partial differential equations jointly with boundary, initial and normalizing conditions has been derived (for analogous system operating in stable environment see [11, 12]. Since the Markov process $\{Z(t)\}_{t\geq 0}$ is a Harris one with positive atom in the states $(i, 0)$, it has steady-state probabilities,

$$\pi_0 = \lim_{t\to\infty} \pi_i(t), \quad \pi_i(x) = \lim_{t\to\infty} \pi_i(t;\, x) \ \ (i = 1, 2).$$

Equations for the stationary regime have been got from the the Kolmogorov equations by taking derivatives with respect to time variable as zero and they have the form, where $\bar{i} = 3 - i$:

$$\begin{aligned}
\Big[\lambda_i + \alpha_i\Big]\pi_{(i,0)} &= \int_0^\infty \pi_{(i,1)}(u)\beta(u)du + \lambda_{\bar{i}}\pi_{(\bar{i},0)}, \\
\left[\frac{d}{dx} + \alpha_i + \lambda_i + \beta_i(x)\right]\pi_{(i,1)}(x) &= \lambda_{\bar{i}}\pi_{(\bar{i},1)}(x), \\
\left[\frac{d}{dx} + \lambda_i + \beta_i(x)\right]\pi_{(i,2)}(x) &= \alpha_i\pi_{(i,1)}(x) + \lambda_{\bar{i}}\pi_{(\bar{i},2)}(x),
\end{aligned} \tag{1}$$

with the boundary conditions for $i = 1, 2$

$$\pi_{(i,1)}(0) = \alpha_i\pi_{(i,0)} + \int_0^\infty \pi_{(i,2)}(u)\beta(u)du, \quad \pi_{(i,2)}(0) = 0, \tag{2}$$

together with the normalizing condition

$$\sum_{i=1}^{2}\left[\pi_{(i,0)} + \sum_{n=1}^{2}\int_0^\infty \pi_{(i,n)}(x)\,dx\right] = 1.$$

In order to explain these equations one should consider the process marked transition graph, represented at the Figure 1.

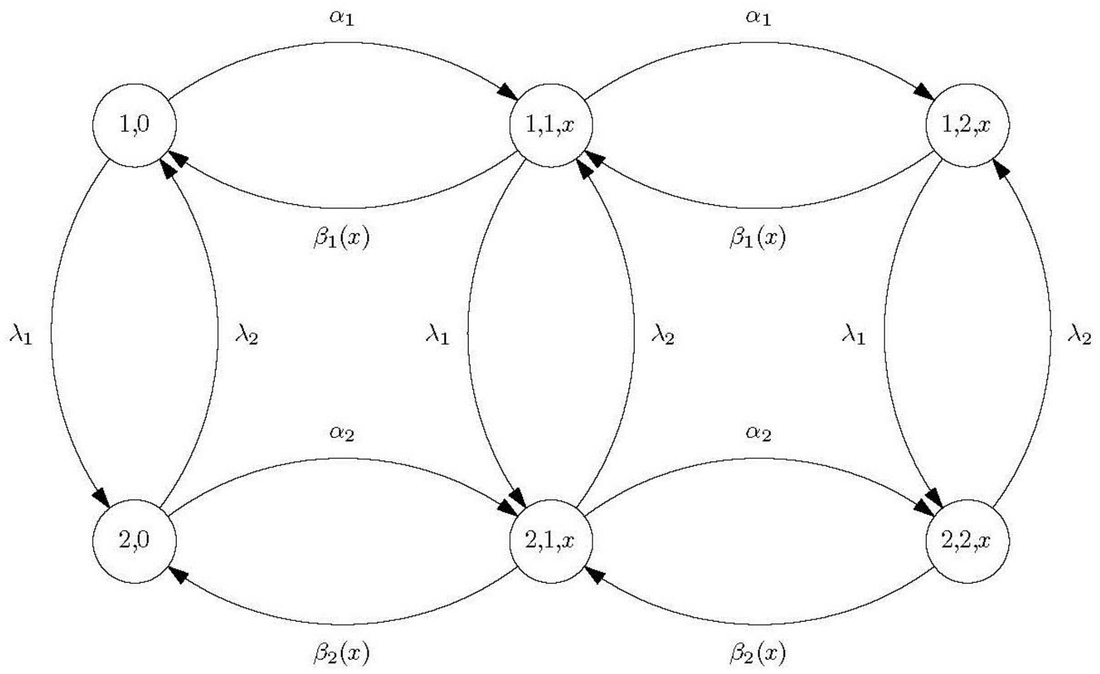

Figure 1. The process transition graph for $< M_2|GI|1(ME) >$ system.

The process passes from one environment state to another and back with intensities λ_1, λ_2. From another side in each environment state i it leaves the states $(i,1,x)$ and $(i,2,x)$ with intensity $\beta_i(x)$, otherwise it stays in these states. The process occurs in the boundary state $(i,1,0)$ in the case if the operating in the state 0 unit fails, or if the repaired in the state $(i,2,x)$ unit restores.

This system of equation does not allow analytical solution and examples of its numerical solutions in the section 5 will be considered.

4 Cold redundancy $< GI_2|M|1(ME) >$ system, operating in Markov environment

Consider now analogous system $< GI_2|M|1(ME) >$ with generally distributed life time of the units and exponentially distributed repair time, operating in two-state Markov environment. Consider an analogous to the previous case stochastic process $\{Z(t)\}_{t\geq 0} = \{I(t),\, N(t),\, X(t)\}_{t\geq 0}$ with the same two first components, but the last one denote now elapsed operational time of the working unit. The reader will not be confused by using the same notations for different processes. It has been done for convenience of the results representation with the same notations.

Define the state probabilities:

(1) $\pi_{(i,n)}(t;\, x)\, dx = \mathbf{P}\{I(t) = i,\, N(t) = n, x < X(t) \leq x + dx\}$ – the joint probability that at time t the system is in i-th environment state $(i = 1,2)$, there are $n\,(n = 0,1)$ failed units and the operational has elapsed working time between x and $x + dx$;

(2) $\pi_{(i,2)}(t) = \mathbf{P}\{I(t) = i,\, N(t) = 2\}$ – the probability of the “bad” state (complete failure state) of the system at time t being in i-th environment state.

Analogously to the previous case for these probabilities the system of forward Kolmogorov partial differential equations jointly with boundary, initial and normalizing conditions has been derived (for analogous system operating in stable environment see [11, 12]). Since the Markov process $\{Z(t)\}_{t\geq 0}$ is a Harris one with positive atom in the states $(i,2)$, it has steady-state probabilities, equations for which have the form

$$\begin{aligned}
\left[\frac{d}{dx}+\alpha_i(x)+\lambda_i\right]\pi_{(i,0)}(x) &= \beta_i\pi_{(i,1)}(x)+\lambda_{\bar{i}}\pi_{(\bar{i},0)}(x),\\
\left[\frac{d}{dx}+\alpha_i(x)+\beta_i+\lambda_i\right]\pi_{(i,1)}(x) &= \lambda_{\bar{i}}\pi_{(\bar{i},1)}(x),\\
(\beta_i+\lambda_i)\pi_{i,2} &= \int\limits_0^\infty \pi_{i,1}(u)\alpha_i(u)du+\lambda_{\bar{i}}\pi_{(\bar{i},2)}. \qquad (3)
\end{aligned}$$

with the boundary and normalizing conditions

$$\begin{aligned}
\int\limits_0^\infty \pi_{(i,0)}(u)\alpha_i(u)du+\beta_i\pi_{(i,2)} &= \pi_{(i,1)}(0), \qquad (4)\\
\sum_{i=1}^{2}\left[\sum_{n=0}^{1}\int\limits_0^\infty \pi_{(i,n)}(x)\,dx+\pi_{(i,2)}\right] &= 1. \qquad (5)
\end{aligned}$$

To explain these equations one can use the analogous to previous case reasonings based on the marked transition graph, represented at the Figure 2.

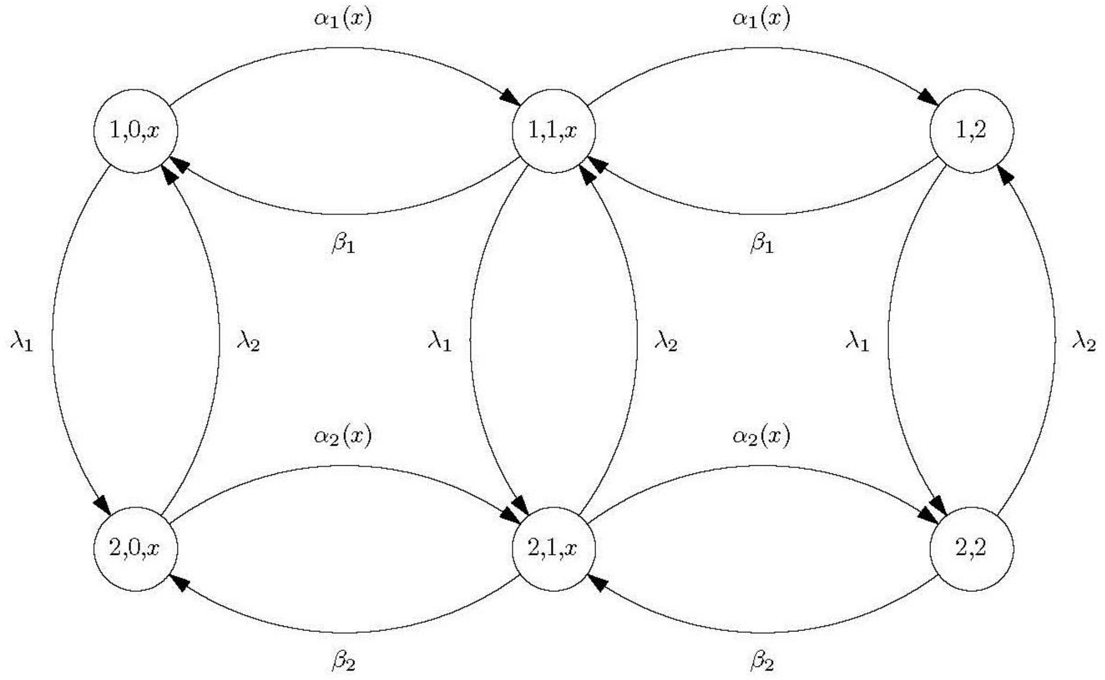

Figure 2. The process transition graph for $< GI_2|M|1(ME) >$ system.

In order to study of environment influence to the reliability characteristics some additional numerical experiment based on the solution of the equations (1) and (3) has been done. Below the results of these investigations will be shown.

5 Numerical investigation of the models

In numerical analysis the Γ-distribution with PDF

$$f(x)=\frac{(x\mu)^{k-1}\,\mathrm{e}^{-x\mu}\mu}{\Gamma(k)}$$

will be used as an example of general distribution. This choice is motivated by the reason that this distribution allows to model the situations with variations grater and smaller than one. As a reliability characteristic an availability coefficient

$$K=\pi_{(1,0)}+\pi_{(2,0)}+\pi_{(1,1)}+\pi_{(2,1)}$$

will be considered. It will be studied depending on mean relative repair rate

$$\rho=\frac{\lambda_1^{-1}\rho_1+\lambda_2^{-1}\rho_2}{\lambda_1^{-1}+\lambda_2^{-1}},\quad \text{where}\quad \rho_i=\frac{\bar{a}_i}{\bar{b}_i}\quad \text{and}$$

$\bar{a}_i,\ \bar{b}_i\ (i=1,2)$ have been defined before.

Availability coefficient K also depends on environment relative variability index

$$u=\frac{\lambda_1^{-1}u_1+\lambda_2^{-1}u_2}{\lambda_1^{-1}+\lambda_2^{-1}},\quad \text{where}\quad u_i=\lambda_i\,\bar{a}_i,$$

and pure environment variability index (ratio of environment change states intensities)

$$z=\frac{\lambda_2}{\lambda_1}.$$

We consider 3 variants of environment variability: slow ($u=0.01$), moderate, when mean time between changing states of environment is equivalent mean life time ($u=1$) and quick ($u=100$). Also we consider 3 variants of values of parameter z: $0.01, 1$ and 100.

In all numerical experiments the elements are more reliable in first state of environment than in the second one and their mean repair times coincides,

$$\frac{\bar{a}_1}{\bar{a}_2}=100\quad \text{and}\quad \frac{\bar{b}_1}{\bar{b}_2}=1.$$

In all numerical experiments parameters k, μ of Γ-distribution has been chosen in such manner that these ratios are fulfilled.

In order to model different types of the life and repair time distributions the variation V of Γ-distribution has been varied at seven levels: 0.1, 0.4, 0.8, 1, 1.2, 1.6 and 1.9.

5.1 Model $<M_2|\Gamma|1>$

For the case when Γ-distribution is used as a repair distribution the parameters k,μ of Γ-distribution are determined based on variation V and mean repair time $\bar{b}_j$,

$$k=\frac{1}{V^2},\quad \mu_j=\frac{1}{\bar{b}_jV^2}\quad (j=1,2).$$

Remark 5.1. *For the calculation it is necessary to take into account that for* $x >> 1$ *the expression* $\beta_j(x) = \frac{b_j(x)}{1-B_j(x)}$ *became non-stable, and in this case asymptotic representation* $\beta_j(x) \approx \mu_j - \frac{k-1}{x}$ *has been used for* $x > \frac{k}{\mu_j} + 6\frac{\sqrt{k}}{\mu_j}$.

Moreover, in the case of $V < 1$, *for* Γ*-distribution* $\lim\limits_{x\to 0} \beta_j(x) = \infty$, *therefore in this case for* $x = 0$ *it is used the expression* $\beta_j(0) = b_j\left(\frac{k}{100\mu_j}\right)$.

The results of numerical experiments as the Figure 3 are presented, where the continuous line corresponds to the case of exponential distribution with variation equal one $V = 1$, the point-wise curve corresponds to the case with variation less than one $V < 1$, and the dash-wise line corresponds to the case with variation grater than one $V > 1$.

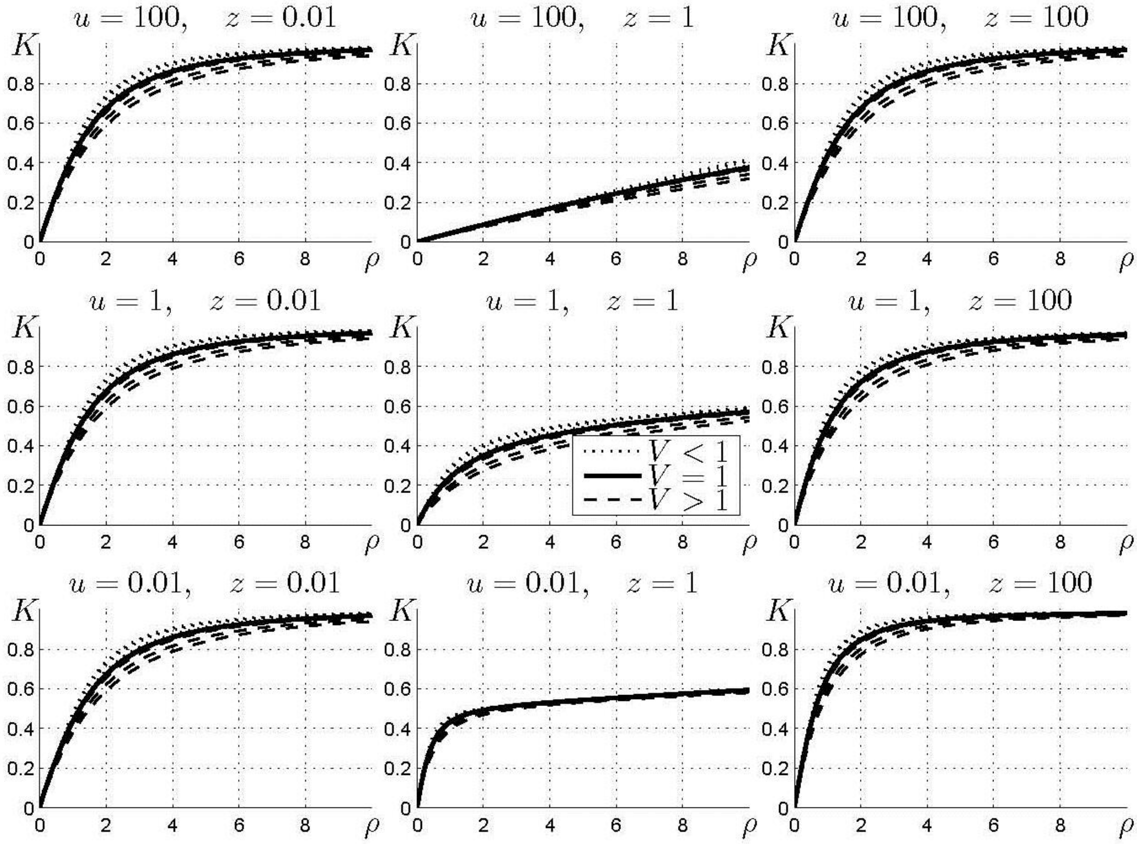

Figure 3. Numerical results for the model $< M_2|\Gamma|1 >$.

The graphs show that the availability coefficient K only a weak sensitive to the shape of the repair time distribution and the sensitivity vanishes with increasing of the relative repair rate coefficient ρ. However, different types of lines demonstrate dependence of the availability coefficient K on variation V: big variation decreases availability while small variation increases it. From another site the influence of the environment relative variability index u to the system availability K is not enough significant, while the pure environment variability index z influences on its convergence rate to one.

5.2 Model $< \Gamma_2|M|1 >$

Consider now the model, when the Γ-distribution is used as elements life time distribution while the repair time distribution is the exponential one. In this case the parameters k, μ of the Γ-distribution are determined based on variation coefficient V and mean life time $\bar{a}_j$ by the following way

$$k = \frac{1}{V^2}, \quad \mu_j = \frac{1}{\bar{a}_j V^2} \quad (j = 1, 2).$$

The graphs of the availability coefficient K versus relative repair rate coefficient ρ for different cases are shown at the Figure 4, where the same type of curves for different values of variations are used.

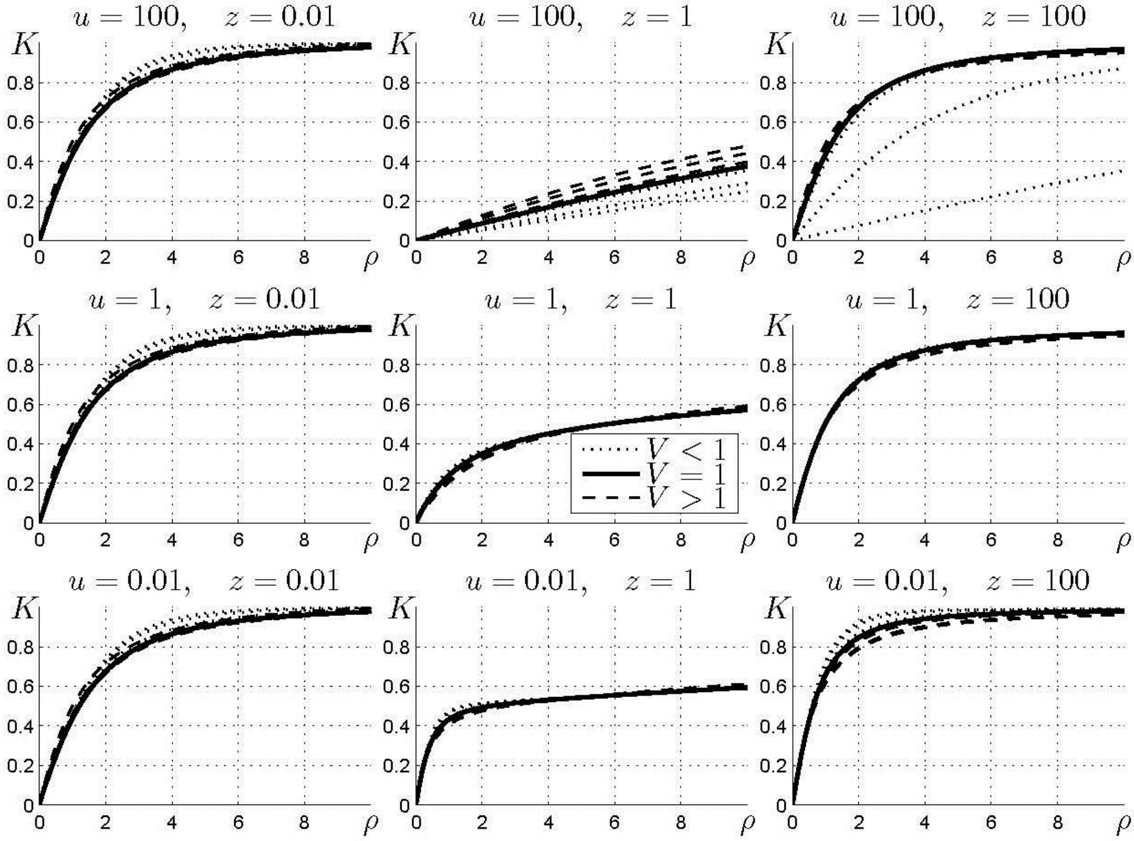

Figure 4. Numerical results for the model $< \Gamma_2|M|1 >$.

Also as before the graphs demonstrate the weak dependence of the system availability coefficient K on the shape of units life time distributions that is vanish under fast restoration (when the relative repair rate coefficient ρ grows to infinity) and the different type of its dependence on the relative and pure environment variability indices u and z. The difference consists only in the variation influence to the availability coefficient K in this case: big variation increases availability while small variation decreases it.

6 Conclusion and gratitude

The sensitivity of availability coefficient for double redundant removable system operating in Markov environment to the shapes of its life and repair time distribution has been considered. It is shown that the sensitivity vanishes with increasing of the elements relative repair rate. From another site the influence of the environment variability to the availability coefficient K is not enough significant and depends on both: relative and pure environment variability indices.

The calculations also show the influence of the elements life and repair time variation to the system availability: the system availability increases when elements life time variation increases and it decreases with increasing the element repair time variation.

The investigation of another cases of redundancy (hot redundant systems, several redundant elements) and another kind (non Markovian) of environment randomness are the problems for further investigations.

The paper has been presented at the MMR-2015 Conference in Tokyo and submitted to ORSJ journal. One of referees gave several remarks and recommend the paper for publication after some modification. Another one “did not see” that the system is modelled by three-dimensional Markov process and reject the paper as too simple. We are grateful the first referee for his remarks that has been used in the present version of the paper.

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