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ABOUT TRIGONOMETRIC DISTRIBUTIONS TO DESCRIBE THE FAILURE OF TECHNICAL DEVICES

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Abstract

There has been proposed and investigated the non-traditional trigonometric distribution to describe the gradual failure of technical devices.

1. The starting statements

At present the calculations of indicators of the technical devices (TD) reliability are carried out, as a rule, with the assumption of the constant failure rate for their component elements. It corresponds to the case when the component elements are subjected only to sudden failures because of external influences. The gradual failures of the component elements connected with internal processes of wear and aging at the same time aren't considered. This doesn't correspond to reality. For example, in [1, 2] there were described the degradation processes which cause wear and aging of the component elements of railway systems power supply, automatic equipment, telemechanics and communication. These processes lead to the gradual failure of the elements and they are described in the theory of reliability by the class of distributions having the increasing failure rate function that is so-called VFI – distributions [3]. We will call the elements with gradual failures as the growing old type elements.

By the collection and processing the data about the failures of these component elements which were restored while performing there were obtained only the indices of the constant values of ω failure stream parameter [1] or the time between failures as $T = 1/\omega$ [2]. However it doesn't mean that failure rate of component elements of the growing old type is a constant value.

According to the definition the failure stream parameter is the relation of the number of the failed products during the time interval of n(dt) to the number of the tested products during this interval of dt provided that the failed products are replaced be the serviceable ones (new or repaired), that is $\omega(t) = n(dt)/Ndt$ where N – the number of tested products which remains to be constant. From the theory of reliability it is known that the failure stream parameter at any kind of distribution under operation strives for the stationary value equal to $\omega = 1/T$. It is also evident when collecting statistical data on TU failures under real conditions of operation.

According to the definition the failure rate is the relation of the number of the failed products during the time interval of n(dt) to the average of the products Ncp which have regularly worked during this time interval of dt, that is $\lambda(t)$ = n(dt)/Ncp dt. By doing so the Ncp decreases because of failures of products with each interval, and $\lambda(t)$ elements of the growing old type increases.

The laws of distribution for the time between failures of component elements of the growing old type being, as a rule, unknown, the problem of the TU reliability indicators calculation has to be solved under uncertain conditions.

The purpose of the article is to offer and to investigate nonconventional trigonometrical distributions for the description of gradual failures of technical devices.

When it is possible to estimate only the value of the time between failures, for example, from expression of $T = 1/\omega$, one could suggest two following methods of the approximate description of the technical devices reliability indicators.

2. Distribution of a cosine

First, as the failure stream parameter at t = T approaches the stationary value equal to 1/T, it is offered to approximate the ω (t) dependence by the piecewise and linear function of the type [4]:

$$\text{at } t < T \quad \omega(t) = t/T2; \quad \text{at } t \geq T \quad \omega(t) = 1/T. \tag{1}$$

Other indicators of the reliability are defined with the use of transformation by Laplace. We will find the density of distribution of f(t) from the equation connecting it in an operator form with the failure stream parameter of $f(s) = \omega(s) / (1+\omega(s))$ as

$$f(t) = (1/T)\sin(t/T). \tag{2}$$

Then the probability of no-failure operation of P (t) and the failure rate of $\lambda(t)$ are defined from the equations:

$$P(t) = 1 - \int_0^t f(t)dt = \cos(t/T); \tag{3}$$

$$\lambda(t) = f(t) / P(t) = (1/T)\, \mathrm{tg}\, (t/T). \tag{4}$$

The argument of t/T in formulas for definition of the reliability indicators is measured in radians. We will call the received distribution as the distribution of the cosine which definition range lies in the range of $0 < t/T < \pi/2$.

The not own integral from the distribution density within the distribution definition range according to [5] has to be equal to one unit. We check

$$\int_0^{\pi T/2} (1/T)\, sin(t/T)\, dt = 1.$$

The distribution variation coefficient is defined from expression

$$V = \mu_2^{0.5}/\mu_1\,, \tag{5}$$

where μ_1- the first initial moment;

μ_2 - second central moment.

$$\mu_1 = \int_0^{\pi T/2} tf(t)dt = \int_0^{\pi T/2} (t/T)\sin(t/T)\, dt = T;$$

$$\mu_2 = \int_0^{\pi T/2} (t-\mu_1)^2 f(t)dt = \int_0^{\pi T/2} (t-T)^2 (t/T)\sin(t/T)\, dt = (\pi - 3)\, T^2\,.$$

Having substituted the values of μ_1 and μ_2 into the expression (5), we will receive $V = 0{,}376$.

As the failure rate of this distribution according to (4) is a monotonously increasing function of time, and the value of the variation coefficient is less than one unit, it belongs to the class of the VFI - distributions and can be used for the description of the TU gradual failures. In [4] article there were defined the asymmetry and excess coefficients and it has been noted that the distribution of the cosine can be presented at the Pearson's areas by a point with coordinates of p2 = 0,18 and β = 2,23. It has been shown what according to [3] the cosine function is also the distribution with the increasing average failure rate, the distribution like "the new thing is better than the used one" and the distribution like "the new thing is on average better than the used one".

With use of the equations of (1), (2), (3) and (4) there have been defined the dependences of the reliability indicators on the relative time of t/T operation of the offered distribution. The results of the calculations are shown in table 1.

Table 1

t/T	0	0,2	0,4	0,6	0,8	1,0	1,2	1,4	π/2
Tf(t)	0	0,20	0,39	0,56	0,72	0,84	0,93	0,98	1,0
Tλ(t)	0	0,20	0,42	0,68	1,03	1,56	2,57	5,80	∞
Tω(t)	0	0,20	0,40	0,60	0,80	1,0	1,0	1,0	1,0
P(t)	1	0,98	0,92	0,83	0,70	0,54	0,36	0,17	0

3. Distribution of cosine square

Secondly, it is offered to approximate the dependence of the technical devices failures probability density of f(t) depending on the operation time by t function of the sine of the type

$$f(t) = (1/\mathrm{T})\, sin(2t/T) \quad (6)$$

with the range of definition $0 < t < \pi T/2$.

The not own integral from the distribution density within the distribution definition range according to [5] has to be equal to one unit. We check

$$\int_0^{\pi T/2} (1/\mathrm{T})\, sin(2t/T)\, dt = 1.$$

Considering that $\sin(2t/T) = 2\sin(t/T)\cos(t/T)$, we will present the expression (6) in the form

$$f(t) = (2/T) sin(t/T) cos(t/T). \quad (6a)$$

The probability of non-failure operation of P (t) considering (6) is defined from the expression

$$P(t) = 1 - \int_0^t f(t)dt = (1 + \cos(2t/T))/2\ . \quad (7)$$

Considering that, we will $\cos(2t/T) = cos^2(t/T) - sin^2(t/T)$,
present the expression (7) in the form

$$P(t) = cos^2(t/T). \quad (7a)$$

We will call the received distribution as the distribution of the cosine square.

The failure rate of λ(t) taking into account of (6a) and (7a) is defined as

$$\lambda(t) = f(t)/P(t) = (2t/T)tg(t/T). \quad (8)$$

The failure stream parameter of ω(t) is defined by Laplace's transformation of the type $\omega(s) = f(s)/(1 - f(s))$ taking into account of (6) as

$$\omega(t) = \left(\sqrt{2}/T\right) sin(\sqrt{2}t/T)\ . \quad (9)$$

The variation coefficient is defined with the use of expression (5).

$$\mu_1 = \int_0^{\pi T/2} tf(t)dt = \int_0^{\pi T/2} (t/\mathrm{T}) \sin(2t/T)\, dt = \pi T/4\,;$$

$$\mu_2 = \int_0^{\pi T/2} (t - \mu_1)^2 f(t)dt \int_0^{\pi T/2} \left(t - \frac{\pi T}{4}\right)^2 \sin(2t/T)\, dt = (\frac{\pi^2}{16} - 0.5)\, T^2.$$

Having substituted the values of μ_1 and μ_2 into the expression (5), we will receive $V = 0{,}435$.

With use of the equations of (6), (7a), (8) and (9) there have been defined the dependences of the reliability indicators on the relative time of t/T operation of the offered distribution. The result of the calculation is reduced in table 2 and presented in figures 1 and 2.

Table 2

t/T	0	0,2	0,4	0,6	π/4	1,0	1,2	1,4	π/2
Tf(t)	0	0,39	0,72	0,93	1,0	0,91	0,68	0,33	0
Tλ(t)	0	0,40	0,84	1,36	2,0	3,12	5,14	11,6	∞
Tω(t)	0	0,39	0,76	1,06	1,26	1,40	1,39	1,30	1,10
P(t)	1,0	0,96	0,85	0,68	0,50	0,29	0,13	0,03	0

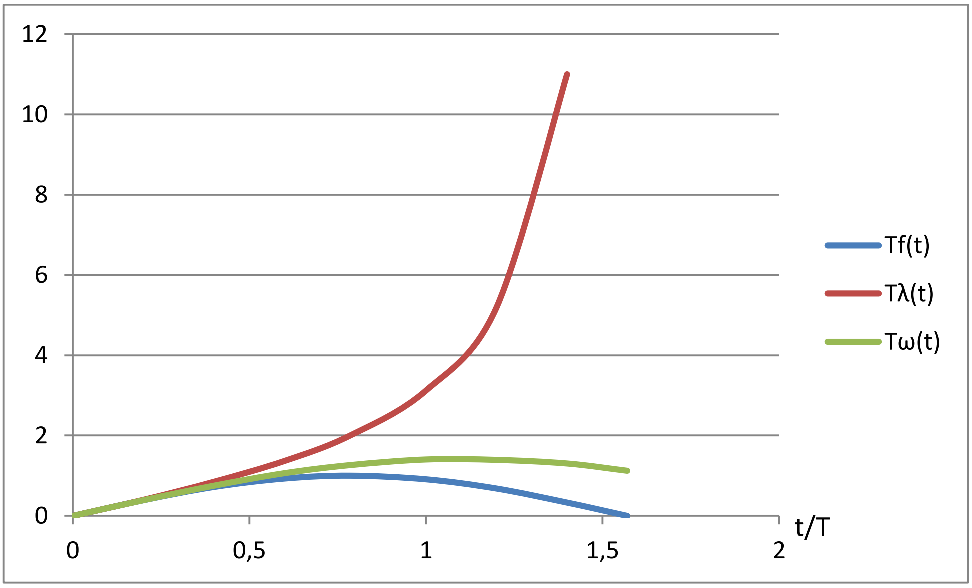

Fig. 1

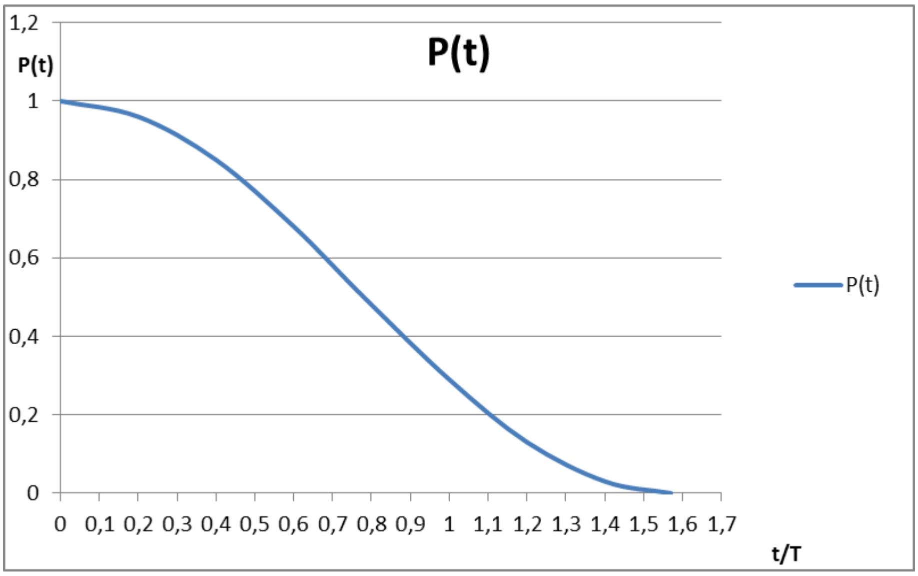

Fig. 2

As it is evident from the formula (8) and the figure 1 the failure rate increases monotonously under operation and considering that the value of variation coefficient is less than one unit, the offered distribution belongs to the class of the VFI - distributions and can be used for the description of the gradual failures of the TU elements. From figure 1 it is also evident that the failure stream parameter strives for the value equal to 1/T. It confirms the famous statement of the theory of reliability that at any distribution the failure stream parameter under operation strives for the established value, the return value of the time between failures.

As it is evident figure 2 the probability of TU non-failure operation under operation decreases, and at value of $t = \pi T/2$ it approaches to zero. And the $P(t)$ curve at first is convex up, and then – down.

For comparison in figures 3 – 5 there are presented the curves of the distribution density, failure rate and failure stream parameter of distributions of the cosine constructed with use of these tables 1 and 2 (are designated in the blue color 1) and the cosine square (are designated in the red color 2). As it is evident from formulas of (4), (8) and figure 4 the failure rate at distribution of the cosine square increases under operation twice as faster than at distribution of the cosine.

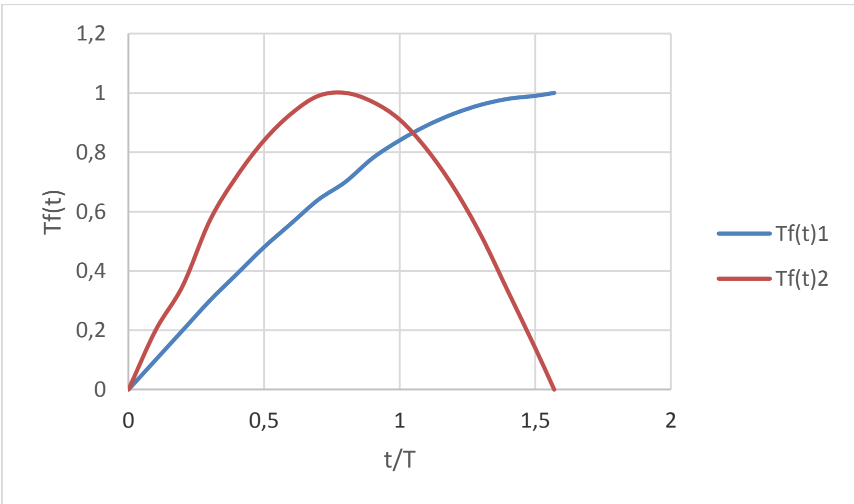

Fig. 3

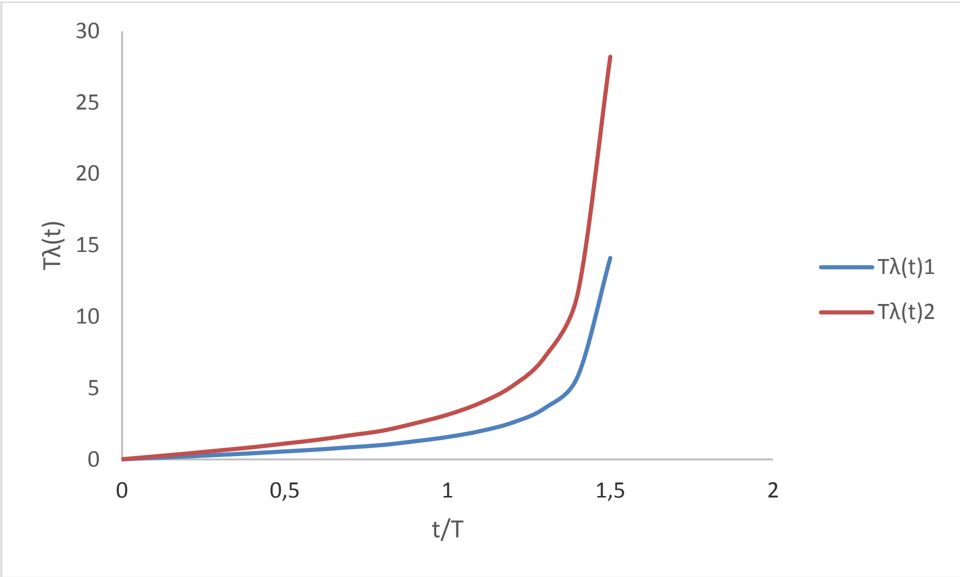

Fig. 4

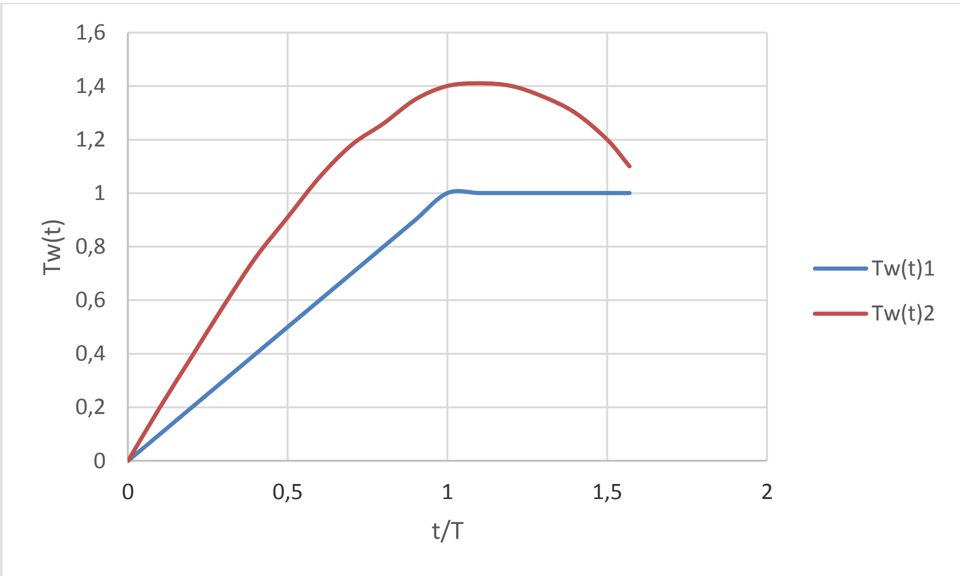

Fig. 5

Conclusion

In the case when only the time between failures is defined, and it is known that elements of technical devices are subjected to wear and aging, for the description of their gradual failures under the conditions of uncertainty it is expedient to use the offered distributions of a cosine and a cosine a square. It is evident from the received results the reliability indicators at these distributions are expressed by the elementary functions that can simplify carrying out calculations of the indicators of systems reliability at different connection of the component elements.

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