 RT&A, No3 (42)
Volume 11, September 2016

A Minimal Repair Model With Imperfect Fault Detection

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Abstract

In this paper we study a model of a system with imperfect repair, where also detection of failure might fail. For such a system we derive the lifetime distribution function of the system and give bounds for the mean life and the mean residual life function.

Keywords: imperfect repair, redundancy, limiting reliability

I. Introduction

In order to improve the reliability of a system there are mainly two possibilities. The first one is to improve the reliability of the components, the second is to implement redundancy. Mainly this is done by using more than one component to fulfill the same function, see e.g. [1]. Redundancy means that in a technical system there are more possibilities present to ensure a function, than the necessary minimum. In a previous paper the authors have studied a model of a redundant system with imperfect switching to the redundant unit. There, we have studied two cases, one of them with hot standby, the other with cold standby, see [5].

These models with hot standby and cold standby describe just two extreme situations, where the redundant units are either completely unused (pure replacement) or used in parallel under full load (hot standby).

There are many other possible models with different replacement or repair strategies.

In this paper, we will study a model, where in place of switching to a redundant unit, a minimal repair is carried out. In addition, the failure detection mechanism and subsequent minimal repair fail with a probability ⊚. Minimal repair is motivated by the use of a large unit, where only one small component is replaced so that the system itself can be seen as unchanged and the effect of using a new small component is negligible for the entire unit.

In this short paper we will derive expressions for the life time distribution of such a system together with results for conservation of properties of the distribution function as the Increasing (Decreasing) Failure Rate Average.

II. The Model

Assume a unit with minimal repair. The original lifetime distribution of the system is denoted by F(x) and hazard function $\circledcirc(x)$ so that we have

$$F(x) = 1 - \exp(-\circledcirc(x)).$$

Let us further denote the hazard rate, i.e. the derivative of $\circledcirc(x)$, by $\circledcirc(x)$.

Assume further that

$$F(0) = 0$$

and

$$\lim_{x \to \infty} F(x) = 1$$

For a unit undergoing minimal repair at the time of failures, the unit is restored during repair to a functioning state, but with the same unit age as before. The unit is therefore used and re-used after repair and the failure times of the unit follow a Nonhomogeneous Poisson Process with cumulative intensity function $\circledcirc(x)$, see [2]. The distribution of the time of the n-th repair is then defined by

$$F_n(x) = e^{-L(x)} \sum_{i=n}^{\infty} \frac{L(x)^n}{n!} \tag{1}$$

with density

$$f_n(x) = \circledcirc(x)\, e^{-\circledcirc(x)}\, \circledcirc(x)^{n-1}/(n-1)! \tag{2}$$

which gives for n=1

$$f_1(x) = \circledcirc(x)\, e^{-\circledcirc(x)}. \tag{3}$$

Assume further that the repair is successful only with probability $1-\circledcirc$, since $\circledcirc$ is the probability that the repair fails. Then the unit fails at the instant of n-th minimal repair with probability

$$(1-\circledcirc)^{n-1}\circledcirc, \tag{4}$$

i.e. n-1 minimal repairs were successful, the n-th not. Combining (2) and (4) and summing over n one gets the density of the lifetime distribution function of a unit with minimal and imperfect repair

$$g(x) = \sum_{n=1}^{\infty} (1\text{-}g)^{n-1} g\, f_n(x) = \sum_{n=1}^{\infty} (1\text{-}g)^{n-1} g\, l(x)\, e^{-L(x)}\, L(x)^{n-1}/(n\text{-}1)! \; = \circledcirc\, \circledcirc(x)\, e^{-(1-\circledcirc)\circledcirc(x)}\, e^{-\circledcirc(x)}.$$

$$= \circledcirc\, \circledcirc(x) \exp(-\circledcirc\, \circledcirc(x)). \quad (5)$$

The distribution function reads

$$G(x) = 1 - \exp(-\circledcirc\circledcirc\circledcirc(x)). \quad (6)$$

III. Results for the lifetime of a unit with imperfect and minimal repair

The mean lifetime of the unit is derived by the distribution (6) by

$$m_G = \int_0^{\infty} \exp(-g\, L(x))\, dx. \quad (7)$$

A distribution function F(x) is said to belong to the IFRA class (increasing failure rate average) / DFRA (decreasing failure rate average), if

$$\circledcirc(x)/x$$

is an increasing / decreasing function, see [1]. Then, also G(x) belongs to the IFRA / DFRA class since

$$\circledcirc\circledcirc(x)/\circledcirc x$$

is also increasing / decreasing, if the property holds for $\circledcirc(x)/x$. If now F is IFRA (DFRA), we have

$$\circledcirc(\circledcirc\, x) / (\circledcirc x) \le (\ge)\, \circledcirc(x) / x, \quad (8)$$

since $\circledcirc \le 1$. This inequality gives

$$\circledcirc(\circledcirc\, x) \le (\ge)\, \circledcirc\, \circledcirc(x).$$

From here, it follows

$$\exp(-\circledcirc(\circledcirc\, x)) \ge (\le) \exp(-\circledcirc\, \circledcirc(x)). \quad (9)$$

Integrating from 0 to ∞ yields

$$\int_0^{\infty} \exp(-L(g\, x))\, dx \ge (\le) \int_0^{\infty} \exp(-g\, L(x))\, dx\, . \quad (10)$$

This is equivalent to

$$m_G \le (\ge)\ m_F/\circledcirc,$$

provided F(x) is IFRA (DFRA). Also, bounds on the residual life function can be derived. Writing

$$T_{RL} = \int_x^{\infty}(1\text{-}G(t))\,dt = \int_x^{\infty}\exp(-g\,L(t))\,dt \le (\ge) \int_x^{\infty}\exp(-L(g\,t))\,dt = (1/\circledcirc)\,T_{RL,F}(\circledcirc\,x), \quad (11)$$

if F is IFRA (DFRA). Here, $T_{RL,F}$ denotes the residual life function of F. Since a distribution function that is IFRA (DRFRA) is also HNBUE (HNWUE), see [3], the result can be extended

$$T_{RL} \le (\ge)\ (1/\circledcirc)\ m_F \exp(-\circledcirc x/m_F), \quad (12)$$

Where we have used the HNBUE (HNWUE) property of F, i..e.

$$\int_x^{\infty}(1\text{-}F(t))dt \le (\ge)\ m_F \exp(-x/m_F).$$

IV. Comparison and Conclusions

In this paper, we have derived results for a model of a unit that undergoes minimal repair with imperfect detection of failures. We have derived the distribution function

$$G(x) = 1 - \exp(-\circledcirc\circledcirc\circledcirc(x)) = 1\text{-}(1\text{-}F(x))^{\circledcirc}, \quad (13)$$

in closed form. This distribution function can be compared with the lifetimes distribution function of a system with hot standby and imperfect switching (repair), see [5]:

$$G_{hs} = \circledcirc F(x)\ /(1\text{-}(1\text{-}\circledcirc)F(x)) \quad (14)$$

One can now observe that

$$G_{hs}(x) \le G(x).$$

This follows from the fact that

$$(1\text{-}F) \le (1\text{-}F)^{\circledcirc}(1\text{-}(1\text{-}\circledcirc)F).$$

This inequality can be proven using that the function

$$h(x) = \circledcirc x^{\circledcirc} + (1\text{-}\circledcirc)x^{1+\circledcirc} - x$$

is nonnegative in the interval [0,1]. This property is a result of the following facts

$$h(0) = 0,\ h(1) = 0,$$

$$h'(x) \text{ tends to infinity as } x \to 0,$$

$$h'(1)=0,$$

$$h''(x) \text{ is negative for } x<x_0=◎/(1+◎) \text{ and}$$

$$h''(x) \text{ is positive for } x>x_0.$$

$$h''(x_0) = 0 \text{ with } h(x_0) = ◎(3-◎) >0.$$

Therefore, $h(x)$ must be nonnegative on the interval $[0,x_0]$, since it is convex there. Furthermore, $h(x)$ has an increasing first derivative on $[x_0,1]$, starts at a positive value at x_0 and decreases to zero at 1. There is no inflection point on the interval $(x_0,1]$, which would be necessary for $h(x)$ to take negative values in $(x_0,1]$, since $h'(1)=0$. Therefore, $h(x)$ is also nonnegative on $[x_0,1]$. Hence we have shown that the failure probability of a unit with minimal repair and imperfect failure detection is larger than for a unit with internal hot standby and imperfect failure detection. Now, we can also compare the bounds. If F belongs to the IFRA class, then the same holds for (13) and the following bounds hold are derived:

$$m_G \leq m_F/◎, \tag{15}$$

$$T_{RL} \leq (1/◎)\ mF \exp(-◎x/m_F), \tag{16}$$

These results can be compared with the ones provided in [5] for hot and cold standby systems. We see immediately that the bounds (15) and (16) are the same as for cold standby systems, which is generally the absolutely upper bound for units of types of redundancies. Therefore, we have derived the distribution function, bounds on the mean lifetime and the residual life function for a unit with minimal repair and imperfect failure detection.

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