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Mean Time To System Failure Assessment Of A Single Unit System Requiring Two Types Of Supporting Device For Operation

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Abstract

This paper studies the mean time to system failure (MTSF) of single unit system operating with the help of two types of external supporting device. Each type of supporting device has two copies I and II. The system is analyzed using differential difference equation to develop the explicit expression for mean time to system failure. Based on assumed numerical values given to system parameters, graphical illustrations are given to highlight important results.

Keywords: *availability, supporting device, probabilistic, single*

I. Introduction

Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order. In real-life situations we often encounter cases where the systems that cannot work without the help of external supporting devices connect to such systems. These external supporting devices are systems themselves that are bound to fail. Such systems are found in power plants, manufacturing systems, and industrial systems. Large volumes of literature exist on the issue relating to prediction of various systems performance connected to an external supporting device for their operations. Yusuf et al (2014) present mathematical modeling approach to analysis of mean time to system failure of two unit cold standby system with a supporting device. Yusuf et al (2015) performed comparative analysis of MTSF between systems connected to supporting device for operation. Yusuf et al (2014) performed reliability computation of a linear consecutive 2-out-of-3 system in the presence of supporting device.

Existing literatures either ignores the impact of multi-supporting device on system performance. Such works laid emphasis on systems connected to one type of an external supporting device whose failure brings about total breakdown. More sophisticated models of systems connected to multi-external supporting device should be developed to assist in reducing operating costs and the risk of a catastrophic breakdown, to maximize output, system availability, and generated revenue, minimize cost, and assure ongoing quality of the parts being produced. The problem considered in this paper is different from discussed authors above. The purpose of this paper is twofold. The first purpose is to develop the explicit expressions for mean time to system failure. The second is to capture the effect of both failure and repair rates on mean time to system failure based on assumed numerical values given to the system parameters.

The organization of the paper is as follows. Section 2 presents model's description and assumptions. Section 3 presents formulations of the models. Numerical examples are presented and discussed in Section 4. Finally, we make a concluding remark in Section 5.

II. Description of the System

In this paper, a single unit system connected to two types of supporting device is considered. It is assumed that each type of supporting has a copy on standby and the switching is perfect. It is also assumed that the system work with either two copies of type I supporting device or two copies of type II supporting device or one copy of both type I and II. Both unit and supporting devices are assumed to be repairable. Each of the primary supporting devices fails independently of the state of the other and has an exponential failure distribution with parameter λ_1 **and** λ_2 **for type I and II respectively.** Whenever a primary supporting device fails, it is immediately sent to repair with parameter μ_1 **and** μ_2 and the standby supporting device is switch to operation. System failure occur when the unit has failed with parameter λ **and service rate with parameter** with parameter μ **or the failure of all copies of type I and type II.**

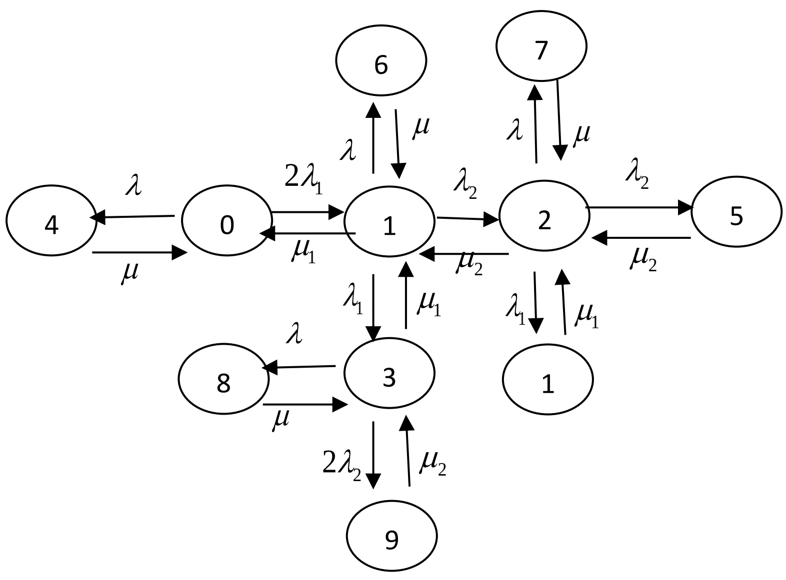

Figure 1: The State transition diagram of System

III. Formulation of the Model

In order to analyze the system availability of the system, we define $P_i(t)$ to be the probability that the system at $t \geq 0$ is in state S_i. Also let $P(t)$ be the row vector of these probabilities at time t. The initial condition for this problem is:

$$P(0)=[p_0(0), p_1(0), p_2(0),..., p_{10}(0)]=[1,0,0,0,0,0,0,0,0,0,0]$$

We obtain the following differential difference equations from Figure 1:

$$p_0'(t)=-(\lambda+2\lambda_1)p_0(t)+\mu_1 p_1(t)+\mu p_4(t)$$
$$p_1'(t)=-(\lambda+\lambda_1+\lambda_2+\mu_1)p_1(t)+2\lambda_1 p_0(t)+\mu_2 p_2(t)+\mu_1 p_3(t)+\mu p_6(t)$$
$$p_2'(t)=-(\lambda+\lambda_1+\lambda_2+\mu_2)p_2(t)+\lambda_2 p_1(t)+\mu_2 p_5(t)+\mu_7 p_7(t)+\mu_1 p_{10}(t)$$
$$p_3'(t)=-(\lambda+2\lambda_2+\mu_1)p_3(t)+\lambda_1 p_1(t)+\mu p_8(t)+\mu_2 p_9(t)$$
$$p_4'(t)=-\mu p_4(t)+\lambda p_0(t)$$
$$p_5'(t)=-\mu_2 p_5(t)+\lambda_2 p_2(t)$$
$$p_6'(t)=-\mu p_6(t)+\lambda p_1(t)$$
$$p_7'(t)=-\mu p_7(t)+\lambda p_2(t)$$
$$p_8'(t)=-\mu p_8(t)+\lambda p_3(t)$$
$$p_9'(t)=-\mu_2 p_9(t)+2\lambda_2 p_3(t)$$
$$p_{10}'(t)=-\mu_1 p_{10}(t)+\lambda_1 p_2(t) \quad \textbf{(1)}$$

This can be written in the matrix form as

$$\dot{P}=TP, \quad \textbf{(2)}$$

where

$$T=\begin{pmatrix} -\delta_1 & \mu_1 & 0 & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\lambda_1 & -\delta_2 & \mu_2 & \mu_1 & 0 & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & -\delta_3 & 0 & 0 & \mu_2 & 0 & \mu & 0 & 0 & \mu_1 \\ 0 & \lambda_1 & 0 & -\delta_4 & 0 & 0 & 0 & 0 & \mu & \mu_2 & 0 \\ \lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 2\lambda_2 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 \\ 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 \end{pmatrix}$$

$$\delta_1=(\lambda+2\lambda_1),\ \delta_2=(\lambda+\lambda_1+\lambda_2+\mu_1),\ \delta_3=(\lambda+\lambda_1+\lambda_2+\mu_2),\ \delta_4=(\lambda_1+2\lambda_2+\mu_1)$$

It is difficult to evaluate the transient solutions, hence following Trivedi (2002), Wang and Kuo (2000), Wang et al. (2006) to develop the explicit for MTSF. The procedures require deleting rows and columns of absorbing states of matrix T and take the transpose to produce a new matrix, say

M . The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)\left(-M^{-1}\right)\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \quad \textbf{(3)}$$

where the initial conditions are given by

$P(0) = [p_0(0), p_1(0), p_2(0), p_3(0)] = [1,0,0,0]$ **and**

$$M = \begin{pmatrix} -(2\lambda_1+\lambda) & 2\lambda_1 & 0 & 0 \\ \mu_1 & -(\lambda+\lambda_1+\lambda_2+\mu_1) & \lambda_2 & \lambda_1 \\ 0 & \mu_2 & -(\lambda+\lambda_1+\lambda_2+\mu_2) & 0 \\ 0 & \mu_1 & 0 & -(\lambda_1+2\lambda_2+\mu_1) \end{pmatrix}$$

The procedure above is successful because of the following relations

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)\int_0^\infty e^{Mt}dt, \quad \textbf{(4)}$$

where $\int_0^\infty e^{Mt}dt = -M^{-1}$ **(5)**

The explicit expression for is given by MTSF

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF = \frac{N}{D} \quad \textbf{(6)}$$

$$N = 2\lambda\lambda_2\mu_2 + \mu_1\mu_2\lambda_1 + 2\mu_1\mu_2\lambda_2 + 2\mu_2\lambda_1\lambda_2 + \lambda_1^3 + 4\mu_1\lambda_1\lambda_2 + \mu_1\lambda_1^2 + 2\mu_1\lambda\lambda_1 + 4\mu_1\lambda\lambda_2 + 4\lambda_1^2\lambda_2 + \mu_2\lambda_1^2 +$$
$$5\lambda_1\lambda_2^2 + \mu_1\mu_2\lambda + \mu_1^2\lambda_2 + 3\mu_1\lambda_2^2 + \mu_2\lambda\lambda_1 + 2\lambda_2^3 + 2\lambda^2\lambda_2 + 6\lambda\lambda_1\lambda_2 + \lambda^2\lambda_1 + \mu_1\lambda^2 + \mu_1^2\lambda_1 + 2\lambda\lambda_1^2 + 4\lambda\lambda_2^2 +$$
$$\mu_1^2\lambda + \mu_1^2\mu_2 + 2\lambda_1(\lambda+\lambda_1+\lambda_2+\mu_2)(2\lambda_2+\lambda_1+\mu_1) + 2\lambda_1\lambda_2(2\lambda_2+\lambda_1+\mu_1) + 2\lambda_1^2(\lambda+\lambda_2+\lambda_1+\mu_2)$$

$$D = 16\lambda\lambda_1^2\lambda_2 + 8\lambda_1^3\lambda_2 + 10\lambda_1^2\lambda_2^2 + 5\lambda\lambda_1^3 + 2\lambda_1^4 + 4\lambda_1\lambda_2^3 + 4\lambda^2\lambda_1^2 + \mu_1\mu_2\lambda^2 + 2\lambda\lambda_2^3 + 2\lambda^3\lambda_2 + \lambda^3\lambda_1 +$$
$$\mu_1\lambda^3 + 4\lambda^2\lambda_2^2 + \mu_1^2\lambda^2 + 2\mu_2\lambda_1^3 + 6\mu_2\lambda\lambda_1\lambda_2 + 4\mu_2\lambda_1^2\lambda_2 + \lambda\lambda_1\lambda_2\lambda_3 + 3\mu_1\mu_2\lambda\lambda_1 + 3\mu_2\lambda\lambda_1^2 + \mu_1^2\mu_2\lambda +$$
$$8\mu_1\lambda\lambda_1\lambda_2 + 2\mu_1\mu_2\lambda\lambda_2 + 3\mu_1\lambda\lambda_1^2 + 2\mu_2\lambda^2\lambda_2 + 4\mu_2\lambda^2\lambda_2 + \mu_2\lambda^2\lambda_1 + 10\lambda^2\lambda_1\lambda_2 + 4\mu_1\lambda^2\lambda_1 + \mu_1^2\lambda\lambda_1 +$$
$$\mu_1^2\lambda\lambda_2 + 3\mu_1\lambda\lambda_2^2 + 2\mu_1\lambda_1^2\lambda_2 + 2\mu_1\lambda_1\lambda_2^2$$

IV. Numerical Examples

Numerical examples are presented to demonstrate the impact of failure and repair rates on **mean time to system failure** based on given values of the parameters. For the purpose of numerical example, the following sets of parameter values are used:

$\mu_1 = 0.3, \mu_2 = 0.5, \mu = 0.5, \lambda_1 = 0.2, \lambda_2 = 0.3, \lambda(0.4, 0.6, 0.8)$ **for Figures 2 – 5.**

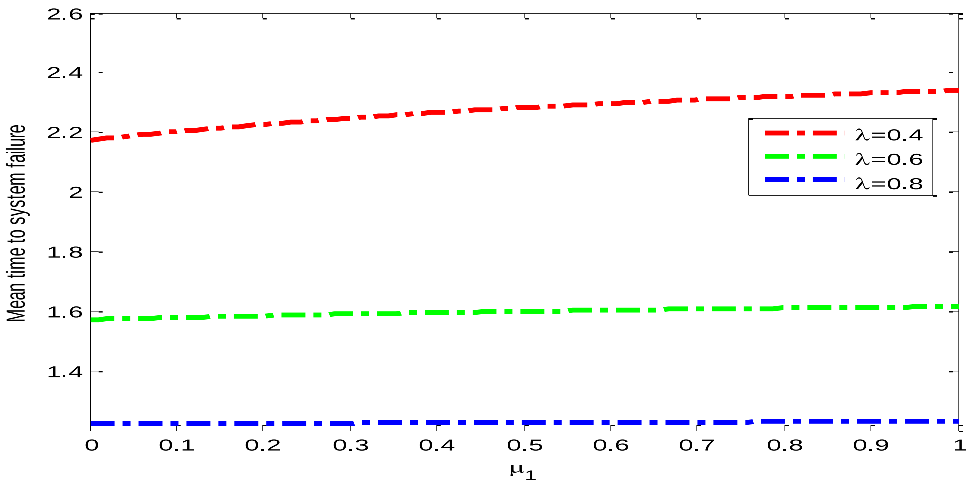

Figure 2: Availability against type I supporting device repair rate μ_1 **for** different values of $\lambda(0.4, 0.6, 0.8)$

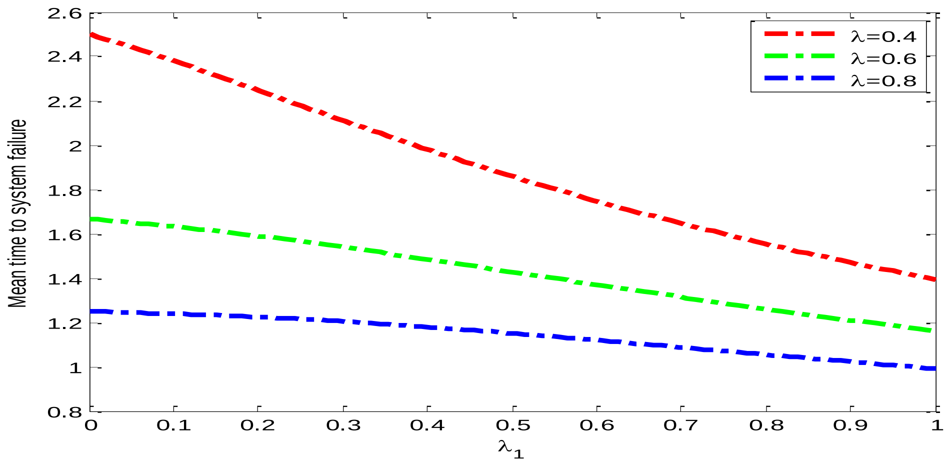

Figure 3: Availability against type I supporting device failure rate λ_1 **for** different values of $\lambda(0.4, 0.6, 0.8)$

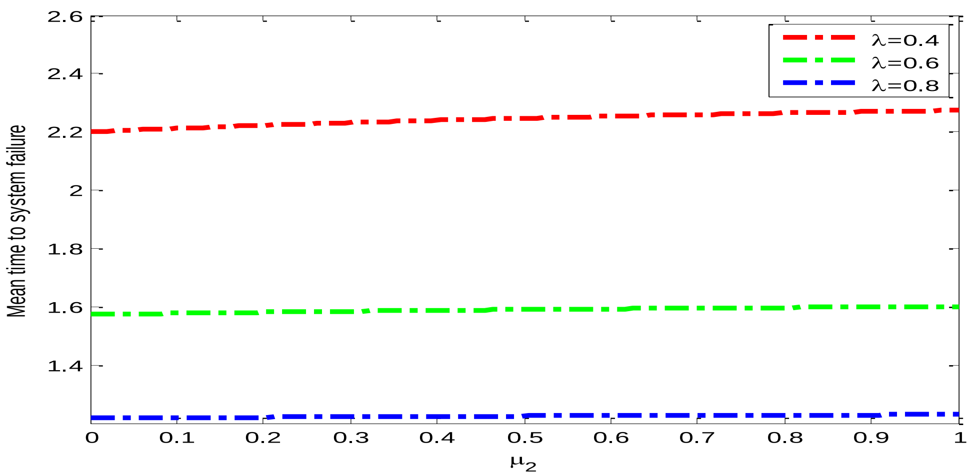

Figure 4: Availability against type II supporting device repair rate μ_2 **for** different values of $\lambda(0.4, 0.6, 0.8)$

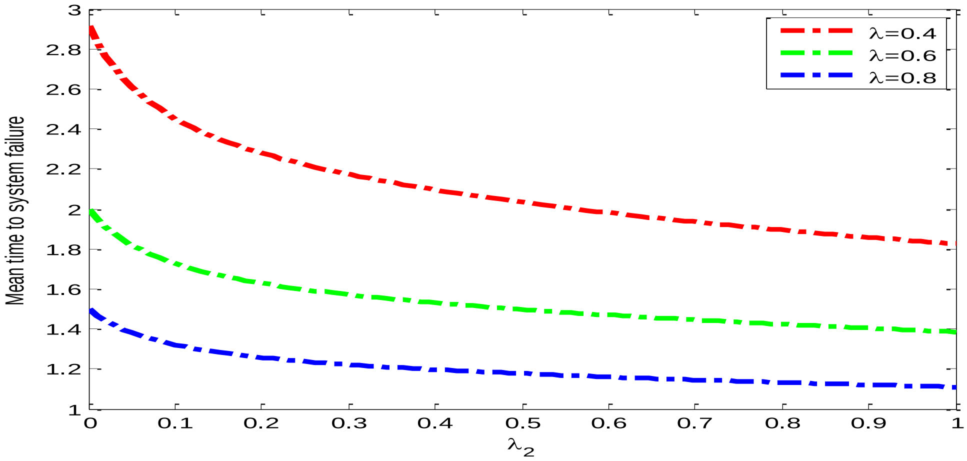

Figure 5: Availability against type II supporting device failure rate λ_2 **for** different values of $\lambda(0.4, 0.6, 0.8)$

IV. Discussion

Numerical results of availability with respect to type k, $k = I, II$ supporting devices repair μ_i and failure rates λ_i, $i = 1, 2$ **for** different values of $\lambda(0.4, 0.6, 0.8)$ **are depicted in Figures 2 - 5 respectively. In Figures 2 and 4, the mean time to system failure** increases as μ_1 and μ_2 for different values of unit failure rate λ. **This sensitivity analysis implies that major maintenance to the unit and supporting devices should be invoked to improve and maximize the mean time to system failure, production output as well as the profit.** On the other hand, Figures 3 and 5 show that the availability decreases as λ_1 and λ_2 increases for different values of unit failure rate λ. **This sensitivity analysis implies that major maintenance should be invoked to the unit and supporting devices to minimize the failure of the system in order to improve and maximize the mean time to system failure, production output as well as the profit.**

V. Conclusion

This paper studied a single system connected to two types of supporting device type I and II for its operation. Explicit expression for the **mean time to system failure** was derived. The numerical simulations presented in Figures 2 – 5 provide a description of the effect of failure rate and repair rate on **mean time to system failure** for different values of unit failure rate λ . On the basis of the numerical results obtained for particular cases, it is suggested that the system mean time to system failure can be improved significantly by:

(i) Adding more cold standby units.
(ii) Increasing the repair rate.
(iii) Reducing the failure rate of the system by hot or cold duplication method.
(iv) Exchange the system when old with new one before failure.

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