Probabilistic Models for Reliability Analysis of a System with Three Consecutive Stages of Deterioration

Ibrahim Yusuf
Department of Mathematical Sciences, Bayero University, Kano, Nigeria
iyusuf.mth@buk.edu.ng
Ramatu Idris Gatawa
Department of Mathematics, North West University, Kano, Nigeria
rigatawa@yahoo.com

Abstract

In this paper we present availability and mean time to failure estimation of a system where the deterioration rates follow the Weibull distribution. The paper presents modeling and evaluation of availability and mean time to system failure (MTSF) of a consecutive three stage deteriorating system. The system has three possible modes: working with full capacity, deterioration and failure mode. The three stages of deterioration are minor, medium and major deteriorations. Minor and major maintenance are allowed at minor and medium deterioration states and replacement at system failure. Explicit expressions for the availability and mean time to failure of the system are obtained analytically. Graphs have been plotted to determine the behavior of availability and mean time to system failure with respect to time for different values of deterioration, maintenance and replacement rates. Also, high values of the shape parameter decreases mean time to system failure and availability. The system is analyzed using differential difference equations.
Keywords: list, keywords, enter, here

I. Introduction

In practical engineering applications, most repairable systems are deteriorative that system failure often cannot be as good as new, it is more reasonable for these deteriorating repairable systems to assume that the successive working times of the system after repair will become shorter and shorter while the consecutive repair times of the system after failure will become longer and longer. Most of these systems are subjected to random deterioration which can result in unexpected failures and disastrous effect on the system availability and the prospect of the economy. Therefore it is important to find a way to slow down the deterioration rate, and to prolong the equipment's life span. Maintenance policies are vital in the analysis of deterioration and deteriorating systems as they help in improving reliability and availability of the systems. Maintenance models can assume minor maintenance, major maintenance before system failure, perfect repair (as good as new), minimal repair (as bad as old), imperfect repair and replacement at system failure.

Several models on deteriorating systems under different conditions have been studied by several researchers such as Bérenguer (2008), Frangopol *et al* (2004), Lam and Zhang (2003), Nicolai *et al* (2007), Rani and Sukumari (2014), Vinayak and Dharmaraja (2012), Yuan *et al* (2012), Yuan and Xu (2011). Analysis of reliability and availability model for deteriorating system have been studied under different conditions such as Liu *et al*. (2011) who investigated reliability analysis of a deteriorating system with delayed vacation of repairman, Tuan *et al* (2013).A Reliability-based Opportunistic Predictive Maintenance Model for k-out-of-n Deteriorating Systems, Xiao *et al* (2013) proposed the Bayesian reliability estimation for deteriorating systems with limited samples using the maximum entropy approach, Yusuf *et al* (2012). Presents modelling the reliability and availability characteristics of a system with three stages of deterioration, Zhang and Wang (2007)

deal with the study of deteriorating cold standby repairable system with priority in use.
This paper considers a system with three consecutive stages of deterioration before failure and derived its corresponding mathematical models. Furthermore, we study mean time to system failure and availability using differential difference equation method. The focus of our analysis is primarily to capture the effect of minor maintenance, minor deterioration and shape parameter on mean time to system failure and availability.
The organization of the paper is as follows. Section 2 contains a description of the system under study. Section 3 presents formulations of the models. The results of our numerical simulations are presented and discussed in section 4. Finally, we make some concluding remarks in Section 5.

II. Description and States of the System

In this paper, one unit system is considered. It is assumed that the system most pass through three consecutive stages of deterioration which are minor, medium and major deterioration before failure. The unit is considered to be non repairable. At early state of the system life, the operating unit is exposed to minor deterioration with rate λ_1 **and this deterioration is rectified through minor maintenance** α_1 **which revert the unit to its earliest position before deterioration.** If not maintained, the unit is allowed to continue operating under the condition of minor deterioration which later changes to medium deterioration with rate λ_2. At this stage, the strength of the unit is still strong that it can rectify to early state with rate α_2. However, the system can move to major deterioration stage with rate λ_3 where the strength of the unit has decreases to the extent that it cannot be reverted to its early state, neither that of minor nor medium deterioration stages. Here the unit is allowed to continue operation until it fails with parameter λ_4 and the system is immediately replaced by with a new one with rate α_3. Deteriorating rates follow Weibull distribution $f(t) = \lambda_k \delta t^{\delta-1}$, $k = 1,2,3$, δ **is the shape parameter.**

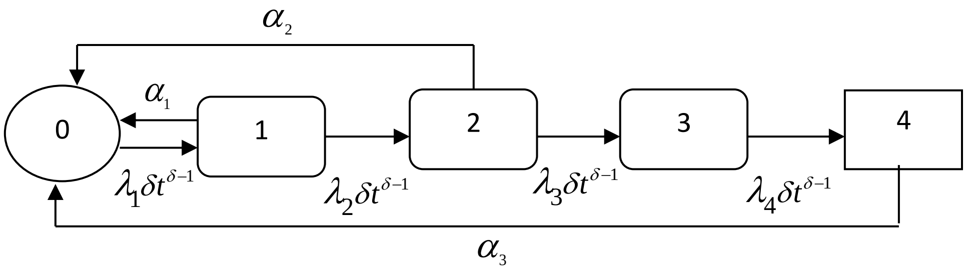

Figure 1: Transition diagram of the system

Table 1: States of the system

State	Description
S_0	Initial state, the system is operative.
S_1	The system is in minor deterioration mode and is under online minor maintenance, and is operative.
S_2	The system is in medium deterioration mode and is under online major maintenance and is operative.
S_3	The system is in major deterioration mode and is operative.
S_4	The system is inoperative.

III. Formulation of the Models

In order to analyze the system availability of the system, we define $P_i(t)$ to be the probability that the system at $t \geq 0$ is in state S_i. Also let $P(t)$ be the row vector of these probabilities at time t. The initial condition for this problem is:

$$P(0) = [p_0(0), p_1(0), p_2(0), p_3(0), p_4(0)] = [1,0,0,0,0]$$

We obtain the following differential difference equations from Figure 1:

$$\frac{d}{dt} p_0(t) = -\lambda_1 \delta t^{\delta-1} p_0(t) + \alpha_1 p_1(t) + \alpha_2 p_2(t) + \alpha_3 p_4(t)$$

$$\frac{d}{dt} p_1(t) = -\left(\alpha_1 + \lambda_2 \delta t^{\delta-1}\right) p_1(t) + \lambda_1 \delta t^{\delta-1} p_0(t)$$

$$\frac{d}{dt} p_2(t) = -\left(\alpha_2 + \lambda_3 \delta t^{\delta-1}\right) p_2(t) + \lambda_2 \delta t^{\delta-1} p_1(t)$$

$$\frac{d}{dt} p_3(t) = -\lambda_4 \delta t^{\delta-1} p_3(t) + \lambda_3 \delta t^{\delta-1} p_2(t)$$

$$\frac{d}{dt} p_4(t) = \alpha_3 p_4(t) + \lambda_4 \delta t^{\delta-1} p_3(t) \qquad \textbf{(1)}$$

This can be written in the matrix form as

$$\dot{P} = TP, \qquad \textbf{(2)}$$

where

$$T = \begin{pmatrix} -\lambda_1 \delta t^{\delta-1} & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 \delta t^{\delta-1} & -\left(\alpha_1 + \lambda_2 \delta t^{\delta-1}\right) & 0 & 0 & 0 \\ 0 & \lambda_2 \delta t^{\delta-1} & -\left(\alpha_2 + \lambda_3 \delta t^{\delta-1}\right) & 0 & 0 \\ 0 & 0 & \lambda_3 \delta t^{\delta-1} & -\lambda_4 \delta t^{\delta-1} & 0 \\ 0 & 0 & 0 & \lambda_4 \delta t^{\delta-1} & -\alpha_3 \end{pmatrix}$$

The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) is given by

$$A_V(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) = \frac{N}{D} \qquad (3)$$

where

$$N = \alpha_3 \lambda_4 \left(\alpha_1 + \delta \lambda_2 t^{(\delta-1)}\right)\left(\alpha_2 + \delta \lambda_3 t^{(\delta-1)}\right) + \alpha_3 \delta \lambda_1 \lambda_4 t^{(\delta-1)} \left(\alpha_2 + \delta \lambda_3 t^{(\delta-1)}\right) + \alpha_3 \delta^2 \lambda_1 \lambda_2 \lambda_3 t^{(\delta-1)} + \alpha_3 \delta^2 \lambda_1 \lambda_2 \lambda_3 \left(t^{(\delta-1)}\right)^2$$

$$D = \delta^3 \lambda_1 \lambda_2 \lambda_3 \lambda_4 t^{(\delta-1)} + \alpha_3 \delta^2 \lambda_1 \lambda_2 \lambda_3 \left(t^{(\delta-1)}\right)^2 + \alpha_3 \delta^2 \lambda_1 \lambda_2 \lambda_4 \left(t^{(\delta-1)}\right)^2 + \alpha_3 \alpha_2 \delta \lambda_1 \lambda_4 t^{(\delta-1)} + \alpha_3 \delta^2 \lambda_1 \lambda_3 \lambda_4 t \left(t^{(\delta-1)}\right)^2 +$$
$$\alpha_1 \alpha_2 \alpha_3 \lambda_4 + \alpha_1 \alpha_3 \delta \lambda_3 \lambda_4 t^{(\delta-1)} + \alpha_2 \alpha_3 \delta \lambda_2 \lambda_4 t^{(\delta-1)} + \alpha_3 \delta^2 \lambda_2 \lambda_3 \lambda_4 \left(t^{(\delta-1)}\right)^2$$

Following Trivedi (2002), Wang and Kuo (2000), Wang et al. (2006) to develop the explicit for MTSF. The procedures require deleting rows and columns of absorbing states of matrix T and take the transpose to produce a new matrix, say M. The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right]=P(0)\left(-M^{-1}\right)\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \tag{4}$$

where the initial conditions are given by
$P(0)=[p_0(0),p_1(0),p_2(0),p_3(0)]=[1,0,0,0]$ **and**

$$M=\begin{pmatrix}-\lambda_1\delta t^{\delta-1} & \lambda_1\delta t^{\delta-1} & 0 & 0\\ \alpha_1 & -\left(\alpha_1+\lambda_2\delta t^{\delta-1}\right) & \lambda_2\delta t^{\delta-1} & 0\\ \alpha_2 & 0 & -\left(\alpha_2+\lambda_3\delta t^{\delta-1}\right) & \lambda_3\delta t^{\delta-1}\\ 0 & 0 & 0 & -\lambda_4\delta t^{\delta-1}\end{pmatrix}$$

The explicit expression for is given by MTSF

$$MTSF=\frac{\lambda_4\left(\alpha_1+\delta\lambda_2 t^{(\delta-1)}\right)\left(\alpha_2+\delta\lambda_3 t^{(\delta-1)}\right)+\delta\lambda_1\lambda_4\left(\alpha_2+\delta\lambda_3 t^{(\delta-1)}\right)+\delta^2\lambda_1\lambda_2\lambda_4\left(t^{(\delta-1)}\right)^3+\delta^2\lambda_1\lambda_2\lambda_3\left(t^{(\delta-1)}\right)^3}{\delta^3\lambda_1\lambda_2\lambda_3\lambda_4\left(t^{(\delta-1)}\right)^3} \tag{5}$$

IV. Discussion

Numerical examples are presented to demonstrate the impact of repair and failure rates on steady-state availability and net profit of the system based on given values of the parameters. For the purpose of numerical example, the following sets of parameter values are used: $\alpha_1=0.1$, $\alpha_2=0.3$, $\alpha_3=0.5$, $\lambda_1=0.4$, $\lambda_2=0.5$, $\lambda_3=0.6$, $\lambda_4=0.2$, $\delta=0.9$, $0\le t\le 10$

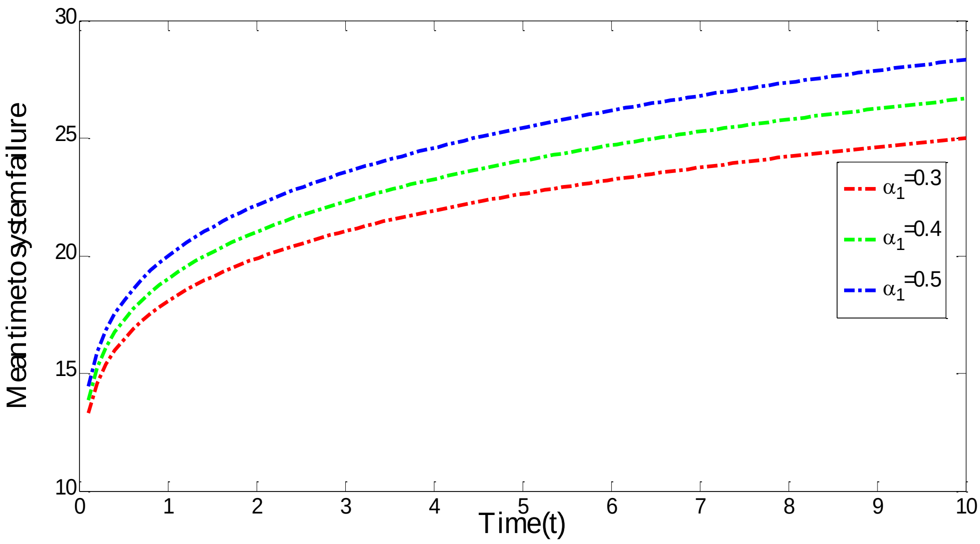

Figure 2: Mean time to system failure against **for** different values of $\alpha_1(0.3,0.4,0.5)$

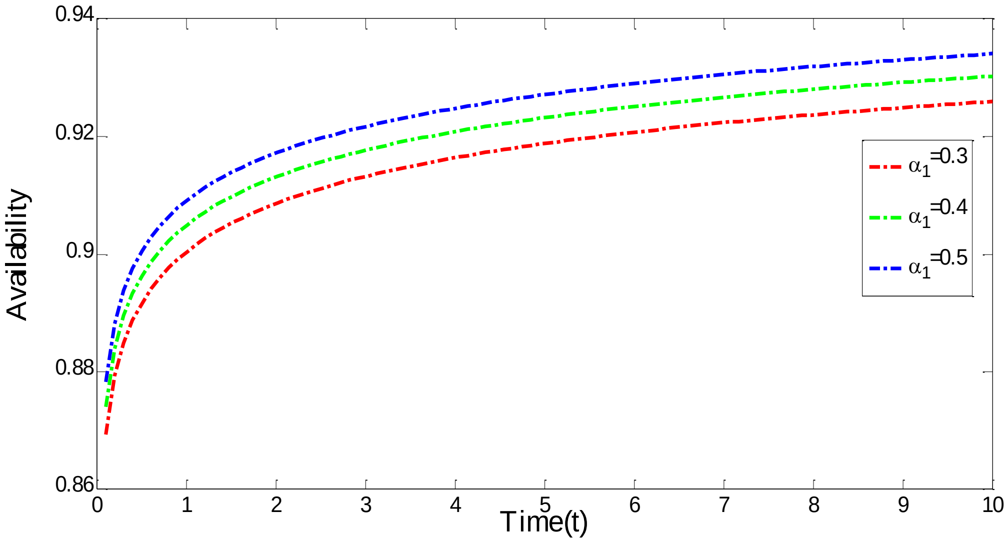

Figure 3: Availability against **for** different values of $\alpha_1 (0.3, 0.4, 0.5)$

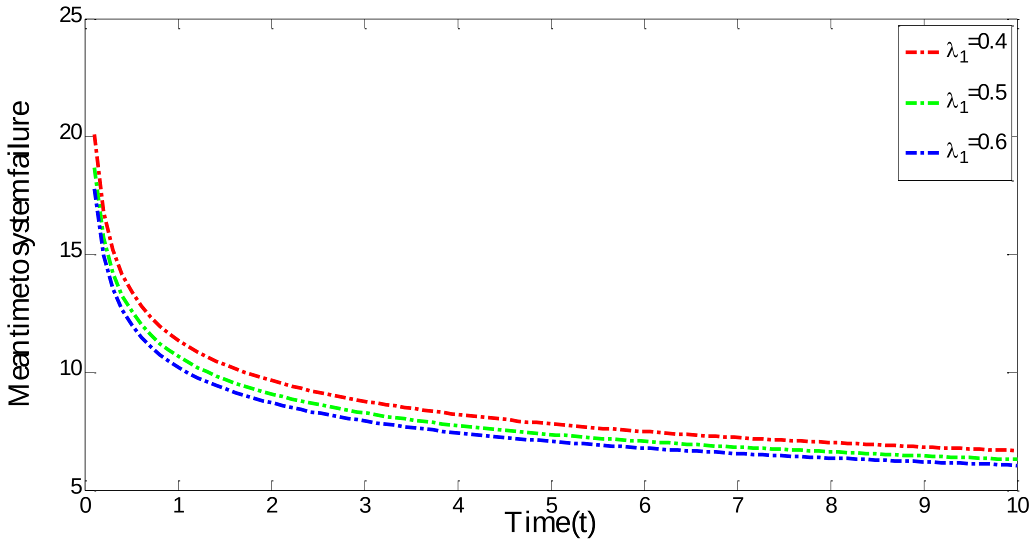

Figure 4: Mean time to system failure against **for** different values of $\lambda_1 (0.4, 0.5, 0.6)$

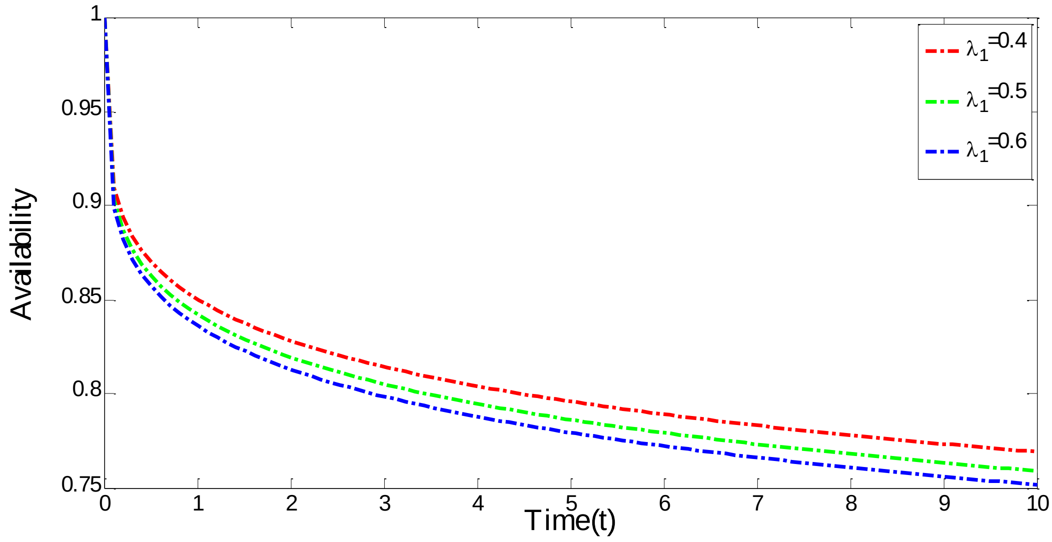

Figure 5: Availability against time for different values of $\lambda_1(0.4, 0.5, 0.6)$

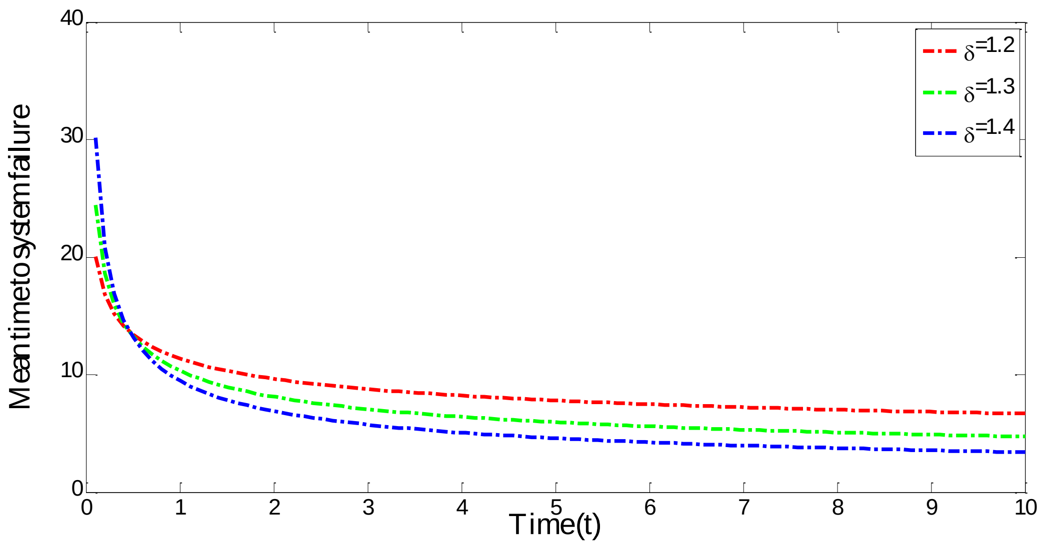

Figure 6: Mean time to system failure against **for** different values of $\lambda_1(0.4, 0.5, 0.6)$

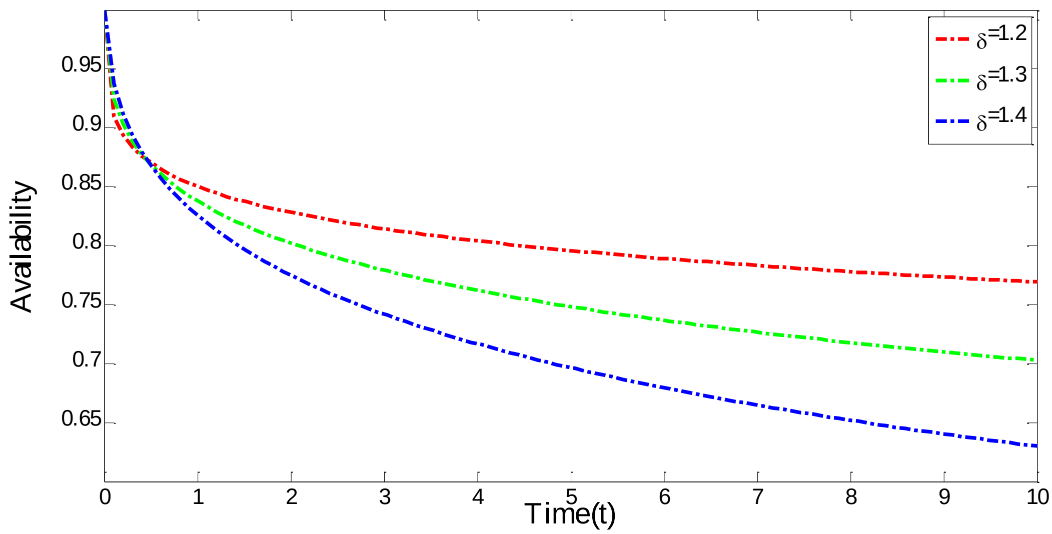

Figure 7: Availability against time for different values of $\delta(1.2,1.3,1.4)$

Numerical results of mean time to system failure and availability with respect to time are depicted in Figures 2 and 3 for different values of minor maintenance rates α_1. In these Figure the mean time to system failure and availability increases as time increases when α_1 increases from 0.3 to 0.5. This sensitivity analysis implies that minor maintenance to the system should be invoked to increase the life span of the system. On the other hand, simulations in Figures 4 and 5, depicts the impact of time on mean time to system failure and availability for different values of minor deterioration λ_1. From these Figures, the mean time to system failure and availability decreases as time increases for different values of λ_1. The above sensitivity analysis depicted the effect of minor maintenance and deterioration rates on mean time to system failure and availability. It can be observe that minor maintenance played a significant role in increasing the mean time to system failure and availability whereas minor deterioration slow down the mean time to system failure. From simulations depicted in Figures 6 and 7, it is evident that the choice of the shape parameter δ influences the time taken for the system to reach the failure state. The higher the value of this shape parameter δ, the less the values of mean time to system failure and availability.

V. Conclusion

This paper studied a one unit system with three consecutive stages of deterioration before failure. Explicit expressions for the mean time to system failure and availability are derived. The numerical simulations presented in Figures 2 – 7 provide a description of the effect of maintenance and deterioration rates on mean time to system failure. On the basis of the numerical results obtained for particular cases, it is suggested that the system availability can be improved significantly by:

- Adding more cold standby units.
- Increasing the maintenance rate.
- Exchange the system at major deterioration with new one before failure.

References

[1] Bérenguer C.(2008). On the mathematical condition-based maintenance modelling for continuously deteriorating systems, International Journal of Materials and Structural Reliability, 6, 133-151.

[2] Frangopol, D.M.; Kallen, M.J. and Van Noortwijk, J.M. (2004). Probabilistic models for life-cycle performance of deteriorating structures: Review and future directions. *Prog. Struct. Eng. Mater*., *6*, 197–212.

[3] Lam,Y and Zhang, Y. L. (2003). A geometric-process maintenance model for a deteriorating system under a random environment, IEEE Trans. Reliability. 52(1), 83-89.

[4] Liu, D., Xu, G and Mastorakis, N. E. (2011).Reliability analysis of a deteriorating system with delayed vacation of repairman, WSEAS Transactions on Systems, 10(12).

[5] Nicolai, R.P. Dekker, R. and Van Noortwijk, J.M. (2007). A comparison of models for measurable deterioration: An application to coatings on steel structures. *Reliab. Eng. Syst. Saf.* , *92*, 1635–1650.

[6] Pandey, M.D.; Yuan, X.X.; van Noortwijk, J.M. The influence of temporal uncertainty of deterioration on life-cycle management of structures. *Struct. Infrastruct. Eng*. 2009, *5*, 145–156.

[7] Rani, T.C and Sukumari, C. (2014). Optimum replacement time for a deteriorating system, International Journal of Scientific Engineering and Research, 2(1), 32-33.

[8] Tuan K. Huynh, Anne Barros, Christophe Bérenguer. (2013).A Reliability-based Opportunistic Predictive Maintenance Model for k-out-of-n Deteriorating Systems, Chemical Engineering Transactions, 33, 493-498.

[9] Vinayak, R and Dharmaraja. S (2012). Semi-Markov Modeling Approach for Deteriorating Systems with Preventive Maintenance, International Journal of Performability Engineering Vol. 8, No. 5, pp. 515- 526.

[10] Wang, K,-H and Kuo, C,-C. (2000). Cost and probabilistic analysis of series systems with mixed standby components, Applied Mathematical Modelling, 24: 957-967.

[11] Wang, K., Hsieh, C and Liou, C. (2006). Cost benefit analysis of series systems with cold standby components and a repairable service station. Journal of quality technology and quantitative management, 3(1): 77-92.

[12] Xiao, T.C., Li, Y, -F., Wang, Z., Peng, W and Huang, H, -Z. (2013). Bayesian reliability estimation for deteriorating systems with limited samples Using the Maximum Entropy Approach, Entropy, *15*, 5492-5509; doi:10.3390/e15125492.

[13] Yuan, W., Z. and Xu, G. Q. (2012). Modelling of a deteriorating system with repair satisfying general distribution, Applied Mathematics and Computation 218, 6340–6350

[14] Yuan, L and Xu, J. (2011).A deteriorating system with its repairman having multiple vacations, Applied Mathematics and Computation. 217(10), 4980-4989.

[15] Yusuf, I., Suleiman, K., Bala, S.I. and Ali, U.A. (2012). Modelling the reliability and availability characteristics of a system with three stages of deterioration, International Journal of Science and Technology, 1(7) , 329-337.

[16] Zhang, Y.L. and Wang, G. J. (2007). A deteriorating cold standby repairable system with priority in use, European Journal of Operational Research, Vol.183, 1, pp.278–295.