Simulation Modelling Of A Sporadic Demand Applying A Bootstraping

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Abstract

This technique bootraping has been successfully used in various applied statistical problems, although not many applications have been reported in the area of time series. In this paper we present a new application of Bootstrap to time series. A fundamental aspect of supply chain management is accurate demand forecasting. We address the problem of forecasting intermittent (or irregular) demand, i.e. random demand with a large proportion of zero values. Items of spare parts with sporadic consumption can make a significant, up to 60% portion of the value of supplies in service and workshop inventory areas of many industrial segments. An understanding of key features of demand data is important when developing computer systems for forecasting and inventory control.

Keywords: Simulation modelling, sporadic demand, bootstraping, forecasting

I. Introduction

So called driving systems are the most common from advanced approaches in management and optimization of inventory management. They are included into stochastic and dynamic inventory models defined by a random demand. As input random variables are generated data on consumed amount of material gained from a statistical probability distribution. The arithmetic, moving average or a weighted moving means. An exponential averaging of the first and the second degree are used in a trend development of a demand or a linear regression. Auto correlation and identification models are used as well. However arrays of empirical data on a sporadic demand include a random variety of null values with no nulls. It might provide for variable results in defining needed amount in forecast of a mean, a standard deviation or dispersion in a very simple parametric way in mathematical operations. Due to deviousness of input data the distribution of random variable (demand) obviously does not meet standard probability distribution. An applicable option, being used, is a non-parametric method using past data on a sporadic demand called bootstrapping. We classify it among MC simulation statistical methods, based on a stochastic forecast of a future demand from data on a recent demand. Numerous methods of bootstrapping work with random data on demand, from which an experimental pdf, cdf functions of a distribution of random variable (demand) are generated through a computer experiment applicable for assignment of parameters for modelling par a level inventory management.

Bootstraping is a method aiming to increase an accuracy value of statistic estimations. The results are dependent only on bootstrapping samples. We do not need to know the basic distribution of a random variable. Bootstraping creates a large amount of random choices from input data of a bootstraping sample and it calculates improved statistics on each of such choice. In addition to numerical characteristics it provides data for statistical characteristics in form of frequency histograms and choices probability histograms. From a data set being reviewed we generate bootstrapping y random choices several thousands times so that we choose with repetition (by a substitution of chosen data) from a data set being reviewed x = (x1, x2,.........,xn), a needed amount of m data y = (y1, y2,.........,ym). The chosen numerical values yi are inter independent and they are chosen for a bootstraping sample with the same probability /uniform distribution/. The samples usually differ from each other and they differ from a base data set being reviewed. As we sample with repetitions, it is possible, that some xi data appear several times in a sample or that we do not choose them ever. In case of a specification of a future demand within a delivery term /a delivery leadtime - LT/ we choose from a bootstrapping sample a number of data corresponding with LT.

II. Simulation model of a sporadic demand

The presented simulation model applies stochastic and dynamic principles of inventories modeling. The simulation model was created based on simple algorithms and MATLAB language commands. It consists of two parts: The first part of a model applies the principle of a bootstrapping aiming to define an optimal stock level for an item with a following sequence:

- Downloading the array and a bootstrapping sample of demand data. Fig. 1.
- Computation of numerical characteristics for a bootstrapping sample of a demand for any term, number of data, min, max, mean, std.
- Specification of simulation input data – number of bootstrapping choices, number of chosen periods for a delivery time period, specification of a quantile of a demanded logistic support of delivery.
- Generating a matrix of indices for bootstrapping samples of uniform distribution.
- Transference of indices matrix into a matrix of demand of bootstrapping choices.
- Sum of values in a row of demand matrix of bootstrapping choices.
- Graphic and statistical processing of output data for a definition of a size of an optimal stock.

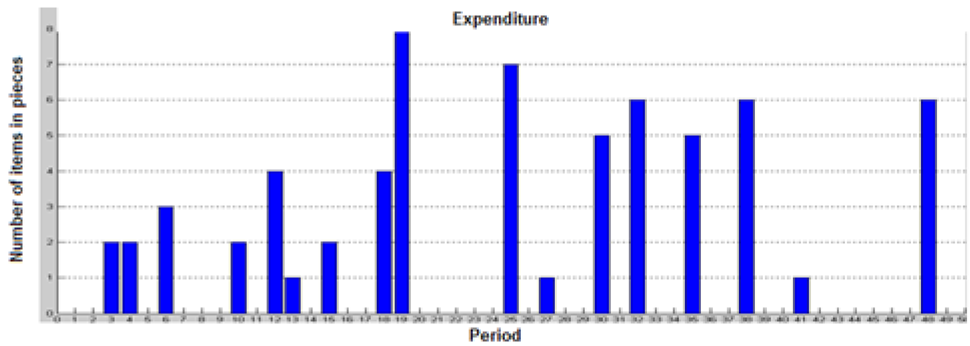


Figure 1: *Sample of demand data for 50 periods*

We simulate choices from a demand sample for 50 time periods. For each choice we randomly choose a number of values corresponding to a lead time. For simplicity LT=2 time periods. Each choice from a sample is represented by a row of a matrix, each column represents a delivery time period. Table1 ... displays a generating of values of bootstrapping indices matrix on 10 choices with a uniform distribution, transformation of the indices matrix into a matrix of bootstrapping choices demand and a sum of values of the rows from a bootstrapping choices demand matrix.

Table 1: *1 Generating 10 choices of indices of an item being reviewed and a demand for LT=2*

Choice number	**Index 1**	**Index 2**	**expenditure1**	**expenditure2**	**SUM expenditure**
1	26	12	0	4	4
2	31	46	0	0	0
3	45	34	0	0	0
4	7	37	0	0	0
5	15	35	2	5	7
6	4	40	2	0	2
7	8	27	0	1	1
8	4	38	2	6	8
9	10	26	2	0	2
10	39	46	0	0	0

Number of choices – simulations has an impact on a provision of a same probability that an item index will be chosen by which we assign a demand in pieces Fig. 2. A requirement of an uniform distribution does not become evident at a small number of simulations, amount of choices of indices lines up with an increasing number of simulations and it confirms an algorithm rightness of a procedure for a generator of a uniform distribution from a choice.

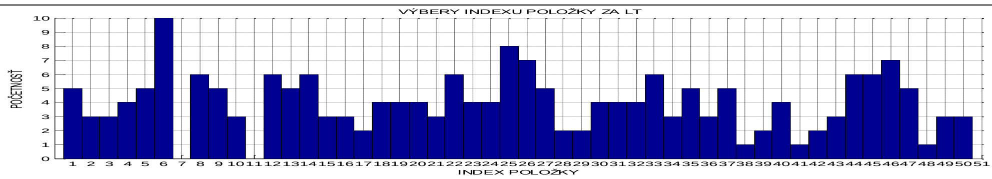
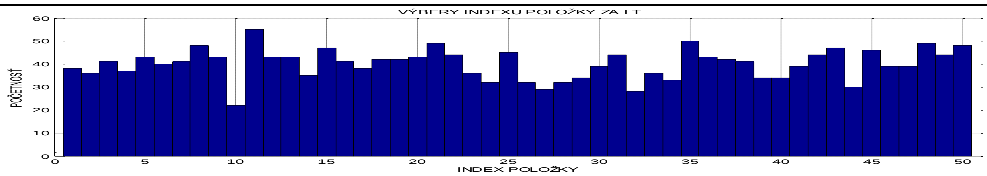
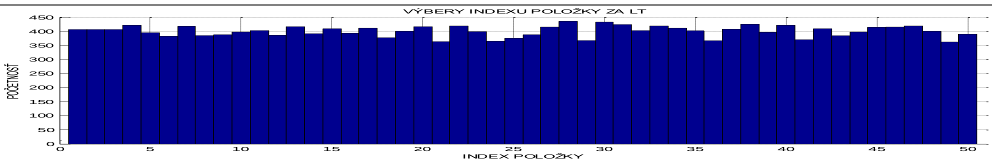
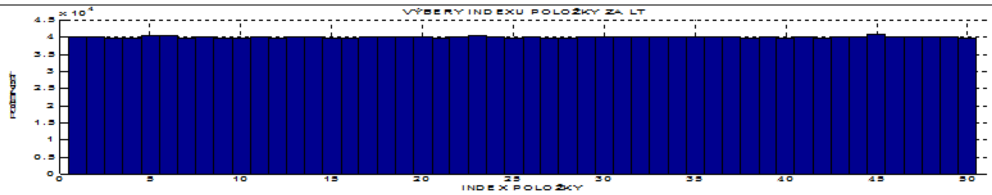
Number of choices	For a delivery time period /LT 2 periods
100	
1000	
10000	
1000000	

Figure 2: *Choice of indices of time periods being modelled and a verification of a uniform distribution*

We develop a histogram from a sum of values from the rows of a demand matrix made based on bootstrapping choices. For 100 choices of the delivery time period 2 periods with a probability 0.95, in the Fig. 3, we see that the intervals with a null demand are represented with the greatest frequency. Statistics of the set: min: 0, max: 13, mean: 2.63, median: 1, std: 3.25

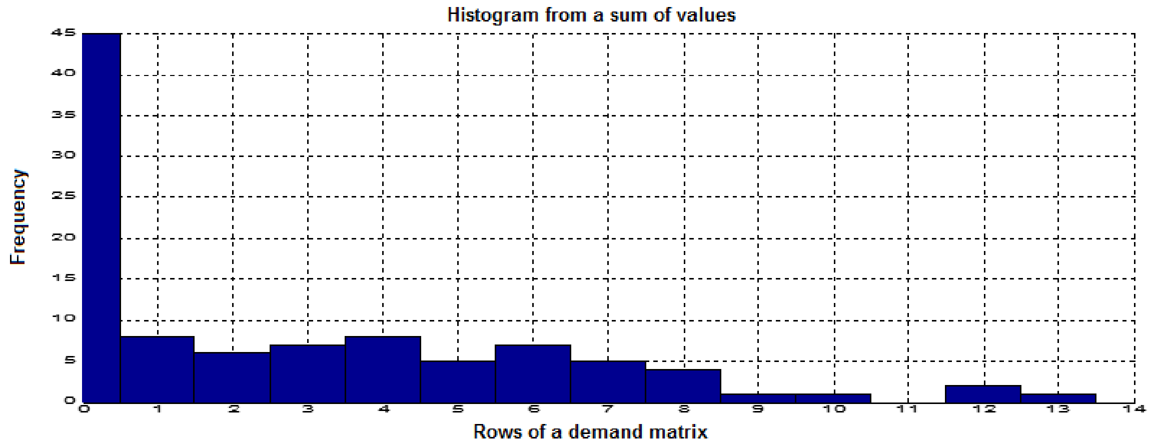


Figure 3: Histogram of a sum of choices frequencies

The second part of a model implements a simulation algorithm of a supplying process with a variable time step.

Characteristics of model parameters:

- Lead Time, a number of time units from sending an order until delivery of an item. A delivery time of an order is defined by contract terms. It may be adjusted by a discharge time of the delivery / a logistic delay.
- Provision probability – Service Level, that a demand will not exceed an offer during implementation with a specified probability. Requested provision probability / Service level is specified ranging from 0.95 - 0.99 by an item criticality.
- Level or stock ordering - Reorder Level is specified as an optimal level with respect to a lead time – is specified as an optimal level with respect to a reorder level and service level. It should ensure that a level of stock during a service level will not drop below zero. Optimal reorder level is specified by a bootstrapping in accordance with a demand forecasting during a lead time of a supplier rounded to the nearest higher ordered amount. Fig. 4. In a moment when a reorder level is intersected, the information system generates an order to a supplier marked with a red asterisk. The above mentioned approach allows a setting of a reorder level and a moment for drawing an offer to refill the stocks in accordance with a specified level of logistic provision.
- Safety stock is created due to an unstable demand / or a lead time as a protection against an item shortage. A safety stock is not created in case of a bootstrapping definition of an optimal stock. A safety factor should be taken into consideration by a Service Level.

Sequence of a simulation algorithm:

- It takes the random data over from bootstrapping choices in order to define a demand of a time period from the first part of the model.
- It monitors a decrease of a stock level / a blue color
- It matches when the stock ordering level reaches a reorder level / green level.

At a moment when a reorder level is reached, or the stock is below the ordering level, it orders an optimal amount of stock, that have been defined in the first part of the model by bootstrapping. Time to draw an order is a random variable.

- It monitors a lead time.
- It carries out a model delivery of an item and it increases a stock level / a red vertical line.
- It collects needed data for computations.

- It computes the costs when a simulation time is shifted.
- It creates graphs of stock and costs courses.
- It repeats a procedure in line with a defined number of time periods in an experiment.

The model allows changing of input values level / delivery time period - LT, number of bootstrapping choices, number of time periods of a demand simulation, a needed level of probability for logistic support, and an initial stock level.
Graphic outputs of a simulation of a short time period are shown in Fig. 4 and Fig. 5.

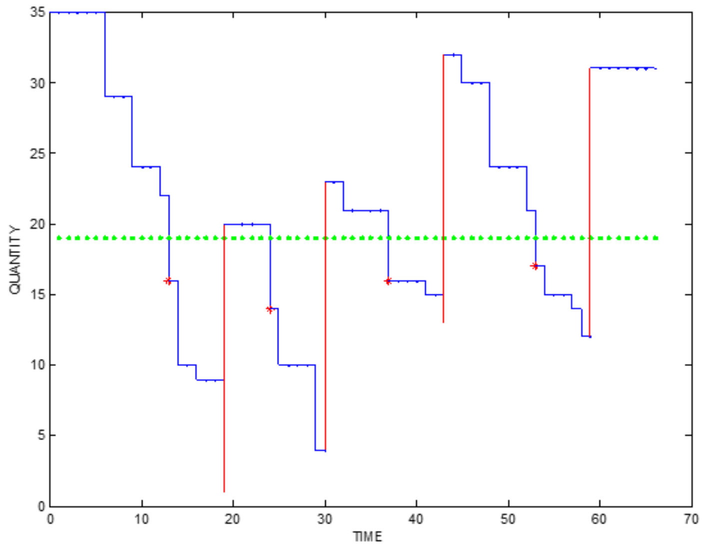


Figure 4: *Course of simulation of stock item movement for 67 time periods at LT=7*

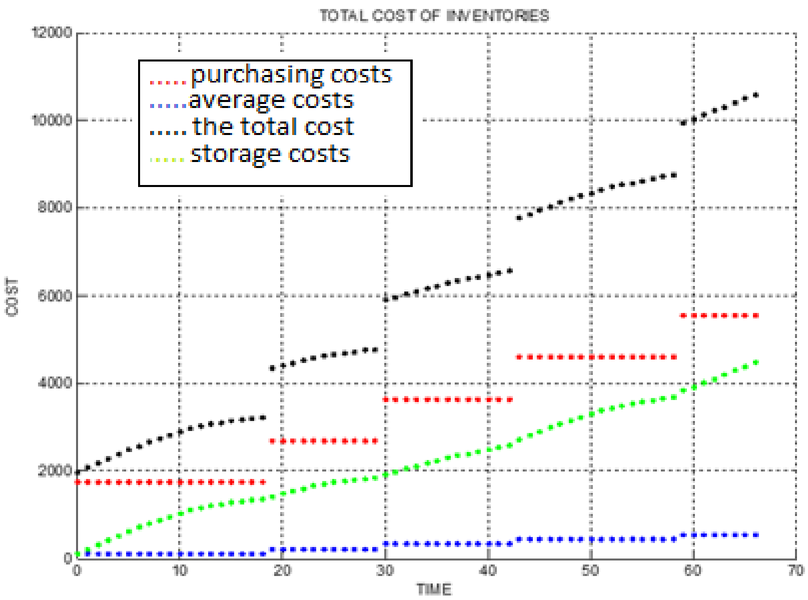


Figure 5: *Course of simulation of stock costs for an item for 67 time periods*

III. Discussion

For evaluation and prediction of the consumption is used arithmetic, moving or weighted moving average. For the trend development of consumption is used exponential equalizing of first and second degree, or linear regression. Auto correlation and identification models are also used. Empirical data arrays of sporadic consumption, however, contain randomly substituting zero values with non-zero. This may, by use of calculations provide variable results for determining the required amount. The bootstrap distribution of a point estimator of a parameter has been used to produce a bootstrapped confidence interval for the parameter's true value, if the parameter can be written as a function of the distribution. Parameters are estimated with many point estimators. A Bayesian point estimator and a maximum-likelihood estimator have good performance when the sample size is infinite, according to asymptotic theory. For practical problems with finite samples, other estimators may be preferable. Asymptotic theory suggests techniques that often improve the performance of bootstrapped estimators; the bootstrapping of a maximum-likelihood estimator may often be improved using transformations related to pivotal quantities [6]. It is obvious from the above mentioned results, that the increased demand for logistic support of an optimum stock / delivery causes an increased level of an optimum stock /Service level and naturally the costs as well. It is interesting, that costs of acquisition are about on the same level, the transportation costs decrease and storage costs increase.

Acknowledgements
This publication was created in the frame of the project Research of a technological base for draft of application of renewable sources of energy in practice, ITMS code 26220220083, of the Operational Program Research and Development funded from the European Fund of Regional Development.

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