X-EXPONENTIAL BATHTUB FAILURE RATE MODEL

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Abstract

The properties of x-Exponantial Bathtub shaped failure rate model are discussed. Estimation process and failure rate behavior is explained.

Keywords: Bathtub failure rate

I. Introduction

There are many distributions for modeling lifetime data. Among the known parametric models, the most popular are the Lindley, Gamma, log-Normal, Exponentiated Exponential and the Weibull distributions. These five distributions are suffer from a number of drawbacks. None of them exhibit bathtub shape for their failure rate functions. The distributions exhibit only monotonically increasing, montonically decreasing or constant failure rates. Most real life system exhibit bathtub shapes for their failure rate functions. Generalized Lindley (GL), Generalized Gamma (GG) and Exponentiated Weibull (EW) distributions are proposed for modeling lifetime data having bathtub shaped failue rate model. In this paper we consider a simple model but exhibiting bathtub shaped failure rate, x-Exponential distribution, and discuss the failure rate behavior of these distributions. The x-Exponential distribution has properties similar to Generalized Lindley, but it is more simple and can be used instead of Generalized Lindley, Generalized Gamma and Exponentiated Weibull. The inference procedure also become simple than these distributions.

Section II, discussed x-Exponential distribution and their properties, Generalized Lindley distribution, Generalised Weibull distribution, discussed Generalized Gamma distribution and conclusions are given at the final section.

II. Bathtub shaped failure rate models

I. X-Exponential Distribution

In this section, consider a simplified form of distribution function,

$$F(x) = (1 - (1 + \lambda x^2) e^\wedge(-\lambda x))^\alpha, x > 0, \lambda > 0, \alpha > 0. \qquad (1)$$

It is an alternative model GL, GG, EW distributions. A life time random variable X has X-Exponential distribution if its cumulative distribution function is (1), [2].

Clearly F(0)=0, $F(\infty) = 1$, F is non-decreasing and right continuous. More over F is absolutely continuous.

The probability density function (pdf) of a x-Exponential random variable X, with scale parameter λ is given by

$$f(x) = \alpha e^{-\lambda x}(\lambda^2 x^2 - 2\lambda x + \lambda)(1-(1+\lambda x^2)e\text{^}(-\lambda x))^{\alpha-1}, x > 0, \lambda > 0, \alpha > 0.$$

It is positively skewed distribution. Failure rate function of x-Exponential distribution is

$$h(x) = \frac{\alpha e\text{^}(-\lambda x)\ (\lambda^2 x^2 - 2\lambda x + \lambda)(1-(1+\lambda x^2)e\text{^}(-\lambda x))\text{^}(\alpha - 1)}{1-(1-(1+\lambda x^2)e\text{^}(-\lambda x))^{\alpha}}, x > 0, \lambda > 0, \alpha > 0.$$

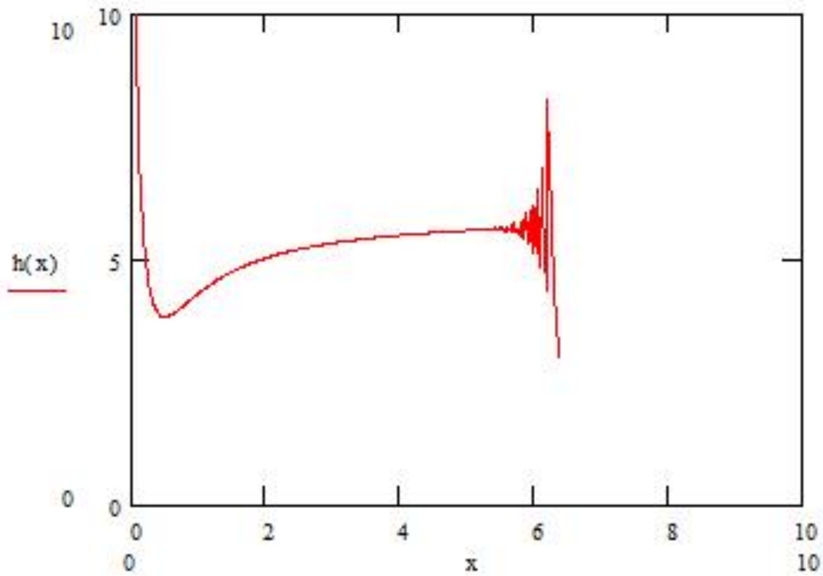

Figure 1. Failure rate function of x-Exponential distribution for **α**=0.01 and λ= 6

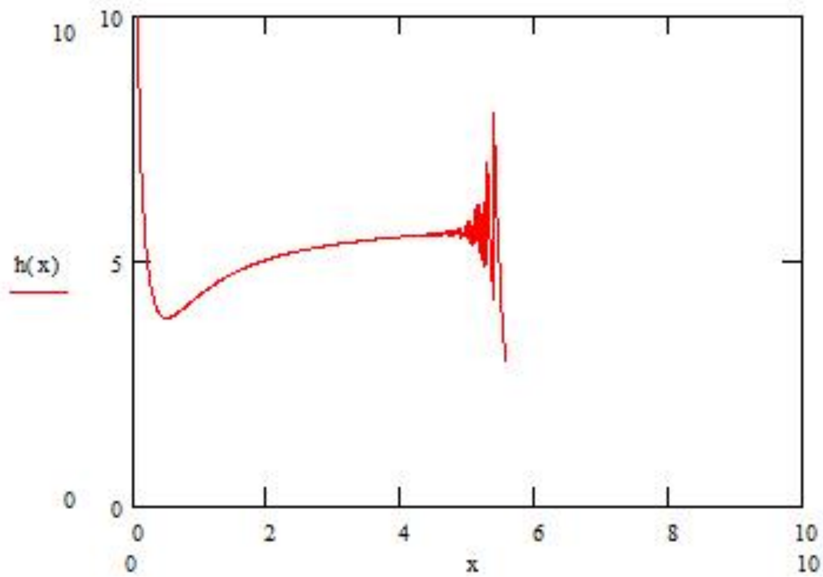

Figure 2. Failure rate function of x-Exponential distribution for **α**=0.0001 and λ= 6

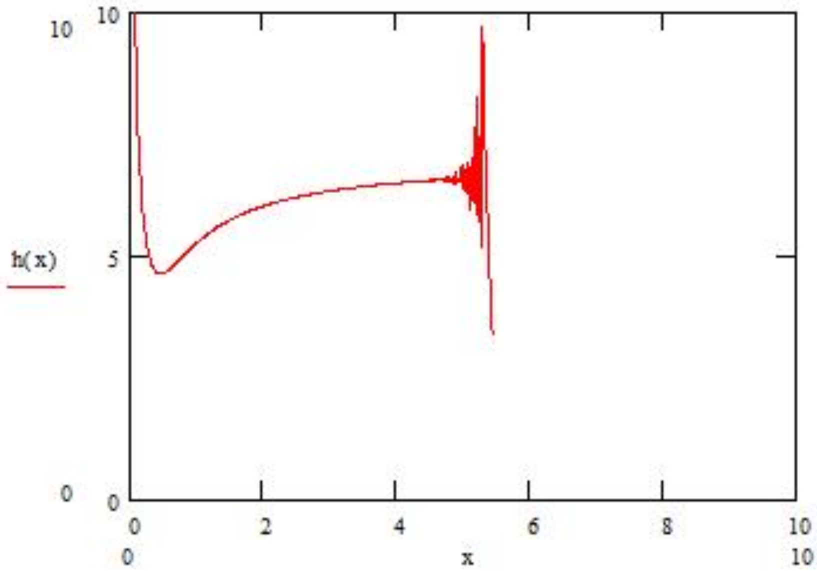

Figure 3. Failure rate function of x-Exponential distribution for **α**=0.001 and λ= 10

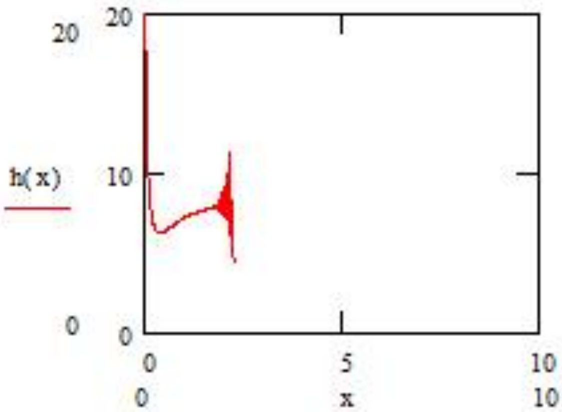

Figure 4. Failure rate function of x-Exponential distribution for **α**=0.000000001 and λ= 9

From Figure 1,2,3 and 4, the shape of the hazard rate function appears monotonically decreasing or to initially decrease and then increase , a bathtub shape if $\alpha < 1$; the shape appears monotonically increasing if $\alpha \geq 1$. So the proposed distribution allows for monotonically decreasing, monotonically increasing and bathtub shapes for its hazard rate function. As **α** decreases from 1 to 0, the graph shift above whereas if λ increases from 1 to ∞ the shape of the graph concentrate near to 0. It is the distribution of the failure of a series system with independent components. The equation (1) has two parameters, α and λ just like the Gamma, log Normal, Weibull and Exponentiated Exponential distributions.

Moments

Calculating moments of X requires the following lemma.

Lemma 2.1: For **α**>0, λ>0, x>0, K(**α**,λ,c)=$\int_0^\infty x^c \left[1-(1+\lambda x^2)e^{-\lambda x}\right]^{\alpha-1} e^{-\lambda x}dx$, Then,

$$K(\alpha,\lambda,c)=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i \sum_{j=0}^{i} C_j^i \lambda^j \left[\int_0^\infty x^{2j+c}\ e^{-i\lambda x}e^{-\lambda x}dx \quad \right]$$

Proof:

We know $(1-z)^{\alpha-1} = \sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i z^i$. Therefore

$$K(\boldsymbol{\alpha},\lambda,c)=\int_0^\infty x^c\left[1-(1+\lambda x^2)e^{-\lambda x}\right]^{\alpha-1}e^{-\lambda x}dx$$

$$=\int_0^\infty x^c \sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i\left[(1+\lambda x^2)e^{-\lambda x}\right]^i e^{-\lambda x}dx$$

$$=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i\int_0^\infty x^c \left[(1+\lambda x^2)e^{-\lambda x}\right]^i e^{-\lambda x}dx$$

$$=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i\int_0^\infty x^c \left[(1+\lambda x^2)\right]^i e^{-i\lambda x}e^{-\lambda x}dx$$

$$=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i\int_0^\infty x^c \sum_{j=0}^{i} C_j^i(\lambda x^2)^j e^{-(i+1)\lambda x}dx$$

$$=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i \sum_{j=0}^{i} C_j^i\lambda^j\int_0^\infty x^c (x^2)^j e^{-(i+1)\lambda x}dx$$

$$=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i \sum_{j=0}^{i} C_j^i\lambda^j\int_0^\infty x^c x^{2j} e^{-(i+1)\lambda x}dx$$

$$=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i \sum_{j=0}^{i} C_j^i\lambda^j\int_0^\infty x^{2j+c} e^{-(i+1)\lambda x}dx$$

$$=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i \sum_{j=0}^{i} C_j^i\lambda^j\left[\frac{\Gamma(2j+c+1)}{((i+1)\lambda)^{2j+c+1}}\right]$$

$$K(\alpha,\lambda,c)=\sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i \sum_{j=0}^{i} C_j^i\lambda^j\left[\frac{\Gamma(2j+c+1)}{((i+1)\lambda)^{j+c+1}}\right]$$

It follows that

$$E(X)=\int_0^\infty x\alpha e^{-\lambda x}(\lambda^2x^2-2\lambda x+\lambda)(1-(1+\lambda x^2)e\verb|^|(-\lambda x))^{\alpha-1}dx$$

$$E(X^n)=\int_0^\infty x^n\alpha e^{-\lambda x}(\lambda^2x^2-2\lambda x+\lambda)(1-(1+\lambda x^2)e\verb|^|(-\lambda x))^{\alpha-1}dx$$

$$E(X^1)=\alpha\lambda^2K(\alpha,\lambda,3)-2\alpha\lambda K(\alpha,\lambda,2)+\boldsymbol{\alpha\lambda}K(\alpha,\lambda,1)$$

The moments are

$$E(X^n)=\alpha\lambda^2K(\alpha,\lambda,n+2)-2\alpha\lambda\ K(\alpha,\lambda,n+1)+\boldsymbol{\alpha\lambda}K(\alpha,\lambda,n),\ n=1,2,3,\ldots$$

Moment Generating Function

$$M_X(t)=\int_0^\infty e^{tx}\ \alpha(\lambda^2x^2-2\lambda x+\lambda)\left[1-(1+\lambda x^2)e^{-\lambda x}\right]^{\alpha-1}e^{-\lambda x}dx$$

$$M_X(t)=\int_0^\infty \alpha(\lambda^2x^2-2\lambda x+\lambda)\left[1-(1+\lambda x^2)e^{-\lambda x}\right]^{\alpha-1}e^{-(\lambda-t)x}dx$$

$$M_X(t)=\alpha\lambda^2K(\alpha,\lambda-t,3)-2\alpha\lambda\ K(\alpha,\lambda-t,2)+\boldsymbol{\alpha\lambda}K(\alpha,\lambda-t,1)$$

Characteristic Function

$$\Phi_X(t)=\int_0^\infty e^{itx}\ \alpha(\lambda^2x^2-2\lambda x+\lambda)\left[1-(1+\lambda x^2)e^{-\lambda x}\right]^{\alpha-1}e^{-\lambda x}dx$$

$$\Phi_X(t)=\int_0^\infty \alpha(\lambda^2x^2-2\lambda x+\lambda)\left[1-(1+\lambda x^2)e^{-\lambda x}\right]^{\alpha-1}e^{-(\lambda-it)x}dx$$

$$\Phi_X(t) = \alpha\lambda^2 \mathrm{K}(\alpha, \lambda - \mathrm{it}, 3) - 2\alpha\lambda \ \mathrm{K}(\alpha, \lambda - \mathrm{it}, 2) + \boldsymbol{\alpha\lambda} \mathrm{K}(\alpha, \lambda - \mathrm{it}, 1)$$

Shape of the density function

Consider probability density function,

$$f(x) = \alpha e^{-\lambda x}(\lambda^2 x^2 - 2\lambda x + \lambda)(1 - (1 + \lambda x^2)e\hat{}(-\lambda x))^{\alpha-1}, x > 0, \lambda > 0, \alpha > 0$$

$$log f(x) = \log(\alpha) - \lambda x + log\,(\lambda^2 x^2 - 2\lambda x + \lambda) + (\alpha - 1)\, log(1 - (1 + \lambda x^2)e\hat{}(-\lambda x)) \quad , x, \lambda, \alpha > 0$$

$$\frac{\mathrm{d}}{\mathrm{dx}} log\, f(x) = -\lambda + \frac{(2\lambda^2 x \ \ - 2\lambda)}{(\lambda^2 x^2 - 2\lambda x + \lambda)} + \frac{(\alpha - 1)}{(1 - (1 + \lambda x^2)e\hat{}(-\lambda x))}((1 + \lambda x^2)(-\lambda)e^{-\lambda x} + 2\lambda x e^{-\lambda x})$$

$$\frac{\mathrm{d}^2}{\mathrm{dx}^2} log\, f(x) = -\frac{1}{x^2} + \frac{(\alpha-1)}{(1 - (1+\lambda x^2)e^{-\lambda x})^1}((-\lambda)((1 + \lambda x^2)(-\lambda)e^{-\lambda x} + 2\lambda x e^{-\lambda x}) \quad - \frac{(\lambda^2 x^2 - 2\lambda x)^2 e^{-2\lambda x}}{(1 - (1+\lambda x^2)e^{-\lambda x})^2}) \quad , x > 0, \lambda > 0, \alpha > 0.$$

Here f(x) first increases and then decreases, it is unimodel.

Mean Deviation about Mean

The amount of scatter in a population is evidently measured to some extent by the totality of deviations from the mean and median. Mean deviation about the mean defined by

$$\mathrm{MD(Mean)}{=}2\mu F(\mu) - 2\mu + 2\int_{\mu}^{\infty} x f(x) dx$$

$$MD(Mean) = 2\mu F(\mu) - 2\mu + 2(\alpha\lambda^2 \mathrm{L}(\alpha, \lambda, 3, \mu) - 2\alpha\lambda \ \mathrm{L}(\alpha, \lambda, 2, \mu) + \boldsymbol{\alpha\lambda} \mathrm{L}(\alpha, \lambda, 1, \mu)$$

where $L(\alpha, \lambda, c, \mu) = \int_{\mu}^{\infty} x^c \left[1 - (1 + \lambda x^2)e^{-\lambda x}\right]^{\alpha-1} e^{-\lambda x} dx$

$$= \sum_{i=0}^{\alpha-1} C_i^{\alpha-1}(-1)^i \ \sum_{j=0}^{i} C_j^i \lambda^j \ [\int_{\mu}^{\infty} x^{2j+c+1} e^{-(j+1)\lambda x}\, dx \quad].$$

Mean deviation about the Median defined by

$$\mathrm{MD(Median)}{=}{-M} + 2\int_{M}^{\infty} x f(x) dx$$

$$MD(Mean) = -M + 2(\alpha\lambda^2 \mathrm{L}(\alpha, \lambda, 3, \mathrm{M}) - 2\alpha\lambda \ \mathrm{L}(\alpha, \lambda, 2, \mathrm{M}) + \boldsymbol{\alpha\lambda} \mathrm{L}(\alpha, \lambda, 1, \mathrm{M})$$

Estimation

Here, we consider estimation by the methods of moments and maximum likelihood. We also consider estimation issues for censored data. Let $X_1, X_2, \ldots, X_n$ are random sample taken from x-Exponential distribution. Let $m_1 = \frac{1}{n}\sum_{i=1}^{n} x_i \quad m_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2$. Equating sample moments to population moments we get moment estimators for parameters.

$$m_1 = \alpha\lambda^2 \mathrm{K}(\alpha, \lambda, 3) - 2\alpha\lambda \mathrm{K}(\alpha, \lambda, 2) + \boldsymbol{\alpha\lambda} \mathrm{K}(\alpha, \lambda, 1)$$

$$m_2 = \alpha\lambda^2 \mathrm{K}(\alpha, \lambda, 4) - 2\alpha\lambda \mathrm{K}(\alpha, \lambda, 3) + \boldsymbol{\alpha\lambda} \mathrm{K}(\alpha, \lambda, 2)$$

The solution of these equations are moment estimators.

To find maximum likelihood estimator, consider likelihood function as,

$$L(\alpha, \lambda) = \prod_{i=1}^{n} f(x_i)$$

$$L(\alpha, \lambda) = \prod_{i=1}^{n} \alpha e^{-\lambda x_i}(\lambda^2 {x_i}^2 - 2\lambda x_i + \lambda)(1 - (1 + \lambda {x_i}^2)e\hat{}(-\lambda x_i))^{\alpha-1}$$

$$L(\alpha, \lambda) = (\alpha)^n \, e^{-\lambda \sum_{i=1}^{n} x_i} \prod_{i=1}^{n} (\lambda^2 {x_i}^2 - 2\lambda x_i + \lambda) \prod_{i=1}^{n} \ (1 - (1 + \lambda {x_i}^2)e\hat{}(-\lambda x_i))^{\alpha-1}$$

$$logL(\alpha, \lambda) = nlog\,(\alpha) \quad -\lambda \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \ log(\lambda^2 {x_i}^2 - 2\lambda x_i + \lambda)$$

$$+ \sum_{i=1}^{n} \ (\alpha - 1) log\ (1 - (1 + \lambda {x_i}^2)e\hat{}(-\lambda x_i))$$

$$= nlog\,(\alpha) \quad -\lambda \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \ log(\lambda^2 {x_i}^2 - 2\lambda x_i + \lambda) + (\alpha - 1)\sum_{i=1}^{n} \ log\ (1 - (1 + \lambda {x_i}^2)e\hat{}(-\lambda x_i))$$

$$\frac{\partial}{\partial\alpha}\, log\, L(\alpha,\lambda) = \frac{n}{\alpha} + \sum_{i=1}^{n} log\,(1-(1+\lambda {x_i}^2)e\hat{}(-\lambda x_i))$$

$$\hat{\alpha} = -1/n \sum_{i=1}^{n} log\,(1-(1+\lambda {x_i}^2)e\hat{}(-\lambda x_i))$$

$$\frac{\partial}{\partial\lambda}\, log\, L(\alpha,\lambda) = -\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \frac{(2\lambda^2 x_i \quad - 2x_i)}{(\lambda^2 {x_i}^2 - 2\lambda x_i + \lambda)} + (\alpha-1)\sum_{i=1}^{n} \frac{1}{(1-(1+\lambda {x_i}^2)e\hat{}(-\lambda x_i))}\,((1+\lambda {x_i}^2)(-x_i)e^{-\lambda x_i} + {x_i}^2 e^{-\lambda x_i})$$

The MLE of λ will be solution of the following non-linear equation.

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} \frac{(2\lambda^2 x_i \quad - 2x_i)}{(\lambda^2 {x_i}^2 - 2\lambda x_i + \lambda)} + (\alpha-1)\sum_{i=1}^{n} \frac{1}{(1-(1+\lambda {x_i}^2)e\hat{}(-\lambda x_i))}\,((1+\lambda {x_i}^2)(-x_i)e^{-\lambda x_i} + {x_i}^2 e^{-\lambda x_i})$$

II. Generalized Lindley Distribution

Suppose $X_1, X_2, \ldots, X_n$ are independent random variables distributed according to Lindley distribution and $T = \min(X_1, X_2, \ldots, X_n)$ represent the failure time of the components of a series system, assumed to be independent, [2]. Then the probability that the system will fail before time x is given by

$$F(x) = [1 - \ (1+\lambda+\lambda x)/(1+\lambda)\; e\hat{}(-\lambda x)]^n, x > 0, \lambda > 0.$$

It is the distribution of the failure of a series system with independent components. The cumulative distribution function and pdf of Generalized Lindley distribution are

$$F(x) = [1 - \ (1+\lambda+\lambda x)/(1+\lambda)\; e\hat{}(-\lambda x)]^\alpha, x > 0, \lambda > 0, \boldsymbol{\alpha} > 0$$

$$f(x) = \frac{\alpha\lambda(1+x)}{1+\lambda}[1 - \ (1+\lambda+\lambda x)/(1+\lambda)\; e\hat{}(-\lambda x)]^{\alpha-1} e^{-\lambda x}, x > 0, \lambda > 0, \boldsymbol{\alpha} > 0$$

The equation has two parameters, λ and $\boldsymbol{\alpha}$ just like the Gamma, log Normal, Weibull and Exponentiated Exponential distribution. For n= 1 it reduces to Lindley distribution.
The failure rate function is

$$h(x) = \frac{\frac{\alpha\lambda(1+x)}{1+\lambda}\left[1 - \frac{1+\lambda+\lambda x}{1+\lambda}e^{-\lambda x}\right]^{\alpha-1} e^{-\lambda x}}{1 - \left[1 - \frac{1+\lambda+\lambda x}{1+\lambda}e^{-\lambda x}\right]^{\alpha}},$$

$$x > 0, \lambda > 0, \boldsymbol{\alpha} > 0$$

The shape of the failure rate function appears monotonically decreasing or to initially decrease and then increase, a bathtub shape if $\boldsymbol{\alpha} < 1$, the shape appears monotonically increasing if $\boldsymbol{\alpha} \geq 1$. So the Generalized Lindley distribution allows for monotonically decreasing, monotonically increasing and bathtub shapes for its failure rate function.

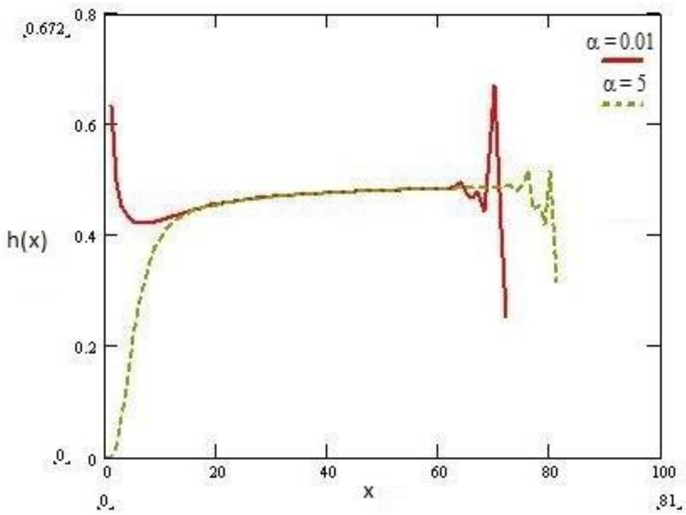

Figure 5. Failure rate function of Generalized Lindley distribution

III. Exponentiated Weibull Distribution

Exponentiated Weibull (EW) distribution has a scale parameter and two shape parameters, [4]. The Weibull family and the Exponentiated Exponential (EE) family are found to be particular cases of this family. The cumulative distribution function of the EW distribution is given by

$$F(x) = \left(1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}}\right)^{\lambda}, \lambda > 0, \boldsymbol{\alpha} > 0, \beta > 0.$$

Here λ and $\boldsymbol{\alpha}$ denote the shape parameters and β is the scale parameter. For When λ= 1, the distribution reduces to the Weibull Distribution with parameters. When β = 1, **α**=1 it represents the EE family. Thus, EW is a generalization of EE family as well as the Weibull family.

Then the corresponding density function is

$$f(x) = \left(\frac{\alpha\theta}{\sigma}\right)[1 - \exp\{-(x/\sigma)^{\alpha}\}]^{\theta-1}\exp\{-(\frac{x}{\sigma})^{\alpha}\}(\frac{x}{\sigma})^{\alpha-1}, x \geq 0.$$

The failure rate function is

$$h(x) = \frac{\left(\frac{\alpha\theta}{\sigma}\right)\left[1 - exp\left\{-\left([\![\frac{x}{\sigma}]\!]^{\alpha}\right\}\right]^{\theta-1} exp\left\{-\left(\frac{x}{\sigma}\right)^{\alpha}\right\}\left(\frac{x}{\sigma}\right)^{\alpha-1}}{1 - \left[1 - exp\left\{-\left([\![\frac{x}{\sigma}]\!]^{\alpha}\right\}\right]^{\theta}}, x \geq 0, \alpha, \theta, \sigma > 0.$$

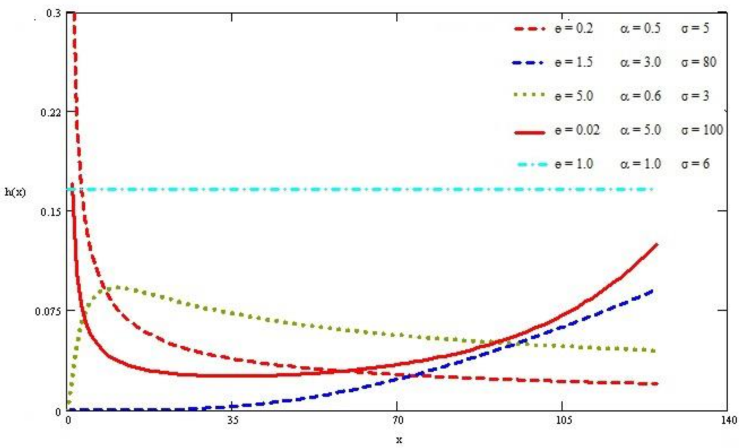

Figure 6: Plot of the failure rate function of EW distribution

The EW distribution is constant for **α**= 1 and = 1. The EW distribution is IFR for **α** > 1 and **θ**≥1. The EW distribution is DFR for **α**< 1 and **θ**≤1. The EW distribution is BT(Bathtub) for **α**> 1 and **θ**< 1. The EW distribution is UBT (Upside down Bathtub) for **α** < 1 and **θ** > 1.

IV. Exponentiated Gamma Distribution

The Gamma distribution is the most popular model for analyzing skewed data and hydrological processes, [3]. This model is flexible enough to accommodate both monotonic as well as non-monotonic failure rates. The Exponentiated Gamma (EG) distribution is one of the important families of distributions in lifetime tests. The EG distribution has been introduced as an alternative to Gamma and Weibull distributions.

The Cumulative Distribution function of the Exponentiated Gamma distribution is given by

$$G(x) = [1 - \exp\{-\lambda x\}(1 + \lambda x)]^{\theta}, x > 0, \lambda, \boldsymbol{\theta} > 0.$$

where λ and **θ** are scale and shape parameters respectively. Then the corresponding probability density function (pdf) is given by

$$g(x) = \theta \lambda^2 x \exp\{-\lambda x\}([1 - \exp\{-\lambda x\}(1 + \lambda x)]^{\theta - 1}, x > 0, \lambda, \boldsymbol{\theta} > 0.$$

The failure rate function is

$$h(x) = \frac{\theta\lambda\hat{}2\, x\, exp\{-\lambda x\}([1 - exp\{-\lambda x\}(1 + \lambda x)]\hat{}(\theta - 1)}{1 - [1 - exp\{-\lambda x\}(1 + \lambda x)]\hat{}\theta}, x > 0, \lambda, \boldsymbol{\theta} > 0.$$

Then the other advantage is that it has various shapes of failure function for different values of . It has increasing failure function when **θ**≥ 1/2 and its failure function takes Bath-tub shape for **θ**<1/2.

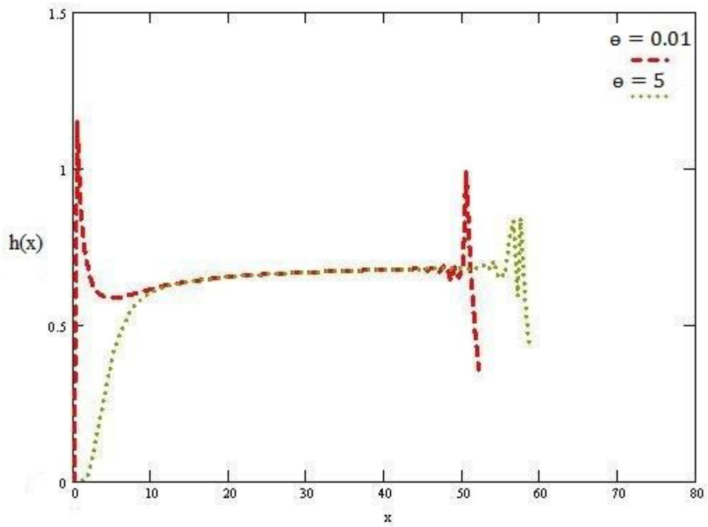

Figure 7: Failure rate function of EG distribution.

III. Generalized X-Exponential Class Distribution

Consider the Distribution function,

$$F(x) = (1 - (\beta + \lambda x^2)e\hat{}(-\lambda x))^{\alpha}, x > 0, \lambda > 0, \alpha > 0, \beta > 0.$$

The failure rate function, provided various Bathtub shaped models as see in Figure 8,9,10. For **α**=0.001, λ=6 and β=5, the failure rate function is

$$h(x) = \frac{\alpha e\hat{}(-\lambda x)\,(\lambda^2 x^2 - 2\lambda x + \lambda)(1 - (\beta + \lambda x^2)e\hat{}(-\lambda x))\hat{}(\alpha - 1)}{1 - (1 - (\beta + \lambda x^2)e\hat{}(-\lambda x))^{\alpha}}, x > 0, \lambda > 0, \alpha > 0, \beta > 0.$$

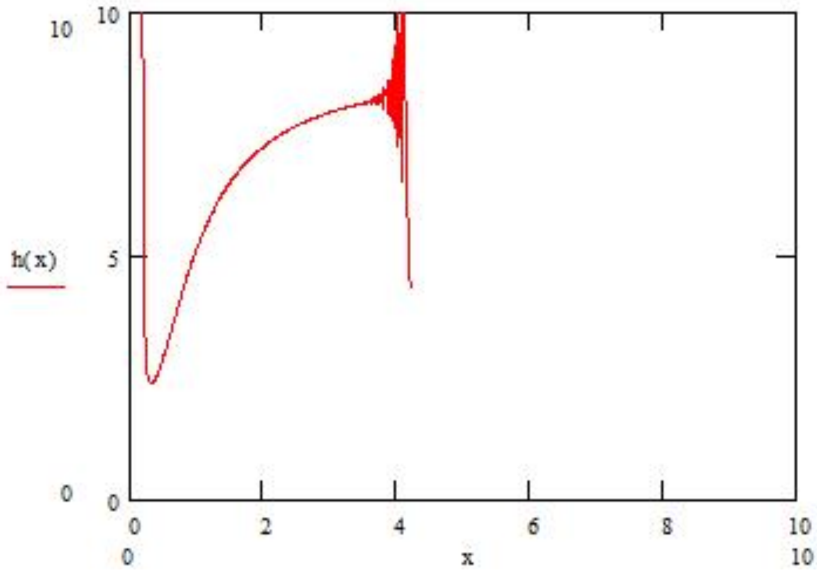

Figure 8. Generalized X-Exponential failure rate function **α**=0.01, λ=9 and β=5

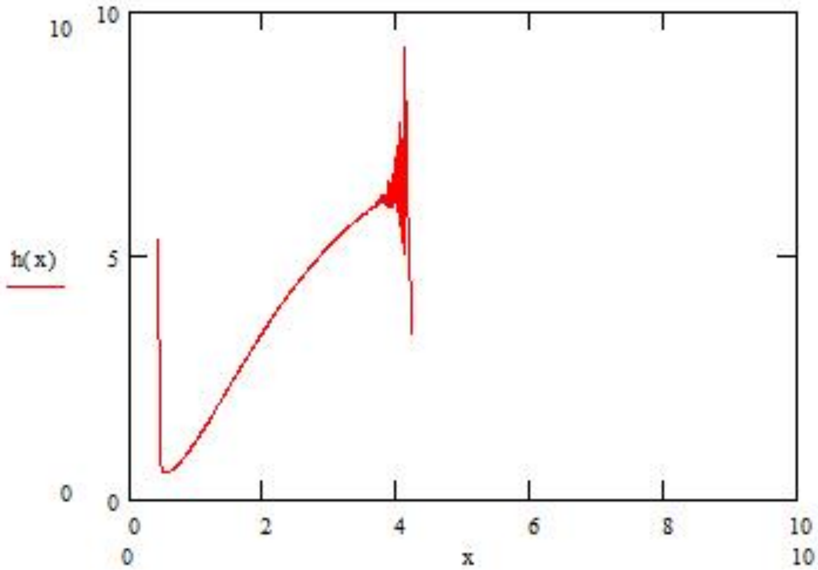

Figure 9. Generalized X-Exponential failure rate function **α**=0.01, λ=9 and β=50

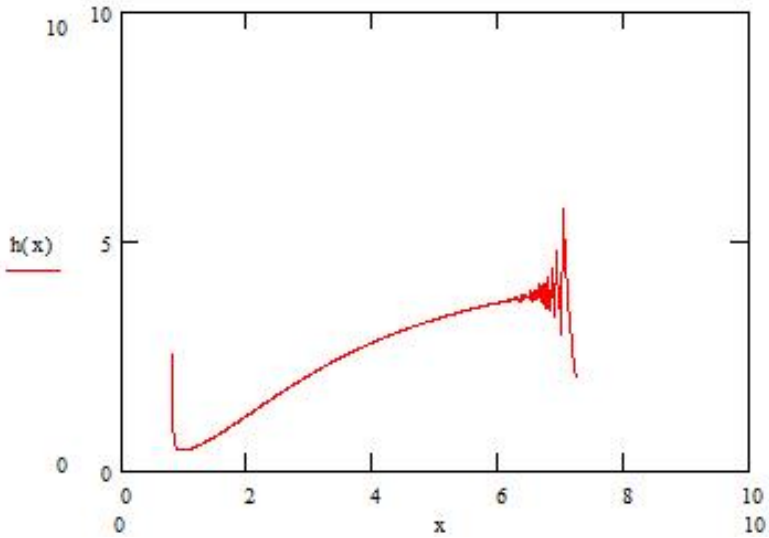

Figure 10. Generalized X-Exponential failure rate function **α**=0.001, λ=5 and β=50

Upside down Bathtub shaped failure rate viewed for $F(x) = (1 - (\beta + \lambda x \ )e\text{^}(-\lambda x))^{\alpha}, x > 0, \lambda > 0, \alpha > 0, \beta > 0$ **α**=0.001, λ=6 and β<1, see figure 11.

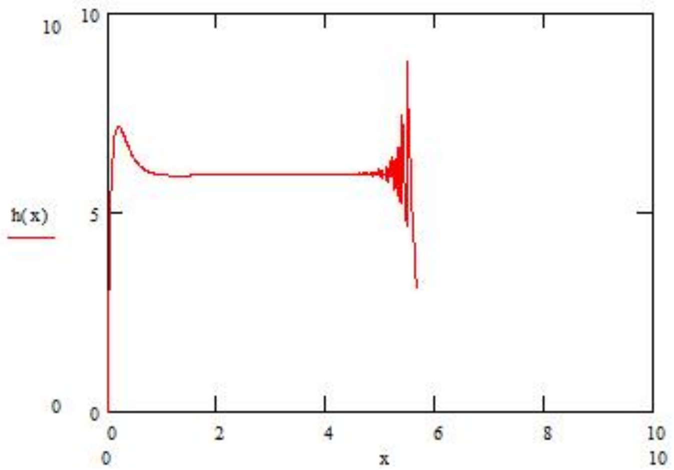

Figure 11. Generalized X-Exponential failure rate function **α**=0.001, λ=6 and β=0.1

All the procedure for finding moments, moment generating function, characteristic function, and estimation are same as that of X-Exponential distribution. If we insert one more parameter **θ** in the model still we get beautiful Bathtub and Upside down bathtub shapes for its failure rate functions as seen below. For $F(x) = (1 - (\beta + \lambda x + \theta x^2)e\text{^}(-\lambda x))^{\alpha}, x > 0, \lambda > 0, \alpha > 0, \beta > 0, \theta > 0$.

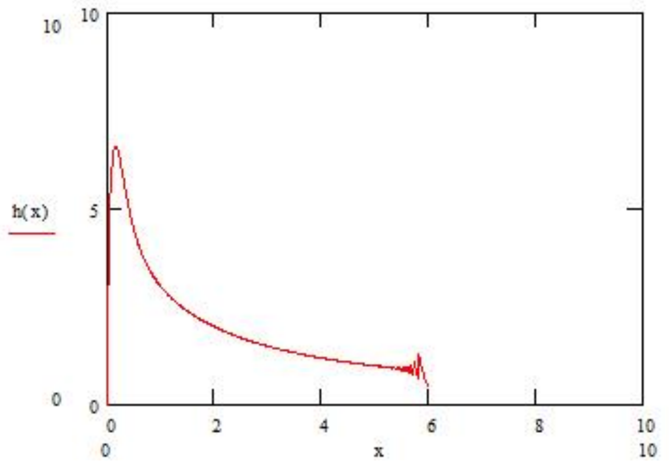

Figure 11. Generalized X-Exponential failure rate function **α**=0.001, λ=6, β=0.1, **θ**=6.

Generalized Lindely distribution is a special case of Generalized X-Exponential distribution.

IV. Conclusions

There are many distributions in reliability which exhibit Bathtub shaped failure rate model, but most of them are complicated in finding the moments, reliability etc. Moreover the increased number of parameters make complication and difficulty in estimation process. The proposed model is similar to Generalized Lindley, so all the computational procedures are like GL distribution. The complication in using GL,GG,GE distributions is reduced in the proposed model. Moreover MLE of **α** is readily available and that of λ can be computed numerically. Generalized X-

Exponential distribution provided various Bathtub shaped and Upside down Bathtub shaped failure rates.

References

[1] Chacko, V.M. (2016). A new bathtub shaped failure rate model, *Reliability: Theory and Applications*, RT&A, No.1(40), Vol.11, March 2016.

[2] Nadarajah, S., Bakouch, H.S., and R. Tahmasbi. 2011. *A generalized Lindley distribution*. Technical Report, School of Mathematics, University of Manchester, UK.

[3] Nadarajah, S., and A.K. Gupta. 2007. The exponentiated gamma distribution with application to drought data. *Calcutta Statistical Association Bulletin* 59:233–234.

[4] Pal, M. M. Ali, J.Woo- Exponentiated Weibull Distribution, Statistica, anno LXVI, n.2,2006.