Parameter Estimation of Mukherjee-Islam Model under Step Stress Partially Accelerated Life Tests with Failure Constraint

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Abstract:

In this paper, we have studied the estimation of parameters under failure censored data using step stress partially accelerated life testing. The lifetimes of test items are assumed to follow Mukherjee-Islam distribution. The estimation of different parameters and acceleration factor are obtained by Maximum Likelihood Method. Relative absolute bias (RAB), mean squared error (MSE), relative error (RE), standard deviation and confidence intervals are also obtained. Asymptotic variance-covariance matrix and also test method are given. Simulation studies have been introduced to illustrate the performance of all the statistical assumptions.

Keywords: Mukherjee-Islam distribution, Step-stress partially accelerated life test, Maximum likelihood, failure censoring.

Introduction:

The present era is the era of high reliability. The products and items made nowadays are too much reliable. Usually they do not fail early at normal use condition. So it is not easy to get reasonable amount of failure data under use condition for a given period of time. For this reason, Accelerated Life Testing is the modest procedure to be applied. By using it, products would fail early and at the end of test we have sufficient failure data to study the behaviour of products. ALT quickens the procedure that's why it costs less money and consume less time.

In ALT, the relationship between lifetime stress is known either in the form of acceleration factor or there exists a mathematical model. But in many situations neither acceleration factor is known nor there exist any such model. Then partially accelerated life test is the better option to use. In PALT, the acceleration factor and mathematical model which sustain the relationship between the life time and stress are not known and cannot be assumed any type of such model.

Nelson [19] introduced that the stress can be applied on test item in various ways, commonly used are step-stress and constant-stress. Under step-stress PALT, a test unit is first subjected to run at normal use condition and if it does not fail for a specified time, then it is run at accelerated use condition until failure occurs or the observation is censored. But in constant stress PALT each item runs at daily use condition or at accelerated condition.

DeGroot and Goel [12] have introduced the concept of step-stress PALT in which a

test item is first run at use condition and, if it does not fail for a specified time 'τ', then it is run at accelerated condition until failure.
A lot of literature is available on SS-PALT analysis, for example, see Goel [13], Bhattacharyya and Soejoeti [11], Bai and Chung [10], Abdel-Ghani [8] and Abdel-Ghaly et al. [6,7], Abdel-Ghani [9]. Ismail [15] studied the estimation and optimal design problems for the Gompertz distribution in SS-PALT with type I censored data. P.W. Srivastava and N. Mittal [20] considered optimum step stress partially accelerated life tests for the truncated logistic distribution with censoring. This article include type I and type II both censoring. S. Hyun and J. Lee [21] used constant stress partially accelerated life testing for log logistic distribution with censored data. F. K. Wang et al [22] have studied partially accelerated life tests for the Weibull distribution under multiply censored data. Recently, Showkat Ahmad Lone et al [23] studied estimation in step stress partially accelerated life tests for the Mukherjee-Islam distribution using time constraint. For a brief knowledge of step-stress ALT, one should go through [14, 17, 16, and 18].

II. Test Methods and Model

Mukherjee-Islam failure model is introduced by Mukherjee and Islam [1]. It is finite range distribution which is one of the most important property of it in recent time in reliability analysis. Its mathematical form is simple and can be handled easily, that is why, it is preferred to use over more complex distribution such as normal, Weibull, beta etc. The pdf of the distribution is given as

$$f(x,\alpha,\lambda)=\frac{\alpha}{\lambda^{\alpha}}x^{\alpha-1},\quad 0\le x\le\lambda,\ \alpha>0,\ \lambda>0$$

Where λ is the scale parameter and α is the shape parameter.
The cdf is given as

$$F(x)=\left(\frac{x}{\lambda}\right)^{\alpha},\ 0\le x\le\lambda,\ \alpha>0,\ \lambda>0$$

And the Reliability function of finite range model is given as

$$R(x)=1-\left(\frac{x}{\lambda}\right)^{\alpha}$$

In SS-PALT, all of the n units are tested first under normal condition, if the unit does not fail for a pre-specified time τ , then it runs at accelerated condition until failure. This means that if the item has not fail by some pre-specified time τ , the test is switched to the higher level of stress and it is continued until items fail. The effect of this switch is to multiply the remaining lifetime of the item by the inverse of the acceleration factor β. In this case, switching to the higher stress level will shorten the life of the test item. Thus the total lifetime of a test item, denoted by Y, passes through two stages, which are the normal and accelerated conditions.
The lifetime of the test unit in SSPALT is given as follows

$$Y=\begin{cases} T & if\quad T\le\tau \\ \tau+\beta^{-1}(T-\tau) & if\ T>\tau \end{cases}$$

Where T is the lifetime of item at use condition.
Therefore, the probability density function of total lifetime Y of an item is given by

$$f(y)=\begin{cases}0 & y<0\\ f_1(y) & 0<y\le\tau\\ f_2(y) & y>\tau\end{cases}$$

Where

$$f_1(y)=\frac{\alpha}{\lambda^{\alpha}}y^{\alpha-1} \qquad \alpha>0,\lambda>0$$

$$f_2(y)=\frac{\beta\alpha}{\lambda^{\alpha}}[\tau+\beta(y-\tau)]^{\alpha-1} \qquad \beta>1,\quad \alpha>0$$

III. Estimation Procedure

The maximum likelihood estimation method is used here because it is very robust and gives the estimates of the parameters with good statistical properties such as consistency, asymptotic unbiasedness, asymptotic efficiency and asymptotic normality. In this section, point and interval estimation for the parameters and acceleration factor of Mukherjee-Islam distribution based on type II censoring are evaluated using this method.

3.1. Point estimates

In type II censoring scheme, we set the number of units or subject to the experiment and stop the experiment at a predetermined number of failure. The observed values of the total lifetime Y are $y_{(1)}<...<y_{(n_u)}\le\tau<y_{(n_u+1)}<...<y_{(n_u+n_a-1)}\le y_{(r)}$, where n_u and n_a are the number of subjects or items failed at normal conditions and accelerated conditions respectively. Let φ_{1i} and φ_{2i} be indicator functions, such that

$$\varphi_{1i}=\begin{cases}1 & y_{(i)}\le\tau\\ 0 & elsewhere\end{cases} \quad i=1,2,...,n$$

And,

$$\varphi_{2i}=\begin{cases}1 & \tau<y_{(i)}\le y_{(r)}\\ 0 & elsewhere\end{cases} \quad i=1,2,...,n$$

For our convenience, $y_{(i)}$ is written as y_i. The likelihood function of independent and identically distributed random variables $y_1,..., y_n$, the life times of the items is given by

$$L(y;\beta,\lambda,\alpha)=\prod_{i=1}^{n}\left\{\frac{\alpha}{\lambda^{\alpha}}y_i^{\alpha-1}\right\}^{\varphi_{1i}}\left[\frac{\beta\alpha}{\lambda^{\alpha}}\{\tau+\beta(y_i-\tau)\}^{\alpha-1}\right]^{\varphi_{2i}}\left[1-\left\{\frac{\tau+\beta(y_{(r)}-\tau)}{\lambda}\right\}^{\alpha}\right]^{\bar{\varphi}_{1i}\bar{\varphi}_{2i}} \qquad (3.1.1)$$

Where $\bar{\varphi}_{1i}=1-\varphi_{1i}$ and $\bar{\varphi}_{2i}=1-\varphi_{2i}$, We take the logarithm of the likelihood function and write it as follows;

$$\ln L=r\log\alpha-r\alpha\log\lambda+(\alpha-1)\left\{\sum_{i=1}^{n}\varphi_{1i}\log y_i+\sum_{i=1}^{n}\varphi_{2i}\log A\right\}+n_a\log\beta+(n-r)\log(1-B^{\alpha}) \qquad (3.1.2)$$

Where, $A=[\tau+\beta(y_i-\tau)]$, $B=\left[\frac{\tau+\beta(y_{(r)}-\tau)}{\lambda}\right]$, $\sum_{i=1}^{n}\varphi_{1i}=n_u$, $\sum_{i=1}^{n}\varphi_{2i}=n_a$,

$$\sum_{i=1}^{n}\varphi_{1i}\varphi_{2i}=n-n_u-n_a \quad \& \quad r=n_u+n_a$$

The maximum likelihood estimates of β, α and λ can be obtained by solving the system of equations which are the first partial derivatives of the above log likelihood equation, and equate to zero with respect to β, α and λ respectively.

Thus the system of solutions are given as:

$$\frac{\partial InL}{\partial\beta}=\frac{n_a}{\beta}+(\alpha-1)\sum_{i=1}^{n}\varphi_{2i}A^{-1}(y_i-\tau)-(n-r)rB^{\alpha-1}\left(1-B^{\alpha}\right)^{-1}\frac{(y_{(r)}-\tau)}{\lambda}=0 \qquad (3.1.3)$$

$$\frac{\partial InL}{\partial\alpha}=\frac{r}{\alpha}-rIn\lambda+\sum_{i=1}^{n}\phi_{1i}In(y_i)+\sum_{i=1}^{n}\phi_{2i}InA-\frac{(n-r)}{(1-B^{\alpha})}B^{\alpha}InB=0 \qquad (3.1.4)$$

$$\frac{\partial InL}{\partial\lambda}=-\frac{r\alpha}{\lambda}+\frac{(n-r)}{\lambda}\left(1-B^{\alpha}\right)^{-1}\alpha B^{\alpha}=0 \qquad (3.1.5)$$

The solution of above equations cannot be obtained in closed form because the equations are non linear in three unknown parameters β, α and λ. Therefore, to find numerical solution we use an iterative method. Newton-Raphson Method is used to obtain the numerical estimate of the parameters β, α and λ.

3.2. Interval estimates

If $L_\omega=L_\omega(y_1,y_2,...,y_n)$ and $U_\omega=U_\omega(y_1,y_2,...,y_n)$ are functions of the sample data $y_1,...,y_n$, then the confidence interval for a population parameter ω is given by

$$p[L_\omega\le\omega\le U_\omega]=\pi \qquad (3.2.1)$$

Where, L_ω and U_ω are the lower and upper confidence limits which enclose ω with probability π. The interval $[L_\omega,U_\omega]$ is called a $100\pi\%$ confidence interval for ω. For large sample size, the maximum likelihood estimates, under appropriate regularity conditions, are consistent and asymptotically normally distributed. Therefore, the approximate $100\pi\%$ Confidence limits for the maximum likelihood estimate $\hat{\omega}$ of a population parameter ω can be constructed, such that

$$p\left[-Z\le\frac{\hat{\omega}-\omega}{\sigma(\hat{\omega})}\le Z\right]=\pi \qquad (3.2.2)$$

Where, Z is the $\left[\frac{100(1-\pi)}{2}\right]$ standard normal percentile. Therefore, the approximate $100\pi\%$ confidence limits for a population parameter ω can be obtained, such that

$$p[\omega-Z\sigma(\hat{\omega})\le\hat{\omega}\le\omega+Z\sigma(\hat{\omega})]=\pi \qquad (3.2.3)$$

Then, the approximate confidence limits for β,α and λ will be constructed using Eq. (3.2.3) with confidence levels 95% and 99%.

IV. Asymptotic variances and covariance of Estimates

The asymptotic variances of maximum likelihood estimates are the elements of the inverse of the Fisher information matrix $I_{IJ}(\varphi)\cong E\{-\partial^2 InL/\partial\varphi_i\partial\varphi_j\}$. The exact mathematical expressions for the above expectation are very difficult to obtain. Therefore, the observed Fisher information matrix is given by $I_{IJ}(\varphi)\cong\{-\partial^2 InL/\partial\varphi_i\partial\varphi_j\}$ which is obtained by dropping the expectation on operation E . The approximate (observed) asymptotic variance–covariance matrix F for the maximum likelihood estimates can be written as follows

$$F = \left\lfloor I_{ij}(\varphi) \right\rfloor \quad i, j = 1,2,3 \qquad (\varphi) = (\beta, \alpha, \lambda) \tag{4.1}$$

The second partial derivatives of the maximum likelihood function are given by the following

$$\frac{\partial^2 InL}{\partial \beta^2} = \frac{-n_a}{\beta^2} - (\alpha - 1)\sum_{i=1}^{n} A^{-2}\phi_{2i}(y_i - \tau)^2 - \alpha(n-r)\frac{(y_{(r)} - \tau)^2}{\lambda^2}\left[\alpha B^{2\alpha-2}(1-B^{\alpha})^{-2} + (\alpha-1)B^{\alpha-2}(1-B^{\alpha})^{-1}\right] \tag{4.2}$$

$$\frac{\partial^2 InL}{\partial \alpha^2} = \frac{-r}{\alpha^2} - (n-r)B^{\alpha}\left(\frac{InB}{1-B^{\alpha}}\right)^2 \tag{4.3}$$

$$\frac{\partial^2 InL}{\partial \lambda^2} = \frac{r\alpha}{\lambda^2} - \frac{\alpha(n-r)}{\lambda^2}\left[\alpha B^{2\alpha}(1-B^{\alpha})^{-2} + \alpha B^{\alpha}(1-B^{\alpha})^{-1} + B^{\alpha}(1-B^{\alpha})^{-1}\right] \tag{4.4}$$

$$\frac{\partial^2 InL}{\partial \beta \partial \alpha} = \sum_{i=1}^{n} \phi_{2i} A^{-1}(y_i - \tau) - (n-r)\left(\frac{y_{(r)} - \tau}{\lambda}\right)\left[\alpha B^{2\alpha-1}(1-B^{\alpha})^{-2} InB + (1-B^{\alpha})^{-1} B^{\alpha-1}(\alpha InB + 1)\right] \tag{4.5}$$

$$\frac{\partial^2 InL}{\partial \beta \partial \lambda} = \alpha^2 (n-r)B^{\alpha-1}(1-B^{\alpha})^{-1}\left(\frac{y_{(r)} - \tau}{\lambda^2}\right)\left[1 + B^{\alpha}(1-B^{\alpha})^{-1}\right] \tag{4.6}$$

$$\frac{\partial^2 InL}{\partial \alpha \partial \lambda} = -\frac{r}{\lambda} + \left(\frac{n-r}{\lambda}\right)B^{\alpha}(1-B^{\alpha})^{-1}\left[1 + \alpha InB + \alpha InB(1-B^{\alpha})^{-1} B^{\alpha}\right] \tag{4.7}$$

Consequently, the maximum likelihood estimators of β, α and λ have an asymptotic variance–covariance matrix defined by inverting the Fisher information matrix F and by substituting $\hat{\beta}$ for β, $\hat{\alpha}$ for α and $\hat{\lambda}$ for λ.

V. Simulation studies

Simulation studies are very important part of the study. It has been performed to illustrate the precision and consistency of the theoretical results of estimation parameters. R software is used in simulation studies. Absolute relative bias (RAB), mean square error (MSE) and relative error (RE) are the main measure to check the performance of resulting estimators. The detailed steps of procedures are presented below:

Step 1. 1000 random samples of sizes 50, 75, 100, 125 and 150 were generated from the Mukherjee-Islam distribution. The data generation of the Mukherjee-Islam distribution is very simple, if U has a uniform (0, 1) random number, and then $Y = [\lambda.u\wedge(1/\alpha)]$ follows a Mukherjee-Islam distribution. The true parameter values are selected as $(\alpha = 1.6,\ \lambda = 2,\ \beta = 1.05)$ and $(\alpha = 1.5,\ \lambda = 2,\ \beta = 1.1)$.

Step 2. Choosing the censoring time τ at the normal condition to be τ=1 and the total number of failure in PALT is to be r=0.75*n

Step 3. For each sample and for the two sets of parameters, the acceleration factor and the parameters of distribution were estimated in SS-PALT under type II censored sample.

Step 4. The Newton–Raphson method was used for solving the nonlinear equations given in (3.1.3), (3.1.4) and (3.1.5).

Step 5. The RABias, MSEs, and REs of the estimators for acceleration factor and other

parameters for all sample sizes were tabulated.
Step 6. The confidence limit with confidence level γ=0.95 and γ =0.99 of the acceleration factor and other parameters were constructed.
The results are summarized in Tables 1 and 2. Table 1 presents the RABias, MSEs, and REs of the estimators. The approximated confidence limits at 95% and 99% for the parameters and acceleration factor are presented in Table 2.
Following are the observations can be made from the tabulated data on the performance of SS-PALT parameter estimation of the above used lifetime distribution:

1. For the second set of parameters $(\alpha = 1.5,\ \lambda = 2,\ \beta = 1.1)$, the maximum likelihood estimators have good statistical properties than the first set of parameters $(\alpha = 1.6,\ \lambda = 2,\ \beta = 1.05)$ for all sample sizes (see Table 1)
2. As the acceleration factor increases the estimates have smaller MSE, and RE. As the sample size increases the RABias and MSEs of the estimated parameters decreases. Hence the estimates provide asymptotically normally distributed and consistent estimators for the acceleration factor and other parameters.
3. The interval of the estimators decreases when the sample size is increasing. Also, the range of the interval estimate at γ=0.95 is smaller than the range of the interval estimate at γ=0.99 (see Table 2).

Table 1 The RABias, MSE and RE of the parameters (α, λ, β) for different sample sizes under type II censoring

N	Parameters (α, λ, β)	(1.6,2,1.05)			(1.5,2,1.1)		
		RABias	MSE	RE	RABias	MSE	RE
50	α	0.0102	0.0616	0.0385	0.0008	0.0634	0.0422
	λ	0.0576	0.0829	0.0414	0.0157	0.0861	0.0430
	β	0.0013	0.0655	0.0013	0.0702	0.0850	0.0773
75	α	0.0094	0.0591	0.0369	0.0019	0.0411	0.0274
	λ	0.0484	0.1006	0.0503	0.1139	0.0448	0.0569
	β	0.0471	0.0747	0.0711	0.0148	0.0761	0.0692
100	α	0.0280	0.0451	0.0282	0.0211	0.0328	0.0219
	λ	0.0371	0.0674	0.0337	0.0448	0.1139	0.0569
	β	0.0184	0.0550	0.0524	0.0289	0.0643	0.0584
125	α	0.0061	0.0392	0.0245	0.0066	0.0458	.0305
	λ	0.0641	0.1070	0.0535	0.0331	0.0951	0.0475
	β	0.0807	0.1287	0.1225	0.0325	0.0755	0.0687
150	α	0.0125	0.0185	0.0115	0.0425	0.0568	0.0379
	λ	0.0304	0.0284	0.0142	0.0065	0.0748	0.0374
	β	0.0012	0.0264	0.0251	0.0255	0.1080	0.0982

Table 2 Confidence bounds of the estimates at confidence levels 0.95 and 0.99

N	Parameters (α, λ, β)	(1.6,2,1.05)			(1.5,2,1.1)		
		Standard Deviation	Lower Bound	Upper Bound	Standard Deviation	Lower Bound	Upper Bound
50	α	0.2547	1.0842	2.0829	0.2584	0.9948	2.0078
			0.9263	2.2409		0.8346	2.1680
	λ	0.2955	1.5359	2.6944	0.3011	1.4413	2.6218
			1.3527	2.8776		1.2546	2.8085
	β	0.2627	0.5364	1.5664	0.2991	0.4363	1.6091
			0.3735	1.7293		0.2508	1.7946
75	α	0.2495	1.0956	2.0739	0.2080	1.0950	1.9107
			0.9409	2.2287		0.9660	2.0397
	λ	0.3255	1.4588	2.7349	0.3463	1.4110	2.7685
			1.2570	2.9367		1.1962	2.9832
	β	0.2804	0.5498	1.6491	0.2830	0.5615	1.6712
			0.3759	1.8230		0.3860	1.8467
100	α	0.2179	1.1277	1.9823	0.1860	1.1035	1.8329
			0.9926	2.1174		0.9881	1.9482
	λ	0.2664	1.5521	2.5966	0.3463	1.4110	2.7685
			1.3869	2.7618		1.1962	2.9832
	β	0.2408	0.5973	1.5414	0.2602	0.6218	1.6419
			0.4480	1.6907		0.4604	1.8033
125	α	0.2032	1.2115	2.0081	0.2199	1.0789	1.9411
			1.0855	2.1341		0.9425	2.0774
	λ	0.3356	1.4705	2.7862	0.3169	1.4449	2.6875
			1.2624	2.9943		1.2484	2.8840
	β	0.3680	0.4132	1.8562	0.2824	0.5105	1.6179
			0.1850	2.0844		0.3354	1.7930
150	α	0.1397	1.3060	1.8537	0.2446	1.0842	2.0433
			1.2194	1.9404		0.9325	2.1950
	λ	0.1731	1.7214	2.4003	0.2806	1.4629	2.5631
			1.6140	2.5077		1.2889	2.7372
	β	0.1667	0.7218	1.3754	0.3372	0.4109	1.7328
			0.6184	1.4788		0.2018	1.9419

Conclusion

Here, we have used type II censored data to obtain the likelihood estimation for performance of Mukherjee-Islam distribution and the acceleration factor under SSPALT. The censoring scheme along with SSPALT will be feasible to the experimenter and he/she will have a fixed number of failure prior to the experiment. That will save cost as well as money. The maximum likelihood estimation technique is used to estimate the parameters and the estimators of it are consistent and asymptotically normally distributed. The current research shows that the second set of parameters has good statistical properties than the first set of parameters. As the sample size increases, the confidence interval become narrower. It can also be noted that interval of the estimators at γ=0.99 is greater than the corresponding at γ=0.95. Therefore, it can be inferred that the model and test method used in this study works satisfactorily for step stress partially accelerated life testing under the certain assumptions.

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