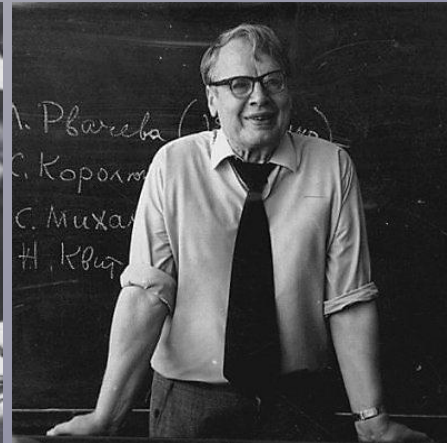


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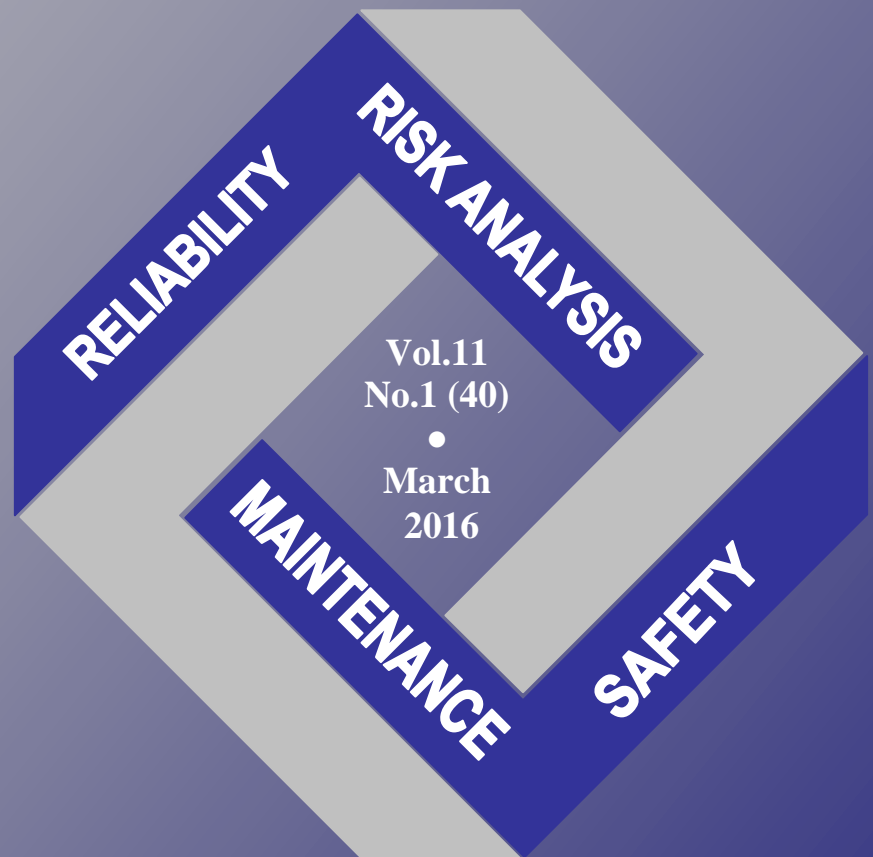


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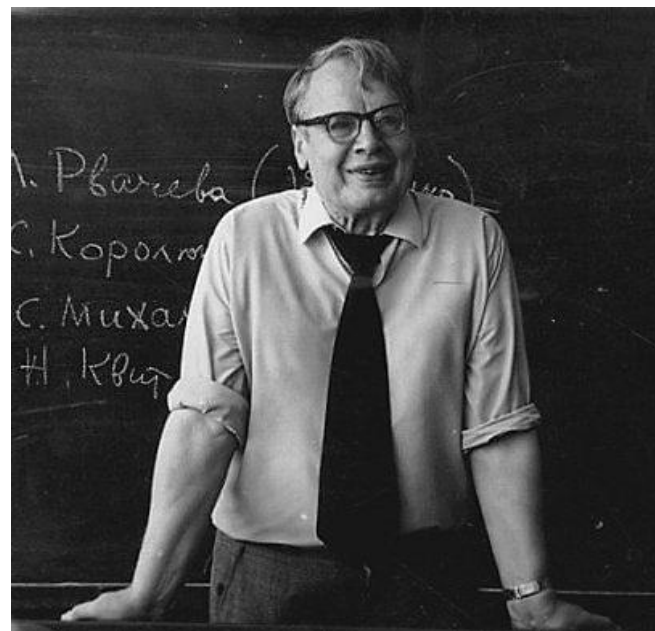
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ASSESSMENT OF QUALITY OF SERVICE IN BANK OFFICES BY METHODS OF THE THEORY OF MASS SERVICE

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Abstract

In work the assessment of quality of service is given in bank offices by methods of the theory of mass service on the basis of which the comparative analysis of real indicators of functioning of branches of the bank is made. Offers on increase in efficiency of service of the lagging behind offices are formulated, the developed scenarios of further possible strengthening and development of overall performance of all branches of the bank are step by step considered.

Keywords: quality assessment, efficiency of service, functioning indicators, branch of the bank, service stream, analysis

I. Introduction

Increase of efficiency of service, aspiration to correspond to the set level of quality, are the priority directions in work with the personnel, practically each financial institution. Especially great interest to the matter is shown by the commercial organizations which aren't provided with the state dating and working in severe conditions of the competition when fight actually goes for each real and potential client.

In such situation, especially actual is a question of preservation already received financial investments, the income and aspiration to pursue the considered, weighed economic policy of development of the enterprise. Within the solution of the matters, the greatest interest causes use of mathematical methods for the analysis of current situation, the forecast of possible decisions and creation of the corresponding scenario of development. These methods are actual and demanded as allow to organize an individual approach in the analysis of a situation that leads to more effective improvement of quality of service and work of the personnel. Thereby allowing to avoid sample actions which are characterized by high expenses, lack of optimization, short-term, unstable improvement of a situation.

Quality of service is a set of the mechanisms, actions, rules and attributes influencing satisfaction of clients at contact with bank [1]. The modern client of bank – skilled man, sophisticated in the requirements also demands a large number of services since it has a right and possibility of a wide choice in the market of banking services. It is possible to distinguish from such requirements: receiving high-quality consultation from bank workers; not the high cost of services of bank; faultless work of employees or technical devices of bank; comfort at service;

existence in bank of modern services and technologies, etc. Each client of bank chooses the indicator of quality of service, but there is the main indicator which each client puts at the head: holding time and turn, to be exact its absence.

The most widespread methods of an assessment of quality which are used at the organization of monitoring and marketing of banking institutions are based on the comparative analysis following the results of supervision or poll. Then purposeful work on elimination of defects which result is defined by the subsequent monitoring is carried out. Unlike this approach, the theory of mass service allows not only to improve separate indicators, but also to optimize process in general. In addition, existence of probabilistic indicators, allows to make already on the basis of the received values preliminary forecasts and according to them to build the plan of action, and to plan estimated results.

In this work, work on an assessment of quality of service in bank offices on the basis of methods of the theory of mass service is carried out. By results, which offers on increase in efficiency of service of the lagging behind offices were formulated and proved. The possible scenarios of further development in strengthening and development of overall performance of branches of the bank leading to increase of number of potential clients and the corresponding economic benefit are counted.

II. Main results

We will consider analytical model of system of mass service with expectation, not the limited arriving flow of requirements of clients and the following characteristics: the system has n of the serving channels, each of which can serve at the same time only one requirement [3]. The simplest (Poisson) flow of requirements with parameter λ comes to it,

$$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad (1)$$

If at the time of receipt of the next requirement in system on service already there is not less n of requirements, the requirement stands in line and expects the beginning of service [3]. A holding time of each requirement of t – a random variable which submits to the exponential law of distribution with parameter μ ,

$$F(t) = 1 - e^{-\mu t} \quad (2)$$

Parameter $\alpha = s/\mu$ – average of experts which needs to be had to serve in unit of time all coming clients is entered, where s – average of the clients arriving for a unit of time, $\bar{t} = 1/\mu$ – the average time of service by one expert of one client.

And the necessary condition $\alpha/n < 1$ which assumes that turn can't grow infinitely, so, the number of the serving experts has to be more average of the experts necessary in order that for a unit of time to serve all arriving clients.

On the basis of this model four real branches of the bank with the following entrance data presented in table 1 were considered. In this case understand additional offices of bank within one settlement as bank office. Using tools of the theory of mass service it is easy to receive the following indicators of functioning of office (table 2) which accept the corresponding values for each branch of the bank (table 3).

Table 1: Basic data on customer service in branches of the bank

Branch of the bank	Number of experts, n	Average time of service by one expert of one client, \bar{t} (min.)	Average of the clients coming to bank within an hour, s
1	10	4,2	124
2	6	5,8	55
3	7	5,3	70
4	8	5,0	94

Table 2: Settlement formulas

Probability of that all experts are free	$P_0 = \left[\sum_{k=0}^{n-1} \frac{\alpha^k}{k!} + \frac{\alpha^n}{n!(1-\alpha/n)} \right]^{-1}$
Probability of that all experts are occupied with service	$P_n = \frac{\alpha^n}{n!(1-\alpha/n)} P_0$
Average time of expectation of clients beginning of service (min.)	$\tilde{t} = \frac{P_n}{\mu(n-\alpha)}$
Average length of turn (persons)	$\bar{L} = \frac{\alpha P_n}{n(1-\alpha/n)}$
Average of free experts (persons)	$\bar{N} = \sum_{k=0}^{n-1} \frac{n-k}{k!} \alpha^k P_0$
Coefficient of loading of experts	$K = 1 - \bar{N} / n$

Table 3: Indicators of quality of functioning of branches of the bank

Branch of the bank	1	2	3	4
Probability of that all experts are free	0,000114	0,002567	0,001153	0,000055
Probability of that all experts are occupied with service	0,577765	0,706923	0,677664	0,93
Average time of expectation of clients beginning of service (min.)	1,83834	4,90395	3,74957	17,3329
Average length of turn (persons)	3,7992	5,5002	5,1309	44,000
Average of free experts (persons)	1,320	0,683	0,817	0,167
Coefficient of loading of experts	0,868	0,886	0,883	0,979

Primary analysis, the obtained data, reflects excellent and harmonious work of staff of office No. 1, moderate work of staff of office No. 2, 3, and weak work in office No. 4.

Owing to the lowest probability of that all experts are free, the highest probability of employment and the corresponding coefficient of loading, it is possible to claim that office No. 4 on the verge of an overload, both physical, and psychological. In too time, from a position of a psychological factor, at an average stream 94 persons an hour and the average length of turn 44 persons, it is possible to claim that soon in office recession of clients and as a result loss of potential profit will be observed.

In this situation, the offer on increase in overall performance of this office will be pertinent. Considering that the average time of service of the client in it is rather low, on the second place on all offices, it makes sense to add working unit. Addition only of one working unit (table 4, the

Stage 1), reduces probability of employment of all experts by a third, average expectation of the client practically by 7 times, and average the turn length 10 times.

The most important point for collective and the enterprise, in this case, is decreases of coefficient of loading and emergence of "free hands", average of free experts – 1,17. In such situation, it is possible to assume increase in speed of service, at the expense of more favorable situation, quiet work of experts and reduction of psychological and moral pressure from the present turn. As a result the increase in average time of service by one expert of one client at 2 seconds, leads to indicators when the average time of service doesn't exceed also 3 minutes (table 4, the Stage 2). That will increase efficiency of service, will keep and, perhaps, will increase average of clients and visitors in general.

Table 4: *Improvement of quality of functioning of branches of the bank No. 4*

Index	Office 4	Stage 1	Stage 2
Probability of that all experts are free	0,000055	0,000255	0,000399241
Probability of that all experts are occupied with service	0,93	0,60	0,514533
Average time of expectation of clients beginning of service (min.)	17,3329	2,47613	1,87383
Average length of turn (persons)	44,000	4,0	2,61439
Average of free experts (persons)	0,167	1,17	1,48
Coefficient of loading of experts	0,979	0,870	0,835556

The picture observed in branches of the bank No. 2, 3 is most widespread and universal (tab. 3). In too time, rather high coefficients of loading of experts and absence free, testifies that any hitch of one of operators, can instantly lead to formation of more long line and beat out office from stable work.

In this case, increase the working units not urgently, but development of a situation develops in this direction. In too time the increase at one established working post in offices No. 2, 3, in the circumstances will lead to more effective results, than procrastination of time to a situation when it is comparable with that occurs in office No. 4.

However, it should be noted that not always there is an opportunity to fill up staff since it demands also economic expenses. In that case, one of options of increase of overall performance of banks is increase of professionalism of workers which can consist not only in passing of courses of vocational training, retraining, etc., but also in debuggings of the process of work, and on a place, in concrete working situations. An indispensable condition of this debugging is the analysis of how there is a work in office and introduction of changes on places, for each office individually. Are for this purpose necessary simply observers, and participants of process, i.e. specialists operators.

Improvement of quality in this case consists, not in "piece" strengthening of permanent members of staff, and in creation of mobile group of experts 3-4 persons who will be serially thrown between branches of the bank, and to be engaged in increase of level of overall performance.

For example, we will consider the branch of the bank No. 2 in which the situation is stable and moderate. Introduction to this division of 3 new experts, will naturally significantly increase all indicators (table 5, the Stage 1). By results of the analysis, it is obvious that the highest average speed of service of the client on all branches of the bank (table 1) is the main reason for low efficiency. After carrying out planned actions on a place, the average time of service of the expert,

perhaps to raise till 4,5 min. This indicator isn't maximum (4,2 min.) therefore it can be quite reached.

In a result, after such actions, with initial number of experts in 6 people, but more with a high speed of service of 4,5 min., this office is essential to raise the indicators (table 5, the Stage 2), in comparison with initial (table 5). As a result, such changes, the branch of the bank No. 2, will be able to increase average, coming to bank with 55 to 70 people (table 5, the Stage 3), without essential loss of quality and efficiency, having kept some gain, in comparison with initial situation.

Table 5: *Mobile improvement of quality of functioning of branches of the bank No. 2*

Index	Office 2	Stage 1	Stage 2	Stage 3
Probability of that all experts are free	0,002567	0,00480593	0,0144418	0,00292659
Probability of that all experts are occupied with service	0,706923	0,109843	0,316216	0,680888
Average time of expectation of clients beginning of service (min.)	4,90395	0,909783	1,38663	3,46659
Average length of turn (persons)	5,5002	0,158551	0,695674	4,76622
Average of free experts (persons)	0,683	3,68333	1,875	0,75
Coefficient of loading of experts	0,886	0,590741	0,6875	0,875

After that the mobile group of experts can be transferred to other branch of the bank, to increase the skill, to train others and to develop new ways of individual improvement of quality of work and efficiency of service. In such approach, obligatory the condition will act entry into mobile group of experts of the business and existence at them the corresponding qualification, and experience.

Financial validity of this offer is confirmed over time since creation of mobile group to allow:

- Systematically to raise and modernize all branches of the bank in general;
- To carry out the analysis of current situations on a place;
- To carry out "live" monitoring of branches of the bank;
- To eliminate system errors and defects in work;
- To transfer experience of the leading branches of the bank to other offices.

As a result, even in the analysis of an initial situation, use of this group on all four offices, on average will increase total number of clients of the people coming to bank on 35-40, without loss of quality, growth of turns and invariable number of experts in offices. In the same conditions, pointed addition of the same number of experts in various offices will increase this number on 18-20 of people, without any further dynamics.

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A NEW BATHTUB SHAPED FAILURE RATE MODEL

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Abstract

In this paper, we introduce a new Bathtub shaped failure rate model named as x -Exponential Model and present a comparative study with Generalized Lindley, Generalized Gamma and Exponentiated Weibull distributions.

I. INTRODUCTION

There are many distributions for modeling lifetime data. Among the known parametric models, the most popular are the Lindley, Gamma, log Normal, Exponentiated Exponential and the Weibull distributions. These five distributions suffer from a number of drawbacks. None of them exhibit bathtub shape for their failure rate functions. The distributions exhibit only monotonically increasing, monotonically decreasing or constant failure rates. This is a major weakness because most real life systems exhibit bathtub shapes for their failure rate functions. At least three of the four distributions exhibit constant failure rates. This is a very unrealistic feature because there are hardly any real life systems that have constant failure rates. This is a major weakness because most real life systems exhibit bathtub shapes for their failure rate functions. Secondly at least three of the four distributions exhibit constant failure rates. This is a very unrealistic feature because there are hardly any real life systems that have constant failure rates.

Generalized Lindley, Generalized Gamma and Exponentiated Weibull distributions are proposed for modeling Lifetime data having bathtub shaped failure rate model. In this paper we introduce a simple model but exhibiting bathtub shaped failure rate and discuss the failure rate behavior of these distributions. A comparative study is carried out.

Section 2, discussed the Lindley Distribution, Section 3 discussed Generalized Lindley distribution, section 4 discussed Generalised Weibull distribution, section 5 discussed Generalized Gamma distribution, section 6 introduced new model, called x -Exponential and conclusions are given at the final section.

II. LINDLEY DISTRIBUTION

Lindley distribution was introduced by Lindley (1958) in the context of Bayesian statistics, as a counter example of fiducial statistics. Ghitany et al. (2008) observed that this distribution can be quite effectively used in lifetime experiments, particularly as an alternative of exponential distribution, as it also has only scale parameter. More so, in real world, we rarely encounter the engineering systems which have constant failure rate through their life span. Therefore, it seems practical to assume failure rate as a function of time. Lindley distribution is one of the distributions, having time-dependent failure rate.

The probability density function (pdf) of a Lindley random variable X , with scale parameter λ is given by

$$f(x) = \frac{\lambda}{1 + \lambda} (1 + x)e^{-\lambda x}, x > 0, \lambda > 0$$

The cumulative distribution function is

$$F(x) = 1 - \frac{1 + \lambda + \lambda x}{1 + \lambda} e^{-\lambda x}, x > 0, \lambda > 0$$

Lindley distribution is positively skewed distribution.

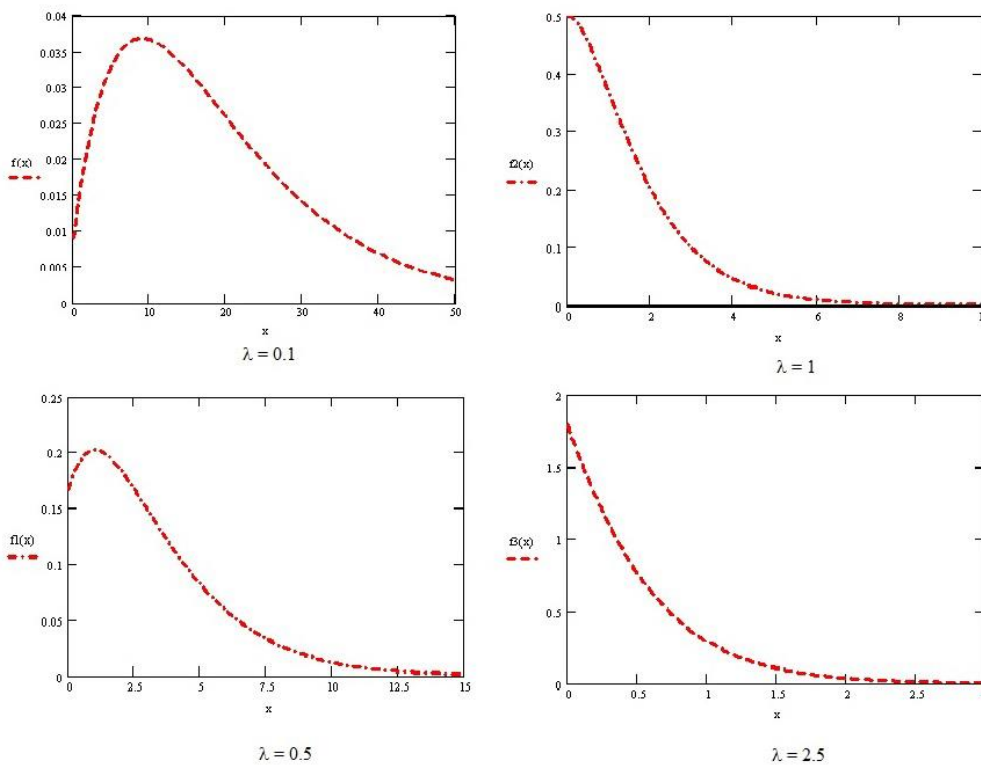


Figure 2.1: Probability density function of Lindley for $\lambda = 0.1; 0.5; 1.0$ and 2.5 .

The Failure Rate Function of Lindley distribution is

$$h(x) = (\lambda^2(1 + x))/(1 + \lambda(1 + x)), x > 0, \lambda > 0.$$

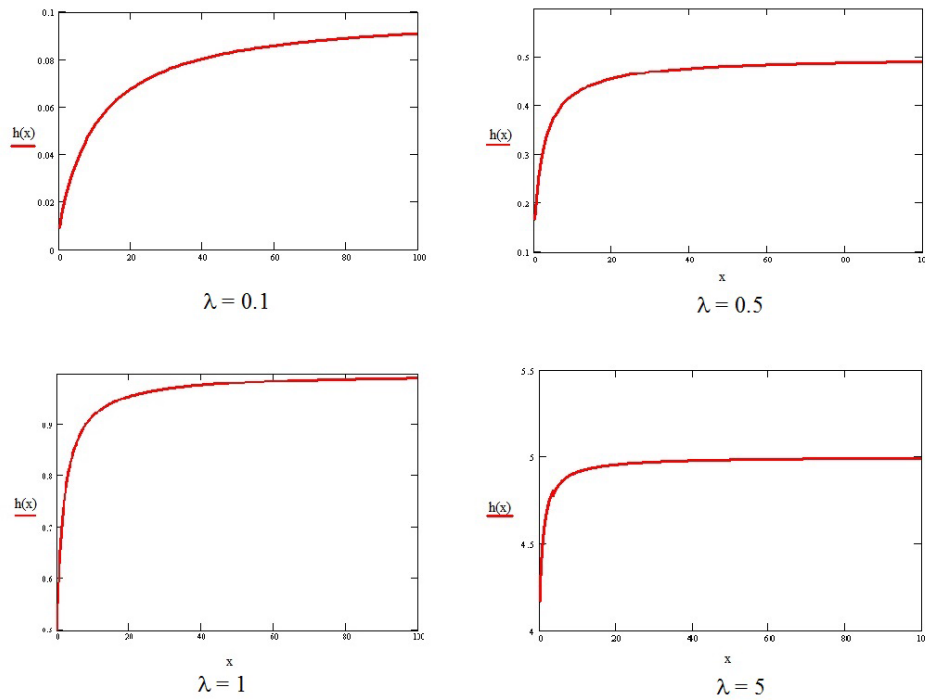


Figure 2.2: Failure rate function of Lindley distribution for $\lambda = 0.1; 0.5; 1.0$ and 5.0 .

III. GENERALIZED LINDLEY DISTRIBUTION

Suppose X_1, X_2, \dots, X_n are independent random variables distributed according to Lindley distribution and $T = \min(X_1, X_2, \dots, X_n)$ represent the failure time of the components of a series system, assumed to be independent. Then the probability that the system will fail before time x is given by

$$F(x) = [1 - (1 + \lambda + \lambda x)/(1 + \lambda) e^{-\lambda x}]^\alpha, x > 0, \lambda > 0.$$

It is the distribution of the failure of a series system with independent components. The cumulative distribution function and pdf of Generalized Lindley distribution are

$$F(x) = [1 - (1 + \lambda + \lambda x)/(1 + \lambda) e^{-\lambda x}]^\alpha, x > 0, \lambda > 0, \alpha > 0$$

$$f(x) = \frac{\alpha \lambda (1 + x)}{1 + \lambda} [1 - (1 + \lambda + \lambda x)/(1 + \lambda) e^{-\lambda x}]^{\alpha-1} e^{-\lambda x}, x > 0, \lambda > 0, \alpha > 0$$

The equation has two parameters, λ and α just like the Gamma, log Normal, Weibull and exponentiated Exponential distribution. For $\alpha = 1$ it reduces to Lindley distribution.

The failure rate function is

$$h(x) = \frac{[(\alpha \lambda (1 + x))/(1 + \lambda) [1 - (1 + \lambda + \lambda x)/(1 + \lambda) e^{-\lambda x}]]^{\alpha-1} e^{-\lambda x}}{1 - [1 - (1 + \lambda + \lambda x)/(1 + \lambda) e^{-\lambda x}]^\alpha}, x > 0, \lambda > 0, \alpha > 0$$

The shape of the failure rate function appears monotonically decreasing or to initially decrease and then increase, a bathtub shape if $\alpha < 1$; the shape appears monotonically increasing if $\alpha \geq 1$. So the Generalized Lindley distribution allows for monotonically decreasing, monotonically

increasing and bathtub shapes for its failure rate function.

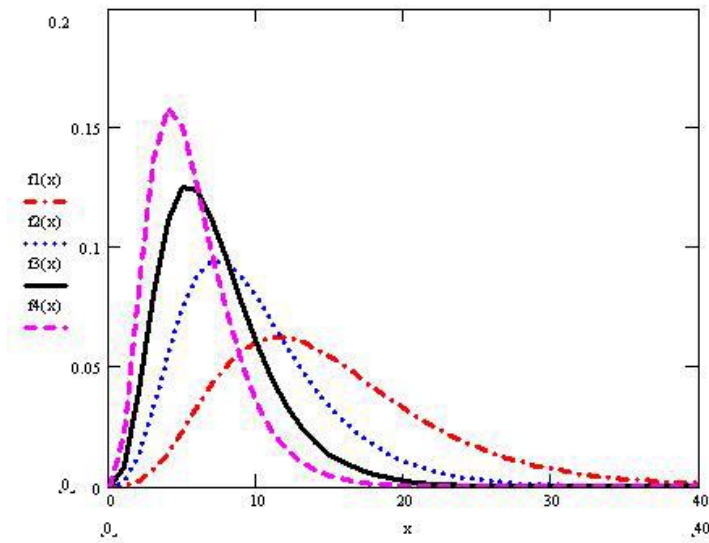


Figure 3.1. Probability density function of Generalized Lindley distribution.

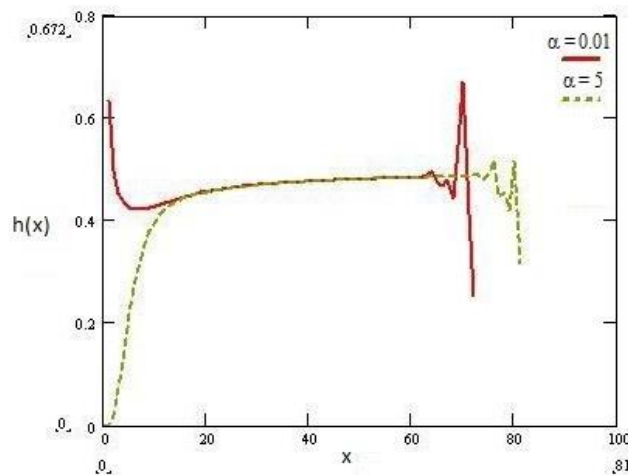


Figure 3.2. Failure rate function of Generalized Lindley distribution

IV. Exponentiated Weibull Distribution

We consider the Exponentiated Weibull (EW) distribution which has a scale parameter and two shape parameters. The Weibull family and the Exponentiated Exponential (EE) family are found to be particular cases of this family. The cumulative distribution function of the Exponentiated Weibull distribution is given by

$$F(x) = \left(1 - e^{-\left(\frac{x}{\beta}\right)^\alpha} \right)^\lambda, \lambda > 0, \alpha > 0, \beta > 0.$$

Here λ and α denote the shape parameters and β is the scale parameter. For When $\lambda = 1$, the distribution reduces to the Weibull Distribution with parameters. When $\beta = 1, \alpha = 1$ it represents

the (EE) family. Thus, EW is a generalization of EE family as well as the Weibull family.

Then the corresponding density function is

$$f(x) = \left(\frac{\alpha\theta}{\sigma}\right) [1 - \exp\{-(x/\sigma)^\alpha\}]^{\theta-1} \exp\{-\left(\frac{x}{\sigma}\right)^\alpha\} \left(\frac{x}{\sigma}\right)^{\alpha-1}, x \geq 0.$$

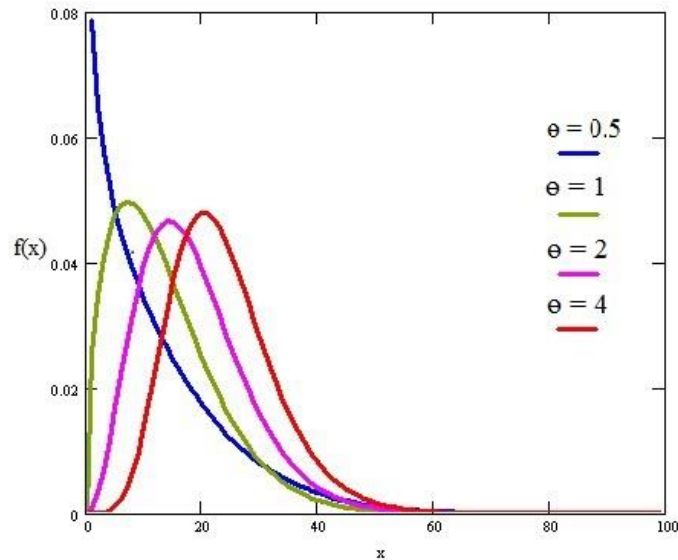


Figure 4.1 Probability density function of Exponentiated Weibull distribution

$$f(x) = \frac{\left(\frac{\alpha\theta}{\sigma}\right) [1 - \exp\{-\left(\frac{x}{\sigma}\right)^\alpha\}]^{\theta-1} \exp\{-\left(\frac{x}{\sigma}\right)^\alpha\} \left(\frac{x}{\sigma}\right)^{\alpha-1}}{1 - [1 - \exp\{-\left(\frac{x}{\sigma}\right)^\alpha\}]^\theta},$$

$$x \geq 0, \quad \alpha, \theta, \sigma > 0.$$

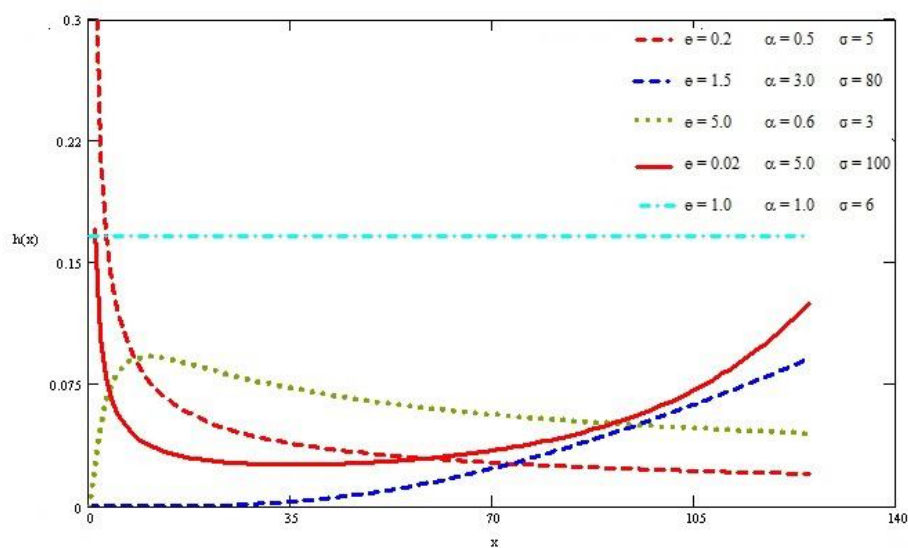


Figure 4.2: Plot of the failure rate function of EW distribution

The EW distribution is constant for $\theta = 1$ and $\lambda = 1$. The EW distribution is IFR for $\lambda > 1$ and $\theta \geq 1$. The EW distribution is DFR for $\lambda < 1$ and $\theta \leq 1$. The EW distribution is BT(Bathtub) for $\lambda > 1$ and $\theta < 1$. The EW distribution is UBT (Upside down Bathtub) for $\lambda < 1$ and $\theta > 1$.

V. Exponentiated Gamma Distribution

The Gamma distribution is the most popular model for analyzing skewed data and hydrological processes. This model is flexible enough to accommodate both monotonic as well as non-monotonic failure rates. The Exponentiated Gamma (EG) distribution is one of the important families of distributions in lifetime tests. The Exponentiated Gamma distribution has been introduced as an alternative to Gamma and Weibull distributions.

The Cumulative Distribution function of the Exponentiated Gamma distribution is given by

$$G(x) = [1 - \exp\{-\lambda x\} (1 + \lambda x)]^\theta, x > 0, \lambda, \theta > 0.$$

where λ and θ are scale and shape parameters respectively.

Then the corresponding probability density function (pdf) is given by

$$g(x) = \theta \lambda^2 x \exp\{-\lambda x\} ([1 - \exp\{-\lambda x\} (1 + \lambda x)]^{\theta-1}), x > 0, \lambda, \theta > 0.$$

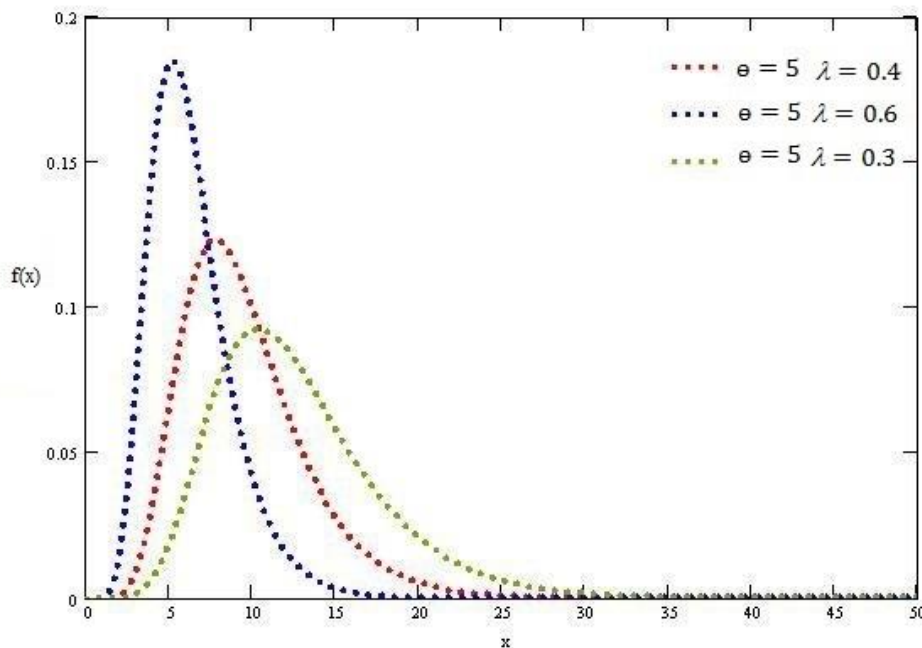


Figure 5.1. Probability density function of EG distribution.

The failure rate function is

$$h(x) = \frac{\theta \lambda^2 x \exp\{-\lambda x\} ([1 - \exp\{-\lambda x\} (1 + \lambda x)]^{\theta-1})}{1 - [1 - \exp\{-\lambda x\} (1 + \lambda x)]^\theta}, x > 0, \lambda, \theta > 0.$$

Then the other advantage is that it has various shapes of failure function for different values of θ . It has increasing failure function when $\theta \geq 1/2$ and its failure function takes Bath-tub shape for

$\alpha < 1/2$.

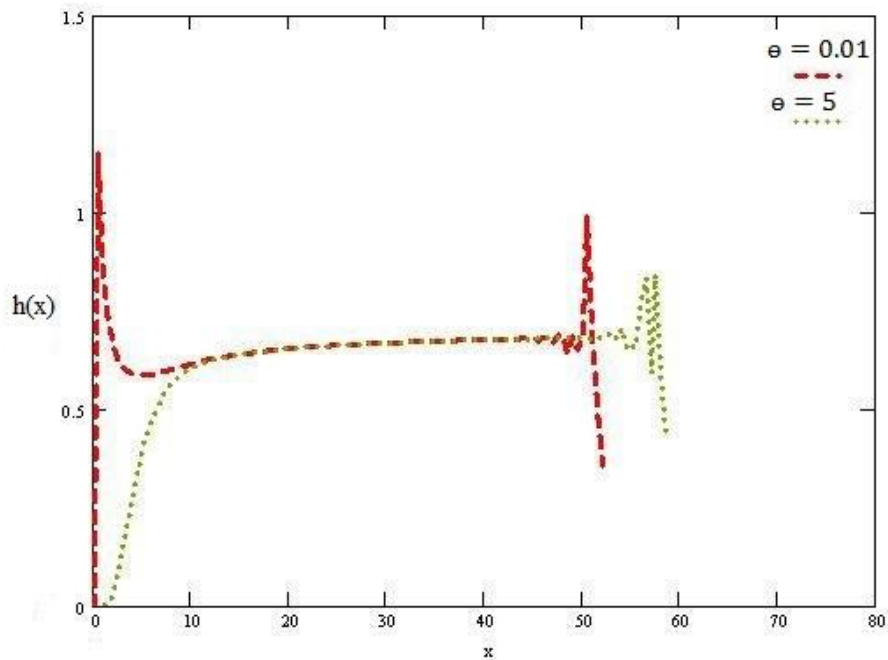


Figure 5.2: Failure rate function of EG distribution.

VI. x-Exponential Distribution

We introduce a new distribution, call it as x-Exponential, as an alternative to Generalized Lindley, Generalized Gamma and Exponentiated Weibull distributions. It is a very simple model than these GL, GG, EW distributions.

A life time random variable X is called x-Exponential distribution if its cumulative distribution function is

$$F(x) = (1 - (1 + \lambda x)e^{-\lambda x})^\alpha, x > 0, \lambda > 0.$$

Clearly $F(0)=0$, $F(\infty) = 1$, F is non-decreasing and right continuous. More over F is absolutely continuous.

The probability density function (pdf) of a x-Exponential random variable X , with scale parameter λ is given by

$$f(x) = \alpha \lambda e^{-\lambda x} (\lambda x) (1 - (1 + \lambda x)e^{-\lambda x})^{\alpha-1}, x > 0, \lambda > 0$$

It is positively skewed distribution.

Failure rate function of x-Exponential distribution is

$$h(x) = \frac{\alpha \lambda e^{-\lambda x} (\lambda x) (1 - (1 + \lambda x)e^{-\lambda x})^{\alpha-1}}{1 - (1 - (1 + \lambda x)e^{-\lambda x})^\alpha}, x > 0, \lambda > 0$$

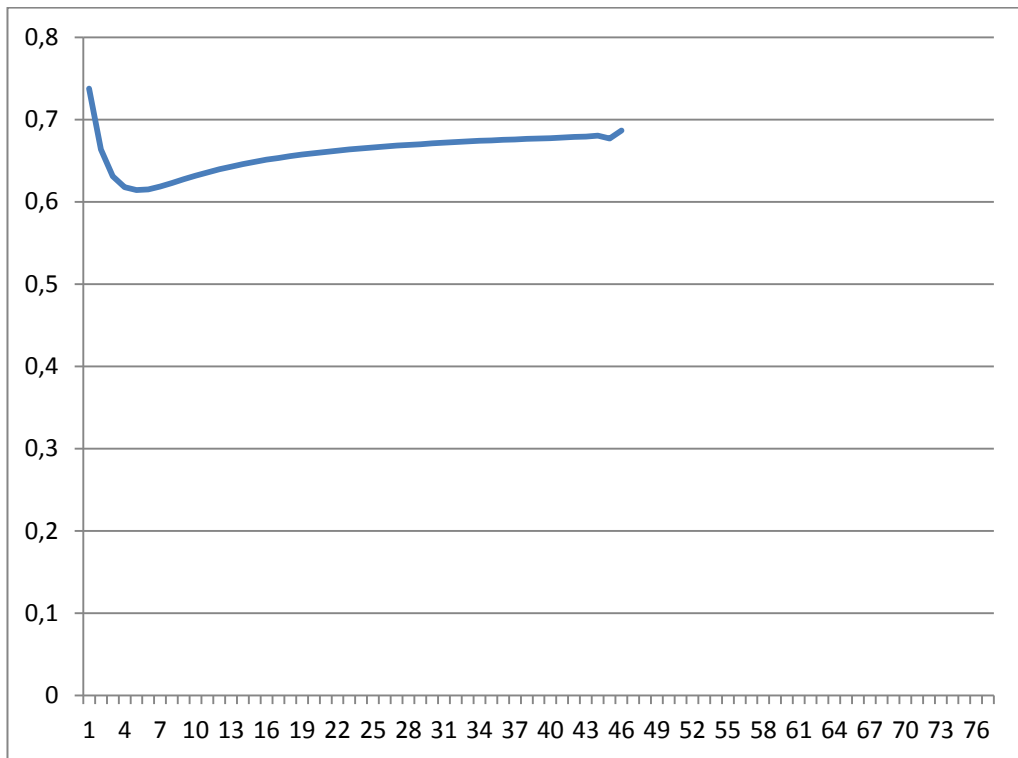


Figure 6.1. Failure rate function of x-Exponential distribution for $\theta=0.01$ and $\lambda=.6$

VII. Conclusions

There are many distributions in reliability which exhibit Bathtub shaped failure rate model, but most of them are complicated in finding the moments, reliability etc. Moreover the increased number of parameters make complication and difficulty in estimation process. The proposed model is similar to Generalized Lindley, so all the computational procedures are like GL distribution. So I am not trying to provide a rigorous proof for that. This distribution can be viewed as distribution of $\text{Min}(X_1, \dots, X_n)$ where X_i is having i.i.d distribution with d.f. $1 - (1 + \lambda x)e^{-\lambda x}$, $X > 0$, $\lambda > 0$, a linear failure rate model.

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ABOUT OPTIMIZING OF LINES AND NETWORKS MAINTENANCE

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Abstract

The result of analysis of the issue having been done, there have been proposed the strategy and mathematical model of optimization of extended object maintenance characterized by the extent of resource recovery depth during the repair of lines and networks. The proposed mathematical model of maintenance optimization would allow to determine the optimal frequency of preventive capital repairs and replacement of lines and networks, as well as the optimal number of overhauls during the service life of extended objects at the given depth of resource recovery.

I. THE ESSENCE OF THE PROBLEM

Lines and networks for various purposes are considered to be extended objects which are the variety of similar type elements connected in series from the point of reliability. For example, these objects include the contact network of electrified sections, overhead and cable power lines, communications lines, data transfer networks in automated control systems and devices of automatics and remote control at railway transport. Identification, localization and recovery of damaged areas of lines and networks are the characteristic feature when there is failure of extended objects. We would imply a set of measures aimed at maintaining and restoring the valid state of objects and at the recovery of their resource by the term "maintenance" according to [1].

We will consider optimization of maintenance at the example of contact networks (CN). During the operation of CN there are done technical service (TS), current repairs (CR) and overhauls (O), as well as reconstruction which is equivalent to the preventive replacement [2,3]. There is defined the technical state of CN only while performing the technical service through examinations, inspection, tests and measures [3]. According to [4] when doing CR there is only the performance recovery, but when doing O there is the recovery up to the certain level of object resource as well. Full recovery of object resource takes place while replacing.

At present in the theory of reliability [5,6] there have been developed the methodological issues of optimization of preventive replacements (PR) with emergency replacements (ER) or with minimum emergency repairs (MER) at failures. In the publications mentioned there have been considered only two extreme cases of the resource recovery depth: no update when MER is done and full update when ER or PR is performed. But they are the intermediate values of the resource recovery depth of devices within these two extreme cases which are of practical significance.

The purpose of the article is to propose and to study a mathematical model of maintenance optimization of extended objects characterized by the extent of the resource recovery depth.

II. STRATEGY AND MATHEMATICAL MODEL OF MAINTENANCE OPTIMIZATION

To account for the depth of resource recovery it is proposed to use the parameter $a = T_{pr} - T_{ir}$ according to [7] which means “age” of an object after performing the preventive overhaul. T_{pr} and T_{ir} are pre-repair and inter-repair resource correspondently [7]. In the future, when developing mathematical models for maintenance optimizing to assess the resource recovery depth it is advisable to use the dimensionless parameter like $\alpha = a/T_{pr}$. If $\alpha = 0$, it means that replacement has been done. If overhaul has been done, for example, in τ time, then “age” decreases from τ to $\alpha\tau$.

Contact network is considered to be an extended object with many different connected in series elements from the point of reliability. In the process of troubleshooting there is restored only a separate damaged section of CN which, practically, would not modify the current reliability of the contact network as a whole. In this regard, we would consider the strategy of maintenance when failure is eliminated by minimum emergency repair, and after n of preventive overhauls the replacement of the contact networks is done.

The character of changes in the failure intensity (FI) connected with the operation life under this strategy is presented in Fig.1. After minimum emergency repairs the failure intensity is not changed. After overhaul repairs (OR) with x frequency and α as resource recovery depth FI is reduced to $\lambda(\alpha)$, and after PR with x_p frequency – to zero level. FI is $\lambda(x+\alpha)$ at the time of OR and PR. Here x and x_p are measured in units of the resource.

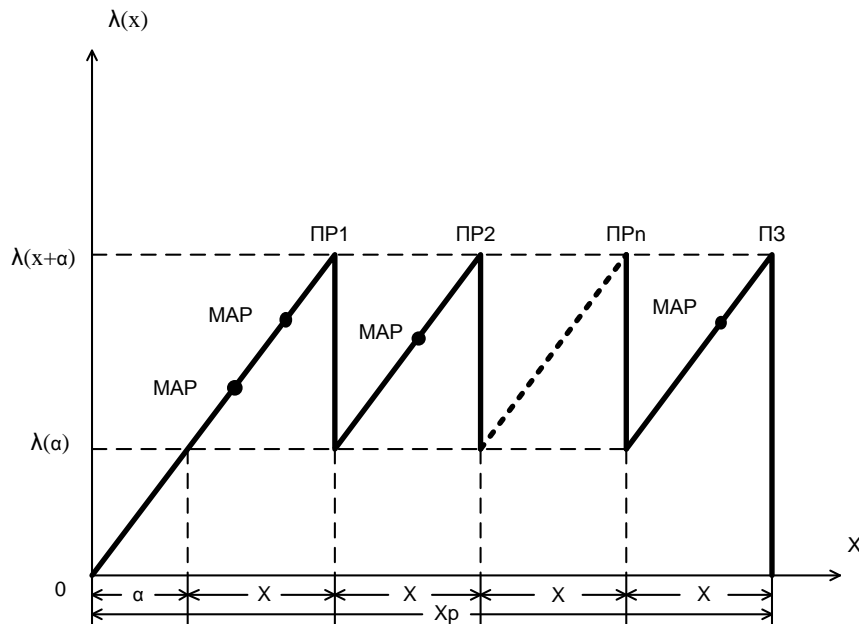


Figure 1. Change of failure intensity in preventive overhaul and replacement with minimum emergency repair

Mathematical model of maintenance optimization for extended objects under this strategy is defined from the expression

$$y = (1 + n\gamma) + \varepsilon \int_0^{x_p} \lambda(x) dx / x_p, \quad (1)$$

where y is the relative specific operating costs;
 γ is parameter of the cost of overhaul repair;
 ε is parameter of the cost of minimum emergency repair;
 λ is failure intensity;

The number of failures at the $0 - x_p$ interval is defined as

$$\int_0^{x_p} \lambda(x) dx = \int_0^{\alpha} \lambda(x) dx + (n+1) \int_{\alpha}^{x+\alpha} \lambda(x) dx = n \ln P(\alpha) - (n+1) \ln P(x+\alpha). \quad (2)$$

Here P is the probability of non-failure operation.

Substituting the values $\int_0^{x_p} \lambda(x) dx$ from (2) into (1) and given that $x_p = \alpha + (n+1)x$, we obtain the mathematical model as

$$y = \frac{1 + n\gamma + \varepsilon(n \ln P(\alpha) - (n+1) \ln P(x+\alpha))}{\alpha + (n+1)x}, \quad (3)$$

Let us consider two particular cases of the model (3):

when $n = 0$ and $\alpha = 0$ (there are done only the replacements fully restoring the initial resource) we obtain the mathematical model as

$$y = (1 - \varepsilon \ln P(x)) / x,$$

which is known as the model of preventive replacements with minimum emergency repair at failure [5];

when $n \rightarrow \infty$ (there are done only overhauls) after the disclosure of the uncertainty in the expression (3), we obtain the mathematical model as

$$y = (\gamma - \varepsilon(\ln P(x+\alpha) - \ln P(\alpha))) / x,$$

which corresponds to the strategy of preventive overhauls with minimum emergency repair at failure [7].

Using the expression (3) with given values of n and α the optimal frequency of preventive overhaul x_0 and minimum specific operating costs y_0 could be defined from the condition $\partial y / \partial x = 0$ as

$$(\alpha + (n+1)x_0) \lambda(x_0 + \alpha) + (n+1) \ln P(x_0 + \alpha) - n \ln P(\alpha) = (1 + n\gamma) / \varepsilon;$$

$$y_0 = \varepsilon \lambda(x_0 + \alpha).$$

The frequency of overhaul could be defined from the expression

$$x = (x_p - \alpha) / (n+1). \quad (4)$$

Then

$$x + \alpha = (x_p + n\alpha) / (n+1). \quad (5)$$

Substituting the expressions obtained x and $x + \alpha$ from (4) and (5) into the expression (3), we

transform it into the form

$$y = \left(1 + n\gamma + \varepsilon \left(n \ln P(\alpha) - (n+1) \ln P\left(\frac{x_p + n\alpha}{n+1}\right) \right) \right) / x_p \quad (6)$$

Using the expression (6) with given values of n and α the optimal frequency of preventive replacements x_{p0} and minimum specific operating costs y_0 could be defined from the condition $\partial y / \partial x_p = 0$ as

$$x_{p0} \lambda \left(\frac{x_{p0} + n\alpha}{n+1} \right) + (n+1) \ln P\left(\frac{x_{p0} + n\alpha}{n+1}\right) - n \ln P(\alpha) = (1+n\gamma) / \varepsilon;$$

$$y_0 = \varepsilon \lambda \left(\frac{x_{p0} + n\alpha}{n+1} \right).$$

Using the expression (6) with given values of x_p and α the optimal number of overhauls n_0 could be defined from the condition $\partial y / \partial n = 0$ as

$$\frac{x_p - \alpha}{n_0 + 1} \lambda \left(\frac{x_p + n_0\alpha}{n_0 + 1} \right) + \ln P\left(\frac{x_p + n_0\alpha}{n_0 + 1}\right) - \ln P(\alpha) = \gamma / \varepsilon .$$

CONCLUSION

To take into account the depth of the resource recovery after overhaul it is advisable to use the parameter which is defined as the difference between pre-repair resource and inter-repair resource related to the pre-repair resource of an extended object.

The proposed mathematical model of maintenance optimization would allow to define the optimal frequency of preventive capital repairs and replacement of lines and networks, as well as the optimal number of overhauls during the service life of extended objects at the given depth of resource recovery.

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ON 'SAFETY' AND 'SECURITY' TERMS AND THEIR RELATIONSHIP

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Abstract

The paper deals with such notions as object safety, subject security, production safety, environmental safety, energy security, and national security. Issues of ensuring the subject security are not limited by problems to be investigated in conjunction with the issues of ensuring the object safety. Considered are the relations between object safety, subject security, production safety, environmental safety, energy security, and national security issues. National security problem includes all the listed above safety/security issues.

Key words: object safety; subject security; production safety; environmental safety; energy security; national security; electric power systems safety

I. Introduction

'Security' and 'safety' are known to be different terms. They refer both to objects (e.g., facilities of energy systems) and to subjects (individuals, state, society). Diversity of safety/security meanings is due to different mix of threats taken into account when safety/security is considered. There is an objective demand to differentiate such notions as object safety, subject security, production safety, environmental safety, energy security, and national security.

II. Object safety and subject security

The object under study includes such energy facilities as power, gas, oil, heat and coal supply systems, as well as nuclear power systems. Safety of the object implies its ability to eliminate circumstances hazardous for people and environment. Object safety is a unique feature of complex nature, namely, reliability [3]. Studies on the object safety, as a rule, consider the so-called 'safety in emergency' when circumstances hazardous for people and environment may be caused by different internal (equipment failures, mistakes of personnel, etc.) and external (hurricanes, earthquakes, military actions, sabotage, etc.) disturbances that lead to the object destruction.

Subject security implies protection of vital and personal interests of an individual, interests of the society and the state from internal and external threats [4]. The issue of ensuring the subject

security includes issues to be investigated in conjunction with the issues of ensuring the object security (Fig. 1). Subject security cannot be ensured without ensuring the object security. An example can be the largest (in terms of victims) man-induced catastrophe in the modern history that happened early morning on December, 3, 1984. It was caused by a severe emergency at a chemical plant owned by the American chemical company in India, in the City of Bhopal. A cloud of toxic gas covered the nearby houses and a railway. Minimum 18 thousand people perished, 3000 of which perished on the day of emergency, and 15 thousand died in the subsequent years.

Subject security assurance is not limited by issues to be investigated in conjunction with the object security assurance issues. This problem is much more diverse. First, incomplete safety can demonstrate itself in normal operating conditions of the energy system. For example, negative impact of hydro power plants (HPP) may show up in constant or temporary flooding of fertile lands. Occurrence of stagnation zones in the water reservoirs due to low water flowage and turbulence may cause changes in the hydrochemical water composition and, as a result, water may become inadequate for vital human needs. Higher air humidity, higher cloudiness, wind force and rate in the water reservoir area result in the higher morbidity rate due to changed climatic conditions.

Second, means for ensuring the subject security are not limited by capabilities for ensuring the object safety. Therefore, the problem of ensuring the subject security differs from the problem of ensuring the object safety. Fig. 1 shows relations between those problems.

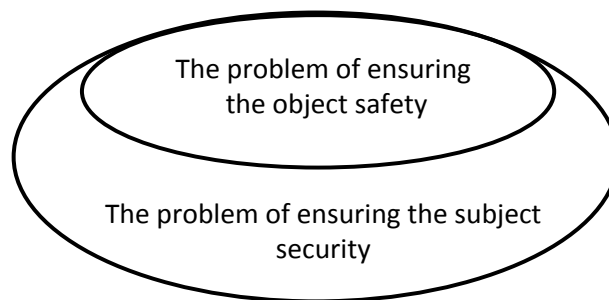


Figure 1. Relations between the energy facility safety and the subject security

III. Object safety, environmental safety, and energy security issues

Environmental safety implies protection of an individual, society and the state from the consequences of technological and natural impacts on the environment. For example, electromagnetic pollution of habitat became so considerable that World Healthcare Organization called it a top priority challenge for a human being. Energy equipment, power lines, power transformers, high-voltage switchgears in particular make their contribution into total electromagnetic pollution of the environment. The problem of ensuring the ecological safety (unlike the object safety assurance problem) takes into account all the man-induced impacts rather than those caused by emergencies at different energy facilities.

The difference lies in the fact that the problem of ensuring the energy facility safety takes into account situations risky for people but not risky for environment, e.g., power supply (as a target product) interruption due to emergencies in the power systems. Fig. 2 shows relations between the object safety and environmental safety assurance problems.

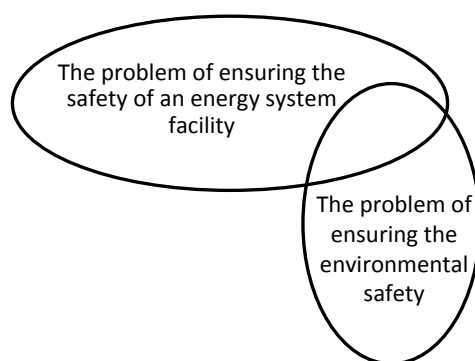


Figure 2. Relations between the energy object safety and the environmental safety assurance problems

The problem of ensuring the labor safety implies protection of operating personnel from hazardous impact of technological processes, power, working environment, equipment items and labor conditions at a production facility. The problem of ensuring the production safety suggests the ability to control production-related hazardous factors impacting the operating personnel. Improper production safety contributes to the increased number of incidents, higher rate of occupational diseases and even death of people during emergencies at energy facilities. Chernobyl catastrophe on April 26, 1986 is a sorrowful example. Personnel of the plant and rescuers got lethal doses of radiation. Emergency at Sayano-Shushenskaya HPP on August 17, 2009 is the largest man-induced catastrophe in the Russian history. 75 people of the plant personnel perished. The emergency had a negative impact on the adjacent aquifer environment. Fig. 3 shows relations between environmental and production safety assurance problems.

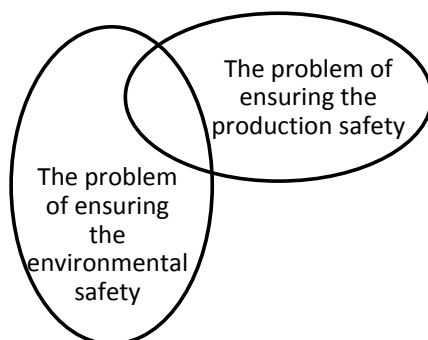


Figure 3. Relations between the environmental and production safety assurance problems

Analysis of incidents and fatalities shows that the major share of emergencies is caused by the so-called 'human factor', i.e., incorrect actions of personnel, responsible persons and officials, violation of technological procedures (about 80-85% of incidents), as well as by drawbacks in the legislative control and safety standards. According to United Nations Organization data, every eight (8) violations of safety rules result in an incident, and one out of 228 violations result in fatality [1].

Unlike the production safety the object safety assurance problem includes only impacts that may result in emergencies at different energy facilities that are hazardous for people and environment. Fig. 4 shows relations between the object safety and production safety assurance problems.

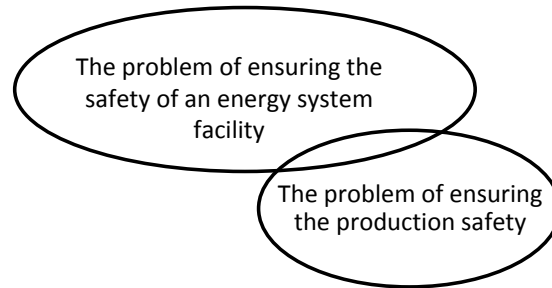


Figure 4. Relations between the energy object safety and production safety assurance problems.

IV. Object safety, environmental safety, and energy security

The problem of ensuring the energy security (with account of recommendations given in [4]) implies protection of citizens, society, state and economy from the threat of the shortage of power supply from economically efficient energy sources of acceptable quality, from the threat of power supply interruption and occurrence of circumstances risky for people and environment [2]. The problem of ensuring the energy security is of more general nature than the problems of ensuring the energy facilities safety as it takes into account a large range of threats, namely, economic, social, foreign economic and political relations, technological, natural and managerial threats [4]. For example, to ensure energy security one should take into account risks of possible power supply interruption due to putting the nuclear power plant out of operation following people's protests, equipment aging and impossibility of its timely replacement. There are cases of successive disconnections of power consumers, for example, at Ukraine in winter 2014 due to lack of finances to pay against the fuel supply contracts.

Energy facilities safety assurance problems are an inherent component of the energy security assurance problem as a whole. Methods for ensuring the energy systems safety (unlike energy security) are limited by capabilities of the systems themselves and do not allow offsetting the consequences of all the listed threats. Fig. 5 shows relations between the energy system safety assurance problem and the problem of energy security assurance as a whole.

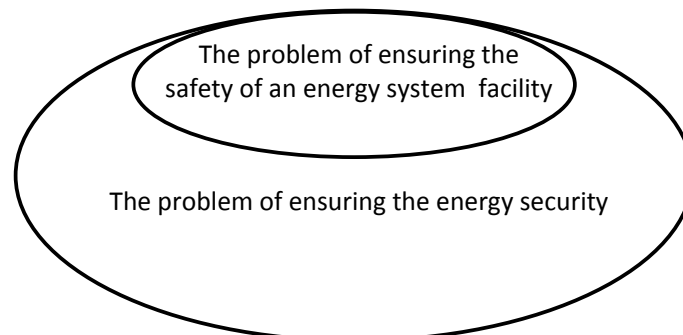


Figure 5. Relations between the problems of ensuring the energy facility safety and energy security as a whole

The issue of energy security assurance does not cover all the challenges related to environmental safety as technological impact on the environment is not caused by the energy facilities only. Motor and railway transport, chemical and other industries have an adverse impact on the environment as well. Fig. 6 shows relations between problems of ensuring the energy security, environmental safety and energy facilities safety.

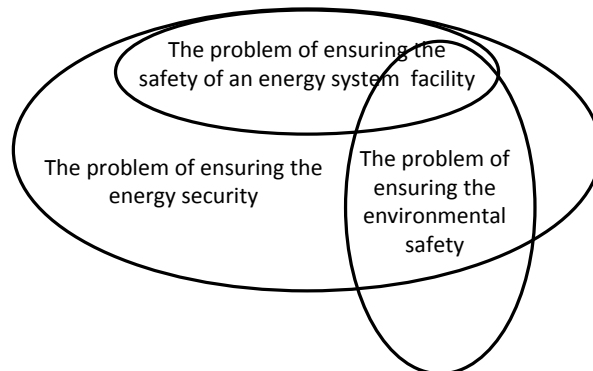


Figure 6. Relations between the energy facilities safety, environmental safety and energy security assurance problems

The problem of ensuring the energy security does not cover all the issues related to production safety assurance. For example, the problem of ensuring the energy security does not take into account labor conditions and modes. Fig. 7 shows relations between the energy security, environmental and production safety, and energy facilities safety assurance problems.

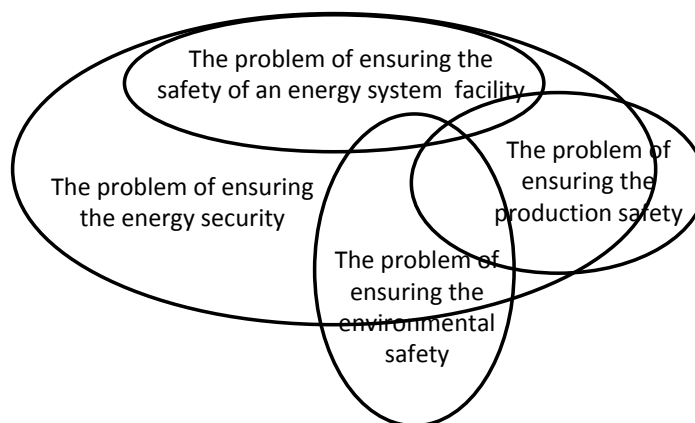


Figure 7. Relations between energy facilities safety, environmental safety, and production safety assurance problems

V. Object safety, environmental safety, and national security problems

National security implies protection of vital interests of the country, society, and its citizens, as well as protection of national interests in the economic, social, international, defense, cultural, religious and other spheres [4]. It is the most general notion of security. The problem of ensuring the national security includes all the above listed safety/security problems. Fig. 8 shows relations between the problems of ensuring the safety of objects, environmental safety, production safety, energy and national security.

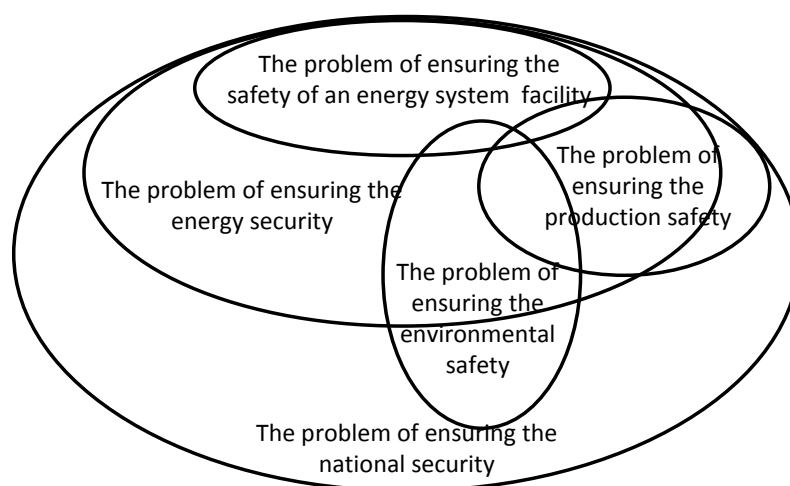


Figure 8. Relations between the problems of ensuring the security/safety

CONCLUSIONS

1. The problem of ensuring the subject security is not limited by issues to be investigated in conjunction with the object security assurance. This problem is much more diverse.

2. Unlike the problem of ensuring the production safety the problem of ensuring the object safety includes only impacts that may result in emergencies at different energy facilities that are hazardous for people and environment.

3. The problem of ensuring the ecological safety takes into account all the technological impacts rather than those caused by emergencies at different energy facilities.

4. The problem of ensuring the energy security is of more general nature than the problem of ensuring the energy facility safety, but at the same time it does not cover all the issues related to environmental safety assurance.

5. The problem of ensuring the national security includes all the above listed safety/security issues.

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EFFICIENCY ASSESSMENT OF THE MAIN EQUIPMENT OF ELECTRIC POWER SYSTEMS

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Abstract

The paper deals with the issues related to efficiency of the main equipment of electric power systems (EPS). An efficiency indicator has been selected that takes into account relationships between efficiency, economic component, technological advance and reliability. We show the possibility for pilot test (or validation) of the efficiency indicator, and for comparison of the main EPS equipment with account of its efficiency.

Key words: efficiency, reliability, availability factor, failure-free operation, maintainability, durability, durability factor, electric power systems.

Efficiency is a complex property that includes degree of technological advance of the object, its reliability, and its economic characteristics. It is obvious that the higher the technological advance and reliability, and the lower the costs, the higher the object's efficiency. Efficiency can be evaluated using integrated indicators that take into account economic efficiency, technological advance and reliability, or a system of indicators each characterizing the corresponding efficiency component. Selection of the efficiency indicator is influenced by certain requirements that, with account of recommendations (1,2), can be worded as follows:

- common physical sense;
- minimum scope of indicators (if they are more than one);
- possibility for pilot tests or for validation;
- sufficient sensitivity of indicators towards impact of different factors on the efficiency.

Recommendations given in (3) propose two indicators for the efficiency evaluation. The first one is total costs for the considered time period (C), and the second one is technological efficiency factor (\mathfrak{E}) that is a product of the performance factor (η), availability factor (K_{Γ}), and the service life factor (K_{Δ}):

$$\mathfrak{E} = \eta \times K_{\Gamma} \times K_{\Delta}.$$

The present paper considers the option of using one complex indicator for the efficiency evaluation that takes into account economic characteristics, technological advance and reliability taken together. The ratio between the total costs for the considered period and the amount of

useful energy (Wr) output by the facility with account of its technological advance and reliability at the same time period is offered to be used as such an indicator.

$$E = C/Wr . \tag{1}$$

Ability to test (or validate) the selected efficiency indicator can be ensured by availability of data on the useful power supplied to consumers for the considered time period and actual total costs for the same period.

Efficiency of new equipment should be assessed using calculations based on the available data on the technological advance and reliability of the facilities, rather than on the results of pilot tests (that are, as a rule, time-consuming). Economic component of the efficiency should comply with the recommendations given in (4).

Evaluation of the equipment efficiency for the forthcoming period of its operation

Performance factor is an indicator of technological advance of the facility (3). Capacity is the energy transfer rate (energy/time ratio), therefore, the performance factor equals to the ratio between the useful energy output and total energy input.

$$\eta = W/W_0 ,$$

where W = useful energy output, provided that the object is absolutely reliable (Fig. 1); W_0 = the amount of energy that can be supplied for the considered time period, i.e., marginal output of the facility.

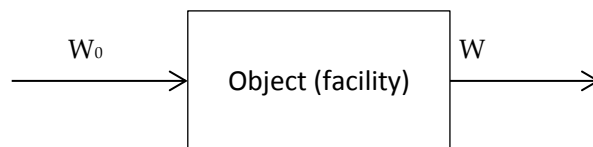


Figure 1. An absolutely reliable facility

Main EPS equipment is not an absolutely reliable object (Fig.2, where Wr = the amount of energy supplied by the facility with account of technological advance, failure-free operation and maintainability) whose behavior is described by a random process where periods of appropriate operation (when the object is fully operable) and periods of recovery (when the object is under overhaul), alternate (5). The object's operation process is shown in Fig. 3, where $To1$ = time to the first failure, $T\epsilon1$ = period of the first emergency repair.

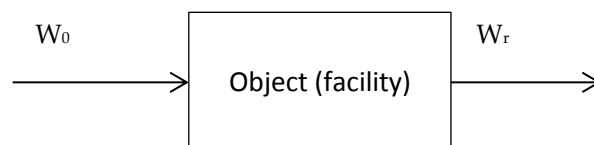


Figure 2. An object whose efficiency is dependent on its technological advance, failure-free operation and maintainability

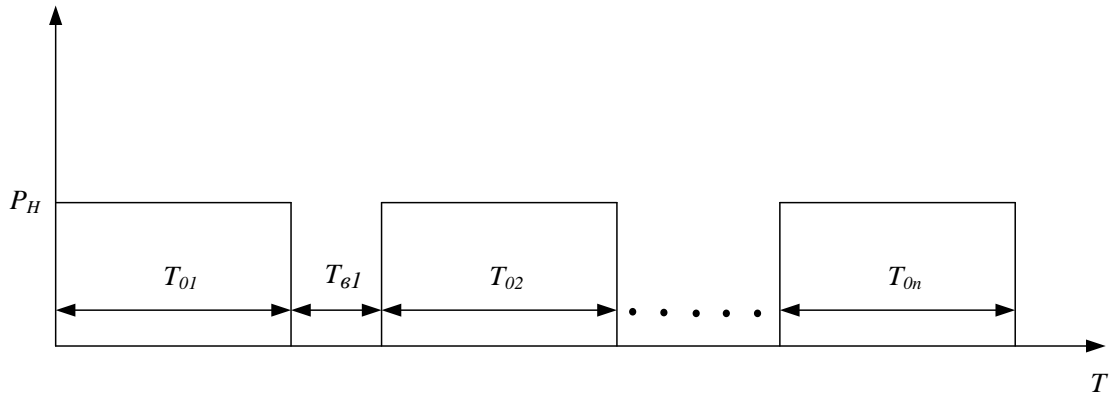


Figure 3. Object operation during considered time period

In formula (1) let us multiply and divide the denominator by W and then by W_o

$$E = C/Wr = C/(Wr \times (W_o/W_o) \times (W/W)) = C/(W_o \times (W/W_o) \times (Wr/W)).$$

Then

$$E = C/(W_o \times \eta \times (Wr/W)). \quad (2)$$

At a sufficiently large number of failures (n) during the considered time period

$$Wr = P_H \times \sum_{i=1}^n T_{O_i}; \quad (3)$$

$$W = P_H \times \left(\sum_{i=1}^n T_{O_i} + \sum_{i=1}^n T_{\theta_i} \right). \quad (4)$$

If we substitute the right-hand parts of equations (3) and (4) into formula (2), and divide the nominator and denominator by n , then an expression for the efficiency factor (E) will take the form:

$$E = C/(W_o \times \eta \times (K_r)). \quad (5)$$

And

$$K_r = T_o / (T_o + T_{\theta}) = Wr / W ,$$

where T_o = mean time of operation between failures.

$$T_o = (1/n) \times \sum_{i=1}^n T_{O_i};$$

T_{θ} = mean time of recovery

$$T_{\theta} = (1/n) \times \sum_{i=1}^n T_{\theta i};$$

Amount of power supplied to consumers in the forthcoming period of operation:

$$W_r = W_o \times \eta \times K_{\Gamma}.$$

Comparison of objects in terms of their efficiency

Compared options must have similar energy effect (4).

Let us first consider the case when similar equipment is compared under similar operating conditions (objects are operated for the same time period (7); load during the period under study is assumed to be equal to the rated load (Pr); the amount of power the object can produce during the considered period is the same for all the objects).

In this case comparison of objects in terms of efficiency is reduced to computing the efficiency factors using formula (5) and to comparison of the values obtained. For example, for two objects:

$$E_1 = C_1 / (W_o \times \eta_1 \times K_{\Gamma 1}) \quad \text{и} \quad E_2 = C_2 / (W_o \times \eta_2 \times K_{\Gamma 2}),$$

where E_1 and E_2 , η_1 and η_2 , $K_{\Gamma 1}$ and $K_{\Gamma 2}$ = efficiency factors, efficiency performances and availability factors for the first and second objects, respectively. As we have already mentioned, the objects under consideration differ in cost, technological advance and reliability. The amount of energy produced by each of them is the same.

Let us now consider the case when objects receive different amount of energy (W_{o1} and W_{o2} , respectively). The efficiency factor for the first object is:

$$E_1 = C_1 / (W_{o1} \times \eta_1 \times K_{\Gamma 1}), \quad (6)$$

The efficiency factor for the second object is:

$$E_2 = C_2 / (W_{o2} \times \eta_2 \times K_{\Gamma 2}). \quad (7)$$

Different objects can be compared if they are reduced to identical energy output. Let us introduce the factor

$$L = W_{o1} / W_{o2}.$$

Now, let us multiply the nominator and denominator of expression (7) by the L factor. Then:

$$E_2 = (C_2 \times L) / ((W_{o2} \times \eta_2 \times K_{\Gamma 2}) \times L) = (C_2 \times L) / (W_{o1} \times \eta_2 \times K_{\Gamma 2}). \quad (8)$$

Comparison of equations (6) and (8) shows that objects were reduced to the required terms of comparison. They receive similar amount of energy. Efficiency of the considered objects can be compared by comparing the efficiency factors computed using equations (6) and (8).

Thus, to be able to assess the object efficiency (for the forthcoming period of its operation) and to compare it to other objects, we need information about total costs and amount of energy the object can obtain in the considered time period, as well as the performance factor, and availability factor.

Comparison of objects one of which has been in operation over a long time period

Equations (6) and (8) can be used for evaluating the efficiency and comparing the new equipment to the equipment that has been in operation over a long time period. It should be kept in mind that the considered time period for equipment that has been in operation over a long time period equals the remaining service life. The amount of energy obtained will correspond to this period. The major share of the main EPS equipment (generating and network equipment) has exceeded the rated service life (due to high wear some equipment cannot be repaired). New equipment will get energy for the period of time that equals the average service life. Results of comparison may be used for justifying the equipment substitution by the new one.

Determination of reliability indicators for equipment that has been in operation over a long time period.

These indicators are usually determined by generalizing the initial statistical data on similar equipment for a certain time period. Time period selected for determining the reliability indicators must be, on the one hand, sufficiently long to ensure more accurate values of the equipment reliability indicators that are based on a large amount of statistical data, and, on the other hand, it must be sufficiently short, as an extended period is inclined to obscure the results of any tendency towards the equipment reliability increase or decline, and, hence, is not desirable.

This period is usually taken equal to 2-5 years. This approach towards determination of the equipment reliability indicators on the base of retrospective data has some drawbacks. It is obvious that reliability of some units can differ considerably due to equipment aging and different operating conditions, as it varies for each piece of equipment as they are impacted by different factors.

Therefore, it is advisable to have data on reliability of individual units that correspond to the forthcoming period of operation (the more so because the decision on their efficiency is made for these particular units).

Accurate determination of the equipment reliability indicators can be made based on the analysis of reliability change for certain units only. This approach to reliability indicators determination was first offered in 80s of the last century when operating conditions of thermal power plants of the Irkutsk Electric Energy System changed. Due water shortage in the reservoirs (the forecast on water inflow into the HPP reservoirs was erroneous) the equipment of thermal power plants was operated at full capacity over a year, scheduled maintenances were not carried out, and accident rate at EPS increased. Adequacy of statistical data used for determination of reliability indicators could not be ensured. For forecasting the reliability indicators it was offered to use the methods of exponential smoothing or a multi-factor model, i.e. availability of data on factors affecting the equipment reliability. Assessed was the validity of relations between equipment reliability in the year under consideration and scope of scheduled maintenances performed in the current year and in the preceding years. Calculations have shown that the scope of scheduled maintenances performed in the current year and in the preceding years has a considerable impact on the accidents rate of generating equipment in the year under consideration. In 90s the equipment reliability was offered to be assessed on the base of the analysis of statistical data represented as time series (7-9).

Identified was the availability of annual cycles of emergencies in EPS depend on the degree of the equipment wear and on a number of other factors. Particularly, existence of multi-year emergency cycles was proven experimentally in electric networks, that are caused by heavy wind and ice loads followed by large-scale failures and damages of supports, and breaks of wires on the transmission lines.

SUMMARY

1. A technique for assessing the efficiency of the main EPS equipment with account of its economic characteristics, technological advance, and reliability has been proposed.

2. For assessing the efficiency of the main EPS equipment, an efficiency factor is proposed that equals the ratio between total costs for the considered time period and amount of useful power supplied to consumers.

3. The amount of useful power supplied to consumers in the forthcoming period is determined as a product of the input energy (the amount of energy the object can receive in the considered time period), performance factor, and availability factor.

4. Ability to check (or validate) the selected efficiency indicator can be ensured by availability of data on the useful power supplied to consumers over the considered time period, and actual total costs for the same period.

5. A technique for assessing the efficiency of the main EPS equipment can be used for evaluating the efficiency and for comparing new equipment with equipment that has been in operation over a long time period. Results of comparison may be used for justifying the equipment substitution by the new one.

6. Equipment aging usually results in higher EPS emergency rate. Equipment reliability for the forthcoming period of operation should be evaluated based on the analysis of its reliability change using appropriate statistical data.

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NUMBER OF CALLS IN A CYCLIC-WAITING SYSTEM

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Abstract

A single-server cyclic-waiting system, i.e. a queuing system, in which a call gets service either at the time of arrival or after some cycle time T , has been studied in the article. The distribution functions of the intervals between arrivals and service times have exponential form; the service discipline is FCFS. Analytical results for the mean number of calls in the system are indicated. Numerical and graphical results have been given.

Keywords: cyclic-waiting system, queuing system, FCFS-service.

1 INTRODUCTION

Queuing systems (Gnedenko 1989) have several applications in different technical and biological systems, especially in telecommunication (Pustova 2010) and network systems. In order to adequately describe a real queuing system, one should consider different aspects of the real system, such as number of servers, availability of buffer and its type, order and duration of service, etc. Retrial queuing systems (Artalejo & Gomez-Corral 2008, Kovalenko & Koba 2010) are one of important tools in research and optimization of characteristics of different systems. Such systems take into account not only the initial flow of calls, but also the repeated ones. Retrial queuing systems have the following behavior. If all servers are busy upon call's arrival the call doesn't get lost, but goes to the virtual queue (an orbit), and after some time (a cycle) repeats its attempt to get service. Ignoring the retrial effect can cause significant accuracy errors in research (Aguir et al. 2004, Aguir et. al. 2008).

Retrial queuing systems may also take into account the service discipline: random service, FCFS (first come first served), LCFS (last come first served), etc. The analysis of such disciplines might increase the complexity of research, so it becomes hard to obtain analytical results.

We consider a so-called cyclic-waiting system (Lakatos 2010) having a special feature comparing to the common retrial systems, namely, calls get service in the order of arrival: if there is any call in the orbit, it will be served earlier than the arriving new ones. The calls from the orbit are served after some time, multiple to a given cycle time T .

The process of airplane landing initiated research of cyclic-waiting systems (Lakatos 1994 & 1998). An airplane lands on a given route. If a runway is available the airplane starts the landing procedure immediately. If there is a threat to the non-compliance of the separation standards or

there already are some planes in the waiting area (in the circle), waiting for permission to land, the aircraft is sent to the queue (to the circle). While on circle, the aircraft waits for permission to land. After the permission is given, the airplane has to get to the corresponding geometric point on the circle route, and start the landing procedure from there. The time period from the moment of landing clearance to the moment until the plane reaches the corresponding geometric position is called "idle": the system is ready to take a plane, but the landing is possible only on reaching the corresponding point on the circle. Thereby, the airplane landing process can be described as a cyclic-waiting system.

Kovalenko (2002) considered the multichannel retrial queuing system with a constant cycle orbit, and gave estimation of the loss probability in light traffic. Rogiest et al. (2006) applied cyclic-waiting system to the research of optical buffers. Lakatos & Zbaganu (2007) investigated different generalizations of the initial model. Karasz (2008) studied some cyclic-models with two different types of calls. Kovalenko (2015) started to study a new class of multiple-cyclic waiting systems, namely, considered a two-cyclic queuing system.

2 ANALYTICAL MODELING

2.1 Problem statement

We consider a single server retrial queuing system with exponentially distributed intervals between arrivals and service times, with corresponding parameters λ and μ (Fig. 1). If the server is free, the arrived call gets service immediately. Otherwise, if the server is occupied, an arrived call goes to the orbit (i.e. a virtual queue), in which it retries its requests with a determined cycle time $T > 0$. The service is defined by FCFS (first come first served) discipline. It means that if there's any call cycling in the orbit, the call from the initial flow doesn't get service immediately, but goes to the orbit instead and after some time, the multiple of T , repeats its attempt to get service.

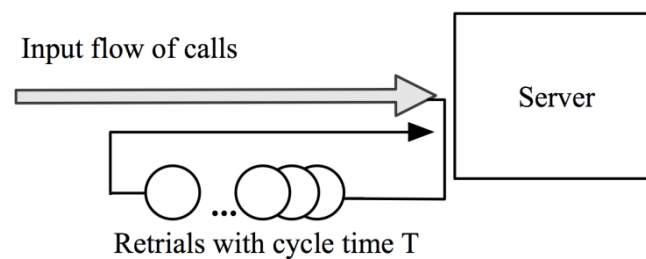


Figure 1. A cyclic-waiting system structure.

2.2 Ergodicity conditions

We will use results, obtained in (Lakatos 1994 & 2010) to find some characteristics of the system.

Let $t_n, n \geq 0$ denote the moment of the service beginning of the n^{th} call. Let's introduce an embedded Markov chain, the states of which are defined as the number of calls in the system at moments $(t_n - 0)$. The matrix of the chain's transition probabilities is defined as

$$\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \cdots \\ a_0 & a_1 & a_2 & a_3 & \cdots \\ 0 & b_0 & b_1 & b_2 & \cdots \\ 0 & 0 & b_0 & b_1 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \ddots \end{bmatrix}, \quad (1)$$

the elements of which are defined by generating functions

$$A(z) = \sum_{i=0}^{\infty} a_i z^i = \frac{\mu}{\lambda + \mu} + \frac{\lambda z}{\lambda + \mu} \frac{(1 - e^{-\mu T}) e^{-\lambda(1-z)T}}{1 - e^{-[\lambda(1-z) + \mu]T}}, \quad (2)$$

(where

$$\frac{\mu}{\lambda + \mu} = \int_0^{\infty} e^{-\lambda x} \mu e^{-\mu x} dx$$

is the probability of calls' non-arrival, and multiplier z in (2) represents the mandatory occurrence of a call), and

$$B(z) = \sum_{i=0}^{\infty} b_i z^i = \frac{1}{(1 - e^{-\lambda T}) [1 - e^{-[\lambda(1-z) + \mu]T}]} \cdot \left\{ \frac{1}{2-z} \left(1 - e^{-\lambda(2-z)T} \right) \left(1 - e^{-[\lambda(1-z) + \mu]T} \right) - \frac{\lambda}{\lambda(2-z) + \mu} \left(1 - e^{-[\lambda(2-z) + \mu]T} \right) \left(1 - e^{-\lambda(1-z)T} \right) \right\}. \quad (3)$$

Let $p_i, i = 0, 1, \dots$ be the ergodic probabilities of the chain. Hence, the generating function for the number of calls in the system is

$$P(z) = \sum_{i=0}^{\infty} p_i z^i = p_0 \frac{(\lambda z + \mu) B(z) - (\lambda + \mu) z A(z)}{\mu [B(z) - z]}, \quad (4)$$

where $A(z)$ and $B(z)$ are defined by (3) and (4) respectively, and p_0 is defined by

$$p_0 = 1 - \frac{\lambda}{\lambda + \mu} \frac{1 - e^{-(\lambda + \mu)T}}{e^{-\lambda T} (1 - e^{-\mu T})}. \quad (5)$$

Then the ergodicity condition is

$$\frac{\lambda}{\mu} \frac{1 - e^{-\lambda T}}{e^{-\lambda T} (1 - e^{-\mu T})} < 1. \quad (6)$$

2.3 Mean number of calls in the system

Let us determine mean number of calls in the system, in other words $P'(1)$. Let us denote

$$g = \frac{p_0}{\mu},$$

$$N(z) = B(z) - z,$$

$$S(z) = \lambda z B(z) + \mu B(z) - \lambda z A(z) - \mu z A(z),$$

then (4) can be written as

$$P(z) = g \frac{S(z)}{N(z)}.$$

If $z = 1$ then

$$N(1) = 1 - 1 = 0,$$

$$S(1) = \lambda + \mu - \lambda - \mu = 0.$$

Let us obtain $P'(z)$:

$$P'(z) = \left(\frac{S(z)}{N(z)} \right)' = \frac{S'(z)N(z) - S(z)N'(z)}{N^2(z)}. \quad (7)$$

If $z = 1$ in (7) the indeterminate form of $0/0$ occurs.

By applying L'Hôpital's rule to (7) we get

$$\begin{aligned} \frac{(S'(z)N(z) - S(z)N'(z))'}{(N^2(z))'} &= \frac{S''(z)N(z) + S'(z)N'(z) - S'(z)N'(z) - S(z)N''(z)}{2N(z)N'(z)} = \\ &= \frac{S''(z)N(z) - S(z)N''(z)}{2N(z)N'(z)}. \end{aligned} \quad (8)$$

Similarly, if $z = 1$ in (8) the indeterminate form of $0 / 0$ occurs. Analogously, for (8) we obtain

$$\frac{(S''(z)N(z) - S(z)N''(z))'}{(2N(z)N'(z))'} = \frac{S'''(z)N(z) + S''(z)N'(z) - S'(z)N''(z) - S(z)N'''(z)}{2[(N'(z)) + N(z)N''(z)]}. \quad (9)$$

If $z = 1$ (9) is transformed to

$$P'(1) = \frac{S''(1)N'(1) - S'(1)N''(1)}{2[N'(1)]^2}. \quad (10)$$

Let us find the unknown derivatives in (10):

$$N'(z) = B'(z) - 1, \quad (11)$$

$$N''(z) = B''(z), \quad (12)$$

$$S'(z) = \lambda B(z) + \lambda z B'(z) + \mu B'(z) - \lambda A(z) - \lambda z A'(z) - \mu A(z) - \mu z A'(z), \quad (13)$$

$$S''(z) = \lambda B'(z) + \lambda B'(z) + \lambda z B''(z) + \mu B''(z) - \lambda A'(z) - \lambda A'(z) - \lambda z A''(z) - \mu A'(z) - \mu A'(z) - \mu z A''(z). \quad (14)$$

If $z = 1$ the (11) – (14) are transformed to

$$N'(1) = B'(1) - 1, \quad (15)$$

$$N''(1) = B''(1), \quad (16)$$

$$S'(1) = B'(1)(\lambda + \mu) - A'(1)(\lambda + \mu) - \mu, \quad (17)$$

$$S''(1) = 2\lambda B'(1) + \lambda B''(1) + \mu B''(1) - 2\lambda A'(1) - \lambda A''(1) - 2\mu A''(1) = \\ = 2\lambda B'(1) + B''(1)(\lambda + \mu) - 2(\lambda + \mu)A'(1) - (\lambda + \mu)A''(1). \quad (18)$$

After some computations the derivatives for $A(z)$ and $B(z)$ that are part of the (15) – (18) can be written as

$$A'(1) = \frac{\lambda(1 - e^{-\mu T} + \lambda T)}{(\lambda + \mu)(1 - e^{-\mu T})}, \quad (19)$$

$$A''(1) = \frac{\lambda^2 T(2 - 2e^{-\mu T} + \lambda T + \lambda T e^{-\mu T})}{(\lambda + \mu)(1 - e^{-\mu T})^2}, \quad (20)$$

$$B'(1) = \frac{1}{(\lambda + \mu)(1 - e^{-\lambda T})(1 - e^{-\mu T})} \left(\lambda + \mu + \lambda^2 T + \lambda \mu T e^{-(\lambda + \mu)T} - (\lambda + \mu)(e^{-\lambda T} + e^{-\mu T}) + \right. \\ \left. + (\lambda + \mu)e^{-(\lambda + \mu)T} - (\lambda + \mu)\lambda T e^{-\lambda T} \right), \quad (21)$$

$$B''(1) = \frac{2\lambda^2 T^2 e^{-2\mu T}}{(1 - e^{-\mu T})^2} + \frac{2\lambda T e^{-\mu T}}{(1 - e^{-\lambda T})(1 - e^{-\mu T})^2} \left((1 - e^{-\lambda T})(1 - e^{-\mu T}) - \right. \\ \left. - \lambda T e^{-\lambda T}(1 - e^{-\mu T}) - \lambda T e^{-\mu T}(1 - e^{-\lambda T}) + \frac{\lambda^2 T(1 - e^{-(\lambda + \mu)T})}{\lambda + \mu} \right) + \\ + \frac{\lambda^2 T^2 e^{-\mu T}}{1 - e^{-\mu T}} + \frac{1}{(1 - e^{-\lambda T})(1 - e^{-\mu T})} \left(2(1 - e^{-\lambda T})(1 - e^{-\mu T}) - \right. \\ \left. - 2\lambda T e^{-\lambda T}(1 - e^{-\mu T}) - 2\lambda T e^{-\mu T}(1 - e^{-\lambda T}) - \lambda^2 T^2 e^{\lambda T}(1 - e^{-\mu T}) + \right. \\ \left. + 2\lambda^2 T^2 e^{-(\lambda + \mu)T} - \lambda^2 T^2 e^{-\mu T}(1 - e^{-\lambda T}) + \frac{2\lambda^3 T(1 - e^{-(\lambda + \mu)T})}{(\lambda + \mu)^2} - \right. \\ \left. - \frac{2\lambda^3 T^2 e^{-(\lambda + \mu)T}}{\lambda + \mu} + \frac{\lambda^3 T^2(1 - e^{-(\lambda + \mu)T})}{\lambda + \mu} \right). \quad (22)$$

Therefore, formulas (11) – (22) define expression (10) entirely. Due to cumbersome form, the final expression of $P'(z)$ (as well as $P'(1)$) isn't given here, though it can be easily obtained with the help of formulas (10), (11) – (22).

2.3 Analysis of the modelling results

Let us denote the left side of (6) as H (we will use this notation for graphical results):

$$H = \frac{\lambda}{\mu} \frac{1 - e^{-\lambda T}}{e^{-\lambda T} (1 - e^{-\mu T})}$$

According to formula (6) we have obtained graphical representation for ergodicity conditions of the system under consideration (see Figs 2-3): everything below the line $H = 1$ qualifies for ergodicity condition (6).

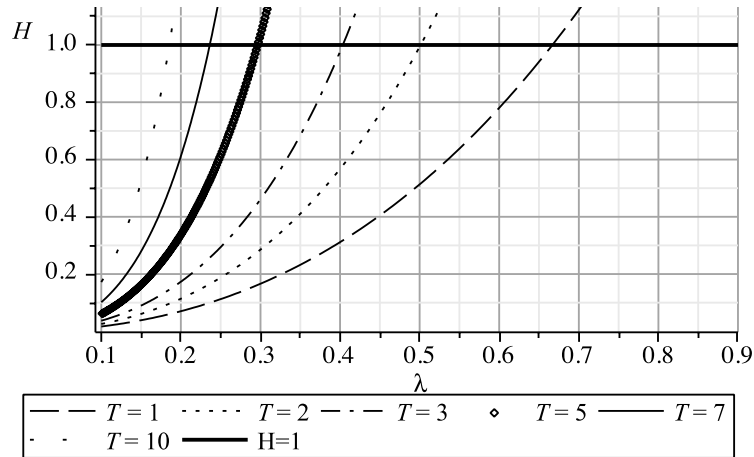


Figure 2. Graphical representation for ergodicity conditions, where $T = \{1, 2, 3, 5, 7, 10\}$, $\lambda = [0.1; 0.9]$, $\mu = 1.0$.

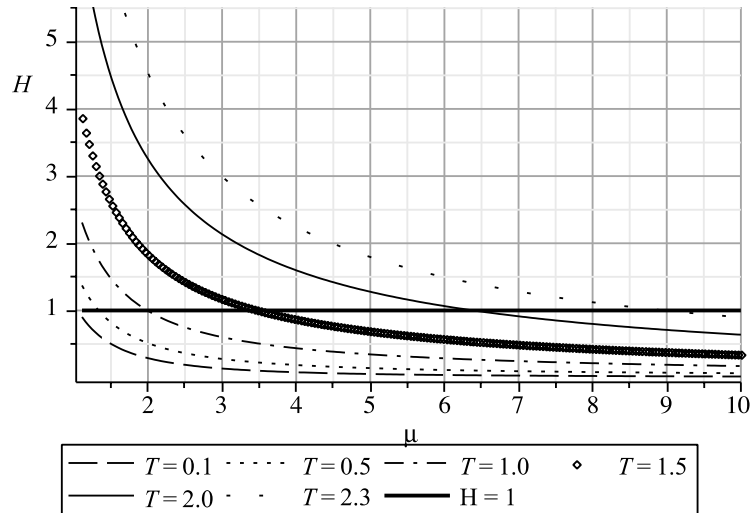


Figure 3. Graphical representation for ergodicity conditions, where $T = \{0.1, 0.5, 1.0, 1.5, 2.0, 2.3\}$, $\lambda = 1.0$, $\mu = [1.11; 10.0]$.

The values of λ and μ for the results on Figures 2-3 have been chosen so that the ratio $\lambda / \mu = [0.1; 0.9]$. From Figure 2 one may see that with the increase of the value of cycle time T the left part of the (6) increases sharply. The same situation persists on Figure 3.

Using results from Figures 2-3, we have also obtained a graphical dependence for the mean number of calls $P'(1)$ from the mean service time $1/\mu$ and cycle time T (see Fig. 4), as well as from the intensity of arrivals λ and cycle time T (see Fig. 5).

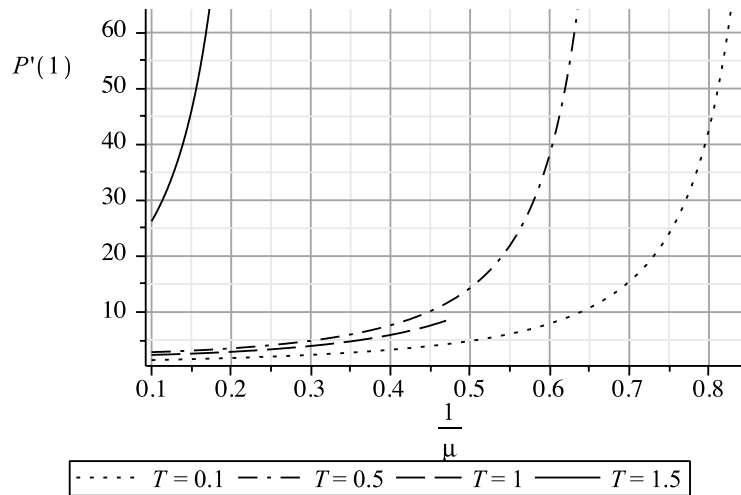


Figure 4. Graph for the mean number of calls ($\lambda = 1.0$).

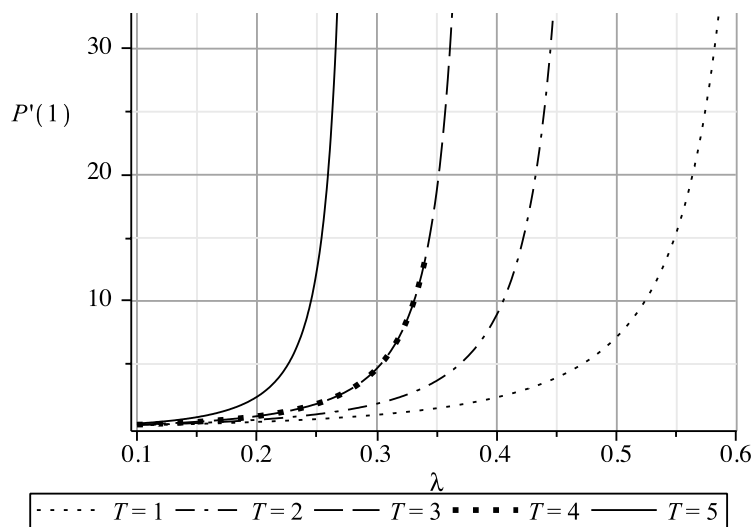


Figure 5. Graph for the mean number of calls ($\mu = 1.0$).

From Figures 4-5 one may see that the mean number of calls rises with the increasing service time as well as with the increasing intensity of arrivals. On both graphs (see Figs 4-5) $P'(1)$ increases with the increasing cycle time T .

3 CONCLUSIONS

In the article we have obtained analytical and graphical results for the mean number of calls in the single-channel cyclic-waiting system. Ergodicity conditions have been shown. The analytical result for $P'(1)$ can be used, for example, to obtain the average time a call spends in a system: from Little's law $W = P'(1) / \lambda$.

The cyclic-waiting system under consideration may have different practical applications (Koba & Pustova 2012, Pustova 2016), such as an airplane landing process, a simple call center, a telephone answering machine, an optical fiber buffer (Rogiest et al. 2006, Langenhorst et al. 1996), an optical delay line, a microring resonator (Bogaerts et al. 2012), a compact optical buffer with ring resonators (IBM, 2006). All these systems have a common feature: a new call, which can't get service until there is at least another one in the system, goes to the orbit and cycles there.

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