

Mathematical Model for Calculating Reliability Characteristics NPP Equipment Under Honhomogeneous Flows Failure

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Abstract

Describes the different mathematical models of nonhomogeneous in time event streams. A review of the literature on the subject of the study. The basic premise models of nonhomogeneous Poisson processes, gamma processes, geometric renewal process, the trend renewal process, the processes Kijima-Sumita. Defines the main features of the model normalizing of the flow function to calculate the required parameters of reliability. A special case of this model is an nonhomogeneous Poisson process. This model will form the basis of calculation methods of NPP equipment reliability indicators change over time and the conditions of their condition. The paper describes a method for estimating the parameters of NPP equipment reliability, which allows to take into account heterogeneity failure flow. It noted the specificity of the incoming statistical data on failures. Noted the specificity of the incoming statistical data on failures. The application of the model normalizing the flow function to calculate the required parameters of reliability. An example of a practical analysis of the failures of some elements of the reactor protection management system (PMS) NPP Bilibino.

Keywords: Failure flow, nonhomogeneous process, normalizing flow function, renewal function, intensity function

I. Introduction

The technical equipment during their deliberations goes through three stages. At each step the intensity of the flow of failures have a certain tendency. For example during normal operation the failure intensity value is approximately constant. In this case it is assumed homogeneity in time equipment operation process. Reliability indexes are calculated by classical methods. At the stage of running the failure intensity decreases with time on the stage of aging increases (there may be more complex regularities). Consequently, at the stages of running and aging operating time between two consecutive failures taken place are not equally distributed random variables. The flow of failures can not be considered recurrence [1-3]. In view of this the use of traditional methods of calculation of reliability characteristics at these stages incorrectly. In the calculations of reliability characteristics necessary to take into account the inhomogeneity of of the failure flow in time.

Consider the literary sources, the authors of which relate to the problem of the failure flow heterogeneity. Separately, we note [2,5-7]. Tutorial [2] is a fairly complete exposition of the current state of the mathematical theory of reliability. The most important issues addressed in the handbook should include the study of various accounting models of aging, degradation, accelerated testing models, etc. The monograph [5] devoted to the review and study of the theory of point processes. In [6] describes some of the models of nonhomogeneous processes of aging accounting models, etc. Manual [7], partly subsumed in [2], is entirely devoted to models of inhomogeneous renewal processes, such as nonhomogeneous Poisson processes, gamma processes, the trend renewal processes, geometric processes, processes and models Kijima, normalizing flow function (NFF).

For the first time the emergence of heterogeneous processes should include nonhomogeneous Poisson process- NHPP-process. Poisson process is called nonhomogeneous if the function of the intensity $\lambda(t)$ of the depends on the time. The intensity may be either a deterministic or random. The most detailed and complete property of this kind of processes were investigated in [2,5,7]. It should be noted that the failure rate is defined as the intensity ratio of the average number of failures – $N[t,t+\Delta t]$ that occurred in the interval $[t,t+\Delta t]$ to the length of the interval Δt:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{E\left(N[t,t+\Delta t]\right)}{\Delta t}.$$

The renewal process is necessary more detailed description, which is carried out such concepts as stochastic intensity and conditional intensity function (CIF), see. [2,5,7].

The next type of nonhomogeneous renewal processes are gamma processes, first appeared in the article [8]. Inhomogeneity of the gamma process (IGP) is called the multiplicity k of the process, formed by the flow of failures points $\{\tau_{kn}\}$, ie points τ_k, τ_{2k}, τ_{3k},... . The mathematical model of the process, which is, in fact, a generalization NHPP-process can be interpreted as follows. Let us consider NHPP-process with intensity function $\lambda(t)$. Assume that there is every k-th event process. At this point the flow is meant as impacts, i.e. failure occurs only on the occurrence of each k-th shock. If, for example, $k=4$ then every fourth load will cause failure. Thus, the IGP-process actually is sparse NHPP-process, each of which is k-th point. In the particular case, $k=1$ the model is reduced to the usual NHPP-processes.

In [9-11] describes a model object changes efficiency, presented equation to calculate availability. Process equipment operation is described by the progressive degradation. The heterogeneity of the flow of events is accounted for as a change in the distribution function of operating time between failures, and recovery time of the distribution function. The study model is a geometric process. The properties of this type of processes described in the books mentioned above [2,6,7]. It is worth noting that the model geometry process is fairly new and many of its properties has not been studied thoroughly. The most interesting seems the problem of determining the asymptotic properties of the failure intensity of this type of processes. In [12, 13] are devoted to the point and interval estimation coefficient degradation of geometrical processes, the study of well-known and little-known features of these processes.

The following model of non-homogeneous recovery processes, in fact, is a "bridge" between the (fully) renewal (like new) and not fully renewal (as before failure) systems. She first appeared in [14].

GRP-inhomogeneous flows pattern is a flow formed by time to failure Δ_n with the following conditional distribution functions:

$$F_{\Delta_n}\left(x \mid V_{n-1}=y\right) = \frac{F(x+y)-F(y)}{1-F(y)};$$

$$V_n = V_{n-1} + q\Delta_n = q\sum_{1}^{n}\Delta_i,\ V_0 = 0 \text{ – Kijima model GRP-1;}$$

$$V_n = q\left(V_{n-1}+\Delta_n\right) = \sum_{1}^{n} q^{n+1-i}\Delta_i,\ V_0 = 0 \text{ – Kijima model GRP-1;}$$

Note that V_n - virtual age of the system, see [6,14,15]. Kioima models allow to take into account the incomplete recovery of the failed element. In fact, this is a very important feature of the model. Until now it was assumed that after a failed repair of the technical system is returned to its original condition - like new. However, even after the overhaul, a number of replacements of old items with new items, the system as a whole can hardly be considered completely new. This is just an idealized assumption allows to simplify the mathematical calculations.

Trend renewal process -TRP is fairly new and is closest to the model of normalizing the function. She first appeared in [16]. We define this type of processes. Let $\lambda(t)$ – non-negative random function defined for $t \in [0, \infty)$, and $\Lambda(t) = \int_0^t \lambda(u)du$. Process $\tau_1, \tau_2, ...$ is a trend renewal process, if $\Lambda(\tau_1)$, $\Lambda(\tau_2)$,... is an ordinary recovery process. The cumulative distribution function of time to failure $F(x)$ and his expectation 1. The function $\lambda(t)$ is called the trend of the process. It should be noted that, generally speaking, $\Lambda(t)$ not a renewal function. Description taken from the source. Later, the requirement on the expectation of the authors of the model refused, saying that such a requirement. Only necessary for the model to be unique.

Let us consider sources of which the authors used in the research model of NFF. The method of accounting of heterogeneity using NFF model is described in [17-21], in which the properties of the resulting processes are studied in sequence. These works are entered asymptotic characteristics similar meaning to the coefficient of readiness, the first results concerning the study of the behavior laws of distribution of the *i*-th time to failure. In [17-19] presented an equation for determining the function of an arbitrary allocation of time to failure under the conditions of the event flow heterogeneity, knowing that you can evaluate, for example, the remaining service life. In [18] obtained the distribution function of the second and subsequent developments up to the power failure pattern NFF. In [19] derived the equations to calculate the average direct and inverse average remaining time based on the event flow heterogeneity. In [20, 21] the model of joint event flow for the calculation of the coefficient of readiness in terms of the event flow heterogeneity, the idea that some overlap with the two-dimensional renewal process [22]. In [20] also presented equation to calculate the resource characteristics and an example of their calculation. In [23] proposed a method for treating inhomogeneous flow of statistical data on failures. The authors present this kind of feedback NFF, which would lead to a heterogeneous flow of failures simple flow. They find an expression for the distribution function of any developments. That is, in fact, the authors used a model of an nonhomogeneous Poisson flow, which is a special case of the model NFF.

The purpose present work is to describe and research methodology for assessing NPP equipment reliability indicators to take into account the possible inhomogeneity of the flow of failures and demonstration of the results of applying methods on real data obtained from operating experience.

II. Initial data

The main sources of information on the operation of NPP facilities are "defects journal" a passport and technical descriptions of the equipment, certificate of technical condition of objects and a number of other documents.

The existing NPP procedure for collecting statistical information on faults reveals the date of failure detection object from the set of the same elements, and the reason why the failure occurred. In this often is not possible to identify a failed object. Suppose that the statistical information delivered for analysis, presented as follows (see table 1):

m- the number of elements in aggregate similar objects;

ν_i - the number of failures in the *i*-th observation interval.

After another failure, repair is of this piece of equipment. The recovery time of the object is assumed to be negligible compared to the time to failure. Broken objects are recovered and returned to the system for later use. Thus, we have grouped expression failure flow. Also, we assume that the failure rates are not equally distributed (generally speaking) and there is a certain pattern in the changing of the law of distribution with changes in the observation interval (index) *i*. For example, we will process the statistics on failures of two elements PMS- compensates neutron camera

(CNC56) and protection amplifier of the speed (PAS). The failure rate of these elements are shown in Figure 1.

Table 1: *Example of presentation of statistical information about failures*

Year of exploitation	1974	1975	1976	…	2015
Failure rate ν_i	5	3	4	…	0
m	25				

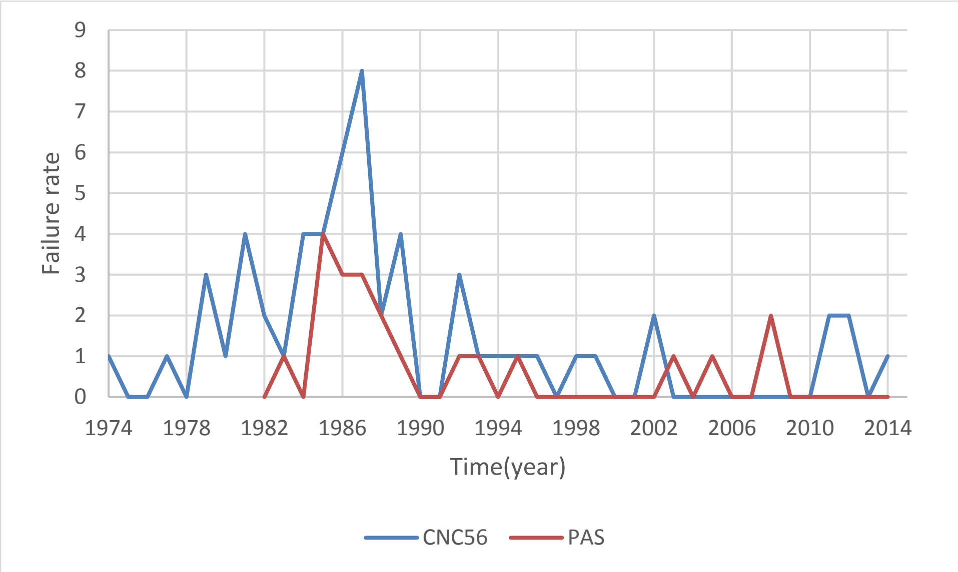

Figure 1: *Failure rate of CNC56 and PAS*

Analysis of the data presented in the form of a group the failure rate is a a nontrivial problem, as the classical methods of calculation of reliability indicators require input data in the form of a known operating time between failures. Methods for calculating the reliability of indicators on statistical information about the grouped failure renewal items worked enough. Classic algorithms outlined in [3, 27, 28], in the presence of the grouped data allow to obtain histogram estimation failure flow parameter. However, the definition of recovery by the equation (see e.g. [1]) on the histogram evaluation of the failure flow parameter distribution density may result in some intervals of negative density values. This is contrary to the basic property of this characteristic. Consequently, this method of calculation should be recognized as incorrect [24-26].

We propose two possible approaches to overcome this problem. The first is based on the assumption that the flow of failures is the simplest. In this case, the failure rate is estimated parameter constant, and the distribution is exponential with a failure rate that is calculated based on statistical information. The second approach is more flexible and is based on the histogram smoothing estimation failure flow parameter, the method of nuclear non-parametric estimates of the flow of the [24-26].

And in fact, and in another case method is based on the assumption of homogeneity of the flow of failures. However, the statistical study of the flow of failures nature suggests that, for example, for Bilibino NPP some of the investigated elements PMS a heterogeneous forms of failure flow of time. In particular, this applies to the CNC56 and PAS.

Appropriate statistical criteria for determining of the hypothesis of a uniform stream of test failures are presented in [4, 20] and we shall not be considered. Nevertheless, by analyzing Fig. 1 can be seen in the failure rate heterogeneity. Over time, the frequency of the a homogeneous flow of failures should stabilize. However, in our case there is a surge of failures in 90 years and the relatively low incidence in the future.

After deciding that the PMS investigated element forms the assumption is made a heterogeneous flow of failures that flow obeys a model normalizing the flow function, the essence of which is set out below.

III. NFF-model

Let us consider mathematical model [2, 7, 17-21], consider the possibility of "distortion" flow of events and allows to determine the parameters of reliability elements, provided that the probability characteristics of the process of change over time. In this model, the actual a heterogeneous flow of failures is the display of a homogeneous flow of events using the monotonic transformation $\Psi(x)$, called normalizing the flow function (NFF) or the function of the inhomogeneity.

The heterogeneity of the flow of events given by $\Psi(x)$ function, the role of which is as follows. Applying this function to hypothetically "abstract" homogeneous flow of failures, we should be getting close to the "real" flow. Using the inverse transformation of "real" flow is roughly homogeneous flow of events. In the "real" flow may be present condensations place (thinning) - when at a certain time interval the number of events will be substantially greater (or less) the number of events in neighboring, similar in duration intervals.

Fig. 2 shows a homogeneous flow of events in the transformation using arbitrary flow function $\Psi(t^*)$. Events homogeneous and inhomogeneous flow of events displayed on the X-axis and Y-axis respectively. The Y-axis of Fig. 2 shows the actual flow of failures with the stages of running and aging when the relatively high failure rate. When the non-linear mapping $\Psi(\cdot)$ of the change will occur over time from cycle to cycle distribution law time between failure. By cycle is meant the work of an element of the system from the beginning of its operation (installation or after repair) to failure, after every repair and installation of the system begins a new cycle of the item. The duration of each cycle of operation is exactly equal to the corresponding time between failures. If a uniform flow of failures in time, his law remains unchanged with the passage of time (from cycle to cycle).

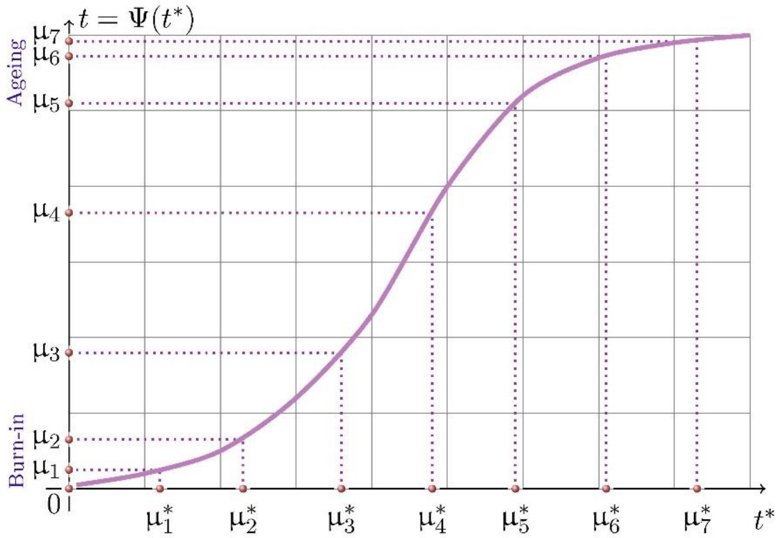

Figure 2: *NFF model*

If the flow is not homogeneous, the law of distribution of operating time will vary depending on the operation cycle. We now turn to the formal description of the essence of the model normalizing the flow function.

As mentioned above, the basic idea is to build a model of NFF continuous strictly monotone increasing mapping abstract recurrent flow of events in the real flow of events [2, 7]. This abstract flow, obviously, will have a dimension of function $\Psi^{-1}(t)$, where t - time. Suppose that system restore is instantaneous.

Definition. Let $\xi_1, \xi_2, \ldots$ *– independent identically distributed (i.i.d) random variables. They are essentially non-negative operating time between failures abstract homogeneous flow with cumulative distribution function (CDF)* $F(x)$. μ_n^* *– time of the n-th event of such a flow, i.e.*

$$\mu_n^* = \sum_{i=1}^{n} \xi_i . \qquad (1)$$

Let $\Psi(t)$ *– be a continuously differentiable strictly increasing function on* $[0;\infty)$*, and* $\Psi(0)=0$.

Then the sequence $\mu_1, \mu_2, \ldots$ *defined by the formula*

$$\mu_n = \Psi\left(\mu_n^*\right); \quad n = 1, 2, \ldots; \quad \mu_0 = 0, \qquad (2)$$

is the renewal process NFF-model: $\mathrm{NFF}\left(F(x), \Psi(\cdot)\right)$.

Obviously, *i-th* operating time between failures will be determined as follows:

$$\zeta_i = \mu_i - \mu_{i-1} = \Psi\left(\mu_i^*\right) - \Psi\left(\mu_{i-1}^*\right). \qquad (3)$$

The value of ζ_i shall call duration *i* cycle of the system. Similarly, the classical theory of renewal, the sequence $\zeta_1, \zeta_2, \ldots$ can also be defined as renewal process.

For this model, renewal function is given by

$$\tilde{\Omega}(t) = \Omega\left(\Psi^{-1}(t)\right), \qquad (4)$$

where $\Omega(t) = F_\xi(t) + \int_0^t \Omega(t-\tau) f_\xi(\tau) d\tau$ - renewal function abstract failure flow (see [2,7]).

It is also proved the asymptotic behavior of the renewal function:

$$\lim_{t\to\infty} \frac{\tilde{\Omega}(t)}{\Psi^{-1}(t)} = \frac{1}{\mathrm{E}\xi}, \quad \tilde{\Omega}(t) \sim \frac{\Psi^{-1}(t)}{\mathrm{E}\xi}, \qquad (5)$$

where ξ - abstract operating time between failures, $\mathrm{E}\xi$ – its expectation.

The intensity failures NFF model will be equal to:

$$\tilde{\omega}(t) = \left[\Psi^{-1}(t)\right]' \omega\left(\Psi^{-1}(t)\right), \qquad (6)$$

where $\omega(t) = f_\xi(t) + \int_0^t \omega(t-\tau) f_\xi(\tau) d\tau$ - intensity failures of abstract flow.

Asymptotically [2,7]:

$$\lim_{t\to\infty} \frac{\tilde{\omega}(t)}{\left[\Psi^{-1}(t)\right]'} = \frac{1}{\mathrm{E}\xi}, \quad \tilde{\omega}(t) \sim \frac{\left[\Psi^{-1}(t)\right]'}{\mathrm{E}\xi}. \qquad (7)$$

A nonhomogeneous Poisson process (NHPP) is a special case of an inhomogeneous flow of events model [2,7]. If we assume that the real flow of failures is described by the model of an nonhomogeneous Poisson process in time - NHPP, then the flow will be abstract usual homogeneous Poisson process (elementary stream) with an intensity of 1. The intensity of the process will be equal

NHPP

$$\lambda(t)=\left[\Psi^{-1}(t)\right]'. \tag{8}$$

Symbolically, this can be written as:

$$\mathrm{NHPP}\left(\lambda(t)\right)=\mathrm{NFF}\left(1-e^{-x},\Psi(t)\right), \tag{9}$$

where $\lambda(t)$ defined by the expression (8).

Within the framework of these two models and the subsequent statistical analysis of baseline information will be made. More specifically, in both cases NFF- model will be applied to the parameter evaluation of function heterogeneity $\Psi(t)$.

In one model, will attend nonparametric density estimation abstract time between failures $f_\xi(x)$, and in the other it will be assumed that $f_\xi(x)=e^{-x}$. In the latter case, we make the assumption that there NHPP process. In view of the foregoing, the first NFF model can be called semi-parametric and parametric second. Build a fully parametric model hard. Firstly, the nonhomogeneity function must satisfy the necessary conditions. Secondly, they must be sufficiently simple calculations not only the function but also the inverse to it and its derivatives.

In terms of mathematical statistics further parameterization of the task is carried out by using an NHPP model. Usually the decision of the parametric task somewhat simpler than nonparametric, while the result has the necessary smoothness. Moreover, the accuracy of the results may be somewhat higher than the results obtained in nonparametric formulation. However, if the prerequisites of parametric models prove to be incorrect, the relative efficiency of the estimates in comparison with the non-parametric counterparts would be extremely low.

Let us consider methods of assessment of heterogeneity functions.

IV. Estimation of heterogeneity function

From (5) it follows that the function heterogeneity $\Psi(t)$ asymptotically uniquely determined from renewal function $\tilde{\Omega}(t)$. Character of interrelation functions next $\tilde{\Omega}(t) \approx \dfrac{m}{\mathrm{E}\xi}\Psi^{-1}(t)$, where m - number of similar objects under observation.

Thus, the problem consists in the qualitative selection (evaluation) functions $\Psi^{-1}(t)$ on the basis of the renewal function. This is a classical problem of mathematical statistics, which can be solved by the method of least squares.

Consider the example of evaluation the reverse of function of the inhomogeneity- $\Psi^{-1}(t)$. As an example, consider the statistics obtained from the experience of operating elements CNC56 and PAS working as a part of standard equipment power EGP-6 NPP Bilibino. Statistical information on faults, refer to Tables 2 and 3. In this case, for the CNC56 the number of flows $m=16$, for PAS- $m=12$.

Table 2: Statistical information on failures elements CNC-56. *m*=16.

Year	№	ν_i	Year	№	ν_i	Year	№	ν_i
1974	0	1	1988	14	1	2002	28	2
1975	1	0	1989	15	2	2003	29	0
1976	2	1	1990	16	0	2004	30	0
1977	3	1	1991	17	3	2005	31	0
1978	4	8	1992	18	3	2006	32	0
1979	5	7	1993	19	1	2007	33	0
1980	6	0	1994	20	1	2008	34	0
1981	7	5	1995	21	2	2009	35	0
1982	8	2	1996	22	2	2010	36	0
1983	9	3	1997	23	0	2011	37	2
1984	10	4	1998	24	1	2012	38	2
1985	11	4	1999	25	1	2013	39	0
1986	12	9	2000	26	0	2014	40	1
1987	13	11	2001	27	0			

We construct a nonparametric estimation averaged renewal function the usual method based on the determination of the ratio of the accumulated frequency of failures $\Omega(t)$ to a given point in time t to the number of observed elements *m*.

$$\tilde{\Omega}(t) = \Omega(t)/m .$$

Visual analysis of nonparametric estimation of the renewal function (points in Fig. 5) shows that superficially reminds function of the probability distribution function. The richest in terms of modeling of various forms of random variable distribution of parametric laws is the Weibull-Gnedenko. The range of renewal function will not be a segment $[0;1]$ and will be semi-infinite straight $[0;\infty)$. It is expedient to multiply the distribution function of the Weibull-Gnedenko by a constant *a*:

$$\Psi^{-1}(x) = \begin{cases} F(x) = a\left(1 - exp\left(-l \cdot x^b\right)\right), x \le T \\ Cx, x > T \end{cases} . \tag{10}$$

Table 3: Statistical information on failures elements PAS. *m*=12.

Year	№	ν_i	Year	№	ν_i	Year	№	ν_i
1982	0	0	1993	11	1	2004	22	0
1983	1	1	1994	12	0	2005	23	1
1984	2	0	1995	13	1	2006	24	0
1985	3	4	1996	14	0	2007	25	0
1986	4	3	1997	15	0	2008	26	2
1987	5	3	1998	16	0	2009	27	0
1988	6	2	1999	17	0	2010	28	0
1989	7	1	2000	18	0	2011	29	0
1990	8	0	2001	19	0	2012	30	0
1991	9	0	2002	20	0	2013	31	0
1992	10	1	2003	21	1	2014	32	0

The model parameters are estimated, as mentioned above, OLS and their values are shown in Table 4.

Table 4: Estimates of the parameters of the nonhomogeneity function

Name	l	a	b	T	C	R^2
CNC56	0.0011	2.5595	3.1990	37	0.1267	0.9859
PAS	0.0338	1.7649	1.1453	21	0.0255	0.9816

Research has shown (see. Figure 3 a, 4 a), which parameterization Weibull-Gnedenko ideal only for a certain time interval $\left[0;T\right]$, where for $T=37$ years about CNC56, and $T=21$ for PAS. We can assume that the heterogeneity of function in the area $\left[T,41\right]$ for CNC56 and $\left[T,32\right]$ for PAS is described by a linear function. Her subsequent behavior can only predict. As the most appropriate forecast model was chosen the same linear growth model. Thus, the parameter T - a point in time immediately preceding the last failure (failure of one or a group). It is determined visually. The angular coefficient is determined by the OLS for evaluating restoration of function in the area $\left[T;41\right]$ for KNK56 and $\left[T;32\right]$ for PAS. Estimation the slope on the "final" phase of operation is clearly more consistent and accurate data available, because it does not clearly overestimated the value of the initial phase of operation. Fig. 3a, 4a is a graph of the reverse NFF – $\Psi^{-1}(t)$ (trend, solid line) and estimation $\tilde{\Omega}(t)$.

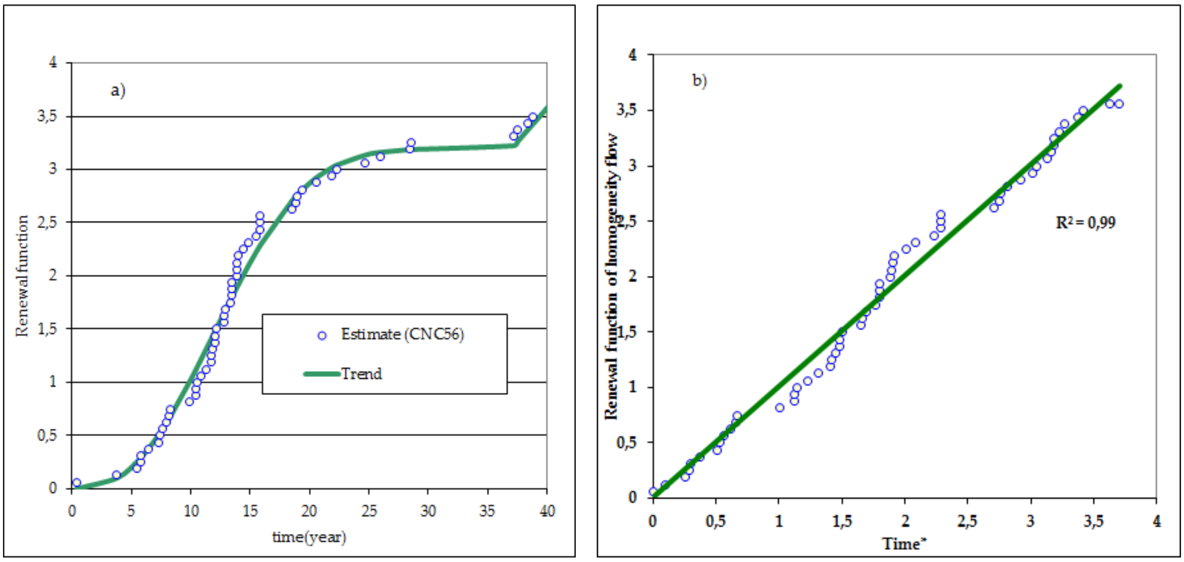

Figure3: *a) Estimate* $\tilde{\Omega}(t)$ *and its trend NFF model* $\Psi^{-1}(x)$ *b) Normalazing estimate* $\tilde{\Omega}(t^*)$ *and its linear trend NFF model* $\Psi^{-1}(x)$

V. Failure flow straightening

The essence of this stage reduced to transformation of an inhomogeneous flow of failures in the abstract homogeneous flow, using the relation

$$t^* = \Psi^{-1}(t).$$

In this case, obviously, t^* has the meaning of "abstract" time, and t - represents the actual operating time on the time axis. Thus, by the time t^* the flow axis to abstract the number of failures occurs, the corresponding point on the "real" axis t. After converting the time axis construct a "normalized" renewal function to abstract time. Fig. 3b, 4b is a graph of the normalized renewal function $\tilde{\Omega}\left(t^{*}\right)$ after conversion to the time axis.

The result was a homogeneous "rectified" failure flow. The figure also shows the approximating function, which is represented as a linear trend also shows the value of the coefficient of determination R^2, which is the usual square of the correlation coefficient. The closer this coefficient is to 1, the greater the proportion of the variance of the dependent variable explained by the considered model of addiction.

The results shown in Fig. 3b, 4b show that the failure rectified flow has a high level agreement with a linear model.

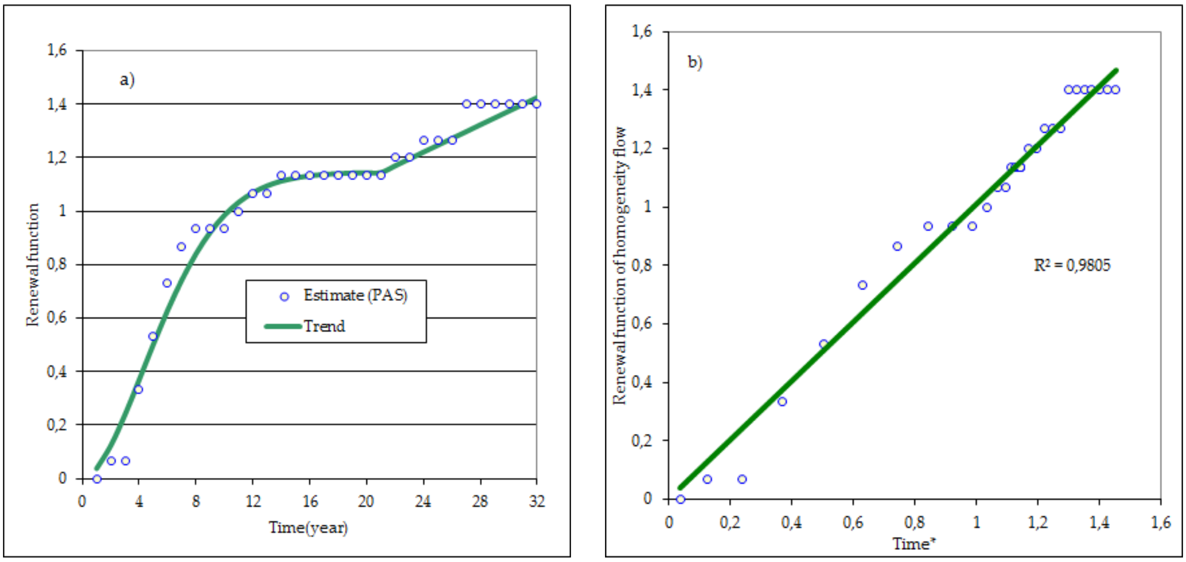

Figure 4: *a) Estimate* $\tilde{\Omega}(t)$ *and its trend NFF model* $\Psi^{-1}(x)$ *b) Normalazing estimate* $\tilde{\Omega}(t^{*})$ *and its linear trend NFF model* $\Psi^{-1}(x)$

VI. Nonparametric estimation of the failure intensity of "rectified" failure flow

Estimation parameter rectified flow of events for the grouped data can be obtained by the following methods (see [2, 4, 24-26].)

1. *Histogram.* Estimation is determined by the formula

$$\hat{\omega}_{hist}(t) = \frac{\nu_t}{m \cdot \Delta},$$

where ν_t – the number of failures recorded in the *i*-th observation interval on the axis of "abstract" time; m – the number of similar objects; $\Delta = t_i - t_{i-1}$ – length of the interval, which are realized on the number of failures ν_t.

2. *Kernel.* Kernel estimation is determined by the formula

$$\hat{\omega}_{kern.}(t,h) = \sum_{i=1}^{s} \frac{\nu_i}{m \cdot (r_i - l_i)} \cdot \left[G\left(\frac{t-l_i}{h}\right) - G\left(\frac{t-r_i}{h}\right) \right] + \varepsilon(t), \qquad (10)$$

where t – "abstract" time; h ($h > 0$) – smoothing parameter; $\vec{\nu}$ – frequency array; l_i and r_i – the left and right boundary of observation interval; $G(x)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{x} e^{-u^2/2}du$ – CDF of the standard normal distribution; $\varepsilon(x)$ – systematic error, which is defined by the formula [2]:

$$\varepsilon(t,n,m) \approx \frac{1}{2a}\operatorname{erfc}\left(\frac{aN-t}{\sqrt{2N}\sigma}\right)+\frac{\sigma}{\sqrt{2\pi}a}\frac{\sqrt{N}}{aN+t}e^{-\frac{(aN-t)^2}{2n\sigma^2}}.$$

Fig. 5a, 6a presents estimates calculated histogram– $\hat{\omega}_{hist}(t)$ and kernel– $\hat{\omega}_{kern.}(t,h)$ techniques.

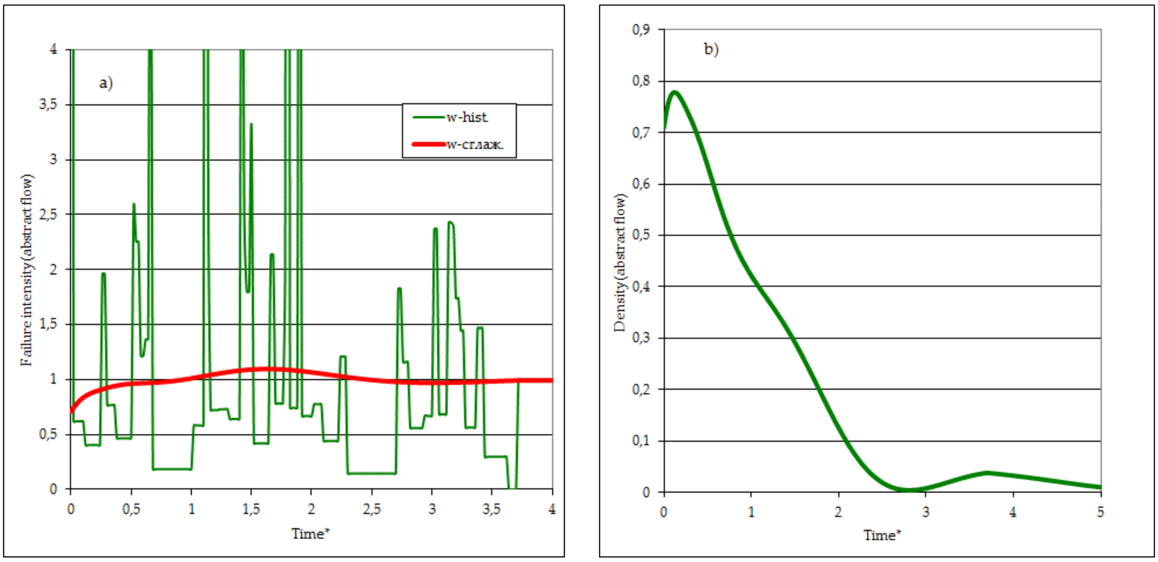

Figure 5: Histogram $\hat{\omega}_{hist}(t)$ and kernel (smoothed) $\hat{\omega}_{kern.}(t,h)$ estimation rectified failure intensity CNC56; b) estimate of the density of the abstract operation times between failures $\hat{f}_{\xi}(t)$ CNC56.

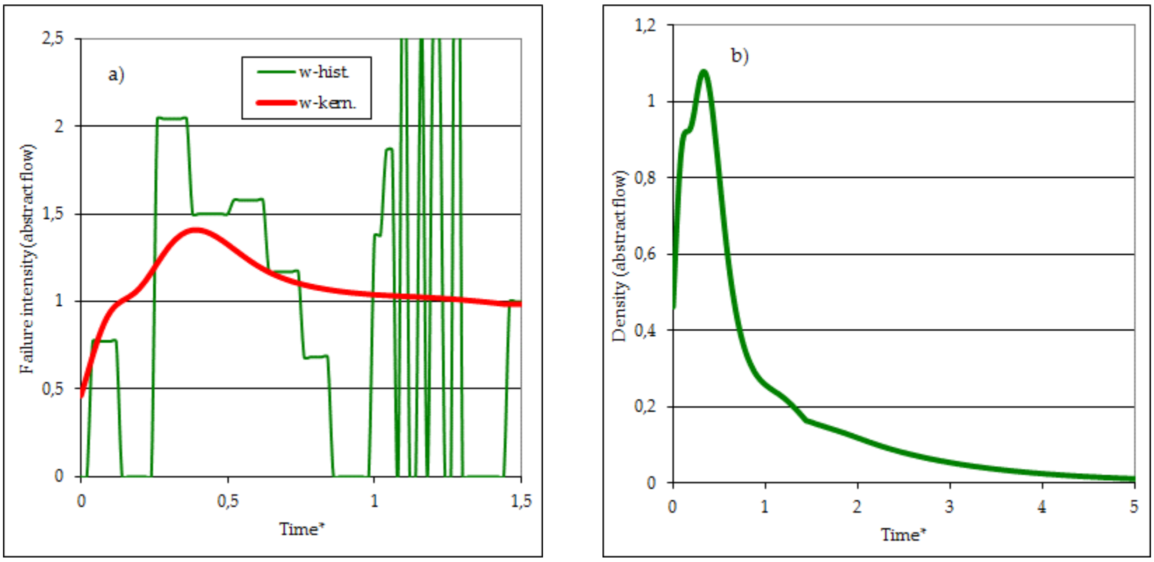

Figure 6: Histogram $\hat{\omega}_{hist}(t)$ and kernel (smoothed) $\hat{\omega}_{kern.}(t,h)$ estimation rectified failure intensity PAS; b) estimate of the density of the abstract operation times between failures $\hat{f}_{\xi}(t)$ PAS.

Direct substitution histogram estimation of failure intensity in the renewal equation may lead to some negative timeslots density distribution. It is contrary to the properties of the density. Therefore, for further calculations will use the kernel estimate of failure intensity, allows to obtain a solution of the renewal equation, which has the necessary properties of density distribution.

VII. Estimating the density distribution of the "abstract" operating time between failures

The density of operating time between failures a homogeneous flow of events in the "abstract" the time axis can be determined by solving the renewal equation – Volterra equation of the second kind [1]:

$$\hat{f}_{\xi}(x) = \omega_{kern.}(x) - \int_0^x \hat{f}_{\xi}(u)\omega_{kern.}(x-u)du\,, \tag{11}$$

where $\omega_{kern.}(x)$ – kernel estimate of failure intensity.

Fig. 5b, 6b shows the estimate of the density of the abstract operating time obtained based on the assessment of kernel rectified the failure intensity. This estimate of density $\hat{f}_{\xi}(x)$ has a non-negative function satisfying the normalization condition. It is because of these reasons was chosen smoothing parameter *h* in (10).

For further calculations on the NFF model will use this estimate distribution density operating time abstract homogeneous stream of refusals. Recall that the model for NHPP

$$\hat{f}_{\xi}(x) = e^{-x} \text{ and } \omega(x) = 1\,. \tag{12}$$

VIII. Estimation of distribution of operating time – ζ_i.

Let the length of the *i*-th cycle of working capacity ζ_i - is the *i*-th operating time between two successive failures of the inhomogeneous flow of events. In [2, 7] gives expression to find the value of the distribution function ζ_i in the framework of NFF model:

$$F_{\zeta_i}(t) = \int_0^{\infty} f_{\mu_{i-1}}(u) F_{\xi}\left(\Psi^{-1}\left(t+\Psi(u)\right)-u\right)du\,, \tag{13}$$

where $\mu_k = \sum_{i=1}^{k} \xi_k$ – the time of the *k*-th event "abstract" homogeneous flow of failures; $F_{\xi}(x)$ – CDF of "abstract" operating time; $f_{\mu_{i-1}}(t) = \int_0^t f_{\mu_{i-2}}(t-u)\, f_{\xi}(u)du$.

The CDF and the density to failure of the first time are determined by the formulas:

$$F_{\zeta_1}(t) = \mathrm{P}\left(\zeta_1 < t\right) = F_{\xi}\left(\Psi^{-1}\left(t\right)\right), \quad f_{\zeta_1}(t) = \left(\Psi^{-1}\left(t\right)\right)' f_{\xi}\left(\Psi^{-1}\left(t\right)\right). \tag{14}$$

By differentiating (13), we find an expression for the density of the duration of the cycle performance:

$$f_{\zeta_i}(t) = \int_0^{\infty} f_{\mu_{i-1}}(u)\, f_{\xi}\left(\Psi^{-1}\left(t+\Psi(u)\right)-u\right)\cdot\left(\Psi^{-1}\left(t+\Psi(u)\right)\right)' du;\ i = 2,3,\ldots, \tag{15}$$

where $f_{\xi}(x)$ – density of the "abstract" operating time.

The CDF of the first time to failure ζ_1 under the NHPP model is defined as follows:

$$F_{\zeta_1}(t)=1-e^{-\Lambda(t)};\ f_{\zeta_1}(t)=\Lambda'(t)e^{-\Lambda(t)}, \tag{16}$$

where $\Lambda(t)=\Psi^{-1}(t)$.

CDF and density of the remaining operating time between failures within NHPP model will be equal

$$F_{\zeta_i}(x)=1-\int_0^{\infty}\frac{u^{i-2}}{(i-2)!}e^{-\Lambda\left(\Lambda^{-1}(u)+x\right)}du, \tag{17}$$

$$f_{\zeta_i}(x)=\int_0^{\infty}\lambda\left(\Lambda^{-1}(u)+x\right)\frac{u^{i-2}}{\Gamma(i-1)}\cdot e^{-\Lambda\left(\Lambda^{-1}(u)+x\right)}du;\ i=2,3,\ldots. \tag{18}$$

On basis of the presented formulas The calculation of these characteristics to the statistics available for the elements CNC56 and PAS. We represent the results of calculations of use density distributions for the first, second, third and fourth performance cycles performed for NFF models (Fig. 7a, 8a) and NHPP model (Fig. 7b, 8b). On the basis of the calculated density of the calculations of durations of *i*-functionality cycle. Fig. 9 shows the mean time to the first, second, third and fourth bounce calculated NFF semi-parametric model and parametric models NHPP. Average values were determined by numerical integration of the respective distribution density.

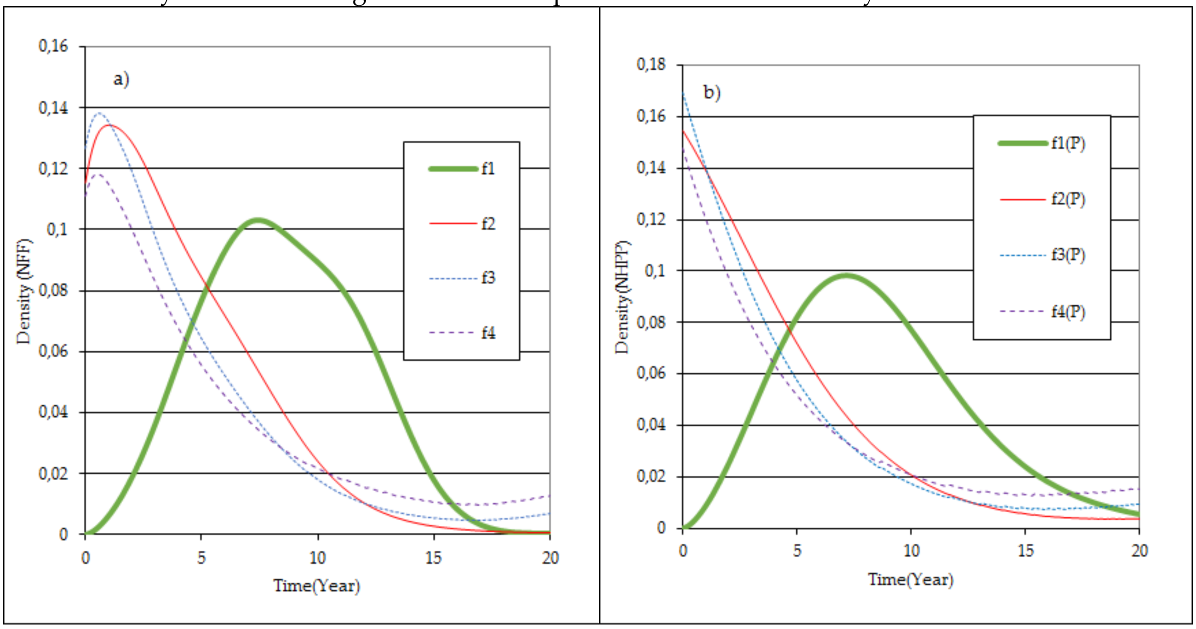

Figure 7: The densities of the four cycles for CNC56: a) NFF, b) NHPP.

By analyzing data on failures CNC56 element shown in Table 1, it can be noted that all failures occurred mainly in the period from 1974 to 1999. Ie for the first 25 years of operation. flow failure $m=16$ form the same elements. By the beginning of 1980 there were 18 failure. Assuming that the operating conditions are the same set of 16 elements, with high probability we can say that each of the elements together broke down at least once. Those. by 1980 for each of the elements together first functionality cycle ended, then the element was replaced and started the second cycle functionality of a particular element. It is advisable to assume that the average first cycle functionality is approximately 8 years. The calculation results are presented in Fig. 7, allow us to estimate the mean value of operating time. You can also note that in the period from 1978 to 1987. There was an increased frequency failure CNC-56. It is equal to about 5 years of failures in a group of similar items in an amount of 16 units. By the beginning of 1985 there were 32 of failure, ie, on average, each element is broken twice. The average value of the second functionality cycle falls on the interval of three to five years (see. Fig. 9a).

The graph functionality density of the second cycle $f_{\zeta_2}(t)$ should shift to the left - the likelihood of small operating time increased, and large declined (Figure 7.). The densities of the 3rd and 4th cycle of about the same and only slightly different from a density of 2 - second cycle. In summary, it can be noted that the behavior of the density of cycle functionality and average operating time adequately describes the input information. Knowing $F_{\zeta_i}(t)$ or $f_{\zeta_i}(t)$, we can find any interesting safety characteristic for an the *i*-th cycle functionality.

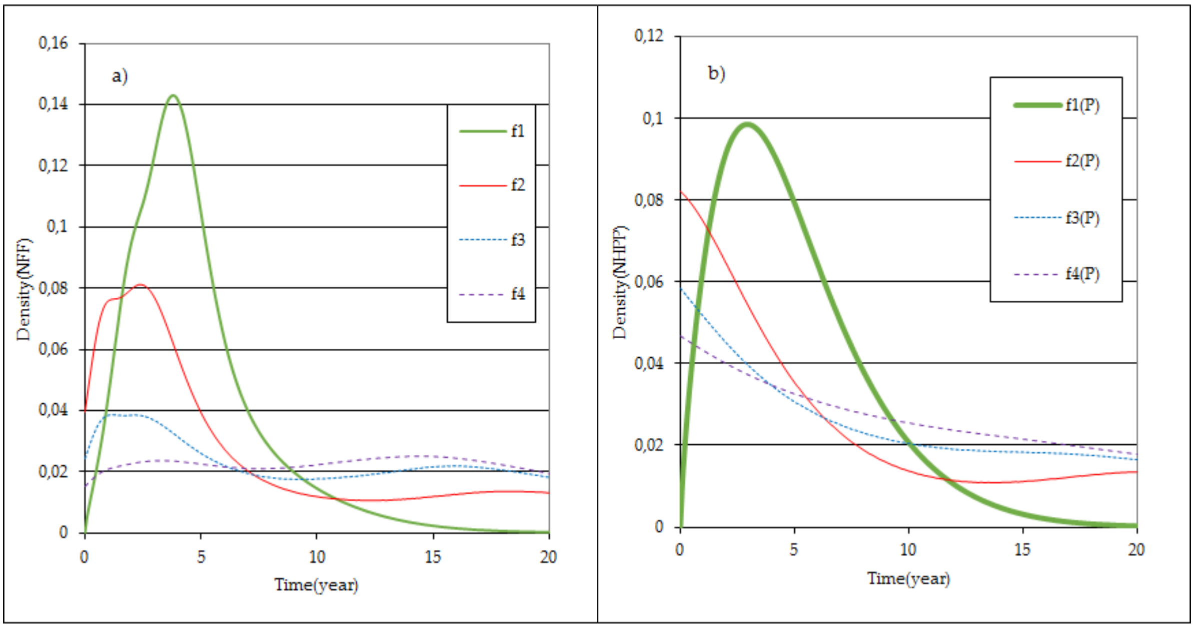

Figure 8: The densities of the four cycles for PAS: a) NFF, b) NHPP.

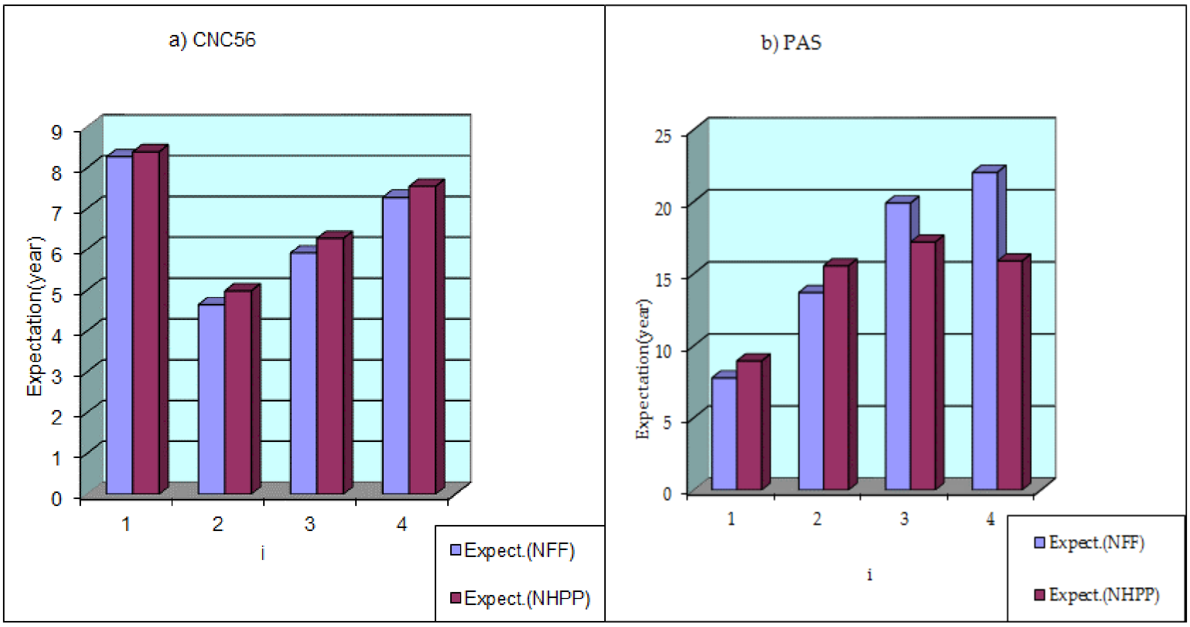

Figure 9: Mean time between failures a) CNC56, b) PAS.

Histograms of mean operating time almost identical, have very different densities and graphics. This suggests that the flow of data elements failure is a NHPP. Therefore, this element can be applied more simple methods of model of an NHPP. In addition, the model of an inhomogeneous Poisson flow is possible to construct confidence intervals.

Now let's analyze the results of calculations for the PAS. This element is in operation since 1982. The failure rate generates $m = 12$ identical elements. There was a total of 21 rejected. The mean number of failure is equal to 1.4. Thus, on average, each of the 15 elements already broke once and stored in the second operation cycle. Failures are mainly observed the first 14 years, then they were not there for 13 years, on the 27th, there was a 2 failure. Since there were no failures. Evaluation PPO differs significantly from the constant 1 (see. Fig. 6a). This explains the gradually emerging a significant difference in the distributions (Fig. 8a) and estimates the average operating time (Fig. 9b) NFF calculated by the model and the model of an NHPP. Those. it can be assumed that the behavior of the flow of failures PSM poorly described by the model of an NHPP. Therefore, further focus on the assessment, obtained by the NFF model. Fig. 9b is a tendency to an increase in the average operating time between failures. In the second cycle of mean time between failures is approximately 13 years on tretem- 19.

IX. Estimating resource characteristics

In [2,7], the expression for the calculation in a failure flow heterogeneity of resource characteristics of reliability, as the average reverse residual time $\mathrm{E}R_t$ and average direct residual time $\mathrm{E}V_t$ the remaining time (method of determining the characteristics R_t and V_t homogeneous flow can be found, for example, in [2]). In the case of an inhomogeneous flow NFF model calculations should be carried out according to the formulas:

$$\mathrm{E}R_t = t \cdot \left(1 - F_\xi\left(\Psi^{-1}(t)\right)\right) + \int_0^\infty g_R(x;t)\, f_\xi(x)dx, \tag{19}$$

$$\mathrm{E}V_t = \int_{\Psi^{-1}(t)}^{\infty} \left(\Psi(x) - t\right) f_\xi(x)dx + \int_0^\infty g_V(x;t)\, f_\xi(x)dx, \tag{20}$$

где $g_R(x;t) = \int_{\left(\Psi^{-1}(t)-x\right)\vee 0}^{\Psi^{-1}(t)} \left(t - \Psi(u)\right)v(u)du$; $v(x) = F_\xi(x) + \int_0^x v(x-u)\, f_\xi(u)du$;

$g_V(x;t) = \int_{\left(\Psi^{-1}(t)-x\right)\vee 0}^{\Psi^{-1}(t)} \left(\Psi(u+x) - t\right)v(u)du$.

Calculation of the resource reliability indices for the case NHPP model is greatly simplified.
Average direct residual time:

$$EV_t = \int_0^\infty e^{-x}\Psi\left(x + \Psi^{-1}(t)\right)dx - t. \tag{21}$$

Average reverse residual time:

$$ER_t = t - \int_0^{\Lambda(t)} e^{-x}\Psi\left(\Psi^{-1}(t) - x\right)dx. \tag{22}$$

Along with these characteristics, you can restore the system to determine the average resource:

$$ED_t = EV_t + ER_t . \tag{23}$$

In fact, this value will be the average cycle time of the system at the time of inspection.

In the above formulas based on statistical information on failures elements CNC56 and PAS have calculated indicators: average residual direct and reverse time, average life. The calculation results are shown in Fig. 10-13.

The behavior of the characteristics of the data leads to the conclusion that :

- To CNC56 average reverse residual time reaches local maximum of 5 years to the eighth year of operation (1982). Therefore, in the previous period in 1982 element failure occurred, most likely in mid-1978. In 1978, finished the stage failure flow dilution, their intensity increased sharply.
- To CNC56 average reverse residual time reaches a local minimum of 4 years We have 15 year of operation (1989). By the early 1987 peak failure observed. In 1987 he completed phase failure rate of condensation.

The subsequent behavior of the indicator ER_t is characterized by almost linear dependence of its operating time. This is due to the fact that in the interval from 1999 to 2011. failure almost was not.

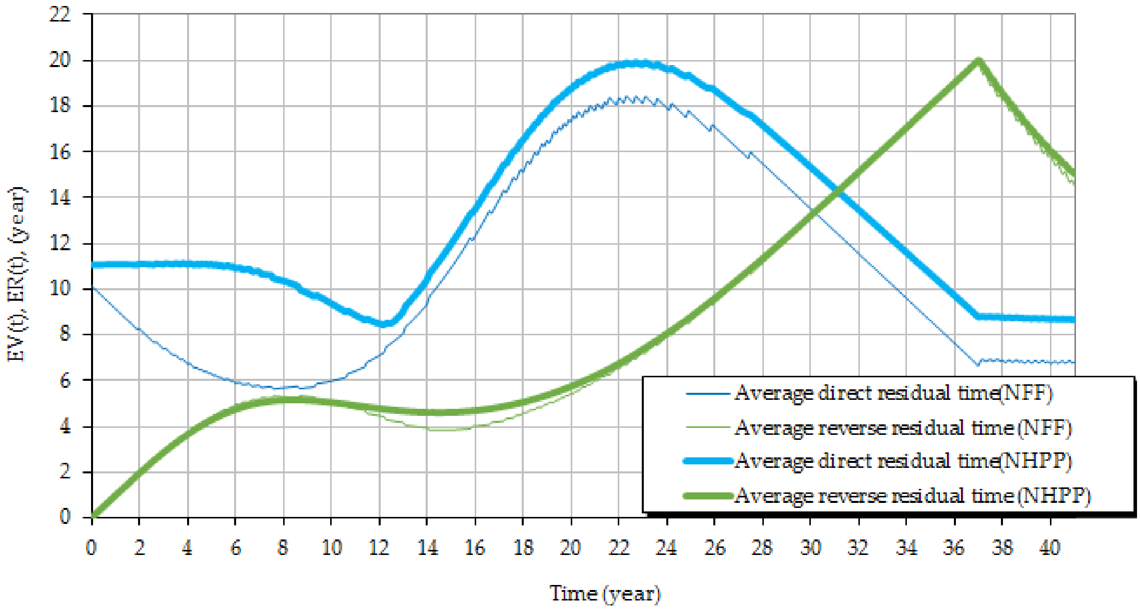

Figure 10: Average reverse residual time ER_t and direct residual time EV_t CNC56

Average direct residual time reaches (by NFF model) local minimum of 5.5 years on the 8 year of operation (1982). Consequently, failure peaked in 1987 (1982 + 5.5). Average direct residual time reaches (by NFF model) a local maximum of 18 years on 22 year of operation (1996). Consequently, the next peak failure can be expected by 2014. Indeed, the failures began to appear in the last 4 years.

In the future, the graph EV_t decreases to 37 years as in the period from 1996 to 2011. there was virtually no failure, and, consequently, less time is left of the current moment of observation before the expected next failure.

The calculations of the direct and reverse residual time allow to predict the remaining service life of products at the time of inspection.

Similar detailed analysis of indicators of resource scheduling can be done for the PAS. But this we will leave to the reader. Consider Fig. 12, which shows the average graphics resource for CNC56. You may notice a local minimum of the characteristics in the 12th year of operation, which is practically the same from 1985 g.- year maximum failure rate.

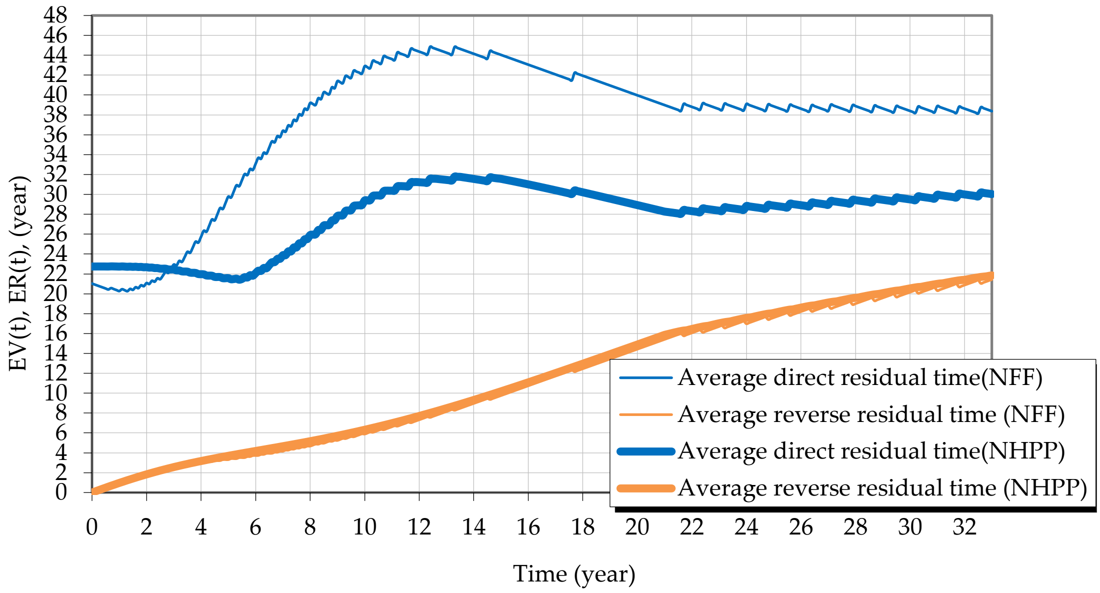

Figure 11: Average reverse residual time ER_t and direct residual time EV_t PAS

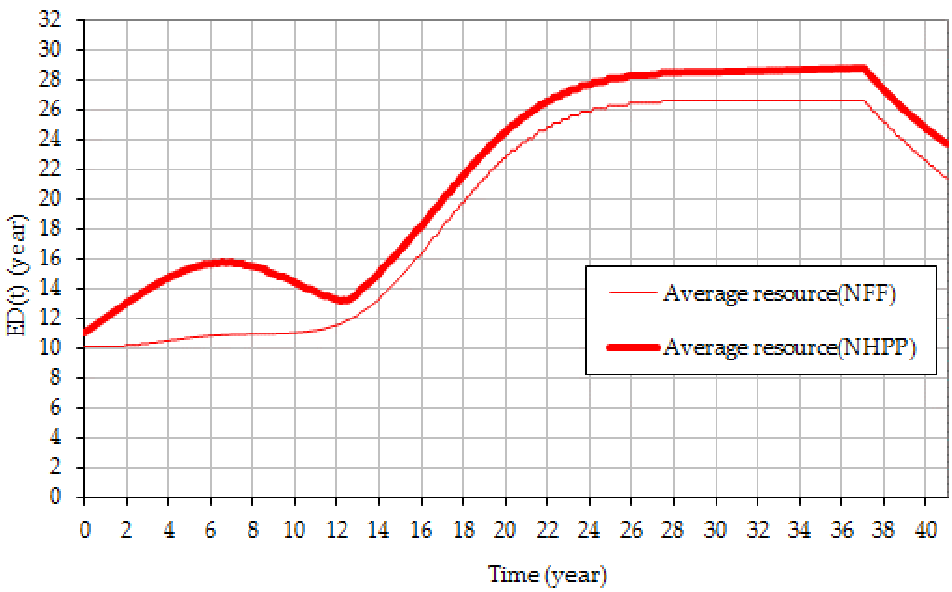

Figure 12: Average resourse ED_t CNC56

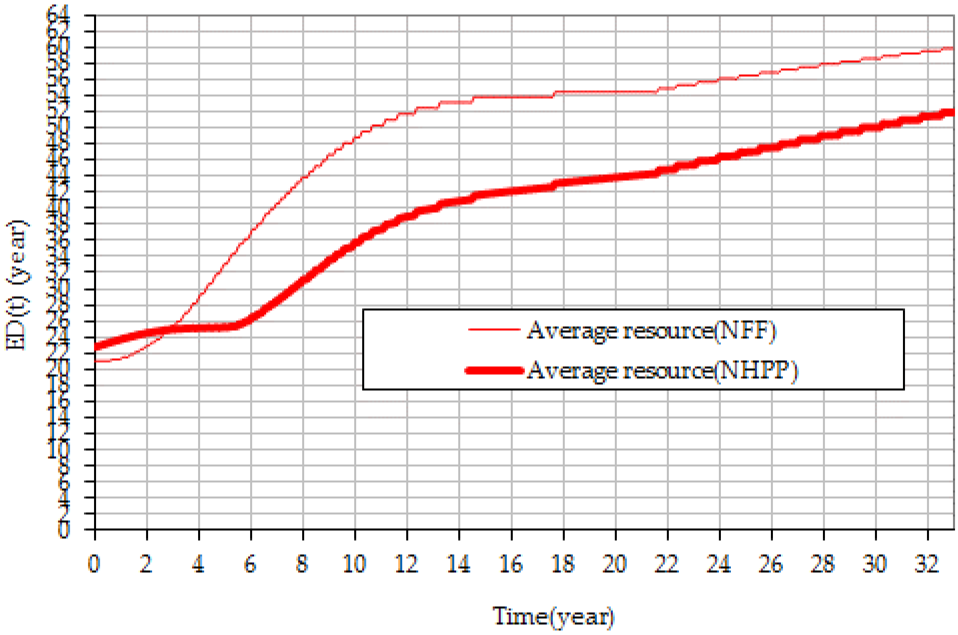

Figure 13: Average resourse ED_t PAS

X. Conclusion

The article describes a new method of analysis of statistical data on failures to estimate the NPP equipment reliability indicators, which allows to take into account the possible heterogeneity of the flow of events. Examples of data analysis at each stage of the study on the basis of statistical information on faults elements CNC56 and PAS derived from operating experience. According to the procedure provided by the calculations of a large group of control components and power protection EGP-6 on the basis of information over a long period of operation (1974 and 2014). The results are presented in [4].

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