Evaluation of System Performance Measures of Multi State Degraded System with Minimal Repair

M. Manoharan & Vidhya G Nair

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Department of Statistics, University of Calicut
mailto:vidhyagn17@gmail.commanumavila@gmail.com,vidhyagn17@gmail.com

Abstract

There is a recent surge of interest in multi state systems mainly due to their wide applications in engineering. Multi state degraded systems have been used in modeling of power generating-supply systems, communication systems and transportation systems etc. In this article we propose a new approach ie, a combination of stochastic process approach and Universal Generating Function(UGF) technique by decomposing system in to several subsystems. Analyzing models through this approach, several system performance measures are evaluated. A real data obtained from a power station modeled as a MSS which has two subsystems with many states of degradation, has been used for illustration to apply the approach presented here.

Keywords: Multi state Systems, Power generating System, Repairable System, System Performance Measures, Universal Generating Function(UGF).

1 Introduction

In binary reliability models the system or its components is assumed to be either in a perfectly functioning state or in a completely failed state. But in most of the real life situations this assumption may not be adequate. There are intermediate states between perfectly functioning state and completely failed state. So we make use of the Multi State system (MSS) reliability model in which the system may rather have more than two states of performance between working perfectly and total failure. The basic concept and further developments of binary system reliability theory were dealt in [2 , 3]. The basic concepts of MSS, tools for MSS reliability assessment and optimization and application problems were discussed in [8]. Multi state with degrading components and concerned with the application of reliability functions to the reliability evaluation of large systems emphasis in [4]. A comprehensive introduction to system reliability theory along with failure models, qualitative system analysis and reliability importance were discussed in [10]. The joint importance measures for multi state reliability systems have been discussed in [13] and[14]. According to [1] Repairable system is a system which after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by a method other than replacement of the entire system. The UGF was first introduced by Ushakov [11] for MSS. The mathematical basics of this technique were available in [12]. An updated version of the UGF with many application was presented in [5, 8]. The combined method using random process and UGF was suggested in [8] and further extended in [6 , 7]. In [9], a new approach was used to evaluate the dynamic reliability of MSS with redundancy.

In this article description of models with assumptions has been presented in section 2. In section 3 the combined stochastic process and UGF technique approach is applied for multi state degraded system for avoiding dimension damnation problem of the stochastic process approach. A new approach of decomposing a system in to two or more sub systems (each sub system consists of the same type of components) has been proposed. Steady state probabilities and system performance characteristics are calculated for subsystems using the random process method and at last reliability indices of the entire system in steady state situation are evaluated using UGF technique. A more realistic system has been taken to validate the applicability of this approach.A power station with two sub systems (each sub system with three generators) has been illustrated in section 4 of this article. Reliability indices of this power station are evaluated in this paper.

2 Multi state degraded system

Any subsystem j of a MSS have k_j different states with performance rates represented by the set $g^j = \{g_1^j, {g_2}^j, g_3^j, \dots, g_{k_j}^j\}$ where g_i^j is the output of subsystem j in state i, $i \in \{1,2, \dots k_j\}$. The output $G_j(t)$ of subsystem j at any instant $t \geq 0$ is a random variable and it takes values from g^j: $G_j(t) \in g^j$. **Assumptions** The system or subsystem may have many levels of degradation which vary from perfect functioning to complete failure. The system or subsystem might fail any 'up' state to its 'down' states and it is minimally repaired. The components of the system might fail independently and they are operated continuous basis. The components of the system are repaired independently.

3 Analysis of Model

Consider a subsystem with m components having $0,1,2, \dots k$ states where k is the best functioning state and 0 is the worst state. The state space of the system is $S = \{0,1,2, \dots k\}$. Components of the system have variable failure rates and variable repair rates . When a component fails a repair action is initiated to bring the component back to its initial up state. The transition probabilities of the Markov process $\{X(t), t \geq 0\}$ with state space $S = \{0,1,2, \dots, k\}$.

$p_{ij}(t) = Pr\{X(t) = j / X_0 = i\}$ for all $i, j \in S$ arranged as a matrix,

$$p(t) = \begin{bmatrix} p_{00}(t) & p_{01}(t) & \dots & p_{0k}(t) \\ p_{10}(t) & p_{11}(t) & \dots & p_{1k}(t) \\ \vdots & \vdots & \vdots & \vdots \\ p_{r0}(t) & p_{r1}(t) & \dots & p_{kk}(t) \end{bmatrix}$$

$0 \leq p_{ij}(t) \leq 1$ for all $t \geq 0 i, j \in S$

$$\sum_{j=0}^{k} p_{ij}(t) = 1 for all i \in S.$$

Specify the transition rates a_{ij} for $i \neq j i, j \in S$. Each transition will usually involve a failure or a repair. The transition rates will therefore be failure rates and repair rates and combinations of these. Hence the infinitesimal generator of the process is

$$A = \begin{pmatrix} a_{00} & a_{01} & \dots & a_{0k} \\ a_{10} & a_{11} & \dots & a_{1k} \\ \vdots & \vdots & \vdots & \vdots \\ a_{k0} & a_{k1} & \dots & a_{kk} \end{pmatrix},$$

where $a_{ii} = -\sum_{j=0, j \neq i}^{k} a_{ij}$.

Let $p(t) = \left[p_0(t), p_1(t), \cdots, p_k(t) \right]$ denote the distribution of Markov process at time t, when we know that the process started in state i at time 0. The distribution $p(t)$ may be found from the Kolmogrov forward equations given in matrix form (see[10]) as

$$p(t)A = \dot{p}(t). \tag{1}$$

Equation (1) is called state equation for the Markov Process. In many application the long run (steady state) probabilities are of interest.

The steady state probabilities $p = \begin{bmatrix} p_0 & p_1 & \dots & p_k \end{bmatrix}$ are given by must therefore satisfy the matrix equation.

$$pA = 0 \tag{2}$$

and

$$\sum_{j=0}^{k} p_j = 1.$$

This can be computed easily using computation algorithms based on MATLAB.

In general, a system consists of n subsystems with each subsystem possessing k states.Here $g^j = \{g_1^j, g_2{}^j, g_3^j, \dots, g_k^j\}$ is the performance level of subsystem j. The steady state probability of j^{th} subsystem is determined by previously described stochastic process approach.

ie,$p^j = \{p_1^j, p_2^j, \dots p_k^j\}$.

The UGF[12] of the j^{th} subsystem is determined as

$$u^j(z) = \sum_{i=1}^{k} p^i z^{g^i}$$

The structure function of a MSS consisting of series and parallel subsystem may be determined by reliability block diagram method ie, iteratively composing the structure functions of the independent subsystems. In order to find u-function for the entire MSS the corresponding operators $\Omega_\Phi$operators should be applied. $\Omega_{\Phi s}$and $\Omega_{\Phi p}$ are used the subsystems connected in series and parallel respectively. For MSS with n subsystem connected in parallel the system structure function is in the form

$$U(z) = \Omega_{\Phi p}\{u^1(z), u^2(z), \dots u^n(z)\}$$

Reliability indices of the system in steady state situation

1. **Steady state MSS availability**

Steady state MSS availability can be obtained for any constant demand w

$$A_\infty(w) = \delta_A(U(z), w) = \sum_{i=1}^{k} (p^i z^{g_i}, w)$$

2. **Mean Steady state MSS performance**

Mean Steady state performance is

$$E_\infty = \sum_{i=1}^{k} p^i g^i$$

3. **Expected steady state MSS performance deficiency**

Expected steady state MSS performance deficiency can be obtained for any constant demand w

$$D_\infty(w) = \sum_{i=1}^{k} p^i max(w - g^i, 0)$$

4 Numerical Example

In Kuttiady Hydro Electric Project , governed by Kerala State Electricity Board(KSEB) under Govt.of Kerala , there are three generators with installed capacity 75MW (each with 25MW) and have same features. States and outputs of Generator 1, 2 and 3(G1, G2 and G3) are respectively 1(0MW), 2(12.5MW) and 3(25mw) and these constitute Subsystem 1. Other three generators with installed capacity 150MW (each with 50MW) have same features. States and outputs of Generators 4, 5 and 6 (G4, G5 and G6) are respectively 1(0MW), 2(25MW)and 3(50MW) and these generators constitute subsystem 2.

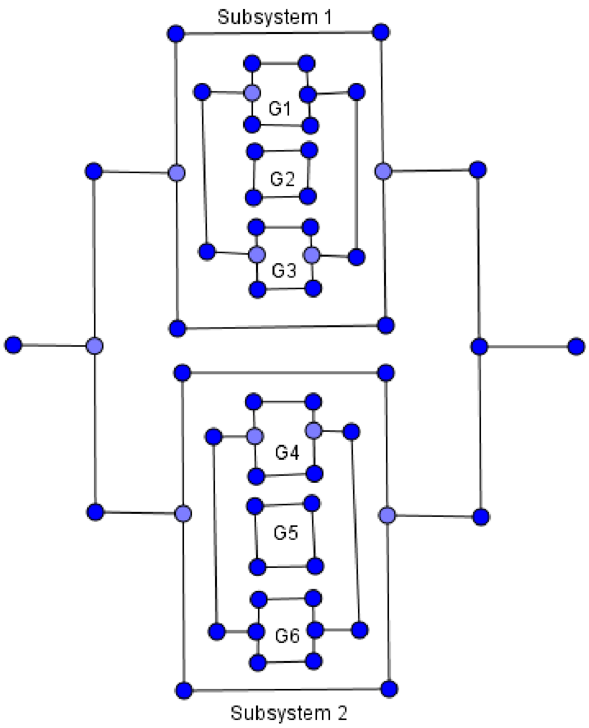

Figure 1: Reliability block diagram of power station

Subsystem 1

Transition rates of the generators G1,G2 and G3 per hour(h^{-1}) are calculated from the collected data and are given in the table below.

Table 1: Transition Rates

Generator	μ_{12}	μ_{23}	λ_{21}	λ_{32}	λ_{31}
G1	6.1×10^{-2}	6.4×10^{-2}	3×10^{-3}	6.7×10^{-2}	3.3×10^{-3}
G2	6.7×10^{-2}	6.5×10^{-2}	3×10^{-3}	6.8×10^{-2}	3.3×10^{-3}
G3	7.1×10^{-2}	5.6×10^{-2}	3×10^{-3}	5.9×10^{-2}	3.3×10^{-3}

The steady state probabilities are obtained using the following system of equations

$$p = [p_1^1 p_2^1 p_3^1 p_4^1 p_5^1 p_6^1 p_7^1]A = [0,0,0,0,0,0,0]$$

$$\sum_{j=1}^{7} p_j = 1.$$

$$-2.1247\times10^{-1}p_1^1 + 9\times10^{-3}p_2^1 + 9.927\times10^{-3}p_3^1 + 5.9427\times10^{-5}p_4^1$$
$$+3.2759\times10^{-5}p_5^1 + 9.801\times10^{-8}p_6^1 + 9.9\times10^{-3}p_7^1 = 0$$
$$1.99\times10^{-1}p_1^1 + -6.4375\times10^{-1}p_2^1 + 2.7821\times10^{-1}p_3^1 + 2.0955\times10^{-2}p_4^1$$
$$+1.3549\times10^{-3}p_5^1 + 3.5986\times10^{-3}p_6^1 + 1.94\times10^{-1}p_7^1 = 0$$
$$1.3175\times10^{-2}p_1^1 + 5.9618\times10^{-1}p_2^1 - 1.3058p_3^1 + 4.8802\times10^{-1}p_4^1$$
$$+1.1254\times10^{-1}p_5^1 + 3.3693\times10^{-3}p_6^1 + 1.2554\times10^{-2}p_7^1 = 0$$
$$2.9018\times10^{-4}p_1^1 + 3.7755\times10^{-2}p_2^1 + 9.6792\times10^{-1}p_3^1 - 1.5005p_4^1$$
$$+4.7544\times10^{-1}p_5^1 + 3.7498\times10^{-2}p_6^1 + 1.5492\times10^{-3}p_7^1 = 0$$
$$8.1484\times10^{-4}p_2^1 + 4.8944\times10^{-2}p_3^1 + 9.5461\times10^{-1}p_4^1 - 1.1705p_5^1$$
$$+3.9899\times10^{-1}p_6^1 + 2.2421\times10^{-2}p_7^1 = 0$$
$$7.5753\times10^{-4}p_3^1 + 3.654\times10^{-2}p_4^1 + 5.6973\times10^{-1}p_5^1 - 6.2489\times10^{-1}p_6^1$$
$$+1.94\times10^{-1}p_7^1 = 0$$
$$2.3296\times10^{-4}p_4^1 + 1.1384\times10^{-2}p_5^1 + 1.85\times10^{-1}p_6^1 - 4.3442\times10^{-1}p_7^1 = 0$$
$$p_1^1 + p_2^1 + p_3^1 + p_4^1 + p_5^1 + p_6^1 + p_7^1 = 1$$

Using MATLAB, we get the steady state probabilities $p_1^1, p_2^1, p_3^1, p_4^1, p_5^1, p_6^1 and p_7^1$ and tabulated below.

Sub system state	Sub system Output	Steady state Probabilities	Average hours in state/year
	0MW	0.015606342403632	136.71
	12.5MW	0.100286202268650	878.51
	25MW	0.128418120076977	1124.94
	37.5MW	0.163690419570730	1433.93
	50MW	0.226938419267982	1987.98
	62.5MW	0.251796810833076	2205.74
	75MW	0.113263685578953	992.19

Subsystem 2

Transition rates of the generators G4 , G5 and G6 per hour(h^{-1}) are calculated from the collected data and are given in the table below.

Table 2: Transition Rates

Generator	μ_{12}	μ_{23}	λ_{21}	λ_{32}	λ_{31}
G4	7.8×10^{-2}	6.6×10^{-2}	3.3×10^{-3}	6.9×10^{-2}	3×10^{-3}
G5	8.9×10^{-2}	2.4×10^{-2}	3.4×10^{-3}	2.7×10^{-2}	3.8×10^{-3}
G6	9×10^{-2}	2.6×10^{-2}	3.3×10^{-3}	2.9×10^{-2}	3.8×10^{-3}

The steady state probabilities are obtained using the following system of equations

$$[p_1^2 p_2^2 p_3^2 p_4^2 p_5^2 p_6^2 p_7^2]A = [0,0,0,0,0,0,0]$$

$$\sum_{j=1}^{7} p_j = 1$$

$$-0.2796p_1^2 + 0.01p_2^2 + 1.0633\times10^{-2}p_3^2 + 7.0677\times10^{-5}p_4^2$$
$$+3.7358\times10^{-5}p_5^2 + 1.2403\times10^{-7}p_6^2 + 4.332\times10^{-8}p_7^2 = 0$$
$$0.257p_1^2 - 0.705p_2^2 + 1.489\times10^{-1}p_3^2 + 2.2074\times10^{-2}p_4^2$$

$$+9.6363\times10^{-4}p_5^2 + 3.9215\times10^{-5}p_6^2 + 1.425\times10^{-6}p_7^2 = 0$$
$$2.1972\times10^{-2}p_1^2 + 6.5197\times10^{-1}p_2^2 - 1.1878p_3^2 + 2.2146\times10^{-1}p_4^2$$
$$+1.0533\times10^{-1}p_5^2 + 8.5445\times10^{-4}p_6^2 + 4.6729\times10^{-3}p_7^2 = 0$$
$$6.2478\times10^{-4}p_1^2 + 4.216\times10^{-2}p_2^2 + 9.8186\times10^{-1}p_3^2 - 1.1305\times10^{-1}p_4^2$$
$$+3.3913\times10^{-1}p_5^2 + 2.9187\times10^{-2}p_6^2 + 9.5923\times10^{-4}p_7^2 = 0$$
$$8.7763\times10^{-4}p_2^2 + 4.6084\times10^{-2}p_3^2 + 8.6276\times10^{-1}p_4^2 - 9.3872\times10^{-1}p_5^2$$
$$+2.6094\times10^{-1}p_6^2 + 1.5243\times10^{-2}p_7^2 = 0$$
$$3.4396\times10^{-4}p_3^2 + 2.4136\times10^{-2}p_4^2 + 4.8914\times10^{-1}p_5^2 - 4.0702\times10^{-1}p_6^2$$
$$+0.125p_7^2 = 0$$
$$4.1184\times10^{-5}p_4^1 + 3.924\times10^{-3}p_5^1 + 0.116p_6^1 - 1.4588\times10^{-1}p_7^1 = 0$$
$$p_1^2 + p_2^2 + p_3^2 + p_4^2 + p_5^2 + p_6^2 + p_7^2 = 1$$

Using MATLAB, we get the steady state probabilities $p_1^2, p_2^2, p_3^2, p_4^2, p_5^2, p_6^2 and p_7^2$ and tabulated below.

Sub system state	Sub system Output	Steady state Probabilities	Average hours in state/year
	0MW	0.00532437363217	46.6415
	25MW	0.07393101898536	327.5636
	50MW	0.114533450326305	1003.313
	75MW	0338786626776639	2967.7709
	100MW	0.341863201329831	2994.7216
	125MW	0.084842959602057	743.2243
	150MW	0.076756286703414	672.3850715

For Subsystem 1

$$g^1 = 0,12.5,25,37.5,50,62.5,75$$
$$p^1 = p_1^1, p_2^1, p_3^1, p_4^1, p_5^1, p_6^1, p_7^1$$
$$u_1(z) = p_1^1 z^0 + p_2^1 z^{12.5} + p_3^1 z^{25} + p_4^1 z^{37.5} + p_5^1 z^{50} + p_6^1 z^{62.5} + p_7^1 z^{75}$$

For Subsystem 2

$$g^2 = 0,25,50,75,100,125,150$$
$$p^2 = p_1^2, p_2^2, p_3^2, p_4^2, p_5^2, p_6^2, p_7^2$$
$$u_2(z) = p_1^2 z^0 + p_2^2 z^{25} + p_3^2 z^{50} + p_4^2 z^{75} + p_5^2 z^{100} + p_6^{21} z^{125} + p_7^2 z^{150}$$

The u-function [12]of the structure of entire system in which two subsystems are connected in parallel(total output of the power station is determined as the outputs of the two sub systems) is

$$U(z) = \Omega_{\phi p}(u_1(z), u_2(z)) = \Omega_{\phi p}(p_1^1 z^0 + p_2^1 z^{12.5} + p_3^1 z^{25} + p_4^1 z^{37.5} + p_5^1 z^{50} + p_6^1 z^{62.5} + p_7^1 z^{75}$$
$$, p_1^2 z^0 + p_2^2 z^{25} + p_3^2 z^{50} + p_4^2 z^{75} + p_5^2 z^{100} + p_6^2 z^{125} + p_7^2 z^{150})$$
$$= p_1 z^0 + p_2 z^{12.5} + p_3 z^{25} + p_4 z^{37.5} + p_5 z^{50} + p_6 z^{62.5} + p_7 z^{75} + p_8 z^{87.5} + p_9 z^{100} + p_{10} z^{112.5}$$
$$+ p_{11} z^{125} + p_{12} z^{137.5} + p_{13} z^{150} + p_{14} z^{162.5} + p_{15} z^{175} + p_{16} z^{187.5} + p_{17} z^{200} + p_{18} z^{212.5} + p_{19} z^{225}$$

where

$$p_1 = p_1^1 p_1^2, p_2 = p_2^1 p_1^2$$
$$p_3 = p_1^1 p_2^2 + p_3^1 p_1^2, p_4 = p_2^1 p_2^2 + p_4^1 p_1^2$$
$$p_5 = p_1^1 p_3^2 + p_3^1 p_2^2 + p_5^1 p_1^2, p_6 = p_2^1 p_3^2 + p_4^1 p_2^2 + p_6^1 p_1^2$$
$$p_7 = p_1^1 p_4^2 + p_3^1 p_3^2 + p_5^1 p_2^2 + p_7^1 p_1^2, p_8 = p_2^1 p_4^2 + p_4^1 p_3^2 + p_6^1 p_2^2$$
$$p_9 = p_1^1 p_5^2 + p_3^1 p_4^2 + p_5^1 p_3^2 + p_7^1 p_2^2$$
$$p_{10} = p_2^1 p_5^2 + p_4^1 p_4^2 + p_6^1 p_3^2$$
$$p_{11} = p_1^1 p_6^2 + p_3^1 p_5^2 + p_5^1 p_4^2 + p_7^1 p_3^2$$

$$p_{12} = p_2^1 p_6^2 + p_4^1 p_5^2 + p_6^1 p_4^2, p_{13} = p_1^1 p_7^2 + p_3^1 p_6^2 + p_5^1 p_5^2$$
$$p_{14} = p_2^1 p_7^2 + p_4^1 p_6^2 + p_6^1 p_5^2, p_{15} = p_3^1 p_7^2 + p_5^1 p_6^2 + p_7^1 p_5^2$$
$$p_{16} = p_4^1 p_7^2 + p_6^1 p_6^2, p_{17} = p_5^1 p_7^2 + p_7^1 p_6^2$$
$$p_{18} = p_6^1 p_7^2, p_{19} = p_7^1 p_7^2$$

Steady state MSS availability for the constant demand $w = 206.44$

$$A_\infty(w) = \delta_A(U(z), w) = \delta_A(\textstyle\sum_{i=1}^{19} p_i z^{g_i}, 206.4)$$
$$= p_{18} + p_{19} = p_6^1 p_7^2 + p_7^1 p_7^2$$
$$= 0.0004126$$

Mean Steady state performance

$$E_\infty = \textstyle\sum_{i=1}^{k} p^i g^i = 91.14MW$$

Expected steady state MSS performance deficiency For w = 206.4 MW

$$D_\infty(w) = \textstyle\sum_{i=1}^{19} p^i max(206.4 - g^i, 0) = 114.97MW$$

5 Conclusion

Here a combination of stochastic process and UGF technique is applied for analysis of a real data of a power station by decomposing the system in to two sub systems. Steady state probabilities of the subsystems and steady state reliability indices of the power station are evaluated. Mathematical model based on straight forward stochastic process is not effective enough for system with several components with huge number of states. A new approach has been introduced in this paper by decomposing the entire MSS in to several subsystems. By using the method of combination of Markov process and UGF technique, analysis of system has been greatly simplified and reliability indices of MSS with minimal repair can be determined easily.

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