Modeling of Fatigue Life of Unidirectional Fibrous Composite by Daniels' Epsilon-Sequence Under Random Loading

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Abstract

The modified Daniels' epsilon sequence (DeS) is used for an analysis of a residual static strength, a fatigue life, a fatigue strength of an unidirectional fiber composite (UFC) considered as some parallel system subjected to cycling loading with a random value of a cycle parameter. Some example of the processing of data set of a carbon-fiber reinforced composite is given.

Keywords: Daniels' sequence, strength, fatigue life, composite

1. Introduction

Here we give a very short review and some corrections of the basic ideas of the application of the definition of the Daniels' sequence (DS) and its modification to study a fatigue life of an unidirectional fibrous composite (UFC) considered as some parallel system subjected to cycling loading. Lately, the composite is widely used, in particular, in aviation. Therefore, a study of a strength and a fatigue life of these materials is very urgent. The first scientific publication appears to be the Peirce's work [1]. Peirce gives an approximate formula for the average strength of a bundle of longitudional items (LIs) (fibers or bundles) forming a foundation of the unidirectional fibrous composite (UFC). A correctness of the normal approximation of the strength distribution law of a LI parallel system was proved by Daniels [2,3]. His result was refined by Smith [4] already with a reference to the series-parallel system, which was earlier proposed in [5]. A lot of the papers are devoted to a reliability of the composite. A detail review, for example, is given in [6]. But here we refer only to the papers connected with the DS and its modification.

The concept of the basic DS with an application to a description of the process of a fatigue failure was first introduced in [7]. The UFC is a composition of the LIs immersed into a composite matrix (CM). Here we suppose that only LIs carry the longitudinal load. In the DS approach a composite specimen for the static strength or the fatigue life tests is considered as a series-parallel system: series system of n_L "links" of the same length but the every link is a parallel system, or more specifically, a bundle of n_C LIs. There is some probability that inside of some link there is some weak micro volume (WMV) the failure of which is a failure of a link and the failure of the tested specimen also. In fact we consider a composite as a random series of the WMVs. This paper is devoted to the fatigue life and tensile strength of one WMV. The relation of the cumulative distribution function (cdf) of the strength of WMV and of the specimen on the whole, the case when the failure of the matrix is considered as failure of the specimen also are studied in [8].

A successful application of the DS approach to the explanation of the relation between the static strength of LI and the fatigue life of UFC and the desciption of mean fatigue curve, SN (stress-number of cycles), is discussed in [8,9]. A more general definition of the DS is considered in [10]. It can be used to study the UFC falure process under any type of the loading process which can be described by a sequence of the load both in the fatigue and the static strength tests. In the paper [11] the definition of *the Daniels' residual strength* was introduced. Generally, the residual strength is described **after the definition of the SN curve**. But here the SN curve is defined **after the calculation of the residual strength**. Finally, in [11, 12] the definitions of the Daniels's epsilon sequence (DeS) was given (it is necessary to note that as consequence of some mistake the definitions of the DeS in [11] is given only in russian version of this paper).

The DS approach describes the structure of the damage accumulation process, gives the explanation of a *fatigue strength* existance (maximum value of a fatigue cycle parameter (cycle maximum, for example) with the infinite fatigue life; it is called also a *fatigue limit*). But the numerical results are very poor. Calculated the DS step number up to failure is very small or just tends to the infinity. More appropriate numerical results can be get using additionaly the theory of semi-Markov process [11, 12]. But the use of this theory requires some additional assumptions. This paper is devoted to some investigation of the results which can be get without the use of this theory. The use of the DeS approuch gives more easier solution.

Usually, only the mean value of fatigue life and the mean SN curve are taken into account. It is supposed also that during the fatigue test the parameter of the cycle load is some constant. But it appears that a great scatter of the fatigue life (particularly, at high level of the fatigue load) can be explaned only by the assumption the randomness of this parameter. This is the reason to develop a specific modification of the DeS when the sequence of loads is a sequence of random variables. This modification is the object of the investigation in this paper. A numerical example is given.

2. The modified Daniel's epsilon sequence

Here we consider a modified definition of DeS. It can be used in section 3 for processing the result of the fatigue test under cycling loading with the random parameter of every cycle.

As it was mentioned already the connection of the static strength of a LI and a strength of a bundel of them was studied by Daniels [2, 3]. Let the strengths of n_C LIs of some WMV, $X_1, X_2, ..., X_{n_C}$, are the independet random variables with the same cumulative distribution function (cdf), $F_X(x)$, and $X_{(1)}, X_{(2)}, ..., X_{(n_C)}$ are the corresponding ordered statstics. Daniels showed that the random variable

$$R_D = \max(X_{(k)}(n_C - (k-1))/n_C:\ 1 \le k \le n_C) \tag{1}$$

has an asymptotically normal distribution with the average and the standard deviation

$$\mu_D = \max x(1 - F_X(x)) = x^*(1 - F_X(x^*)),\quad \sigma_D = (\mu_D\, x^* F_X(x^*)/n_C)^{1/2}. \tag{2}$$

By "unwrapping" this model in time for specific sample $x_{1:n_C} = (x_1, ..., x_{n_C})$ which is a realization of the random vector $X_{1:n_C} = (X_1, ... X_{n_C})$ and assuming that the process of loading is described by sequence $s^+_{0:\infty} = \{s^+_0, s^+_1, s^+_2, ...\}$, where s^+_i is a realization of some random variable, we obtain a sequence of the local stresses $s_{0:\infty} = (s_0, s_1, s_2, ...)$ (in the WMV where the damage develops) described by the equation

$$s_0 = s^+_0,\quad s_{i+1} = s_i + \varepsilon(s^0_{i+1} - s_i) = (1-\varepsilon)s_i + \varepsilon s^0_{i+1},\quad i = 0, 1, 2, ..., \tag{3a}$$

where

$$s^0_{i+1} = \begin{cases} k_C s^+_{i+1}/(1 - \nu(s_i)/n_c) & if\ \ k_C s^+_{i+1}/(1 - \nu(s_i)/n_c) > s_i, \\ s_i\ \ if\ \ k_C s^+_{i+1}/(1 - \nu(s_i)/n_c) \le s_i, \end{cases} \tag{3b}$$

$\nu(s_i)$ is a number of the LIs the strength of which is lower than s_i.

Let us note that the definition of the DeS which was given in [11, 12] is true ONLY for the loading $s^+_{0:\infty} = (s^+_0, s^+_1, s^+_2, ...)$ under condition that $s^+_{i+1} \geq s^+_i$, $i = 1, 2, 3, ...$ In application to the loading of UFC we must take into accout that the number of the LI failures can not decrease. The equation (3b) ensures this condition and in the case when the event $s^+_{i+1} < s^+_i$, $i = 1, 2, 3, ...$ can take place. This event can take place not only in specific program of load but and in the case when the random variables S^+_{i+1} and S^+_i, $i = 1, 2, 3, ...$, have the same cdf. This is the reason to include the equation 3(b) in the general definition DeS .

The new definition of the DeS can be used in any loading $s^+_{0:\infty}$ But its application to the fatigue and the static strength tests is the most important.

1) **The fatigue test.** Later on we make processing of the test data set in order to get SN curve. In this case $s^+_i = s^+$, $i = 0, 1, 2, 3, ...$, where s^+ is the some parameter of the cycling loading. It is some constant. For example, it is the maximum stress of the cycle load.

2) **The static strength test**. In this case $s^+_0 \leq s^+_1 \leq s^+_2, ...$ and the items of sequence $s^+_{0:\infty}$ increase up to infinity.

The sequence of the local stresses is called *the Daniels' sequence for a constant load* (DeS_CL) if s^+_i is some constant. It is called *the Daniels' sequence for an enlarging external load* (DeS_EL) in the loading process of the second type. The realizations of these random processes are defined by the pair $(s^+_{0:\infty}, x_{1:n_C})$.

For **the specific pair** $(s^+_{0:\infty}, x_{1:n_C})$ of the DeS_CL let us consider the following definitions.

1. *The* K *-Daniels' function*

$$r_{DK}(k) = x_{(k)}(1 - \nu(x_{(k)}) / n_C) \tag{4}$$

where $x_{(K)}$ *is the k th ordered statistic of the sample* $x_{1:n_C} = (x_1, ..., x_{n_C})$, $\nu(.)$ *is the function defined in* (3b).

2. *The S-Daniels' function*

$$r_{DS}(s) = s(1 - \nu(s)) / n_C). \tag{5}$$

3. *The Daniels' residual function.*

$$r_{MX}(x) = \max_{s > x} r_{DS}(s) \tag{6}$$

Under condition that the failure of LI does not take place at the preliminary fatigue loading, $s^+_{0:i} = (s^+_0, s^+_1, s^+_2, ..., s^+_i)$, and the DeS reaches the value s_i then this function defines *the Daniels'conditional loading rate-independent residual strength,* $r_{MX}(s_i)$.

Usually, during static strength test we have $s^+_i = (i-1)\delta^+_S$, where δ^+_S, $\delta^+_S > 0$, is the rate of loading. In order to take into account the specific process of a static test loading we must consider the loading defined by two sequences: preliminary fatigue loading, $s^+_{0:i}$, and loading at the static strength test, $s^{++} = (s^+_i, s^{++}_1, s^{++}_2, ...)$ where s^{++}_i tends to infinity.

The corresponding *Daniels ultimate residual strength*

$$r_{DU} = s^{++}_{n^U} \tag{7}$$

where $n^U = 1 + \max(i : r_{MX}(s_i) > s^{++}_i, \ i = 1, 2, ...)$.

If the prelimiunary loading does not exist, then the equation (7) defines *the DeS-ultimate static strength*.

4. The transition from s_i to s_{i+1} we call *a step of the DeS*. At least three definitions of the number of steps , i_f , corresponding to the *failure of all LIs* can be considered. They correspond to three conditions which appears first time: 1) $\bar{\nu}(s_{i_f}) = \nu(s_{i_f}) / n_C = 1$; 2) $s_{i_f} = \infty$; 3) $r_{MX}(s_{i_f}) = 0$. If one (or another) of these conditions is met we say that *failure of the DeS takes place*. Actually, for real calculation we must use some critical values $\bar{\nu}_c$, s_C and r_C close to 1, to the very large or very small

value instead of 1, ∞ and 0 correspondingly.

As a matter of fact, these three conditions give very close *DeS fatigue life* values. Here we use the last condition and the following definition of the *the DeS fatigue lives*

$$n_{DR} = 1 + \max(i : r_{MX}(s_i) > r_C,\ i = 0,1,2,...) . \tag{8}$$

5. For the case of the DeS_CL let $S^+_{ND\infty}$ is a set of s^+ for which there is a solution of the equation

$$x = s^+ / (1 - \nu(x)/n_c) . \tag{9}$$

If $s^+ \in S^+_{ND\infty}$ there is such i^* that $s_{i^*+1} = s_{i^*}$. Then $r_{MX}(s_{i^*+1}) = r_{MX}(s_{i^*})$. The change of the items of the process $\{s_i,\ i = 0,1,2...\}$ and the process $\{r_{MX}(s_i),\ i = 0,1,2,...,\}$ will be stopped. Some LIs never will be destroyed. And we say that the DeS fatigue life will be equal to infinity.

If this set is not empty *the DeS fatigue strength (the fatigue limit) is defined by the equation*

$$s_D = \max(s^+ : s^+ \in S^+_{ND\infty}) . \tag{10}$$

Let us remark that the value $\bar{\nu}(s_i)$ can be considered as a measure of failures accumulated up to moment when the DeS reaches the value s_i. We say that it is a value of the *failure functio. of two sequences*, $x_{1:nc}$ *and* $s^+_{0:i}$: $\bar{\nu}(s_i) = F_{XS^+}(x_{1:nc}, s^+_{0:i})$. *The valueof function* $F_{XS^+}(.)$ *is the measure of the accumulated damages of any loading process.* It chages in interval $[0,1]$. The value unit (or very close to unit) corresponds to the DeS failure.

For the DeS_EL *the intact function*

$$k_R(x) = \begin{cases} k & if \quad r_{MX}(x) > x, \\ 0 & if \quad r_{MX}(x) \le x. \end{cases} \tag{11}$$

is very important also. It defines the number of of ILs which are still intact under steady increasing load x. If we try to increase the load more than $r_{MX}(x)$ then the failure of all the LIs takes place.

The examples of the introduced functions and definitions are given in the next section.

The introduced definitions are connected primarily with fatigue test at the constant cycle parameters. But the equations and definitions (3a, 3b,4,…,8) can be used for calculation the residual strength and the fatigue life for any specific realization of any random process of loading. In numerical example we consider specific random sequence of loads ($S_0^+, S_1^+, S_2^+, ...$) where all random variables have the same cdf. In this case equations (9,10) can not be used. The fatigue strength (the fatigue limit) does not exist because after some event $s_{i+1} = s_i$ the process of increasing of items of DeS can continue and condition of the failure can be reached. But using the Monte Carlo method we can calculate the cdf of the random fatigue life N_{DR} for any cdf of random variable $S_i^+,\ i = 0,1,2,...$:

$$F_{N_{DR}}(x) = \sum_{j=1}^{N_{MC}} e(x - (n_{DR})_j) \tag{12}$$

where $e(x) = 0$ *if* $x < 0$ and $e(x) = 1$ if $x \ge 0$; N_{MC} is the number of Monte Carlo tials; $(n_{DR})_j$ is the n_{DR} for ith specific realization $(x_{1:nc}, s^+_{0:i})_j$, $j = 1,2,...,N_{MC}$, of the random pair $(X_{1:n_C}, S^+_{0:i})$.

3. Application to experimental data

3.1. Some additional assumptions

If $s^+ = Const$ already the DS approuch allows to explain the structure of a fatigue phenomenon: the DeS fatigue life function of s^+ is similar to SN curve; the DS approuch allows to explain the existence of the fatigue strength (fatigue limit); the S-type changes of DS realization are similar to changes of some physical parameter during fatigue loading,… But the numerical results are very pure: the values of DS fatigue limit is too large, the number of the DS steps to the failure is very small even if s^+ is very close to the fatigue strength, The DeS approach allows to get more appropriate value of fatigue life but still we need to make some additional „patches" assumptions in order to make the DeS mathematics useful for the numerical description of the fatigue phenomenon.

1) The accumulation of the damages takes a place only in **local WMV** in which there is some stress concentration. It can be different in the fatigue and in the tension tests. For the fatigue test we assume that the **local external stress** is k_c time more than external load.

2) The cdf of the local static strength does not coincide with the cdf of the static strength of the single LI: the "size" of the WMV and the adjacent LIs have a specific influence on the local static strength cdf which we denote by $F_{X_L}(x)$.

3) Equations (3a, 3b) defines new value of the local stress, s_{i+1}, under assumption that $\nu(s_i)$ LIs are failed already. But really for the destruction of these LIs some accumulation of an energy and some time are needed. For this reason for DeS approach in equations (3) the value ε is introduced. We assume that it is some function of s^+, $\varepsilon_D(s^+)$.

Usually, in order to get the description of the SN curve we suppose that during the fatigue test the parameter of the cycle, s^+, is some constant. But processing of the several fatigue test results shows that in this case it is difficult to explain a very large variance of the fatigue life, particularly at high level of the load (see Fig. 6). Here we suppose that really there is a randomness of the load. We assume that the load in every ith DeS step , s_i^+, is a realization of some random variable, S_i^+, $i = 0,1,2,...$. All the variables have the same cdf $F_S(x)$.

For the Monte Carlo calculation of the realization of the DeS for the specific loading, $(s_o^+, s_1^+, s_2^+,...)$ which is the realization of some random sequence $(S_o^+, S_1^+, S_2^+,...)$, now we should use the equation

$$s_{i+1}^0 = \begin{cases} k_C s_{i+1}^+ / (1-\nu(s_i)/n_c) & if \quad k_C s_{i+1}^+ / (1-\nu(s_i)/n_c) > s_i, \\ s_i \quad if \quad k_C s_{i+1}^+ / (1-\nu(s_i)/n_c) \le s_i, \end{cases} \tag{3c}$$

where $s_0 = s^+$, k_C is a stress concentration factor (let us note that in [12] it was not defined that $s_0 = s^+$).

3.2. Numerical example

We consider the experimental fatigue data taken from Tables 1- 3 in Ref. [14] concerned T300/934 graphite/epoxy laminates with $[[0/45/90-45_2/90/45/0]_2$ lay-up. In Table 1 of this paper the static strength of 25 specimens, in Table 2 in terms of cycles to failure at three different stress levels (namely: for $\sigma_{\max}$ = 400, 380 and 290 MPa), $R = \sigma_{\min}/\sigma_{\max} = 0$; and in Table 3 two sets of residual strength data are reported for 15 and 18 specimens subjected to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 and 345 MPa, respectively. Actually, the previous equations for calculation of the Daniels' fatigue life can not be used for this type of composite structure. It is not structure of the UFC. But we suppose that the failure of this composite takes the place after the failure of some its WMV which is the bundle of the n_C parallel LIs. We make (enough rough) assumption that there is some *strength-equivalent WMV* which has the same distribution of static and fatigue strength for every process of the loading.

Now we need to know the distribution of the static strength of the single LI of this equivalent WMV. There is not this data in [14]. But we get the estimates of the parameters of a Weibull distribution of the static strenth of the LIs of an equivalent bundle which has the same parameters of the static strength (the mean and the standard deviation) as a real specimen. It can be made using the Daniels theorem (see equation (1)). If the strength of the LI has the Weibull distribution with cdf

$$F(x) = 1 - \exp(-\exp((\log(x) - \theta_0) / \theta_1) , \tag{10 }$$

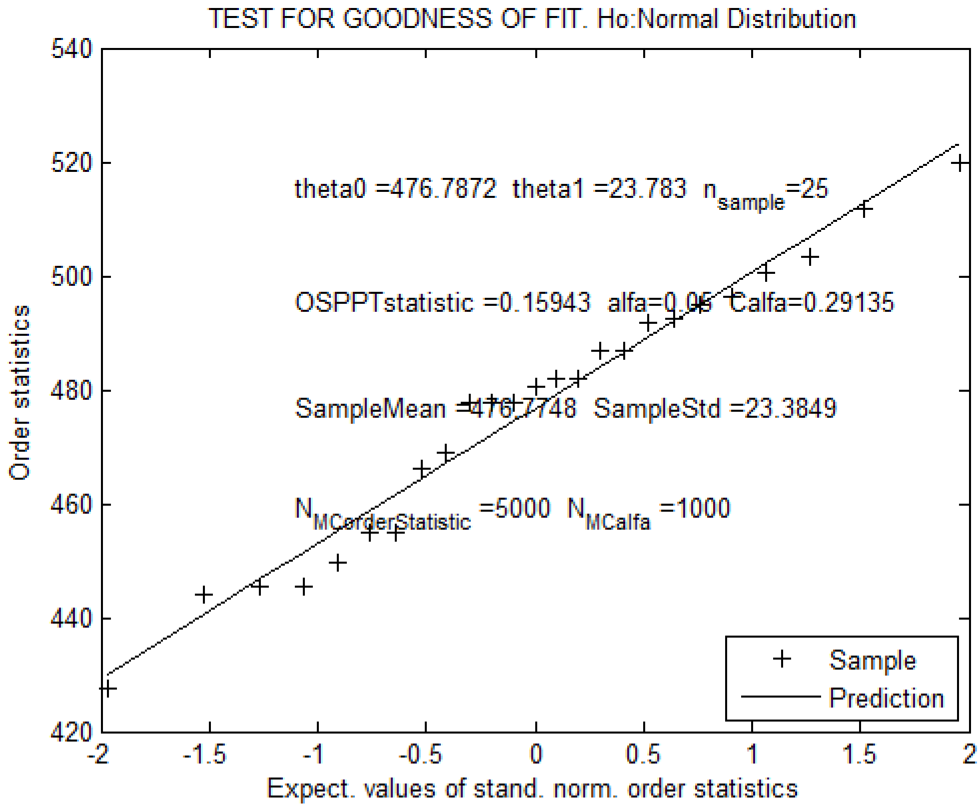


Fig.1. The OSPPT plot. The parameters of the sample of the static strength (Table 1 of [14]); The test for goodness of fit for normal distribution .

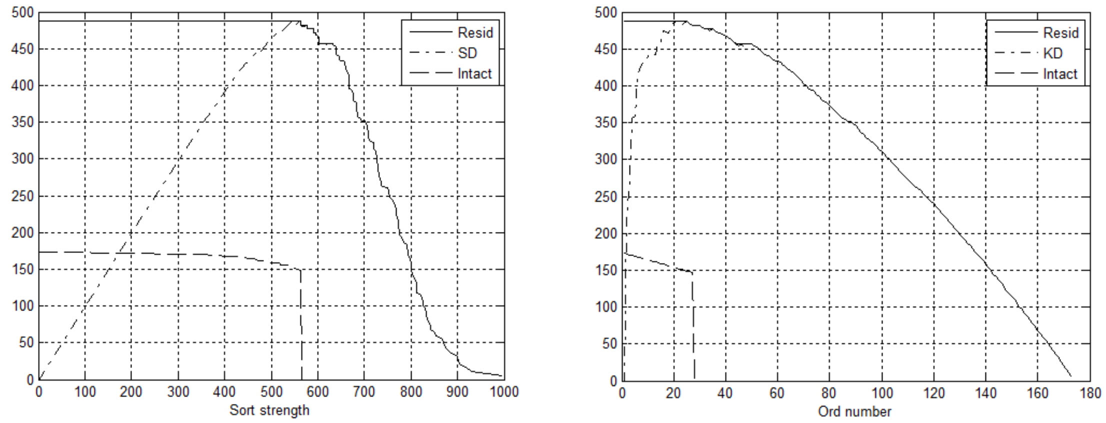
then approximately, in accordance with this theorem, the strength of the bundle has normal distribution with mean $\mu_D = \exp(\theta_0 + \theta_1(\log(\theta_1) - 1))$ and standard deviation $\sigma_D = \mu_D(\exp(\theta_1 - 1) / n_C)^{1/2}$. And vice versa, if μ_D an σ_D are known, and θ_1 is known also, we can get the equations for the θ_0 and the equivalent size of the WMV, n_C :

$$\theta_0 = \log(\mu_D) - \theta_1(\log(\theta_1) - 1), \quad n_C = \exp(2\log(\mu_D) + \theta_1 - 2\log(\sigma_D) - 1) . \tag{ 11}$$

In Fig. 1 the fitting of the static strengths of the 25 specimens (from Table 1 of [14]), the parameter of this sample (the mean, theta0= $\mu_D = 476.8$, and the standard deviation, theta1= $\sigma_D = 23.78$), and data for testing of normal distribution hypothesis are given. The statistic of the OSPPTest [8] is equal to 0.159. The boundary of critical region of the test for goodness of fit is equal to 0.291. So, the normal hypothesis is not rejected. The parameter θ_1 in equation (11) remains unknown but it can be chosen for a best fitting of experimental fatigue data.

Approximate estimate of parameter θ_1 can be found in other publications. In [10] there is a result of fitting of the sample of strength of 64 specimens of some carbon-fiber bundles. Using these data taken from [13] for the Weibull cdf in the form (10) it was found the estimates $\theta_0 = 6.5 \quad \theta_1 = 0.13$. For processing the data set from [14] as the first approximation it was set $\theta_1 = 0.15$ Then using (11) we have $\theta_0 = 6.6, \quad n_C = 172$.

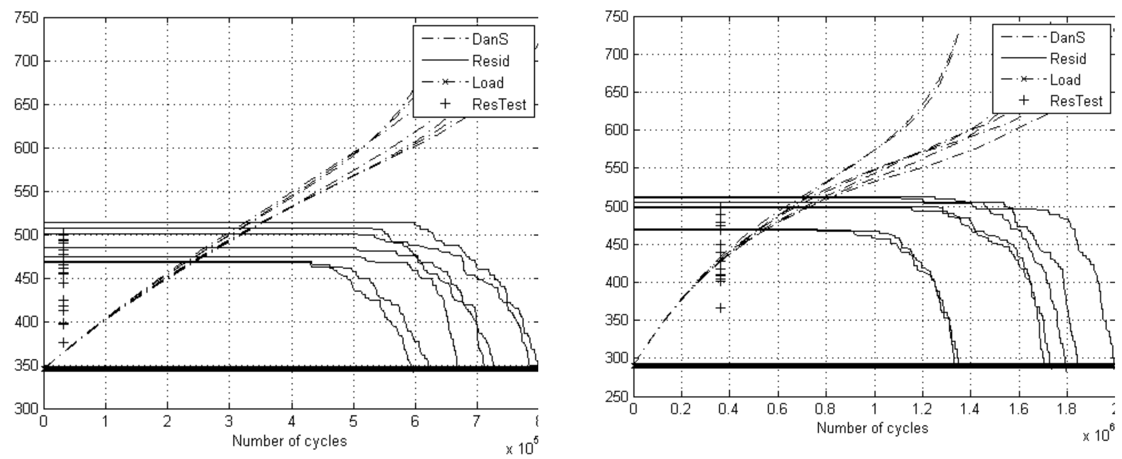
In Fig. 2 we see the Daniels' residual strength (-), the S-Daniels' function (-.-) (a), the Daniels'residual strength (-), the K-Daniels' function (-.-) (b) and the intact number of LIs for continuesly increasing load (--) as a s-function (a) and as a k-function (b). In Fig 2a the corresponding values are the functions of the maximum of strength of the already failed LIs. In Fig 2b they are the function of k, $1 \le k \le n_C$. Let us remind that we can increase the value of the function $r_{DS}(s) = s(1-\nu(s))/n_C)$ only to the value corresponding to the its maximum . Then we need to decreas it because if it remains to be equal to this maximum or more then all the LIs will be destroyed as it is shown by intact function. It worth to note that we see the failure of the WMV after failure of only very small part of the LIs.



a b

Fig.2. The S-Daniels' (a) and The K-Daniels' (b) functions (-.-), the Daniels'residual strength (-) and intact functions (--) .

We make two stages of processing the experimental data. First, we make the processing of the considered fatigue test data assuming that the fatigue test machinery and equipment allow to keep the precise level of the load and the value of the s^+ is a constant. In the Fig. 3 we see the result of the modeling



a b

Fig.3. The modeling of the DeS (-.-), the Daniels'residual strength (-) for $\theta_0 = 6.6$, $\theta_1 = 0.15$ and $n_C = 172$; the level of the load (-x-) and the residual strength experimental data sets (+) corresponding to the cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 (a) and 345 MPa (b), respectively.

of the DeS (-.-), the Daniels'residual strength (-), the test residual strength (+) and level of load (-x-) for $s^+ = 335$ MPa and $s^+ = 290$ MPa. We see that the Daniels' local residual strength does not change a long time but then decreases very drastically. And we see also that the test of the residual strength was made a significant time before the failure time in order to prevent the failure at the preliminary loading.

In Fig. 4 we see the test fatigue life (+), the mean (-), smallest (►) and largest (◄) calculated values using 100 MoneCarlo trials. So the smallest and largest calculated values correspond approximately the probability of the failure equal to 0.01 and 0.99 correspondingly. For the calculation it was used already mentioned parameters: $\theta_0 = 6.6$, $\theta_1 = 0.15$ and $n_C = 172$. Additionally it was used $k_C = 1.9$; the fuction $\varepsilon(s^+) = \varepsilon_{290} + (\varepsilon_{400} - \varepsilon_{290})(s^+ - 290)/(400-290)$ where $\varepsilon_{400} = 0.07$, $\varepsilon_{290} = 0.002$. The residual strength data are shown also for 15 and 18 specimens subjected to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 and 345 MPa, respectively.

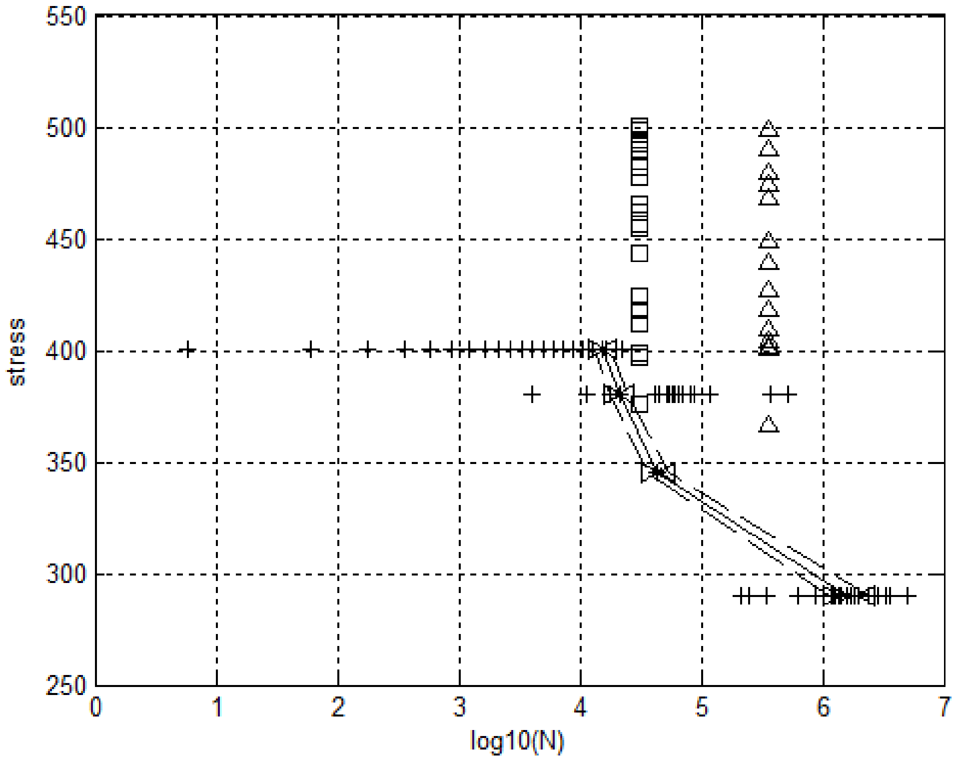
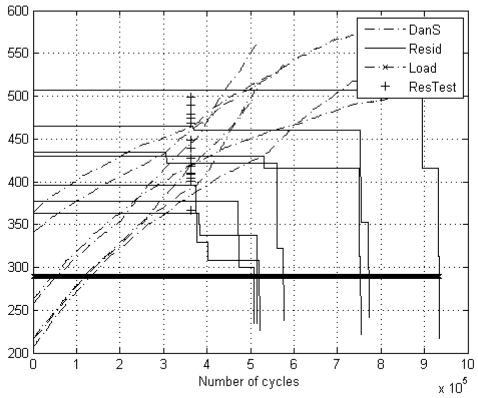


Fig. 4 . The test (+) and calculated mean (-), smallest (►) and largest (◄) fatigue lives for $\theta_0 = 6.6$, $\theta_1 = 0.15$ and $n_C = 172$. The residual strength experimental data sets correspond to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 (▲) and 345 MPa (■), respectively.

In Fig 4 we see the reasonable fitting of the mean fatigue lives and the residual strength. But the scatter of the test fatigue life is much more than calculated one. In order to explain this we make the additional assumption. We assume that the real load in every DeS step, S_i, $i = 0,1,2,...$ is some random variable which has the normal distribution with the cdf $F_{S^+}(x) = \Phi(x - \theta_{0S})/\theta_{1S})$, $\theta_{0S} = s^+$; $s^+ = 400,\ 380,\ 345,\ 290$ MPa for four level of test; θ_{1S} =0.2; $\Phi(.)$ is cdf of the standard normal distribution. It appears also to be necessary to change values θ_0 and n_C and the equation for $\varepsilon(s^+)$. The decreasing of the n_C increses the scatter of fatigue life. But this time even very significant decreasing of the n_C is not enough to explain the real value of this scatter. The change of the n_C is the reason to change another parameters. Now $\theta_0 = 6.375$, $n_C = 5$,

$\varepsilon(s^+) = \varepsilon_{290} + (\varepsilon_{400} - \varepsilon_{290})((s^+ - 290)/(400-290))^{10}$ are used. The final result is shown in the Fig. 5-6 where the same previous notations are used. We see the much better fitting of the residual strength, Fig.5, and of the fatigue life, Fig. 6. It should be noted additionaly that the limitation of the smallest fatigue life by the value 1000 at the $s^+ = 400$ and 380 MPa in Fig. 6 is a 'technical' limitation, because for the initial fatigue life data time unit the 1000 cycles was chosen.



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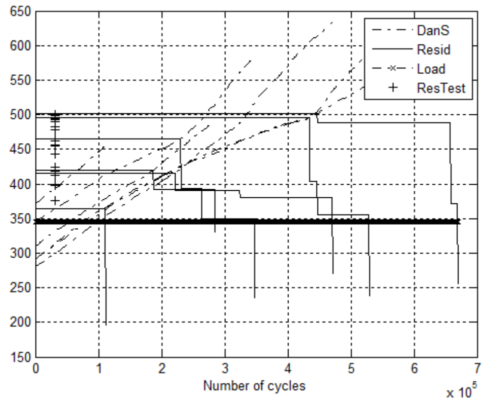

b

Fig. 5 . The modeling of the DeS (-.-), the Daniels'residual strength (-) for $\theta_0 = 6.375$, $\theta_1 = 0.15$ and $n_C = 5$; the level of the load (-x-) and the residual strength experimental data sets (+) corresponding to the cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 (a) and 345 MPa (b), respectively.

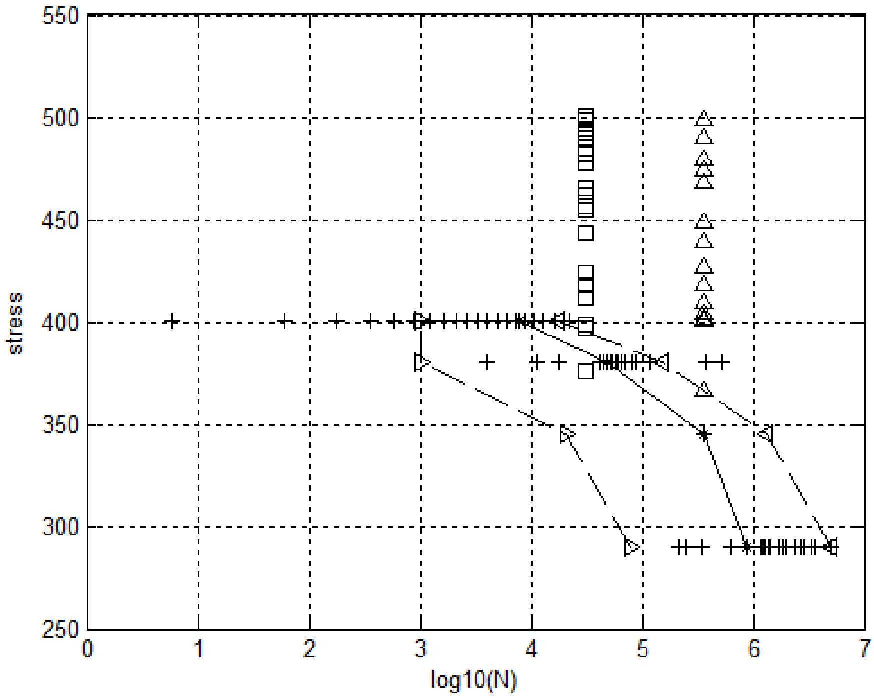

Fig. 6 . The test (+) and calculated mean (-), smalest (►) and largest (◄) fatigue lives for $\theta_0 = 6.375$, $\theta_1 = 0.15$ and $n_C = 5$. The residual strength data corresponding to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max} = 290$ (▲) and 345 MPa (■), respectively.

For the case when s^{+} is some constant from the set 400, 380, 345 and 290 MPa the calculated fatigue life is final with probability one. But now the probability to have final fatigue life for the load level 290 MPa is equal only 0.97. The corresponding part of the cdf of fatigue strength is shown in Fig. 7

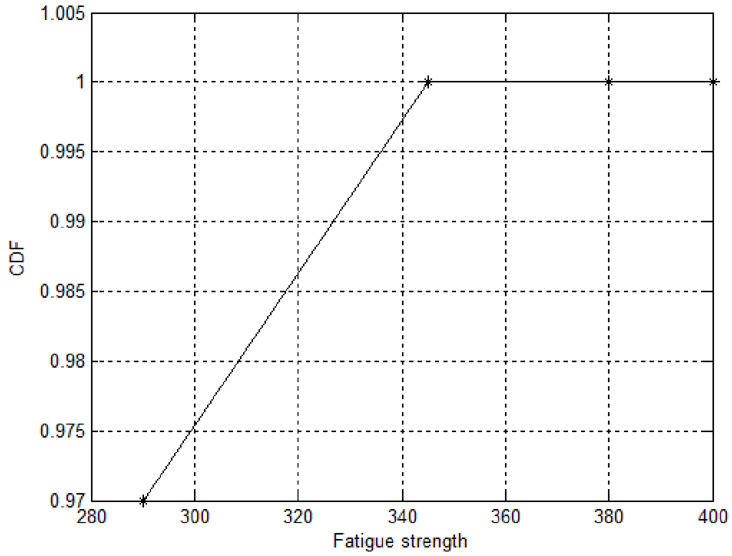

Fig. 7. The cdf of fatigue strength .

Conclusions and areas for further research

It was shown that the DeS approach to the description and modelling of the fatigue life and the fatigue strength of UFC allows:
1) to explain some specific features of the fatigue tests :
a) the condition of the existence of the fatigue strength and the possibility to have the infinite fatigue life for the determined cycling loading, $s^{+} = Const$
b) the specific features of the residual strength behaviour: the strength degrades smoothly in the first time part of the loading with a sudden drop accuring just before the failure. Even the assumption that the residual strength does not change up to failure ("the rectangle hypothesis of the residual strength") is not too far from the truth. It is not the real problem to estimate the "rectangle" residual strength but the estimation of the time to failure is the real problem.
2) to make the interpretation of the parameters of the studied models as the parameters of the local static strength (this is the main difference of models of the DeS approach from the others models). The difference of the parameters of the cdf of the local strength of the LIs in the framework of the UFC and ones of the isolated single LIs do not allows to make prediction of the fatigue parameters of the UFC using the static strength parameters of the LI. These parameters can be used as first approximations of the corresponding parameters of nonliner regression analysis of the data set.
3) to calculate the measure of accumulated fatigue damage $\bar{\nu}(s_i) = F_{XS^{+}}(x_{1:nc}, s_{0:i}^{+})$ for any *cycling loading* $s_{0:i}^{+}$ It worth to note that just as the Palmgren_Mainer measure, $\sum_j n_j / N_j$, the value $\bar{\nu}(s_i)$ changes in interval $[0,1]$, the condition $\bar{\nu}(s) = 1$ is the condition of the DeS failure.

. The considered here the modified version of the DeS model with random loading gives the explanation of the great scatter of the fatigue life. Usually we suppose that the specimen with the higher initial static strength has also longer fatigue life (Strength-Life Equal Rank Assumption). In the studied here model it is not necessary true because different specimens have the different random loading history.

For different levels of an adequacy of an analysis result the different modifications of the DeS models can be used. In the simplest case it can be set $n_C = \infty$ and the $F_X(s)$ can be used instead of $\bar{\nu}(s)$ in equations (3a, 3b). For any realization of load sequence the value $\bar{\nu}(s)$ is easily calculated but the search of the parameters of the corresponding nonlinear regression for the above described models is a difficult task. Against all the odds, we think that, in due course, the structure of the models suggested will be of the interest not only for the graduation theses of the students but also for the engineering applications, in particular, for the prediction of the variations in the parameters of the strength and the fatigue life of the UFC after the changes in the parameters of their components.

The considered approach seems quite promising yet it requires serious stydy of the cdf of local static strength distribution (the Weibull distribution is not appropriate enough) and the connection of the number of DeS steps and the number of the cycles of the fatigue loading.

References

[1] Peirce F.T. The Weakest Link. Theorem on the strength of long and composite specimens // J. of the Text. Inst. transaction. -1926.- 17(7). - P.355 – 68.

[2] Daniels H.E. The statistical theory of the strength of bundles of threads // Proc. Royal Soc. London, Ser. A.-1945.- No. 183.- P. 405-435.

[3] Daniels H.E. The maximum of a Gaussian process whose mean path has a maximum, with an application to the strength of bundles of fibers // Advances in Applied Probability. -1989.- Vol. 21, No. 2. - P. 315-333.

[4] Smith RL. The asymptotic distribution of the strength of a series-parallels system with equal load sharing // Ann. Prob. - 1982. - (10). – P. 137-171.

[5] Gucer D.E., Gurland J. Comparison of the statistics of two fracture modes // J. Mech. Phys.Solids. - 1962.- 10.- P. 365-373.

[6] Philippidis T.P., Passipoularidis V.A. Residual strength after fatigue in composites: Theory vs.exsperiment // International Journal of Fatigue .-2007.- Vol.29.—P. 2104-2116.

[7] Paramonov Yu., Kleinhofs M., Paramonova A. Markov model of connection between the distribution of static strength and fatigue life of a fibrous composite.// Springer Science+Business Media, Inc, Translated from Mekhanika Kompozitnykh Materialov. – 2006. - 42(5) – P. 615-630.

[8] Paramonov Yu, Kleinhofs M. Models of reliability of composite. Lambert. Saarbrucken, Germany. 2014.

[9] Paramonov Yu., Cimanis V., Varickii S. Aproximation of fatigue curve and fatigue limit of fibre composite using random Daniels' sequence and Markov chains // Aviation.-2013.- Vol. 17, No. 3 — P. 91–97.

[10] Paramonov Yu., Cimanis V., Varickii S., Kleinhofs M. Modelling the strength and fatigue life of an unidirectional fibrous composite by using Daniels' sequence and Markov chains // Mechanics of Composite Materials. -2013.-Vol. 49, No. 5.-- P. 821-838.

[11] Paramonov Yu., Cimanis V., Varickii S., Kleinhofs M. Modelling of the residual strength of an fibrous composite using the residual Daniels' function // Mechanics of Composite Materials. -2016.-Vol. 52, No. 4.-- P. 701-716.

[12] Paramonov Yu., Daniels' epsilon-sequence and modelling of reliability of unidirectional fibrous composite // - 2016. RT&A, Vol. 11, - P.29-41.

[13] Kleinhofs M. Investigation of static strength and fatigue of composite material used in aircraft structure. Candidate degree thesis, Riga, 1983.

[14] D' Amore A.,Georgio M.,Grassia L. Modelling the reridual strength of carbon fiber reinforced composites subjected to cycling loading. Int. J. Fatigue 78, 31-37 2015.