# A Queueing Network Model for Delay and Throughput Analysis in Multi-hop Wireless Ad Hoc Networks

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**Abstract**

In this paper we present a queueing network model for computing average end-to-end delay, and maximum throughput that can be attained in random access multi-hop wireless ad hoc networks with stationary nodes under two popular contention resolution schemes namely, Binary Exponential Back-off (BEB) and Double Increment Double Decrement (DIDD) rules. This model takes into consider some realistic features of the system like (i) the generation of different classes of packets at nodes, and (ii) the dependence of the transmission time and transmission probability on the distance between the transmitter and receiver. Probability distributions and the associated measures of characteristics of the time spent by a packet at arbitrary node are analytically derived by using phase type random variate theory, which in turn are used for the computation of average end-to-end delay and maximum achievable throughput. Theoretical results are numerically illustrated.

**Keywords:** queue, network, phase type distribution, Markov chain

## 1 Introduction

A multi-hop wireless ad hoc network is a collection of nodes that communicate with each other without any established infrastructure or centralized control. Due to the limited transmission range of wireless network interfaces, multiple network hops may be needed for one node to exchange data with another across the network. Thus, in this network, the packets may have to be forwarded by several intermediate nodes before they reach their destinations and therefore each node operates not only as a host but also as a router. Hence, each node may act as a source, destination or relay. For a detailed description of some of the situations where ad hoc networks can be used, refer [7].

The wireless medium is shared and scarce. Multiple stations may want to transmit data frames at the same time over the same channel. So, multiple access protocols are needed to coordinate the transmissions. Since ad hoc networks lack infrastructure and centralized control, these protocols should be distributed. IEEE 802.11 protocol has been set up for fixing international standards for Wireless Local Area Networks (WLAN's). In the 802.11 protocol, the fundamental mechanism to access the medium is known as the distributed co-ordination function (DCF). DCF is a random access scheme based on the carrier sense multiple access with collision avoidance (CSMA/CA) protocol.

According to DCF basic access mechanism, each station with a packet, ready for transmission, monitors the channel activity and if the channel is found to be idle for a pre-determined period called distributed inter-frame space (DIFS), transmits the packet. Otherwise, if the channel is sensed busy, the station initializes its back-off timer and defers transmission for a randomly selected back-off period to minimize the collisions. At each time point at which the channel is monitored, the back-off counter is decremented when the medium is idle, and is frozen when the medium is sensed busy. The timer resumes only after the medium has been idle for a period longer than DIFS. The station whose back-off timer expires first begins transmission and the other stations freeze their timers and defer transmission. Once the current transmission gets completed, the back-off process repeats again and the remaining stations reactivate their back-off timers. Upon the successful reception of a packet, the destination sends back an immediate acknowledgement (ACK) after a time interval equal to short inter-frame space (SIFS). In addition to the basic access mechanism, another optional method called ready-to-send/clear-to-send (RTS/CTS) mechanism is also adopted under DCF. According to this, a node operating in RTS/CTS mode, before transmission, reserves the channel by sending a special ready-to-send short frame and the destination node acknowledges the receipt of the same by sending back a clear-to send frame. After this, the normal packet transmission and ACK response occur. Since collision may occur only on the RTS frame, and it is detected by the lack of CTS response, the RTS/CTS mechanism allows to increase the system performance by reducing the duration of a collision when lengthy messages are transmitted. More importantly, to some extent, the RTS/CTS mechanism adopted in the 802.11 protocol is useful to address the so called hidden terminal problem, which was first mentioned by [14]. For more details on hidden node problem, also refer [6].

DCF employs a contention resolution method namely, binary exponential back-off (BEB) rule, to minimize the probability of collisions due to multiple simultaneous transmissions. Under this rule, if a packet is ready for transmission from a node for the first time, contention window size is chosen as $W$ and according to the collision avoidance protocol procedures, a random value for its back-off counter is uniformly selected from 0, 1,2, $W-1$. If the packet meets with a collision in that attempt, the contention window size will be set as $W_1 = 2W$ and a value for back-off counter is selected uniformly from 0, 1,2, $W_1 - 1$ and if it is further included in a collision on its next attempt, the contention window size will be doubled again and this will continue up to a maximum of $m$ collisions. After $m$ unsuccessful attempts, if it is again met with a collision, the contention window size will be fixed as $W_m = 2^m W$. If an attempt results in successful transmission, the contention window size for that node will be reset as $W$. Hence the minimum contention window size $CW_{min} = W$ , and the maximum contention window size $CW_{max} = 2^m W$.

Apart from the BEB scheme, many researchers have proposed different schemes to fix the contention window size in order to enhance the performance of wireless LANs. Of which the DIDD (Double Increment Double Decrement) scheme proposed by [8] deserves special mention. Under this scheme, if a packet meets with collision while it is being transmitted, the contention window size for the next transmission will be doubled as in the case of BEB rule, whereas after a successful transmission it will be halved unlike under BEB scheme where the contention window size is reset to $W$ under the same scenario. For more about other schemes and their detailed performance analysis, refer [8] and the references therein.

Several researchers have attempted to analyse the throughput and packet delay occurring in communication networks. [2] proposed analytical models to learn the IEEE 802.11 protocol under unsaturated traffic conditions for multihop networks. [4] made an attempt to characterize the average end-to-end delay and maximum achievable per-node throughput in random access multi-hop wireless ad hoc network with stationary nodes. They modelled random access multi-hop wireless networks as open $G/G/1$ queueing networks and used the diffusion approximation (see [10]) to derive closed form expressions for the average end-to-end delay. However, none of these aforementioned references has addressed the important problem of finding the probability distribution of the end-to-end delay experienced by the packets in the network.

This article is in the same line with [4]; however, a more detailed and comprehensive delay analysis has been carried out for a multi-hop wireless ad hoc network with stationary nodes under more general and realistic assumptions. More importantly, probability distributions of the time spent by a packet at an arbitrary node from the epoch at which it is ready for transmission till it is successfully transmitted have been derived under both BEB and DIDD rules, as discrete Phase-Type(PH) distributions. Analytical representation of these distributions enable us to compute some important statistical measures like variance and coefficient of variation of the packet waiting time at a node, which in turn could be used in computing the mean total time spent by a packet in the system before it reaches its destination. For more details on PH distributions and their characteristics, see [12] and [11]. Following are some of the highlights of this paper. (i) The analysis aims to capture several salient aspects of wireless networks like the relationship between the probability of successful transmission between two nodes and the distance between them, interferences caused by hidden nodes, generation of different classes of packets at nodes based on the number of hops to be visited etc. (ii) PH- representation of single hop delay under both BEB and DIDD back-off schemes are derived explicitly and the important statistical measures like mean, variance, and coefficient of variation of the single hop delay are computed analytically and are presented in compact form. (iii) By using the diffusion approximation, the important measures like average queue size and mean waiting time of a packet at an arbitrary node are computed.

Though the present paper does not consider the routing algorithms, mobility models, and path length of source-destination pair that are currently applied in ad hoc scenarios, it renders a concrete analytic approach which may be helpful to get approximate solutions to some important measures that decide the performance of ad hoc models. Even though it does not take into account all the features of a practical ad hoc model, it may be treated as an analytical model which help us to get some insight into the performance behavior of a system governed by probabilistic laws. A detailed description of our model is as follows.

## 2 Methods

We consider a wireless ad hoc network with $N$ nodes that are assumed to be uniformly distributed inside a compact set $W \subset R^3$ of unit volume. Each node has an equal transmission range $R$. That is, if a node transmits a packet, it can reach at another node which is at a distance of maximum $R$ units from the source node. Let $r_{ij}$ be the distance between nodes $i$ and $j$. Nodes $i$ and $j$ are called as *neighbours* if they can directly communicate with each other, that is if $r_{ij} \leq R$. The set of neighbours of node $i$ is denoted by $N(i)$ and it is assumed that all neighbours of a node lie inside a sphere of volume $v = \frac{4}{3}\pi R^3 (< 1)$ centered at that node. Since the nodes are distributed uniformly, the number of neighbours is binomially distributed with mean $(N-1)v$. Being an ad hoc network, each node in the network can be a source, destination, or relay of packets. Depending on the number of hops to be traversed by a packet, we classify the packets into $M$ categories. A packet is said to be of class $l$ , $1 \leq l \leq M$, if it has to visit $l$ nodes before reaching its destination. A packet generated at an arbitrary node is assumed to be class $l$, $1 \leq l \leq M$ with probability $c_l$, where $\sum_{l=1}^{M} c_l = 1$. Packets are generated at nodes in the network as a renewal process with rate $\lambda_e$ and coefficient of variation $C_E$ . It is to be noted that, as per our assumption, the process by which an arbitrary node generates class $l$ packets is a renewal process with rate $\frac{\lambda_e c_l}{N}$.

## Computation of forwarding probability

Let $q_{ij}$ be the probability that a packet at node $i$ (either generated at $i$ or received from some other node) is forwarded to node $j$. When node $i$ transmits a packet, any of its neighbours can receive it; however, we assume that the probability that it reaches at a neighbouring node depends on how far the receiving node is from node $i$. More precisely, the probability that the packet reaches at the node which is the $k$th neighbour of $i$ is assumed to be inversely proportional to the average distance between node $i$ and its $k$th neighbour.

By equation (13) in [13] , the average distance between a node and its $k$th neighbour,

$$E(R_k) = R\frac{\Gamma(k+1/3)\Gamma(L+1)}{\Gamma(k)\Gamma(L+4/3)}, \tag{1}$$

where

$$\Gamma(m+1/3) = \Gamma(1/3)\frac{(3m-2)!^{(3)}}{3^m},$$

$L$ is the largest integer less than or equal to $(N-1)v$, $n!^{(l)}$ is the $l$th multifactorial of $n$, and $\Gamma(1/3) \approx 2.6789385347$.

We have

$$q_{ij} = P\{i \to j\} = P\{i \to j | j \in N(i)\} * P\{j \in N(i)\}.$$

By conditioning on the number of neighbours of $i$, we get

$$q_{ij} = \sum_{p=1}^{N-1} P\{i \to j | j \in N(i), |N(i)| = p\} * P\{|N(i)| = p | j \in N(i)\} * P\{j \in N(i)\}. \quad (2)$$

Now

$$P\{i \to j | j \in N(i), |N(i)| = p\} = \sum_{k=1}^{p} P\{i \to j || N(i)| = p, j \ \ is\ the\ \ k\ th\ neighbour\ of\ i\} * P\{j \ \ is\ the\ \ k\ th\ neighbour\ of\ i\ || N(i)| = p\}. \quad (3)$$

We have

$$P\{i \to j || N(i)| = p, j \ \ is\ the\ \ k\ th\ neighbour\ of\ \ i\} = P\{the\ packet\ is\ not\ absorbed\ at\ i\} * \frac{E_p}{E(R_k || N(i)| = p)},$$

where $E(R_k || N(i)| = p)$ is obtained from eqn(1) by replacing $L$ by $p$, and

$$E_p = \left(\sum_{l=1}^{p} \frac{1}{E(R_l || N(i)| = p)}\right)^{-1}$$

is the normalization constant.

Thus

$$P\{i \to j || N(i)| = p, j \ \ is\ the\ \ k\ th\ neighbour\ of\ \ i\} = [1 - P\{the\ final\ destination\ of\ the\ packet\ is\ \ i\}] * U_p(k),$$

where

$$U_p(k) = \frac{E_p}{E(R_k || N(i)| = p)}.$$

So

$$P\{i \to j || N(i)| = p, j \ \ is\ the\ \ k\ th\ neighbour\ of\ \ i\}$$

$$= [1 - \sum_{l=1}^{M} P\{the\ packet\ is\ absorbed\ at\ \ i | the\ packet\ is\ of\ class\ \ l\} * P\{the\ packet\ is\ of\ class\ \ l\}] U_p(k)$$

$$= [1 - \sum_{l=1}^{M} P\{the\ packet\ has\ traversed\ exactly\ \ l\ \ hops\} * c_l] U_p(k).$$

Now $q_{ij}$, the forwarding probability from node $i$ to node $j$ is independent of the particular choice for $i$ and $j$ so that we can remove the suffix to write $q$ instead of $q_{ij}$.

Hence we get

$$P\{i \to j || N(i)| = p, j \ \ is\ the\ \ k\ th\ neighbour\ of\ \ i\} = (1 - \sum_{l=1}^{M} q^l c_l) U_p(k). \quad (4)$$

Substituting eqn (4) in (3) we get

$$P\{i \to j | j \in N(i), |N(i)| = p\} = (1 - \sum_{l=1}^{M} q^l c_l) \sum_{k=1}^{p} P\{j \ \ is\ the\ \ k\ th\ neighbour\ of\ \ i || N(i)| = p\} U_p(k). \quad (5)$$

Now from [9], the average distance between two random points uniformly distributed inside a sphere of radius $r$ is $\frac{72r^2}{35}$.

Therefore

$$P\{j \ \ is\ the\ \ k\ th\ neighbour\ of\ \ i || N(i)| = p\}$$

$$= P\{\ \ among\ \ p-1\ \ neighbours\ (other\ than\ \ j)\ \ of\ \ i\ \ exactly\ \ k-1\ \ lie\ inside\ a\ sphere\ of\ radius\ \ \frac{72R^2}{35}\}$$

$$= p - 1_{k-1} (v')^{k-1} (1-v')^{p-k}, \quad where\ \ v' = \left(\frac{72R}{35}\right)^3.$$

Hence eqn (5) becomes

$$P\{i \to j | j \in N(i), |N(i)| = p\} = (1 - \sum_{l=1}^{M} q^l c_l) \sum_{k=1}^{p} p - 1_{k-1} (v')^{k-1} (1-v')^{p-k} U_p(k). \quad (6)$$

Also

$$P\{|N(i)| = p | j \in N(i)\} = \binom{N-2}{p-1} v^{p-1}(1-v)^{N-p-1}, \tag{7}$$

and

$$P\{j \in N(i)\} = v. \tag{8}$$

By substituting eqns (6), (7), and (8) in eqn (2) we get

$$q = (1 - \textstyle\sum_{l=1}^{M} q^l c_l) \sum_{p=1}^{N-1} \sum_{k=1}^{p} \binom{N-2}{p-1}\binom{p-1}{k-1} v^p (1-v)^{N-p-1}(v')^{k-1}(1-v')^{p-k} U_p(k). \tag{9}$$

## A recursive algorithm to compute $q$

From eqn (9) we get

$$q = \frac{G(1-\sum_{l=2}^{M} q^l c_l)}{1+c_1 G}, \tag{10}$$

where

$$G = \sum_{p=1}^{N-1} \sum_{k=1}^{p} \binom{N-2}{p-1}\binom{p-1}{k-1} v^p (1-v)^{N-p-1}(v')^{k-1}(1-v')^{p-k} U_p(k).$$

As a particular case, if $c_l$ is assumed as uniform (that is, if $c_l = 1/M, l = 1,2 \dots, M$), then we can write

$$q = \frac{MG + q^2 M + q^{M+1} G}{M+G+MG},$$

which in turn gives the recursive algorithm

$$q^{[k+1]} = \frac{MG + (q^{[k]})^2 M + (q^{[k]})^{M+1} G}{M+G+MG}. \tag{11}$$

**Lemma:** The effective arrival rate at an arbitrary node, denoted by $\lambda$, is

$$\lambda = \frac{\lambda_e}{N(1-(N-1)vq)}. \tag{12}$$

*Proof:* Since the effective arrival rate at a node is the sum of the external arrival rate at that node and the average inflow rate to that node from its neighbouring nodes, we have

$$\lambda = \frac{\lambda_e}{N} + (N-1)\lambda v q.$$

Hence the lemma.

## Finding the interfering nodes

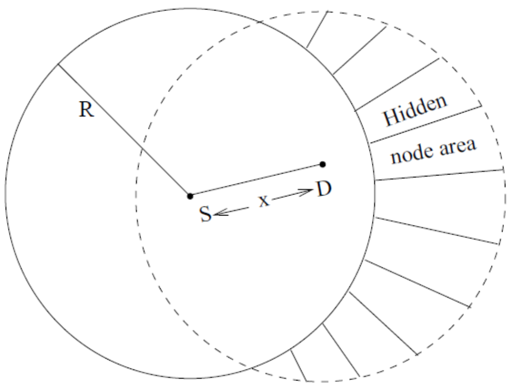


**Figure 1:** Illustration of hidden-terminal area

While a packet is being transmitted from a node to another, all the nodes that are lying in the neighbourhood of the source node can hear the details regarding the transmission by sensing the medium , whereas the ones which are not the neighbours can not. So the nodes which are located within the sensing region of the intended destination and off-range of the source node may make transmission to destination node simultaneously with source node, which may result in collision at the destination node. This is the well known *hidden terminal problem* and the corresponding nodes are termed as *hidden nodes*. These hidden nodes together with the neighbouring nodes of the source node constitute the set of *interfering nodes* of the source node. In order to conduct the waiting time analysis for a packet in the whole network, we need to get a measure of the average number of interfering neighbours an arbitrary node has. For this, we proceed as follows.

Distribution function of the distance $X$ between source and destination nodes is given by

$$F(x) = (\frac{x}{R})^3, 0 < x \leq R.$$

The volume of the solid inside which the hidden nodes lie is a random variable. For a given value of X, this volume ( two-dimensional analogue of this case is shown as shaded portion in Figure 1, the details of which are given in [1]) can be computed as

$$V(x) = v - \frac{1}{12}\pi(4R + x)(2R - x)^2.$$

So the average volume of the solid inside which the hidden nodes lie,

$$v_h = \int_0^R V(x)dF(x) = \frac{17}{24}\pi R^3.$$

Hence the average volume of the solid inside which the interfering nodes lie,

$$v_I = v_h + v = \frac{49}{24}\pi R^3.$$

Now it is easy to see that the probability distribution of the interfering neighbours of a node is binomial with mean $(N-1)v_I$. Hence the average number of interfering neighbours of an arbitrary node,

$$N_I = (N-1)v_I. \tag{13}$$

## Waiting time analysis

In this section, we derive the probability distribution and some important measures of characteristics of waiting time for a packet at an arbitrary node under both BEB and DIDD schemes. Here, by waiting time at a node we actually mean the time spent by a packet at that node from the instant at which it is ready for transmission till it s successfully transmitted. This does not include the time spent by the packet at the buffer before its transmission turn occurs. The objective of this paper is not to compare the efficiency among BEB or DIDD or any other scheme proposed by researchers, rather our focus here is to derive the waiting time distribution of a packet at an arbitrary node analytically, for which BEB and DIDD rules are being used just for theoretical illustration.

## Under the DIDD scheme

[3] analysed the performance of IEEE 802.11 distributed coordination function, where BEB rule is used as contention resolution method, by means of a two dimensional Markov chain and computed the conditional collision probability (that is, the probability of collision seen by a packet while it is being transmitted). By the same approach, in this case, we can compute the conditional collision probability say, $p_D$ by means of the formula

$$p_D = 1 - (1 - \tau_D)^{N_2}, \tag{14}$$

where $\tau_D$, the transmission probability of a node in a random time slot under DIDD rule, is derived as (proof is shown below)

$$\tau_D = \frac{2(1-2a)(1-a^{m+1})}{(1-(2a)^{m+1})(1-a)W+(1-2a)(1-a^{m+1})} \tag{15}$$

with $a = \frac{p_D}{1-p_D}$, and $N_2$ represents the largest integer less than or equal to $N_I$.

Eqns (14) and (15) represent a nonlinear system in unknowns $\tau_D$ and $p_D$, which can be solved numerically (by using fixed point iteration scheme) to get $p_D$.

Now, we derive the probability distribution of the number of time slots spent by an arbitrary packet at a node from the time instant at which it is ready for transmission till it is successfully transmitted, by using the embedded Markov chain technique. For this, consider the system at the end of a time slot at which either the channel is sensed idle by the node or transmission of a packet (which may or may not be successful) from that node is over. More precisely, let $t_i$ be the beginning of a time slot such that the previous slot $[t_{i-1}, t_i)$ ends either with transmission of a packet from the node, or the channel is sensed idle by the node. Then the embedded stochastic process $\{(s(t_i), b(t_i); i \in N\}$, where $s(t_i)$ and $b(t_i)$ respectively denote the backoff stage and backoff time counter of the node at $t_i$, is a Markov chain. Note that whenever $s(t_i) = j$, then $b(t_i)$ can take one of the values uniformly from $\{0,1,2,\dots, W_j - 1\}$, where $W_j = 2^j W, j = 0,1,2,\dots, m$.

The transition probabilities of the Markov chain are denoted by

$$P\{(i_1, k_1)|(i_0, k_0)\} = P\{(s(t_{i+1}), b(t_{i+1})) = (i_1, k_1)|(s(t_i), b(t_i)) = (i_0, k_0)\}.$$

Under the DIDD scheme, it can be seen that

$$P\{(i,k)|(i,k+1)\} = 1, \quad for \quad k = 0,1,\dots, W_i - 2; \quad i = 0,1,\dots, m$$

$$P\{(i-1,k)|(i,0)\} = \frac{1-p_D}{W_{i-1}}, \quad for \quad k = 0,1,\dots, W_{i-1} - 1; \quad i = 1,\dots, m$$

$$P\{(i+1,k)|(i,0)\} = \frac{p_D}{W_{i+1}}, \quad for \quad k = 0,1,\dots, W_{i+1} - 1; \quad i = 0,1,\dots, m-1$$

$$P\{(m,k)|(m,0)\} = \frac{p_D}{W_m}, \quad for \quad k = 0,1,\dots, W_m - 1$$

and

$$P\{(0,k)|(0,0)\} = \frac{1-p_D}{W}, \quad for \quad k = 0,1,\dots, W - 1.$$

Define state vector

$$\bar{\iota} = ((i,0),(i,1),(i,2),\dots,(i,W_i - 1)), \quad for \quad i = 0,1,\dots, m.$$

Then the transition probability matrix of the Markov chain is given by

$$P = \begin{bmatrix} D_0 + C_{0'} & B_1 & 0 & \dots & \dots & \dots & \dots & 0 \\ C_0 & D_1 & B_2 & \ddots & & & & \vdots \\ 0 & C_1 & D_2 & B_3 & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & C_{m-2} & D_{m-1} & B_m \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & C_{m-1} & D_m + B_{m'} \end{bmatrix}$$

where $B_i, i = 1,2,\dots, m$ of dimension $W_{i-1} \times W_i$ , given by

$$B_i = \begin{bmatrix} \frac{p_D}{W_i} & \frac{p_D}{W_i} & \dots & \frac{p_D}{W_i} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 \end{bmatrix};$$

$C_i, i = 0,1,\dots, m-1$, $W_{i+1} \times W_i$ matrix, given by

$$C_i = \begin{bmatrix} \frac{1-p_D}{W_i} & \frac{1-p_D}{W_i} & \dots & \frac{1-p_D}{W_i} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 \end{bmatrix};$$

$D_i, i = 0,1,\dots, m$, of dimension $W_i \times W_i$ , given by

$$D_i = \begin{bmatrix} 0 & 0 \\ I_{W_i - 1} & 0 \end{bmatrix}$$

with $I_{W_i-1}$ as the identity matrix of order $W_i - 1$ and 0 is row(column) vector of appropriate dimension.

$C_{0'}$ has the same structure as $C_0$ with the only distinction that it is a square matrix of order $W$. Similarly $B_{m'}$, a square matrix of order $W_m$, differs from $B_m$ only in dimension.

If we define $\Pi = (\Pi_0, \Pi_1, \dots, \Pi_m)$, where $\Pi_i = (\pi_{i0}, \pi_{i1}, \dots, \pi_{iW_i-1}), i = 0,1, \dots, m,$ as the stationary distribution of the above Markov chain, it can be seen that

$$\pi_{ik} = (\frac{p_D}{1-p_D})^i (\frac{W_i-k}{W_i})\pi_{00}, \quad for \quad i = 0,1, \dots, m; \quad k = 0,1, \dots, W_i - 1$$

and

$$\pi_{00} = 2[W(\frac{1-p_D}{1-3p_D})(1-(\frac{2p_D}{1-p_D})^{m+1}) + (\frac{1-p_D}{1-2p_D})(1-(\frac{p_D}{1-p_D})^{m+1})]^{-1}.$$

Hence $\tau_D$, the probability that a node transmits in a random slot time is given by

$$\tau_D = \sum_{i=0}^{m} \pi_{i0} = \frac{2(1-2a)(1-a^{m+1})}{(1-(2a)^{m+1})(1-a)W + (1-2a)(1-a^{m+1})},$$

which is eqn (15).

Now, let $\alpha_i, i = 0,1,2, \dots, m-1$ be the probability that a packet at the head of the waiting line at a node starts with backoff stage $i$. Then

$$\alpha_i = P\{ \text{ the previous packet which was successfully transmitted from the node left the system at stage } i+1 \}$$

so that

$$\alpha_i = \sum_{l=0}^{i+1} \alpha_l (1-p_D) p_D^{i-l+1}, \quad for \quad i = 1, \dots, m-2$$

with

$$\alpha_{m-1} = \sum_{l=0}^{m-1} \alpha_l p_D^{m-l},$$

and

$$\alpha_0 = \alpha_1(1-p_D) + \alpha_0(1-p_D^2).$$

From this, by recursion we get

$$\alpha_i = \frac{p_D^{i+1}}{(1-p_D)^i}\alpha_0, \quad for \quad i = 1,2, \dots, m-1$$

so that

$$\alpha_0 = \frac{(1-2p_D)(1-p_D)^{m-1}}{(1-p_D)^{m+1} - p_D^{m+1}} \tag{16}$$

by using the normalizing condition $\sum_{i=0}^{m-1} \alpha_i = 1$. Thus we have

$$\alpha_i = \frac{(1-2p_D)p_D^{i+1}}{((1-p_D)^{m+1} - p_D^{m+1})(1-p_D)^{i-m+1}}, \quad for \quad i = 1, \dots, m-1 \tag{17}$$

with $\alpha_0$, given by eqn (16).

Now the definition of a discrete PH random variable (see [12] and [11]) for details on PH distribution and PH renewal theory) leads to the following theorem.

**Theorem 1:** The number of transitions undergone (time slots spent) by a packet say, $S_D$ from the instant at which it is ready for transmission till it is successfully transmitted, is a discrete PH random variable having representation $(\bar{\alpha}_D, T_D)$ with $\bar{\alpha}_D = (\frac{\bar{\alpha}_0}{W}, \frac{\bar{\alpha}_1}{W_1}, \frac{\bar{\alpha}_2}{W_2}, \dots, \frac{\bar{\alpha}_{m-1}}{W_{m-1}}, 0_{W_m}, 0)$, where $\bar{\alpha}_i, \quad i = 0,1,2 \dots, m-1$ is the vector having $W_i$ components with each component $\alpha_i$, $0_{W_m}$ is the vector of zeroes having $W_m$ components, and $T_D$ is the matrix given by

$$T_D = \begin{bmatrix} D_0 & B_1 & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & D_1 & B_2 & \ddots & & & & \vdots \\ \vdots & \ddots & D_2 & B_3 & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \ddots & D_{m-1} & B_m \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & D_m + B_m, \end{bmatrix}.$$

Also, its pmf, $P(S_D = k) = \bar{\alpha}_D T_D^{k-1}(-T_D e), k > 0$, where $e$ is a column vector, having components 1, of appropriate dimension.

**Corollary 1:** Average number of transitions undergone(time slots spent) by a packet at an arbitrary node from the instant at which it is ready for transmission till it is successfully transmitted, is given by

$$E(S_D) = \bar{\alpha}_D (I - T_D)^{-1} e = \frac{\alpha_0}{W}\left(W x_0 + \frac{W(W-1)}{2}\right) + \sum_{i=1}^{m-1} \frac{\alpha_i}{W_i}\left(W_i x_i + \frac{W_i(W_i-1)}{2}\right), \quad (18)$$

where

$$x_i = \frac{2-p_D}{2(1-p_D)} + \frac{W_i p_D (1-(2p_D)^{m-i-1})}{1-2p_D} + \frac{p_D^{m-i} W_{m-1}}{1-p_D}, \quad for \quad i = 0,1,2,\dots,m-1. \quad (19)$$

## Under the standard BEB scheme

Following the same lines as in the case of DIDD scheme, we have

**Theorem 2:** Under the standard BEB scheme, $S_B$, the number of time slots spent by a packet at a node from the instant at which it is ready for transmission till it is successfully transmitted, is a discrete PH variate having representation $(\bar{\alpha}_B, T_B)$ with $\bar{\alpha}_B = (\frac{e_W^T}{W}, 0,0,\dots,0)$, and $T_B$ is the matrix having the same structure as $T_D$ with the only exception that $p_D$ in $T_D$ is replaced by $p_B$ in $T_B$, where the conditional collision probability $p_B$ under BEB scheme is computed by solving the nonlinear system of equations (see [3])

$$p_B = 1 - (1 - \tau_B)^{N_2}, \quad (20)$$

and

$$\tau_B = \frac{2(1-2p_B)}{(1-2p_B)(W+1) + p_B W(1-(2p_B)^m)}. \quad (21)$$

Also, pmf of $S_B$ is given by

$$P(S_B = k) = \bar{\alpha}_B T_B^{k-1}(-T_B e), k > 0.$$

**Corollary 2:** Under the standard BEB scheme, average number of time slots spent by a packet at an arbitrary node from the instant at which it is ready for transmission till it is successfully transmitted, is given by

$$E(S_B) = \bar{\alpha}_B (I - T_B)^{-1} e = \frac{1}{W}\left(W y_0 + \frac{W(W-1)}{2}\right), \quad (22)$$

where $y_0$ has the same expression as $x_0$ (given by eqn (19)), in which $p_D$ is replaced by $p_B$.

### Probability distribution of slot length

We start with this section by summarizing the important steps (with minor changes pertaining to our model) in the derivation of probability distribution of length of an arbitrary slot, as detailed in [3]. A slot is called *active* if at least one transmission takes place in that slot. Let $P_a$ be the probability that the slot is active. Since, on the average $N_2 + 1$ nodes are contending on the channel (we have seen that on the average, a node will have $N_2$ interfering nodes), and each transmits with probability $\tau$

$$P_a = 1 - (1 - \tau)^{N_2+1}, \quad (23)$$

where $\tau = \tau_D$ or $\tau_B$ depending on whether DIDD or BEB scheme is used. Transmission of a packet may result either in success or in collision.

Let

$$P_s = P(\ transmission\ in\ a\ slot\ is\ successful | slot\ is\ active\ ).$$

Then

$$P_s = \frac{(N_2+1)\tau(1-\tau)^{N_2}}{P_a} = \frac{(N_2+1)\tau(1-\tau)^{N_2}}{1-(1-\tau)^{N_2+1}}. \quad (24)$$

At this juncture it may be remembered that a time slot may be (i) a back off time slot $\sigma$ if no transmission takes place in that slot (ii) $T_s$, the average time the channel is sensed busy because of a successful transmission, or (iii) $T_c$, the average time the channel is sensed busy because of a collision.

Thus the probability distribution of the slot length $SL$ is given by

$$SL = \begin{cases} \sigma & with \quad prob \quad 1 - P_a \\ T_s & with \quad prob \quad P_a P_s \\ T_c & with \quad prob \quad P_a(1 - P_s) \end{cases} \quad (25)$$

If we assume that the system is completely managed by the basic access mechanism, then

$$T_s = SIFS + T(E(P)) + DIFS + T(ACK)$$

and

$$T_c = DIFS + T(E(P^*)),$$

where $E(P)$ denotes the expected packet size and $T(E(P))$ represents the average time required for a node to transmit a packet of size $E(P)$ to its neighbouring node. Similarly, $E(P^*)$ stands for the expected value of the largest packet size included in a collision and $T(E(P^*))$ for the corresponding mean transmission time . Also, $T(ACK)$ represents the mean time required to transmit an acknowledgement message from destination to source node. Just in the lines of eqn (15) in [3] it can be seen that

$$E(P^*) = \frac{\sum_{k=2}^{N_I+1} {N_I+1}_k \tau^k (1-\tau)^{N_I+1-k} \int_0^{P_{max}} (1-(F(x))^k) dx}{1-(1-\tau)^{(N_I+1)}-(N_I+1)\tau(1-\tau)^{N_I}}, \quad (26)$$

where $F(.)$ is the packet size distribution function and $P_{max}$ is the maximum value of the packet size. It is to be noted that

$$E(P) = \int_0^{P_{max}} (1 - F(x))dx.$$

Now, for the computation of $T(E(P))$, $T(E(P^*))$, and $T(ACK)$, we assume that the transmission time for a packet from a node to another depends on how far the latter is from the former. Earlier we have seen that if a packet is transmitted from node $i$,

$$P\{\, it \ reaches \ at \ node \ j \mid j \ is \ the \ kth \ neighbour \ of \ i \,\} = \frac{E_L}{E(R_k)},$$

where

$$E_L = (\sum_{l=1}^{L} \frac{1}{E(R_l)})^{-1},$$

and $E(R_k)$ is given by eqn (1). So, if $Z$ represents the transmission time (in $\mu_s$) for a bit to reach the receiving node, which lies at a unit distance from the source node, then the transmission time for a packet to reach the receiver, which is the $k$th neighbour of the source node is $ZE(P)E(R_k)$ so that

$$T(E(P)) = \sum_{l=k}^{L} ZE(P)E(R_k)\frac{W_L}{E(R_k)} = LZW_L E(P). \quad (27)$$

Similar results hold for $T(E(P^*))$, and $T(ACK)$.

Now the expected slot length,

$$E(SL) = (1 - P_a)\sigma + P_a P_s T_s + P_a(1 - P_s)T_c. \quad (28)$$

If $TS$ denotes the time spent by a packet at a node from the instant at which it is ready for transmission till its successful transmission, then

$$E(TS) = E(S)E(SL), \quad (29)$$

where $S = S_D \ or \ S_B$ depending on whether the system is under DIDD or BEB scheme, and $E(S)$ is given by eqn (18) or (22) as the case may be. Note that since $TS$ is the actual time taken for a node to complete a packet transmission since the epoch at which it is ready for transmission, as per the queueing terminology, $TS$ is equivalent to the *effective service time* rendered for a packet at a node in the network. Now, let us compute the variance of $TS$.

$$E(TS^2) = E(E(TS^2|S = k)).$$

Now

$$E(TS^2|S = k) = E((\sum_{j=1}^{k} SL_j)(\sum_{h=1}^{k} SL_h)),$$

where $SL_j$ denotes the length of $j$th time slot. Since $SL_j$ are $iid$ variates with mean $E(SL)$, we have

$$E(TS^2|S = k) = \sum_{j=1}^{k} \sum_{h=1}^{k} (E(SL))^2 + \sum_{j=1}^{k} E(SL^2).$$

Thus

$$Var(TS) = [(1 - P_a)\sigma + P_a P_s T_s + P_a(1 - P_s)T_c]^2 [2\bar{\alpha}(I - T)^{-2} Te$$

$$+\bar{\alpha}(I - T)^{-1}e - (\bar{\alpha}(I - T)^{-1}e)^2] + \bar{\alpha}(I - T)^{-1}e[(1 - P_a)\sigma^2 + P_a P_s T_s^2 + P_a(1 - P_s)T_c^2], (30)$$

where $\bar{\alpha} = \bar{\alpha}_D$ or $\bar{\alpha}_B$, and $T$=$T_D$ or $T_B$ depending on whether the system is under DIDD or BEB scheme. Moreover $C_{TS}$, the coefficient of variation of $TS$, given by

$$C_{TS}^2 = \frac{Var(TS)}{(E(TS))^2} \quad can \ be \ computed \ by \ using \ the \ eqns \ (29) \ and \ (30).$$

These results can be used to get approximate solution to queue size distribution at each node as done in [4]. [10] introduced a vector-valued normal process and its diffusion equation in order to obtain an approximate solution to the joint distribution of queue lengths in a general network of queues. By this approximation, the queue size distribution at node $i$ say, $p_i$ is obtained as

$$p_i(n) = \begin{cases} 1-\rho_i, & n = 0 \\ \rho_i(1-\hat{\rho}_i)\hat{\rho}_i^{\,n-1}, & n > 0 \end{cases} \quad (31)$$

where $\rho_i = \lambda_i E(S_i)$; $\lambda_i$ is the effective arrival rate at node $i$ , and $E(S_i)$ is the mean service time required for a packet at node $i$. Also,

$$\hat{\rho}_i = exp\,(-\frac{2(1-\rho_i)}{C_{A_i}^2\rho_i + C_{S_i}^2}), \quad (32)$$

where $C_{A_i}^2$ and $C_{S_i}^2$ are the squares of coefficients of variation of inter-arrival times and service times respectively, of packets at node $i$. As shown in [5] , $C_{A_i}^2$ is approximated by using the relation

$$C_{A_i}^2 = 1 + \sum_{j=0}^{N}(C_{S_j}^2 - 1)q_{ji}^2 e_j e_i^{-1}, \quad (33)$$

where $C_{S_0}^2 = C_E^2$, and $e_j$ is the average number of visits that a packet makes to node $j$ during its stay in the network. Since all the nodes are considered identical in our model, the eqn, analogues to the one given by(33), associated with our model assumes the form

$$C_A^2 \approx 1 + \frac{(C_E^2-1)}{N^2} + (N-1)(C_{TS}^2 - 1)q^2, \quad (34)$$

where $q$ is given by eqn (11).

Also, mean number of packets at an arbitrary node

$$\bar{K} = \frac{\rho}{1-\hat{\rho}}. \quad (35)$$

By Little's law, average waiting time of a packet at an arbitrary node

$$\bar{W} = \frac{\bar{K}}{\lambda} = \frac{\rho}{\lambda(1-\hat{\rho})}, \quad (36)$$

where $\lambda$, the effective arrival rate at a node is given by eqn (12).

Since a packet generated at an arbitrary node is of class $l, l = 1,2,\dots,M$ with probability $c_l$ and a class $l$ packet visits exactly $l$ hopes before absorption, average number of hops traversed by a packet before absorption

$$\bar{H} = \sum_{l=1}^{M} lc_l = \frac{M+1}{2}, \quad if \quad c_l \quad is\ uniform.$$

Hence, the average end to end delay experienced by a packet in the whole network

$$D = \bar{H}\bar{W} = \frac{(M+1)\rho}{2\lambda(1-\hat{\rho})}, \quad if \quad c_l \quad is\ uniform. \quad (37)$$

Since $\lambda$, given by eqn (12), is the effective arrival rate at an arbitrary node, and $E(TS)$, given by eqn(29), is the actual mean time required for a packet to be successfully transmitted from a node, for the stability of the system

$$\lambda E(TS) < 1.$$

Hence the maximum achievable throughput can be attained when $\lambda_e$ is enhanced to the values near its upper bound, governed by the rule

$$\lambda_e < N(1-(N-1)vq)E(TS) \quad (38)$$

for a selected set of parameters.

## 3 Results

In order to illustrate the performance of the system, we present some numerical results. The values of the system parameters used in this analysis are summarized in Table 1 and Table 2. Most of these parameters are set to comply with the 802.11 MAC specifications. The wireless nodes are assumed to be distributed uniformly inside a compact subset in $R^3$ of volume $10^6\ m^3$, which is taken as 1 $cubic\ unit$. All nodes are considered as identical. Packets are generated independently at nodes as per a renewal process with rate $\lambda_e$ and coefficient of variation $C_E = 0.95$. types of packets generated at each node are assumed as uniform with mean $1/M$, where $M = 15$. Packet size are assumed to be uniformly distributed over an interval [64,1518], measured in bytes, so that the average packet size is 791 bytes. In all numerical illustrations, we have included both BEB and DIDD schemes in order to get a complete picture of the system performance.

Table 1: Physical Parameters

| Parameter | Value |
|---|---|
| $CW_{min}$ | 32 |
| $CW_{max}$ | 1024 |
| No. of classes of packets(M) | 15 |
| Propagation speed | $3\times10^{8} m/s$ |
| Channel bit rate | 1 Mbps |
| Slot time ($\sigma$) | 50 $\mu s$ |
| SIFS | 28 $\mu s$ |
| DIFS | 128 $\mu s$ |
| | |

Table 2: Packet Parameters

| Parameter | Value |
|---|---|
| Average Packet size | 6328 bits |
| COV of the packet arrival process $C_E$ | 0.95 |
| PHY header | 128 bits |
| ACK | 112 bits + PHY header |
| | |

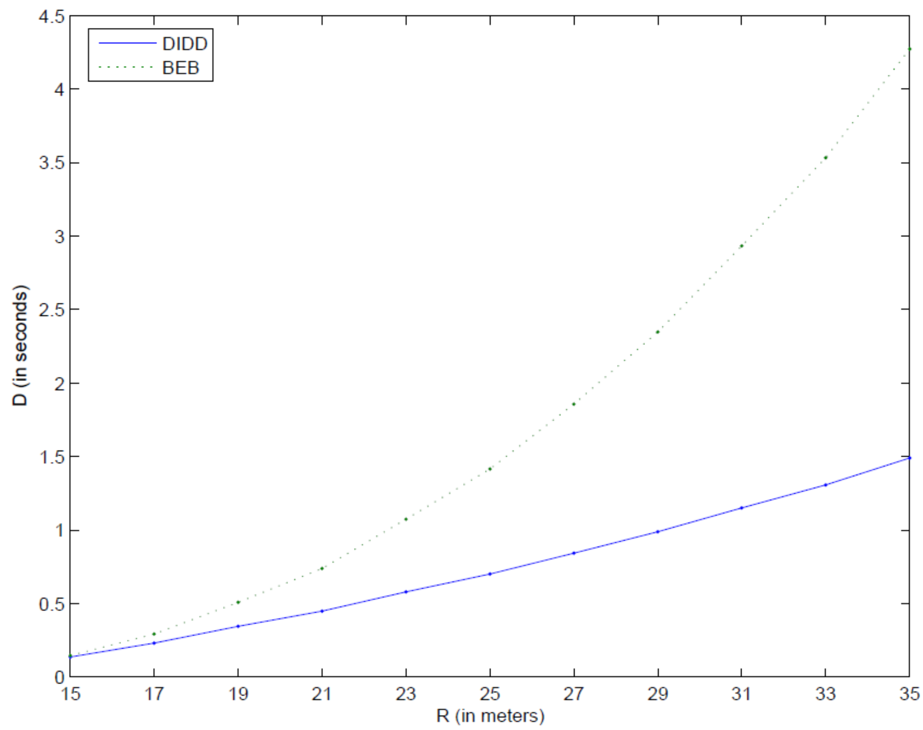


**Figure 2:** Average Delay versus Transmission Range ($N = 300$, $\lambda_e = 2$ $packets/sec$ )

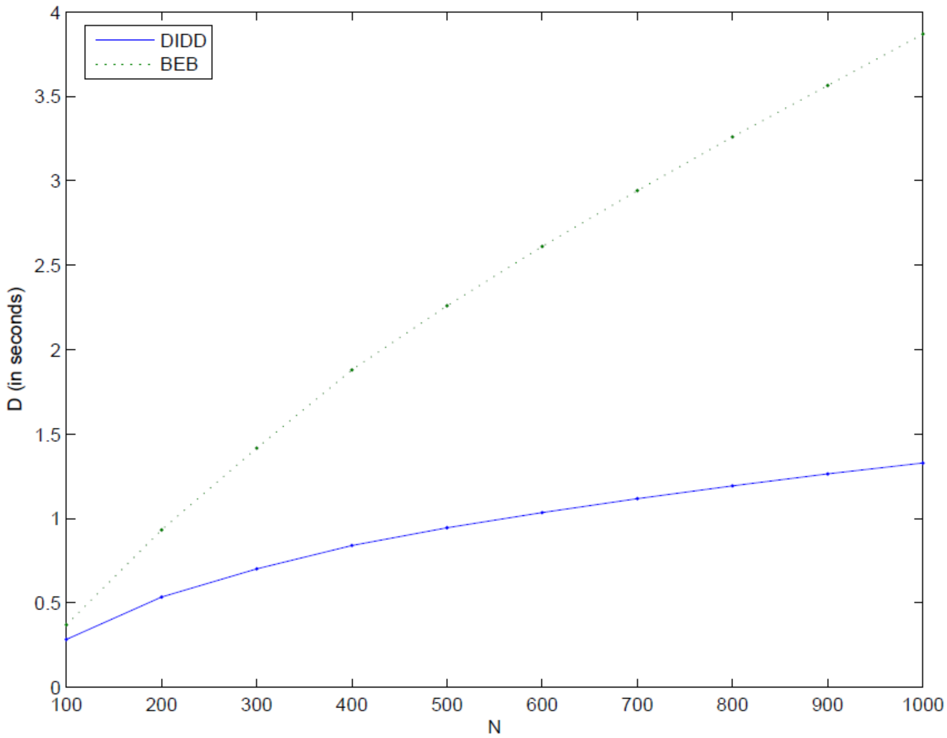


**Figure 3:** Average Delay versus Number of Nodes ($R = 25m$, $\lambda_e = 2packets/sec$)

## 4 Discussion

Figure 2 shows how the average end to end delay $D$ experienced by a packet in the whole network varies with different values of the transmission range $R$. When $R$ increases, average number of interfering neighbours of nodes increases so that the conditional collision probability also increases, which results in more delay for packets at each node. Also, since the conditional collision probability is more for the system under standard BEB scheme than under the DIDD scheme, the average end to end delay for the former is much higher than the latter, as obvious from Figure 2.

In Figure 3, the variation in average end to end delay corresponding to change in values of the the number of nodes $N$, by keeping $R = 25m$, and $\lambda_e = 2packets/sec$, is exhibited. As in the previous case, here also it is seen that $D$ moves in the same direction with $N$, under both schemes.

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