Statistical Analysis for Type-I Progressive Hybrid Censored Data from Burr Type XII Distribution under Step-Stress Partially Accelerated Life Test Model

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Abstract

Progressive hybrid censoring scheme is now quite common in the experiment of life testing and reliability analysis. In this article the data, failure life times of units, is obtained by using type-I progressive hybrid censoring scheme. It is assumed that data follows Burr Type XII distribution. The point and interval estimation of the Burr Type XII distribution parameters and acceleration factor are performed using maximum likelihood estimator under stress partially accelerated life test model. Monte Carlo simulation study is used to obtain the biases and mean square errors of the estimators.

Keywords: Reliability, type I progressive hybrid censoring scheme, Burr Type XII distribution, maximum likelihood function, Monte Carlo simulation, Step-Stress partially accelerated life test model.

1. Introduction

Nowadays, due to quick and rapid advances in technology and increasing global competition, pressure on manufacturers to produce high quality products has increased. Life testing and reliability experiments are often used to gain knowledge about product failure time distribution. But the information of such high reliable products cannot be obtained at usual level of stress (or normal stress). So to collect the quick information of the products accelerated life test (ALT) is used. In ALT, we put the items or products at higher than normal stress. Here the relationship between the stress and lifetime is known or can be assumed or acceleration factor is known. But sometimes we face the situation when neither these relationships are known nor it can be assumed. At this point of time partially accelerated life test (PALT) is used to test the items and to gather information on lifetimes of products. In PALT, first the products or testing items have been put at use condition or normal stress, after a specified time, we increase the stress. Therefore, in PALT the items run at normal as well as accelerated condition, see DeGroot and Goel(1979).

Nelson (1990) described the ways by which stress(es) can be applied into the experiment of life test. The common stresses are constant stress, step stress and progressive or linearly increasing stress. In constant stress test, each unit runs at a prespecified level of stress which does not vary with time that is every unit is put at only one stress level until unit is failed or the experiment is terminated at some specific point of time. In step stress test, the items are being tested at some specified stress level after a certain time the stress level is increased and the test continues until all the items get failed or the experiment is terminated at some pre specified time.

There are many situations in reliability and life testing experiments in which units (or subjects) are lost or removed from the experimentation before failure. Complete information on failure times may not be obtained by the experimenter for all experimental units. The data obtained from the experiment are called censored data and the method is censoring method or censoring scheme. Conventional type-I and type-II censorings are the most common censoring schemes. In type-I censoring scheme, the test will be terminated at a pre specified time T and in type-II censoring scheme, the test will run upto rth failure (where r is pre-specified). The mixture of type-I and type-II censoring schemes is known as the hybrid censoring scheme. The hybrid censoring scheme was first introduced by Epstein(1954). But recently it becomes quite popular in the reliability and life testing experiments, e.g. Chen and Bhattacharya(1988), Childs et al.(2003), Draper and Guttman (1987), Fairbanks, Madason and Dykstra(1982), Gupta and Kundu (1998), Jeong, Park and Yan(1996), Lin, Ng and Chan(2009), Ling, Xu and Li (2009) etc.

The major drawback of these censoring schemes is that they do not allow the removal of the units from the experiment other than the terminal point. To deal with this problem, a more general censoring scheme called progressive type-II right censoring is introduced. It can be described as follows: consider an experiment in which n(>m) units are placed on a life test. At the time of first failure $Y_{1:m:n}, R_1$ units are randomly removed from the remaining (n-1) surviving units. Next at the time of the second failure $Y_{2:m:n}, R_2$ of the remaining $(n-2-R_1)$ units are removed randomly. The test continues until the m-th failure. At the time of m-th failure, all the remaining $R_m = n-m-R_1-\cdots-R_{m-1}$ units are removed from the test. The m integers $(R_1, R_2, \cdots, R_m)$ are fixed prior to the study. They are determined by the experimenter to control the total test time or to observe the more failures which in return have good efficiency in statistical inference. Conventional type-II right censoring is a special case when $R_1 = R_2 = \cdots = R_{m-1} = 0$ and $R_m = n-m$. For further details the reader may refer to Balakrishnan and Aggrawala (2000), Balakrishnan (2007), Balakrishnan and Cramer(2014).

Kundu and Joarder(2006) and Childs et al.(2008) suggested a progressive hybrid censoring (PHC) scheme, named as type-I progressive hybrid censoring scheme which is described as follows: The life test experiment with progressive censoring scheme $(R_1, R_2, \cdots, R_m)$ is stopped at a random time $\min\{y_{m:m:n}, \eta\}$ where $\eta \in (0,\infty)$ *and* $1 \le m \le n$ are fixed prior to the study. The ordered failure times collected from the experiment is $Y_{1:m:n} < Y_{2:m:n} < \cdots Y_{m:m:n}$. If the mth progressively failure occurs before time $\eta\,(i.e.Y_{m:m:n} < \eta)$ the experiments terminates at time $Y_{m:m:n}$ and if m-th failure does not occur before time η, the experiment will be terminated at time point with J failures such as $Y_{j:m:n} < \eta < Y_{j+1:m:n}$ and all the remaining $(n - J - \sum_{i=1}^{J} R_i)$ surviving items are censored at time η. Therefore, J, the number of failures upto time η is the random variable. Lin, Ng and Chan (2009) indicated the purpose of it to control the total test time of the experiment.

Literature available on the PALT has been studied using censoring schemes, for example, see Goel(1971), DeGroot and Goel(1979), Bai, Chung and Chen(1993), Bhattcharya and Soejoeti(1989), Bai and Chung(1992), Abdel-Ghaly et al.(2011), Abdel-Ghani(2004), Ismail(2010), Aly and Ismail (2008), Ismail and Sarhan (2009), Ismail and Aly (2010), Ismail(2012), Lone, Rahman and Islam (2016), Rahman, Lone and Islam(2016), Zarrin et al.(2012), Kamal et al.(2013), also, SSPALT has been studied under hybrid censoring, see Ismail(2012). In addition, Ismail (2012) has considered SSPALT, using the progressive Type-II censoring scheme.

Ismail(2014) has first studied progressive type-I hybrid censoring scheme under SSPALT. After that Ismail (2014) has considered progressive hybrid censored data from Weibull distribution under SSPALT. This scheme under SSPALT will be described in the next section.

This article is arranged as follows. In Section 2, the model and test method are described. Based on the data obtained from section 2, the parameters of the distribution are estimated under SSPALT using maximum likelihood (ML) estimation technique in Section 3. Also, the asymptotic confidence bounds for the model parameters are constructed based on the asymptotic distribution of ML estimators. The simulation study has been performed in Section 4 to check and evaluate the performance of the estimators based on the PHC scheme. Conclusion and suggestion for future work on the PHC is described in Section 5.

2. Description of the Model

It is assumed that the random variable Y representing the lifetimes of the product has Burr Type XII distribution with parameter (c, k). The pdf of the distribution is given as follows:

$$f(y,c,k) = kcy^{c-1}(1+y^c)^{-(k+1)} \qquad y>0, c>0 \text{ and } k>0 \tag{2.1}$$

Where c and k are the shape parameters of the distribution.
The cumulative distribution function is

$$F(y,c,k) = 1-(1+y^c)^{-k} \quad y>0, c>0 \text{ and } k>0 \tag{2.2}$$

The reliability function of the Burr Type XII distribution

$$R(y,c,k) = (1+y^c)^{-k} \tag{2.3}$$

The hazard function of the Burr Type XII distribution

$$h(y,c,k) = kcy^{c-1}(1+y^c)^{-1} \tag{2.4}$$

The Burr (c, k) distribution was first introduced as a lifetime model by Dubey (1972,1973). Evans and Simons (1975) studied further the distribution as a failure model and they also derived maximum likelihood estimators as well as moments of the Burr (c, k) probability density function. Lewis (1981) noted that the Weibull and exponential distributions are special limiting cases of the parameter values of the Burr (c, k) distribution. She proposed the use of the Burr(c, k) distribution as a model in accelerated life test data.

Assumptions

(a) The lifetimes of the items follow Burr type-XII distribution with parameters (c, k).
(b) The total lifetime Y of an item is defined as

$$Y = \begin{cases} T, & 0 < T \le \tau \\ \tau + \beta^{-1}(T-\tau) & T > \tau \end{cases}$$

Where T is the lifetime of the items at normal stress, τ is the time at what stress is to be increased and β is the acceleration factor which is the ratio of the lifetime at normal stress to that at accelerated condition.

(c) The lifetimes of test items are independent and identically distributed random variable.
(d) Under type I progressive hybrid censoring, the test is terminated at min $\{Y_{m:m:n}, \eta\}$.

Test procedure

(a) All n identical and independent items are placed on life test and run at used condition.
(b) Change the level of experiment at time τ to accelerated condition and observe the lifetimes of the items before the test is terminated at min $\{Y_{m:m:n}, \eta\}$.
(c) Once experiment is started, the items begin to fail. At the time of the ith failure we remove the R_i units from the remaining units. Finally at the time of min $\{Y_{m:m:n}, \eta\}$, all the remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ or $R_J = n - J - \sum_{i=1}^{J-1} R_i$ units are removed accordingly from the test and test is terminated.

The description of progressively Type-I hybrid censoring scheme is as follows. Suppose that n identical and independent units are placed on a life test. All of them are run first under the normal stress (use condition). The normal stress level is changed to an accelerated condition at time τ, put all the remaining units at accelerated condition and the test is continued. At the time of first failure Y_1, R_1 of the units are removed randomly from the remaining (n-1) units, when second failure occurs Y_2, R_2 units from the remaining $(n-2-R_1)$ units are removed randomly. If the m-th failure (m < n) occurs at a time $Y_{m:m:n}$ before a prefixed time η > τ, the experiment terminates at the time point $Y_{m:m:n}$. But if $Y_{m:m:n} > \eta$, then all the remaining units are removed and the experiment terminates at the time η. This censoring scheme is called the progressively Type-I hybrid censoring scheme. It is noted that compared to the conventional Type-I censoring scheme, the termination time of the progressively Type-I hybrid censoring scheme is at most η. Let n_u be the number of units that fail before time τ, n_a be the number of units that fail before time η at accelerated condition and n_t be the total number of units that fail on the experiment. So we have

$$n_t = \begin{cases} n_u + n_a = m, & if\ \tau < y_{m:m:n} \le \eta \\ n_u + n_a = J, & if\ y_{m:m:n} > \eta \end{cases}$$

We observe the following samples under type I progressively hybrid censoring scheme

Set 1: $y_{1:m:n} < y_{2:m:n} < \ldots < y_{n_u:m:n} \le \tau < y_{n_u+1:m:n} < \ldots\ldots < y_{J:m:n} < \eta, \quad if\ y_{m:m:n} > \eta$

Set 2: $y_{1:m:n} < y_{2:m:n} < \ldots < y_{n_u:m:n} \le \tau < y_{n_u+1:m:n} < \ldots\ldots < y_{m:m:n} \le \eta, \quad if\ \tau < y_{m:m:n} \le \eta$

The pdf of Y under step stress partially accelerated life test is given by

$$f(y) = \begin{cases} 0 & y \le 0 \\ f_1(y) = f_Y(y; c, k) & 0 < y \le \tau \\ f_2(y) & y > \tau \end{cases} \tag{2.5}$$

Where

$$f_2(y) = kc\beta[\tau + \beta(y-\tau)]^{c-1}\left[1 + \{\tau + \beta(y-\tau)\}^c\right]^{-(k+1)} \tag{2.6}$$

3. **Estimation Process**

In this section, the process of obtaining the estimates of the parameters and acceleration factor based on the data observed by progressively type I hybrid censoring scheme under SSPALT model have been discussed. Also, consider both point and interval estimates of the parameters. Maximum likelihood estimation technique is used to estimate the parameters.

3.1. Point Estimation

In this section the likelihood function for the data observed based on the progressively type I hybrid censoring scheme are constructed under SSPALT.

The likelihood function is given by

$$L(x : c, k, \beta) \propto \prod_{i=1}^{n_u} f_1(x_i)[S_1(\tau)]^{R_i} \prod_{i=n_u+1}^{J} f_2(x_i)[S_2(\eta)]^{R_J^*} \tag{3.1}$$

$$L(x : c, k, \beta) \propto \prod_{i=1}^{n_u} \{k\, c\, x_i^{\,c-1}(1 + x_i^{\,c})^{-(k+1)}[(1+\tau^c)^{-k}]^{R_i}\} \times$$

$$\prod_{i=n_u+1}^{J} k\, c\, \beta\{\tau + \beta(x_i - \tau)\}^{c-1}[1 + \{\tau + \beta(x_i - \tau)\}^c]^{-(k+1)}\{[1 + \{\tau + \beta(\eta - \tau)\}^c]^{-k}\}^{R_J^*} \tag{3.2}$$

where, $R_J^* = n - J - \sum_{i=1}^{J} R_i$.

The log likelihood function is maximized. The natural logarithm of the likelihood function is as follows.

$$\ln L = J\ln k + J\ln c + n_a \ln\beta + (c-1)\{\sum_{i=1}^{n_u}\ln x_i + \sum_{i=n_u+1}^{J}\ln\phi_i\} - (k+1)\left[\sum_{i=1}^{n_u}\ln(1+x_i{}^c) + \sum_{i=n_u+1}^{J}\ln(1+\phi_i)\right]$$
$$-\sum_{i=1}^{n_u} R_i\, k\ln(1+\tau^c) - kn_a\, R_J^*\ln(1+\phi_\eta{}^c) \tag{3.3}$$

The first order partially derivatives of Eq.(3.3) with respect to *c*, *k* and *β* are obtained and are equated to zero.

$$\frac{\partial \ln L}{\partial c} = 0 = \frac{J}{c} + \sum_{i=1}^{n_u}\ln x_i + \sum_{i=n_u+1}^{J}\ln\phi_i - k\frac{\tau^c\ln\tau}{1+\tau^c}\sum_{i=1}^{n_u}R_i - (k+1)\left(\sum_{i=1}^{n_u}\frac{x_i{}^c\ln x_i}{1+x_i{}^c} + \sum_{i=n_u+1}^{J}\frac{\phi_i{}^c\ln\phi_i}{1+\phi_i}\right)$$
$$-kn_aR_J^*\frac{\phi_\eta{}^c\ln\phi_\eta}{1+\phi_\eta} \tag{3.4}$$

$$\frac{\partial \ln L}{\partial k} = 0 = \frac{J}{k} - \left(\sum_{i=1}^{n_u}\ln(1+x_i{}^c) + \sum_{i=n_u+1}^{J}\ln(1+\phi_i{}^c)\right) - \ln(1+\tau^c)\sum_{i=1}^{n_u}R_i - n_aR_J^*\ln(1+\phi_\eta{}^c) \tag{3.5}$$

$$\frac{\partial \ln L}{\partial \beta} = 0 = \frac{n_a}{\beta} + (c-1)\sum_{i=n_u+1}^{J}\frac{(x_i-\tau)}{\phi_i} - c(k+1)\sum_{i=n_u+1}^{J}\frac{\phi_i^{c-1}(x_i-\tau)}{(1+\phi_i{}^c)} - \frac{kcn_aR_J^*(\eta-\tau)\phi_\eta{}^{c-1}}{(1+\phi_\eta{}^c)} \tag{3.6}$$

Obviously, it is very difficult to obtain the closed form solution for three nonlinear equations (3.4)-(3.6). Newton-Raphson iterative process is used to get the MLE solutions $(\hat{c},\hat{k},\hat{\beta})$.

3.2.Interval Estimation

The most common method to construct the approximate confidence interval of parameters is based on the asymptotic distribution of the ML estimators of the unknown parameters $\Omega = (c,k,\beta)$. The asymptotic distribution of the ML estimators of Ω is given as

$$\left((\hat{c}-c),(\hat{k}-k),(\hat{\beta}-\beta)\right) \to N(0, I^{-1}(c,k,\beta))$$

where $I^{-1}(c,k,\beta)$ is the variance covariance matrix of the unknown parameters $\Omega = (c,k,\beta)$. The matrix is of 3×3 dimension and its elements $I_{ij}(c,k,\beta),\ i,j = 1,2,3$ can be approximated by $I_{ij}(\hat{c},\hat{k},\hat{\beta})$.

where,
$$I_{ij}(\Omega) = -\left.\frac{\partial^2 \ln L(\Omega)}{\partial\Omega_i\partial\Omega_j}\right|_{\Omega=\hat{\Omega}}$$

So the elements of the matrix are given as follows

$$\frac{\partial^2 \ln L}{\partial c^2} = -\frac{J}{c^2} - (k+1)\sum_{i=1}^{n_u}x_i{}^c\left(\frac{\ln x_i}{1+x_i{}^c}\right)^2 - (k+1)\sum_{i=n_u+1}^{J}\phi_i{}^c\left(\frac{\ln\phi_i}{1+\phi_i{}^c}\right)^2 - k\tau^c\left(\frac{\ln\tau}{1+\tau^c}\right)^2\sum_{i=1}^{n_u}R_i$$
$$-kn_a\phi_\eta{}^c\left(\frac{\ln\phi_\eta}{1+\phi_\eta{}^c}\right)^2 R_J^*$$

$$\frac{\partial^2 \ln L}{\partial k^2} = -\frac{J}{k^2}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{n_a}{\beta^2} - (c-1)\sum_{i=n_u+1}^{J}\left(\frac{x_i-\tau}{\phi_i}\right)^2 - c(k+1)\sum_{i=n_u+1}^{J}\phi_i^{\,c-2}(c-1-\phi_i^{\,c})\left(\frac{x_i-\tau}{1+\phi_i^{\,c}}\right)^2$$
$$-kcn_a R_J^* \phi_\eta^{\,c-2}(c-1-\phi_\eta^{\,c})\left(\frac{\eta-\tau}{1+\phi_\eta^{\,c}}\right)^2$$

$$\frac{\partial^2 \ln L}{\partial k \partial c} = -\sum_{i=1}^{n_u}\frac{x_i^{\,c}\ln x_i}{1+x_i^{\,c}} - \sum_{i=n_u+1}^{J}\frac{\phi_i^{\,c}\ln\phi_i}{1+\phi_i^{\,c}} - \frac{\tau^c \ln\tau}{1+\tau^c}\sum_{i=1}^{n_u}R_i - n_a R_J^*\frac{\phi_\eta^{\,c}\ln\phi_\eta}{1+\phi_\eta^{\,c}}$$

$$\frac{\partial^2 \ln L}{\partial k \partial \beta} = -c\sum_{i=n_u+1}^{J}\frac{\phi_i^{\,c-1}(x_i-\tau)}{1+\phi_i^{\,c}} - \frac{n_a c R_J^*(\eta-\tau)\phi_\eta^{\,c-1}}{1+\phi_\eta^{\,c}}$$

$$\frac{\partial^2 \ln L}{\partial c \partial \beta} = \sum_{i=n_u+1}^{J}\frac{(x_i-\tau)}{\phi_i} - (k+1)\sum_{i=n_u+1}^{J}\frac{(x_i-\tau)\phi_i^{\,c-1}(1+c\ln\phi_i+\phi_i^{\,c})}{(1+\phi_i^{\,c})^2} - kn_a R_J^*(\eta-\tau)\frac{\phi_\eta^{\,c-1}(1+c\ln\phi_\eta+\phi_\eta^{\,c})}{(1+\phi_\eta^{\,c})^2}$$

Thus, the approximate *100(1-ε)%* two sided confidence intervals for *c*, *k* and *β* are respectively given by

$$\hat{c} \pm z_{\varepsilon/2}\sqrt{I_{11}^{-1}(\hat{c},\hat{k},\hat{\beta})}\,,\quad \hat{k} \pm z_{\varepsilon/2}\sqrt{I_{22}^{-1}(\hat{c},\hat{k},\hat{\beta})}\,,\quad \hat{\beta} \pm z_{\varepsilon/2}\sqrt{I_{11}^{-1}(\hat{c},\hat{k},\hat{\beta})}$$

Where $z_{\varepsilon/2}$ is the upper $(\varepsilon/2)^{th}$ percentile of a standard normal distribution.

4. **Simulation Studies**

In this section simulation study is performed to evaluate the performance of the MLEs in terms of their mean squared errors(MSEs) for various choices of n, m, τ and η values. Also, 95% asymptotic confidence bounds are made based on the asymptotic distribution of the ML estimators. It is performed using the R software.

The considered schemes are as follows:

$$Scheme1: R_1 = \cdots = R_{m-1} = 0 \; and \; R_m = n-m$$
$$Scheme2: R_1 = n-m \;\; and \;\; R_2 = \cdots = R_m = 0$$
$$Scheme3: R_1 = \cdots = R_{m-1} = 1 \; and \, R_m = n-2m+1$$

The algorithm of the simulation study is given as

(1) Specify the values of n, m, τ and η.

(2) Choose values of c, k and β.

(3) To generate the data from the Burr type XII distribution,a random sample of size n from uniform random variable [0,1]. Then we use iverse cdf in eq(2.2) to generate data from the distribution $y = \left[\exp\left(-\ln(1-u)/k\right) - 1\right]^{1/c}$.

(4) The data set can be considered to generate progressively type I hybrid censored data for the given values of n, m, τ, η(η>τ), c, k and β.

(5) Parameters are estimated using the above data. Newton-Raphson iterative method is used for solving the system of nonlinear equations.

(6) Replicate step 3-5, 10,000 times to avoid randomness.

(7) Compute the average values of biases and MSEs associated with the ML estimators of the parameters.

Table 1: The average of MLEs and its Biases and MSEs at the values of parameters (c=1.3, k=1.4, β=1.1) for different sample sizes under different schemes of type-I progressive hybrid censoring

		Estimate of c			Estimate of k			Estimate of β		
(n,m)	Schemes	MLE	Bias	MSE	MLE	Bias	MSE	MLE	Bias	MSE
(30,15)	1	1.251	0.453	0.438	1.393	0.431	0.404	1.121	0.378	0.343
	2	1.261	0.496	0.451	1.524	0.463	0.440	1.072	0.417	0.386
	3	1.352	0.467	0.449	1.436	0.451	0.429	1.182	0.385	0.370
(30,20)	1	1.285	0.389	0.354	1.387	0.410	0.395	1.142	0.336	0.304
	2	1.271	0.425	0.382	1.426	0.443	0.428	1.213	0.369	0.337
	3	1.286	0.403	0.368	1.408	0.436	0.416	1.128	0.362	0.317
(50,30)	1	1.318	0.381	0.352	1.387	0.313	0.286	0.978	0.295	0.254
	2	1.273	0.412	0.379	1.478	0.407	0.348	1.218	0.331	0.286
	3	1.306	0.396	0.360	1.386	0.353	0.303	1.017	0.318	0.275
(50,40)	1	1.279	0.347	0.304	1.385	0.296	0.228	1.005	0.273	0.228
	2	1.347	0.423	0.374	1.437	0.341	0.285	1.193	0.309	0.263
	3	1.286	0.369	0.325	1.404	0.318	0.259	1.247	0.297	0.237
(70,50)	1	1.317	0.276	0.239	1.392	0.243	0.198	1.204	0.247	0.190
	2	1.330	0.328	0.283	1.378	0.328	0.263	1.185	0.283	0.234
	3	1.282	0.292	0.244	1.413	0.277	0.221	0.996	0.268	0.210
(70,60)	1	1.313	0.202	0.176	1.408	0.185	0.129	1.135	0.193	0.155
	2	1.338	0.283	0.217	1.389	0.238	0.183	1.217	0.249	0.192
	3	1.316	0.227	0.183	1.418	0.208	0.167	0.952	0.241	0.178

Table 2: The average of MLEs and its Biases and MSEs at the values of parameters (c=1.3, k=1.4, β=1.25) for different sample sizes under different schemes of type-I progressive hybrid censoring

		Estimate of c			Estimate of k			Estimate of β		
(n,m)	Schemes	MLE	Bias	MSE	MLE	Bias	MSE	MLE	Bias	MSE
(30,15)	1	1.263	0.427	0.387	1.372	0.408	0.382	1.118	0.354	0.326
	2	1.258	0.463	0.418	1.496	0.445	0.398	1.289	0.423	0.352
	3	1.326	0.448	0.392	1.419	0.437	0.387	1.127	0.361	0.333
(30,20)	1	1.274	0.394	0.327	1.361	0.382	0.349	1.187	0.318	0.273
	2	1.267	0.418	0.353	1.418	0.436	0.384	1.221	0.349	0.318
	3	1.283	0.386	0.342	1.426	0.408	0.327	1.162	0.327	0.280
(50,30)	1	1.313	0.344	0.302	1.438	0.305	0.276	1.402	0.279	0.247
	2	1.259	0.382	0.328	1.464	0.374	0.317	1.481	0.303	0.268
	3	1.338	0.365	0.317	1.373	0.329	0.281	1.183	0.284	0.252
(50,40)	1	1.262	0.317	0.273	1.357	0.298	0.242	1.386	0.267	0.217
	2	1.320	0.384	0.328	1.446	0.352	0.263	1.282	0.283	0.243
	3	1.265	0.349	0.294	1.429	0.321	0.259	1.350	0.295	0.225

(70,50)	1	1.276	0.255	0.218	1.387	0.240	0.186	1.228	0.225	0.173
	2	1.321	0.293	0.237	1.364	0.331	0.238	1.452	0.267	0.217
	3	1.277	0.270	0.225	1.436	0.288	0.207	1.197	0.239	0.184
(70,60)	1	1.325	0.216	0.167	1.427	0.174	0.115	1.183	0.178	0.138
	2	1.326	0.273	0.224	1.356	0.204	0.153	1.237	0.203	0.162
	3	1.309	0.230	0.176	1.433	0.194	0.129	1.376	0.187	0.147

Table 3: Confidence intervals of the parameters (c=1.3, k=1.4, β=1.1) at confidence level 0.95.

(n,m)	Schemes	Estimate of c		Estimate of k		Estimate of β	
		LCL	UCL	LCL	UCL	LCL	UCL
(30,15)	1	0.758	1.820	0.736	2.002	0.585	1.987
	2	0.791	1.836	0.717	2.253	0.562	2.014
	3	0.773	1.847	0.749	1.995	0.571	1.973
(30,20)	1	0.784	1.816	0.758	1.974	0.606	1.954
	2	0.759	1.838	0.733	2.117	0.579	1.997
	3	0.803	1.782	0.746	1.981	0.593	1.941
(50,30)	1	0.836	1.748	0.772	1.939	0.628	1.885
	2	0.825	1.785	0.748	1.972	0.582	1.916
	3	0.841	1.753	0.753	1.946	0.643	1.894
(50,40)	1	0.842	1.742	0.815	1.895	0.662	1.833
	2	0.812	1.763	0.786	1.938	0.604	1.876
	3	0.828	1.737	0.799	1.887	0.677	1.828
(70,50)	1	0.849	1.724	0.842	1.858	0.690	1.784
	2	0.827	1.756	0.801	1.896	0.651	1.839
	3	0.861	1.747	0.857	1.840	0.683	1.768
(70,60)	1	0.863	1.718	0.895	1.818	0.727	1.730
	2	0.842	1.750	0.864	1.854	0.686	1.782
	3	0.875	1.746	0.887	1.809	0.744	1.741

Table 4: Confidence intervals of the parameters (c=1.3, k=1.4, β=1.25) at confidence level 0.95.

(n,m)	Schemes	Estimate of c		Estimate of k		Estimate of β	
		LCL	UCL	LCL	UCL	LCL	UCL
(30,15)	1	0.778	1.809	0.747	2.093	0.524	1.782
	2	0.815	1.817	0.738	2.187	0.503	1.814
	3	0.794	1.883	0.763	2.066	0.548	1.763
(30,20)	1	0.795	1.792	0.778	1.986	0.580	1.747
	2	0.830	1.781	0.754	1.947	0.552	1.764
	3	0.821	1.799	0.768	1.974	0.594	1.726
(50,30)	1	0.854	1.756	0.838	1.968	0.603	1.718
	2	0.862	1.743	0.823	1.946	0.578	1.746
	3	0.857	1.767	0.882	1.903	0.639	1.689
(50,40)	1	0.835	1.828	0.901	1.881	0.626	1.693
	2	0.804	1.782	0.889	1.853	0.617	1.720
	3	0.817	1.753	0.917	1.819	0.654	1.671
(70,50)	1	0.867	1.773	0.952	1.807	0.651	1.660
	2	0.836	1.791	0.937	1.792	0.637	1.682
	3	0.859	1.762	0.961	1.799	0.668	1.627
(70,60)	1	0.883	1.745	0.995	1.716	0.705	1.638
	2	0.862	1.767	0.968	1.704	0.678	1.664
	3	0.894	1.738	0.983	1.738	0.714	1.609

Findings:

Simulation study has been performed and the results are summarized in table 1-4. To get the smooth results and to avoid the randomness, the procedures are replicated 10000. From table 1 & 2, it is observed that the biases and MSEs are decreasing as the values of sample size are increased for all cases. When number of failures increases, the RABs and MSEs also are decreased. Only Scheme 2 has a slightly larger biases and MSEs than scheme 1 and scheme 3, since the experiment is censored heavily in the beginning of it. It can also be observed that Table 2 has RABs and MSEs less than Table 1 because when the acceleration factor increases the errors and biases decreases. Additionally, the confidence intervals also get narrower as the sample size and acceleration factor increases.

4.5. Conclusion

In this study, it is considered that the lifetimes of the units follow Burr Type XII distribution. To estimate the acceleration factor and parameters of the distribution maximum likelihood estimation technique is used under step-stress partially accelerated life test method using type-I progressive hybrid censored data. Newton-Raphson method is used to obtain the point estimates of the parameters and tampering coefficient. Their performances are analysed and discussed in terms of biases and MSEs. It has been observed that all the statistical assumptions are fulfilled. This shows that the assumptions of experiment, model considered and data used are correct. Bayesian inferences under the SSPALT assuming the same censoring proposed in this article can be considered as future work plan.

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