Reliability Assessment of Deteriorating System

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Abstract

Industrial equipment and technical systems are exposed various degree of deterioration ranging from minor deterioration, medium to major deterioration and subsequently failed thereafter and are replaced at failure. Such deteriorations can slightly reduce system performance and will ultimately lead to random failure. This paper presents modelling and evaluation of reliability characteristics such as availability, profit and mean time to failure (MTTF) of a system subjected to three consecutive stages of deterioration (minor, medium and major) before failure. Markov models of the system are derived through the system state transition probabilities and differential equations which are further used to evaluate the system availability, busy period, profit and mean time to failure (MTTF). Based on assumed numerical values given to system parameters, graphical illustrations are given to highlight important results.

Keywords: , Deterioration, Reliability, Availability, profit.
Mathematical Subject Classification: 90B25

Introduction

The process industry comprises of large complex engineering systems, subsystems arranged in standby, series, parallel or a combination of them. For efficient and economical operation of a process plant, each system or the subsystem should work failure free under the existing operative plant conditions. Most of these systems are subjected to random deterioration which can result in unexpected failures and disastrous effect on the system availability and the prospect of the economy. Therefore, it is important to find a way to slow down the deterioration rate, and to prolong the equipment's life span. Maintenance policies are vital in the analysis of deterioration and deteriorating systems as they help in improving reliability and availability of the systems. Maintenance models can assume minor maintenance, major maintenance before system failure, perfect repair (as good as new), minimal repair (as bad as old), imperfect repair and replacement at system failure.

Several models on deteriorating systems under different conditions have been studied by several researchers. Liu *et al.* [1] presented the reliability analysis of a deteriorating system with delayed vacation of repairman. Pandey *et al.*[2] discussed the influence of temporal uncertainty of deterioration on life-cycle management of structures. Rani *et al.* [3] discussed the replacement time for a deteriorating system. Tuan *et al.* [4] dealt with reliability-based predictive maintenance modelling for k-out-of-n Deteriorating Systems. Vinayak and Dharmaraja [5] presented semi-Markov modeling approach for deteriorating systems with preventive maintenance. Xiao *et al.* [6] studied the Bayesian reliability estimation for deteriorating systems with limited samples. Yuan *et* *al.* [7] analyzed modeling of a deteriorating system with repair satisfying general distribution. Yuan and Xu [8] studied deteriorating system with its repairman having multiple vacations. Yusuf *et al* [9] presented modeling the reliability and availability characteristics of a system with three stages of deterioration.

In this paper, a single system with four consecutive modes minor, medium and major deterioration failure modes is considered and derived its corresponding mathematical models. Furthermore, we study availability of the system using linear first order differential equations. The focus of our analysis is primarily to capture the effect of minor and medium deterioration rate, minor and major maintenance rates on steady-state availability and profit.

The organization of the paper is as follows. Section 2 contains a description of the system under study. Section 3 presents formulations of the models. The results of our numerical simulations are presented in section 4. Finally, we make some concluding remarks in Section 5.

Description and States of the System

In this paper, a single system with three consecutive modes of deterioration: a minor, medium and major deterioration and failure mode is considered. At early state of the system life, the unit is exposed to minor deterioration with rate λ_1 **and this deterioration is rectified through minor maintenance** α_1 **which revert the unit to its earliest position before deterioration**. If not maintained, the unit is allowed to continue operating under the condition of minor deterioration which later results to medium deterioration with rate λ_2 . At this stage, the strength of the unit still strong that it can rectified to early state with major maintenance with rate α_2 . However, the system can move to major deterioration stage with rate λ_3 where the and subsequently failed with parameter λ_4 and replaced by with a new one with rate α_3 .

Table 1: Transition rate table

	s_0	s_1	s_2	s_3	s_4
s_0		λ_1			
s_1	α_1		λ_2		
s_2	α_2			λ_3	
s_3					λ_4
s_4	α_3				

Table 2: States of the System

State	Description
S_0	Initial state, the unit is working. The system is working.
S_1	The working unit is in minor deterioration mode and is under online minor maintenance. The system is working.
S_2	The working unit is in medium deterioration mode and is under online major maintenance. The system is working.
S_3	The working unit is in major deterioration mode. The system is working.
S_4	The working unit has failed. The system is inoperative. .

Formulation of the Models

In order to analyze the system availability of the system, define $P_i(t)$ to be the probability that the system at $t \geq 0$ is in state S_i. Also let $P(t)$ be the row vector of these probabilities at time t. The initial condition for this problem is:

$$P(0) = [p_0(0), p_1(0), p_2(0), p_3(0), p_4(0)]$$
$$= [1, 0, 0, 0, 0]$$

the following differential difference equations are obtained from Figure 1:

$$\left.\begin{aligned}
\frac{d}{dt}p_0(t) &= -\lambda_1 p_0(t) + \alpha_1 p_1(t) + \alpha_2 p_2(t) + \alpha_3 p_4(t) \\
\frac{d}{dt}p_1(t) &= -(\alpha_1 + \lambda_2)\, p_1(t) + \lambda_1 p_0(t) \\
\frac{d}{dt}p_2(t) &= -(\alpha_2 + \lambda_3)\, p_2(t) + \lambda_2 p_1(t) \\
\frac{d}{dt}p_3(t) &= -\lambda_4 p_4(t) + \lambda_3 p_2(t) \\
\frac{d}{dt}p_4(t) &= -\alpha_3 p_4(t) + \lambda_4 p_3(t)
\end{aligned}\right\} \quad \mathbf{(1)}$$

This can be written in the matrix form as

$$\dot{P} = MP\,, \quad \mathbf{(2)}$$

where

$$M = \begin{pmatrix} -\lambda_1 & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\ 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\ 0 & 0 & 0 & \lambda_4 & -\alpha_3 \end{pmatrix}$$

Equation (2) is expressed explicitly in the form

$$\begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{pmatrix} = \begin{pmatrix} -\lambda_1 & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\ 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\ 0 & 0 & 0 & \lambda_4 & -\alpha_3 \end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{pmatrix}$$

The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) and busy periods (the sum of the probabilities of states involving minor and major maintenance and replacement) are given by

$$A_V(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) \quad (3)$$

$$B_{P1}(\infty) = p_1(\infty) \quad (4)$$

$$B_{P2}(\infty) = p_2(\infty) \quad (5)$$

$$B_{P3}(\infty) = p_8(\infty) \quad (6)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (2) become

$$MP = 0 \quad (7)$$

this is in matrix form

$$\begin{pmatrix} -\lambda_1 & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\ 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\ 0 & 0 & 0 & \lambda_4 & -\alpha_3 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Subject to following normalizing conditions:

$$p_0(\infty)+p_1(\infty)+p_2(\infty)+p_3(\infty)+p_4(\infty)=1 \quad (8)$$

Substitute (8) in the last row of (7) to compute the steady-state probabilities $p_0(\infty), p_1(\infty), p_2(\infty), p_3(\infty), p_4(\infty)$

$$\begin{pmatrix} -\lambda_1 & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\ 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The expressions for the steady-state availability and busy periods involving minor and major maintenance and replacement given in equations (3), (4), (5) and (6) above are given by

$$A_T(\infty)=\frac{\alpha_3\lambda_4(\lambda_2\lambda_3+\alpha_2\lambda_2+\alpha_1\lambda_3+\alpha_1\alpha_2)+\alpha_3\lambda_1\lambda_4(\alpha_2+\lambda_3)+\alpha_3\lambda_1\lambda_2\lambda_4+\alpha_3\lambda_1\lambda_2\lambda_3}{\lambda_2\lambda_3(\alpha_3\lambda_4+\lambda_1\lambda_4+\alpha_3\lambda_1)+\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_1)+\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_1)+\alpha_2\alpha_3\lambda_4(\alpha_1+\lambda_1)}$$

$$B_{P1}(\infty)=p_1(\infty)=\frac{\alpha_3\lambda_1\lambda_4(\alpha_2+\lambda_3)}{\lambda_2\lambda_3(\alpha_3\lambda_4+\lambda_1\lambda_4+\alpha_3\lambda_1)+\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_1)+\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_1)+\alpha_2\alpha_3\lambda_4(\alpha_1+\lambda_1)}$$

$$B_{P2}(\infty)=\frac{\alpha_3\lambda_1\lambda_2\lambda_4}{\lambda_2\lambda_3(\alpha_3\lambda_4+\lambda_1\lambda_4+\alpha_3\lambda_1)+\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_1)+\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_1)+\alpha_2\alpha_3\lambda_4(\alpha_1+\lambda_1)}$$

$$B_{P3}(\infty)=\frac{\lambda_1\lambda_2\lambda_3\lambda_4}{\lambda_2\lambda_3(\alpha_3\lambda_4+\lambda_1\lambda_4+\alpha_3\lambda_1)+\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_1)+\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_1)+\alpha_2\alpha_3\lambda_4(\alpha_1+\lambda_1)}$$

From Figure 1, the system is under minor and major maintenance due to minor and medium deterioration and replacement due to failure as can be observed in the states 1, 2,5, 6 and 8 respectively. Let C_0, C_1, C_2 and C_3 be the revenue generated when the system is in a working state , equivalently loss of income when in an inoperative/failed state and the cost of each maintenance and replacement respectively. The expected total profit per unit time generated by the system in the steady-state is

Profit=total revenue generated – (total maintenance and replacement cost).

$$PF=C_0A_V(\infty)-C_1B_{P1}(\infty)-C_2B_{P2}(\infty)-C_3B_{P3}(\infty) \quad \textbf{(9)}$$

It is difficult to evaluate the transient solutions, hence we follow Wang and Kuo (2000) and Wang et al. (2006) and delete the rows and columns of absorbing state of matrix M and take the transpose to produce a new matrix, say Q. The expected time to reach an absorbing state is obtained from

$$MTTF=P(0)(-Q^{-1})[1,1,1,1]^T=\frac{\lambda_4(\alpha_1+\lambda_2)(\alpha_2+\lambda_3)+\lambda_1\lambda_4(\alpha_2+\lambda_3)+\lambda_1\lambda_2(\lambda_3+\lambda_4)}{\lambda_1\lambda_2\lambda_3\lambda_4} \quad (10)$$

Where

$$Q = \begin{pmatrix} -\lambda_1 & \lambda_1 & 0 & 0 \\ \alpha_1 & -(\alpha_1+\lambda_2) & \lambda_2 & 0 \\ \alpha_2 & 0 & -(\alpha_2+\lambda_3) & \lambda_3 \\ 0 & 0 & 0 & -\lambda_4 \end{pmatrix}$$

Numerical Examples and Discussion

Numerical examples are presented to demonstrate the impact of deterioration and maintenance rates on steady-state availability and net profit of the system based on given values of the parameters. For the purpose of numerical example, the following sets of parameter values are used: $\lambda_1 = 0.1$, $\lambda_2 = 0.5$, $\lambda_3 = 0.9$, $\lambda_4 = 0.1$, $\alpha_1 = 0.1$, $\alpha_2 = 0.3$, $\alpha_3 = 0.3$, $C_0 = 500,000$, $C_1 = 10,000$, $C_2 = 12,000$, $C_3 = 15,000$.

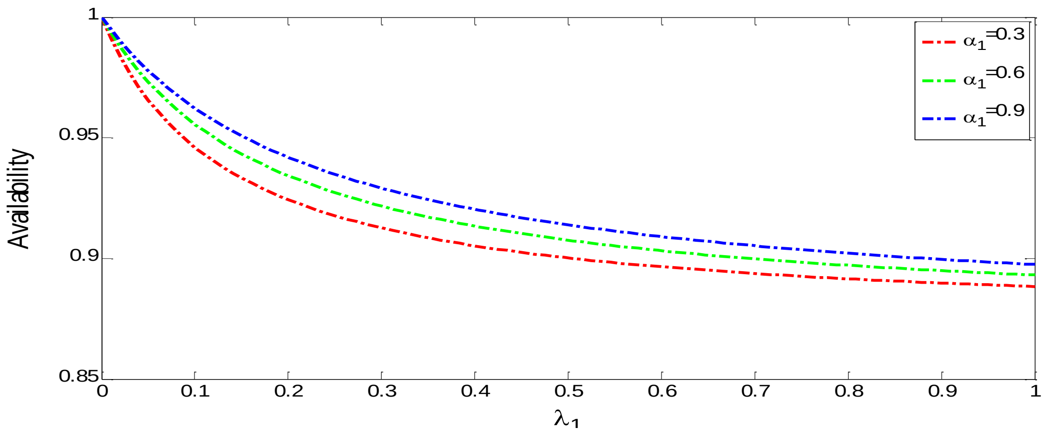

Figure 1: Availability against minor deterioration rate λ_1 **for** different values of $\alpha_1 (0.3, 0.6, 0.9)$

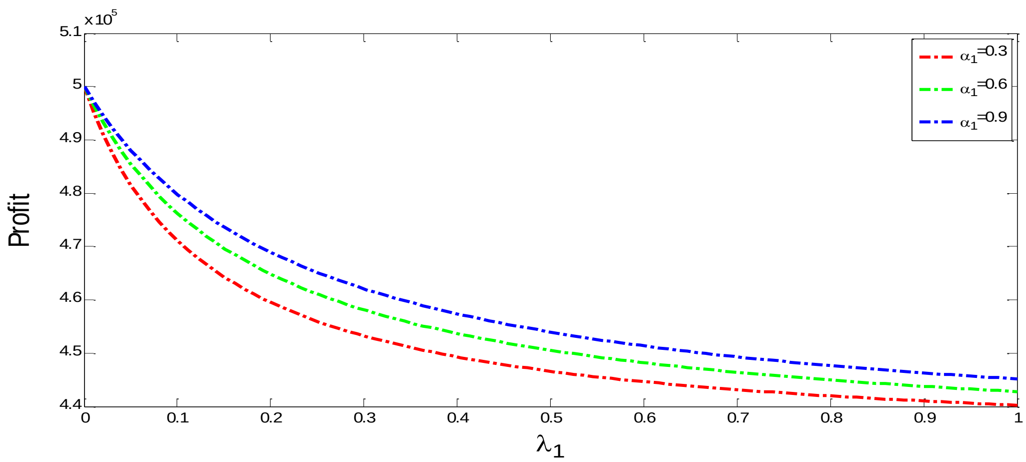

Figure 2: Profit against minor deterioration rate λ_1 **for** different values of $\alpha_1 (0.3, 0.6, 0.9)$

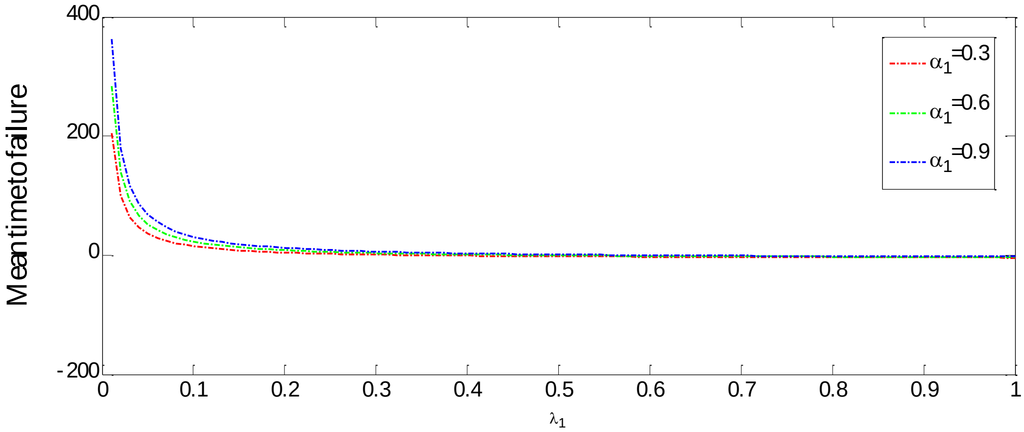

Figure 3: MTTF against minor deterioration rate λ_1 **for** different values of $\alpha_1(0.3, 0.6, 0.9)$

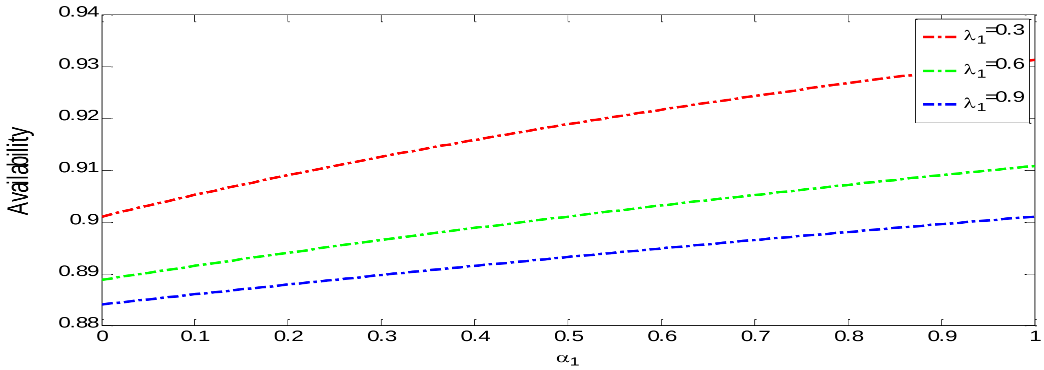

Figure 4: Availability against minor maintenance rate α_1 **for** different values of $\lambda_1(0.3, 0.6, 0.9)$

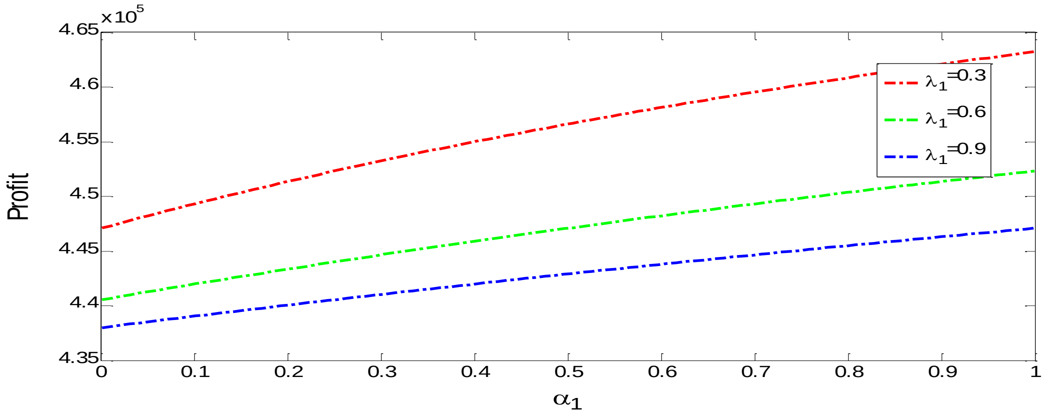

Figure 5: Profit against minor maintenance rate α_1 **for** different values of $\lambda_1(0.3, 0.6, 0.9)$

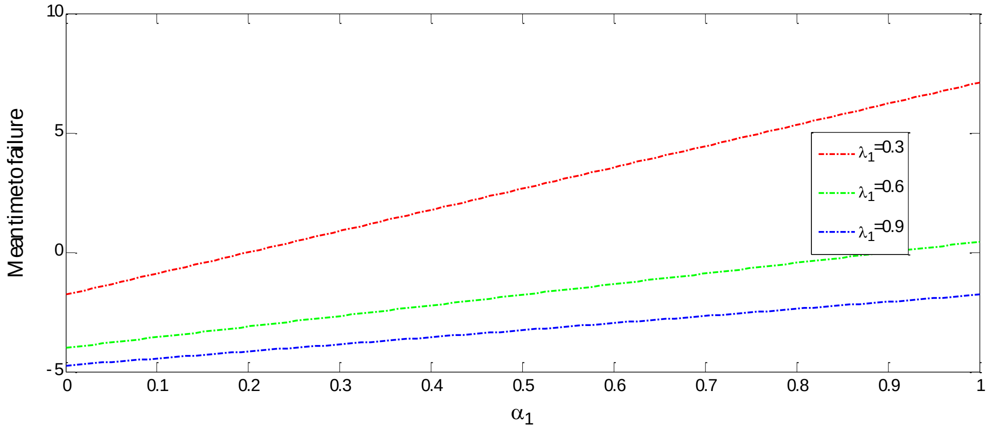

Figure 6: MTTF against minor maintenance rate α_1 **for** different values of $\lambda_1(0.3, 0.6, 0.9)$

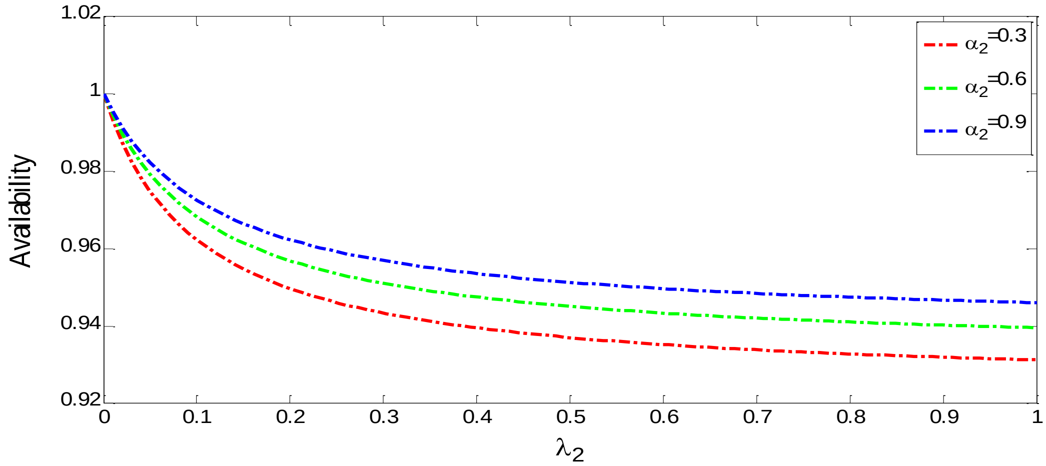

Figure 7: Availability against medium deterioration rate λ_2 **for** different values of $\alpha_2(0.3, 0.6, 0.9)$

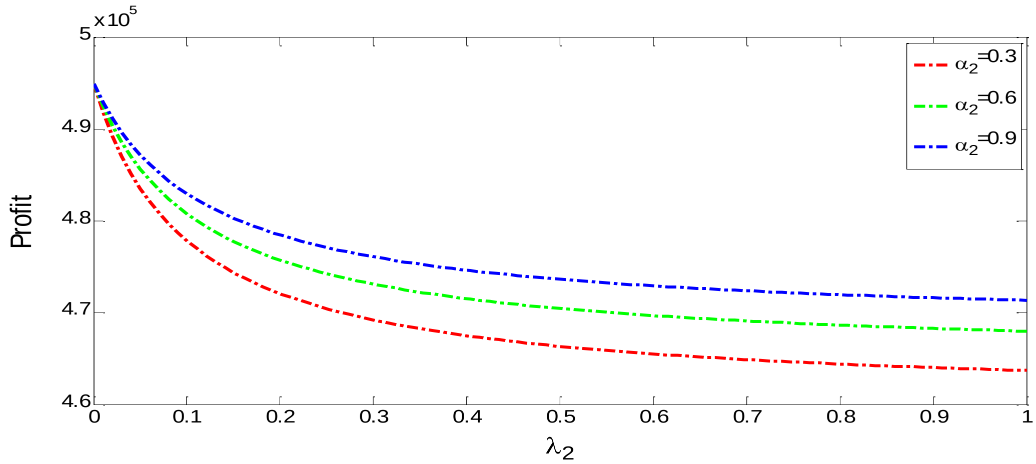

Figure 8: Profit against medium deterioration rate λ_2 **for** different values of $\alpha_2(0.3, 0.6, 0.9)$

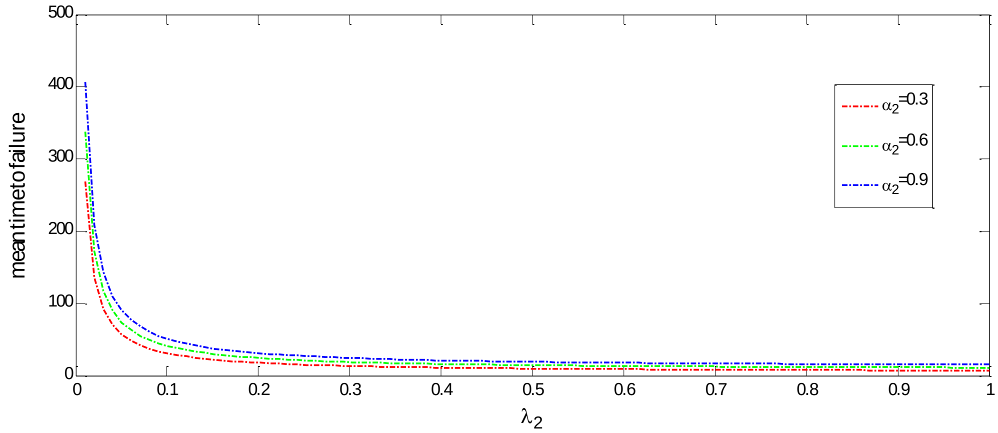

Figure 9: MTTF against medium deterioration rate λ_2 **for** different values of $\alpha_2(0.3, 0.6, 0.9)$

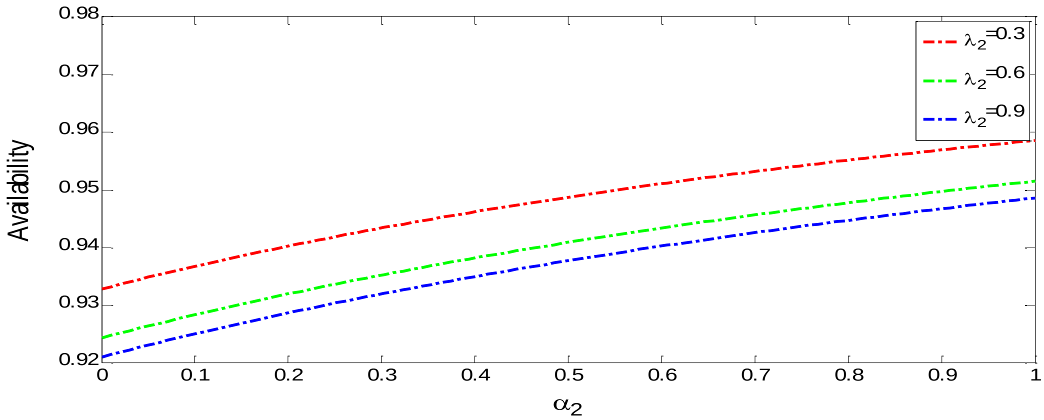

Figure 10: Availability against major maintenance rate α_2 **for** different values of $\lambda_2(0.3, 0.6, 0.9)$

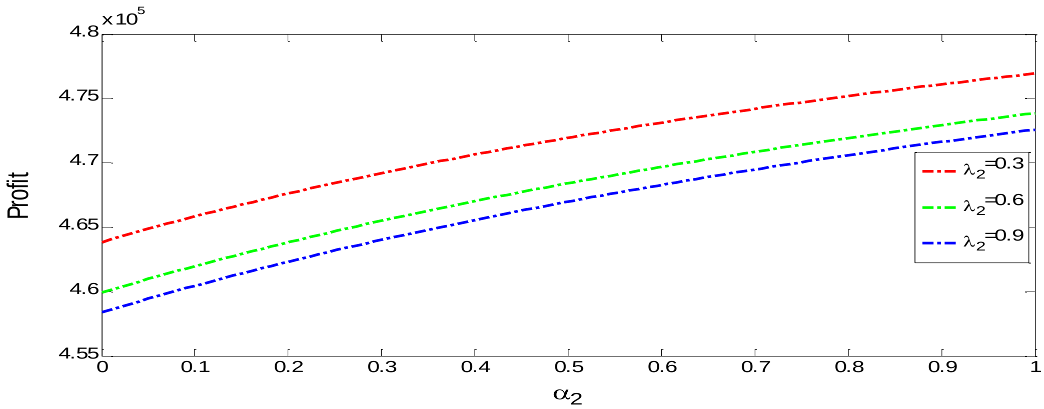

Figure 11: Profit against major maintenance rate α_2 **for** different values of $\lambda_2(0.3, 0.6, 0.9)$

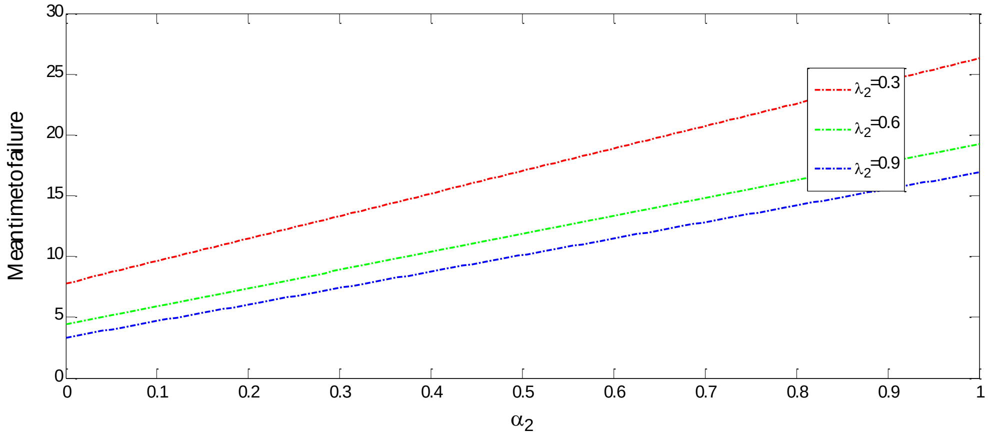

Figure 12: MTTF against major maintenance rate α_2 **for** different values of $\lambda_2(0.3, 0.6, 0.9)$

The results which compare the steady state availability and profit with respect to λ_1 for different values of α_1, are depicted in Figures 1-3. From these Figures, it is evident that system availability, profit and MTTF decrease as λ_1 increases. It is clear from these that availability and profit tend to be higher when $\alpha_1 = 0.9$. This shows that minor maintenance could make a significant difference to the system availability and profit. Numerical results of availability, profit and MTTF with respect to minor maintenance α_1 for different values of λ_1 are depicted in Figures 4-6. It is clear from these Figures that availability and profit increases as α_1 increases. It is evident from these Figures that minor maintenance significantly slow down minor deterioration from 0.9 to 0.3 as depicted in the Figures. Thus, availability, profit and MTTF are higher when $\lambda_1 = 0.3$. Similar observations can be seen in Figures 7-9 with respect to medium deterioration λ_2 and major maintenance α_2. Availability, profit and MTTF displayed decreasing pattern as λ_2 increases for different values of α_2 in Figures 7 and 8. It is clear from these that availability and profit tend to be higher when $\alpha_2 = 0.9$. This shows that higher major maintenance could make a great impact to the system availability and profit. On the other hand, Results of availability, profit and MTTF with respect to α_2 for different values of λ_2 are depicted in Figures 10-12. It is clear from these Figures that availability, profit and MTTF increases as α_2 increases. It is evident from these Figures that major maintenance significantly slow down minor deterioration from 0.9 to 0.3 as depicted in the Figures. Thus, availability and profit are higher when $\lambda_2 = 0.3$.

Conclusion

In this paper, a single unit system with four modes: minor deterioration, medium deterioration, and major deterioration and failure modes is studied. The paper presents modelling and evaluation of reliability characteristics such as availability, profit and MTTF of the system. Explicit expressions for the steady-state availability, busy period for minor and major maintenance, and replacement, profit function and MTTF have been developed.

On the basis of the numerical examples presented in Figures 2-13, it is suggested that the availability, profit and MTTF of a system can be enhance by

(i) By taking emphasis to maintenance (preventive maintenance) before or at early stage of deterioration.
(ii) By increasing maintenance rate.
(iii) Adding more spares/ cold standby units

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