How Entropy Infiltrates the Reliability Domain: Two Theoretical Inquiries

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Abstract

The reliability theory includes an enormous amount of works but does not yet become a mature discipline. Many factors account for this fail; as first the reliability domain is lacking the comprehensive and unifying frame.
The entropy concept demonstrates to be capable of offering unified views and providing integrated mathematical tools in various scientific sectors. The present paper presents a concise review of two recent theoretical studies that introduce the entropy in the reliability domain in order to treat broad issues. The first of these applies thermodynamic entropy to degradation mechanisms and provides a model for a wide range of degradation processes. The second employs the Boltzmann-like entropy to describe the spontaneous decay of systems during the entire lifespan.

Keywords: The present state of the reliability theory, thermodynamic degradation, entropy generation theorem, Boltzmann-like entropy, bathtub curve, reparability function.

I. Introduction

I.1 The present state of the reliability theory

Reliability theory was boosted after the Second World War when experts of the US and the USSR elaborated effective answers to practical issues arising from companies and institutions. Reliability studies continued to progress in various directions in the following decades. However, those mathematicians who were mostly attracted by applied problems paid little attention to broad issues and general principles. Nowadays, reliability theory looks like a 'heap' of works addressing an assortment of topics that are sometimes not clearly connected in terms of logic due to the missing unifying conceptual frame.
There are several small theories and specialist approaches under the umbrella of the reliability domain, as examples we mention the following self-explanatory parts of modern reliability studies: physics-of-failure methods, the cumulative and extreme shock theory, availability calculations, human factors and reliability-centered maintenance. They provide a rather effective aid to practitioners, though the large number of data-driven inquiries about system dependability can be compared to a collection of things placed randomly one on top of the other because of the missing comprehensive frame.

I.2 The unifying concept of entropy

By the end of the seventeenth century, steam engines triggered the Industrial Revolution, although those engines were somewhat inefficient. Rudolf Clausius tackled this kind of issues and stated the second law of thermodynamics by means of the entropy H_C that is a function of the exchanged amount of heat Q and temperature T

$$dH_C = \frac{\delta Q}{T} \leq 0. \tag{1}$$

The quantity dH_C is capable of explaining the irreversibility of real systems that are steam engines and also living beings, nuclear plants, stars etc. [1]. Ludwig Boltzmann fixed on the entropy H_B as a measure of the statistical disorder caused by the number Ω of complexions typical of the equilibrium state of S [2]. When the temperature reaches absolute zero, a perfect lattice of molecules has only one complexion, and H_B illustrates the third law of thermodynamics, which is a universal rule

$$H_B = k_B \ln \Omega = 0. \tag{2}$$

Claude Shannon defined this entropy for a source of discrete signals

$$H_S = -\sum_{i}^{n} P_i \log_2 P_i \ . \tag{3}$$

Where P_i is the probability of the generic signal emitted by the source. He established that the average code-word length L for any distortionless source coding is bounded as

$$L \geq H_S. \tag{4}$$

The entropy H_S proves the source coding theorem that is a fundamental statement in information theory.

Scholars devised other entropy equations, yet the present summary should be sufficient to recall how the entropy is able to express general laws that belong to a variety of sectors. The entropy unifies several empirical expressions in steam engineering and also in areas that are rather distant from thermodynamics, such as telecommunications, quantum mechanics, computing, economics, and biology. The entropy notion enhances experts' knowledge and at the same time improves the effectiveness of practitioners. Once a comprehensive equation has been formulated with the aid of entropy, then convenient specializations to particular structural elements proceed on a more informed basis.

I.3 An assortment of specialist issues

A group of theorists employ the concept of entropy to solve specific issues in the reliability sector. For example, some put information redundancy near to operation redundancy and use the Shannon entropy to optimize the parallels between systems [4], [5]; others aim at controlling the complexity of intelligent systems [6]. The present paper makes a short review of two inquiries whose authors address broad issues. They seek a unified view and integrated mathematical instruments namely they tackle the issue presented in Section I.1. The first study looks at degradation processes, and the second answers some problems that lay at the basis of the reliability theory.

II. Entropy and Energy Dissipation

Degradation processes affect several systems and have a notable impact on modern economies. Speaking in general, degradation processes involve different mechanisms with distinctive features, types, and rates. Experts use collective terms such as corrosion, erosion, wear, fatigue, thermal degeneration, and plasticity to group together the varieties of failure mechanisms. For example, by the term 'corrosion' experts mean galvanic corrosion, pitting, dealloying, crevice corrosion, microbiologically-influenced corrosion, stress corrosion cracking, intergranular corrosion, fretting, and hydrogen damage [7].
Normally a degradation process evolves over a certain period of time, and the path to failure depends on an assortment of factors that requires systematic inspections of the system components and the collection of data from different sources such as photographic documentation, microscopic scanning, chemical analysis etc. The dynamics of each degeneration are based on particular physical principles, and often a formal model is limited primarily to a single type of failure. The complex and specific traits of each degradation process require engineers to use a non-negligible set of variables. The science of materials employs a large apparatus of concepts, formal definitions and mathematical tools, while instead the agile theory of degenerative phenomena should be derived from fruitful generalization and uniform principles. The present complicated intellectual situation stimulated the search for a unifying frame capable of facilitating the work of practitioners

II.1 Entropy generation theorem

A group of researchers started by observing how different degeneration processes share a common and intriguing feature: *the loss of energy*. Progressive decay consists of dissipative transformations, and the entropy H_C offers the appropriate lens to investigate these transformations with unifying mathematical support, as an alternative to the partial and heterogeneous models currently in use. The change in irreversible entropy can be calculated via the thermodynamic equations when the heat or energy dissipated is known.
The early attempts in this new direction of study may be ascribed to Klamecki, who developed the thermodynamic analysis of friction and wear for bodies in sliding contact [8], [9]. Another pioneer, Zmitrowicz, conducted a complex enquiry to predict the friction and wear of bodies in contact [10]. Progressively, research went in various directions: for instance, Dai and others analyzed the production of entropy associated with fretting wear [11].
Cemal Basaran set up a ponderous inquiry assuming the corrosion process as an irreversible entropy generating chemical process. According to second law of thermodynamics, irreversible entropy generation gradually increases as system evolves toward the final failure. Entropy production and its rate are used as the sole measures of system evolution in the place of a large number of specialist measures [13], [14]. Basaran and other scientists experimentally verified his unified scheme for several degradation factors such as mass transport, electromigration, thermomigration, creep, thermo-mechanical degradation, phase change, fatigue tribology and loading agents [12].
Sosnovskiy and Sherbakov following the ideas of Basaran examine a mechano-thermodynamic system which they conceive as a continuum including scattered solids which interact with each other and with the continuum. Sosnovskiy and Sherbakov prepare a generalized theory of system evolution which is based on the concept of tribo-fatigue entropy, in this way they make attempts to place under a single umbrella classical mechanics and thermodynamics [15]. Essence of the proposed approach is that tribo-fatigue entropy is determined by thermodynamic and mechanical effects causing to the change of states of the system.
Finally, we mention Bryant and others who develop a thermodynamic characterization of degradation dynamics using H_C as the fundamental measure. The paper [16] formulate the *entropy generation theorem* (EGT) which relate the entropy production and decay, via generalized thermodynamic forces and degradation forces. The authors assume that n dissipative processes $p_i = p_i \ \xi_i^j$ (i = 1,2,...n) are characterized by a set of time-dependent variables ξ_i^j (j = 1, 2,...m_i), while

the 'degradation measure' $w = w\{p_1, p_2,..., p_i,.., p_k\}$ determines the characters of the collective, including k processes. The results of the EGT can be summarized in the following three points:

(i) The generalized 'degradation forces' $Y_i^{\,j}$ are linear functions of the generalized 'thermodynamic forces' $X_i^{\,j}$.

(ii) The degradation component $\dot{w}_i = \sum_j Y_i^j J_i^j$ proceeds at the rate $J_i^{\,j}$ that is determined by the entropy production.

(iii) The degradation rate $\dot{w} = \sum_i \dot{w}_i$ is a linear combination of the components that produce the overall entropy.

The entropy generation theorem shows how linear expressions, which are consistent with the laws of thermodynamics, govern the decadence of systems. EGT suggests a simplified approach for degradation analysis and speeds up a methodology for the accelerated testing of degradation.

II.2 Multiple degradation

Some researchers are aiming to explain the damage mechanism for single materials as well as the synergistic effect amongst different mechanisms. Amiri and Modarres [17], [18] exploit EGT and write the total entropy of S as the sum of the entropies of reversible and irreversible changes

$$dH_C = dH_C^r + dH_C^d. \tag{5}$$

The reversible entropy dH_C^r is caused by the transfer of mass and heat

$$\frac{dH_C^r}{dt} = -\int_\Omega J_s d\Omega. \tag{6}$$

Where J_s is a vector of the total entropy flow per unit area. The entropy of irreversible processes dH_C^d is produced by the physical components of the system when σ is the entropy dissipated per unit volume and unit time

$$\frac{dH_C^d}{dt} = \int_V \sigma dV. \tag{7}$$

Using the conservation of energy, mass, and momentum, Amiri and Modarres calculate various dissipative phenomena pertaining to (6) and (7) in this way

$$J_s = -\nabla\left(\frac{J_q - \sum_k c_m\zeta + \mu_k \;\; J_k}{T}\right). \tag{8}$$

$$\sigma = \frac{1}{T^2} J_q \nabla T - \sum_k J_k\left(\nabla\frac{\mu_k}{T}\right) + \frac{1}{T}\;\; \tau : \varepsilon_p \;\; + \frac{1}{T}\left(\sum_i v_i D_i\right) + \frac{1}{T}\left(\sum_m c_m J_m \;\; - \nabla\zeta\right). \tag{9}$$

This entropic frame illustrates the multifold degeneration dynamics and their interrelations; more precisely, eqn. (8) is the result of the exchange of heat and mass with the surroundings. Eqn. (9) includes five terms that, in order, calculate internal heat conduction, diffusion dynamics, plastic deformation, chemical reactions, and external forces. More precisely, T is the temperature, J_q the heat flux, J_k the diffusion flow, c_m the coupling constant, ζ the potential of the external field (magnetic or electric), μ_k is the chemical potential, J_m the magnetic or electrical flux, v_i the chemical reaction rate, D_i the chemical reaction potential difference, τ the stress tensor, and ε_p the plastic strain rate tensor. In conclusion, this approach, which could be labeled as 'thermodynamic degradation' or 'thermodynamic reliability', provides a few equations for multiple degradation processes. The authors calculate the competing mechanisms that contribute to damage and can also account for the synergistic effects.

III. In Search of Ideal Models

Boris V. Gnedenko – often recognized as the 'father' of the reliability theory – conceives the reliability domain as a new science and lays the first theoretical stone for building up this scientific edifice in [19]. He defines the *hazard rate* $\lambda(t)$ (also called the *failure* or *mortality rate*) as the instantaneous incidence rate

$$\lambda\,(t) = -\,P'\,(t)/P(t). \tag{10}$$

And deduces the following exponential function through mathematical proof

$$P(t) = e^{-\int_0^t \lambda(t)dt}\ . \tag{11}$$

Where $P(t)$ – normally termed the *reliability* of S – is the probability of functioning with no failure in the interval from time 0 to time t. Several writers in the reliability sector assume that $\lambda(t)$ complies with the *bathtub curve*, however numerous evidence disproves this curve [20], [21], [22], which was set up through generic intuition and not after mathematical demonstration. A rigorous study that relates $\lambda(t)$ in all particulars to the system lifespan is lacking and the construction inaugurated by Gnedenko is no longer progressing because of the discrepancy between theory and practice.

III.1 The Boltzmann-like entropy

The preparation of a unified construct is a challenging duty, but a new form of entropy – the *Boltzmann-like entropy* – offers aid. The *irreversibility* and the *reversibility* (I/R) of the generic state A_i (i=1,2,..n) of the stochastic system S are coupled properties, and the Boltzmann-like entropy qualifies the I/R of the state A_i in accordance to the criteria adopted by Boltzmann [23]

$$H(A_i) = H(P_i) = \log\,(P_i). \tag{12}$$

Where P_i is the probability of the state A_i. Let us confine our attention to the *functioning state* A_f and the *recovery state* A_r during which S works steadily and is repaired/maintained respectively. The following intuitive remarks can help the reader to grasp the physical significance of the *functional entropy* $H_f = H(A_f) = H(P_f)$ and the *recovery entropy* $H_r = H(A_r) = H(P_r)$. In consequence of the coupled properties I/R we obtain the following paired remarks a and b:

1.a) When H_f is 'high', the functioning state is irreversible and the system works steadily. In particular, the higher H_f is, the more irreversible A_f is, and S is capable of working.
1.b) On the other hand, when H_f is low, S often abandons A_f and switches to A_r, and we say that S is incapable of working.

2.a) When H_r is 'high', the recovery state is irreversible, and the workers operate on S with effort. In particular, the higher H_r is, the more stable A_r is, and in practice S is hard to repair and/or maintain in the world.

2.b) On the other hand, when H_r is low, S leaves A_r and we say that S can be easily restored or maintained.

In summary, the Boltzmann-like entropy qualifies the behavior of the systems this way: *the reliability entropy expresses the aptitude of S to work without failures; the recovery entropy illustrates the disposition of S toward reparation.*

III.2 Ideal tripartite function

The reliability entropy furnishes an aid for detailing (11). If one assumes that the capability of good functioning decreases regularly with time, [24] demonstrates

$$\lambda(t) = c, \quad c > 0. \tag{13}$$

When the system's components have a certain degree of deterioration, a damaged part spoils a neighboring part and this in turn can affect another part and so on. If this cascade effect is linear, H_f leads to

$$\lambda(t) = at^n, \quad a, n > 1. \tag{14}$$

When the cascade effect evolves in various directions, H_f leads to

$$\lambda(t) = d\exp(t), \quad d > 1. \tag{15}$$

The precise assumptions that lead to (13), (14) and (15) enable an expert to deduce the position of these results within the system lifespan. The cascade effect occurs if the system components have a certain level of deterioration and this hypothesis is appropriate for aging. More precisely, (14) is suitable for machines and (15) for living systems that have intricate structures. The cascade effects do not work during system maturity when the constant hazard rate (13) is occurring. Generally, manufacturers' burn-in and infant mortality cause a decreasing distribution of $\lambda(t)$, and hence one can depict the hazard rate in linear terms during the system infancy

$$\lambda(t) \propto -mt, \quad m > 0. \tag{16}$$

Let t_1 and t_2 be the extreme times of the useful lifespan, then eqns. from (13) to (16) can be joined into the following tripartite function

$$\lambda(t) \begin{cases} \propto -mt, & 0 < t < t_1. \\ = c, & t_1 < t < t_2. \\ = at^n, \ \text{OR } d\exp(t), & t > t_2. \end{cases} \tag{17}$$

This result can be classified as the *ideal model* of systems' hazard rate since it is based on exact hypotheses. An ideal model is different from a *statistical model*: the first is developed through *deductive logic* and the second through *inductive logic*. The ideal model is true provided that the hypotheses are true, and this means that (17) is not a general rule but is true as long as the hypotheses are plausible. When the conditions for (13), (14), (15) and (16) do not occur in the real world, then the hazard rate function $\lambda(t)$ does not conform to (17) and in practice presents peaks, troughs and humps. The special features of the ideal model (17) conform with the wide usage of the bathtub curve in the literature and at the same time justify the exceptions that derive from the assumptions of (17) that do not occur in the physical reality.

III.3 Reparability function

When one maps the reliability and the recovery entropies, one obtains *the reparability function*

$$H_f = f\left(H_r\right) = \ln\left(1-\exp\left(H_r\right)\right) \tag{18}$$

An assortment of empirical criteria and heuristic methods regulate the repair and maintenance of systems, while some achievements even seem contradictory. The mathematical development of (18) qualifies the results that have until now been obtained by trial and error. For example, eqn. (18) confirms that generally the state of a system after repair will be *as-good-as-new* (AGAN) or *as-bad-as-old* (ABAO), while an intermediate quality outcome is not readily achievable. The reparability function explains also that the amelioration of a repaired system depends on the initial state of *S*. An identical intervention in two systems – e.g. the replacement of an old component with a new one – can bring either great benefit or trivial progress. Ultimately, function (18) proves how the working capacity of a reparable system follows a *saw-shaped curve*, which is the *ideal model for describing repair cycles.*

IV. Discussion

This paper reminds the readers that the concept of entropy is used in various fields and everywhere it supports unifying theories and comprehensive frames in the place of heterogeneous empirical statements. This paper makes a concise survey of two entropy functions introduced in the reliability domain: the well-known thermodynamic entropy and the new Boltzmann-like entropy. Both functions seek to explain self-generated phenomena typical of degraded systems in an exhaustive manner. This pair of theoretical frames derive all their results from rigorous assumptions, and they furnish intriguing answers in the areas of interest. The first applies thermodynamic entropy to degradation mechanisms and provides a model for various degradation processes that can even interfere one another. The second deduces two distinct ideal models for functioning systems and repaired systems.
The theoretical achievements supported by the entropy functions cover a broad range of situations and thus can help the reliability theory to evolve and reach the status of 'science' on a par with disciplines such as mechanics and chemistry.

The inquiries – briefly reviewed in this paper – are relatively recent and need experimental validation. The exhaustive intent of the authors implies that testing will not be trivial, and the corroboration phase will have the last word in the present argument.

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