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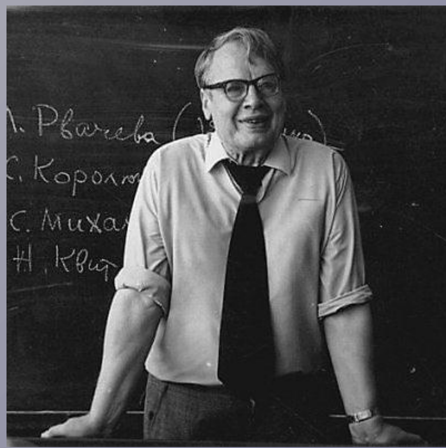

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THEORY&APPLICATIONS

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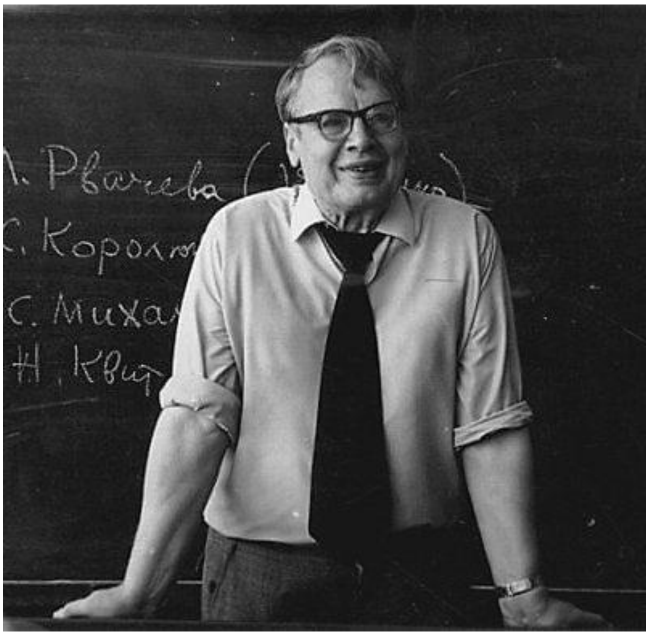

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THEORY & APPLICATIONS

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Table of Contents

The present paper represents a review of the Controllable Queueing Systems theory development from the very beginning up to nowadays. The main stages of this theory development are considered. Some new problems are mentioned. The review is devoted to those, who are interested in the creation and the development of the Controllable Queueing Systems theory from its generation up to nowadays, and who want to understand the tendency of its development and the new directions and problems of its study.

Keywords: Controllable queueing systems, Markov decision processes, Optimality principle, monotonicity of optimal policies

We propose a method for building confidence intervals for percentiles with application to quality control of the random properties of composite polymer materials strength. The basises (i.e. lower confidence limits for percentiles) are analyzed. The new developed method employs the statistical bootstrap modeling method. For the explanation of bootstrap procedure, the problem of building confidence interval for the mean value for the random values of composite material strength is presented. The result is compared with classical one. Due to bootstrap, it is possible to overcome some problems of classical statistical constraints. There is no need in postulating some type of distribution (normal or Weibull). Some real problems are presented with the comparison of the results by standard procedures and by the new method. It is shown that the agreement is satisfactory.

Keywords: percentile, bootstrap-modeling, composite material, strength

In this paper we present a queueing network model for computing average end-to-end delay, and maximum throughput that can be attained in random access multi-hop wireless ad hoc networks with stationary nodes under two popular contention resolution schemes namely, Binary Exponential Back-off (BEB) and Double Increment Double Decrement (DIDD) rules. This model takes into consider some realistic features of the system like (i) the generation of different classes of packets at nodes, and (ii) the dependence of the transmission time and transmission probability on the distance between the transmitter and receiver. Probability distributions and the associated measures of characteristics of the time spent by a packet at arbitrary node are analytically derived by using phase type random variate theory, which in turn are used for the computation of average end-to-end delay and maximum achievable throughput. Theoretical results are numerically illustrated.

Keywords: queue, network, phase type distribution, Markov chain

Gnedenko, E., Rice, D.

Nuclear energy safety is a critical issue as the world increasingly turns to nuclear energy to meet our growing needs for power and decreasing dependence on fossil fuels. Major nuclear disasters like Chernobyl (1986) and Fukushima Daiichi (2001) demonstrate the need for improved controls of these systems. The nuclear power supply chain is a critical component of the highly interconnected nuclear energy system. This paper introduces industry best practices and how these relate to the nuclear energy industry safety and reliability.

Keywords: Nuclear energy, safety, supply chain, reliability, operations costs

Gnedenko, D.

The bibliography, which we present below, includes everything that could be found from the written and published B.V. Gnedenko, starting with books and ending with newspaper articles. This includes also published interviews. The bibliography is broken down by years, and for each year everything that was published during this year, including reprints, is presented. The work for each year is arranged in one definite order: books, scientific articles, articles on various aspects of teaching, reviews, articles from general journals, newspaper publications, interviews. If the same article was published during the year in different places, then it is indicated under the same number, listing the output of all publications. Sometimes during the year there are articles with the same name, but different content. They are listed under different numbers, if possible, nearby.

Keywords: Gnedenko, bibliography, memory, history

Why We Need Probability Distributions With Periodic Failure Rates In Reliability And Risk

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Abstract

We discuss situations in real life where probability distributions with periodic failure rates should be considered. This discussion leads us to a new class, called Almost-Lack-of-Memory (ALM) probability distributions. We explain the structure of these distributions, and list some of their important properties. One of the main properties is its periodic failure rate. Throughout this article we notice some areas of possible applications of these distributions, and relate applications to the properties of these distributions. However, periodic variability also is observed. This may be another interesting continuation of this study.

Keywords: ALM class of probability distributions; Periodic failure rate functions; Non-homogeneous Poisson Process; periodic random environment

I. Introduction

A positive r.v. X (life time) is uniquely determined by its *failure rate function* (FRF) $\lambda(t)$. The function $\lambda(t)\Delta t$ is presenting the conditional probability that the object will fail within the nearest time interval $[t, t+\Delta t)$ given that it did not fail before t. For X continuous with c.d.f. $F(x)$ and p.d.f. $f(x)$, it holds

$$\lambda(t) = f(t)/\ 1 - F(t), \text{ for all } t \geq 0, \text{ where } 1 - F(t) \neq 0. \tag{1}$$

The function

$$\Lambda(t) = -\ln[1 - F(t)],\ t \geq 0 \tag{2}$$

is known as *hazard function* (HF) of the object.
The relationship

$$\Lambda(t) = \int_0^t \lambda(x)dx \tag{3}$$

allows to understand that either of the four functions $f(x)$, $F(x)$, $\Lambda(t)$, or $\lambda(t)$ uniquely determines the other three. In demography and survival analysis the FRF $\lambda(t)$ is known as *mortality rate*. In [7] for the needs of age comparison, we proposed to call $\lambda(t)$ risk function (risk to fail, risk to die, risk of something to happen at age t since the aging process, has started).

We proposed to call $\lambda(t)$ with numerous of appropriate names, e.g. *risk function* (risk to fail, risk to die, risk of something to happen at age t since the aging process has started). Another suitable terminology for $\lambda(t)$ is *stress function*. $\Lambda(x)$ is the *accumulated stress* (or *accumulated risk*) during the life up to age x. The FRF $\lambda(t)$ varies over the time. It reflects the impact of the environment and its interaction with the working object. In Risk analysis the FRF usually is related to weariness, fatigue, maintenance to stabilization and improvement, or other internal properties of the working objects. Traditionally, the FRF are considered increasing, decreasing, bathtub, or arc shaped. It is reasonable to consider also periodic FRFs in reliability and in other applications too!

Periodic intensity rate FRF $\lambda(t)$ with period c equal to the periodicity in the environmental changes. The function FRF $\lambda(t)$ should satisfy the equations $\lambda(t + nc)= \lambda(t)$ for any $t \geq 0$, and for any n = 1, 2, . . . An appropriate choice of the function $\lambda(t)$ on the interval [0, c) solves the problem of determination of the distributions from this class, and any related characteristics.

In this work we give several examples where periodic FRF takes place. These examples also pinpoint areas of application for our models. Then we discuss the class of ALM probability distributions (first Introduced in [11]) which best suits (by physical and analytical properties) to model the phenomena described. We also focus some attention on random processes related to the ALM probability distributions and on their unique relationships. Each discussion briefly notices areas of possible applications.

II. Examples where periodic failure rates appear naturally

Here we describe some examples of life time distributions where distributions with periodic FRF can be expected. By the way, periodicity is in front of our eyes: We have daily periodicity in our habits every 24 hours; we have the weekly periodicity in our weekly schedules; we have some kind of monthly periodicity, at least while we pay our bills; we have the quarterly periodicity in some payments or other dues; we have half a year periodicity by season changes, or vacation opportunity; we have yearly periodicity in many means (season changes, insurance or subscription renewals, etc.). Some of these are described in more detail in the following examples.

Example 1: **Periodic Reliability Maintenance with Replacements**. The system requires use of items with strictly limited workload. S.

- The airplane motor must work 300 hours, and then replaced by a new one;
- In military operations this is very strict that an ammunition can be replaced after reaching certain age;
- Medical prescription require medication intake after expiration of certain fixed time;
- In the food industry the expiration date is strictly enforced. Food supply works as a periodic inventory system.

Assume that a failed item before the expiration date is replaced instantaneously with a working item at the same age as the just failed.

At the times of compulsory maintenance, made periodically any c time units, the operating item is replaced by a completely new one and the process continues under the same rules. The life time of such system has a FRF of period c.

Analogous behavior can be noticed in the inventory systems with periodic refill of the storage rooms, as considered in [16] and [17].

Aerospace engineering uses satellites for various purposes for some fixed periods of time 3-8 years. This example may fit some of their reliability problems too.

In social life and politics the periodic elections form processes of constant periodic nature. Numerous interesting variables with periodic FRFs can be associated with such processes elsewhere.

Example 2: **Service on non-reliable server** (It is borrowed from [9], [11, [16], [18]).
Consider a job, processed on an unreliable server, according to "preemptive – repeat - different" service discipline.

- The server may fail during service of a job. After *each failure it gets an instantaneous repair, and becomes as good as new. The server' life times T_n* are i.i.d. r.v.'s.;
- The job service time has duration Y if not interrupted;
- The interrupted job is continued upon the server's recovery, as new with other independent realization of the r.v. Y;

- • The service will be complete when the new service time Y_n is less than the server's life time T_n;
- • *The r.v.'s* Y_n *and* T_n are mutually independent.

Let X be the total service time of the job, and

$$N = \inf \{n;\ Y_n < T_n,\ n \geq 1\} . \tag{4}$$

Then

$$X = \sum_{n=1}^{N-1} T_n + Y_N \tag{5}$$

is the structure of the total service time X. If assume that the server life times T_n are all equal to one and the same constant c, i.e. if $P(T_n = c)=1$ for $n=1,2, \ldots$, and the required service time Y is a continuous r.v., then X has periodic FRF of period c. The authors of [9] and [16] derived the mathematical expression for the distribution of the total service time.

Example 3: **The time to first event in a Non-Homogeneous Poisson Process (NHPP**) The NHPP $\{N_t : t > 0\}$ is a counting process the random number of events (arrivals) N_t in $[0, t)$, $t \geq 0$ (discussed in [2], [3]). The arrival rate $\lambda(t)$ is a non-negative function with finite integral on any finite interval of integration. The integrated rate function $\Lambda(t)$ of the NHPP, given by (3), represents the expected number of events $E(N_t)$, within $[0, t)$, $t \geq 0$. For any $t \geq 0$ the r.v. N_t has Poisson distribution of parameter $\Lambda(t)$ The arrivals on any non-overlapping intervals are independent r.v.'s. One can always associate a non-negative r.v. X with every NHPP. X is defined by the integrated rate function $\Lambda(t)$ declaring $F_X(x) = 0$ for $x < 0$, and $F_X(x) = 1 - e^{-\Lambda(t)}$, $x \geq 0$, according to relationships (1) - (3). This r.v. represents the waiting time up to the first event since the process starts: $F(x) = 1 - P\{X \geq x\} = 1 - P\{N[0,x) = 0\}$, $x \geq 0$. Reversely, any non-negative r.v. X generates a NHPP $\{N_t : t > 0\}$ whose intensity function coincides with the FRF of X, treated as life time. The times of the events in N_t can be interpreted as the flow of minimal repairs for an operating item whose life time is X (used in [3], [4] and [8]). If the intensity rate $\lambda(t)$ of the NHPP is periodic of period c, so is the FRF of the r.v. X .

Example 4: **Periodicity in Earth' life generates periodic NHPPs.** Periodicity in surrounding environmental conditions may have a considerable impact on the chances of random events to occur, on the random variables and the processes these events generate. As a natural descriptor of the effects of the periodic random environment on the rates of related random events, we suggest to use NHPP with periodic intensity rate $\lambda(t)$ with period c equal to the periodicity in the environmental changes. The function $\lambda(t)$ should satisfy the equations

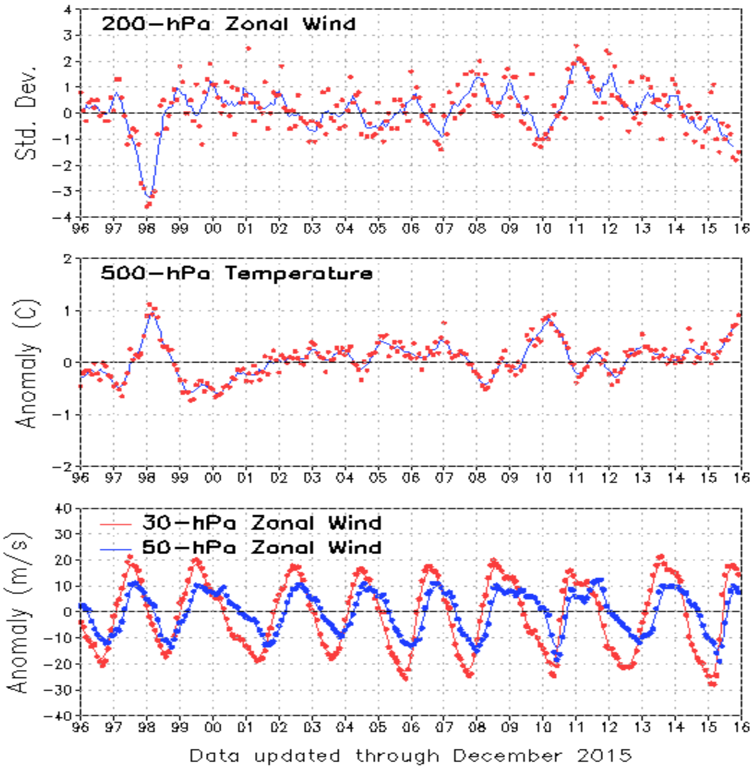
$$\lambda(t + nc) = \lambda(t) \text{ for any } t \geq 0, \text{ and for any } n = 1, 2, \ldots . \tag{6}$$

An appropriate choice of the function $\lambda(t)$ on the interval $[0, c)$ solves the problem of determination of the NHPP and any related characteristics.

Car accidents: The winter intensity rates due to wet, snowy or foggy weather are higher compare to what is in dry summer months; A bathtub-shape for the claims rate can be repeatedly expected every year in the northern countries. Natural periodicity of 1 year is reasonable, but periodic behavior of several years also seems admissible.

Hurricane activities: More than hundred years of records show that "the hurricane season starts by the end of May, has a season of high activity from second half of August to the first half of October and vanishes by the second half of December". The hurricane activity at coastal U.S. is modeled with a bell-shaped intensity function with a mode about the second half of the year. Natural periodicity seems to be 1 year; More realistic however, is the 6 year periodic alternation "El Ni˜no - La Ni˜na" seasons. In the South Hemisphere the shape of hurricane's intensity function should be bathtub (in a yearly interval starting in January) Sinusoidal up/down shifts with depression in the yearly behavior on a 3 year segment are reported and clearly visible on the site

http://www.cpc.ncep.noaa.gov/products/CDB/Tropics/figt3.gif
Go there, and you'll see the picture



We do believe that you clearly see periodicity and agree to our suggestions. But, *this variability is observed in the standard deviation too. This is a deep challenge to see what class of life time distributions it may call.*

Forest fires: Their intensity is low during wet seasons and high during dry seasons of the year. Usually, wet are Fall, Winter and Spring, and dry is the Summer. Respectively, a symmetric bell-shaped intensity function for the counts of the forest fires could be an appropriate model.

Flooding related events: Their intensity obviously goes high with snow melting when Winter ends and Spring comes. Somewhere the flooding may be related to hurricanes, or Summer thunderstorms. The respective behavior of $\lambda(t)$ may depend on the local specifics. Generally, $\lambda(t)$ is expected to have a bell shape with maximum shifted to the first third of the period. For some areas $\lambda(t)$ may be a multi-modal shaped flooding intensity.

House and car sales: Here everything can be expected. Most likely, there are two picks of the sale' intensity function, in the Spring and in the Fall. In such cases a mixture of two one pick functions, or a polynomial function of third degree may fit an appropriate model for the sale' intensity function $\lambda(t)$.

The examples may assume that the intensity at the yearend smoothly matches the value of this intensity at the beginning of the next year. There are possible situations with picks either at the beginning of the year (e.g. intensity of new loan contracts, purchase intensity of certain consumer's goods), or picks located near the end of the year (e.g. donations, spending funds related to taxes, and similar reasons in financial models). Respective $\lambda(t)$.can be decreasing in t, increasing, any, and no need to match the starting values with the values at the end of a cycle, and is a periodic function. The waiting time up to the occurrence of the first event in either of these processes is having periodic FRF.

Example 5: **Extended in time Bernoulli Trials (BTs) generate periodic NHPP** Numerous processes with established periodicity may be related to certain events (”successes”) in an artificially created sequence of extended in time BTs.

The traditional sequence of Bernoulli trials is based on the following conditions: – The consecutive trials are independent;

– Each trial has only two possible outcomes conditionally called “success” (S) and ”failure” (F)

– The probability $P(S)=1-\alpha$ does not depend on the current trial (so does the probability for failure $P(F) = \alpha$), and is the same for each trial.

Let us add three more components (requirements to the sequence of BTs as in [9]) due to the facts that: Each trial essentially takes some considerable time to be conducted. As soon as success is observed it can be immediately recognized and recorded.

Hence:

– There is a need of considerable time $c > 0$ to complete each one of these BT;

– If success occurs within a trial, the time from the start of this trial until the success occurs, is a r.v. Y. Its distribution has support on the interval $[0, c)$.

– The fact that a particular trial fails can be confirmed only when it is completed.

We list some of the potential contenders to the BTs constructions.

– Periodic alternations between cold and warm eras on Earth (of yet to be established periodicity);

– The 24 hours rotation of the Earth around its axis;

– The 11 years periodic cycles in solar activity;

– Other periodic configurations between planets or other spatial objects;

– The 24 hours and 50 minutes in length cycle in the tidal events (tides are cyclic rises and falls of seawater);

– The mentioned yearly changing climatic and meteorological conditions. Numerous pollution characteristics (chemical concentrations, emissions) can be related to this periodic process;

Equidistant check-points (measured in time, in miles, or in other workload units) in the maintenance procedures, associated with the normal work of a production system, a technical item, or a social system, are good sources for consideration of embedded extended ”in time” BTs;

Risk-associated events, such as house fires, diseases, fertility, mutation, bankruptcy, consumption rates, currency exchange rates, investment’ revenue, etc. may be involved in extended in time BTs of appropriate periodicity (time to perform a trial);

Human bio-rhythmic cycles like breathing cycles, heartbeats, eating cycles, use of prescription drugs, and others;

The waiting time until the occurrence of the first success in extended in time BTs has a periodic FRF (Khalil and Dimitrov [9]).

III. The role of origin in the time count

In the definition of periodic NHPP the origin is fixed as the beginning of a period, e.g. 1st of January each year. Imagine, the time origin t_0 is moving within the year, and can be located at any other day on the calendar. For instance

– April 15th as the last day to submit tax documents in the United States

. – July 1st as start of the new Academic year for most American

– A 12-month fiscal year, begins on October 1 and ends on September 30 next year,

In the sequel it will be important to have the same fixed start of any next period of time of duration c, as shown in [8] and [12]. The flows of events related to such determination of the time origin may change the analytical expression that fits best. Such variability creates some challenging statistical problems, namely to find the best position of the time origin that might produce the best goodness of fit to given observations.

The nature of the waiting time up to the occurrence of the first event of the process is not changed no matter what is the time origin. The properties of the r.v.'s related with periodic NHPPs, remain stable despite of some variability in the explicit mathematical forms in their description.

IV. Presentations of the distributions with periodic failure ratesons with periodic failure rates: equivalent analytical presentations

In this section we give a summary of results found by various authors during last 20 years on establishing properties and analytical forms of the distributions with periodic failure rates (see [6], [7], [8], [10], [15]).

Theorem 1: **(A)** The cdf of the waiting time X up to the first event in a periodic NHPP has the form

$$F_X(t) = 1 - \alpha^{\left\lfloor \frac{x}{c} \right\rfloor}\left(1-(1-\alpha)F_Y(t-\left[\frac{t}{c}\right]c)\right), \quad t \geq 0, \tag{7}$$

where $\alpha \in [0, 1]$, and $F_Y(y)$ is a cdf with support on the interval $[0, c)$. These are determined by the equations

– $\alpha = e^{-\int_0^c \lambda_X(u)du}$;

– The r.v. Y is defined either by its cdf

$$F_Y(y) = \frac{1}{1-e^{-\int_0^c \lambda_X(u)du}}\left[1-e^{-\int_0^y \lambda_X(u)du}\right], \text{ for } y \in [0, c].$$

Its pdf is

$$f_Y(y) = \frac{\lambda_X(y)}{1-e^{-\int_0^c \lambda_X(u)du}} \; e^{-\int_0^y \lambda_X(u)du}, \text{ with } y \in [0, c).$$

It is said that a r.v. X with a cdf of the form (7) belongs to the class of ALM(α, c, $F_Y(y)$) distributions. α, c , and $F_Y(y)$ are parameters of this class of probability distributions;

(B) In the continuous case the pdf $f_X(t)$ exists and has the form

$$f_X(t) = \alpha^{\left\lfloor \frac{x}{c} \right\rfloor}(1-\alpha)\, f_Y(t-\left[\frac{t}{c}\right]c), \qquad t \geq 0\,; \tag{8}$$

(C) If a r.v. X has the distribution given by (7), then it satisfies the conditional probability property

$$P\{X - nc \geq t \mid X \geq nc\} = P\{X \geq t\} \tag{9}$$

for all $t \geq 0$, and for arbitrary integer $n = 0, 1, 2, \ldots$.

Namely, this property is called "Almost-Lack-of-Memory (ALM) property", first in [13].

(D) Let a non-negative r.v. **X** have the lack of memory at a point c, $c > 0$, i.e. let

$$P\{X - c \geq t \mid X \geq c\} = P\{X \geq t\} \tag{10}$$

holds for all values of $t \geq 0$ and for the given value of the constant c. Then it holds:

(Di) X possesses the LM(nc) property with the value of c, for arbitrary integer $n = 1, 2, \ldots$;

(Dii) The cdf $F_X(t)$ is given by equation (7), where $F_Y(y)$ is a cdf with support on the interval $[0, c)$ if and only if (10) holds;

(E) A r.v. X has the LM property at some moment $c > 0$ if and only if X belongs to the class of ALM(α, c, $F_Y(y)$) distributions;

(F) A r.v. X belongs to the class of ALM(α, c, $F_Y(y)$) distrib*utions if and only if it can be represented as a sum* $X = Y + cZ$ *of two independent r.v.'s* Y *and* Z, where

– Y is located on the interval [0, c) with probability 1, and

– Z has the geometric distribution $P\{X = k\} = \alpha^k (1 - \alpha)$, $k = 0, 1, 2, \ldots$ with some $\alpha \in (0, 1)$;

(G) The Laplace-Stieltjes transform of the life time X is given by the formula

$$\varphi_X(s) = \varphi_Y(s) \frac{1-\alpha}{1-\alpha e^{-sc}} ;$$

(H) The Hazard Distribution Function (HDF), defined as a two argument function by the relation

$$\Lambda_X(x, t) = P\{X - x < t \mid X \geq x\} = \frac{F_X(x+t) - F_X(x)}{1 - F_X(x)} \tag{11}$$

is periodic with respect to first argument x, i.e. it satisfies

$$\Lambda_X(x + c, t) = \Lambda_X(x, t),\ x \geq 0, \text{ for any } t \geq 0 ;$$

(I) If a life time r.v. X has periodic HDF of period $c > 0$ then its cdf $F_X(t)$ has the form (7), where $\alpha \in [0, 1]$, and $F_Y(y)$ is a cdf with support on the interval [0, c). Moreover, it is fulfilled:

(Ia) The FRF of X has the form

$$\lambda_X(t) = \frac{(1-\alpha) f_Y(t - \left\lfloor \frac{t}{c} \right\rfloor c)}{1-(1-\alpha)F_Y(t - \left[\frac{t}{c} \right] c)}, \quad t \geq 0;$$

(Ib) The FRF $\lambda_X(t)$ is periodic function of period c;

(J) If a r.v. $X \sim ALM(\alpha, c, F_Y(y))$ distribution, then

$$P\{X - (nc + y) < t \mid X \geq nc + y\} = P\{X - y < t \mid X \geq y\} \tag{12}$$

holds for all $t \geq 0$, for arbitrary $y \in [0, c)$, and for arbitrary integer $n = 1, 2, \ldots$. This property explains that the ALM property is irrelevant in respect of the location of the origin within the interval [0, c);

(K) If $X \sim ALM(\alpha, c, F_Y(y))$ with $\alpha > 0$, and $c > 0$, then the r.v. X is unbounded, i.e. for arbitrary $M > 0$ it is fulfilled

$$P\{X \geq M\} > 0 .$$

Precise expressions for cases of discrete distributions and discrete times are also feasible. Some of these are shown in [11] and [12].

V. Distributions with periodic failure rates: equivalent analytical presentations

In this section we give a summary of results found by various authors during last 20 years on establishing properties and analytical forms of the distributions with periodic failure rates (see [6], [7], [8], [10], [15]).

Theorem 1: **(A)** The cdf of the waiting time X up to the first event in a periodic NHPP has the form

$$F_X(t) = 1 - \alpha^{\left\lfloor \frac{x}{c} \right\rfloor} \left(1-(1-\alpha)F_Y(t - \left[\frac{t}{c} \right] c) \right), \quad t \geq 0, \tag{7}$$

where $\alpha \in [0, 1]$, and $F_Y(y)$ is a cdf with support on the interval [0, c). These are determined by the equations

– $\alpha = e^{-\int_0^c \lambda_X(u)du}$;

– The r.v. Y is defined either by its cdf

$$F_Y(y) = \frac{1}{1-e^{-\int_0^c \lambda_X(u)du}}\left[1-e^{-\int_0^y \lambda_X(u)du}\right], \text{ for } y \in [0, c].$$

Its pdf is

$$f_Y(y) = \frac{\lambda_X(y)}{1-e^{-\int_0^c \lambda_X(u)du}} \; e^{-\int_0^y \lambda_X(u)du}, \text{ with } y \in [0, c).$$

It is said that a r.v. X with a cdf of the form (7) belongs to the class of ALM(α, c, $F_Y(y)$) distributions. α, c , and $F_Y(y)$ are parameters of this class of probability distributions;

(B) In the continuous case the pdf $f_X(t)$ exists and has the form

$$f_X(t) = \alpha^{\left\lfloor \frac{x}{c} \right\rfloor}(1-\alpha) f_Y(t-\left[\frac{t}{c}\right]c), \qquad t \geq 0 ; \tag{8}$$

(C) If a r.v. X has the distribution given by (7), then it satisfies the conditional probability property

$$P\{X - nc \geq t \mid X \geq nc\} = P\{X \geq t\} \tag{9}$$

for all $t \geq 0$, and for arbitrary integer $n = 0, 1, 2, ...$

Namely, this property is called "Almost-Lack-of-Memory (ALM) property", first in [13].

(D) Let a non-negative r.v. **X** have the lack of memory at a point c, $c > 0$, i.e. let

$$P\{X - c \geq t \mid X \geq c\} = P\{X \geq t\} \tag{10}$$

holds for all values of $t \geq 0$ and for the given value of the constant c. Then it holds:

(Di) X possesses the LM(nc) property with the value of c, for arbitrary integer $n = 1, 2, ...$;

(Dii) The cdf $F_X(t)$ is given by equation (7), where $F_Y(y)$ is a cdf with support on the interval $[0, c)$ if and only if (10) holds;

(E) A r.v. X has the LM property at some moment $c > 0$ if and only if X belongs to the class of ALM(α, c, $F_Y(y)$) distributions;

(F) A r.v. X belongs to the class of ALM(α, c, $F_Y(y)$) distrib*utions if and only if it can be represented as a sum* $X = Y + cZ$ *of two independent r.v.'s* Y *and* Z, where

– Y is located on the interval $[0, c)$ with probability 1, and

– Z has the geometric distribution $P\{X = k\} = \alpha^k(1-\alpha)$, $k = 0, 1, 2, ...$ with some $\alpha \in (0, 1)$;

(G) The Laplace-Stieltjes transform of the life time X is given by the formula

$$\varphi_X(s) = \varphi_Y(s)\, \frac{1-\alpha}{1-\alpha e^{-sc}} \;;$$

(H) The Hazard Distribution Function (HDF), defined as a two argument function by the relation

$$\Lambda_X(x, t) = P\{X - x < t \mid X \geq x\} = \frac{F_X(x+t) - F_X(x)}{1 - F_X(x)} \tag{11}$$

is periodic with respect to first argument x, i.e. it satisfies

$$\Lambda_X(x + c, t) = \Lambda_X(x, t),\; x \geq 0, \text{ for any } t \geq 0 ;$$

(II) If a life time r.v. X has periodic HDF of period $c > 0$ then its cdf $F_X(t)$ has the form (7), where $\alpha \in [0, 1]$, and $F_Y(y)$ is a cdf with support on the interval [0, c). Moreover, it is fulfilled:

(Ia) The FRF of X has the form

$$\lambda_X(t) = \frac{(1-\alpha) f_Y(t-\left\lfloor \frac{t}{c} \right\rfloor c)}{1-(1-\alpha)F_Y(t-\left[\frac{t}{c}\right]c)}, \quad t \geq 0;$$

(Ib) The FRF $\lambda_X(t)$ is periodic function of period c;

(J) If a r.v. $X \sim$ ALM(α, c, $F_Y(y)$) distribution, then

$$P\{X - (nc + y) < t \mid X \geq nc + y\} = P\{X - y < t \mid X \geq y\} \quad (12)$$

holds for all $t \geq 0$, for arbitrary $y \in [0, c)$, and for arbitrary integer $n = 1, 2, \ldots$. This property explains that the ALM property is irrelevant in respect of the location of the origin within the interval $[0, c)$;

(K) If $X \sim ALM(\alpha, c, F_Y(y))$ with $\alpha > 0$, and $c > 0$, then the r.v. X is unbounded, i.e. for arbitrary $M > 0$ it is fulfilled

$$P\{X \geq M\} > 0 .$$

Precise expressions for cases of discrete distributions and discrete times are also feasible. Some of these are shown in [11] and [12].

VI. Properties of periodic NHPP

The only process which can be uniquely associated to the r.v.' s with periodic failure rates are the NHPPs with periodic intensity functions and with same interval of periodicity [2], [3], [5], [8]. They possess some interesting and important properties which can be used in applications. These can also be treated as an indivisible part of the properties of the life times. It holds

Theorem 2: An integer valued random process $\{N_t\}$ with periodic intensity rate $\lambda(t)$ of period $c > 0$ is a NHPP if and only if the following two properties hold:

(A) **(i)** For some constant c and for arbitrary $t \geq 0$ it is true that

$$P\{N[c,c+t) = m\} = P\{N[0,t) = m\} \text{ for any integer } m = 0, 1, 2, \ldots;$$

(ii) The r.v.'s $N[c,c+t)$ and $N[0,c)$ are independent for any $t \geq 0$;

(B) A counting process $\{N_t,\ t \geq 0\}$ is generated by random environment with periodic behavior of period $c > 0$

$$N_t =^d M_1 + M_2 + \ldots + M_{[t/c]} + N_{t - [t/c]c}\,, \quad (13)$$

where $\{M_n\}_{n=0}^{\infty}$ are i.i.d. Poisson r.v.'s of parameter $\Lambda(c) = \int_0^c \lambda(x)dx$, independent of the component $N_{t - [t/c]c}$, and $\lambda(x) \geq 0$ is some periodic function of period $c > 0$, presenting the intensity of occurrence of events.

The notation "$=^d$" means equality in distribution;

(C) A NHPP $\{N_t,\ t \geq 0\}$ of intensity rate $\lambda(t)$ is periodic of period $c > 0$ if and only if the associated random variable X belongs to the class of $ALM(\alpha, c, F_Y(y))$ distributions with parameters defined as in Theorem 1 (A);

(D) If each of the K mutually independent point processes $N_t^{(1)}, \ldots, N_t^{(K)}$ is a periodic NHPP with integrated intensity functions $\Lambda_1(t), \ldots, \Lambda_K(t)$, and all of the K processes have a common period c^*, then their superposition is a periodic NHPP with integrated intensity function $\Lambda(t) = \Lambda_1(t) + \ldots + \Lambda_K(t)$, and its period is c^*.

The random sum

$$Z_t = \sum_{n=0}^{N_t} \xi_n \text{ , with } \xi_0 = 0$$

is called Process of accumulation;

(E) If the sequence of sum components $\{\xi_n,\ n\geq 1\}$ consists of i.i.d. random variables, the aggregate claim process $\{Z_t\,;\ t \geq 0\}$ driven by a periodic NHPP can be decomposed into the form

$$Z_t =^d Z^{(1)} + Z^{(2)} + \ldots + Z^{([t/c])} + Z_{t-[t/c]c}\,. \quad (14)$$

The variables $\{Z^{(n)},\ n\geq 0\}$ are i.i.d. r.v.'s distributed as the compound Poisson sum

$$Z^{(1)} = \sum_{n=0}^{M_1} \xi_n \,,$$

and M_1 is a Poisson r.v. as defined in part (A). The last term $Z_{t-[t/c]c}$ in (14) is also a compound Poisson term, independent of the other components $Z^{(n)}$, and with parameter $\Lambda(t)$, for $t \leq c$. Some applications of these results in modeling of processes in reliability and environmental studies can be seen in [2],[3] and [14].

VII. Conclusions

It is natural to consider life times with periodic FRF in reliability, risk, insurance, environmental modeling, financial mathematics, political theory, climate changes and various other applications. The reason is that the periodicity in surrounding environmental conditions has a considerable impact on generated random events, variables and processes.

An easy to understand description of these effects is suggested in the present article, and simple analytical relationships between its mathematical models are presented.

NHPP's with periodic failure rates provide good models for counting processes in any periodic random environment.

The periodic nature of the intensity $\lambda(t)$ in modeling risk processes could become a powerful tool in the study of cost models and financial problems related to various maintenance policies. Statistical challenges for best estimation of this function fir varying origin and fixed length of periodicity are raised.

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Reliability of the Estimate of Interrelation of Parameters Efficienty Their Work of Objects of Ees

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Abstract

Given is computational method of statistical distribution function of possible realizations of coefficients of linear correlation of Pearson and grade correlation of Spirmen for dependent random quantities. Dependence provided with ranging of random quantities of selections. Random quantities are modeled and have uniform distribution in the range of [0,1]. These distributions allow to evaluate the error of II type and to compare it with the error of I type. The error of II type established on statistical distribution function of coefficient correlation of independent selections. These distributions are in a basis of criteria for estimate of the importance of coefficients of correlation of technical and economic parameters of objects of electrical power systems.

Keywords: Correlation coefficient, selection, ranging, distribution, technical and economic indicator

I. Introduction

The indispensable condition of objectivity of ranging of objects of electric energy systems (EES) by comparison of integral parameters (IP) of efficiency (reliability, profitability and non-failure operation) their works is reliability of IP. In turn, reliability of the estimate of IP depends on the level of statistical communications of the consisting IP of technical and economic indicators (TEI). Reliability of IP is possible at independence TEI the objects of the same name. If the number of dependent TEI more, then the reliability of IP will be is lower and the risk of the wrong decision will is higher. Need of ranging of objects arises systematically at the organization of their maintenance and repair, load distribution, carrying out tests. And, first of all, - for objects, perioditon recovery of wear is not reglamented and is defined by their technical condition. Main characteristic of technical condition of objects of EES serves the set of formalized TEI [1].

At classification of statistical data passport data and data on service conditions are not less important. A set of indicators characterizing of objects selected expressly consciously, since the personnel of the power supply system should solve operational problems, being guided by technical condition of specific objects. If besides to add that many of these indicators differ with units, the scale and the scale of measurement, then difficulties of ensuring reliability of ranging of objects with calculation methods become obvious. For this reason in practice of the decision, following from ranging of objects, are accepted intuitively, in many respects heuristically. The risk of the wrong decision at decrease in qualification of personnel increases. The multidimensionality noted above and natural difficulty of accounting of all information is not less important reason of risk of the wrong decision.

In these conditions the possibility of the automated methodical and information support of personnel is very tempting. Thus:

- application of modern methods of the analysis and synthesis of statistical data, bulkiness of calculations, knowledge volumity, cause the high probability of "mechanical" mistakes by production of calculations of IP manually;
- methodical support of personnel consists in the recommendations based on objective comparison and ranging of objects;
- information support consists in justification of some decisions in the form of specialized calculation tables [2].

It should be noted that it is inexpedient and to absolutize the importance of methodical support of personnel as IP not always consider all factors on which objectivity of the decision depends.

To such factors:

- availability of the defective nodes of object demanding replacement;
- materials, relevant repair crews, possibility of shutdown of object, etc.

Estimate of the importance of statistical communication TEI in practice it is carried out by comparison of design values of the correlation factors (CF), with the CF critical value for the set error of I Type α. The CF critical values are tabulated for the number of values α and the sample size of n_v. They can be also determined by formulas of calculation of boundary values of the reliability interval provided that distribution function of CF corresponds to the normal law. Among the set of possible CF we will use the most often used by coefficient of linear correlation of Pearson (γ) and the index of rank of Spirmen (r). Let's remind that Pearson's CF (γ) characterizes only statistical interrelation of selections which implementations are set in the quantitative scale, and Spearmen (ρ) CF characterizes statistical interrelation of selections of ranks (ordinal values) of implementations. Critical values γ and ρ are considered equal, i.e. $\gamma_\alpha=\rho_\alpha$ [3].

Calculation formulas γ and ρ are received for the assumption of compliance of implementations of selections to the normal distribution law. The premises above noted when calculating γ_α and ρ_α for TEI objects of EES, as a rule, are not carried out. And if to be more exact, distribution of implementations TEI it is unknown, changes and depends on the large number of factors. To such, volumes of selection are small.

The preference of strategy of minimization of the error of I Type of α, when effects of these errors are accepted much more dangerously than effects of the error of II Type β is not justified. Explicit preference of one of two assumptions by comparison TEI is absent. However computational methods of risk of the wrong decision for the assumption "statistical communication between TEI it is significant" and the known CF are not developed.

The present article is devoted to development of the computational method and accounting of the error of II Type.

II. Results of the analysis of fiducially distributions of correlation factors of independent random quantities

Fiducial distribution is understood as distribution of possible implementations of complex indicators. As complex it is considered to be indicators which implementations can be received as a result of calculations. Complex indicators the arithmetic average (harmonious, geometrical), dispersion indicators (dispersion, the average quadratic deviation, scope), maintenance coefficient, the availability quotient are among random quantities. CF also belongs to their number. Method and algorithm of modeling of possible implementations of CF, creation of statistical function of fiducial distributions (s.f.f.d.) and the assessment of the CF critical values for the set α are brought in [4].

The carried-out calculations allowed establishing:

2.1. The requirement of compliance of selections of random quantities ξ to the normal law (for the reliable assessment of coefficient of linear correlation of Pearson (γ) at small selections ($n_v \leq 15$) and uniform distribution of random quantities ξ in the range of [0,1] was not confirmed. At number modeled γ_M equal N=10000 discrepancy of critical values $\gamma_{M,\alpha}$, an and tabular values given in reference books γ_α for α equal 0,1; 0,005; 0,02; 0,01; 0,005; and 0,001 did not exceed 1,5%. In conditions when for practical calculations value α is accepted, as a rule, equal 0,05 and occasionally 0,01, such discrepancy of the quantile of distribution can be considered admissible.

2.2. Irrespective of n_v and α equality of critical values γ_α and ρ_α is confirmed. At the same time, for the same modeled (м) couples of selections $\{\xi_1\}_{nv}$ and $\{\xi_2\}_{nv}$ CF $\gamma_м$ and $\rho_м$ can significantly differ. We met this phenomenon also by comparison of criteria of check of the hypothesis of nature of discrepancy of statistical distribution functions (s.f.d.) random quantities [5]. Such discrepancy explained by distinction of estimates of statistical parameters of selections there. By comparison of the CF experimental values γ_e and ρ_e for the same selections, by analogy, influences observed discrepancy distinction of physical essence γ and ρ. But, and this is important, the recommendation [5] remains invariable: the importance of statistical connection has to be established on the basis *of the principle of the addition* considering independence of properties γ and ρ [6]. In other words, the decision is made by results of comparison of experimental and critical value for both CF. Statistical communication is accepted significant if at least one of CF is significant;

2.3. If implementations TEI change in the quantitative scale, then it is recommended to calculate CF γ_{e1}, γ_{e2} both ρ_{e2} on one and oh to the formula for Pearson's CF:

- CF γ_{e1} – on selections $\{\tau_1\}_{nv}$ and $\{\tau_2\}_{nv}$;
- CF γ_{e2} – on selections of points $\{b_1\}_{nv}$ and $\{b_2\}_{nv}$;
- CF ρ_{e2} - on selections of ranks (ordinal values) $\{r_1\}_{nv}$ and $\{r_2\}_{nv}$

For γ_2 and ρ_2 as mean value is accepted the median. If implementations TEI are measured in the serial scale, then it is recommended to calculate CF γ_{e2} and ρ_{e2}

2.4. S.f.f.d. F*(ρ) have discrete character. Maximum step of discretization at n_v=5. In the absence of identical implementations of selections the step is equal 0,1 and not linearly decreases with growth of n_v. Discrete character of $F^*(\rho)$ excludes possibilities of the reliable assessment of critical ρ_α values. Use of the tabular ρ_α values recommended in reference books is connected with essential risk of the wrong decision which increases at reduction of n_v. The new criterion of identification of statistical communications offered. The difference with the existing criterion is that not compared ρ_e and ρ_α, but are compared $R^*(\rho_e)=1-F^*(\rho_e)$ and α.

2.5. The new way of decrease in influence of discrete nature of distribution $F^*(\rho_1)$ is offered.
This way is based on transformation of continuous random quantities to discrete (points) and taking note of identical implementations of points on their ranks. And the invariance of critical $\rho_\alpha=\gamma_\alpha$ values was the most surprising at these calculations.

2.6. Statistical analysis of CF $\gamma_м$ testifies to full symmetry of distribution of positive and negative values. This feature allows passing to s.f.f.d. absolute values of CF.

III. Method and algorithm of calculation of fiducial distributions of correlation factors of dependent random quantities

Transition from boundary values of the confidential interval to boundary values of the fiducial interval completely changes physical essence of the KK critical values, makes understanding of intervals more available and physically clear. The matter is that all implementations of the variational series of the range $\gamma_м$ and $\rho_м$ are real, and, in fact, as critical value we have to choose the greatest importance of implementations. Each value of this row both when modeling and according to operation perhaps, but differs in probability of emergence on the set of implementations. And only comparison of these probabilities of risk of the wrong decision allows preferring one or the other assumptions (H). Namely: compared TEI are independent (H_1) or statistical communication between TEI it is significant (H_2). And before passing to such type of criterion it is necessary to be able to build fiducial distributions of CF of statistically dependent selections of random quantities. Let's notice that the set of possible fiducial communications is infinite and significant statistical communication - only one. It or is (for example, $\gamma_e>\gamma_\alpha$), or it is absent ($\gamma_e<\gamma_\alpha$).

Modeling method essence s.f.f.d. CF of statistically dependent selections of random quantities is reduced to the way of forming of statistically dependent selections. All other calculations are similar to algorithm of modeling s.f.f.d. CF of independent random quantities [5].

To receive two statistically dependent selections of statistically independent selections of random quantities with positive CF it is offered to range these independent selections in ascending order of random quantities. At this CF will change in the range of [0,1]. We will receive two dependent selections with negative CF if we execute ranking of the first selection in ascending order, and the second - in decreasing order. Or on the contrary. The enlarged algorithm of calculation s.f.f.d. CF of dependent selections is given in fig. 1.

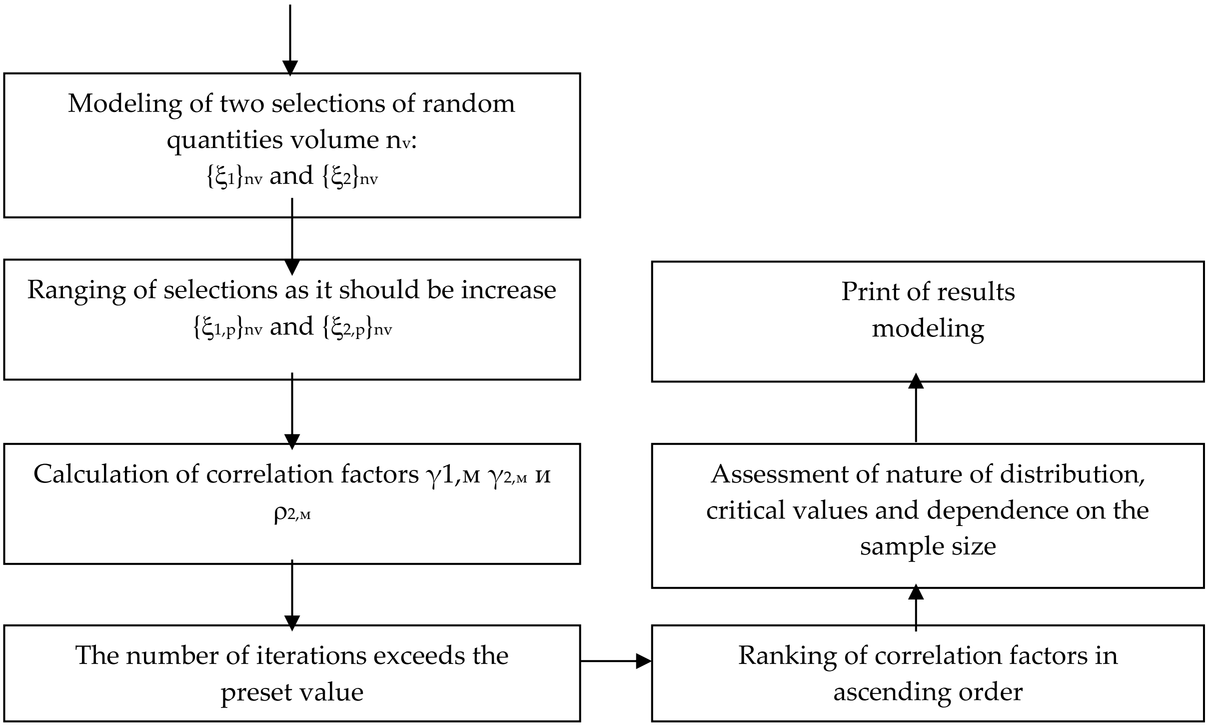


Fig.1. Enlarged flowchart of algorithm of modeling s.f.f.d. CF

Let's notice that CF $\rho_{1,м}$, to m for the ranged implementations of selections $\{\xi_1\}_{nv}$ and $\{\xi_2\}_{nv}$ will be equal to unit and therefore it is excluded from consideration. Results of modeling $\gamma_{1,м}$ $\gamma_{2,м}$ and $\rho_{2,м}$, for dependent selections $\{\xi_1\}_{nv}$ and $\{\xi_2\}_{nv}$ allowed to establish:

3.1. By analogy with independent selections for any couple of dependent selections of implementation $\gamma_{1,м}$ $\gamma_{2,м}$ and $\rho_{2,м}$ are also different. Some implementations of these CF for n_v=10 are given in the illustrative purposes in table 1.

3.2. Unlike the CF critical values of independent selections the CF critical values for dependent selections and small n_v significantly differ. With increase in n_v it is a divergence decreases. For confirmation the CF critical values, respectively, $\gamma_{1,м,\beta}$, $\gamma_{2,м,\beta}$ and $\rho_{2,м,\beta}$ are given in tables 2, 3 and 4

3.3. Comparison of critical values of coefficients of linear correlation of Pearson $\gamma_{1,м}$ for errors of I Type and II Type is given in table 5. This table confirms a possibility of consent with both assumptions (H_1 and H_2). In other words a number of realization $\gamma_{1,м}$, belongs to set of values $\{\gamma_{1,n,\alpha}\}_N$ and to a set of values $\{\gamma_{1,n,\beta}\}_N$

Table 1. Realization of coefficients of correlation of dependent selections

N	coefficients of correlation			Note
	$\gamma_{1,м}$	$\gamma_{2,м}$	$\rho_{2,м}$	
1	0,988	0,946	0,939	$n_v=10$
2	0,756	0,747	0,890	$\gamma_{1,м}=\frac{\sum_{j=1}^{n_v}[\Delta\xi_{1,j}\cdot\Delta\xi_{2,j}]}{\sqrt{\sum_{j=1}^{n_v}\Delta\xi_{1,j}^2}\cdot\sqrt{\sum_{j=1}^{n_v}\Delta\xi_{2,j}^2}}$
3	0,923	0,879	0,928	$\gamma_{2,м}=\frac{\sum_{j=1}^{n_v}[\Delta b_{1,j}\cdot\Delta b_{2,j}]}{\sqrt{\sum_{j=1}^{n_v}\Delta b_{1,j}^2}\cdot\sqrt{\sum_{j=1}^{n_v}\Delta b_{2,j}^2}}$
4	0,956	0,970	0,990	$\rho_{2,м}=\frac{\sum_{j=1}^{n_v}[\Delta r_{1,j}\cdot\Delta r_{2,j}]}{\sqrt{\sum_{j=1}^{n_v}\Delta r_{1,j}^2}\cdot\sqrt{\sum_{j=1}^{n_v}\Delta r_{2,j}^2}}$
5	0,931	0,937	0,980	$\Delta\xi_j=\xi_j-\bar{\xi}$; $\Delta b_j=b_j-\bar{b}$;
6	0,879	0,830	0,895	$\Delta r_j=r_j-\bar{r}$ $\bar{r}$ - median
7	0,354	0,894	0,916	

Table 2. Critical values of coefficients of linear correlation of Pearson $\gamma_{1,м,\beta}$ for selections of dependent random variables.

Selection volume	Error II Type β					
	0,5	0,1	0,05	0,01	0,005	0,001
5	0,919	0,787	0,743	0,646	0,602	0,538
8	0,935	0,848	0,815	0,738	0,694	0,625
10	0,945	0,874	0,845	0,776	0,746	0,679
15	0,967	0,913	0,892	0,851	0,832	0,787
30	0,979	0,954	0,943	0,919	0,908	0,883

Table 3. Critical values of coefficients of linear correlation of Pearson $\gamma_{2,м,\beta}$ for discrete selections of random variables.

Selection volume	Error II Type β					
	0,5	0,1	0,05	0,01	0,005	0,001
5	0,877	0,690	0,612	0,408	0,375	0
8	0,897	0,778	0,733	0,643	0,612	0,520
10	0,906	0,809	0,768	0,673	0,638	0,575
15	0,923	0,858	0,833	0,777	0,750	0,663
30	0,941	0,902	0,890	0,856	0,823	0,783

Table 4. Critical values of coefficients of rank correlation of Spirmen $\rho_{2,м,\beta}$ for selections of dependent random variables.

Selection volume	Error II Type β					
	0,5	0,1	0,05	0,01	0,005	0,001
5	0,894	0,740	0,646	0,408	0,335	0
8	0,918	0,833	0,803	0,713	0,679	0,589
10	0,926	0,864	0,835	0,759	0,729	0,648
15	0,938	0,899	0,883	0,849	0,817	0,764
30	0,950	0,929	0,921	0,898	0,895	0,868

Table 5. Comparison $\gamma_{1,м,\alpha}$ and $\gamma_{1,м,\beta}$ at $\alpha=\beta$ and $n_v=5$

Errors Types	Wrong decision risk					
	0,5	0,1	0,05	0,01	0,005	0,001
α	0,403	0,812	0,885	0,962	0,976	0,989
β	0,913	0,787	0,749	0,646	0,602	0,538

3.4. Some characteristic combinations of realization $\gamma_{1,м}$ $\gamma_{2,м}$ и $\rho_{2,м}$ and confirmation of statistical communication of selections for β=0,05 are given in table 6. As appears from these data:

- selections $\{\xi_1\}$ and $\{\xi_2\}$for which statistical communication is confirmed all three CF, two of three CF ($\gamma_{2,м}$ and $\rho_{2,м}$) and only one CF are observed ($\gamma_{1,м}$).
- *CF $\gamma_{1,м}$ $\gamma_{2,м}$ и $\rho_{2,м}$, supplement each other*

Table 6. Illustration of results of verification of the assumption of statistical communication of dependent selections

i	$\gamma_{1,м,i}$	H_2	$\gamma_{2,м,i}$	H_2	$\rho_{2,м,i}$	H_2	Note
1	0.7790	+	0.4082	-	0.4082	-	$\gamma_{1,м,0,05}$=0,6456
2	0.8068	+	0.5929	+	0.6250	+	$\gamma_{2,м,0,05}$0,4082
3	0.5799	-	0.4841	+	0.6250	+	$\rho_{2,м,0,05}$=0,4082
4	0.7676	+	0.4082	-	0.4082	-	
5	0.6065	-	0.3750	-	0.3953	-	
6	0.9618	+	0.5625	+	0.5441	+	
7	0.7427	+	0.5601	+	0.6455	+	
8	0.7439	+	0.6022	+	0.6250	+	
9	0.9437	+	0.6667	+	0.6455	+	

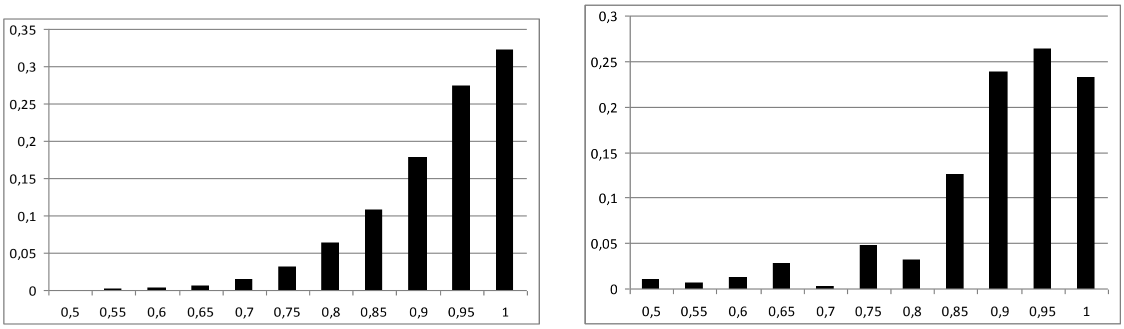
3.5 Criteria of decision-making for selections with a quantitative scale of measurement have an appearance:

$$\left.\begin{array}{l} \text{if } \gamma_{1,м} \le \gamma_{1,м,\beta}, \text{ then } H \Rightarrow H_1 \to \text{exit} \\ \text{if } \gamma_{2,м} \le \gamma_{2,м,\beta}, \text{ then } H \Rightarrow H_1 \to \text{exit} \\ \text{if } \rho_{2,м} \le \rho_{2,м,\beta}, \text{ then } H \Rightarrow H_1 \to \text{exit} \\ \text{otherwise} \quad H \Rightarrow H_2 \end{array}\right\} \quad (1)$$

At a serial scale of measurement the criterion of decision-making (H) has an appearance:

$$\left.\begin{array}{l} \text{if } \gamma_{2,м} \le \gamma_{2,м,\beta}, \text{ then } H \Rightarrow H_1 \to \text{exit} \\ \text{if } \rho_{2,м} \le \rho_{2,м,\beta}, \text{ then } H \Rightarrow H_1 \to \text{exit} \\ \text{otherwise} \quad H \Rightarrow H_2 \end{array}\right\} \quad (2)$$

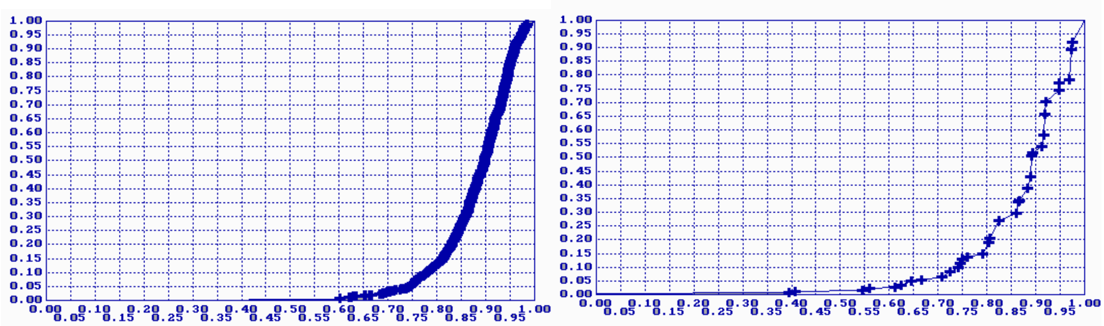
3.6. In fig. 2. Histograms of fiducial distribution of CF $\gamma_{1,м}$ and $\rho_{2,м}$ of dependent selections $\{\xi_1\}_{nv}$ and $\{\xi_2\}_{nv}$ with n_v=5 are provided



a) b)

Fig. 2. The histogram of distribution of possible realization of coefficients of correlation of Pearson $\gamma_{1,м}$ (a) and Spearmen $\rho_{2,м}$, (b) of dependent selections with n_v=5.

And in fig. 3. Statistical functions of fiducial distribution of CF $\gamma_{1,м}$ и $\rho_{2,м}$ for selections of random variables ξ with n_v=5 are given. These drawings demonstrate difference of distribution from normal, a real possibility of an operational assessment of critical values and distinction of distributions of these CF. The dispersion of realization of $\rho_{2,м}$ of fiducial distribution of $F^*(\rho_{2,м}/H_2)$ is caused by discrete nature of change of $\rho_{2,м}$ at small and the discretization step changing on value.



a) b)

Fig. 3. Statistical functions of fiducial distribution $\gamma_{1,м}$ (a) and $\rho_{2,м}$ (b) of dependent selections with n_v=5

Conclusion

1. The method of an assessment of critical values of coefficients of correlation is developed provided that selections are statistically connected;
2. The method based on fiducial approach. The assessment of statistical function of distribution is carried out on possible realization of coefficients of correlation;
3. Realization of coefficients of correlation is calculated on the modeled selections of random variables;
4. The interrelation of these selections is reached by ranging of random variables as their increase with uniform distribution in the range of [0,1];
5. Unlike coefficients of correlation of the independent selections changing within [-1,1] coefficients of correlation of dependent selections change in the range of [0,1] at the ranging type noted above. If ranging of realization of selections is opposite, then coefficients of correlation change in the range of [-1,0]

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Modeling of Fatigue Life of Unidirectional Fibrous Composite by Daniels' Epsilon-Sequence Under Random Loading

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Abstract

The modified Daniels' epsilon sequence (DeS) is used for an analysis of a residual static strength, a fatigue life, a fatigue strength of an unidirectional fiber composite (UFC) considered as some parallel system subjected to cycling loading with a random value of a cycle parameter. Some example of the processing of data set of a carbon-fiber reinforced composite is given.

Keywords: Daniels' sequence, strength, fatigue life, composite

1. Introduction

Here we give a very short review and some corrections of the basic ideas of the application of the definition of the Daniels' sequence (DS) and its modification to study a fatigue life of an unidirectional fibrous composite (UFC) considered as some parallel system subjected to cycling loading. Lately, the composite is widely used, in particular, in aviation. Therefore, a study of a strength and a fatigue life of these materials is very urgent. The first scientific publication appears to be the Peirce's work [1]. Peirce gives an approximate formula for the average strength of a bundle of longitudional items (LIs) (fibers or bundles) forming a foundation of the unidirectional fibrous composite (UFC). A correctness of the normal approximation of the strength distribution law of a LI parallel system was proved by Daniels [2,3]. His result was refined by Smith [4] already with a reference to the series-parallel system, which was earlier proposed in [5]. A lot of the papers are devoted to a reliability of the composite. A detail review, for example, is given in [6]. But here we refer only to the papers connected with the DS and its modification.

The concept of the basic DS with an application to a description of the process of a fatigue failure was first introduced in [7]. The UFC is a composition of the LIs immersed into a composite matrix (CM). Here we suppose that only LIs carry the longitudinal load. In the DS approach a composite specimen for the static strength or the fatigue life tests is considered as a series-parallel system: series system of n_L "links" of the same length but the every link is a parallel system, or more specifically, a bundle of n_C LIs. There is some probability that inside of some link there is some weak micro volume (WMV) the failure of which is a failure of a link and the failure of the tested specimen also. In fact we consider a composite as a random series of the WMVs. This paper is devoted to the fatigue life and tensile strength of one WMV. The relation of the cumulative distribution function (cdf) of the strength of WMV and of the specimen on the whole, the case when the failure of the matrix is considered as failure of the specimen also are studied in [8].

A successful application of the DS approach to the explanation of the relation between the static strength of LI and the fatigue life of UFC and the desciption of mean fatigue curve, SN (stress-number of cycles), is discussed in [8,9]. A more general definition of the DS is considered in [10]. It can be used to study the UFC falure process under any type of the loading process which can be described by a sequence of the load both in the fatigue and the static strength tests. In the paper [11] the definition of *the Daniels' residual strength* was introduced. Generally, the residual strength is described **after the definition of the SN curve**. But here the SN curve is defined **after the calculation of the residual strength**. Finally, in [11, 12] the definitions of the Daniels's epsilon sequence (DeS) was given (it is necessary to note that as consequence of some mistake the definitions of the DeS in [11] is given only in russian version of this paper).

The DS approach describes the structure of the damage accumulation process, gives the explanation of a *fatigue strength* existance (maximum value of a fatigue cycle parameter (cycle maximum, for example) with the infinite fatigue life; it is called also a *fatigue limit*). But the numerical results are very poor. Calculated the DS step number up to failure is very small or just tends to the infinity. More appropriate numerical results can be get using additionaly the theory of semi-Markov process [11, 12]. But the use of this theory requires some additional assumptions. This paper is devoted to some investigation of the results which can be get without the use of this theory. The use of the DeS approuch gives more easier solution.

Usually, only the mean value of fatigue life and the mean SN curve are taken into account. It is supposed also that during the fatigue test the parameter of the cycle load is some constant. But it appears that a great scatter of the fatigue life (particularly, at high level of the fatigue load) can be explaned only by the assumption the randomness of this parameter. This is the reason to develop a specific modification of the DeS when the sequence of loads is a sequence of random variables. This modification is the object of the investigation in this paper. A numerical example is given.

2. The modified Daniel's epsilon sequence

Here we consider a modified definition of DeS. It can be used in section 3 for processing the result of the fatigue test under cycling loading with the random parameter of every cycle.

As it was mentioned already the connection of the static strength of a LI and a strength of a bundel of them was studied by Daniels [2, 3]. Let the strengths of n_C LIs of some WMV, $X_1, X_2, ..., X_{n_C}$, are the independet random variables with the same cumulative distribution function (cdf), $F_X(x)$, and $X_{(1)}, X_{(2)}, ..., X_{(n_C)}$ are the corresponding ordered statstics. Daniels showed that the random variable

$$R_D = \max(X_{(k)}(n_C - (k-1))/n_C:\ 1 \le k \le n_C) \tag{1}$$

has an asymptotically normal distribution with the average and the standard deviation

$$\mu_D = \max x(1 - F_X(x)) = x^*(1 - F_X(x^*)),\quad \sigma_D = (\mu_D\, x^* F_X(x^*)/n_C)^{1/2}. \tag{2}$$

By "unwrapping" this model in time for specific sample $x_{1:n_C} = (x_1, ..., x_{n_C})$ which is a realization of the random vector $X_{1:n_C} = (X_1, ... X_{n_C})$ and assuming that the process of loading is described by sequence $s^+_{0:\infty} = \{s^+_0, s^+_1, s^+_2, ...\}$, where s^+_i is a realization of some random variable, we obtain a sequence of the local stresses $s_{0:\infty} = (s_0, s_1, s_2, ...)$ (in the WMV where the damage develops) described by the equation

$$s_0 = s^+_0,\quad s_{i+1} = s_i + \varepsilon(s^0_{i+1} - s_i) = (1-\varepsilon)s_i + \varepsilon s^0_{i+1},\quad i = 0, 1, 2, ..., \tag{3a}$$

where

$$s^0_{i+1} = \begin{cases} k_C s^+_{i+1}/(1 - \nu(s_i)/n_c) & if\quad k_C s^+_{i+1}/(1-\nu(s_i)/n_c) > s_i, \\ s_i & if\quad k_C s^+_{i+1}/(1-\nu(s_i)/n_c) \le s_i, \end{cases} \tag{3b}$$

$\nu(s_i)$ is a number of the LIs the strength of which is lower than s_i.

Let us note that the definition of the DeS which was given in [11, 12] is true ONLY for the loading $s_{0:\infty}^{+}=(s_0^{+},s_1^{+},s_2^{+},...)$ under condition that $s_{i+1}^{+}\geq s_i^{+},\ i=1,2,3,...$ In application to the loading of UFC we must take into accout that the number of the LI failures can not decrease. The equation (3b) ensures this condition and in the case when the event $s_{i+1}^{+}<s_i^{+},\ i=1,2,3,...$ can take place. This event can take place not only in specific program of load but and in the case when the random variables S_{i+1}^{+} and $S_i^{+},\ i=1,2,3,...$, have the same cdf. This is the reason to include the equation 3(b) in the general definition DeS .

The new definition of the DeS can be used in any loading $s_{0:\infty}^{+}$ But its application to the fatigue and the static strength tests is the most important.

1) **The fatigue test.** Later on we make processing of the test data set in order to get SN curve. In this case $s_i^{+}=s^{+},\ i=0,1,2,3,...$, where s^{+} is the some parameter of the cycling loading. It is some constant. For example, it is the maximum stress of the cycle load.

2) **The static strength test**. In this case $s_0^{+}\leq s_1^{+}\leq s_2^{+},...$ and the items of sequence $s_{0:\infty}^{+}$ increase up to infinity.

The sequence of the local stresses is called *the Daniels' sequence for a constant load* (DeS_CL) if s_i^{+} is some constant. It is called *the Daniels' sequence for an enlarging external load* (DeS_EL) in the loading process of the second type. The realizations of these random processes are defined by the pair $(s_{0:\infty}^{+},x_{1:n_C})$.

For **the specific pair** $(s_{0:\infty}^{+},x_{1:n_C})$ of the DeS_CL let us consider the following definitions.

1. *The* K *-Daniels' function*

$$r_{DK}(k)=x_{(k)}(1-\nu(x_{(k)})/n_C) \tag{4}$$

where $x_{(K)}$ *is the* k *th ordered statistic of the sample* $x_{1:n_C}=(x_1,...,x_{n_C})$, $v(.)$ *is the function defined in* (3b).

2. *The S-Daniels' function*

$$r_{DS}(s)=s(1-\nu(s))/n_C). \tag{5}$$

3. *The Daniels' residual function.*

$$r_{MX}(x)=\max_{s>x} r_{DS}(s) \; . \tag{6}$$

Under condition that the failure of LI does not take place at the preliminary fatigue loading, $s_{0:i}^{+}=(s_0^{+},s_1^{+},s_2^{+},...,s_i^{+})$, and the DeS reaches the value s_i then this function defines *the Daniels'conditional loading rate-independent residual strength,* $r_{MX}(s_i)$.

Usually, during static strength test we have $s_i^{+}=(i-1)\delta_S^{+}$, where δ_S^{+}, $\delta_S^{+}>0$, is the rate of loading. In order to take into account the specific process of a static test loading we must consider the loading defined by two sequences: preliminary fatigue loading, $s_{0:i}^{+}$, and loading at the static strength test, $s^{++}=(s_i^{+},s_1^{++},s_2^{++},...)$ where s_i^{++} tends to infinity.

The corresponding *Daniels ultimate residual strength*

$$r_{DU}=s_{n^U}^{++} \tag{7}$$

where $n^U=1+\max(i:r_{MX}(s_i)>s_i^{++},\ i=1,2,...)$.

If the prelimiunary loading does not exist, then the equation (7) defines *the DeS-ultimate static strength*.

4. The transition from s_i to s_{i+1} we call *a step of the DeS*. At least three definitions of the number of steps , i_f , corresponding to the *failure of all LIs* can be considered. They correspond to three conditions which appears first time: 1) $\bar{\nu}(s_{i_f})=\nu(s_{i_f})/n_C=1$; 2) $s_{i_f}=\infty$; 3) $r_{MX}(s_{i_f})=0$. If one (or another) of these conditions is met we say that *failure of the DeS takes place*. Actually, for real calculation we must use some critical values $\bar{\nu}_c$, s_C and r_C close to 1, to the very large or very small

value instead of 1, ∞ and 0 correspondingly.

As a matter of fact, these three conditions give very close *DeS fatigue life* values. Here we use the last condition and the following definition of the *the DeS fatigue lives*

$$n_{DR} = 1 + \max(i : r_{MX}(s_i) > r_C,\ i = 0,1,2,...) . \tag{8}$$

5. For the case of the DeS_CL let $S^+_{ND\infty}$ is a set of s^+ for which there is a solution of the equation

$$x = s^+ / (1 - \nu(x) / n_c) . \tag{9}$$

If $s^+ \in S^+_{ND\infty}$ there is such i^* that $s_{i^*+1} = s_{i^*}$. Then $r_{MX}(s_{i^*+1}) = r_{MX}(s_{i^*})$. The change of the items of the process $\{ s_i,\ i = 0,1,2... \}$ and the process $\{ r_{MX}(s_i),\ i = 0,1,2,..., \}$ will be stopped. Some LIs never will be destroyed. And we say that the DeS fatigue life will be equal to infinity.

If this set is not empty *the DeS fatigue strength (the fatigue limit) is defined by the equation*

$$s_D = \max(s^+ : s^+ \in S^+_{ND\infty}) . \tag{10}$$

Let us remark that the value $\bar{\nu}(s_i)$ can be considered as a measure of failures accumulated up to moment when the DeS reaches the value s_i. We say that it is a value of the *failure functio. of two sequences* , $x_{1:nc}$ *and* $s^+_{0:i}$: $\bar{\nu}(s_i) = F_{XS^+}(x_{1:nc}, s^+_{0:i})$. *The valueof function* $F_{XS^+}(.)$ *is the measure of the accumulated damages of any loading process.* It chages in interval $[0,1]$. The value unit (or very close to unit) corresponds to the DeS failure.

For the DeS_EL *the intact function*

$$k_R(x) = \begin{cases} k & if \quad r_{MX}(x) > x, \\ 0 & if \quad r_{MX}(x) \le x. \end{cases} \tag{11}$$

is very important also. It defines the number of of ILs which are still intact under steady increasing load x. If we try to increase the load more than $r_{MX}(x)$ then the failure of all the LIs takes place.

The examples of the introduced functions and definitions are given in the next section.

The introduced definitions are connected primarily with fatigue test at the constant cycle parameters. But the equations and definitions (3a, 3b,4,…,8) can be used for calculation the residual strength and the fatigue life for any specific realization of any random process of loading. In numerical example we consider specific random sequence of loads ($S^+_0, S^+_1, S^+_2, ...$) where all random variables have the same cdf. In this case equations (9,10) can not be used. The fatigue strength (the fatigue limit) does not exist because after some event $s_{i+1} = s_i$ the process of increasing of items of DeS can continue and condition of the failure can be reached. But using the Monte Carlo method we can calculate the cdf of the random fatigue life N_{DR} for any cdf of random variable $S^+_i,\ i = 0,1,2,...$:

$$F_{N_{DR}}(x) = \sum_{j=1}^{N_{MC}} e(x - (n_{DR})_j) \tag{12}$$

where $e(x) = 0$ *if* $x < 0$ and $e(x) = 1$ if $x \ge 0$; N_{MC} is the number of Monte Carlo tials;

$(n_{DR})_j$ is the n_{DR} for ith specific realization $(x_{1:nc}, s^+_{0:i})_j$, $j = 1,2,...,N_{MC}$, of the random pair $(X_{1:n_C}, S^+_{0:i})$.

3. Application to experimental data

3.1. Some additional assumptions

If $s^+ = Const$ already the DS approuch allows to explain the structure of a fatigue phenomenon: the DeS fatigue life function of s^+ is similar to SN curve; the DS approuch allows to explain the existence of the fatigue strength (fatigue limit); the S-type changes of DS realization are similar to changes of some physical parameter during fatigue loading,… But the numerical results are very pure: the values of DS fatigue limit is too large, the number of the DS steps to the failure is very small even if s^+ is very close to the fatigue strength, The DeS approach allows to get more appropriate value of fatigue life but still we need to make some additional „patches" assumptions in order to make the DeS mathematics useful for the numerical description of the fatigue phenomenon.

1) The accumulation of the damages takes a place only in **local WMV** in which there is some stress concentration. It can be different in the fatigue and in the tension tests. For the fatigue test we assume that the **local external stress** is k_c time more than external load.

2) The cdf of the local static strength does not coincide with the cdf of the static strength of the single LI: the "size" of the WMV and the adjacent LIs have a specific influence on the local static strength cdf which we denote by $F_{X_L}(x)$.

3) Equations (3a, 3b) defines new value of the local stress, s_{i+1}, under assumption that $\nu(s_i)$ LIs are failed already. But really for the destruction of these LIs some accumulation of an energy and some time are needed. For this reason for DeS approach in equations (3) the value ε is introduced. We assume that it is some function of s^+, $\varepsilon_D(s^+)$.

Usually, in order to get the description of the SN curve we suppose that during the fatigue test the parameter of the cycle, s^+, is some constant. But processing of the several fatigue test results shows that in this case it is difficult to explain a very large variance of the fatigue life, particularly at high level of the load (see Fig. 6). Here we suppose that really there is a randomness of the load. We assume that the load in every ith DeS step , s_i^+, is a realization of some random variable, S_i^+, $i = 0,1,2,...$. All the variables have the same cdf $F_S(x)$.

For the Monte Carlo calculation of the realization of the DeS for the specific loading, $(s_o^+, s_1^+, s_2^+,...)$ which is the realization of some random sequence $(S_o^+, S_1^+, S_2^+,...)$, now we should use the equation

$$s_{i+1}^0 = \begin{cases} k_C s_{i+1}^+ / (1-\nu(s_i)/n_c) & if \quad k_C s_{i+1}^+ / (1-\nu(s_i)/n_c) > s_i, \\ s_i \quad if \quad k_C s_{i+1}^+ / (1-\nu(s_i)/n_c) \le s_i, \end{cases} \tag{3c}$$

where $s_0 = s^+$, k_C is a stress concentration factor (let us note that in [12] it was not defined that $s_0 = s^+$).

3.2. Numerical example

We consider the experimental fatigue data taken from Tables 1- 3 in Ref. [14] concerned T300/934 graphite/epoxy laminates with $[[0/45/90-45_2/90/45/0]_2$ lay-up. In Table 1 of this paper the static strength of 25 specimens, in Table 2 in terms of cycles to failure at three different stress levels (namely: for $\sigma_{\max}$ = 400, 380 and 290 MPa), $R = \sigma_{\min}/\sigma_{\max} = 0$; and in Table 3 two sets of residual strength data are reported for 15 and 18 specimens subjected to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 and 345 MPa, respectively. Actually, the previous equations for calculation of the Daniels' fatigue life can not be used for this type of composite structure. It is not structure of the UFC. But we suppose that the failure of this composite takes the place after the failure of some its WMV which is the bundle of the n_C parallel LIs. We make (enough rough) assumption that there is some *strength-equivalent WMV* which has the same distribution of static and fatigue strength for every process of the loading.

Now we need to know the distribution of the static strength of the single LI of this equivalent WMV. There is not this data in [14]. But we get the estimates of the parameters of a Weibull distribution of the static strenth of the LIs of an equivalent bundle which has the same parameters of the static strength (the mean and the standard deviation) as a real specimen. It can be made using the Daniels theorem (see equation (1)). If the strength of the LI has the Weibull distribution with cdf

$$F(x) = 1 - \exp(-\exp((\log(x) - \theta_0)/\theta_1), \qquad (10)$$

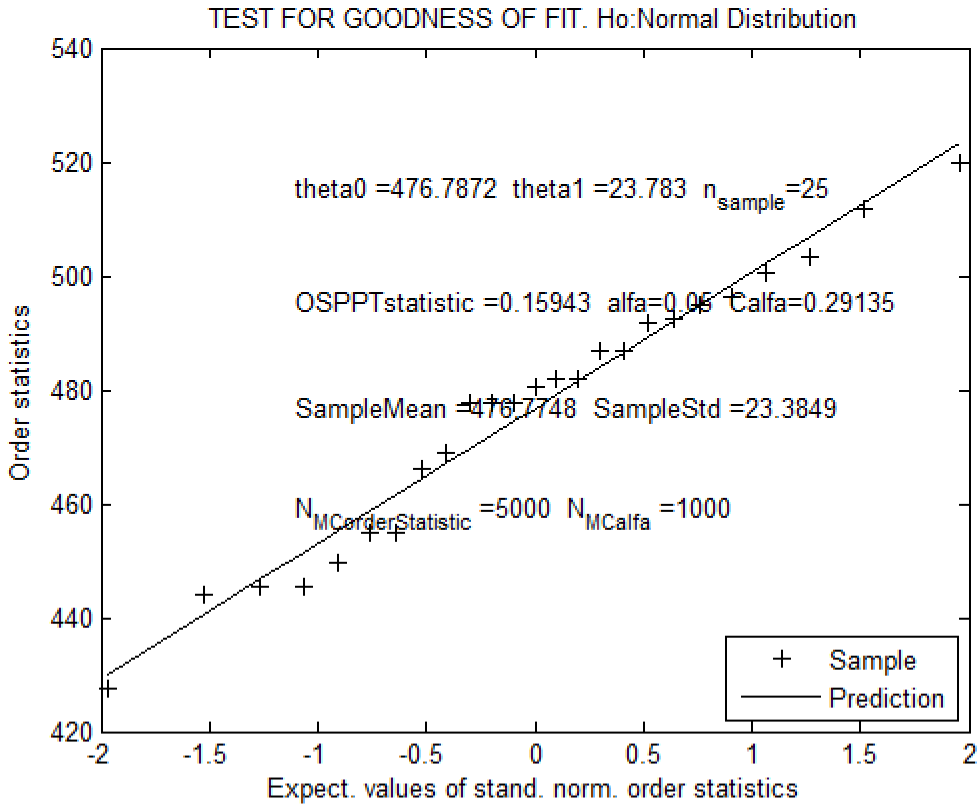


Fig.1. The OSPPT plot. The parameters of the sample of the static strength (Table 1 of [14]); The test for goodness of fit for normal distribution .

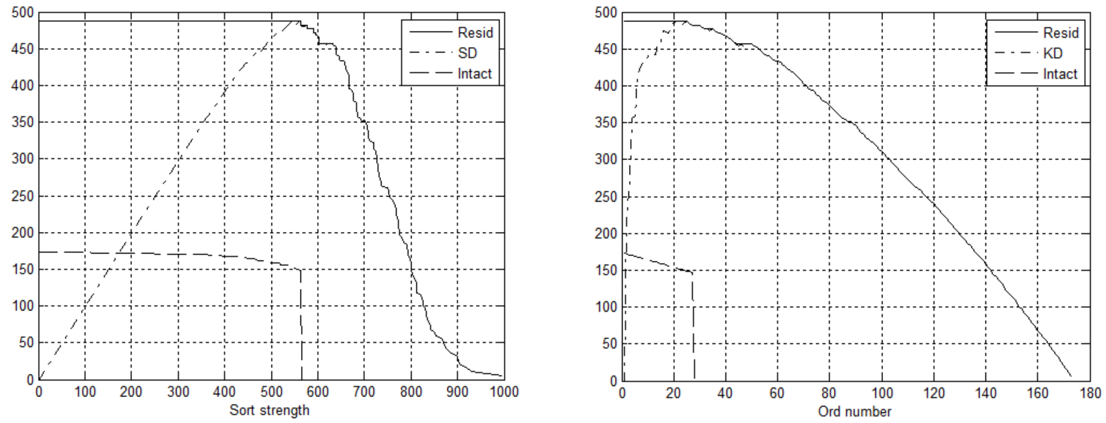
then approximately, in accordance with this theorem, the strength of the bundle has normal distribution with mean $\mu_D = \exp(\theta_0 + \theta_1(\log(\theta_1) - 1))$ and standard deviation $\sigma_D = \mu_D(\exp(\theta_1 - 1)/n_C)^{1/2}$. And vice versa, if μ_D an σ_D are known, and θ_1 is known also, we can get the equations for the θ_0 and the equivalent size of the WMV, n_C:

$$\theta_0 = \log(\mu_D) - \theta_1(\log(\theta_1) - 1), \quad n_C = \exp(2\log(\mu_D) + \theta_1 - 2\log(\sigma_D) - 1). \qquad (11)$$

In Fig. 1 the fitting of the static strengths of the 25 specimens (from Table 1 of [14]), the parameter of this sample (the mean, theta0= $\mu_D = 476.8$, and the standard deviation, theta1= $\sigma_D = 23.78$), and data for testing of normal distribution hypothesis are given. The statistic of the OSPPTest [8] is equal to 0.159. The boundary of critical region of the test for goodness of fit is equal to 0.291. So, the normal hypothesis is not rejected. The parameter θ_1 in equation (11) remains unknown but it can be chosen for a best fitting of experimental fatigue data.

Approximate estimate of parameter θ_1 can be found in other publications. In [10] there is a result of fitting of the sample of strength of 64 specimens of some carbon-fiber bundles. Using these data taken from [13] for the Weibull cdf in the form (10) it was found the estimates $\theta_0 = 6.5$ $\theta_1 = 0.13$. For processing the data set from [14] as the first approximation it was set $\theta_1 = 0.15$ Then using (11) we have $\theta_0 = 6.6$, $n_C = 172$.

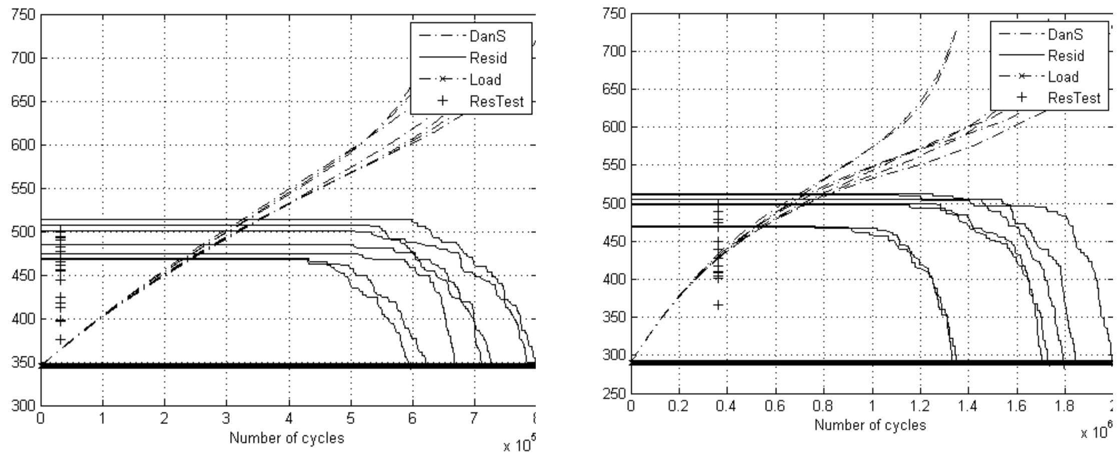
In Fig. 2 we see the Daniels' residual strength (-), the S-Daniels' function (-.-) (a), the Daniels'residual strength (-), the K-Daniels' function (-.-) (b) and the intact number of LIs for continuesly increasing load (--) as a s-function (a) and as a k-function (b). In Fig 2a the corresponding values are the functions of the maximum of strength of the already failed LIs. In Fig 2b they are the function of k, $1 \le k \le n_C$. Let us remind that we can increase the value of the function $r_{DS}(s) = s(1-\nu(s))/n_C)$ only to the value corresponding to the its maximum . Then we need to decreas it because if it remains to be equal to this maximum or more then all the LIs will be destroyed as it is shown by intact function. It worth to note that we see the failure of the WMV after failure of only very small part of the LIs.



a b

Fig.2. The S-Daniels' (a) and The K-Daniels' (b) functions (-.-), the Daniels'residual strength (-) and intact functions (--) .

We make two stages of processing the experimental data. First, we make the processing of the considered fatigue test data assuming that the fatigue test machinery and equipment allow to keep the precise level of the load and the value of the s^+ is a constant. In the Fig. 3 we see the result of the modeling



a b

Fig.3. The modeling of the DeS (-.-), the Daniels'residual strength (-) for $\theta_0 = 6.6$, $\theta_1 = 0.15$ and $n_C = 172$; the level of the load (-x-) and the residual strength experimental data sets (+) corresponding to the cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 (a) and 345 MPa (b), respectively.

of the DeS (-.-), the Daniels'residual strength (-), the test residual strength (+) and level of load (-x-) for $s^+ = 335$ MPa and $s^+ = 290$ MPa. We see that the Daniels' local residual strength does not change a long time but then decreases very drastically. And we see also that the test of the residual strength was made a significant time before the failure time in order to prevent the failure at the preliminary loading.

In Fig. 4 we see the test fatigue life (+), the mean (-), smallest (►) and largest (◄) calculated values using 100 MoneCarlo trials. So the smallest and largest calculated values correspond approximately the probability of the failure equal to 0.01 and 0.99 correspondingly. For the calculation it was used already mentioned parameters: $\theta_0 = 6.6$, $\theta_1 = 0.15$ and $n_C = 172$. Additionally it was used $k_C = 1.9$; the fuction $\varepsilon(s^+) = \varepsilon_{290} + (\varepsilon_{400} - \varepsilon_{290})(s^+ - 290)/(400-290)$ where $\varepsilon_{400} = 0.07$, $\varepsilon_{290} = 0.002$. The residual strength data are shown also for 15 and 18 specimens subjected to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 and 345 MPa, respectively.

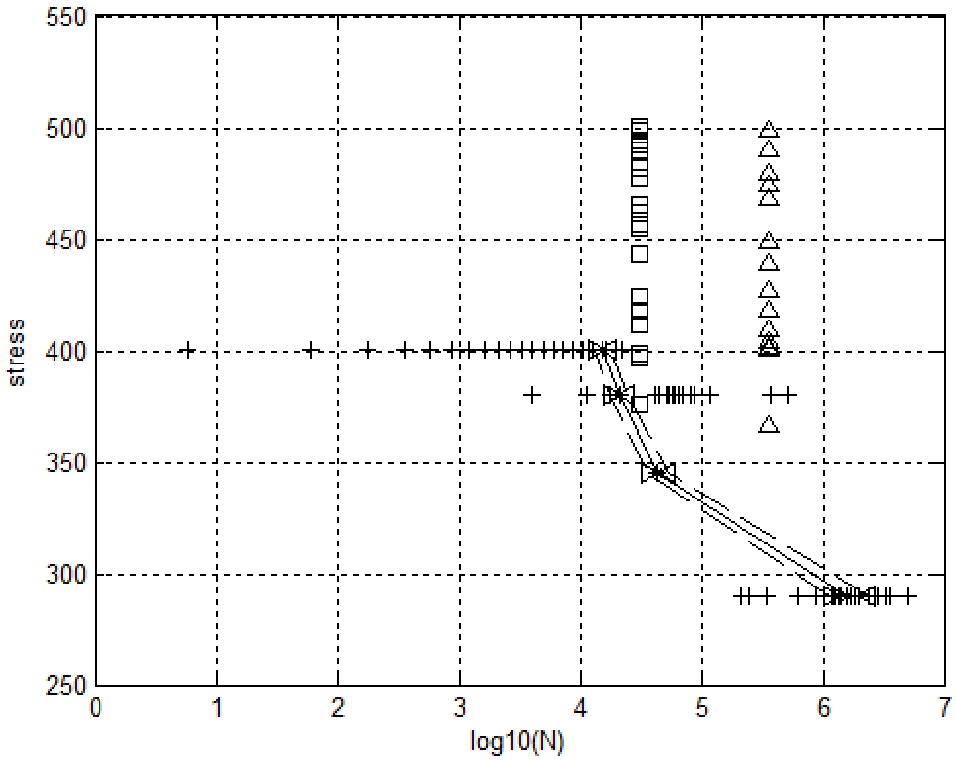


Fig. 4 . The test (+) and calculated mean (-), smalest (►) and largest (◄) fatigue lives for $\theta_0 = 6.6$, $\theta_1 = 0.15$ and $n_C = 172$. The residual strength experimental data sets correspond to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 (▲) and 345 MPa (■), respectively.

In Fig 4 we see the reasonable fitting of the mean fatigue lives and the residual strength. But the scatter of the test fatigue life is much more than calculated one. In order to explain this we make the additional assumption. We assume that the real load in every DeS step, S_i, $i = 0,1,2,...$ is some random variable which has the normal distribution with the cdf $F_{S^+}(x) = \Phi(x - \theta_{0S})/\theta_{1S})$, $\theta_{0S} = s^+$; $s^+ = 400,\ 380,\ 345,\ 290$ MPa for four level of test; θ_{1S} =0.2; $\Phi(.)$ is cdf of the standard normal distribution. It appears also to be necessary to change values θ_0 and n_C and the equation for $\varepsilon(s^+)$. The decreasing of the n_C increses the scatter of fatigue life. But this time even very significant decreasing of the n_C is not enough to explain the real value of this scatter. The change of the n_C is the reason to change another parameters. Now $\theta_0 = 6.375$, $n_C = 5$,

$\varepsilon(s^+) = \varepsilon_{290} + (\varepsilon_{400} - \varepsilon_{290})((s^+ - 290)/(400-290))^{10}$ are used. The final result is shown in the Fig. 5-6 where the same previous notations are used. We see the much better fitting of the residual strength, Fig.5, and of the fatigue life, Fig. 6. It should be noted additionaly that the limitation of the smallest fatigue life by the value 1000 at the $s^+ = 400$ and 380 MPa in Fig. 6 is a 'technical' limitation, because for the initial fatigue life data time unit the 1000 cycles was chosen.

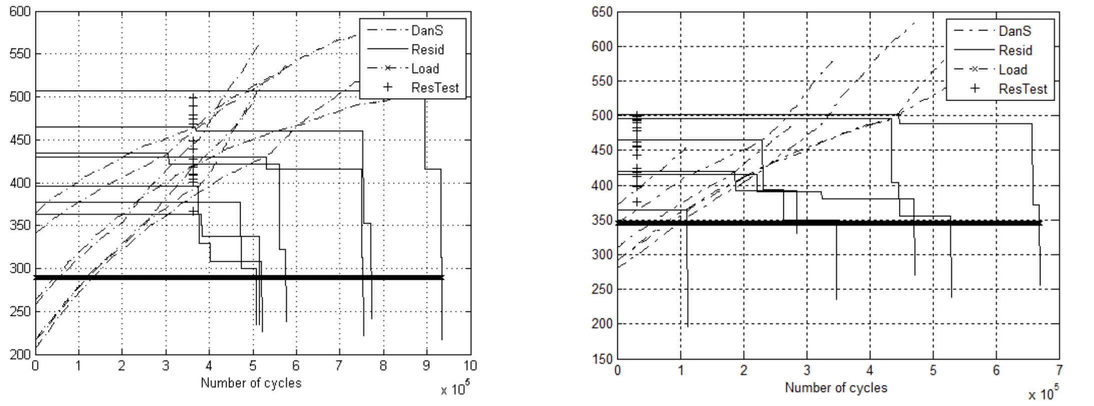


Fig. 5 . The modeling of the DeS (-.-), the Daniels'residual strength (-) for $\theta_0 = 6.375$, $\theta_1 = 0.15$ and $n_C = 5$; the level of the load (-x-) and the residual strength experimental data sets (+) corresponding to the cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 (a) and 345 MPa (b), respectively.

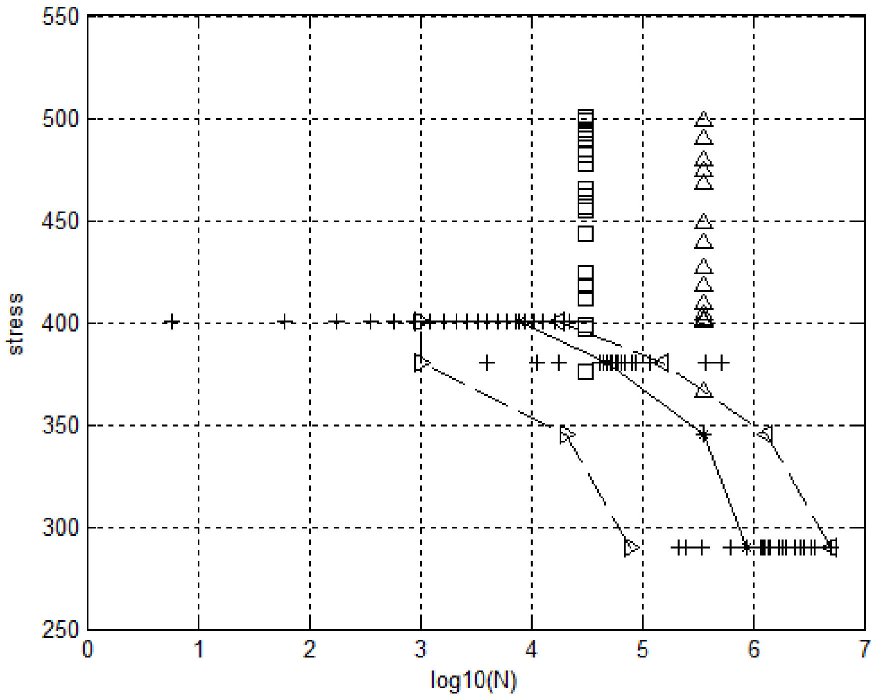


Fig. 6 . The test (+) and calculated mean (-), smalest (►) and largest (◄) fatigue lives for $\theta_0 = 6.375$, $\theta_1 = 0.15$ and $n_C = 5$. The residual strength data corresponding to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 (▲) and 345 MPa (■), respectively.

For the case when s^+ is some constant from the set 400, 380, 345 and 290 MPa the calculated fatigue life is final with probability one. But now the probability to have final fatigue life for the load level 290 MPa is equal only 0.97. The corresponding part of the cdf of fatigue strength is shown in Fig. 7

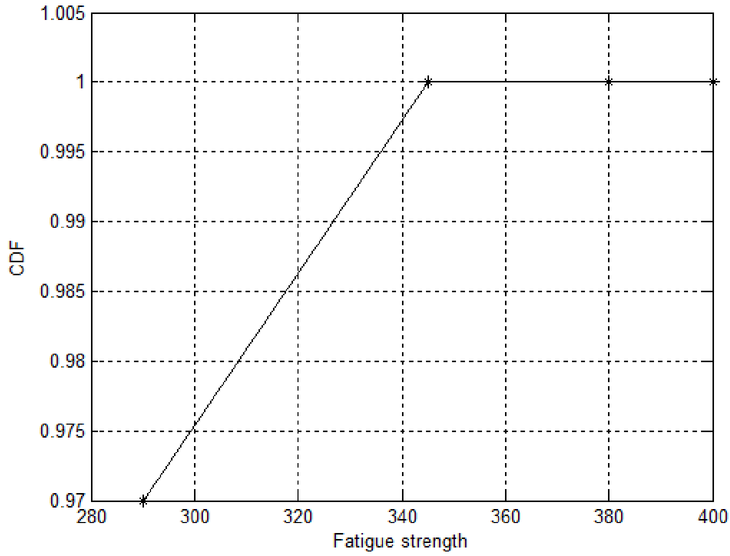


Fig. 7. The cdf of fatigue strength .

Conclusions and areas for further research

It was shown that the DeS approach to the description and modelling of the fatigue life and the fatigue strength of UFC allows:

1) to explain some specific features of the fatigue tests :

a) the condition of the existence of the fatigue strength and the possibility to have the infinite fatigue life for the determined cycling loading, $s^+ = Const$

b) the specific features of the residual strength behaviour: the strength degrades smoothly in the first time part of the loading with a sudden drop accuring just before the failure. Even the assumption that the residual strength does not change up to failure ("the rectangle hypothesis of the residual strength") is not too far from the truth. It is not the real problem to estimate the "rectangle" residual strength but the estimation of the time to failure is the real problem.

2) to make the interpretation of the parameters of the studied models as the parameters of the local static strength (this is the main difference of models of the DeS approach from the others models). The difference of the parameters of the cdf of the local strength of the LIs in the framework of the UFC and ones of the isolated single LIs do not allows to make prediction of the fatigue parameters of the UFC using the static strength parameters of the LI. These parameters can be used as first approximations of the corresponding parameters of nonliner regression analysis of the data set.

3) to calculate the measure of accumulated fatigue damage $\bar{v}(s_i) = F_{XS^+}(x_{1:nc}, s^+_{0:i})$ for any *cycling loading* $s^+_{0:i}$ It worth to note that just as the Palmgren_Mainer measure, $\sum_j n_j / N_j$, the value $\bar{v}(s_i)$ changes in interval $[0,1]$, the condition $\bar{v}(s) = 1$ is the condition of the DeS failure.

. The considered here the modified version of the DeS model with random loading gives the explanation of the great scatter of the fatigue life. Usually we suppose that the specimen with the higher initial static strength has also longer fatigue life (Strength-Life Equal Rank Assumption). In the studied here model it is not necessary true because different specimens have the different random loading history.

For different levels of an adequacy of an analysis result the different modifications of the DeS models can be used. In the simplest case it can be set $n_C = \infty$ and the $F_X(s)$ can be used instead of $\bar{\nu}(s)$ in equations (3a, 3b). For any realization of load sequence the value $\bar{\nu}(s)$ is easily calculated but the search of the parameters of the corresponding nonlinear regression for the above described models is a difficult task. Against all the odds, we think that, in due course, the structure of the models suggested will be of the interest not only for the graduation theses of the students but also for the engineering applications, in particular, for the prediction of the variations in the parameters of the strength and the fatigue life of the UFC after the changes in the parameters of their components.

The considered approach seems quite promising yet it requires serious stydy of the cdf of local static strength distribution (the Weibull distribution is not appropriate enough) and the connection of the number of DeS steps and the number of the cycles of the fatigue loading.

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Controllable Queueing Systems: From the Very Beginning up to Nowadays[1]

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Abstract

The present paper represents a review of the Controllable Queueing Systems theory development from the very beginning up to nowadays. The main stages of this theory development are considered. Some new problems are mentioned. The review is devoted to those, who are interested in the creation and the development of the Controllable Queueing Systems theory from its generation up to nowadays, and who want to understand the tendency of its development and the new directions and problems of its study.

Keywords: *Controllable queueing systems, Markov decision processes, Optimality principle, monotonicity of optimal policies*

1 Introduction

The theory of Controllable Queueing Systems (CQS) is a special direction of investigations of a general theory of controllable stochastic processes from one side, and of a Queueing Theory (QT) from another side. The theory of controllable stochastic processes is a special topic, which we will not touch here, and will fix on CQS. Some papers devoted to the problems of Queueing Systems (QS) control have been arisen almost simultaneously with the first works about QS, but the special approach to CQS has been done by Rykov [48] in 1975. Several monographs devoted to this problem arisen thereafter [25, 26, 31, 64, 67].

The preliminary results of this theory development one can find in [48]. In this paper, we concentrate our attention on the new results and approaches in this theory, nevertheless some initial principal results, on which the theory is based also should be reminded a little bit.

The paper is organized as follows.

In the next section the definition of CQS will be done and some examples of CQS will be proposed. The elements of the theory of Discrete Time Controllable Semi-regenerative Processes (DTCSRP), which serves as a base for the CQS study, will be considered then. The optimality principle for CQS as a result of this theory development and the problems of the real optimal rules for CQS calculation including numerical methods of the optimal policies calculation will be discussed in the next two sections. The qualitative properties of the optimal policies in this section also considered. Sections 5 – 8 are devoted to special CQS. At least in the Conclusion some new problems, approaches and new problems settings will be discussed.

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2 CQS. Definition and main properties

2.1 Definitions

In this review, we will use a little bit modified Kendall's system of notations for QS [29] and will consider QS as a mathematical object, consist of four components $\alpha \mid \beta \mid \gamma \mid \delta$, where

- α —input flow,
- β —service mechanism,
- γ —system structure,
- δ —service discipline.

The symbols α and β take usual values M, GI etc., for Markov, recurrent and others for input flow, and distributions of recurrent service mechanism, the system structure γ consists of two numbers $\gamma = (n, m)$, where the first symbol means the number of servers, and the second means size of the buffer, and it is omitted for buffer size equal to infinity. The last symbol δ is used for service discipline and it is omitted for FIFO (first in first out) discipline.

Each of these components is also a complex mathematical object, which is usually studied in details in QT and is determined by some parameters. QS investigation usually could be divided into three directions:

- **analysis** that means the calculation of some output system Quality of service (QoS) characteristics for completely determined system;
- **synthesis** that means finding of some of the input characteristics in order to provide needed QoS indexes, and
- **control** that means operating of the system during its work with a goal of its behavior optimization with respect to given criteria.

In the review we focus on the last problems that is most important for applications. Based on the above, the following definition of a CQS is reasonable.

Definition 1 *CQS is a QS, for which some parameters of its components admit dynamic variation during its operation. Naturally, this variation is admitted in some domain and serves to some goals, determined with some QoS functionals.*

2.2 Classification of CQS

Accordingly to this definition CQS's can be classified as systems with:

- controllable input flow,
- controllable service mechanism,
- controllable system structure,
- controllable service discipline and
- complex CQS, for which some parameters of several components admit control.

For the control problem setting one needs in the goal of control and control rules.

2.3 Control times, goals and rules

Note firstly that in most practical situation the change of control (decision making) is possible only in special times, for example in times of customers arrival or their service completion.

We will call these times as *control* or *decision times* (DT). From another side the control problem solution is usually accompanied with the *goal of control* and besides of control parameters also depends on the admissible domain of their variation, rules of their use as well as of process observation possibility.

Concerning the goals of control they should be given by some *control quality functionals* optimization. It might be some QoS characteristics optimization in a steady state regime of the system operation or optimization (minimization or maximization) of a time of some state or set of states attainment. In some other cases the goal of control could be formulated as an optimization of some economic indexes, connected with the system operation. In the last case some *structure of losses and rewards* (Loss-Reward Structure—LRS) connected with the system operation should be done. Following to the tradition we will consider as an optimality criterion the minimization of some *Loss Functional* (LF). Losses or rewards could be connected both with the system stay in different states and with transition from one state to another. The LRS usually includes:

- random reward R_n of the system manager for the service of n-th customer with finite mean value m_R (*service cost*),
- penalty $C_w(l)$ for sojourn time of l customers in the system (*waiting* or *holding cost*),
- cost $C_u(a_k)$ for using k-th service mode (*using cost),*
- penalty $C_s(k,k')$ for switching from k-th service mode to the k'-th one (*switching cost*).

Using these data the LF should be constructed. The general form of the LF will be considered in the next section, and its special form jointly with concrete examples will be represented.

The control rules are usually determined with the help of *control strategies*, which define the manner to take the decisions by *a Decision Maker* (DM) and depends, generally speaking, on system behavior observability and can be realized in several ways:

- taking into account whole history of the process,
- taking into account only the system last state, or
- without any information;
- also the decision can be made randomly or not.

Specification of these strategies will be introduced in the section 3.2.2.

2.4 Examples

Consider some examples of CQS that will be studied in details later.

2.4.1 Arrival control

Consider a $M/GI/1/\delta$–QS with controllable input. The customers arrive accordingly to Poisson flow with intensity λ and are served during random times that are i.i.d. with general Cumulative Distribution Function (CDF) $B(t)$ with mean value $m_B = \mu^{-1}$ and variance σ_B^2. The LRS includes:

- the random reward R_n of the system manager from the service of n-th customer with finite mean value m_R (service cost),
- the linear penalty $C_w(l) = C_w l$ for sojourn time of l customers in the system (waiting cost),

The control times are the arrival times, and the decisions consist in the admission or rejection of an arriving customer into the system. The problem consists in the admission of customers in the system organization aimed reward maximization (or loss minimization).

2.4.2 Control of service mechanism

Consider a $M/M/1/\delta$ –QS with controllable service rate. Customers arrive accordingly to Poisson input with intensity λ, and are served with exponentially distributed service time having one of finite number K values of parameter μ_k $(k = 1,2,\ldots K)$. The LRS includes:

- the random cost R_n for the service of n-th customer with finite mean value m_R (service cost),
- the linear penalty $C_w(l) = C_w l$ for sojourn time of l customers in the system (waiting cost),
- cost $C_u(\mu_k)$ of using server in k-th regime with service rate μ_k (using cost), and
- penalty $C_s(\mu_k, \mu_l)$ for switching from k-th regime to the l-th one (switching cost).

Control times are both: arrival and service completion times, and the decision consist in the choice of the service regime (service rate) aiming at long run expected reward per unit of time maximization.

2.4.3 Control of system structure

Consider a $M/GI/1/\delta$–QS with controllable system structure. Customers arrive accordingly to Poison input with intensity λ, and are served accordingly to recurrent service mechanism with generally distributed with CDF $B(t)$ service time. The LRS includes:

- penalties C_1 for switching on a server, and C_0 for its switching off,
- cost C_s of server using per unit of time (using cost),
- the linear penalty $C_w(l) = C_w l$ for sojourn time of l customers in the system (waiting, cost).

Control times are both: arrival and service completion times, and the decision consists in possibility to switch on a server in customer arrival time and switch off it in service completion time aiming long ran expected loss per unit of time minimization.

2.4.4 Service discipline control

Consider s $M_N/GI_N/1/\delta$ –QS with several (N) types of customers and controllable service discipline among the priority disciplines. Inside the classes the customers are served accordingly to the FIFO discipline.

The customers arrive from Poison input with intensity $\lambda_i\,(i = \overline{1,N})$ for i-th type of customer, and are served accordingly to the recurrent mechanism with general CDF $B_i(t)(i = \overline{1,N})$. The LRS includes:

- the linear penalty $C_i(l) = C_i l$ for sojourn time of l customers of i-th type in system (waiting cost).

The control consists in the choice of a customer for service in any service completion (decision) time, aiming the minimization of a long ran expected loss per unit of time.

Consider firstly the common model for investigation those and many others CQS.

3 Discrete time controllable semi-regenerative processes

As it has been mentioned above in the section 2.3 in the most practical situation the change of control (decision making) is possible only at special times, for example in times of customers arrival or their service completion. We will call these times as *decision times* (DT) and denote by $\{S_n, n = 0,1,\ldots, S_0 = 0\}$ the sequence of decision times. Of course they are some measurable functionals of the process, describing the system behavior. For modelling CQS it is possible to use so called *Discrete Time Controllable Semi-regenerative Process* (DCSRP). Detailed information about these processes and its applications one can find in [31]. Remind here some needed definition and properties of these processes.

3.1 Semi-regenerative processes

3.1.1 Definitions and main properties

Semi-regenerative processes are some mixture of regenerative and semi-Markov processes. Several authors (**G .Klimov, E .Nummelin, V. Rykov, M. Yastrebenetsky**) introduced them under different names. Remind here its contemporary definition. Let

- $X = \{X(t), t \in R\}$ be a stochastic process with measurable state space (E, E),
- $F_t^X = \sigma\{X(v), v \le t\}$ be generated flow of σ-algebras, and
- $\{S_n, n = 0,1,2,\ldots\}$ be a sequence of its Markov times with $S_0 = 0$.

7

Definition 2 (Rykov, Yastrebenetsky (1971)) *A pair $\{X(t), S_n\}$ is called a (homogeneous) Semi-Regenerative Process (SRPr), if for any subset $\Gamma \subset E$ and for all $n = 1,2,\ldots$ takes place*

$$P\{X(S_n + t) \in \Gamma | F_{S_n}\} = P\{X(S_n + t) \in \Gamma | X(S_n)\} = \\ = P\{X(S_1 + t) \in \Gamma | X(S_1)\}. \quad (1)$$

Here

- r.v.'s S_n are called *Regeneration Times* (RT's),
- intervals $T_n = (S_n, S_{n+1}]$ and their lengths $T_n = S_{n+1} - S_n$ are called *Regeneration Periods* (RP's),
- functional random elements W_n, where

$$W_n = \{(X(S_n + t), T_n), t \le T_n, n = 1,2,\ldots\}$$

are called *Regeneration Cycles* (RC), and

- random elements $X_n = X(S_n)$ are called *Regeneration States* (RS).

Remark 1 *Intuitively clear that the SRP behavior is fully determined with its regeneration cycles W_n that form a Markov Chain in functional space, and under an additional condition of regularity any SRP is reconstructed (up to equivalence) by it. However the real determination of a generator of Markov chain in functional space is not a simple procedure.*

Therefore, we focus on the most important consideration of the one-dimensional characteristics of SRP. For this consider also some complementary processes

$$Y_n = (X_n, T_n), \quad N(t) = \max\{n : S_n \le t\}, \ and \quad Y(t) = X(S_{N(t)}).$$

Theorem 3 (Jolkof, Rykov (1981) and Rykov (1997)) *Let* $\{(X(t), S_n), t \geq 0, n = 1,2,\ldots\}$ *is an SRP. Then the sequences* $\{X_n, n = 1,2,\ldots\}$ *and* $\{Y_n, n = 1,2,\ldots\}$ *are homogeneous Embedded Markov (EMCh) and Semi-Markov chains (SMCh), and* $\{N(t), t \geq 0\}$ *is a Markov Renewal Process (MRP) while* $Y(t)$ *is a semi-Markov process (SMP).*

Proof of the theorem can be found in the remind papers (see also [20, 45] for generalization). Denote

- the Transition Matrix (TM) of MCh $\{X_n, n = 1,2,\ldots\}$ and Semi-Markov Matrix (SMM) of SMCh $\{Y_n, n = 1,2,\ldots\}$ as

$$P(x, y) = P\{X_{n+1} = y \mid X_n = x\},$$
$$Q(x, t, y) = P\{X_{n+1} = y, T_{n+1} \leq t \mid X_n = x\};$$

- one-dimensional SRP distribution of separate regeneration periods (*SRP transition function*) by

$$\phi(x, t, \Gamma) = P\{X(S_n + t) \in B\}, t < T_n \mid X(S_n) = x\}, \quad n \geq 1;$$

- one-dimensional SRP distribution given an initial state x by

$$\pi(x, t, B) = P\{X(t) \in B \mid X(0) = x\} \equiv P_x\{X(t) \in B\};$$

- Markov Renewal Matrix (MRM) [**Korolyuk, Turbin** (1972), **Jolkof, Rykov** (1981)] and [**Rykov** (1997)] by

$$K(x, t, y) = M_x \sum_{n \geq 0} 1_{\{([0,t], y)\}}(S_n, X_n).$$

The behavior of a SRP $\{(X(t), S_n), t \geq 0, n = 1,2\ldots\}$ does not fully determined by its transition function. But many useful properties and characteristics of SRP can be represented in terms of appropriate characteristics at its separate cycle and its MRM. Especially, for a most interesting in practice one-dimensional distributions of a SRP, the following theorem can be proved with the help of complete probability formulae:

Theorem 4 *One dimensional distributions of SRP satisfy to the following relations*

$$\pi(x, t, B) = \phi(x, t, B) + \sum_{y \in E} \int_0^t Q(x, du, y) \pi(y, t-u, B0, \tag{2}$$

$$\pi(x, t, B) = \phi(x, t, B) + \sum_{y \in E} \int_0^t K(x, du, y) \phi(y, t-u, B) = \tag{3}$$

$$= \phi(x, t, B) + K \star \phi(t, B), \tag{4}$$

where $\star$ denotes the matrix-functional convolution..

Remark 2 *One can see that the equality (4) is the solution of the Markov renewal equation (2).*

3.1.2 Renewal, Limit and Ergodic Theorems

A well- known Key Renewal, Limit and Ergodic Theorems are generalized for SRP's as follows.
Theorem 5 Key renewal theorem, *[**Rykov, Yastrebenetsky** (1971), **Jolkoff, Rykov** (1981)] Under usual for the renewal theorem conditions, the following limiting formula holds*

$$\lim_{t\to\infty} K\star g(t,x) \equiv \lim_{t\to\infty}\int_0^t \sum_{y\in E} K(x,du,y)g(y,t-u)dt =$$

$$= m^{-1}\int_0^\infty \sum_{y\in E}\hat{\pi}(y)g(y,t)dt,$$

where $\hat{\pi} = \{\hat{\pi}(x), x\in E\}$ is the invariant distribution of the EMCh and

$$m = \int_0^\infty \sum_{x\in E}\hat{\pi}(x)Q(x,[s,\infty),E)ds$$

is a stationary mean of RP length.

The last statement provides the calculation of SRP stationary probability distribution in terms of its distributions at separate RP's and invariant measure of EMCh.

Theorem 6 Limit theorem *Under some addition assumption of uniform regularity, SRP with ergodic (positively recurrent) EMCh steady state probabilities does exist and is equal to*

$$\pi(\Gamma) = \lim_{t\to\infty}\pi(x,t,\Gamma) = m^{-1}\int_0^\infty \sum_{x\in E}\alpha(x)\phi(x,t,\Gamma)dt \tag{5}$$

Theorem 7 Ergodic theorem *For uniformly regular SRP with ergodic (positively recurrent) EMCh,*

$$\lim_{t\to\infty} 1t\int_0^t g(X(u))du = \frac{M_{\tilde{\pi}}\int_0^{T_1} g(X(u))du}{M_{\tilde{\pi}}T_1} =$$
$$= m^{-1}\sum_{x\in E}\hat{\pi}(y)g(x)q(x), \tag{6}$$

where $q(x) = \int_0^\infty (1-Q(x,u,E)du$ is the mean time of the semi-Markov process $Y(t)$ staying in the state x.

The **proof** of these theorems one can find in [20, 22, 45, 61].

3.2 Discrete time controllable SRP

3.2.1 Definition

In controllable stochastic processes usually two factors should be considered: actions of Nature and will of the Decision Maker (DM). As a system behavior modeling process, we consider the SRP and suppose that the decision times coincide with the regeneration times that leads to the definition of a discrete time controllable process (DTCSRP).

Definition 8 *DTCSRP is a triple* $\{(X(t), S_n, U_n), n = 1,2,\ldots\}$*, where* $\{(X(t), S_n), n = 1,2,\ldots\}$ *is a SRP for which Regeneration Times (RT's)* $\{S_n\}$ *are also Decision Times (DT's), and* U_n *denotes the decision in time* S_n*. As before for DTSRP the intervals* $T_n = (S_n, S_{n+1}]$ *and their lengths* $T_n = S_{n+1} - S_n$ *are called Regeneration Periods (RP's).*

Many concrete CQS could be modeled with DTCSRP, including those that has been proposed in the section 2.4. We will return to them in the sections 5 – 8. See also [31].

As for usual SRP the main role for DTSRP play the controllable embedded Markov (CMCh) $\{(X_n, U_n), n = 1,2,\ldots\}$ and semi-Markov $\{(Y_n, U_n), n = 1,2,\ldots\}$ (CSMCH) chains. Its family (with respect to the decision $a \in A$) of transition matrices (described the Nature actions) is denoted as

$$P(x, y; a) = P\{X(S_n) = y \mid X(S_{n-1}) = x, U(S_n) = a\},$$
$$Q(x, t, y; a) = P\{X(S_n) = y, T_n \le t \mid X(S_{n-1}) = x, U(S_n) = a\}.$$

Remark 3 *Besides these two processes one could consider also controllable functional random elements* $\{(W_n, U_n), n = 1,2,\ldots\}$ *where* $W_n = \{(X(S_n + t), T_n), t \le T_n, n = 1,2,\ldots\}$ *is a Regeneration Cycle (RC), and* U_n *is the appropriate decision in time* S_n.

The behavior of the controllable process determines not only its transition probabilities but also by the controller or Decision Maker (DM). The control rules are determined by the strategies.

3.2.2 Strategies

Definition 9 *The manner of decision making is called strategy.*

As it was mentioned in the section 2.3, the strategy depends on system behavior, observability and can be realized by several ways. In order to formalize mentioned there possibilities we need to consider the history of the DTCSRP. Denoting the decision at the time S_n by U_n and putting $T_n = S_n - S_{n-1}$ the random history of the process up to time S_{n+1} can be represented as a sequence

$$H_n = \{X_0, U_0, T_1, X_1, \ldots U_{n-1}, T_n, X_n\}$$

and their realization with appropriate small letters.

A trajectory of DTCSRP is presented in the diagram

(-25, 10) X_0

The mostly general decision rule in the case of fully observable controllable process trajectory is determined by the *random measurable strategy*

$$\delta = \{d_i(u_i \mid h_i), i = 0,1,\ldots\},$$

where $d_i(u_i \mid h_i)$ is a distribution of the DTCSRP decisions admissible at the trajectory. This class of strategies is denoted by Δ. Beside this class, there exists another classes, and a mostly popular one is a class of *simple Markov strategies*, for which decisions depends only on the last state of the process and appropriate distribution is degenerated one at some decision

$$d_i(u_i \mid h_i) = \varepsilon(f(x_i))$$

where x_i is the last state of the trajectory h_i and $f(x_i) = u_i^\star$ determines an optimal non-randomized decision in the state x_i. The appropriate strategy can be represented as $\delta = \{f, f, \ldots f, \ldots\} = f^\infty$, and appropriate class of simple Markov strategies as Δ_0.

Suppose, for simplicity that the initial time $S_0 = 0$ is a decision and regeneration time. Then, with an initial distribution α of the process and given strategy δ on the set of process trajectories, there exists some probability measure P_α^δ. Appropriate to this measure expectation will be denoted by E_α^δ. For degenerated at the state x initial distribution $\alpha = \varepsilon(x)$ appropriate probability and expectation will be denoted as P_x^δ and E_x^δ, respectively.

3.2.3 Quality functional and optimization problem

The loss functional associated with the system RLS will be denoted as $Z(t)$ and it is supposed that it can be represented in the form

$$Z(t) = \sum_{n\geq 0} 1_{\{S_n \leq t\}} Z_n(t - S_n) \tag{7}$$

where $Z_n(t)$ represents the appropriate loss functional resulting only from the decision time S_n and which does not depend on another decisions. It is calculated based on the LRS for the concrete models.

There are several approaches for control of the process. The most popular are two of them.

- Expected discounted loss minimization,

$$w(x,\delta) \equiv E_x^\delta \int_0^\infty e^{-st} Z(t)dt \Rightarrow \text{infinum}; \tag{8}$$

- Expected long run loss minimization,

$$g(x,\delta) \equiv \lim_{t\to\infty} 1tE_x^\delta Z(t) \Rightarrow \text{infinum}. \tag{9}$$

For both cases the optimal strategy δ^* is that, for which

$$w(x,\delta^*) = \inf\{w(x,\delta), \delta \in \Delta\} = w(x), \tag{10}$$

$$g(x,\delta^*) = \inf\{g(x,\delta), \delta \in \Delta\} = g(x). \tag{11}$$

3.2.4 Optimality equations

One of the main results of the Markov Decision Processes theory, which also holds for DTCSRP is (see [31]) that the functions (10) satisfy to so called *optimality*, or *Bellman equations* that have the forms:

- For the discounted loss minimization

$$w(x) = \inf_{a\in A_x} \left\{ \tilde{c}(x,a) + \sum_{y\in E} \tilde{q}(x,a,y) w(y) \right\}, \tag{12}$$

where $\tilde{c}(x,a)$ is one step discounted expected lost function with the initial state x and decision a,

$$\tilde{c}(x,a) = E_x^a \int_0^\infty e^{-st} Z_n(t);$$

and $\tilde{q}(x,a,y)$ is a probability generating function of a RP (inter decision times) under decision a for initial state x at given DT S_n,

$$\tilde{q}(x,a,y) = \int_0^\infty e^{-st} Q_{x,y}(dt;a).$$

- For long run criterion optimization, the optimality equation besides the *model price* $g(x)$ includes also so called *value function* $v = \{v(x), x \in E\}$,

$$v(x) = \inf_{a \in A_x} \left\{ c(x,a) - g(x)m(x,a) + \sum_{y \in E} P(x,a,y)v(y) \right\}, \quad (13)$$

where $c(x,a)$ is a one step expected lost function with an initial state x and decision a,

$$c(x,a) = E_x^a Z_n(\infty),$$

and $P(x,a,y)$ is a transition probability during a RP (inter decision times) under decision a for initial state x at given DT S_n,

$$P(x,a,y) = Q_{x,y}(\infty; a).$$

For given functions $w(x)$ or $g(x)$ and $v(x)$, the right side hand of the equations (12, 13) are known as a *Bellman function* $b(x,a)$. Another name of these equations is the *Bellman equations*, which in general case could be represented as

$$v(x) = \inf_{a \in A_x} b(x,a). \quad (14)$$

4 Optimal strategy construction

4.1 Optimality Principle

The main result of the DTCSRP theory, as well as the theory of DMP, consists in the validity of *optimality principle* in the framework of some assumptions that takes place for mostly applicable situations. In [31] it was shown that under enough reasonable from application point of view conditions for DTCSRP, the optimality principle holds. i.e. there exists a simple Markov optimal strategy, and appropriate policies can be found from optimality equations. The optimality principle means that:

- an optimal strategy exists and belongs to the class of simple Markov strategies,

therefore it is determined by the policy $f = \{ f(x : x \in E)$, and

- it could be found as the solution of optimality equation

$$f(x) = argmin\{b(x,a) : a \in A(x)\}.$$

As a result of these investigations the problem of QS control is reduced to the problems of

- solution of Bellman equation (14) and
- the Bellman function $b(x,a)$ minimization with respect to $a \in A(x)$.

4.2 Numerical methods

There are two main methods for the solution of Bellman equation

- iteration algorithm due to **Howard** [28]
- linear programming algorithm due to Wolf and Danzig .

4.2.1 Iteration algorithm

The iteration algorithm has been proposed firstly by **R.A.Howard** for MDP [28]. For both discounted and long run cost minimization, it consists of two procedures.

The algorithm

- **Beginning:** Choose some (for example, one-step optimal policy f_0)
- **Policy evaluation:** For a given policy f_k , solve the Bellman equations (12) or (13) in

order to find value functions $w_k = \{w_k(x), x \in E\}$ or $g_k = \{g_k(x), x \in E\}$ and $v_k = \{v_k(x), x \in E$

- **Policy improvement:** For the given value functions w_k or g_k and v_k , find for each state x the decision $f_{k+1}(x)$ that minimizes the value of Bellman function

$$f_{k+1}(x) = argmin\{b(x, a; v_k) : a \in A(x)\}.$$

Construct the new policy $f_{k+1} = \{f_{k+1}(x); x \in E\}$, leaving in each state x the previous decision $f_k(x)$ if it coincides with the new one: $f_{k+1}(x) = f_k(x)$.

- **End:** Compare two successive policies f_{k+1} and f_k. If they coincide STOP and the last policy is optimal, if no go to the step **Policy evaluation**.

It has been proved in [28] that at each step k of the algorithm the value functions w_k or g_k and v_k

do not increase and therefore for the case of the decision process with finite number of states, the algorithm stops for the finite number of steps, and for the case of system with denumerable states space the algorithm converges.

There are different improvements and specification of the algorithm (see, for example, **Puterman** [46], **Rykov** [49], and others.

4.2.2 Linear programming algorithm

Consider firstly LPA for the discounted losses model. For an event

$$B_n(x,u) = \{\ldots, X(S_n) = x, U(S_n) = u, \ldots\}$$

denote by $\tilde{\pi}_n(x,u)$ the quantity

$$\tilde{\pi}_n(x,u) = E_\mu^\delta e^{-sS_n} 1_{\{B_n(x,u)\}}$$

for simple Markov strategy $\delta = f^\infty$ and for initial process distribution μ. Than the functional (8) with the process initial distribution μ due to (7) can be represented as follows

$$\begin{aligned} w(\mu,\delta) &\equiv E_\mu^\delta \int_0^\infty e^{-st} Z(t)dt = \\ &= E_\mu^\delta \int_0^\infty e^{-st} \sum_{n\ge 0}\sum_{x\in E}\sum_{u\in A(x)} 1_{\{B_n(x,u), S_n \le t\}} Z_n(t - S_n)dt = \\ &= E_\mu^\delta \sum_{n\ge 0}\sum_{x\in E}\sum_{u\in A(x)} e^{-sS_n} 1_{\{B_n(x,u)\}} E_x^u \int_0^\infty e^{-sv} Z_n(v)dv = \\ &= \sum_{n\ge 0}\sum_{x\in E}\sum_{u\in A(x)} \tilde{\pi}_n(x,u)\tilde{c}(x,u), \end{aligned} \tag{15}$$

where $\tilde{c}(x,u) = E_x^u \int_0^\infty e^{-sv} Z_n(v)dv$.

Using the backward Kolmogorov equation for DTCSRP, one can show that the function $\tilde{\pi}_n(x,u)$ satisfies the equation

$$\sum_{u\in A_x} \tilde{\pi}_n(y,u) = \begin{cases} \mu(y) & \text{for } n = 0 \\ \sum_{x\in E}\sum_{u\in A_x} \tilde{\pi}_{n-1}(x,u)\tilde{q}(x,u,y) & \text{for } n > 0, \end{cases} \tag{16}$$

If we introduce the variables $\xi(x,u) = \sum_{n\geq 0}\tilde{\pi}_n(x,u)$, then the problem of the discounted cost minimization can be represented as

$$\sum_{x\in E}\sum_{u\in A(x)} \tilde{c}(x,u)\xi(x,u) \Rightarrow \text{infinum} \qquad (17)$$

under the restrictions that arise from the equations (16)

$$\sum_{x\in E}\sum_{u\in A(x)} (\delta_{x,y} - \tilde{q}(x,u,y))\xi(x,u) = \mu(y), \quad y\in E, \qquad (18)$$

and

$$\sum_{x\in E}\sum_{u\in A_x} \xi(x,u) = 1 + \tilde{\tau}_1(s)(1+\tilde{\tau}_2(s)(1+\ldots)\ldots, \quad \xi(x,u)\geq 0. \qquad (19)$$

The connection of the Linear Program solution and optimal control policy to an appropriate DTCSRP is contained in the following theorem:

Theorem 10 (Wolf Danzig) *For DTSRP with any simple Markov strategy* $\delta = f^{\infty}$ *the variables* $\xi(x,u)$ *of the Linear Program (17–rest-lp-discopt-2) have the following property:* $\xi(x,u) > 0$ *only for one, say* $u^*(x)$ *value* $u\in A(x)$ *for any* $x\in E$, *being* $\xi(x,u) = 0$ *for all another* $u\in A(x)$ *and vice versa. To to any solution y of Linear Program with this property corresponds a simple Markov strategy* $f = \{f(x) = u^*(x), x\in E\}$.

Let us turn now to the problem of long run loss minimization. The Linear Programming Algorithm (LPA) for the criteria of long run expected loss minimization looks like a little bit complicated. For any simple Markov strategy $\delta = f^{\infty}$, denote by $\Pi(\delta)$ and $H(\delta)$ the limiting and the fundamental matrices of the embedded Markov chain (MC) $X = \{X_n\}$,

$$\Pi(\delta) = 1n \sum_{0\leq k\leq n-1} P^k(\delta) = [\pi(x,y;f)], H(\delta) = (I - P(\delta) + \Pi(\delta))^{-1} = [h(x,y;f)].$$

The LPA for long run expected loss minimization looks like the following

$$\sum_{x\in E}\sum_{u\in A(x)} c(x,u)\xi(x,u) \Rightarrow \text{infinum} \qquad (20)$$

with respect to variables

$$\xi(y,u) = \sum_{x\in E}\mu(x)\pi(x,y;\delta)d(u\mid y), \quad \text{and} \quad \eta(y,u) = \sum_{x\in E}\mu(x)h(x,y;\delta)d(u\mid y).$$

under the restrictions

$$\sum_{x\in E}\sum_{u\in A(x)} (\delta_{x,y} - p(x,u,y)\xi(x,u) = 0,$$

$$\sum_{x\in E}\sum_{u\in A(x)} \xi(x,u) + \sum_{x\in E}\sum_{u\in A(x)} (\delta_{x,y} - p(x,u,y)\eta(x,u) = \mu(y)\,\forall\, y\in E. \qquad (21)$$

Connections of the LPA solution to optimal policy is established with the help of Wolf and Danzig theorem.

Remark 4 *Additional attractive feature of the LPA consists in the possibility to use the dual linear programming algorithms for constructing of the optimality domains of some given simple control rules, as it will be shown in the section 8.2.*

4.3 Qualitative properties of optimal policies

The knowledge of some qualitative properties of optimal policies allows significantly simplify their calculation. For example, for monotone policies it is enough to find only the levels for switching the policy from one regime to another. The optimality principle validation allows to investigate some qualitative properties of optimal policies, for example its monotonicity. Because the optimal policy is the optimizer of the Bellman function $b(x,u)$,

$$f(x) = argmin\{b(x,u) : u \in A(x)\},$$

one can investigate the conditions for it, which allow to provide the monotonicity of the optimal policy $f = \{f(x) : x \in E\}$.

Because for CQS the states and decision sets E and A are usually multi-dimensional, the problem of the optimal policy monotonicity investigation consists in finding the conditions for monotonicity of the multidimensional optimization problem solution. These conditions usually have a form of sub- or super-modularity for the optimizing function [69]. For the special case of QS optimal control an appropriate condition has been proposed by **Rykov** in [53]. In order to explain the optimal policy monotonicity conditions, consider firstly the problem of a smooth function $b(x,a)$ minimization in an enough good domain $G \subset E \times A$.

$$\min\{b(x,a) : a \in A_x\} \tag{22}$$

The necessary condition of a local minimum is

$$b_a(x,a) = 0, \quad b_{aa}(x,a) > 0, \tag{23}$$

where the notations are used:

$$b_a(x,a) = \frac{\partial b(x,a)}{\partial a}, \quad b_{aa}(x,a) = \frac{\partial^2 b(x,a)}{(\partial a)^2}, \quad b_{ax}(x,a) = \frac{\partial^2 b(x,a)}{\partial a \partial x}.$$

Therefore the problem solution is a function $a = f(x)$ from the first equation of (23). Its derivative can be found from the equation

$$b_{ax}(x,a) + b_{aa}(x,a) f'(x) = 0,$$

or

$$f'(x) = -\frac{b_{ax}(x,a)}{b_{aa}(x,a)}.$$

Thus, the monotonicity condition of the smooth optimization problem can be formulated as a theorem

Theorem 11 *, If the minimizer $f(x)$ of a smooth function $b(x,a)$ minimization is monotone, then $b_{ax}(.,.)$ preserve the sign along this solution. At that it is non-positive for non-decreasing solution $f(x)$ and it is non-negative for non non-increasing solution. The inverse conversation also holds: if the second mixed derivative of smooth function $b(.,.)$ in enough good domain G preserves the sign, then the solution of the minimization problem (22) can be chosen monotone.*

It is necessary to note that because the solution of problem (22) may be **non-unique,** the **monotone choice** should be used.

For the discrete problem of minimization, appropriate conditions are almost similar, however instead of the second mixed derivative now the second difference of the Bellman function with respect to variables of state and control spaces is used. If it will be denoted the same as before by , $b(x,a)$ the conditions of the discrete monotonicity problem look like in the previous theorem. For detailed formulation and proof see [53].

It succeeds not too often to prove the monotonicity property of an optimal policy. Nevertheless, because for queueing systems monotonicity leads to the threshold type of strategy, and the attraction of this kind simple strategies and the simplicity of their realization leads to many investigations of CQS under strategies with monotone (threshold) policies. In a series of papers of Dudin and his colleagues [11] – [16], the systems with multi-threshold control policies for QS with Markov Arrival (MAP) and Batch Markov Arrival Processes (BMAP) have been investigated. The numerical investigation of optimal control of a multi-server system with heterogeneous servers has been done in the paper [17]. In **Rykov & Efrosinin** [57], the optimal control policy of systems on their lifetime has been found in the class of threshold policies.

In the next sections we demonstrate the above considered methods for some of CQS examples.

5 Arrival control

Control of arrivals to QS is a traditional problem in QT and has a long story. An excellent review of the earliest works about the problem one can find in **Stidhan Jr**. [65], to which we will follow in some parts of this section. Consider a model of arrival control $M/GI/1$, proposed in the example of the section 2.4.1. In this model the customers arrive accordingly to Poisson flow with intensity λ and are served during random times that are i.i.d. with CDF $B(t)$, mean value $m_B = \mu^{-1}$ and variance σ_B^2. The LRS usually includes: (a) reward R_n for each served customer with mean value m_R and (b) penalty $C_w(l)$ for l customers staying in the system per unit of time (holding or waiting cost). The control times are the arrival times, and the decisions consist in the admission or rejection of an arriving customer into the system.

The problem consists in admission of the arriving to the system customers (jobs) for service. There are different possibilities for control of arrivals to QS: (a) static, or (b) dynamic for (c) single-server, (d) multi-server, or (e) network queueing models.

5.1 Static flow control

5.1.1 Single-server static arrival control

In [65] a single-server static flow control model $M/GI/1$ has been reviewed. For this model the DM admits each job with probability p and rejects it with additional probability $1-p$. Therefore the real admitted flow of jobs has an intensity λp. Under this control rule for any control parameter p the system is a usual $M/GI/1$ QS with Poisson input intensity λp and generally distributed service time $B, B(t) = P\{B \le t\}$ with mean service time b and variance σ_b^2.

The problem consists in admission of jobs to the system in order to maximize the value of long run mean reward that due to proposed LRS has the form

$$\lambda p m_R - \lambda p C_w w(p) \quad \text{under the condition} \quad 0 \le \lambda p \le b, \tag{24}$$

where $w(p)$ is a stationary customer sojourn time in the system. Therefore, for the considered system one has

$$\lambda p w(p) = l(p) = \lambda p b + \frac{\lambda^2 p^2 (b^2 + \sigma_b^2)}{2(1-\lambda p b)}.$$

In terms of $\lambda b = \rho$ and $V_b = \sigma_b b^{-1}$ the problem (24) can be represented in the form

$$\lambda p m_R - \lambda p C_w w(p) = \lambda m_R p - c_w \left\lfloor \rho p + \frac{\rho^2 (1+V_b^2) p}{2(1-\rho p)} \right\rfloor \Rightarrow \max \tag{25}$$

under the condition $0 \le p \le \rho^{-1}$.

This is a simple optimization problem that could be simply solved numerically. Moreover, the last equation allows to obtain some interesting theoretical results proposed in [65].

5.1.2 Multi-server static arrival control

In this case the r-server system $M/G/r$ is considered. A Poisson input flow of customers with the intensity λ arrives into the system . The customers are served by servers during random times with general distribution $B_i(\cdot)$ with mean value b_i and variance σ_i^2 for i-th server $(i = \overline{1,r})$. In order to provide the stationary regime's existence, it is supposed that $\lambda\sum_{1\le i\le r} b_i < 1$ and the servers are ordered such that

$$b_1 \le b_2 \le \ldots \le b_r.$$

The (static) control rule is determined by the probability vector $p_1, p_2, \ldots, p_r$, where p_i is the probability to admit the arrived job to the i-th server.

In this case in [65] the problem of the static optimization is considered with respect to minimization of summary number of customers in the queue

$$\text{minimize} \sum_{1\le i\le r} l_i(p_i) \quad \text{subject to} \quad \sum_{1\le i\le r} p_1 = 1,\ 0 \le p_i < \rho_i^{-1},$$

where $l_i(p_i)$ is the stationary number of customers in the buffer of i-th server, and $\rho_i = \lambda p_i$.

5.2 Dynamic arrival control

The dynamic arrival control of this model also in [65] has been considered. Some different generalization of the dynamic flow control model in **Kitaev & Rykov** (1995) [31] has been done.

The Bellman equation for the problem has been proposed and it was shown that the Bellman function satisfies to the theorem 11, from which it follows that the optimal policy belongs to the class of monotone polices and therefore the optimal strategy has a threshold property. Thus, there exists the threshold level, say l^* such that all arriving customers, which find in te

queue more than l^* another customers should be rejected. The optimal threshold level l^* can be found by investigation of the loss functional for the system $M/GI/(1,l)$ with finite buffer.

6 Service mechanism control

Control of the system service mechanism is the most frequently considered area of control. There are many diverse settings of the problem. The distinguish consists in input and service mechanism as well as in LRS.

One of the earliest work devoted to the service mechanism control has been proposed by **Sabeti** [62]. He considers the $M/M/(1,n)$ system with controllable service rate without waiting cost C_w and proves the monotonicity property of the optimal policy. For the $M/GI/(1,n)$ system, this result has been generalized in [63]. For closed queueing systems with controllable service rate the analogous results have been done in [7, 8]. More detailed review of the earliest works on the topic one can find in [9, 66, 48]. In the framework of DTCSRP, the system has been considered in [31]. The numerical investigation of the optimal service rate policies has been proposed in [73].

The hysteresis phenomena of optimal policy arise when the switching costs are taken into account. The $M/M/(1,n)$ system, with $n \le \infty$, with controllable both input and service rate and a switching cost in [42] has been considered. As a control parameter the pair $a = (\lambda, \mu)$ is considered and the total order of the control parameters is supposed to be: $a \le a'$ iff both $\lambda \le \lambda', \mu \le \mu'$. The control epochs for the model are both arrival and service completion times. The switching cost $C_s(a, a')$ has been included into the model. The conditions that provide optimality of monotone hysteresis policy are studied. One step optimization problem of the Bellman function for the same model was studied in [27]. The optimality of monotone hysteresis policy under a little bit more general conditions is proved in [50]. The same mode under more general conditions in the framework of DTCSRP has been considered in [31]. The optimality of monotone hysteresis policy has been established there.

Remark 5 *This system also can be considered as a complex system, in which two parameters input and service rates are controllable.*

The oore general case of a $M/GI/(1,n)$ system with controllable service rate, however without switching cost] has been studied in [73]. Customers arrive with Poisson flow of intensity λ and are served with one of finite numbers of rates (service modes) $a \in A = \{a_k : k = 0,1,2,\ldots r\}$, such that the service time has a CDF

$$B_k(t) = B(t;a_k) = B(a_k t), k = \overline{0,r},$$

where $B(t)$ is a given CDF with finite mean m_B. As the control times (epochs) the service beginning times are supposed. At that the service delay it is convenient to consider as service mode with rate $a_0 = 0$. The control consists in choice in the control epochs, one of the service modes in order to maximize the discounted or long run expected reward or minimize appropriated loses. The monotonicity of the optimal policy has been studied numerically.

For multi-server systems the control rules consist usually in choice of servers for service. This setting is closed to the problems of the system structure control and will be considered in the next section.

7 System structure control

7.1 Servers switching on and off problem

The problems of system structure control are very closed to the service mechanism control and consist in switching on and off servers depending on the system state with the goal to optimize some given functional. Such type of systems have been considered by **Heyman** [24] and **Deb** [10].

Consider the system $M/GI/1$ with controllable system structure, which consists in the possibility to switch on and off the server. Customers arrive accordingly to Poison flow of the intensity λ and are served during random service time with general CDF $B(t)$. Reward structure includes:

- penalties C_1 and C_0 for switching on and off the server;
- penalty c_s per unit of server operating time (using cost); and
- penalty $c_w(i)$ for waiting i customers per unit of time.

Control epochs are both the arrival and service completion times and the control consist in switching on and off the server with the goal long run average losses minimization.

The monotonicity of the optimal policy has been proved that leads to its threshold property.

7.2 Slow server problem

So called slow server problem also can be considered as a system structure control problem. The problem is the following. For the $M/M/r$ QS with Poisson input intensity λ and r heterogeneous servers with service intensities $\mu_i\ (i = \overline{1,r})$, ordered such that

$$\mu_1 > \mu_2 > \ldots > \mu_r \tag{26}$$

the conservative discipline, for which any idle server should be used, will be not expedient for example, with respect to minimization of the mean sojourn (holding) time (or mean number) of customers in the system. Therefore, the problem of optimal servers using arise.

The problem firstly has been considered for two servers in the static regime by **B. Krishnamoorthy** in [39], then also for two servers in the dynamic regime it is studied by **Hajek, Lin&Kumar and Koole** in [23, 41, 37]. It was shown that the optimal rule for using of the slow server has a threshold character and the optimal level of queue length q^* for using the slow server is that for which

$$q^* = \left[\mu_1 \mu_2^{-1}\right] + 1.$$

For the system with several heterogeneous servers the analogous rule has been proved by **Rykov** in [54], see also [53]. For the problem investigation consider a Markov decision process with set of space states $E = \{x = (q, d_1, \ldots d_r)\}$, where q is the queue length and $d_k\ (k = \overline{1, r})$ is an indicator of the k-th server state: $d_k = 0$ if the k th server free, and $d_k = 1$ if it is busy. A system of sets

$$A(x) = \{\text{set of indexes of free serves in the state } x\}$$

will be used as action sets.

It was proved that the optimal servers using rule also have the threshold structure. For any system state $x = (x_1, \ldots, x_r)$, there exists a threshold level $q^*(x)$ such that the new server should be used iff only the queue length q is greater than $q^*(x), q > q^*(x)$. At that the server of the most intensity among the free servers should be switched on. However the levels of the servers switching on and off should be calculated numerically, and one example of this calculation one can be found in [17].

For the generalized setting of the problem, which include also the waiting cost c_0 for any customer waiting in queue per unit of time and the using servers cots $c_k\ (k = \overline{1, r})$ for k-th server using per unit of time, the analogous optimal service rule has been found in [56, 55]. In this case the monotonicity of the optimal policy preserves if for servers ordering accordingly to (26) also the following condition holds:

$$c_1^{-1}\mu_1 > c_2^{-1}\mu_2 > \ldots > c_r^{-1}\mu_r. \tag{27}$$

It should be noted that the optimal rule does not depend on the input intensity λ and waiting cost c_0. Also the threshold levels $q^*(x)$ should be found numerically.

8 Service discipline control

One of the first investigations on CQS does not touch the delicate problems regarding the MDP and deals only with the optimization problems in the framework of simple priority systems.

8.1 Priority optimization

The problem of priority system optimization can be considered as the problem of service discipline optimization in small class of decision rules that does not depend on the process trajectory observation and on the system state.

One of the first papers, devoted to QS optimal control, was the problem of optimal priority assignment [2]. The problem is the following. Consider a single server $M_r/GI_r/1$ queuing system with r independent Poison inputs of intensity $\lambda_k, (k = \overline{1, r})$. Random service time B_k of k-th type customers has CDF $B_k(t), (k = \overline{1, r})$ with mean value $b_k = \mu_k^{-1}$ and its waiting unit of time penalties with c_k units. The problem consists in choice of the service priorities in order to minimize the expected long run loses

$$\lim_{t\to\infty} 1tE\int_0^t \sum_{1\le k\le r} c_k L_k(s)ds = \sum_{1\le k\le r} c_k l_k,$$

where $L_k(t)$ is the k-th type of customers queueing length in time t, l_k its stationary mean value, w_k is the stationary waiting time of the k-th type customers. The priority systems has been investigated in detail by **Klimov** [32, 35], **Gnedenko** and others [19], **Jaiswal** [21] and others, where the steady state system characteristics have been calculated. Based on the stationary characteristics of the priority systems, the solution has been proposed by **Bronstein and Rykov** in [2] (see also [6]) with the help of a simple method that many years after got the name *perturbation method* [3], and the rule now known as the $c\mu$-rule. It is the following. The priority should be organized in a such manner that

$$b_1 c_1 \le b_2 c_2 \le \ldots \le b_r c_r$$

or in the order

$$c_1\mu_1 \ge c_2\mu_2 \ge \ldots \ge c_r\mu_r,$$

which gives the name $c\mu$ *-rule* to this discipline.

Further, it was shown [59] that this discipline is also optimal inside essentially awide class of so called dynamic priorities, where the idea is the following: the whole set of states E is divided into classes $E_i,(i=0,1,\ldots r)$, in which the decision $a=i$ in the decision epoch should be taken. This class of decisions in fact represents a class of Markov decision strategies, and therefore the results show that the priority is the optimal discipline in the class of Markov decisions. These results have been developed further for system with dynamic preemptive resume priorities in [70, 71, 72] and others.

8.2 System with feedback

For more general settings the problem of service discipline control has been proposed by **Klimov** [33] and **Kitaev&Rykov** [30]. Remind more general system $M_r/GI_F 1/\delta$ with feedback, proposed in [30]: r types of customers arrive into the single-server system from Poisson flows with intensities λ_i for i-th type of customers (i-customer); the customers are served with i.i.d. service times distributed accordingly to the CDF $B_i(t)$ with mean value b_i. Each served i-customer leaves the system with probability $q_i(0)$, or generates the set of the same type of customers $(n_1,\ldots,n_r)$ with probability $q_i(n_1,\ldots,n_r)$ that should be served also in the system. The LRS includes only a linear waiting (holding) cost $C_w(l,i)=c_i l_i$ for the i-th type of customer unit of time spent in the system.

The problem consists in service discipline construction that minimize the long run expected loss for the system exploitation. In [31] the existence of a simple Markov optimal strategy has been proved that gave the possibility to consider the problem in the framework of dynamic priority setting. As the result, the problem has been reduced to the linear program

$$\sum_{1\le i,j\le r} b_i c_j x_{ij} \Rightarrow \min \tag{28}$$

with respect to x_{ij}, under restrictions

$$\sum_{1\le i\le r} (a_{ij}x_{ik} + (a_{ik}x_{ij}) = \gamma_{jk} \quad (j,k=\overline{1,r}). \tag{29}$$

Here $x_{ij} = E_i L_j$ is the stationary mean value of j-th type of customers in the i-th type decision epochs, $a_{ij} = \delta_{ij} - b_i\lambda_j - q_{ij}$, and q_{ij} is mean number of j-th type of customers generated by the i-th one. The model constant $\gamma_{jk} = \gamma_{kj} > 0$ does not depend on the control rule.

The linear problem (28, 29) solution is attained as usual in the extreme points of manifold, determined by the restrictions (29). This statement allows to show that the optimal policy determines the priority discipline. The dual linear program has been used in order to find the optimal priority rule. It is not enough simple and it is determined with an algorithm, which one can find in [30, 31].

Remark 6 *It is well known the connection between iteration algorithm of optimal policy construction and some linear program. In this problem, using a dual linear program allows to construct the optimal control policy. The possibilities of the dual linear program does not fully used up today. Really using the dual linear program allows to search parameters of the model, in which some given simple rule (or service discipline) will be optimal (see also remarks in the Conclusion). This approach has not yet exhausted itself and should be developed in future.*

The further investigation of the priority queues has been done by **Miscoy** and others [43], where especially numerical methods for generalized Kendall equation has been proposed. It also can be used as a method for optimal priority construction for the priority queues with switching times.

8.3 Closed queuing systems

The optimality of the priority rule for closed queueing system has been proved by **Koole** in [38]. A closed system $< M_r/M/1/\delta >$ with r sources, one server and controllable service discipline is considered. Sources have exponentially distributed life and repair times with parameters $\lambda_i \mu_i \ (i = \overline{1,r})$.. The system includes a waiting (holding) cost $c_k \ (k = \overline{1,r})$ for every unit of time that the k-th source is not functioning. The preemptive service discipline that minimizes the total average waiting cost is the goal of investigation.

The problem is formulated in the framework of the decision Markov model. In [38] it is proved that if the sources are ordered in such a manner that the conditions hold

$$\lambda_1 \leq \ldots \leq \lambda_r \quad \text{and} \quad c_1\mu_1 \geq \ldots \geq c_r\mu_r \tag{30}$$

then the priority rule: *serve the source with minimal index* is optimal.

In the case when all $c_k = 1$ this result shows that this rule is also optimal for minimizing the average queue length. For the case when $\lambda_k = \lambda$ for all $k = \overline{1,r}$, the results show that the $c\mu$-rule also holds for the closed system with homogeneous sources. It is possible to see that the results also coincide with the analogous results for the open queueing system with heterogeneous servers considered in subsection 7.2.

9 Conclusion

To model CQS in the paper the DTCSRP is used. Of course, the assumption about the possibility to change the control in special control epochs is a some restriction. However, this restriction is natural for many practical situations. Moreover, for MDP due to the memoryless property of exponential distribution the epochs of any states changing are the natural decision times. However, there are some examples (and works), where the choose of control epochs is a subject of investigation. So in [4, 72, 58] the problem of optimal service interrupting has been investigated.

Another approach that could be mentioned as a subject for further investigation consists of construction of the domains into system parameters set such that some natural control rule would be optimal if the system parameters lie in the domain (*domains of optimality*). Besides the practical importance of such approach, it allows in some cases to find an optimal policy for all possible values of the system parameters (example of such approach has been proposed in [30, 31].

Further development of the above approach for optimal control of QS is the following. Because for CQS the states and decision sets E and A are usually multi-dimensional, the problem of the optimal rules qualitative properties investigation should be stated as follows: Is it possible to introduce into sets E and A such a partial order, in which the optimal policy would be monotone?

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Percentiles Confidence Intervals Building Using Bootstrap-Modeling: Application for High-Tech Production Quality Control

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Abstract

We propose a method for building confidence intervals for percentiles with application to quality control of the random properties of composite polymer materials strength. The basises (i.e. lower confidence limits for percentiles) are analyzed. The new developed method employs the statistical bootstrap modeling method. For the explanation of bootstrap procedure, the problem of building confidence interval for the mean value for the random values of composite material strength is presented. The result is compared with classical one. Due to bootstrap, it is possible to overcome some problems of classical statistical constraints. There is no need in postulating some type of distribution (normal or Weibull). Some real problems are presented with the comparison of the results by standard procedures and by the new method. It is shown that the agreement is satisfactory.

Keywords: percentile, bootstrap-modeling, composite material, strength

I. Introduction

During the certification of high-tech products, which include composites made by autoclave molding of prepreg HexPly brand, it is necessary to conduct the experiment to determine some strength properties. The values being investigated are: 1) the ultimate strength (σ_B, MPa); 2) the modulus of elasticity in tension (E, GPa); 3) strength interlaminar shear at normal temperature (τ20, MPa); 4) strength interlaminar shear at elevated temperature (τ120, MPa). Since the experimental values are random, it is necessary to conduct the statistical analysis. For this purpose 1) the percentiles γ% are estimated. (These values are almost the same as quintiles q, γ%=q/100); 2) for 1% and 10% percentiles the α=95% confidence intervals are estimated. The lower boundaries of those confidence intervals are called basises: namely A- basis for 1% percentile and B- basis for 10% percentile. Although the percentiles are themselves at some extend the interval characteristics, the necessity to build the confidence intervals for them makes the researchers face new challenges.

Let us dwell on the characteristics of percentiles. The percentile γ % is the characteristics of the sample, which express the ranges of the elements in the array as the numbers from 1 to 100, and indicate what percentage of the values are below a certain level. More generally, define the quantile q= γ %/100 is been used. Mathematically, the quantile is determined as follows. Suppose there are independent and identically distributed random variables, for which there exists a distribution function F with density distribution f=F′. Define q-th quantile of the population, such that $F^{-1}(q) \equiv \inf\{x \in R: F(x) \geq q\}$. Quantile q=0.1 (or, equivalently, the γ%=10th percentile) indicates that 10% of all values are below this level. Quantile (percentile) is a random value, which is determined by the sample, so it requires the assessment of their variability.

To calculate confidence intervals for the values of the percentiles obtained for the random sample the basises are used. Two types of basises: A-basis and B-basis [1] are investigated. They are the lover limits with confidence 95% for the percentiles 1% and 10% respectively. Before now it was obligatory to choose the appropriate type of distribution for solving this problem. For each type of distribution the complex dependencies are developed. For example, for the calculation basises with the assumption of normal distribution of a random variable in [1] the formulae are proposed:

$$B = \bar{x} - k_B s, \qquad (1)$$
$$A = \bar{x} - k_A s,$$

where x is the average; s – is the mean square deviation and k_B и $k_A$are the coefficients of tolerance appropriate to the sample size. The values of these coefficients are given in tables or can be calculated with an error of not more than 0.2% by the following formulas:

$$k_B = 1{,}282 + \exp\left(0{,}958 - 0{,}520\ln(n) + \frac{3{,}19}{n}\right), \qquad (2)$$
$$k_A = 2{,}326 + \exp\left(1{,}340 - 0{,}522\ln(n) + \frac{3{,}87}{n}\right).$$

where n is a sample size.

II. Methods

In the present work as an alternative for methods [1] a method of constructing basises with the use of statistical bootstrap is proposed.

We will briefly review the description of the method of the statistical bootstrap. It was introduced in 1977, by the mathematician Bradley Efron [2]. The statistical bootstrap is a way of obtaining robust estimates of standard errors and confidence intervals, but not only this. It is being used to evaluate the variability of different characteristics. The method is based on the repeated simulation of the so-called bootstrap samples which constructed on the basis of the original sample with the replacement and is based on intensive use of computers. The number of bootstrap samples (denoted by R) should be large: in the present study were used R=100 and R= 1000. At the nowadays computers speed those number is not a problem. It will be only the fractions of a second calculations. The size of each bootstrap sample corresponds to the size of source sample, namely n, and the elements of the bootstrap samples are formed from the elements of the original sample, this is a random choice with replacement. For statistics, for which the exact mathematical expressions of variability exist, a number of studies have shown satisfactory agreement of the estimates based on bootstrap with the classical estimates (see also Appendix). To date, already has significant experience of applying statistical bootstrap to engineering problems, see for example [3]. On the other hand, mathematicians warn of excessive enthusiasm to this method: where the statistic theory is well developed and where the methods of data analysis in some sense close to optimal were found, the bootstrap has nothing to do.

In our case, for such statistics as, for example, γ% percentile, the mathematical expressions for the variance are complex and their optimality is not strictly proven. In this regard, it is interesting to compare the interval estimates of the bootstrap γ% percentile with ones, constructed by the used nowadays methods and to consider the possibility of new method introduction in the practice of engineering design. The evaluation of confidence intervals for quantiles using the bootstrap simulation was considered also in [4,5]. In [4] smoothing method confidence intervals for quantiles was proposed, in particular, nuclear assessment. In [5] the accuracy of bootstrap estimates of confidence intervals of quantiles, depending on the distribution of random variables was investigated.

Let's take for example h-th delivery of the s-th random value. In the investigated pool h=7,8…11 (total 5 deliveries) and s=1,2,3,4 (total 4 characteristics). We accept R=1000 bootstrap samples which seems to be sufficient. Separately for each h and s we perform R=1000 random choices with replacement. There is a good function for it in [6] – "sample" function. It be metioned that it is necessary to include parameter "replace=true" among the others, or else the choice will be without replacement, which contradict the main bootstrap rule. [6] provide several algorithms for estimation of quantiles. We employed the algorithm i=3, which is taken by default. Sample quantiles of the algorithm type i are defined by:

$$Q[i](p) = (1 - \gamma)\, x[j] + \gamma\, x[j+1] \qquad (3),$$

where $1 \le i \le 9$, $(j-m)/n \le p < (j-m+1)/n$, x[j] is the jth order statistic, n is the sample size, the value of γ is a function of j = floor(np + m) and g = np + m - j, and m is a constant determined by the sample quantile type. For i=3 $\gamma = 0$ if g = 0 and j is even, and 1 otherwise. Type 3 is discontinuous sample quantile, as well as the types i=1,2.

After the algorithm of calculating quantiles has been chosen, the bootstrap modeling is performed. For the building confidence limits the random values of bootstrap samples are arranged into the variational series. The members of variational series with indexes LOW and UP form the α% confidence interval for the statistic of interest. Here are the expressions:

$$\text{LOW=integer part} \left[\frac{1-\alpha.100}{2} \text{R}\right] \qquad (4)$$

$$\text{UP=integer part} \left[\frac{1+\alpha.100}{2} \text{R}\right] \qquad (5)$$

For building basises only the low limits are necessary. If number of bootstrap samples is R=1000 and confidence level as it is defined for basises α=95%, the lower index is: LOW=25.

III. Results

For the delivery #8 of carbon fiber specimens for the random value of ultimate strength σ_B, Mpa in Fig.1 the histogram of q=0.1 quantile (or that is the same γ=10% percentile) is shown.

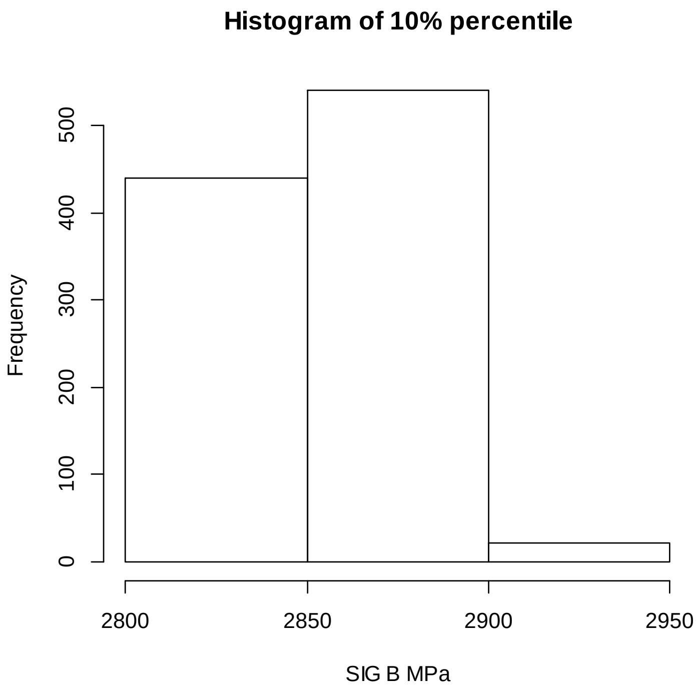

Figure 1: *Histogram of percentiles*

It can be seen, that the distribution is far from being normal, so we have all the reasons to employ the bootstrap, which is free from requirements of the normal distribution. In the Figure 2 the cumulative distribution function of the bootstrap estimations of percentile ϒ=10% is shown ("ecdf" function in [6]). The random value of ultimate strength σ_B is shown in MPa.

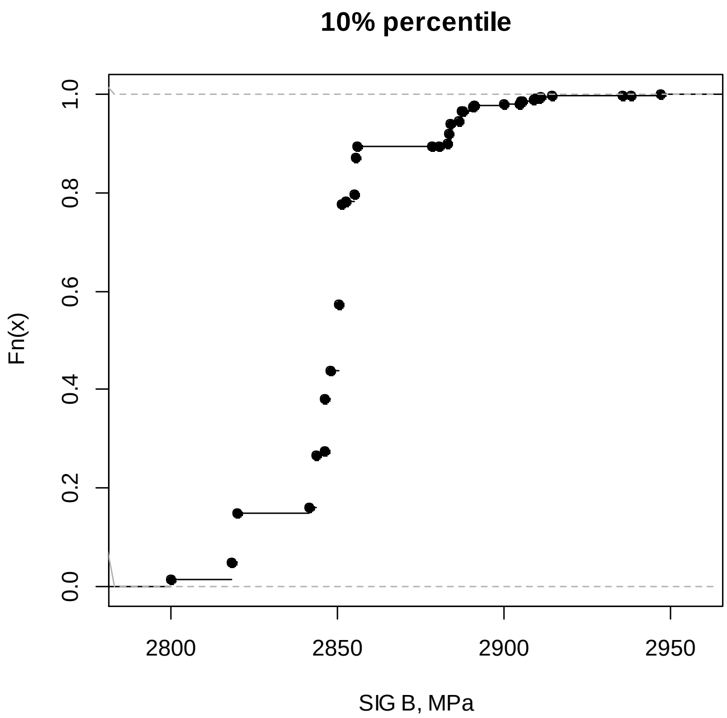

Figure 2: *Percentiles cumulative distribution*

Following the rules for building the bootstrap confidence interval (4,5) we estimate the indexes of variational series: LOW=25; UP=975. For the basises we need, actually, only the lower index. For deliveries the data of basises are presented in Table 3.

Table 3: *Ultimate strength σ_B, МПа for some deliveries of HexPlay*

Delivery index	Mean	Standard deviation	10% percentile	B-basis	
				bootstrap	[2]
7	3013.3	142.6	2817.0	2789	2693
8	2962.5	84.9	2850.5	2818	2780
9	2719.3	154.6	2512.2	2580	2361
10	2848.9	172.3	2630.2	2580	2463
11	2729.7	134.7	2579.3	2501	2403

IV. Discussion

Because of standard requirements in industry it is important to estimate basises of some statistical characteristics. The proposed method was applied to the quality control of the strength characteristics of carbon fiber composite specimens. The comparison has been made with the results, obtained by nowadays applied methods. The agreement is shown to be satisfactory. On the other hand the new developed method possesses some advantages, for example, it is free from distribution. It all refers to B-basises (the lower 95% limit of 10% percentile). As for A-basises (the lower 95% limit of 1% percentile) there are some problems. They are due to very small samples size (up to n=30). For building A-basises for small samples it might be necessary to develop new methods, based on numerical imitation.

Appendix
Example of applying bootstrap for average value confidence intervals building.

To explain the method of construction confidence intervals using bootstrap-modeling and to make reader more familiar with bootstrap procedure, we employ the problem of building confidence intervals for the average value of the normal set. This problem was chosen because in has a good classical decision. As the object of investigation the shear strength under room temperature tau20, MPa of the composite samples has been taken. The delivery number was 7, because for this set a good agreement with the normal distribution was obtained [7]. Here the number of bootstrap trials was chosen as R=100.

The initial set is presented in Table A1.

Table A1: *Shear strength under room temperature τ20 , MPa, delivery number 7, initial sample*

Order index	1	2	3	4	5	6	7	8	9	10
τ20 , МПа	86.1	89.7	97.1	95.9	93.7	94.4	90.6	93.6	96.4	88.8
Order index	11	12	13	14	15	16	17	18	19	20
τ20, МПа	101	89.3	85.9	92.8	94.3	91.5	90.6	91.5	92.8	90.1
Order index	21	22	23	24	25	26	27	28	29	30
τ20, МПа	97.2	94.9	91.5	93.8	96	95.9	89.3	99.8	95.9	107.8

Initial sample parameters (Table A1)

$$\text{Mean value}(\tau 20) = 93.61 \text{ MPa} \quad (6)$$

Standard deviation (τ20) = 4.502 MPa

According to the bootstrap rules R bootstrap samples are simulated. In Table A2 an example of simulated k-th bootstrap sample is shown, constructed based on the source sample shown in Table A1:

Table A2: *k – th bootstrap sample (example) modeled on the base of initial sample (Table A1)*

	1	2	3	4	5	6	7	8	9	10
τ20, MPa	107.8	93.6	90.1	99.8	92.8	97.1	89.3	90.5	91.5	89.3
Order index	11	12	13	14	15	16	17	18	19	20
τ20, MPa	107.8	96.4	93.6	94.4	93.8	90.1	91.5	89.3	101.0	92.8
Order index	21	22	23	24	25	26	27	28	29	30
τ20, MPa	89.3	88.8	89.7	101.0	97.2	93.7	93.8	89.3	89.7	93.8

It can be seen that some random values are repeated in k-th bootstrap sample (in Table A2, for example, the elements 1 and 11 are the same: τ20=107.8 MPa). Some elements of the Table A1 was not included even once in the k –th sample (for example, item number 1: τ20=86.1 MPa), but it might be included in the bootstrap sample number k+1.

k-th sample parameters (Table A2):

$$\text{Mean value}(\tau 20) = 93.99 \text{ MPa} \quad (7)$$

Standard deviation (τ20) = 5.137 MPa

It is seen that the characteristics at the initial sample (6) and k-th sample (7) differ, albeit only slightly. For each k=1,2...R bootstrap sample it is necessary determine the characteristics of interest, namely average. The set of estimations characterize the variability of the point estimate.

The histogram shape looks like the shape of the normal distribution. The average value for the bootstrap samples is the bootstrap mean(τ20)=93.603 MPa. This value is close to the average for the original sample (6). For building the confidence intervals with probability α=90% for the average the formulae (4,5) are applied. For the given parameters α=90% and R=100 the values in indexes are: LOW=3 and UP=97. The standard statistical estimation [8] for the same purpose provides very close values. The confidence limits built by two methods are shown in Table A3.

Table A3: α=95% *confidence limits for the mean value of shear strength value [MPa]*

LOW 90%, Student's	LOW 90%, bootstrap	UP 90%, Student's	UP 90%, bootstrap
92.22	92.38	95.0	95.10

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A Queueing Network Model for Delay and Throughput Analysis in Multi-hop Wireless Ad Hoc Networks

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Abstract

In this paper we present a queueing network model for computing average end-to-end delay, and maximum throughput that can be attained in random access multi-hop wireless ad hoc networks with stationary nodes under two popular contention resolution schemes namely, Binary Exponential Back-off (BEB) and Double Increment Double Decrement (DIDD) rules. This model takes into consider some realistic features of the system like (i) the generation of different classes of packets at nodes, and (ii) the dependence of the transmission time and transmission probability on the distance between the transmitter and receiver. Probability distributions and the associated measures of characteristics of the time spent by a packet at arbitrary node are analytically derived by using phase type random variate theory, which in turn are used for the computation of average end-to-end delay and maximum achievable throughput. Theoretical results are numerically illustrated.

Keywords: queue, network, phase type distribution, Markov chain

1 Introduction

A multi-hop wireless ad hoc network is a collection of nodes that communicate with each other without any established infrastructure or centralized control. Due to the limited transmission range of wireless network interfaces, multiple network hops may be needed for one node to exchange data with another across the network. Thus, in this network, the packets may have to be forwarded by several intermediate nodes before they reach their destinations and therefore each node operates not only as a host but also as a router. Hence, each node may act as a source, destination or relay. For a detailed description of some of the situations where ad hoc networks can be used, refer [7].

The wireless medium is shared and scarce. Multiple stations may want to transmit data frames at the same time over the same channel. So, multiple access protocols are needed to coordinate the transmissions. Since ad hoc networks lack infrastructure and centralized control, these protocols should be distributed. IEEE 802.11 protocol has been set up for fixing international standards for Wireless Local Area Networks (WLAN's). In the 802.11 protocol, the fundamental mechanism to access the medium is known as the distributed co-ordination function (DCF). DCF is a random access scheme based on the carrier sense multiple access with collision avoidance (CSMA/CA) protocol.

According to DCF basic access mechanism, each station with a packet, ready for transmission, monitors the channel activity and if the channel is found to be idle for a pre-determined period called distributed inter-frame space (DIFS), transmits the packet. Otherwise, if the channel is sensed busy, the station initializes its back-off timer and defers transmission for a randomly selected back-off period to minimize the collisions. At each time point at which the channel is monitored, the back-off counter is decremented when the medium is idle, and is frozen when the medium is sensed busy. The timer resumes only after the medium has been idle for a period longer than DIFS. The station

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whose back-off timer expires first begins transmission and the other stations freeze their timers and defer transmission. Once the current transmission gets completed, the back-off process repeats again and the remaining stations reactivate their back-off timers. Upon the successful reception of a packet, the destination sends back an immediate acknowledgement (ACK) after a time interval equal to short inter-frame space (SIFS). In addition to the basic access mechanism, another optional method called ready-to-send/clear-to-send (RTS/CTS) mechanism is also adopted under DCF. According to this, a node operating in RTS/CTS mode, before transmission, reserves the channel by sending a special ready-to-send short frame and the destination node acknowledges the receipt of the same by sending back a clear-to send frame. After this, the normal packet transmission and ACK response occur. Since collision may occur only on the RTS frame, and it is detected by the lack of CTS response, the RTS/CTS mechanism allows to increase the system performance by reducing the duration of a collision when lengthy messages are transmitted. More importantly, to some extent, the RTS/CTS mechanism adopted in the 802.11 protocol is useful to address the so called hidden terminal problem, which was first mentioned by [14]. For more details on hidden node problem, also refer [6].

DCF employs a contention resolution method namely, binary exponential back-off (BEB) rule, to minimize the probability of collisions due to multiple simultaneous transmissions. Under this rule, if a packet is ready for transmission from a node for the first time, contention window size is chosen as W and according to the collision avoidance protocol procedures, a random value for its back-off counter is uniformly selected from 0, 1,2, $W-1$. If the packet meets with a collision in that attempt, the contention window size will be set as $W_1 = 2W$ and a value for back-off counter is selected uniformly from 0, 1,2, $W_1 - 1$ and if it is further included in a collision on its next attempt, the contention window size will be doubled again and this will continue up to a maximum of m collisions. After m unsuccessful attempts, if it is again met with a collision, the contention window size will be fixed as $W_m = 2^m W$. If an attempt results in successful transmission, the contention window size for that node will be reset as W. Hence the minimum contention window size $CW_{min} = W$, and the maximum contention window size $CW_{max} = 2^m W$.

Apart from the BEB scheme, many researchers have proposed different schemes to fix the contention window size in order to enhance the performance of wireless LANs. Of which the DIDD (Double Increment Double Decrement) scheme proposed by [8] deserves special mention. Under this scheme, if a packet meets with collision while it is being transmitted, the contention window size for the next transmission will be doubled as in the case of BEB rule, whereas after a successful transmission it will be halved unlike under BEB scheme where the contention window size is reset to W under the same scenario. For more about other schemes and their detailed performance analysis, refer [8] and the references therein.

Several researchers have attempted to analyse the throughput and packet delay occurring in communication networks. [2] proposed analytical models to learn the IEEE 802.11 protocol under unsaturated traffic conditions for multihop networks. [4] made an attempt to characterize the average end-to-end delay and maximum achievable per-node throughput in random access multi-hop wireless ad hoc network with stationary nodes. They modelled random access multi-hop wireless networks as open $G/G/1$ queueing networks and used the diffusion approximation (see [10]) to derive closed form expressions for the average end-to-end delay. However, none of these aforementioned references has addressed the important problem of finding the probability distribution of the end-to-end delay experienced by the packets in the network.

This article is in the same line with [4]; however, a more detailed and comprehensive delay analysis has been carried out for a multi-hop wireless ad hoc network with stationary nodes under more general and realistic assumptions. More importantly, probability distributions of the time spent by a packet at an arbitrary node from the epoch at which it is ready for transmission till it is successfully transmitted have been derived under both BEB and DIDD rules, as discrete Phase-Type(PH) distributions. Analytical representation of these distributions enable us to compute some important statistical measures like variance and coefficient of variation of the packet waiting time at a node, which in turn could be used in computing the mean total time spent by a packet in the system before it reaches its destination. For more details on PH distributions and their characteristics, see [12] and [11]. Following are some of the highlights of this paper. (i) The analysis aims to capture several salient aspects of wireless networks like the relationship between the probability of successful transmission between two nodes and the distance between them, interferences caused by hidden nodes, generation of different classes of packets at nodes based on the number of hops to be visited etc. (ii) PH- representation of single hop delay under both BEB and DIDD back-off schemes are derived explicitly and the important statistical measures like mean, variance, and coefficient of variation of the single hop delay are computed analytically and are presented in compact form. (iii) By using the diffusion approximation, the important measures like average queue size and mean waiting time of a packet at an arbitrary node are computed.

Though the present paper does not consider the routing algorithms, mobility models, and path length of source-destination pair that are currently applied in ad hoc scenarios, it renders a concrete analytic approach which may be helpful to get approximate solutions to some important measures that decide the performance of ad hoc models. Even though it does not take into account all the features of a practical ad hoc model, it may be treated as an analytical model which help us to get some insight into the performance behavior of a system governed by probabilistic laws. A detailed description of our model is as follows.

2 Methods

We consider a wireless ad hoc network with N nodes that are assumed to be uniformly distributed inside a compact set $W \subset R^3$ of unit volume. Each node has an equal transmission range R. That is, if a node transmits a packet, it can reach at another node which is at a distance of maximum R units from the source node. Let r_{ij} be the distance between nodes i and j. Nodes i and j are called as *neighbours* if they can directly communicate with each other, that is if $r_{ij} \leq R$. The set of neighbours of node i is denoted by $N(i)$ and it is assumed that all neighbours of a node lie inside a sphere of volume $v = \frac{4}{3}\pi R^3 (< 1)$ centered at that node. Since the nodes are distributed uniformly, the number of neighbours is binomially distributed with mean $(N-1)v$. Being an ad hoc network, each node in the network can be a source, destination, or relay of packets. Depending on the number of hops to be traversed by a packet, we classify the packets into M categories. A packet is said to be of class l , $1 \leq l \leq M$, if it has to visit l nodes before reaching its destination. A packet generated at an arbitrary node is assumed to be class l, $1 \leq l \leq M$ with probability c_l, where $\sum_{l=1}^{M} c_l = 1$. Packets are generated at nodes in the network as a renewal process with rate λ_e and coefficient of variation C_E . It is to be noted that, as per our assumption, the process by which an arbitrary node generates class l packets is a renewal process with rate $\frac{\lambda_e c_l}{N}$.

Computation of forwarding probability

Let q_{ij} be the probability that a packet at node i (either generated at i or received from some other node) is forwarded to node j. When node i transmits a packet, any of its neighbours can receive it; however, we assume that the probability that it reaches at a neighbouring node depends on how far the receiving node is from node i. More precisely, the probability that the packet reaches at the node which is the kth neighbour of i is assumed to be inversely proportional to the average distance between node i and its kth neighbour.

By equation (13) in [13] , the average distance between a node and its kth neighbour,

$$E(R_k) = R\frac{\Gamma(k+1/3)\Gamma(L+1)}{\Gamma(k)\Gamma(L+4/3)}, \quad (1)$$

where

$$\Gamma(m+1/3) = \Gamma(1/3)\frac{(3m-2)!^{(3)}}{3^m},$$

L is the largest integer less than or equal to $(N-1)v$, $n!^{(l)}$ is the lth multifactorial of n, and $\Gamma(1/3) \approx 2.6789385347$.

We have

$$q_{ij} = P\{i \to j\} = P\{i \to j | j \in N(i)\} * P\{j \in N(i)\}.$$

By conditioning on the number of neighbours of i, we get

$$q_{ij} = \sum_{p=1}^{N-1} P\{i \to j | j \in N(i), |N(i)| = p\} * P\{|N(i)| = p | j \in N(i)\} * P\{j \in N(i)\}. \quad (2)$$

Now

$$P\{i \to j | j \in N(i), |N(i)| = p\} = \sum_{k=1}^{p} P\{i \to j || N(i)| = p, j \quad is\ the \quad k\ th\ neighbour\ of\ i\} *$$

$$P\{j \quad is\ the \quad k\ th\ neighbour\ of\ i\ || N(i)| = p\}. \quad (3)$$

We have

$$P\{i \to j || N(i)| = p, j \quad is\ the \quad k\ th\ neighbour\ of\ i\} = P\{the\ packet\ is\ not\ absorbed\ at\ i\} *$$

$$\frac{E_p}{E(R_k || N(i)| = p)},$$

where $E(R_k || N(i)| = p)$ is obtained from eqn(1) by replacing L by p, and

$$E_p = \left(\sum_{l=1}^{p} \frac{1}{E(R_l || N(i)| = p)}\right)^{-1}$$

is the normalization constant.

Thus

$$P\{i \to j || N(i)| = p, j \quad is\ the \quad k\ th\ neighbour\ of\ i\} = [1 - P\{the\ final\ destination\ of\ the\ packet\ is\ i\}] * U_p(k),$$

where

$$U_p(k) = \frac{E_p}{E(R_k || N(i)| = p)}.$$

So

$$P\{i \to j || N(i)| = p, j \quad is\ the \quad k\ th\ neighbour\ of\ i\}$$

$$= [1 - \sum_{l=1}^{M} P\{the\ packet\ is\ absorbed\ at\ i | the\ packet\ is\ of\ class\ l\} * P\{the\ packet\ is\ of\ class\ l\}] U_p(k)$$

$$= [1 - \sum_{l=1}^{M} P\{the\ packet\ has\ traversed\ exactly\ l\ hops\} * c_l] U_p(k).$$

Now q_{ij}, the forwarding probability from node i to node j is independent of the particular choice for i and j so that we can remove the suffix to write q instead of q_{ij}.

Hence we get

$$P\{i \to j || N(i)| = p, j \quad is\ the \quad k\ th\ neighbour\ of\ i\} = (1 - \sum_{l=1}^{M} q^l c_l) U_p(k). \quad (4)$$

Substituting eqn (4) in (3) we get

$$P\{i \to j | j \in N(i), |N(i)| = p\} = (1 - \sum_{l=1}^{M} q^l c_l) \sum_{k=1}^{p} P\{j \quad is\ the \quad k\ th\ neighbour\ of\ i || N(i)| = p\} U_p(k). \quad (5)$$

Now from [9], the average distance between two random points uniformly distributed inside a sphere of radius r is $\frac{72r^2}{35}$.

Therefore

$$P\{j \quad is\ the \quad k\ th\ neighbour\ of\ i || N(i)| = p\}$$

$$= P\{\ among\ p-1\ neighbours\ (other\ than\ j)\ of\ i\ exactly\ k-1\ lie\ inside\ a\ sphere\ of\ radius\ \frac{72R^2}{35}\}$$

$$= p-1_{k-1} (v')^{k-1} (1-v')^{p-k}, \quad where \quad v' = \left(\frac{72R}{35}\right)^3.$$

Hence eqn (5) becomes

$$P\{i \to j | j \in N(i), |N(i)| = p\} = (1 - \sum_{l=1}^{M} q^l c_l) \sum_{k=1}^{p} p-1_{k-1} (v')^{k-1} (1-v')^{p-k} U_p(k). \quad (6)$$

Also

$$P\{|N(i)| = p | j \in N(i)\} = \binom{N-2}{p-1} v^{p-1}(1-v)^{N-p-1}, \tag{7}$$

and

$$P\{j \in N(i)\} = v. \tag{8}$$

By substituting eqns (6), (7), and (8) in eqn (2) we get

$$q = \left(1 - \sum_{l=1}^{M} q^l c_l\right) \sum_{p=1}^{N-1} \sum_{k=1}^{p} \binom{N-2}{p-1} \binom{p-1}{k-1} v^p (1-v)^{N-p-1} (v')^{k-1} (1-v')^{p-k} U_p(k). \tag{9}$$

A recursive algorithm to compute q

From eqn (9) we get

$$q = \frac{G\left(1 - \sum_{l=2}^{M} q^l c_l\right)}{1 + c_1 G}, \tag{10}$$

where

$$G = \sum_{p=1}^{N-1} \sum_{k=1}^{p} \binom{N-2}{p-1} \binom{p-1}{k-1} v^p (1-v)^{N-p-1} (v')^{k-1} (1-v')^{p-k} U_p(k).$$

As a particular case, if c_l is assumed as uniform (that is, if $c_l = 1/M, l = 1,2 \ldots, M$), then we can write

$$q = \frac{MG + q^2 M + q^{M+1} G}{M + G + MG},$$

which in turn gives the recursive algorithm

$$q^{[k+1]} = \frac{MG + (q^{[k]})^2 M + (q^{[k]})^{M+1} G}{M + G + MG}. \tag{11}$$

Lemma: The effective arrival rate at an arbitrary node, denoted by λ, is

$$\lambda = \frac{\lambda_e}{N(1-(N-1)vq)}. \tag{12}$$

Proof: Since the effective arrival rate at a node is the sum of the external arrival rate at that node and the average inflow rate to that node from its neighbouring nodes, we have

$$\lambda = \frac{\lambda_e}{N} + (N-1)\lambda v q.$$

Hence the lemma.

Finding the interfering nodes

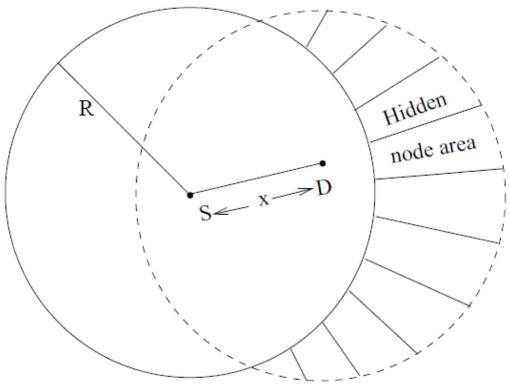

Figure 1: Illustration of hidden-terminal area

While a packet is being transmitted from a node to another, all the nodes that are lying in the neighbourhood of the source node can hear the details regarding the transmission by sensing the medium , whereas the ones which are not the neighbours can not. So the nodes which are located within the sensing region of the intended destination and off-range of the source node may make transmission to destination node simultaneously with source node, which may result in collision at the destination node. This is the well known *hidden terminal problem* and the corresponding nodes are termed as *hidden nodes*. These hidden nodes together with the neighbouring nodes of the source node constitute the set of *interfering nodes* of the source node. In order to conduct the waiting time analysis for a packet in the whole network, we need to get a measure of the average number of interfering neighbours an arbitrary node has. For this, we proceed as follows.

Distribution function of the distance X between source and destination nodes is given by

$$F(x) = (\frac{x}{R})^3, 0 < x \leq R.$$

The volume of the solid inside which the hidden nodes lie is a random variable. For a given value of X, this volume (two-dimensional analogue of this case is shown as shaded portion in Figure 1, the details of which are given in [1]) can be computed as

$$V(x) = v - \frac{1}{12}\pi(4R + x)(2R - x)^2.$$

So the average volume of the solid inside which the hidden nodes lie,

$$v_h = \int_0^R V(x)dF(x) = \frac{17}{24}\pi R^3.$$

Hence the average volume of the solid inside which the interfering nodes lie,

$$v_I = v_h + v = \frac{49}{24}\pi R^3.$$

Now it is easy to see that the probability distribution of the interfering neighbours of a node is binomial with mean $(N-1)v_I$. Hence the average number of interfering neighbours of an arbitrary node,

$$N_I = (N-1)v_I. \tag{13}$$

Waiting time analysis

In this section, we derive the probability distribution and some important measures of characteristics of waiting time for a packet at an arbitrary node under both BEB and DIDD schemes. Here, by waiting time at a node we actually mean the time spent by a packet at that node from the instant at which it is ready for transmission till it s successfully transmitted. This does not include the time spent by the packet at the buffer before its transmission turn occurs. The objective of this paper is not to compare the efficiency among BEB or DIDD or any other scheme proposed by researchers, rather our focus here is to derive the waiting time distribution of a packet at an arbitrary node analytically, for which BEB and DIDD rules are being used just for theoretical illustration.

Under the DIDD scheme

[3] analysed the performance of IEEE 802.11 distributed coordination function, where BEB rule is used as contention resolution method, by means of a two dimensional Markov chain and computed the conditional collision probability (that is, the probability of collision seen by a packet while it is being transmitted). By the same approach, in this case, we can compute the conditional collision probability say, p_D by means of the formula

$$p_D = 1 - (1 - \tau_D)^{N_2}, \tag{14}$$

where τ_D, the transmission probability of a node in a random time slot under DIDD rule, is derived as (proof is shown below)

$$\tau_D = \frac{2(1-2a)(1-a^{m+1})}{(1-(2a)^{m+1})(1-a)W + (1-2a)(1-a^{m+1})} \tag{15}$$

with $a = \frac{p_D}{1-p_D}$, and N_2 represents the largest integer less than or equal to N_I.

Eqns (14) and (15) represent a nonlinear system in unknowns τ_D and p_D, which can be solved numerically (by using fixed point iteration scheme) to get p_D.

Now, we derive the probability distribution of the number of time slots spent by an arbitrary packet at a node from the time instant at which it is ready for transmission till it is successfully transmitted, by using the embedded Markov chain technique. For this, consider the system at the end of a time slot at which either the channel is sensed idle by the node or transmission of a packet (which may or may not be successful) from that node is over. More precisely, let t_i be the beginning of a time slot such that the previous slot $[t_{i-1}, t_i)$ ends either with transmission of a packet from the node, or the channel is sensed idle by the node. Then the embedded stochastic process $\{(s(t_i), b(t_i); i \in N\}$, where $s(t_i)$ and $b(t_i)$ respectively denote the backoff stage and backoff time counter of the node at t_i, is a Markov chain. Note that whenever $s(t_i) = j$, then $b(t_i)$ can take one of the values uniformly from $\{0,1,2,\dots,W_j - 1\}$, where $W_j = 2^j W, j = 0,1,2,\dots,m$.

The transition probabilities of the Markov chain are denoted by

$$P\{(i_1,k_1)|(i_0,k_0)\} = P\{(s(t_{i+1}), b(t_{i+1})) = (i_1,k_1)|(s(t_i), b(t_i)) = (i_0,k_0)\}.$$

Under the DIDD scheme, it can be seen that

$$P\{(i,k)|(i,k+1)\} = 1, \quad for \quad k = 0,1,\dots,W_i - 2; \quad i = 0,1,\dots,m$$

$$P\{(i-1,k)|(i,0)\} = \frac{1-p_D}{W_{i-1}}, \quad for \quad k = 0,1,\dots,W_{i-1} - 1; \quad i = 1,\dots,m$$

$$P\{(i+1,k)|(i,0)\} = \frac{p_D}{W_{i+1}}, \quad for \quad k = 0,1,\dots,W_{i+1} - 1; \quad i = 0,1,\dots,m-1$$

$$P\{(m,k)|(m,0)\} = \frac{p_D}{W_m}, \quad for \quad k = 0,1,\dots,W_m - 1$$

and

$$P\{(0,k)|(0,0)\} = \frac{1-p_D}{W}, \quad for \quad k = 0,1,\dots,W - 1.$$

Define state vector

$$\bar{\iota} = ((i,0),(i,1),(i,2),\dots,(i,W_i - 1)), \quad for \quad i = 0,1,\dots,m.$$

Then the transition probability matrix of the Markov chain is given by

$$P = \begin{bmatrix} D_0 + C_{0\prime} & B_1 & 0 & \dots & \dots & \dots & \dots & 0 \\ C_0 & D_1 & B_2 & \ddots & & & & \vdots \\ 0 & C_1 & D_2 & B_3 & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & C_{m-2} & D_{m-1} & B_m \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & C_{m-1} & D_m + B_{m\prime} \end{bmatrix}$$

where $B_i, i = 1,2,\dots,m$ of dimension $W_{i-1} \times W_i$, given by

$$B_i = \begin{bmatrix} \frac{p_D}{W_i} & \frac{p_D}{W_i} & \dots & \frac{p_D}{W_i} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 \end{bmatrix};$$

$C_i, i = 0,1,\dots,m-1$, $W_{i+1} \times W_i$ matrix, given by

$$C_i = \begin{bmatrix} \frac{1-p_D}{W_i} & \frac{1-p_D}{W_i} & \dots & \frac{1-p_D}{W_i} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 \end{bmatrix};$$

$D_i, i = 0,1,\dots,m$, of dimension $W_i \times W_i$, given by

$$D_i = \begin{bmatrix} 0 & 0 \\ I_{W_i - 1} & 0 \end{bmatrix}$$

with $I_{W_i - 1}$ as the identity matrix of order $W_i - 1$ and 0 is row(column) vector of appropriate dimension.

$C_{0\prime}$ has the same structure as C_0 with the only distinction that it is a square matrix of order W. Similarly $B_{m\prime}$, a square matrix of order W_m, differs from B_m only in dimension.

If we define $\Pi = (\Pi_0, \Pi_1, \dots, \Pi_m)$, where $\Pi_i = (\pi_{i0}, \pi_{i1}, \dots, \pi_{iW_i-1}), i = 0,1,\dots,m$, as the stationary distribution of the above Markov chain, it can be seen that

$$\pi_{ik} = (\frac{p_D}{1-p_D})^i (\frac{W_i-k}{W_i})\pi_{00}, \quad for \quad i = 0,1,\dots,m; \quad k = 0,1,\dots,W_i - 1$$

and

$$\pi_{00} = 2[W(\frac{1-p_D}{1-3p_D})(1-(\frac{2p_D}{1-p_D})^{m+1}) + (\frac{1-p_D}{1-2p_D})(1-(\frac{p_D}{1-p_D})^{m+1})]^{-1}.$$

Hence τ_D, the probability that a node transmits in a random slot time is given by

$$\tau_D = \sum_{i=0}^{m} \pi_{i0} = \frac{2(1-2a)(1-a^{m+1})}{(1-(2a)^{m+1})(1-a)W + (1-2a)(1-a^{m+1})},$$

which is eqn (15).

Now, let $\alpha_i, i = 0,1,2,\dots,m-1$ be the probability that a packet at the head of the waiting line at a node starts with backoff stage i. Then

$$\alpha_i = P\{ \text{ the previous packet which was successfully transmitted from the node left the system at stage } i+1 \}$$

so that

$$\alpha_i = \sum_{l=0}^{i+1} \alpha_l (1-p_D) p_D^{i-l+1}, \quad for \quad i = 1,\dots,m-2$$

with

$$\alpha_{m-1} = \sum_{l=0}^{m-1} \alpha_l p_D^{m-l},$$

and

$$\alpha_0 = \alpha_1(1-p_D) + \alpha_0(1-p_D^2).$$

From this, by recursion we get

$$\alpha_i = \frac{p_D^{i+1}}{(1-p_D)^i}\alpha_0, \quad for \quad i = 1,2,\dots,m-1$$

so that

$$\alpha_0 = \frac{(1-2p_D)(1-p_D)^{m-1}}{(1-p_D)^{m+1} - p_D^{m+1}} \tag{16}$$

by using the normalizing condition $\sum_{i=0}^{m-1} \alpha_i = 1$. Thus we have

$$\alpha_i = \frac{(1-2p_D)p_D^{i+1}}{((1-p_D)^{m+1} - p_D^{m+1})(1-p_D)^{i-m+1}}, \quad for \quad i = 1,\dots,m-1 \tag{17}$$

with α_0, given by eqn (16).

Now the definition of a discrete PH random variable (see [12] and [11]) for details on PH distribution and PH renewal theory) leads to the following theorem.

Theorem 1: The number of transitions undergone (time slots spent) by a packet say, S_D from the instant at which it is ready for transmission till it is successfully transmitted, is a discrete PH random variable having representation $(\bar{\alpha}_D, T_D)$ with $\bar{\alpha}_D = (\frac{\bar{\alpha}_0}{W}, \frac{\bar{\alpha}_1}{W_1}, \frac{\bar{\alpha}_2}{W_2}, \dots, \frac{\bar{\alpha}_{m-1}}{W_{m-1}}, 0_{W_m}, 0)$, where $\bar{\alpha}_i, \quad i = 0,1,2\dots,m-1$ is the vector having W_i components with each component α_i, 0_{W_m} is the vector of zeroes having W_m components, and T_D is the matrix given by

$$T_D = \begin{bmatrix} D_0 & B_1 & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & D_1 & B_2 & \ddots & & & & \vdots \\ \vdots & \ddots & D_2 & B_3 & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \ddots & D_{m-1} & B_m \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & D_m + B_m \end{bmatrix}.$$

Also, its pmf, $P(S_D = k) = \bar{\alpha}_D T_D^{k-1}(-T_D e), k > 0$, where e is a column vector, having components 1, of appropriate dimension.

Corollary 1: Average number of transitions undergone(time slots spent) by a packet at an arbitrary node from the instant at which it is ready for transmission till it is successfully transmitted, is given by

$$E(S_D) = \bar{\alpha}_D(I - T_D)^{-1}e = \frac{\alpha_0}{W}(Wx_0 + \frac{W(W-1)}{2}) + \sum_{i=1}^{m-1} \frac{\alpha_i}{W_i}(W_i x_i + \frac{W_i(W_i-1)}{2}), \quad (18)$$

where

$$x_i = \frac{2-p_D}{2(1-p_D)} + \frac{W_i p_D(1-(2p_D)^{m-i-1})}{1-2p_D} + \frac{p_D^{m-i} W_{m-1}}{1-p_D}, \quad for \quad i = 0,1,2,\dots,m-1. \quad (19)$$

Under the standard BEB scheme

Following the same lines as in the case of DIDD scheme, we have

Theorem 2: Under the standard BEB scheme, S_B, the number of time slots spent by a packet at a node from the instant at which it is ready for transmission till it is successfully transmitted, is a discrete PH variate having representation $(\bar{\alpha}_B, T_B)$ with $\bar{\alpha}_B = (\frac{e_W^T}{W}, 0,0,\dots,0)$, and T_B is the matrix having the same structure as T_D with the only exception that p_D in T_D is replaced by p_B in T_B, where the conditional collision probability p_B under BEB scheme is computed by solving the nonlinear system of equations (see [3])

$$p_B = 1 - (1 - \tau_B)^{N_2}, \quad (20)$$

and

$$\tau_B = \frac{2(1-2p_B)}{(1-2p_B)(W+1) + p_B W(1-(2p_B)^m)}. \quad (21)$$

Also, pmf of S_B is given by

$$P(S_B = k) = \bar{\alpha}_B T_B^{k-1}(-T_B e), k > 0.$$

Corollary 2: Under the standard BEB scheme, average number of time slots spent by a packet at an arbitrary node from the instant at which it is ready for transmission till it is successfully transmitted, is given by

$$E(S_B) = \bar{\alpha}_B(I - T_B)^{-1}e = \frac{1}{W}(Wy_0 + \frac{W(W-1)}{2}), \quad (22)$$

where y_0 has the same expression as x_0 (given by eqn (19)), in which p_D is replaced by p_B.

Probability distribution of slot length

We start with this section by summarizing the important steps (with minor changes pertaining to our model) in the derivation of probability distribution of length of an arbitrary slot, as detailed in [3]. A slot is called *active* if at least one transmission takes place in that slot. Let P_a be the probability that the slot is active. Since, on the average $N_2 + 1$ nodes are contending on the channel (we have seen that on the average, a node will have N_2 interfering nodes), and each transmits with probability τ

$$P_a = 1 - (1 - \tau)^{N_2+1}, \quad (23)$$

where τ=τ_D or τ_B depending on whether DIDD or BEB scheme is used. Transmission of a packet may result either in success or in collision.

Let

$$P_s = P(\ transmission\ in\ a\ slot\ is\ successful | slot\ is\ active\ ).$$

Then

$$P_s = \frac{(N_2+1)\tau(1-\tau)^{N_2}}{P_a} = \frac{(N_2+1)\tau(1-\tau)^{N_2}}{1-(1-\tau)^{N_2+1}}. \quad (24)$$

At this juncture it may be remembered that a time slot may be (i) a back off time slot σ if no transmission takes place in that slot (ii) T_s, the average time the channel is sensed busy because of a successful transmission, or (iii) T_c, the average time the channel is sensed busy because of a collision.

Thus the probability distribution of the slot length SL is given by

$$SL = \begin{cases} \sigma & with \quad prob \quad 1 - P_a \\ T_s & with \quad prob \quad P_a P_s \\ T_c & with \quad prob \quad P_a(1 - P_s) \end{cases} \quad (25)$$

If we assume that the system is completely managed by the basic access mechanism, then

$$T_s = SIFS + T(E(P)) + DIFS + T(ACK)$$

and

$$T_c = DIFS + T(E(P^*)),$$

where $E(P)$ denotes the expected packet size and $T(E(P))$ represents the average time required for a node to transmit a packet of size $E(P)$ to its neighbouring node. Similarly, $E(P^*)$ stands for the expected value of the largest packet size included in a collision and $T(E(P^*))$ for the corresponding mean transmission time . Also, $T(ACK)$ represents the mean time required to transmit an acknowledgement message from destination to source node. Just in the lines of eqn (15) in [3] it can be seen that

$$E(P^*) = \frac{\sum_{k=2}^{N_I+1} {N_I+1}_k \, \tau^k (1-\tau)^{N_I+1-k} \int_0^{P_{max}} (1-(F(x))^k) dx}{1-(1-\tau)^{(N_I+1)}-(N_I+1)\tau(1-\tau)^{N_I}}, \quad (26)$$

where $F(.)$ is the packet size distribution function and P_{max} is the maximum value of the packet size. It is to be noted that

$$E(P) = \int_0^{P_{max}} (1 - F(x))dx.$$

Now, for the computation of $T(E(P))$, $T(E(P^*))$, and $T(ACK)$, we assume that the transmission time for a packet from a node to another depends on how far the latter is from the former. Earlier we have seen that if a packet is transmitted from node i,

$$P\{ it \ reaches \ at \ node \ j \,|\, j \ is \ the \ kth \ neighbour \ of \ i \} = \frac{E_L}{E(R_k)},$$

where

$$E_L = \left(\sum_{l=1}^{L} \frac{1}{E(R_l)}\right)^{-1},$$

and $E(R_k)$ is given by eqn (1). So, if Z represents the transmission time (in μ_s) for a bit to reach the receiving node, which lies at a unit distance from the source node, then the transmission time for a packet to reach the receiver, which is the kth neighbour of the source node is $ZE(P)E(R_k)$ so that

$$T(E(P)) = \sum_{l=k}^{L} ZE(P)E(R_k)\frac{W_L}{E(R_k)} = LZW_L E(P). \quad (27)$$

Similar results hold for $T(E(P^*))$, and $T(ACK)$.

Now the expected slot length,

$$E(SL) = (1 - P_a)\sigma + P_a P_S T_s + P_a(1 - P_s)T_c. \quad (28)$$

If TS denotes the time spent by a packet at a node from the instant at which it is ready for transmission till its successful transmission, then

$$E(TS) = E(S)E(SL), \quad (29)$$

where $S = S_D \ or \ S_B$ depending on whether the system is under DIDD or BEB scheme, and $E(S)$ is given by eqn (18) or (22) as the case may be. Note that since TS is the actual time taken for a node to complete a packet transmission since the epoch at which it is ready for transmission, as per the queueing terminology, TS is equivalent to the *effective service time* rendered for a packet at a node in the network. Now, let us compute the variance of TS.

$$E(TS^2) = E(E(TS^2|S = k)).$$

Now

$$E(TS^2|S = k) = E\left(\left(\sum_{j=1}^{k} SL_j\right)\left(\sum_{h=1}^{k} SL_h\right)\right),$$

where SL_j denotes the length of jth time slot. Since SL_j are iid variates with mean $E(SL)$, we have

$$E(TS^2|S = k) = \sum_{j=1}^{k}\sum_{h=1}^{k} (E(SL))^2 + \sum_{j=1}^{k} E(SL^2).$$

Thus

$$Var(TS) = [(1 - P_a)\sigma + P_a P_s T_s + P_a(1 - P_s)T_c]^2[2\bar{\alpha}(I - T)^{-2}Te$$

$$+\bar{\alpha}(I - T)^{-1}e - (\bar{\alpha}(I - T)^{-1}e)^2] + \bar{\alpha}(I - T)^{-1}e[(1 - P_a)\sigma^2 + P_a P_s T_s^2 + P_a(1 - P_s)T_c^2], \quad (30)$$

where $\bar{\alpha} = \bar{\alpha}_D$ or $\bar{\alpha}_B$, and T=T_D or T_B depending on whether the system is under DIDD or BEB scheme. Moreover C_{TS}, the coefficient of variation of TS, given by

$$C_{TS}^2 = \frac{Var(TS)}{(E(TS))^2} \quad can \ be \ computed \ by \ using \ the \ eqns \ (29) \ and \ (30).$$

These results can be used to get approximate solution to queue size distribution at each node as done in [4]. [10] introduced a vector-valued normal process and its diffusion equation in order to obtain an approximate solution to the joint distribution of queue lengths in a general network of queues. By this approximation, the queue size distribution at node i say, p_i is obtained as

$$p_i(n) = \begin{cases} 1-\rho_i, & n=0 \\ \rho_i(1-\hat{\rho}_i)\hat{\rho}_i^{\,n-1}, & n>0 \end{cases} \quad (31)$$

where $\rho_i = \lambda_i E(S_i)$; λ_i is the effective arrival rate at node i , and $E(S_i)$ is the mean service time required for a packet at node i. Also,

$$\hat{\rho}_i = exp\,(-\frac{2(1-\rho_i)}{C_{A_i}^2\rho_i + C_{S_i}^2}), \quad (32)$$

where $C_{A_i}^2$ and $C_{S_i}^2$ are the squares of coefficients of variation of inter-arrival times and service times respectively, of packets at node i. As shown in [5] , $C_{A_i}^2$ is approximated by using the relation

$$C_{A_i}^2 = 1 + \sum_{j=0}^{N} (C_{S_j}^2 - 1) q_{ji}^2 e_j e_i^{-1}, \quad (33)$$

where $C_{S_0}^2 = C_E^2$, and e_j is the average number of visits that a packet makes to node j during its stay in the network. Since all the nodes are considered identical in our model, the eqn, analogues to the one given by(33), associated with our model assumes the form

$$C_A^2 \approx 1 + \frac{(C_E^2 - 1)}{N^2} + (N-1)(C_{TS}^2 - 1)q^2, \quad (34)$$

where q is given by eqn (11).

Also, mean number of packets at an arbitrary node

$$\overline{K} = \frac{\rho}{1-\hat{\rho}}. \quad (35)$$

By Little's law, average waiting time of a packet at an arbitrary node

$$\overline{W} = \frac{\overline{K}}{\lambda} = \frac{\rho}{\lambda(1-\hat{\rho})}, \quad (36)$$

where λ, the effective arrival rate at a node is given by eqn (12).

Since a packet generated at an arbitrary node is of class $l, l = 1,2,\ldots, M$ with probability c_l and a class l packet visits exactly l hopes before absorption, average number of hops traversed by a packet before absorption

$$\overline{H} = \sum_{l=1}^{M} lc_l = \frac{M+1}{2}, \quad if \quad c_l \quad is\ uniform.$$

Hence, the average end to end delay experienced by a packet in the whole network

$$D = \overline{H}\,\overline{W} = \frac{(M+1)\rho}{2\lambda(1-\hat{\rho})}, \quad if \quad c_l \quad is\ uniform. \quad (37)$$

Since λ, given by eqn (12), is the effective arrival rate at an arbitrary node, and $E(TS)$, given by eqn(29), is the actual mean time required for a packet to be successfully transmitted from a node, for the stability of the system

$$\lambda E(TS) < 1.$$

Hence the maximum achievable throughput can be attained when λ_e is enhanced to the values near its upper bound, governed by the rule

$$\lambda_e < N(1-(N-1)vq)E(TS) \quad (38)$$

for a selected set of parameters.

3 Results

In order to illustrate the performance of the system, we present some numerical results. The values of the system parameters used in this analysis are summarized in Table 1 and Table 2. Most of these parameters are set to comply with the 802.11 MAC specifications. The wireless nodes are assumed to be distributed uniformly inside a compact subset in R^3 of volume $10^6\ m^3$, which is taken as 1 $cubic\ unit$. All nodes are considered as identical. Packets are generated independently at nodes as per a renewal process with rate λ_e and coefficient of variation $C_E = 0.95$. types of packets generated at each node are assumed as uniform with mean $1/M$, where $M = 15$. Packet size are assumed to be uniformly distributed over an interval [64,1518], measured in bytes, so that the average packet size is 791 bytes. In all numerical illustrations, we have included both BEB and DIDD schemes in order to get a complete picture of the system performance.

Table 1: Physical Parameters

Parameter	Value
CW_{min}	32
CW_{max}	1024
No. of classes of packets(M)	15
Propagation speed	$3\times10^8 m/s$
Channel bit rate	1 Mbps
Slot time (σ)	50 μs
SIFS	28 μs
DIFS	128 μs

Table 2: Packet Parameters

Parameter	Value
Average Packet size	6328 bits
COV of the packet arrival process C_E	0.95
PHY header	128 bits
ACK	112 bits + PHY header

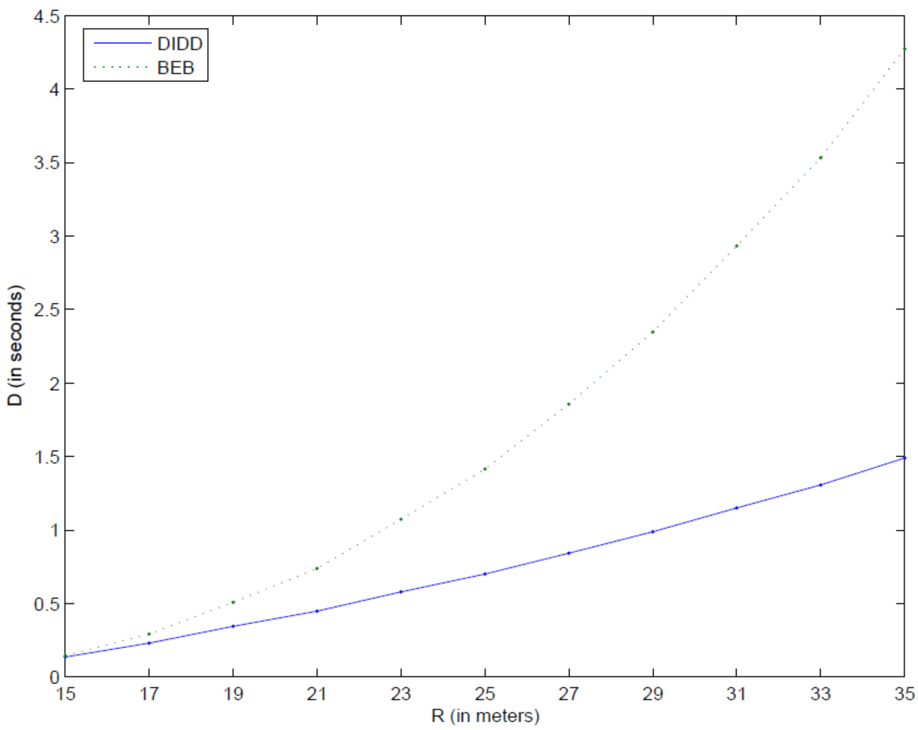

Figure 2: Average Delay versus Transmission Range ($N = 300$, $\lambda_e = 2$ $packets/sec$)

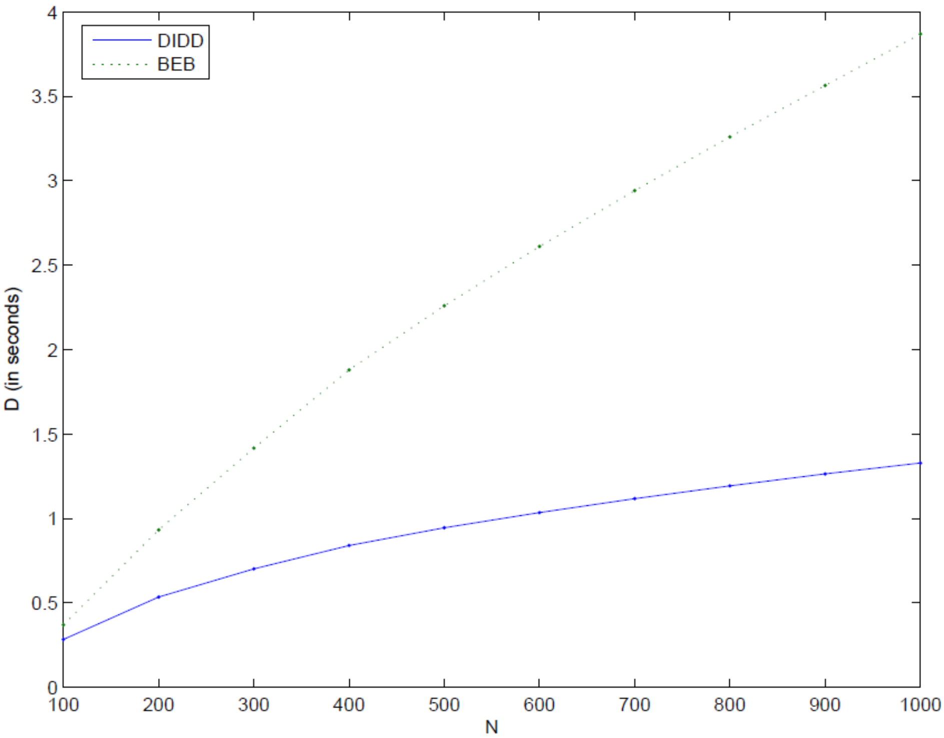

Figure 3: Average Delay versus Number of Nodes ($R = 25m$, $\lambda_e = 2packets/sec$)

4 Discussion

Figure 2 shows how the average end to end delay D experienced by a packet in the whole network varies with different values of the transmission range R. When R increases, average number of interfering neighbours of nodes increases so that the conditional collision probability also increases, which results in more delay for packets at each node. Also, since the conditional collision probability is more for the system under standard BEB scheme than under the DIDD scheme, the average end to end delay for the former is much higher than the latter, as obvious from Figure 2.

In Figure 3, the variation in average end to end delay corresponding to change in values of the the number of nodes N, by keeping $R = 25m$, and $\lambda_e = 2packets/sec$, is exhibited. As in the previous case, here also it is seen that D moves in the same direction with N, under both schemes.

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The Economics and Management of Nuclear Energy Industry: Supply Chain Reliability and Safety

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Abstract

Nuclear energy safety is a critical issue as the world increasingly turns to nuclear energy to meet our growing needs for power and decreasing dependence on fossil fuels. Major nuclear disasters like Chernobyl (1986) and Fukushima Daiichi (2001) demonstrate the need for improved controls of these systems. The nuclear power supply chain is a critical component of the highly interconnected nuclear energy system. This paper introduces industry best practices and how these relate to the nuclear energy industry safety and reliability.

Keywords: Nuclear energy, safety, supply chain, reliability, operations costs

I. Introduction

Reliability is considered to be a branch of theory of probability and statistics. However, there is an important role that social sciences (economics, geography) can play in the development of the field of reliability. For example, a simple cost-benefit analysis of a new (contemplated) nuclear plant location will likely indicate that such a plant should be built closer to major population centers, especially those that lack local fossil fuel resources. Another economic cost-reducing feature would be to build such a plant next to the water (river, lake) in order to have a readily available source of cooling. Introducing the notion of reliability in this economic analysis would, however, indicate that such a location can be very costly in case of emergency: a nuclear disaster in a densely-populated area may result in a loss of many lives, whereas the location near the water body my result in ever-lasting pollution and loss of valuable source of drinking and irrigation water.

Although very unlikely, such disasters do occur from time to time. To list a few: Fukushima Daiichi (Japan, 2011), Chernobyl (Ukraine, 1986), Three Mile Island (USA, 1979). Located in ecologically vulnerable areas, nuclear industry receives a harsh attitude from the public. For example, the Fessenheim nuclear power plant (France) is located in a potentially seismic region, causing public safety concerns in the center of Europe. As another example, the majority of nuclear power plants in Russia are located in the densely-populated European part of the country, often in the upper reaches of Volga and other rivers. The location next to major rivers may potentially cause much more damage because the significant portion of released radiation can be spread by the strong currents over larger areas (as it was the case during the Chernobyl disaster when the radioactive pollution in the Dnieper river has affected the water basins of the adjacent rivers).

In light of development of climate-friendly technologies, economists claim that the resurgence of nuclear power would be beneficial in terms of carbon-free electricity generation and job creation. The 2009 estimate by the US National Commission on Energy Policy was about 14,360 man-years per GW installed (Har, 2009). Another immediate benefit of nuclear power is that it is very cheap comparatively to the other means of base-load electricity generation2. However, the potential danger of the nuclear power generation may well exceed its economic benefits, both in monetary terms and the cost of lost human lives. Still, caused by concern about future energy supplies, the IAEA projects the growth of the world's nuclear power generating capacity by up to 88% (IAEA, 2012; Chudakov, 2015).

The important lessons are being learned, however. For example, after Fukushima accident, Ken Buesseler together with the researchers at Woods Hole Oceanographic Institution (USA) have formed the Center for Marine and Environmental Radioactivity to help share up-to-date information about radiation from human and natural sources (Pacchioli, 2013). To have a rigorous account of the vulnerability and reliability issues, a new branch in economics has been developed several decades ago: environmental and natural resource economic analysis. However, a strong link between these two science fields is still missing. The need for bridging reliability, economics and management was brought up during the recent event organized by the Gnedenko Forum and its associates.

In April 2016 Dr. Michael Yastrebenetsky, the Honored Scientist of Ukraine State Scientific and Technical Center for Nuclear and Radiation Safety, gave a talk organized by the Gnedenko Forum at a joint Boston Chapter American Statistical Association (BCASA), IEEE Reliability Society, INFORMS Boston Chapter, and Northeastern University INFORMS Student Chapter event at Northeastern University. Dr. Yastrebenetsky began the talk by commemorating 30 years since Chernobyl nuclear power plant disaster (26 April 1986). He introduced several important nuclear energy safety and reliability concepts about instrumentation and control (I&C) systems for Nuclear Power Plants (NPP). Also, Dr. Yastrebenetsky emphasized the series of mistakes that have led to the disaster which may have been avoided if sufficient I&C systems were in place and operated correctly.

This talk highlighted the importance of I&C in the nuclear energy industry and motivated the investigation of important issues related to the reliability of the nuclear energy supply chain. The nuclear supply chain is critical to the safe operation of the world's nuclear energy providers. Sometimes, However, nuclear suppliers can be unreliable and even unsafe such as the recent investigators findings of Areva SA and Le Creusot Forge, two major French nuclear suppliers. These companies have been cited by a team of international inspectors and the French Nuclear Safety Authority for extensive management weaknesses, safety failings, and a decades long cover-up of major manufacturing problems, underscoring the extent of problems in the supply chain of the world's crucial nuclear power components. This highlights the need for improved management and reliability of the worldwide nuclear supply chain and the nuclear industry. (Dalton, 2017).

Supply chains require close coordination of various components to reliably deliver value. A supply chain, as defined by the American Production and Inventory Control Society (APICS), is a system of organizations, people, technologies, activities, information and resources involved in moving materials, products and services all the way through the manufacturing process, from the original supplier of materials supplier to the end customer. (http://www.apics.org/) Integrated supply chains increase the confidence and trust between organizations, provide closer linkage and better communications, and boost the overall reliability of the supply chain. (Heizer, Render, & Munson, 2017). The supply chain in the nuclear industry is characterized by several very specific features. The extreme level of health risk is, of course, one of them. The other feature of the nuclear industry supply chain is its tendency to power plant consolidation under one ownership (in the market economies). The inherent reason for that is the need for some infrequent maintenance services such as refueling outages that need to be done once in 1.5 years and require a company to hire a contractor team for short periods of time.

[2] "The 104 nuclear reactors operating in the United States provide the lowest-cost baseload electricity, averaging 1.83 cents per kWh," (Har, 2009)

When instead a company manages several power plants, it may hire a single permanent team of highly skilled employees to service its various power plants throughout the year to make maintenance and refueling operations less costly and more reliable (Davis and Wolfram, 2012). In economics, this phenomenon is referred to as the economies of scale. Even with the recent creation of nuclear reactors of a smaller scale, this feature of nuclear supply chain will remain an important part of increasing worldwide competitiveness of nuclear power generation firms and therefore will persist in the long run.

Besides these recent positive economic trends to consolidate and deregulate nuclear industry, driven by market forces, there have been infrequent but highly significant failures that have led to disasters causing entire nations rethinking the prospects of the future of nuclear energy. These failures are perplexing as the nuclear energy industry is compelled to put reliability and safety first. This includes the requirement for an ultra reliable global sourcing of materials while meeting strict international (e.g., World Nuclear Association) and national (e.g., U.S. Nuclear Regulatory Commission, French Nuclear Safety Authority, Australian Radiation Protection and Nuclear Safety Agency, etc.) guidelines and regulations leading to an extremely complex supply chain. Nuclear supply chain reliability is often a major obstacle for developing new nuclear energy capabilities; however, by applying Integrated Supply Chain best practices and by measuring and monitoring supply chain operations using tools such as the Supply Chain Operations Reference (SCOR) model, these obstacles can be greatly reduced increasing the visibility and reliability of the nuclear supply chain.

II. Overview - Nuclear Energy Industry Supply Chains

Building new nuclear energy power plants requires a robust supply chain due to the large number of components and sub-components and depend on nuclear manufacturers to deliver the high-quality supplies needed to include concrete, pumps, electronics, wiring, instrumentation, piping, and specific equipment. A “cradle to grave” approach demands that each new build nuclear energy plant have multiple supply chain elements include pre-build, construction, operation, and decommissioning components. Moreover, reliability and safety considerations drive emerging measures for the global standardization for the raw material supply chain.

The use of uranium, required for creating plutonium needed for use as fuel in nuclear reactors, introduces intricacies into the supply chain for the operation of nuclear energy plants. Additionally, spent uranium and plutonium fuel needs to be recovered and recycled for reuse in the creation of fresh reactor fuel or, in the case that nuclear materials that are not recycled, they must be responsibly disposed. Safety and reliability of the supply chain is an important issue at every point in the nuclear energy life cycle.

Moreover, shipping and delivery of nuclear materials is under strict international standards and controls. International shipping standards are in place for the transport of these materials, including rules for packaging and marking the materials, and shipping practices must be closely monitored and enforced. Shipping containers are monitored throughout the shipping process (e.g., inspected and weighed) and these measures must comply with the International Atomic Energy Agency (IAEA) requirements as well as the European Atomic Energy Community.

III. Reliability of the Nuclear Energy Supply Chain

While many today take the reliability of energy supply for granted in the major nuclear nations, this has not always been the case in many parts of the world. In this highly interdependent and interconnected world, we are becoming increasingly more dependent on the availability of electrical power. Power to heat and cool our homes, store and prepare the food we eat, purify and deliver the water we drink, transport ourselves and the goods we come to depend on being delivered, power up the devices we use to work, communicate, and entertain ourselves. Nuclear power generates about 11 percent of the worldwide electrical power, with France generating 72.3 percent of its electrical power (Nuclear Energy Institute, https://www.nei.org/Knowledge-Center/Nuclear-Statistics/World-Statistics).

Nuclear power also operates at an extremely high capacity (92.2 percent in the US nuclear power generation in 2015) in comparison to other fuel types. Operating at this level of capacity requires an extremely reliable system with minimal scheduled downtime for maintenance and repair without unexpected interruptions and this system must be continuously maintained if we are to expect the energy supply to be there to meet our needs.

U.S. Capacity Factors by Fuel Type
2015

Fuel Type	Average Capacity Factors (%)
Nuclear	92.2
Geothermal	71.7
Gas (Combined Cycle)	56.3
Coal	54.6
Hydro	35.9
Wind	32.5
Solar PV	28.6
Oil (Steam Turbine)	14.7
Gas (Steam Turbine)	11.7

NEI NUCLEAR ENERGY INSTITUTE
Source: Energy Information Administration
Updated: 4/16
nuclear. clean air energy.

Figure 1 - Average Energy Capacity Factors by Fuel (U.S. only)

The nuclear energy supply chains, like other similar supply chains including medical use nuclear materials, "have unique features and characteristics due to the products' time sensitivity along with their hazardous nature" necessitating improved modeling to include "multicriteria decision-making and optimization to capture the operational and waste management costs as well as risk management." (Nagurney, Yu, Masoumi, & Nagurney, 2013). Optimized supply chains are needed but difficult to achieve with the unique features and complexities of the nuclear energy chain of supplies (for various reasons).

Therefore, nuclear energy best practices include the "establishment and reinforcement of the supply chain" to ensure viability, reliability, and safety throughout the nuclear power plant (NPP) life cycle from construction to decommissioning. (van der Hoeven & Magwood, 2015) The NPP supply chain has a large impact on the economics of nuclear power because it has a very high contribution to NPP capital costs. For example, equipment supply has been estimated to "constitute around 48% of overnight costs (i.e. the cost of construction excluding financing costs)." (NEA and OECD, 2015) NPP viability is threatened when supply chain costs are high and the danger to the industry is clear as recently evidenced in the bankruptcy of Westinghouse Electric Co., the U.S. nuclear unit of Japan's Toshiba Corporation. There continue to be key questions raised about the future of four nuclear reactors under construction in Georgia and South Carolina, and specifically about at Plant Vogtle in Georgia. (Foody, 2017) Industry trends indicate a localizing NPP supply chains to control costs; however, this brings into focus safety and reliability issues that can never be compromised. (Beutier, 2013)

Therefore, one of the key recommendations in the Technology Roadmap for Nuclear Energy is that "safety culture needs to be enhanced and monitored across the nuclear sector" including the NPP supply chain. "Safety culture" can be defined as a "set of characteristics and attitudes in organizations and individuals that ensures that nuclear safety issues receive appropriate attention as an overriding priority over other considerations." In this roadmap, the need for promoting a safety culture across organizations is emphasized through a case study of the Fukushima Daiichi NPP accident. (van der Hoeven & Magwood, 2015)

Controlling nuclear energy supply chain costs, safety, and reliability requires the application of industries best practices, as well as new approaches, to model, analyze, implement, and then measure supply chain performance. New analytical methods are needed to model and understand the intricacies of nuclear energy supply chain networks. Safety considerations must be a primary consideration in all decisions made about the supply chain. Finally, metrics for the measurement of nuclear supply chain performance must be in place to monitor and react to changes in the supply chain, marketplace, and regulatory environment. The next section will discuss industry best practices for measuring nuclear energy supply chain performance.

IV. Measuring Nuclear Energy Supply Chain Reliability

Industry best practices should be adopted for measuring supply chain performance first by customizing the methods for nuclear energy supply chain performance measurement and monitoring. The Supply Chain Operations Reference (SCOR) model is an excellent reference model that is widely used by firms to capture and compare metrics on how they compare to other firms and other industries, and SCOR provides a useful reference point to understand the nuclear energy supply chain performance. There are five primary components to the SCOR model as depicted in Figure 2: namely (1) Plan; (2) Source; (3) Make; (4) Deliver; and (5) Return with another component, Buy, indicated. These are addressed with a focus on the nuclear energy supply chain in the next paragraphs.

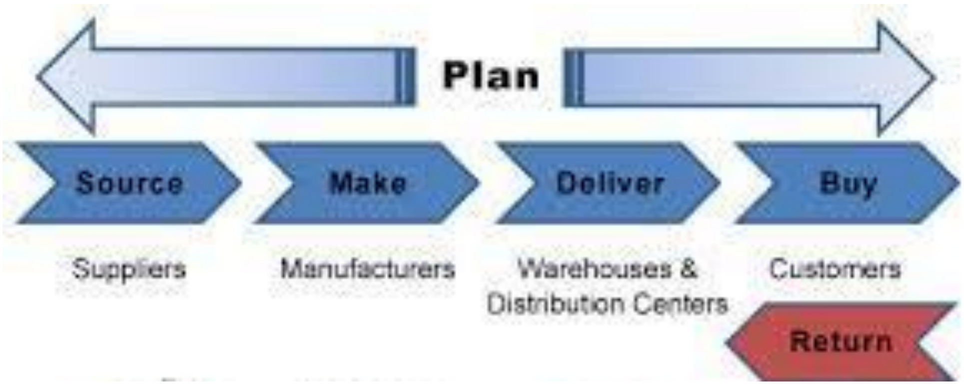

Figure 2 -A depiction of the Supply Chain Operations Reference (SCOR) Model

Plan - Planning activities

Planning for the supply and demand requirements of the nuclear power system is critical for reliability of the system, existing facilities should be able to reliably meet the demand and future construction of nuclear facilities to meet growing demand for power. The planning elements include the need to balance resources with requirements and enabling communication throughout the supply chain. Business rules should be established to measure the efficiency of the supply chain to make improvements in inventory, logistics, asset management, and compliance with federal regulations.

Source - Sourcing and purchasing activities

Nuclear power sourcing a procurement of necessary materials for the construction and continuous operation of nuclear power plants includes the sourcing of infrastructure and material acquisition, and describes how to manage inventory, supplier network, agreements, and performance including supplier payments and timing, verify, and transfer of receipts.

Make - Production activities

Production of nuclear power involves all power generation and plant operations activities that depend heavily on nuclear power supply chain reliability. Production activities includes managing the production network, equipment and facilities, and transportation.

Deliver. Distribution activities

Delivery includes order management, warehousing, and transportation of nuclear power supplies, plant, and equipment to include customers' power demands and invoicing for delivery of power. This step includes energy distribution to customer.

Return. Closed-loop supply chain activities.

Nuclear power plant supply chain reliably handles spent fuel and materials related to the safe disposal of spent fuel, containers, packaging, and other items. Handling of these involves the management of business rules, inventory handling, assets, transportation, and regulatory requirements.

The SCOR model enables understanding of the nuclear energy supply chain from "cradle to grave", that is from pre-construction planning through decommissioning of plants and everything in between such as ongoing disposal and recycling of nuclear materials that includes nuclear waste. NPPs have an operating life of approximately 25 years and the decommissioning may take more than twenty years. NPP decommissioning is an extremely complex process that includes the removal and disposal of radioactive materials and elimination of radioactive hazards. All materials must be removed and properly disposed. Frequently, the nuclear waste disposal involves crossing national borders and operating in different legal environments. The SCOR model can give NPPs a framework for managing all aspects of the nuclear energy supply chain and SCOR metrics, such as those shown provided by the Supply Chain Council shown in Figure 3, provide the measurements needed to understand nuclear energy supply chain performance. SCOR metrics should be tailored for the nuclear energy industry to provide solid supply chain measures in various dimensions including speed, flexibility, cost, and reliability.

Performance Attribute	Performance Attribute Definition	Level 1 Metric
Supply Chain Reliability	The performance of the supply chain in delivering: the correct product, to the correct place, at the correct time, in the correct condition and packaging, in the correct quantity, with the correct documentation, to the correct customer.	Perfect Order Fulfillment
Supply Chain Responsiveness	The speed at which a supply chain provides products to the customer.	Order Fulfillment Cycle Time
Supply Chain Flexibility	The agility of a supply chain in responding to marketplace changes to gain or maintain competitive advantage.	Upside Supply Chain Flexibility
		Upside Supply Chain Adaptability
		Downside Supply Chain Adaptability
Supply Chain Costs	The costs associated with operating the supply chain.	Supply Chain Management Cost
		Cost of Goods Sold
Supply Chain Asset Management	The effectiveness of an organization in managing assets to support demand satisfaction. This includes the management of all assets: fixed and working capital.	Cash-to-Cash Cycle Time
		Return on Supply Chain Fixed Assets
		Return on Working Capital

Figure 3 - SCOR Metrics (provided by the Supply Chain Council)

V. Discussion

This paper has introduced some of the challenges of managing the nuclear energy supply chain to account for the complexities introduced by regulatory and other factors as well as critical cost, reliability, and safety considerations. In summary, we recommend three challenges that can be addressed by the reliability community; namely:

1. Develop new supply chain models and methods to improve nuclear energy supply chain modeling by introducing multi-criteria decision modeling to include cost, reliability, and safety considerations
2. Create and maintain a "safety culture" in global and local nuclear energy supply chains.
3. Introduce new cost, reliability, and safety metrics into industry supply chain best practices such as the SCOR model providing new ways to measure, understand, and improve nuclear energy supply chain performance.

Dr. Yastrebenetsky brought up many ideas to consider about what could have been done differently that may have spared the nuclear power industry and world the Chernobyl and Fukushima disasters, including nuclear energy safety and reliability concepts that should have been well understood and under control. However, it appears that building safety and reliability into the nuclear energy processes is difficult at best. The nuclear energy supply chain is a substantial part of the world's nuclear energy processes, and a critical spot to rethink nuclear energy safety. By continuously building reliability and safety into the nuclear energy supply chain, perhaps we can help to avoid future disasters while ensuring the enduring viability of nuclear power as an alternative energy solution that will provide the citizens of the world with safe and dependable power they need to live their lives.

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B. V. Gnedenko. Bibliography

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Abstract

The bibliography, which we present below, includes everything that could be found from the written and published B.V. Gnedenko, starting with books and ending with newspaper articles. This includes also published interviews. The bibliography is broken down by years, and for each year everything that was published during this year, including reprints, is presented. The work for each year is arranged in one definite order: books, scientific articles, articles on various aspects of teaching, reviews, articles from general journals, newspaper publications, interviews. If the same article was published during the year in different places, then it is indicated under the same number, listing the output of all publications. Sometimes during the year there are articles with the same name, but different content. They are listed under different numbers, if possible, nearby.

Keywords: Gnedenko, bibliography, memory, history

Boris Vladimirovich Gnedenko was born on January 1 (according to a new style) in 1912 in Simbirsk (now Ulyanovsk).

His grandfather, Vasily Ksenofontovich Gnedenko (born 1850) and grandmother Anastasia Izotovna (born 1854) (both on his father's side) are peasants from the Poltava province who moved to Kazan Province in the seventies of the 19th century, where they received land in the village Bazarny Matak Spassky district. They had four children: Mikhail (born 1879), Vladimir (1886-1939), Sergei (born 1889) and Anna (born 1893).

Boris Vladimirovich's father - Vladimir Vasilyevich Gnedenko - graduated from the land management school and worked as a land surveyor. Mother - Maria Stepanovna (1886-1961) - was born in Kostroma, she graduated from the gymnasium (seven-year college), in which she received a musical specialization (piano playing), which gave her the right to teach music. Brother - Gleb Vladimirovich - was born on November 1, 1909, was killed on October 27, 1943 when crossing the Dnieper in the Dnipropetrovsk region.

In 1915 the family moved to Kazan, where simultaneously with the work of the land surveyor Vladimir Vasilievich from the autumn of 1916 he became a student of the physics and mathematics faculty of the university. In the spring of 1918, on the false denunciation of one of his colleagues, Vladimir Vasilievich was arrested and spent more than six months in a concentration camp near Kazan. His health was severely undermined, and upon his return home he was forced to leave the student bench.

In the same autumn of 1918, Boris Vladimirovich (BV) entered the school. As he himself writes in his memoirs: "Everything would be fine if there were no arithmetic. I really did not like arithmetic, although I added, subtracted, multiplied and divided quite well. I was fond of poetry. "

April 4, 1922, Vladimir Vasilyevich again arrested, and he spent more than three months in prison GPU. It is released on July 12. Staying in Kazan was dangerous, and the family moved to Galich in September, where Vladimir Vasilievich began to work as a senior land surveyor. By the arrival of the family in Galich, the recruitment to the schools was completed, and this year Mom deals with Boris and his brother Gleb. "Mom learned the program and began to engage with us so that we did not fall behind. Got a textbook of grammar, Kiselev's arithmetic, Ivanov's textbook of geography. I read with special pleasure the textbook of geography and taught the rules of grammar of the Russian language. <...> In the summer we were enrolled with my brother to school in the same sixth grade. "

In April 1925 the family moved to Saratov. This was due to the fact that parents began to worry about the further education of their children, who two years later had to finish school (at that time secondary education was nine years old).

In Saratov, the brothers were enrolled in school No. 3, a former real school. It turned out that they were seriously behind in chemistry and mathematics. In the autumn, they were assigned re-examinations in these subjects. This proved to be very useful. "We were able to think through all the material on mathematics and chemistry, solve a lot of dozens of problems, and in the autumn, thanks to this, the re-examination was successful. Moreover, chemistry and mathematics began to be perceived completely freely, the tasks did not cause any difficulties, and I began to solve problems immediately in my mind as soon as I recognized the condition. In mathematics and chemistry, I moved to the top of the class. Classmates started to contact me for help. Mathematics began to please me. <...> I liked to study, to read books in addition, to solve non-standard problems. <...> I got a collection of competitive tasks offered at the entrance exams to the Petrograd Institute of Railway Engineers. No problem from this collection caused me difficulties ... <...> I realized that I want to study further and I will seek this right. I carefully studied the rules for admission to universities in the country and everywhere I came across one requirement that I did not satisfy - the incoming student must be 17 years old, I was only 15. <...> My brother wanted to become either an engineer or a physicist, and I dreamed of shipbuilding . I even sent a letter to the Leningrad Shipbuilding Institute asking me to admit me to entrance examinations in my fifteen years. "

From the city on the Neva on this letter BV received a refusal. Then he sends a letter to the People's Commissar for Education A.V. Lunacharsky with a request to allow him to enter Saratov University. By the beginning of the entrance examinations permission was obtained.

Since the autumn of 1927, BV - student of the Physics and Mathematics Faculty of the Saratov University. "In May 1930 we were told that we would be engaged all summer long, so that in September we would go to work places. It was decided to organize an accelerated release. <...> The exams were handed over, and in mid-August we received documents about the graduation from Saratov University. I felt neither joy nor satisfaction from this. I understood that a flawed education had been received and many efforts must be made to rectify the situation. "

One of the university teachers BV - Professor Georgy Petrovich Boev - at this time was invited to manage the chair of mathematics in the Textile Institute, organized in Ivanovo-Voznesensk and, in turn, invited BV to the post of assistant of this department.

In Ivanovo-Voznesensk BV taught and dealt with the application of mathematical methods in the textile business. Here he wrote his first work on the theory of mass service, here BV was carried away by the theory of probability. This period of activity played a huge role in its formation as a scientist and teacher.

Realizing the need to deepen their mathematical knowledge, BV in 1934 entered the graduate school of the Mechanics and Mathematics Faculty of Moscow State University. Its scientific supervisors are A.Ya. Khinchin and A.N. Kolmogorov.

In postgraduate study BV was carried away by limit theorems for sums of independent random variables. June 23, 1937 he defended his thesis on the topic "On some results on the theory of infinitely divisible distributions," and from September 1 of the same year he is a junior research fellow at the Institute of Mathematics of the Moscow State University.

In the works of A.Ya. Khinchin and G.M. Bavli established that the class of possible limit distributions for sums of independent random variables coincides with the class of infinitely divisible distributions. It remained to clarify the conditions for the existence of limit distributions and the conditions for convergence to each possible limiting distribution. The merit of setting and solving these problems belongs to B.V. Gnedenko. To solve the problems that have arisen, BV proposed an original method, called the method of accompanying infinitely divisible laws (the idea of the method appeared in October 1937 and published in the "Reports of the Academy of Sciences of the USSR" in 1938). He allowed a single method to obtain all the results previously found in this field, as well as a number of new ones.

On the night of the 5th to the 6th of December 1937, Boris Vladimirovich was arrested. He was shown a far-fetched charge of counter-revolutionary activity and participation in a counterrevolutionary group headed by Professor A.N. Kolmogorov. He was taken to interrogation, during one of which he was not allowed to sleep for eight days. They demanded to sign papers confirming the accusations. Boris Vladimirovich did not sign anything that could be blamed on him, A.N. Kolmogorov or anyone else. In late May 1938, he was released a number of new ones.

Since the autumn of 1938, BV - Associate Professor of the Department of Probability, Faculty of Mechanics and Mathematics, Moscow State University, Academic Secretary of the Institute of Mathematics of Moscow State University. By this period are the works of B.V. Gnedenko, in which two important problems are solved. The first one concerned the construction of asymptotic distributions of the maximal term of the variational series, elucidation of the nature of the limiting distributions and the conditions for convergence to them. The second problem concerned the construction of a theory of corrections to the indications of Geiger-Muller counters used in many fields of physics and technology.

May 28, 1941 BV defended his doctoral dissertation, consisting of two parts: the theory of summation and the theory of the maximal term of the variational series.

During the Great Patriotic War, BV took an active part in solving numerous tasks related to the defense of the country.

In February 1945, Boris Vladimirovich was elected a Corresponding Member of the Academy of Sciences of the Ukrainian SSR and sent by the Presidium of the Academy of Sciences of the Ukrainian SSR to Lviv to restore the work of the Lviv University.

In Lviv, BV reads a variety of lecture courses: mathematical analysis, calculus of variations, theory of analytic functions, probability theory, mathematical statistics, etc., in the final formulation proves the local limit theorem for independent, identically distributed lattice summands (1948), begins research on nonparametric methods of statistics . In Lviv, they were brought up talented students - E.L. Rvacheva (Yushchenko), Yu.P. Studnev, I.D. Quit et al.

The course of lectures on probability theory served as a basis for Boris Vladimirovich to write the textbook "Course of the theory of probability" (1949). This book has been published many times in different countries and is one of the main textbooks on probability theory in our days. In the same years he, together with A.N. Kolmogorov wrote the monograph "Limit distributions for sums of independent random variables" (1949), for which the authors were awarded the Prize of the USSR Academy of Sciences. P.L. Chebysheva (1951). Together with A.Ya. Khinchin BV writes "Elementary introduction to the theory of probability" (1946), which, in turn, withstood many publications in the USSR and abroad (the 12th Russian-language edition was published in 2012 by the publishing house "Editorial URSS"). In addition, Boris Vladimirovich wrote a remarkable book "Essays on the History of Mathematics in Russia" (1946) (the 4th edition of this book was published in 2009 by the Publishing House "Librocom").

In 1948, BV elected academician of the Academy of Sciences of the Ukrainian SSR, and in 1950 the Presidium of the Academy of Sciences of the Ukrainian SSR transferred him to Kiev. Here he heads the department of probability theory, which was just created at the Institute of Mathematics of the Academy of Sciences of the Ukrainian SSR, and at the same time starts to head the chair of probability theory and algebra at the Kiev University. Very soon a group of young people, interested in probability theory and mathematical statistics, formed around him. The first Kiev students BV were V.S. Korolyuk, V.S. Mikhalevich and A.V. Skorokhod.

At this time BV was carried away by himself and carried away many of his students and colleagues with tasks connected with checking the homogeneity of the two samples. V.S. Korolyuk, V.S. Mikhalevich, E.L. Rvacheva (Yushchenko), Yu.P. Studnev et al received serious results in this field.

At the end of 1953 BV was sent to the GDR for lecturing at the University. Humboldt (Berlin). He spent the whole of 1954 there. During this time, BV managed to interest a large group of young German mathematicians (D. Koenig, I. Kirstan, K. Mattes, V. Richter, G.- I. Rossberg, etc.) with problems of probability theory and mathematical statistics. The Government of the GDR awarded Boris Vladimirovich with the Order of Merit for the Fatherland in silver (1968), and the University of Humboldt elected him an honorary doctor.

Returning at the end of 1954 in Kiev, BV on behalf of the Presidium of the Academy of Sciences of the Ukrainian SSR, led the work on the organization of the Computing Center. A collective was created, which included employees of the laboratory of Academician S.A. Lebedev, the author of the first computer in continental Europe, known as MESM (small electronic computer). The laboratory by this time was headed by its oldest employees – E.A. Shkabara and L.N. Dashevsky. S.A. Lebedev has already moved to Moscow, where he was entrusted with the organization of the Institute of Precision Mechanics and Computing. This team also included mathematicians, among whom, in the first place, should be called V.S. Korolyuk, E.L. Yushchenko and I.B. Pogrebyskogo. Work began on designing a universal machine "Kiev" and a specialized machine for solving systems of linear algebraic equations.

Simultaneously BV began to read a computer programming course at the university and headed the work on writing a textbook on programming. This course (the first in the USSR book on programming in the open press) was published in Moscow in 1961 (authors – B.V. Gnedenko, V.S. Korolyuk, E.L. Yushchenko). At the same time (1955) the Presidium of the Academy of Sciences of the Ukrainian SSR assigned to B.V. Gnedenko duties of the director of the Institute of Mathematics of the Academy of Sciences of the Ukrainian SSR and the chairman of the bureau of the physico-mathematical branch of the Academy of Sciences of the Ukrainian SSR.

During this period, Boris Vladimirovich began to develop two new areas of applied scientific research - the theory of mass service (TEM) and the application of mathematical methods in medicine.

To the first he attracted I. N. Kovalenko, T.P. Maryanovich, N. V. Yarovitsky, S. M. Brody, and others. Applied methods of TEM to the calculation of electrical networks of industrial enterprises. In 1959, Lectures on the theory of mass service (issue 1) were published, read by BV in the Quilting in 1956 about 57 years. Then came the issues 2 (1960), issues 3 (1963, together with I.N. Kovalenko). These books served as the basis for the monograph "Introduction to the theory of mass service" (1966), written by B.V. Gnedenko and I.N. Kovalenko.

The second direction is connected with the development of an electronic diagnostician of heart diseases. B.V. Gnedenko, N.M. Amosov, E.A. Shkabara and M.A. Kulikov worked on this problem. In early 1960, the assembly of the world's first diagnostician was completed.

Having moved to Moscow in July 1960, Boris Vladimirovich resumed his work at the Mechanics and Mathematics Faculty of Moscow State University. Work again completely captured him: reading a variety of lecture courses, new students, new duties.

In the year of Boris Vladimirovich's fiftieth birthday (1962), Andrei Kolmogorov wrote in the journal "Theory of Probability and its Applications": "Academician of the Academy of Sciences of the Ukrainian SSR Boris Gnedenko is one of the most outstanding mathematicians currently working in the field of probability theory. He combines an exceptionally subtle possession of the methods of classical analysis with an understanding of the broad contemporary problems of probability theory and with a constant interest in its applications. "

In 1961, BV together with Ya.M. Sorin, Yu.K. Belyaev, A.D. Soloviev, Ya.B.Shore, L.Ya. Shuhgalter organizes a seminar on reliability at the Polytechnic Museum, which has worked effectively for many years. Soon there is a need to organize a separate seminar specifically on the mathematical methods of reliability theory. This seminar begins to work at the Faculty of Mechanics and Mathematics of Moscow State University under the direction of B.V. Gnedenko, A.D. Solovyev, Yu.K. Belyaev and I.Kovalenko, who at that time worked in Moscow. The seminar on mathematical methods in reliability theory worked regularly until the end of the eighties. He helped in the scientific sense to stand up to many of its participants, now widely known to experts in the field of reliability, such as E.Yu. Barzilovich, V.A. Kashtanov, I.A. Ushakov, etc. This seminar influenced, in turn, on its leaders, and encouraged B.V. Gnedenko, Yu.K. Belyaev and A.D. Solovyov to write the monograph "Mathematical methods in theory widely known in our country and abroad" Reliability "(1965). For a series of works in the field of reliability BV together with the closest assistants was awarded in 1979 the USSR State Prize.

In connection with the reliability problems of BV again returned to the study of limit theorems for sums of independent random variables, but already in a random number. To this line of research BV attracts many of his students. For these works in 1982 he was awarded the Prize. M.V. Lomonosov first degree, and in 1986 - the prize of the Ministry of Higher Education of the USSR.

BV did not cease to be interested in the history of mathematics, having connected his students to this line of work. In various domestic and foreign journals, his articles on this line of research were published, and his "Essay on the History of Probability" gives the most complete picture of his views on the history of this science.

Together with AI. Markushevich BV supervised the seminar on pre-education in secondary school. He closely cooperated with the editions of the journals "Bulletin of Higher School" and "Mathematics in School". In these and many foreign journals, in the collections of the Scientific and Methodological Council of the USSR Ministry of Higher Education, he published a large number of articles on various aspects of teaching. On the same issues, BV in those years he also wrote several books.

In January 1966, A. Kolmogorov gave B.V. Gnedenko leadership of the Department of Probability Theory of the Faculty of Mechanics and Mathematics of Moscow State University. Headed until the last days of his life.

While still working in Lviv, BV a lot of time and effort gave work to the society "Knowledge". Since 1949 he has been consistently elected chairman of the regional board of the society, he headed the republican physico-mathematical section of society, he was a member of the Presidium of the Board of the All-Union Society "Znanie", chairman of the "Knowledge" of the Moscow State University.

BV was a member of the editorial boards of a number of domestic and foreign journals, was a member of the Royal Statistical Society (Great Britain), was elected Honorary Doctor of Berlin University, Honorary Doctor of the University of Athens, University of Economics and Business.

In the last years of life, knowing the severe sentence of doctors, BV continues to lead the department, proposes and implements the idea of creating in the Faculty of Mechanics and Mathematics an economic specialization and training in its framework for specialists in the field of actuarial and financial mathematics. In addition, he outlines a list of books that you need to write in the remaining time. And he writes. Finally blinded, dictates, but fulfills the intended.

December 27, 1995 Boris Vladimirovich was gone. He is buried in the Kuntsevo cemetery in Moscow.

David Kendall and Yu.M. Suhov in the obituary "Boris Vladimirovitch Gnedenko" ("Bernoulli", 1997, 3 (1), 121-122) wrote:

"His death marks the end of a magnificent and fruitful era that forever transformed the theory of probability and significantly expanded its horizons and the number of its applications."

B.V. Gnedenko left a lot of students. Among them are academicians and corresponding members of various academies, professors and associate professors. In their memory, unforgettable days of familiarizing themselves with science and independent creativity under the guidance of a great scientist and teacher, the hours of direct communication with a Man of great erudition and high culture remain.

The Bibliography

1933 год

1. Методика составления эмпирических формул (совм. с Г.П.Боевым. «Бюллетень ИВНИТИ», N3, 24-37; № 6, 46-65).
2. О связи коэффициента неровноты с вариационным коэффициентом (совм. с Г.П.Боевым. «Бюллетень ИВНИТИ», N7, 48-53).
3. К вопросу о распределении обратных величин («Бюллетень ИВНИТИ», № 8-9, 55 - 62).
4. О статистическом распределении степенных функций (Сборник научно-исследовательских работ ИВНИТИ, 18 - 22).
5. О нормировании методом станкообходов (совм. с Г.В.Соколовым. Сборник научно-исследовательских работ ИВНИТИ, 157 - 182).

1934 год

6. К методике и технике установления потерь в производительности оборудования текстильной промышленности из-за неуспеваемости («Бюллетень ИВНИТИ», № 3-4, 114 -117).
7. Вычисление среднего перехода между станками («Бюллетень ИВНИТИ», № 1-2, 117-122).
8. О вычислении среднего перехода между станками конечных размеров («Бюллетень ИВНИТИ», № 10-12, 118-122)
9. О среднем простое станков при многостаночном обслуживании из-за неуспеваемости рабочего («Известия хлопчато-бумажн. промышленности», № 11, 15 - 18).

1936 год

10. Методика составления эмпирических зависимостей и номограмм в текстильном деле (совм. с Г.П.Боевым и Ю.С.Виноградовым. Гизлегпром, 1-128).
11. О Θ в формуле Лагранжа («Математическое просвещение», № 7, 31 - 35).

1937 год

12. Об единственности системы ортогональных функций, инвариантной относительно дифференцирования (Доклады АН СССР, т.14, N4, 159 - 161).
13. Об одном характеристическом свойстве безгранично делимых законов распределения («Бюллетень МГУ. Математика и механика», т.1, вып. 5, 10-16).
14. О характеристических функциях («Бюллетень МГУ. Математика и механика», т.1, вып. 5, 17 - 18).

1938 год

15. О сходимости законов распределения сумм независимых слагаемых (Доклады АН СССР, т. 18, № 4-5, 231 - 234).

1939 год

16. О предельных теоремах для сумм независимых слагаемых (Доклады АН СССР, т. 22, № 2, 61 - 64).
17. К теории предельных теорем для сумм независимых случайных величин («Известия АН СССР», серия математическая, № 2, 181-232).

18. К теории предельных теорем для сумм независимых случайных величин (исправления к статье под тем же названием) («Известия АН СССР», серия математическая, № 6, 643-647).
19. Об областях притяжения устойчивых законов («Доклады АН СССР», т. 24, № 7, 640 - 642).
20. К теории областей притяжения устойчивых законов («Ученые записки МГУ», вып. 30, 61 - 81).
21. О предельных законах теории вероятностей («Доклады АН СССР», т. 23, № 9, 868 - 871).
22. О сходимости законов распределения нормированных сумм независимых случайных величин (совм. с А.В.Грошевым. «Математический сборник», т. 6, (48), № 3, 521 - 541).
23. Обзор современного состояния теории предельных законов для сумм независимых слагаемых («Ученые записки Тюменского пединститута», вып. 1, 5 - 28).
24. Андрей Николаевич Колмогоров (совм. с П.С.Александровым, В.В.Степановым, И.Г.Петровским, А.Я.Хинчиным. Статья напечатана под рубрикой «Кандидаты в действительные члены Академии Наук СССР»). («Правда», 19 января).

1940 год

25. Несколько теорем о степенях функций распределения. («Ученые записки МГУ», вып. 45, 61 - 71).

1941 год

26. К теории счетчиков Гейгер-Мюллера («Журнал экспериментальной и теоретической физики», т. 11, вып. 1, 101 - 106).
27. Предельные теоремы для максимального члена вариационного ряда («Доклады АН СССР», т. 32, № 1, 7-9).
28. Зона действия истребителя-перехватчика (отчет сдан заказчику).
29. Исследование влияния ветра на траекторию самолета, летящего на радиостанцию с постоянным курсовым углом и постоянной скоростью (отчет сдан заказчику).
30. О некоторых задачах теории стрельбы (отчет сдан заказчику).
31. Гордость советской математики (совм. с Н.В. Смирновым. «Лауреаты Сталинской премии ученые МГУ». Издание МГУ, 26-27).
32. Замечательный воспитатель молодежи («Лауреаты Сталинской премии ученые МГУ». Издание МГУ, 27-28).

1942 год

33. О локально устойчивых законах распределения («Доклады АН СССР», т.35, № 9, 295 -298).
34. Локально устойчивые законы распределения («Известия АН СССР», серия математическая, № 6, 291 - 308).
35. Исследование роста однородных случайных процессов с независимыми приращениями («Доклады АН СССР», т. 36, № 1, 3-4).
36. Средний расход снарядов в случае поражения цели одним попаданием (отчет сдан заказчику).

1943 год

37. О росте однородных случайных процессов с независимыми приращениями («Известия АН СССР», серия математическая, т.7, 89 - 110).
38. Sur la distribution limite du terme maximume d'une serie aleatoire (Annals of Mathematics v.44, № 3, 423-453).

39. О росте однородных случайных процессов с независимыми однотипными приращениями («Доклады АН СССР», т.11, № 3, 103-107).
40. О законе повторного логарифма для однородных случайных процессов с независимыми приращениями («Доклады АН СССР», т. 11, № 7, 291 - 293).
41. Решение одной задачи теории ошибок механизмов от торцевого биения (отчет сдан заказчику).

1944 год

42. Элементы теории функций распределения случайных векторов («Успехи математических наук», т. X, 230 - 244).
43. Предельные законы для сумм независимых случайных величин («Успехи математических наук», т. X, 115 - 165).
44. Теория ошибок одного аэрокартографического прибора (совм. с инженер-майором М.П.Бордюковым; отчет сдан заказчику).

1946 год

45. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. ГИТТЛ, 1 - 128).
46. Очерки по истории математики в России (ГИТТЛ, 1 - 247).
47. Краткие беседы о зарождении и развитии математики (Издательство Академии педагогических наук РСФСР, 1 - 40).
48. Русская школа теории вероятностей (М. Институт истории естествознания АН СССР. «Тезисы докладов к совещанию по истории естествознания, 24-26 декабря 1946», 39-40).
49. Видатний математик (до 145-річчя з дня народження М.В. Остроградського) (Киевский университет, газета «За радянські кадри», 14 октября, № 29-30).
50. О работе университетского издательства (Газета «Московский Университет», 23 апреля).

1947 год

51. Как математика изучает случайные явления (Издательство АН УССР, 1 - 75).
52. К вопросу об ошибках рычажного пантографа (совм. с М.П.Бордюковым. «Вестник Военно-инженерной Академии им. Куйбышева», т. 48, вып. 6, 68 - 79).
53. Об эллипсоидах рассеивания («Ученые записки Львовского университета», т. 5, вып. 2, 116 - 120).
54. О функциях от случайных величин («Известия Львовского университета», т. 5, вып. 2, 121 - 128).
55. Стефан Банах («Ученые записки Львовского университета», т. 5, вып. 1, 5 - 9).

1948 год

56. Об одной теореме С.Н.Бернштейна («Известия АН СССР», серия математ., т. 12, 97 - 100).
57. К теории роста однородных случайных процессов с независимыми приращениями («Труды Института математики АН УССР», № 10, 60 - 82).
58. Про одну характеристичну властивість нормального закону розподілу (совм. с Е.Л.Рвачевой. «Доповіді АН УРСР», № 3, 3 - 5).
59. Об одном характеристическом свойстве нормального закона распределения (совм. с Е.Л.Рвачевой. «Труды Института математики АН УССР», № 11, 36 - 42).
60. О локальной предельной теореме теории вероятностей («Успехи математических наук», т. 3, вып 8, 187 - 194).

61. Развитие теории вероятностей в России («Труды Института истории естествознания», т. 2, 390 - 425).
62. Русская школа теории вероятностей (М.Л. Изд-во АН СССР. Институт истории естествознания. "Труды совещания по истории естествознания 24-26 декабря 1946 г. Тезисы", 192 - 193).
63. Теория вероятностей (совм. с А.Н.Колмогоровым. «Математика в СССР за 30 лет», Гостехиздат, 701 – 727, 739-756).
64. Математика в Московском университете в XX в (до 1940г) (совм. с П.С.Александровым и В.В.Степановым. «Историко-математические исследования», 1-я серия, вып. 1, 9 - 42).
65. Михаил Васильевич Остроградский (Сборник "Люди русской науки", ГИТТЛ, т. 1, 99-104).
66. Пафнутий Львович Чебышев (Сборник "Люди русской науки", ГИТТЛ, т. 1, 111-121).
67. Андрей Андреевич Марков (Сборник "Люди русской науки", ГИТТЛ, т. 1, 179-185).
68. С.С.Банах (В книге Банах С.С. «Курс функціонального аналізу». Киев, изд-во «Радянська школа», 1-2).
69. Рецензия на книгу Г. Крамера «Случайные величины и распределения вероятностей» («Успехи математических наук», т. 3, вып. 6, 220-221).
70. Рецензия на книгу Г. Крамера «Математические методы статистики» («Успехи математических наук», т. 3, вып. 4, 184-186).

1949 год

71. Предельные распределения для сумм независимых случайных величин (совм. с А.Н.Колмогоровым. ГИТТЛ, 1 - 264).
72. Курс теорії імовірностей («Радянська школа», Київ, Львів, 1949, 1-128).
73. О локальной предельной теореме для случая бесконечной дисперсии («Труды Института математики АН УССР», № 12, 22 - 30).
74. О некоторых свойствах предельных распределений для нормированных сумм («Украинский математический журнал», т.1, 3-8).
75. О локальной теореме для областей нормального притяжения устойчивых законов («Доклады АН СССР», т. 66, № 3, 325 - 326).
76. О локальной теореме для предельных устойчивых распределений («Украинский математический журнал», № 4, 3-15).
77. Последовательный анализ (Ташкент. Изд-во АН УзССР. «Труды второго Всесоюзного совещания по математической статистике», 5 - 23).
78. О работах Н.И.Лобачевского по теории вероятностей («Историко-математические исследования», 1-я серия, вып. 2, 129 - 136).

1950 год

79. Курс теории вероятностей (ГИТТЛ, 1 - 360).
80. Курс теорії імовірностей («Радянська школа», Київ, Львів, 1 - 387).
81. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. ГИТТЛ, 2-е издание, 1 - 144).
82. Об области притяжения нормального закона («Доклады АН СССР», т.71, № 3, 425 - 428).
83. Несколько замечаний к теории областей притяжения устойчивых законов (совм. с В.С.Королюком. «Доповіді АН УРСР», № 4, 275 - 278).
84. Теория вероятностей и познание реального мира («Успехи математических наук», т. 5, вып. 1, 3 - 23).
85. Theorie pravdepodobnosti a poznavani realneho sveta («Otazky statistiky», 12 (32), 65).
86. Про деякі питання викладання математики в університеті (Киевский университет, газета «За радянські кадри», 18 ноября, № 27).

87. «Арифметика» Магницького (еженедельная газета «Зірка», 3 февраля, № 6).

1951 год

88. Fuggetlen valoszinusegi valtozok osszegeinek hatareloszlasai (es A.N.Kolmogorov. Akademiai Kiado, Budapest, 1-256).
89. Limit theorems for sums of independent random variables (American Mathematical Society, 1 - 82). (Смотри «Успехи математических наук», 1944, № 10, 115-165).
90. О максимальном расхождении двух эмпирических распределений (совм. с В.С. Королюком. «Доклады АН СССР», т.80, № 4, 525 - 528).
91. Про імовірне відхилення («Доповіді Академії наук УРСР», 1951, № 2).
92. Несколько замечаний о локальной предельной теореме теории вероятностей («Ученые записки Киевского университета», т. X, вып. 1, «Математический сборник», № 5, 21 - 28).
93. О работах М.В.Остроградского по теории вероятностей («Историко-математические исследования», 1-я серия, вып. 4, 99 - 123).
94. М.В.Остроградський (совм. с Е.Я.Ремез. «Вісник АН УРСР», № 9, 61 - 70).
95. Михаил Васильевич Остроградский («Успехи математических наук», т.6, вып. 5, 3 - 25).
96. Попередне повідомлення про рукописи М.В.Остроградського (совм. с Е.Я.Ремез. «Вісник АН УРСР», № 8, 52 - 63).
97. Михаил Васильевич Остроградский («Украинский математический журнал», т.3, № 3, 235 - 239).
98. Михаил Васильевич Остроградский (Общество по распространению политических и научных знаний УССР, Киев, 1 - 41).
99. Історія математики як дисципліна викладання і як предмет наукового дослідження (Методічний збірник "Математика в школі", вип.5, 14 - 31).
100. Про бесіди з історії науки на уроках математики (Журнал "Радянська школа", № 3, 44 - 49).
101. Механіко-математичний (Киевский университет, газета «За радянські кадри», 11 июня, № 15).
102. М.В.Остроградський (Киевский университет, газета "За радянські кадри", 28 сентября, № 20).
103. Выдающийся ученый и педагог («Учительская газета», 22 сентября, № 76).
104. Видатний учений і педагог (совм. с Е.Л. Рвачевой. «Київська правда», 23 сентября, № 189).
105. Выдающийся русский ученый (газета «Защитник Отечества», 26 мая, № 121).
106. Выдающийся русский ученый («Красная звезда», 23.09., № 224).

1952 год

107. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. ГИТТЛ, 3-е изд., 1 -- 144).
108. Elementy rachunku pravdopodobienstwa (совм. с А.Я. Хинчиным. Warsawa, Panstwowe wydawnictwo naukowe, 1 - 155).
109. М.В.Остроградский (Очерки жизни, научной и педагогической деятельности, ГИТТЛ, 1 - 331).
110. З історії математики в Россії (Радянська школа, 1 -- 40).
111. Декілька зауважень до статей О.А.Ілляшенка і Й.І. Гіхман («Доповіді АН УРСР», № 1, 10 - 12).
112. Об одной задаче сравнения двух эмпирических распределений (совм. с Е.Л.Рвачевой. «Доклады АН СССР», т. 82, № 4, 513-516).
113. Некоторые результаты о максимальном расхождении между двумя эмпирическими распределениями («Доклады АН СССР», т. 82, № 5, 661 - 663).

114. О распределении числа выходов одной эмпирической функции распределения над другой (совм. с В.С. Михалевичем. «Доклады АН СССР», т. 82, № 6, 841 - 843).
115. Две теоремы о поведении эмпирических функций распределения (совм. с В.С. Михалевичем. «Доклады АН СССР», т. 85, № 1, 25 - 27).
116. Порівняння эфективності деяких методів перевірки однорідності статистичного матеріалу (совм. с Ю.П.Студневым. «Доповіді АН УРСР», № 5, 359 - 363).
117. Зависимость неровноты пряжи от длины образца («Текстильная промышленность», № 3, 27 - 31).
118. О полных ортогональных системах тригонометрических функций («Вопросы элементарной и высшей математики», вып.1, изд. Харьковского гос. университета, 24 - 34).
119. Выдающийся русский ученый М.В.Остроградский (Изд-во общества "Знание", Москва, 1 - 24).
120. Про філософські проблеми математики в зв'язку з ii викладанням (Методичний збірник "Математика в школі", вип.7, 7-23).
121. Выдающийся русский ученый (Газета "Защитник Отечества", № 121, 26.5).

1953 год

122. Introducere elementara in calculul probabilitatilor (si A.Hinchin. Bucuresti, Editure Tehnica, 1 - 116).
123. О роли максимального слагаемого при суммировании независимых случайных величин («Украинский математический журнал», т. 5, № 3, 291 - 298).
124. О некоторых свойствах срединного уклонения («Труды Института математики и механики АН УзССР», вып. 10, ч. 1, 26-35).
125. Об одной работе П.Л.Чебышева, не вошедшей в полное собрание сочинений («Историко-математические исследования», вып. 6, 215-222).
126. Две лекции по философским вопросам математики (Киевский гос. университет. «Научные записки», т. XII, вып. VI, Математический сборник, № 7, 5 - 36).
127. 250 років "Арифметики" Магницького (совм. с И.Б.Погребысским. «Вісник АН УРСР», № 7, 53 - 63).
128. Про боротьбу матеріалізму з ідеалізмом в математиці («Вісник АН УРСР», № 11, 27 - 37).
129. Pafnutij Lvovic Cebysev (Prelozeno z knihy «Prehled dejin matematiky v Rusku», str 112-125, a dodatek 3, str. 232-239) (Casop. pestov. mat., 78, № 1, 89-103).
130. Aleksandr Michajlovic Ljapunov (Prelozeno z knihy «Prehled dejin matematiky v Rusku», str. 133-143) (Casop. pestov. mat., 78, № 1, 105-112).
131. Колмогоров Андрей Николаевич (Большая Советская Энциклопедия, т. 22, стр. 13).
132. Советская школа теории чисел (Deutscher Verlag der Wissenschaften, Berlin. «Mathematik», 1, Russische fachtexte fur den Hochschulunterricht, heft 6, 3 - 20).
133. Советская школа теории вероятностей (Deutscher Verlag der Wissenschaften, Berlin. «Mathematik», 1, Russische fachtexte fur den Hochschulunterricht, heft 6, 21 - 62).

1954 год

134. Курс теории вероятностей (ГИТТЛ, 2-ое изд., 1 - 411).
135. Limit Distributions for the Sums of Independent Random Variables (with A.N. Kolmogorov. Addison-Wesley, USA, 1 - 264).
136. Limit Distributions for the Sums of Independent Random Variables (Translated and annotated by K.L. Chung. With an Appendix by J.L. Doob. Addison-Wesley Publishing Company, Inc., Cambridge, Mass. IX + 264 pp.).

137. Bevezetes a valoszinugszamitasba (совм. с А.Я. Хинчиным. Budapest, Müvelt nép Konivkiado, 1 - 141).
138. Elementarny wstep do rachunku prawdopodobienstwa (совм. с А.Я. Хинчиным. Warszawa, Panstwowe Wydawnictwo Naukowe, 1 - 158).
139. Elementarni uvod do theorie pravdepodobnosti (совм. с А.Я. Хинчиным. Praga, Statni nakladatelstvi technike literatury, 1 - 115).
140. Предельные теоремы для сумм независимых слагаемых и цепей Маркова («Украинский математический журнал», т. 6, № 1, 5 - 20).
141. Локальная предельная теорема для плотностей («Доклады АН СССР», т. 95, № 1, 5 - 7).
142. Проверка неизменности распределения вероятностей в двух независимых выборках (с добавлением «Kriterien fur die Unveranderlichkeit der Wahrscheinlichkeitsverteilung von zwei unabhungigen Stichprobenreihen» («Mathematische Nachrichten», Band 12, Heft 1/2, 29 - 66).
143. О локальной предельной теореме для одинаково распределенных независимых слагаемых «Wissenschaftliche Zeitschrift der Humboldt Universitat zu Berlin», № 4, 287 - 293).
144. Розвиток теорії імовірностей на Україні (совм. с И.И.Гихманом. «Праці Київського ун-та, Природничі науки», № 11, 59 - 94).
145. Wahrscheinlichkeitsrechnung und Mathematische Statistik (und L.Kalujnin. «Das Hochschulwesen», № 8-9, 50 - 54).
146. Über die Formen der Kollektiven Wissenschaftlichen Arbeit in sowietischen Hochschulwesen («Das Hochschulwesen», № 3, 1 - 6).
147. Über den Kampf zwischen Materialismus und den Idealismus in der Mathematik (zusam. L.A.Kalujnin. «Wissenschaftliche Zeitschrift der Technischen Hochschule Drezden», B. 3, heft 5, 631 - 638).
148. Despre lupta dintre materialism si idealism in matematica («Analele Romino-Sovietice, matematica-fizica», № 3, 68 - 80).
149. О математической жизни в ГДР (совм. с Л.А.Калужниным. «Успехи математических наук», т. 9, вып., 4, 133 - 154).
150. Подготовка математиков в Дрезденской высшей технической школе («Вестник высшей школы», № 11, 59 - 61).
151. Рецензия на полное собрание сочинений П.Л.Чебышева («Успехи математических наук», т.9, вып. 4, 263 - 266).
152. Die Wahrscheinlichkeitsrechnung und die Erkenntnis der realen Welt (Naturwissenschaftliche Reiche, 22 oktober 1954, № 38, Wissenschaftliche Beilage des Forum).
153. Wahrscheinlichkeitsrechnung und ihre praktische Anwendung («Neues Deutschland», 24.X, № 250).
154. Wahrscheinlichkeitsrechnung und Produktion («Tagliche Rundschau», 16.X, № 240 (2871)).

1955 год

155. Курс теории вероятностей (Пекин, 1 - 426).
156. Предельные распределения для сумм независимых слагаемых (совм. с А.Н.Колмогоровым. Пекин, Кюсюэ Чубаньше, 1 - 280).
157. Elementare Einfurung in die Wahrscheinlichkeitsrechnung (und A.Chintschin. Berlin, Deutscher Verlag der Wissenschaften, 1 - 136).
158. О популярных лекциях по математике и ее истории (Киев, общество "Знание", 1 - 44).
159. Александр Яковлевич Хинчин (к 60-летию со дня рождения) («Успехи математических наук», т.X, вып.3, 197 - 212).
160. Остроградский Михаил Васильевич (Большая Советская Энциклопедия, т. 31, 346 - 347).
161. Über die Ausbildung der Mathematik und Physiklehrer in der Sowietunion («Mathematik und Physik in der Schule», № 11, 489 - 497).

162. Walka między materialismem a idealismem w matematyce (совм.с Л.А.Калужниным. Warszawa. «Matematika. Czasopismo dla nauczicieli», № 5-6, 1 - 8).
163. Букви замість цифр (Киев, «Зірка», 8 апреля, № 15).

1956 год

164. Курс теории вероятностей (пер. на китайский со 2-ого издания, Пекин, 1 - 449).
165. Über die Nachprüfung statistischer Hipotesen mit Hilfe der Variationsreihe (Berlin. Deutscher Verlag der Wissenschaften. «Bericht über die Tagung Wahrscheinlichkeitsrechnung und die Mathematische Statistik in Berlin vom. 19 bis 22 october 1954», 97 - 107).
166. Непараметрические задачи статистики. (Совместно с И.И.Гихманом и Н.В.Смирновым. Москва, изд-во АН СССР. Труды третьего Всесоюзного математического съезда (Москва, июнь-июль 1956), т. II. Краткое содержание обзорных и секционных докладов, 47-48).
167. The main stages in the history of the theory of probability («Actes du VIII-e Congres International d'histoire des sciences» (Florence 3-9 September 1956), p. 128 - 131).
168. A.I.Hincin («Analele Romino-Sovietice, matematica-fizica», anul X -- seria III-a, № 3, 115 - 124).
169. Masinile electronice de calculat («Analele Romino-Sovietice, matematica-fizica», anul X – seria III-a, № 4, 5 - 15).
170. О развитии математики на Украине (совм. с И.Б.Погребысским. «Историко-математические исследования», 1-я серия, вып.9, 403 - 426, вып.10, 766).
171. Развитие теории вероятностей на Украине (совм. с И.И.Гихман. «Историко-математические исследования», 1-я серия, вып.9, 477 - 536).
172. О работах Гаусса по теории вероятностей ("Карл Фридрих Гаусс", сборник статей к 100-летию со дня смерти. Издательство АН СССР, 217 - 240).
173. О некоторых задачах истории математики (Москва, изд-во АН СССР. Труды третьего Всесоюзного математического съезда (Москва, июнь-июль 1956), т.II, краткое содержание обзорных и секционных докладов, 100 - 101,).
174. Евгений Яковлевич Ремез (совм. с И.Б. Погребысским. «Украаинский математический журнал», т. 8, № 2, 218-222).
175. Begrussungsansprache (Berlin. Deutscher Verlag der Wissenschaften. «Bericht über die Tagung Wahrscheinlichkeitsrechnung und die Mathematische Statistik in Berlin vom. 19 bis 22 october 1954», 3-5).
176. Сучасні швидкодіючі обчислювальні машини (газета "Радянська Україна", 19 декабря, № 295).
177. 20 000 обчислень на секунду (журнал "Україна", № 12, стр. 28).
178. Геніальний математик («Наука и життя», № 2, 29 - 30).
179. Activitate ştiinţifică legată de practică (Румыния, «Scinteia», 11.05., № 3592).

1957 год

180. Курс теории вероятностей (Япония. 3-505).
181. Lehrbuch der Wahrscheinlichkeitsrechnung (Akademie-Verlag, Berlin, 1 - 387).
182. Rozklady graniczne sum zmiennych losowych niezaleznych (i A.Kolmogorov. PWN, Warszawa, 1 - 262).
183. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. ГИТТЛ, 4-е изд., 1 -- 144).
184. О некоторых задачах истории математики (Труды третьего Всесоюзного математического съезда (июнь-июль 1956 г., Москва), т. III, обзорные доклады, секция истории математики, 579 - 583, Москва, изд-во АН СССР).
185. О некоторых задачах истории математики (совм.с И.Б.Погребысским. «Украинский математический журнал», т.IX, № 4, 359 - 368).

186. О некоторых советских работах по теории информации (Prague. Publishing house of the Czechoslovak Academy of Sciences. «Transactions of the First Prague Conference on Information Theory, Statistical Decision Functions, Random Processes held at Liblice near Prague from November 28 to 30, 1956», 21-28).
187. О некоторых задачах теории вероятностей («Украинский математический журнал», т.9, N4, 359 - 368).
188. К новым успехам советской школы теории вероятностей (Изд-во «Наука», «Теория вероятностей и ее применения», т. 2, вып. 3, 289-291).
189. Електронні цифрові машини (совм. с В.М.Глушковым. «Вісник АН УРСР», № 9, 3 - 10).
190. Радянська математика за сорок років («Вісник АН УРСР», № 11, 29 - 41).
191. Чебышев Пафнутий Львович (Большая Советская Энциклопедия, т. 47, 81 - 82).
192. Über die Arbeiten von C.F.Gauss zur Wahrscheinlichkeitsrechnung (C.F.Gauss Gedenkband, Teubner, Leipzig, 194 - 204).
193. Mathematical Education in the USSR («The American Mathematical Monthly», v. 64, № 6, 389 - 408).
194. Despre formarea notiunilor matematice («Analele Romino-Sovietice, matematicā–fizicā», № 1, 89 - 100).
195. Перша нарада математиків України (Вісник АН УРСР, № 10, 69 - 72).
196. Первое совещание математиков Украины («Успехи математических наук», т. XII, вып.6, 215 - 220).
197. Два совещания по теории информации («Украинский математический журнал», т. IX, № 3, 345 - 347).
198. Месяц у математиков Румынской Народной Республики («Украинский математический журнал», т.9, № 1, 111 - 112).
199. Вклад українських математиків (журнал «Наука і життя», № 8, 3-5).
200. Майбутне обчислювальних машин («Робітнича газета», 23 марта, № 70).
201. Пути улучшения научной работы (газета «Правда Украины», 24 апреля, № 98).
202. Про математичну освіту (газета «Радянська Освіта», 14 сентября, № 37).
203. За дальший розвиток математичної науки на Україні (совместно с И.Ф.Тесленко. газета «Радянська Освіта», 20 июля, № 29).
204. Будущее науки необозримо («Правда Украины», 18 августа, № 193).
205. Мощь и зрелость нашей науки и техники («Правда Украины», 17 октября, № 244).
206. Успіхи українських математиків («Літературна газета», 30 липня, № 59).
207. Вивчайте іноземні мови (совм. с Л.А. Калужниным, газета Киевского университета «За радянські кадри», 26 января, № 7).
208. Мир потрібний усім народам («Радянська Україна», 1 мая, № 103).

1958 год

209. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. Пекин, 1 -- 132).
210. On limit theorems of the theory of probability (Киев. АН УССР. Институт математики. 1 - 35).
211. Про одну задачу масового обслуговування («Доповіді АН УРСР», № 5, 477 - 479).
212. Непараметрические задачи статистики (совм. с И.И. Гихманом и Н.В.Смирновым. Изд-во АН СССР. «Труды Третьего Всесоюзного Математического съезда. Москва, июнь-июль 1956», т. III, 320 - 334).
213. О критерии Вилкоксона сравнения двух выборок («Бюллетень Польской Академии Наук, серия математических, астрономических и физич. наук», т.6, № 10, 611 - 614).
214. Education Cientifico-Matematica, en la Union de Republicas Socialistas Sovieticas (Peru, Lima. «Nueva Educacion», , 23-30).
215. O nekterych ukolech historie matematiky (a I.B. Pogrebysskiy. Praha. Československa Akademie VĚD. «Pokroku Matematiky, Fisiky a Astronomie», t. III, N5, 526- 535).

216. О некоторых задачах истории математики («Историко-математические исследования», 1-я серия, вып. 11, 47 - 62).
217. Об истории математики и ее значении для математики и других наук (совм. с И.Б.Погребысским, «Историко-математические исследования», 1-я серия, вып. 11, 441-460).
218. О работах Л.Эйлера по теории вероятностей, теории обработки наблюдений, демографии и страхованию (Изд-во АН СССР. Сборник "Л.Эйлер, 250-летие со дня рождения", 184 - 209).
219. Очерк жизни, научного творчества и педагогической деятельности М.В.Остроградского (совм. с И.А.Мароном. Изд-во АН СССР. В книге «Избранные труды М.В. Остроградского», 380 - 457).
220. Десять лет "Историко-математических исследований" (совместно с И.Б.Погребысским. «Успехи математических наук», т. X, № 2, 229 - 230).
221. Десять лет "Историко-математических исследований" (совместно с И.Б.Погребысским. «Успехи математических наук», т. 13, № 5, 229 - 233).
222. Про міжнародні зв'язки Інституту математики АН УССР (совм. с В.Т. Гаврилюк. «Вісник АН УРСР», № 3, 66 - 67).
223. Республиканская конференция по вопросам статистических методов анализа и контроля производства («Украинский математический журнал», т.X, № 1).
224. Рецензия на книгу Л.Шметтерер «Введение в математическую статистику» (Изд-во «Наука», "Теория вероятностей и ее применения", т.III, вып. 1, 118 - 120; «Новые книги за рубежом», серия А, № 9, 13 – 15).
225. Рецензия на книгу О.Онические, Г.Михок, Ч.Ионеску Тульча "Теория вероятностей и ее приложения" (Изд-во «Наука», "Теория вероятностей и ее применения", т.III, вып. 1, 117 - 118; «Новые книги за рубежом», серия А, № 8, 20 – 23).
226. Рецензия на книгу А.Реньи "Исчисление вероятностей" (Изд-во «Наука», "Теория вероятностей и ее применения", т.III, вып. 1, 115 - 116).
227. Рецензия на книгу У.Гренандер и М.Розенблат "Статистический анализ стационарных временных рядов" (совм. со Н.П. Слободенюком) (Изд-во «Наука», "Теория вероятностей и ее применения", т.III, вып.4, 475 - 477, «Новые книги за рубежом», серия А, № 8, 24 – 27).
228. Рецензия на книгу Saunders L., Fleming R. «Математика и статистика для фармацевтов, биологов и химиков («Новые книги за рубежом», серия А, № 10, 9 – 10).
229. Рецензия на книгу Myslivec V. «Статистические методы в сельскохозяйственных и лесоводческих исследованиях» («Новые книги за рубежом», серия А, № 10, 11).
230. Наука, яка творить чудеса (Николаев, «Південна правда», 14 июня, № 117).
231. Математическая статистика и производство («Известия», 18 июня, № 145).
232. К итогам Всесоюзного совещания по теории вероятностей и математической статистике (Ереван, газета «Коммунист», 30 сентября, № 230).
233. Математическое образование («Известия», 21 декабря, № 303).
234. Посланец дружбы («Правда Украины», 17 июня, № 114).

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235. Лекции по теории массового обслуживания (Изд-во КВИРТУ, вып. I, 1 - 104).
236. Lehrbuch der Wahrscheinlichkeitstheorie (Berlin. Academy Verlag. Zweite auflage).
237. Grenzverteilungen von Summen unabhangiger Zufallsgrossen (und A.Kolmogorov. Berlin. Academie Verlag. 1 - 279).
238. Про одне узагальнення формул Ерланга («Доповіді АН УРСР», № 4, 347 - 360).
239. Математическая статистика (совм. с И.И.Гихманом. ГИФМЛ, «Математика за 40 лет», т. I, 797 - 808).

240. О методике определения расчетных нагрузок промышленных предприятий (совм. с Б.С. Мешелем. «Электричество», № 2, 13 – 15).
241. Об оценке эффективности уточнения расчетов электрических нагрузок промышленных сетей (совм. с Б.С. Мешелем. «Электричество», № 11, 70 - 72).
242. Несколько замечаний к двум работам Д.Баррера («Buletinul Institutului Politehnic», din iasi, seria noua, tomul 5(IX), fasc. 1-2, 111 - 118).
243. Математика (МСЭ, т. 5, 1018 - 1024).
244. O nekterych ulohach teorie pravdepodobnosti (Praha. Československa Akademie VĚD. «Pokroky Matematiky, Fisiky a Astronomie», t. IV, № 1, 1 - 12,).
245. Исследования по теории вероятностей и математической статистике в системе АН УССР («Украинский математический журнал», т. II, № 2, 123 - 136).
246. О работах А.М.Ляпунова по теории вероятностей («Историко-математические исследования», вып.12, 135 - 160).
247. Розвиток теорії ймовірностей у роботах О.М.Ляпунова (АН УРСР, Інститут математики, «Історіко-математичний збірник», № 1, 133 - 139).
248. Про дослідження Л.Ейлера з теорії ймовірностей, теорії обробки спостережень, демографії та страхування (АН УРСР, Інститут математики, «Історіко-математичний збірник», № 1, 71 - 76).
249. O historii matematyki i jej znaczeniu dla matematyki i innych nauk (i I.Pogrebyski. «Roczniki Polskiego Towarzystwa mattmatycznego», seria II, Wiadomosti matematyczne, t. III, № 1, 49 - 64).
250. О роли математических методов в биологических исследованиях (АН СССР, «Вопросы философии», № 1, 85 - 97).
251. Роль математики в биологических исследованиях (Изд-во АН СССР. "Философские проблемы современного естествознания", Труды Всесоюзного совещания по философским вопросам математики, , 494 - 498).
252. О математическом образовании в высших военно-учебных заведениях («Вестник противовоздушной обороны», № 12, 14 - 19).
253. Sur l'enseignement superieur des mathematiques (Lisboo. «Gaseta de Matematica», v. XX, № 76-77, 15-25).
254. The formation of mathematical concept (Actes IX Congr. internat. Histoire Scinces, Barcelona-Madrid, 1-7 Sept. 1959, p. 472).
255. "Материализм и эмпириокритицизм" В.И.Ленина и философские вопросы математики (Изд-во АН УРСР. Збірник «Геніальний філософський твір. В.І.Ленина», 300 - 318).
256. Про об'єктивний характер математичних понять (Державне видавництво політичної літератури УРСР. Збірник «Теоретична зброя комунізму», 249- 271).
257. Despre lupta dintre materialism si idealism in matematica (совм. с Л.А. Калужниным. Бухарест. «Gaseta matematica si fisica», seria A, vol. XI, № 7, 385-397).
258. Рецензия на книгу Т.В.Андерсона «Введение в многомерный статистический анализ» (Изд-во «Наука», "Теория вероятностей и ее применения", т. IV, вып. 2, 247 - 248; «Новые книги за рубежом», серия А, № 4, 14 – 16).
259. Рецензия на книгу М.Фиш "Теория вероятностей и математическая статистика" (Изд-во «Наука», "Теория вероятностей и ее применения", т. IV, вып. 3, 365 - 367; «Новые книги за рубежом», серия А, № 2, 16 – 20).
260. Рецензия на книгу McCarthy P.J. «Введение в статистические рассуждения» («Новые книги за рубежом», серия А, № 4, 17 – 19).
261. Рецензия на книгу Quenoille M.A. «Основа статистических рассуждений» («Новые книги за рубежом», серия А, № 5, 17 – 18).
262. Рецензия на книгу Vincze István «Статистический контроль качества» (совм. с М.Г. Фариничем, «Новые книги за рубежом», серия А, № 6, 35 –37).

263. Рецензия на книгу Ionescu H.M. «Элементы математической статистики» («Новые книги за рубежом», серия А, № 7, 8 –9).
264. Рецензия на книгу Sahleanu V. «Математические методы в медико–биологических исследованиях («Новые книги за рубежом», серия А, № 7, 9 – 11).
265. Рецензия на книгу Gumbel E.J. «Статистика крайних» («Новые книги за рубежом», серия А, № 9, 14 – 15).
266. Рецензия на книгу Dugué D. «Трактат по теоретической и прикладной статистике» («Новые книги за рубежом», серия А, № 9, 19 – 21).
267. Рецензия на книгу Mises R.V. «Вероятность, статистика и истина» (2-ое изд.) («Новые книги за рубежом», серия А, № 10, 3 – 4).
268. Рецензия на книгу Pollaczek F. «Стохастические проблемы, возникающие при изучении формирования очереди у кассы и аналогичных явлений» («Новые книги за рубежом», серия А, № 11, 8 - 10).
269. Математика у природознавстві (журнал "Наука і життя", № 4, 17 - 19).
270. Мрія стала дійсністю (журнал "Наука і життя", № 10, 5).
271. Кібернетика (совм. с Е.А. Шкабара. "Наука і життя", № 12, 9 - 11).
272. Стрибок у космос («Радянська Україна», 4 января, № 3).
273. Этого требует сердце (совм. с Н.М.Амосовым и М.Вепринцевым. «Комсомольская правда», 5 апреля).
274. Объеднати зуссиля («Вечірній Київ», 3 июня, № 129).
275. Всесоюзное совещание по теории вероятностей («Советское Закарпатье», 8 октября, № 237).
276. Вікопомна подія («Вечірній Київ», 14 сентября, № 217).
277. На благо народов («Правда Украины», 21 октября, № 245).

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278. Introduction a la theorie des probabilites (совм. с А.Я.Хинчиным. Dunod, Paris, 1 - 157).
279. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (und A.Chintschin. Berlin, Deutscher Verlag der Wissenschaften. Zweite, verbessert Auflage).
280. Grenzverteilungen von Summen unabhängiger Zufallsgrößen (met Kolmogorov. Zweite Auflage. Akademie Verlag, Berlin).
281. Лекции по теории массового обслуживания (КВИРТУ вып.1- 2, 1 - 207).
282. О предельных теоремах теории вероятностей (Cambridge University Press, New Jork. «Proceedings of International Congress of Mathematicians, 14 - 21 August 1958»; 518 – 528).
283. Über einige Aspekte der Entwiklung der Warteschlangen (Zeitschrift fur modern Rechentechnik und Automation «Mathematik, Technik, Wirtschaft», heft 4, 162 - 166).
284. Об одной задаче теории массового обслуживания (Pragua. Publishing house of the Czechoslovak Academy of Sciences. «Transactions of the Second Prague Conference on Information Theory, Statistical Decision Functions, Random Procecces held at Liblice near Prague from June 1 to 6, 1959», 177 - 183).
285. О некоторых задачах теории массового обслуживания (Ереван. Изд-во АН Армянской ССР. «Труды Всесоюзного совещания по теории вероятностей и математической статистике», 15 - 24).
286. К проекту "руководящих указаний" по расчету электрических нагрузок промышленных предприятий (совм. с Б.С.Мешелем. «Промышленная энергетика», № 3, 41 - 44).
287. О математических методах в экономических исследованиях («Экономика Советской Украины», № 4, 74 - 81).
288. О некоторых разделах теории вероятностей, имеющих непосредственное отношение к проблемам биологии и медицины (Изд-во ЛГУ. «Применения математических методов в биологии», 6 - 16).

289. Математическая статистика и задачи практики («Вестник АН СССР», № 2, 38 - 43; «Analele Romino-Sovietice», № 3, 43 - 49).
290. The teaching of probability theory and mathematical statistics («Report of the second conference on mathematical education in South Asia, Tata Institute of Fundamental Research, Bombay», 1 - 32).
291. О работах Н.В.Смирнова по математической статистике (совм. с А.Н.Колмогоровым и др., «Теория вероятностей и ее применения», т. V, вып. 4, 437 - 440).
292. Комментарии к статьям Е.Е. Слуцкого «К вопросу о логических основах исчисления вероятностей» и «О стохастических асимтотах и пределах» (Москва. Изд-во АН СССР. Е.Е. Слуцкий «Избранные труды», 283-286).
293. Евгений Евгеньевич Слуцкий (биографический очерк) (Москва. Изд-во АН СССР, Е.Е. Слуцкий «Избранные труды», 5 - 11).
294. Александр Яковлевич Хинчин (некролог) (совм. с А.Н. Колмогоровым. «Успехи математических наук», т.XV, вып. 4, 97 - 110).
295. Александр Яковлевич Хинчин (некролог) (Изд-во «Наука», «Теория вероятностей и ее применения», т.V, вып.1, 3 - 6).
296. Георгий Петрович Боев (некролог) (совм. с Н.Г. Чудаковым, «Известия ВУЗов, Математика», № 1(14), 245 - 246).
297. Предисловие к книге Д.Я.Стройка "Коротка історія математики" (Київ. Изд-во «Радянська школа», 5 - 7).
298. Рецензия на книгу "Математика в СССР за сорок лет" (совм. с И.Б. Погребысским. «Успехи математических наук», т.XV, вып.5, 235 - 236).
299. Рецензия на книгу Zemanek Heinz «Элементарная теория информации» («Новые книги за рубежом», серия А, № 2, 17).
300. Рецензия на книгу Borel E. «Курс теории вероятностей», т. I, вып. 1 («Новые книги за рубежом», серия А, № 3, 11 – 12).
301. Рецензия на книгу Mittenecker E. «Планирование экспериментов и статистические выводы из них», (изд. 2) («Новые книги за рубежом», серия А, № 3, 12).
302. Рецензия на книгу Lindgren B.W., McElrath G.W. «Введение в теорию вероятностей и статистику» («Новые книги за рубежом», серия А, № 5, 15 – 16).
303. Рецензия на книгу Lukacs E. «Характеристические функции» («Новые книги за рубежом», серия А, № 11, 8 – 10).
304. Рецензия на книгу Derman C., Klein M. «Теория вероятностей и статистические выводы для инженеров» («Новые книги за рубежом», серия А, № 12, 9).
305. Математика і сучасність (газета «Радянська Освіта», 18 червня, № 49).
306. Мрії стають дійсністю («Радянська Україна», 17 травня, № 113).
307. Про кибернетику (совм. с Е.А.Шкабара. журн. "Дніпро", № 4, 138 - 142).
308. Життя і математика («Літературна газета», 8 июля, № 54).
309. Об юношеских математических школах (совм. с К.И.Швецовым. газета «Радянська Освіта», 20 февраля).
310. Чудово («Вечірній Київ», 23 января, № 19).

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311. Элементы программирования (совместно с В.С.Королюком и Е.Л.Ющенко. Москва, Физматгиз. 3-348).
312. Курс теории вероятностей (Физматгиз, Изд. 3-е, 1-406).
313. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. ГИФМЛ, изд. 5-ое, 1 - 144).
314. An Elementary introduction to the theorie of probability (and A.Khintchin. S.Francisco and London, Freeman and Co., 1 - 139).

315. Эхтимоллар незариясидан бошлангич маълумотлар (совм. с Хинчиным. Ташкент, 1 - 126).
316. Теоретико-вероятностные основы статистического метода расчета электрических нагрузок промышленных предприятий («Известия ВУЗов. Электромеханика», № 1, 90 - 99).
317. О статистических методах расчета и исследования электрических нагрузок промышленных предприятий (совм. с Б.С. Мешелем. «Электричество», № 2, 81 - 85).
318. О статистической оценке режимов сетей населенных пунктов (совм. с Б.С. Мешелем. «Электричество», № 6, 71 - 74).
319. Теория вероятностей и некоторые ее применения («Морской сборник», № 9, 31 - 41).
320. Некоторые вопросы кибернетики и статистики (Сборник «Кибернетику на службу коммунизму», т.I, 55 - 70).
321. Asymptotic expansions in probability theory (with W. Koroluk and A. Skorochod. «Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability», v. 2, 153 – 170. Univ. California Press).
322. Математические параметры универсальной цифровой автоматической машины "Киев" (совм. с В.М.Глушковым и Е.Л.Ющенко. Изд-во АН УССР. «Збірник праць з обчислювальної математики і техніки», том II, 7 - 14).
323. Імовірностей теорія (Українська Радянська Енциклопедія, т. 5, 396-397).
324. Математическая статистика - мощное орудие в работе заводской лаборатории («Заводская лаборатория», т. 27, № 10, 1251 - 1253).
325. Каждому специалисту нужно знать математическую статистику («Вестник высшей школы», № 12, 29 - 30).
326. Александр Яковлевич Хинчин («Математическое просвещение», № 6, 3 - 6).
327. О статье А.Я.Хинчина "Частотная теория Р.Мизеса и современные идеи теории вероятностей" («Вопросы философии», № 1, 91 - 92).
328. A.Ia.Khinchin («Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability», v. 2, 1 - 15).
329. Михаил Васильевич Остроградский (Москва. Физматгиз. «Люди русской науки», 104 - 110).
330. Пафнутий Львович Чебышев (Москва. Физматгиз. «Люди русской науки», 129 - 140).
331. Андрей Андреевич Марков (Москва. Физматгиз. «Люди русской науки», 193 - 199).
332. Комментарии к работам М.В. Остроградского по теории вероятностей (Киев. Изд-во Академии наук Украинской ССР. М.В. Остроградский «Полное собрание трудов», 343-344, 347-348, 365, 371-372).
333. Некоторые экономические проблемы технического прогресса ("Коммунист", № 10, 23 - 34).
334. Что такое теория надежности? («Математика в школе», № 6, 8 - 14).
335. Про математичну освіту в радянській школі (Киев. Изд-во "Радянська школа". «Викладання математики в школі», вип.1, 5 - 12).
336. Предисловие к 3-му изданию книги А.Я.Хинчина "Цепные дроби". (Москва, Физматгиз).
337. Рецензия на книгу Ю.В.Линника "Разложения вероятностных законов" (Изд-во «Наука», «Теория вероятностей и ее применения», т.VI, вып. 2).
338. Рецензия на книгу Lehmann E. «Проверка статистических гипотез» («Новые книги за рубежом», серия А, № 1, 11).
339. Рецензия на книгу Avondo Bodini G., Brambilla F. «Теория очередей» (часть I. Статистика) («Новые книги за рубежом», серия А, № 2, 6 – 8).
340. Рецензия на книгу Diamond S. «Информация и ошибка» («Новые книги за рубежом», серия А, № 2, 8 – 9).
341. Рецензия на книгу Scheffй H. «Дисперсионный анализ» («Новые книги за рубежом», серия А, № 3, 22).

342. Это нужно внедрять (О применении математических методов при решении прозводственно-экономических проблем) (журнал «Наука и жизнь», № 9, 26-29).
343. Машина ставит диагноз (газета «Ленинское знамя», 8 октября).

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344. The theory of probability (USA, Chelsea publ. Co., 1 - 459).
345. Курс теории вероятностей («Giao trinh ly thuyet xac suat») (Hanoi, 1 - 389).
346. Lehrbuch der Wahrscheinlichkeitsrechnung (Academie-Verlag, Berlin, 3 erweiterte Auflage, 1 - 393).
347. Limit Distributions for the Sums of Independent Random Variables (with A.N. Kolmogorov. Translated and annotated by K.L. Chung. With an appendix by G.L. Doob. Addison-Wesley Publishing Company, Inc.,Cambridge, Mass. IX + 264 pp. 2-nd print).
348. An elementary introduction to the theory of probability (and A.Khinchin. New York, Dover Publications, Inc., 1 - 130).
349. О понятии надежности (совм. с Ю.К.Беляевым. «Вопросы радиоэлектроники», серия 12, вып. 13, 3 - 11).
350. Основные направления исследований в теории массового обслуживания (совм. с Ю.К.Беляевым и И.Н.Коваленко. «Труды 6-го Всесоюзного совещания по теории вероятностей и математической статистике. Вильнюс», 341 - 355).
351. Математика (Українська Радянська Енциклопедія, т.8, 536-537).
352. Математична статистика (Українська Радянська Енциклопедія, т.8, 537-538).
353. Математичні машини та прилади (Українська Радянська Енциклопедія, т.8, 538-539).
354. Языком математики (Изд-во «Знание», IX серия, Физика и химия, № 7, 1 - 30).
355. Einige Fragen der Kybernetik und Statistik (Sowietwissenschaft, Verlag Kultur und Fortschritt, 10, 1071 - 1090).
356. Применение математических методов при обработке результатов биологических наблюдений (совм. с С.В.Фоминым и Я.И.Хургиным. Изд-во АН СССР. Сборник "Биологические аспекты кибернетики", 103 - 111).
357. Роль математики в развитии техники и производства («Математика в школе», № 1, 25 - 35).
358. Замечания к статье С.И.Петухова "Решение одной задачи теории массового обслуживания" («Морской сборник», № 2, 44 - 47).
359. А.И.Фетисов (к семидесятилетию со дня рождения) (совм. с А.Я. Маргулисом. «Математика в школе», № 1, 76 - 77).
360. Прекрасный памятник выдающемуся ученому и педагогу («Математика в школе», № 3, 87).
361. Рецензия на книгу «Труды второй Пражской конференции по теории информации, статистическим решающим функциям и теории случайных процессов» («Новые книги за рубежом», серия А, № 1, 10 – 12).
362. Рецензия на книгу «Вероятность и статистика» (том, посвященный Г. Крамеру) («Новые книги за рубежом», серия А, № 4, 9 – 11).
363. Рецензия на книгу Mosteller F., Rourke R.E.K., Thomas G.B. «Теория вероятности с приложениями к статистике» («Новые книги за рубежом», серия А, № 5, 8 – 10).
364. Рецензия на книгу Plackett R.L. «Принципы регрессионного анализа» («Новые книги за рубежом», серия А, № 7, 12 – 13).
365. Рецензия на книгу Rashevsky N. «Математические принципы в биологии и их применения» («Новые книги за рубежом», серия А, № 11, 18 – 20).
366. Современная математика и строительство коммунизма («Народное образование», № 5, 24 - 28).
367. Математика вокруг нас («Наука и человечество», т.I, 106 - 117).

368. Математика вокруг нас (журнал «Наука и жизнь», № 8, 18 - 23).
369. Слово теории вероятностей (журнал «Знание – сила», № 5, 14 - 15).
370. Математика и надежность («Московская правда», 13 июня, № 137).
371. На уровне XIX века («Учительская газета», 21 июня, № 73).
372. Надежность – это наука («Московская правда», 4 сентября, № 207).
373. Математика вторгается в жизнь («Московский комсомолец», 18 октября, № 229).
374. Патриарх русской математики («Восточно-сибирская Правда», 04.01., № 3).
375. Кафедра намечает планы («Московский университет», 13.02, № 8).
376. Книжные полки науки (совм. с академиком АН СССР А.А. Баландиным, член-корреспондентом АН СССР О.А. Реутовым и др., еженедельник «Неделя», 25.11-1.12, № 48).

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377. The theory of probability (New York, Chelsea publ. Co., 1 - 471).
378. Елементарно въведение в теорията на вероятностите (совм. с А.Я.Хинчиным. Малка математическа библиотека, София, изд-во "Техника", 1 - 128).
379. Elementarny wstȩ̀p do rachunku prawdopodobienstwa (совм. с А.Я. Хинчиным. Warszawa, PWN, 1 - 199).
380. Teoria de las probabilidades (совм. с А.Я. Хинчиным. Buenos - Aires, 1 - 153).
381. Лекции по теории массового обслуживания (совм. с И.Н.Коваленко. КВИРТУ. Вып. 1 - 3, 1 - 316).
382. Элементы программирования (совм. с В.С.Королюком и Е.Л.Ющенко. Физматгиз. 2-ое изд., 1 - 348).
383. Михаил Васильевич Остроградский (Жизнь и работа. Научное и педагогическое наследство) (совм. с И.Б. Погребысским. Москва. Изд-во АН СССР, 5-270).
384. Niektore zagadnienia cybernetyki i statystyki («Roczniki Polskiego Towarzystwa Matematycznego», seria II: Wiadomosci Matematyczne VII, 65 - 85).
385. Надежность (совм. с Я.Б.Шором. Госуд. научное изд-во «Советская Энциклопедия». Энциклопедический справочник "Автоматизация производства и промышленная электроника", т. 2, 348-353).
386. О проблемах истории математики в России и СССР и о работах в этой области за 1956 - 1961 гг. (совм. с И.Б. Погребысским, И.З. Штокало и А.П. Юшкевичем. «Историко-математические исследования», 1-я серия, вып. 15, 11 - 36).
387. Проблемы истории математики Нового времени (совм. с К.А.Рыбниковым и Н.И.Симоновым. «Историко-математические исследования», 1-я серия, вып. 15, 73 - 96).
388. О работах А.Н.Колмогорова по теории вероятностей («Успехи математических наук», т. 18, вып. 5, 5 - 11).
389. О работах А.Н.Колмогорова по теории вероятностей (совм. с Н.В.Смирновым. Изд-во «Наука», «Теория вероятностей и ее применения», т. VIII, № 2, 167 - 174).
390. А.Н.Колмогоров как педагог (совм. с П.С.Александровым. «Успехи математических наук», т. 18, вып. 5, 115 - 120).
391. Андрей Николаевич Колмогоров (к шестидесятилетию со дня рождения) («Математика в школе», № 2, 67 - 68).
392. Первые шаги в развитии счета («Математика в школе», № 4, 5 - 10).
393. Лекции по истории математики. Вводная лекция («Математика в школе», № 1, 3 - 13).
394. Математика древних народов Двуречья («Математика в школе», № 6, 3 - 12).
395. Математика в биологии и медицине («Биология в школе», № 5, 73-79).
396. О программированном обучении («Морской сборник», № 9, 13 - 20).
397. Предисловие редактора и заключительная статья в книге А.Я.Хинчина «Педагогические статьи» (Изд-во Акад. пед. наук РСФСР, 3 - 12, 180 - 203).

398. Предисловие редактора и заключительная статья «О некоторых постановках задач и результатах теории массового обслуживания» в книге А.Я.Хинчина "Работы по теории массового обслуживания" (Физматгиз, 4 - 7, 221 - 235).
399. За борбата между материализма и идеализма в математиката (совм. с Л.А. Калужниным, «Математика и физика», № 1, 1 - 9, № 2, 1 - 5, София).
400. Предисловие к книге В.А.Вышенского, М.И.Ядренко "Збірник задач для учасників математичних олімпіад" (Киев, изд-во "Радянська школа").
401. Предисловие к книге А.М.Кондратова "Числа вместо интуиции" (изд-во "Знание", IX-ая серия "Физика и химия", № 8).
402. Предисловие к книге Р.Х.Зарипова "Кибернетика и музыка" (изд-во "Знание", IX-ая серия "Физика и химия", № 18).
403. Рецензия на книгу Cox D.R. «Теория восстановления» («Новые книги за рубежом», серия А, № 4, 14 – 17).
404. Рецензия на книгу Saaty Th.L. «Элементы теории очередей» («Новые книги за рубежом», серия А, № 7, 15 – 18).
405. Рецензия на книгу Parratt L.G. «Вероятность и экспериментальные ошибки в естествознании» («Новые книги за рубежом», серия А, № 8, 25 – 27).
406. Рецензия на книгу Beneљ J. «Статистическая динамика теории регулирования» («Новые книги за рубежом», серия А, № 10, 15 – 17).
407. Рецензия на книгу Bailey N.T. «Введение в математическую теорию генетического сцепления» («Новые книги за рубежом», серия В, № 10, 18 – 19).
408. Рецензия на книгу Rosenblutt M. «Случайные процессы» («Новые книги за рубежом», серия А, № 11, 27 – 30).
409. Рецензия на книгу О.Б.Шейнина «К истории оценок непосредственных наблюдений и закона распределения случайных ошибок» (Реферативный журнал, математика, 8А30).
410. Разказва за кибернетиката (София, газета «Отечествен фронт», 9 и 10 апреля).
411. Надежность - ключ автоматизации («Известия», 3 июля, № 157).
412. Ключевая проблема современной техники («Учительская газета», 1 октября, № 116).
413. Выдающийся математик современности («Известия», 24.04., № 98).
414. Поздравляем выдающегося математика («Московский университет», 28.04)
415. Рисовать картины будущего ("Искусство кино", № 6, 108-116).

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416. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. Изд-во «Наука», изд. 6-е, 1 - 146).
417. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (zum A.J. Chintschin. Berlin, Deutscher Verlag der Wissenschaften, 6 Auflage, 3 - 174).
418. Elementen der programmirung (mit Koroluk und Justenko, Teubner, Leipzig, 1 - 327).
419. Bevezetes a programozasba (Koroluk und Juscenko, Budapest, v.1, 1 - 228 und v.2, 1 - 204).
420. О критерии знаков («Доклады Болгарской АН», т.17, № 9, 793 - 796).
421. О ненагруженном дублировании (Известия АН СССР, «Техническая кибернетика», № 4, 3 - 12).
422. О дублировании с восстановлением (Известия АН СССР, «Техническая кибернетика», № 5, 111 - 118).
423. Об определении оптимального числа причалов (совм. с М.Н. Зубковым. «Морской сборник», № 6, 30 - 39).
424. Статистические методы в теории надежности (Москва. Изд-во «Стандартизация», № 6, 16 – 22), (изд-во «Знание», 1-25).
425. Роль математики в развитии современного естествознания (Изд-во «Мысль». Сборник "Диалектика в науках о неживой природе", 45 - 85).

426. Статистически методи в теория на сигурността (София. «Физико-математическое списание», т.7, вып. 2, 120 - 134).
427. Шо е теория на масовото обслужване (София. «Физико-математическое списание», т.7, вып.3, 200 - 211).
428. За подготовка на учителя по математика (София. «Математика и физика», № 4, 4 - 11, № 5, 1 - 9).
429. Математически страни на теорията на сигурността (София. «Новости в автоматиката и телемеханиката», кн. 4, 5 - 34).
430. Из история на математиката (София. «Математика», № 2, 6 - 9,).
431. Mathematik in Biologie und Medizin («Biologie in der Schule», B.13, H.3, 107 - 111).
432. О теории массового обслуживания («Математика в школе», № 3, 10 - 20).
433. Наука о случайном (Детская энциклопедия, 2-ое изд., 452 - 461).
434. О математических методах теории надежности (Детская энциклопедия, 2-ое изд., 461 - 465).
435. О воспитании учителя математики («Математика в школе», № 6, 8 - 20).
436. Предисловие к книге Эмиля Бореля "Вероятность и достоверность" (изд. 2-ое, М. "Наука").
437. Рецензия на книгу M.Rosenblutt "Random Processes" (Annals of Mathematical Statistics, v.35, no.4, 1832 - 1833).
438. Рецензия на книгу Walsh J.E. «Справочник по непараметрическим статистикам» («Новые книги за рубежом», серия А, № 1, 18 – 20).
439. Рецензия на книгу «Математические проблемы в биологических науках» (под редакцией R.E. Bellman) (совм. с И.Б.Загорской. «Новые книги за рубежом», серия А, № 3, 24 – 28).
440. Рецензия на книгу Takacs L. «Введение в теорию очередей» («Новые книги за рубежом», серия А, № 4, 15 – 18).
441. Рецензия на книгу Freeman H. «Введение в теорию статистических выводов» («Новые книги за рубежом», серия А, № 10, 14 – 17).
442. Рецензия на книгу Bodion G. «Диалектическая теория вероятностей, включающая ее классическое и квантовое исчисление» (совм. с Г.А.Зайцевым. «Новые книги за рубежом», серия А, № 12, 9 – 11).
443. Рецензия на книгу «Исследования по порядковым статистикам» (под редакцией A.E. Sarhan и B.G. Greenberg) («Новые книги за рубежом», серия А, № 12, 11 – 13).
444. Возможности и нужды молодой науки (Журнал «Научно-технические общества СССР», № 12, 2-5).
445. Стандарт высокого качества (совм. с А.И.Бергом, Я.М.Сориным и Я.Б.Шором. «Известия», 9 января, № 8).
446. Великий рыцарь науки (к 400-летию со дня рождения Галилео Галилея) («Красная звезда», 15 февраля, № 39).
447. Надежность и математика («Неделя», 10 - 16 мая 1964, № 20).
448. Сколько стоит нагнетатель? («Неделя», 5-11 июля, № 28).
449. Качеством можно управлять (совм. с Я.М.Сориным. «Известия», 20 июля, № 172).
450. По някои въпроси на училищното образование (Болгария. София. «Учительско дело», , 23 октомври, № 84).
451. Продор у тајне мишлена (СФРЮ. «Борба», 13 января).

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452. Курс теории вероятностей («Наука», изд. 4-ое, 1 - 400).
453. Математические методы в теории надежности (совм. с Ю.К. Беляевым и А.Д. Соловьевым. «Наука», 1 - 524).
454. Lehrbuch der Wahrscheinlichkeitsrechnung (Berlin, Academie-Verlag, 4 Auflage, 1-393).
455. Об одной задаче теории дублирования («Морской сборник», № 1, 14 - 23).

456. Об одном аспекте проблемы оператор-машина («Вопросы радиоэлектроники», вып. 25, серия XII, 3 - 11).
457. Математика (Київ. Українська Радянська Енциклопедія, т. 17, 484-487).
458. Обеспечение качества, надежности и долговечности массовой продукции и статистические методы исследования (Москва. Изд-во «Стандартизация», № 5, 4 - 6).
459. К вопросу надежности сельскохозяйственной техники (совм. с В.П.Поповым. «Тракторы и сельскохозяйственные машины», № 6, 21 - 24).
460. O teorii obslugi (Warszawa. «Matematika. Czasopismo dla nauczycieli», № 1(85), 1 - 9, № 2(86), 50 - 54).
461. Bedienungstheorie (Volk und Wissen Volkseigener Verlag Berlin. «Mathematik in der Schule», № 5, 325 - 340).
462. С езика на математиката (София. «Народна просвета», 1 - 47).
463. Математиката на древните народи от Месопотомия (София. «Математика», № 2, 1 – 4; № 3, 6-10).
464. Математические модели и программированное обучение (совм. с Ю.И.Берилко. «Советская педагогика», № 10, 140 - 142).
465. Механико-математическое образование (Москва. Изд-во «Советская энциклопедия». «Педагогическая энциклопедия», т. II, 822-824).
466. Символ прогрессивных идей и методов в педагогике («Вестник Высшей Школы», № 5, 13 - 20).
467. О перспективах математического образования («Математика в школе», № 6, 2- 11).
468. Perspektiven der mathematischen Ausbildung (Volk und Wissen Volkseigener Verlag Berlin. "Probleme des Mathematikunterrichts. Diskussionsbtitrage sowjetischer Wissenschaftler", 28 - 59).
469. Meine Idealvorstellung vom Mathematiklehrer (Volk und Wissen Volkseigener Verlag Berlin. «Probleme des Mathematikunterrichts. Diskussionsbtitrage sowjetischer Wissenschaftler», 88 - 104).
470. Mathematical education in the USSR («The Australian Mathematics Teacher», v.21, № 3, 49 - 59).
471. Итоги открытого конкурса на учебники по математике (совм. с И.С.Петраковым. «Математика в школе», № 2, 4 - 9).
472. К столетию Московского математического общества («Математика в школе», № 2, 95 - 96).
473. Предисловие к книге Т.Саати "Элементы теории массового обслуживания и ее приложения" (М. Изд-во "Советское Радио", 5 - 14).
474. Предисловие к книге Э. Гумбель «Статистика экстремальных значений» (М. «Мир», 5-7).
475. Рецензия на книгу Христов Вл. «Основы теории вероятностей и математической статистики с приложениями в технике и экономике» («Новые книги за рубежом», серия А, № 2, 15 – 18).
476. Рецензия на книгу Bergstrom H. «Предельные теоремы для сверток» («Новые книги за рубежом», серия А, № 8, 20 – 22).
477. Рецензия на книгу Spitzer F. «Принципы случайного блуждания» («Новые книги за рубежом», серия А, № 9, 32 – 33).
478. Рецензия на книгу «Концепция информации в современных науках» («Новые книги за рубежом», серия А, № 11, 5 – 7).
479. Теория массового обслуживания («Экономическая газета», 3 февраля, № 5).
480. Слагаемые надежности машин (совм. с А.Башкирцевым и В.Поповым. «Правда», 25 февраля, № 56).
481. Статистические методы - в промышленость (совм. Я.Б.Шором. «Известия», 14 апреля, № 88).
482. Проблемы повышения надежноости («Труд», 2 июля, № 153).
483. О некоторых вопросах кибернетики («Советская Аджария», 1 сентября, № 171).

484. Статистика у пракси (СФРЮ. «Вечерни експрес»,17 сентября).
485. Брак в солидных обложках (совм. с А.Н. Колмогоровым, А.А. Дородницыным, А. Вайнштейном. «Известия», 23 января, № 19).

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486. The theory of probability (Chelsea publ., New York, 1 - 479).
487. Введение в теорию массового обслуживания (совм. с И.Н.Коваленко. «Наука», 1- 431).
488. Математическая теория надежности и ее применения в машиностроении (Москва. Министерство станкостроительной и инструментальной промышленности СССР, НИИ информации по машиностроению. «Труды Первой научно-технической конференции по повышению надежности и долговечности машин, оборудования и приборов, проведенной в Москве в 1965 году», вып. 1, 73 - 80).
489. Математические вопросы теории надежности (совм. с Ю.К.Беляевым и И.Н.Коваленко. Москва. ВИНИТИ. «Итоги науки», серия математика, «Теория вероятностей. Математическая статистика. Теоретическая кибернетика. 1964», 7 - 53).
490. Об эффективности восстановления резервированных устройств (АН Узбекской ССР, Институт математики им. В.И. Романовского. Изд-во «Фан». Ташкент. Сборник "Предельные теоремы и статистические выводы", 46 - 59).
491. О некоторых предельных теоремах в теории резервирования с восстановлением («Вопросы радиоэлектроники», сер.12, вып.13, 3 - 15).
492. Роль истории физико-математических наук в развитии современной науки (Изд-во МГУ. Сборник "История и методология естественных наук", вып.5, 5 - 14).
493. Об алгоритмическом подходе к обучению (совм. с Б.В.Бирюковым. Изд-во «Просвещение», предисловие к книге Л.Н.Ланда "Алгоритмизация в обучении", 11 - 33).
494. О методах комбинаторики в теории вероятностей и математической статистике («Математика в школе», № 5, 11 - 18).
495. О математических моделях в педагогике («Вестник высшей школы», № 9, 25 - 31).
496. Mathematik rund um uns («Wissenschaft und Menschheit», 78 - 89).
497. Ташмухамед Алиевич Сарымсаков (к пятидесятилетию со дня рождения) («Успехи математических наук», т.XXI, вып.3, 248 - 253).
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499. О вступительных экзаменах в МГУ в 1966 г. (совм. с М.К.Потаповым. «Математика в школе», № 6, 37 - 49).
500. Предисловие к книге Ж. Мот «Статистические предвидения и решения на предприятии» (Москва. Изд-во «Прогресс». 5-9).
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502. Рецензия на «Историко-математические исследования», вып. 16 («Успехи математических наук», т.XXI, вып.3, 262 - 264).
503. Рецензия на книгу Drooyan J., Hadel W. «Программированное введение в числовые системы» («Новые книги за рубежом», серия А, № 2, 6).
504. Рецензия на книгу Barlow R.E., Proschan F. «Математическая теория надежности» («Новые книги за рубежом», серия А, № 8, 24 – 26).
505. Аннотации к книгам, выпущенным издательством «Academic Press»: Richard von Mises «Mathematical Theory of Probability and Statistics» and Solomon Lefschetz «Stability of Nonlinear Control Systems» («Математика в школе», № 3, 89 – 90); «Physical Acoustics» (Principles and Methods) (vol. I, II, III) and James Singer «Elements of Numerical Analysis» («Математика в школе», № 4, стр. 90, 93).
506. Шостаковичу – 60 лет («Советская музыка», № 9, 15).

507. Алгебра прогресса («Московская правда», 20 апреля, № 92).
508. Генеральная задача («Горьковский рабочий», 9 июня, № 133).
509. Математика и современность («Красная звезда», 16 августа).
510. Молоді – математичний журнал (совм. с М.Чайковским, М.Ядренко, А.Конфоровичем, Г.Саковичем. «Радянська освіта», 15 октября, № 83).

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512. On some stochastic problems of reliability theory (together with Yu.Beliaev and A.Soloviev. «Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability», v. 3, 259 - 270).
513. Some theorems on standbys («Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability», v. 3, 285 - 291).
514. О связи теории суммирования независимых случайных величин с задачами теории массового обслуживания и теории надежности («Revue Roumaine de Mathematiques pures et appliquees», t.XII, № 9, 1243 - 1253).
515. О некоторых задачах теории массового обслуживания (совм. с И.Н.Коваленко. «Известия АН СССР. Техническая кибернетика», № 5, 88 - 100).
516. Асимтотические методы в теории надежности (Доклады Советской делегации на II Чехословацкой конференции по контролю качества и надежности (г. Брно, 19 – 21 сентября 1967 г.)). (Комитет стандартов, мер и измерительных приборов при Совете Министров СССР, Всесоюзный научно-исследовательский институт стандартизации, заказ № 749, 3-10).
517. Асимптотические методы в вопросах исследования операций (АН СССР. Научный Совет по философским вопросам естествознания. Центральное бюро философских (методологических) семинаров. Материалы к симпозиуму "Исследование операций и анализ развития науки", ч. II (Вопросы исследования операций), 23 - 37).
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519. Применение теории массового обслуживания к задачам больших систем (совм. с И.Н.Коваленко. Москва. Редакционно-издательская группа Главного вычислительного центра Госплана СССР. Материалы научно-технической конференции «Научные и практические проблемы больших систем», часть II, 54 - 55).
520. О системном подходе к проблеме создания автоматизированных систем управления технологическими процессами (совм с А.Н.Леонтьевым, И.Н.Коваленко и др. Москва. Редакционно-издательская группа Главного вычислительного центра Госплана СССР. Материалы научно-технической конференции «Научные и практические проблемы больших систем», часть III, 59 – 66).
521. Теория вероятностей и задачи стандартизации («Стандарты и качество», № 1, 75 – 77).
522. Основные понятия теории вероятностей («Стандарты и качество», № 2, 72-75).
523. Статистические методы в вопросах приемочного контроля («Стандарты и качество», № 4, 76-78).
524. Вопросы теории испытаний изделий на качество и надежность («Стандарты и качество», № 5, 77-79).
525. Стандартизация и математика («Стандарты и качество», № 6, 86 - 92).
526. Элементы теории случайных процессов («Стандарты и качество», № 7, 31-34).
527. Развитие теории вероятностей в Московском университете («Вестник МГУ», Серия «Математика, Механика», № 6, 30 - 51).

528. Математика в СССР за 50 лет («Математика в школе», № 6, 5 - 13).
529. О проблемах школьного математического образования (София. «Физико - математическо списание», т. X, кв. 4, 311 - 324).
530. О книге Б.Е.Бердичевского "Оценка надежности аппаратуры автоматики" (Москва. «Стандарты и качество», № 8, 59).
531. Предисловие к книге Д.Кокс, В.Смит «Теория восстановления» (М., «Советское радио», 3-4).
532. Предисловие к книге Э.Николау "Введение в кибернетику" (М., изд-во "Мир". 5-6).
533. Предисловие к книге Л.Феликса "Элементарная математика в современном изложении" (М., "Просвещение", 3-5).
534. Предисловие к книге Дж. Букан, Э. Кенигсберг «Научное управление запасами» (М., «Наука», 6-7).
535. Рецензия на книгу Walsh J.E. «Справочник по непараметрическим статистикам» (том II) («Новые книги за рубежом», серия А, № 5, 29 – 31).
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540. Проблемы больших систем (совм. с Л. Дудкиным и Б. Коробочкиным. Еженедельник «Экономическая газета», ноябрь, № 48).
541. На търсещите пътя към знанието (София. «Студентска трибуна», бр. 2 (542), год.XIX, 3 октября).
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547. Lehrbuch der Wahrscheinlichkeitsrechnung (Academy Verlag, Berlin, 5 erweiterte Auflage, 1 - 399).
548. The theory of probability (Chelsea Publishing Company, New York).
549. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (zum A.J. Chintschin. Berlin, Deutscher Verlag der Wissenschaften, 7 Auflage, 3 - 174).
550. Osnovi teorije vjerojatnosti (совм.с А.Я. Хинчиным. Zagreb, Technicka kniga, 1 - 114).
551. Introduccion a la teoria de las probabilidades (совм. с А.Я. Хинчиным. Barcelona, Montaner y Simon, S.A., 1 - 182).
552. Metode matematice in teoria sigurantei (совм. с Ю.К.Беляевым, А.Д.Соловьевым. Bucuresti, 1 - 560).
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555. Introduction to Queuing theorie (with I.N.Kovalenko. Ierusalem, 1 - 281).

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557. Применение вероятностных методов (совм. с Е.Ю.Барзиловичем и Е.В.Чепуриным. «Известия АН СССР. Техническая кибернетика», № 6, 27 - 40).
558. Асимптотические методы в теории надежности («Стандарты и качество», № 2, 63 - 64).
559. О ненагруженном резервировании с восстановлением (совм. с Ю.Насром. «Автоматика и телемеханика», № 7, 105 - 111).
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561. О математических методах в здравоохранении и медицинском приборостроении (Москва, ВНИИ медицинского приборостроения. «Новости приборостроения», 5 - 7).
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563. Die Mathematik in der UdSSR wahrend der letzen 50 Jahre (Mathematik in der Schule, Jahrgang 6, N.7, 483 - 495).
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565. Задачи стандартизации и математика (Москва. Всесоюзный НИИ технической информации, классификации и кодирования. Изд-во стандартов. Сборник «Актуальные проблемы стандартизации», 16 - 31).
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569. О месте риторики в преподавании математики («Математика в школе», № 3, 89).
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574. Рецензия на книгу «Избранные работы Эгона Пирсона» («Новые книги за рубежом», серия А, № 6, 21-22).
575. Рецензия на книгу Moran P.A.P. «Введение в теорию вероятностей» («Новые книги за рубежом», серия А, № 8, 25-27).

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578. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. Пер. на японский, Токио, 6 - 161).
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582. Об одной теореме переноса (совм. с Х.Фахимом. «Доклады АН СССР», т. 187, № 1, 15 - 17).

583. Резервирование с восстановлением и суммирование случайного числа слагаемых (Венгрия, «Труды совещания по теории надежности в Тихани, 16 – 19 сентября 1969 года», 1-9)
584. О связи задачи суммирования случайных величин с теорией резервирования (Рига. РВВИАУ им. Алксниса. «Сборник научных статей», вып. № 4, 23 - 34).
585. О статистических методах в теории надежности (МАТИ. Сборник «Основные вопросы надежности и долговечности машин», 22 - 42).
586. Об основных направлениях математических исследований в теории надежности (Ташкент. Изд-во АН УзССР. «Труды совещания по матемематической теории надежности», 3 - 18).
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588. О применении нормального распределения при обработке опытных данных в машиностроении (Изд-во «Вестник машиностроения», № 2, 12 - 13).
589. О роли и месте теории надежности в процессе создания сложных систем (совм. с И.А.Ушаковым, Б.А.Козловым. Изд-во «Наука». Сборник «Теория надежности и массовое обслуживание», 14 - 32).
590. Всесторонне развивать науку о надежности (совм. с Я.М. Сориным. Москва. Изд-во стандартов. «Надежность и контроль качества», № 1, 3-9).
591. Математические методы – основа контроля качества и надежности промышленной продукции (Москва. Изд-во стандартов. «Надежность и контроль качества», № 2, 3 – 13). (Сборник «Надежность и контроль качества» (по материалам Всесоюзного научно-технического семинара-совещания по совершенствованию работы служб технического контроля на промышленных предприятиях, Пермь, 17-20 июня 1968 года), 1(2), 111-119).
592. Random processes and their application to Demography and Insurance («8-th ASTIN Colloquium, Sopot, september 1969», 1-25).
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594. Несколько замечаний к одной теореме И.Н.Коваленко (совм. с Б.Фрайером. «Литовский математический сборник», т. IX, № 3, 463 - 470).
595. Вопросы теории испытаний изделий на качество и надежность (Москва. Изд-во стандартов. «Стандарты и качество», № 5, 77-79).
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597. Об одной математической модели в задачах инженерной психологии (Warszawa. «Zastosowania matematyki» (юбилейный том, посвященный Хуго Штейнхаусу), v. X, 205-211).
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600. Wahrscheinlichkeitsrechnung und Kombinatorik (zus. I.G. Shurbenko. «Mathematik in der Schule», № 3, 170 - 210, № 4, 284 - 295).
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602. О преподавании биологии и математической статистики (Комментарии к статье Л.К. Андреевой и Л.С. Ровкиной «Элементы математической статистики в теме «Органы движения». «Биология в школе», 1969, № 5, 39-42) («Биология в школе». № 5, 42 - 43).
603. Об образовании математических понятий (Изд-во «Знание». «Математика в современном мире», серия «Математика и кибернетика», № 9, 3 - 10).
604. О пропаганде математических знаний («Слово лектора», № 1, 92 - 95).

605. О формирану наставника математике (Beograd. «Nastava matematike i fizike», serija B, XVII – XVIII (1968-1969), 49-68).
606. О математике во ВТУЗе (Рига. РВВИАУ им. Алксниса. «Сборник научнных статей», № 4, 5 - 16).
607. Ленинская теория познания и вопросы математизации современного знания («Вестник АН СССР», № 5, 53 - 60).
608. Леонтий Магницкий и его «Арифметика» (совм. с И.Б.Погребысским. «Математика в школе», № 6, 78 - 82).
609. Сергей Натанович Бернштейн (некролог) (совм. с А.Н.Колмогоровым. Изд-во «Наука», «Теория вероятностей и ее применения», т.12, вып. 3, 532-535).
610. Сергей Натанович Бернштейн (некролог) (совм. с А.Н.Колмогоровым, П.С. Александровым, Н.И.Ахиезером. УМН, т. 24, вып. 3, 211-218).
611. Виктор Иосифович Левин (к 60-летию со дня рождения) (совм. с А.Я.Маргулисом. «Математика в школе», № 6, 65).
612. Предисловие к сборнику «Теория надежности и массовое обслуживание» (Москва, «Наука», 7 – 13).
613. Предисловие к книге Барлоу, Прошан "Математическая теория надежности" (Москва, изд-во «Советское радио», 5-7).
614. Предисловие и примечания редактора к книге Э. Бореля «Вероятность и достоверность» (Москва, «Наука», 5-6, 105-110).
615. Предисловие к книге А.Реньи "Диалоги о математике" (изд-во «Мир», 5 - 19).
616. Предисловие к книге С.С. Демидова «Проблемы Гильберта» (Москва, изд-во «Знание», 3).
617. Рецензия на книгу А.П.Юшкевича «История математики в России» (совм. с И.Г. Башмаковой. «Математика в школе», № 4, 86 –87).
618. Рецензия на книгу Pal Revesz «The laws of large numbers» («Успехи математических наук», т. XXIV, вып. 2, 260 –261).
619. Профессор Московского университета – юным математикам ЗЮМШ (газета «Брянский комсомолец», 4 июля, № 77).
620. Технический прогресс и математическое образование (газета «Социалистическая индустрия», 1 августа, № 28).
621. В борьбе за качество (интервью, газета «Горьковский рабочий», 26 августа, № 199).

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622. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. изд-во «Наука», изд. 7-е, 1-167).
623. Lehrbuch der Wahrscheinlichkeitsrechnung (Academie-Verlag, Berlin, 6 Auflage, 1-399).
624. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (zum A.J. Chintschin. Berlin, Deutscher Verlag der Wissenschaften, 8 Auflage, 3 - 174).
625. A megbizhatōsāgelmelet matematikai modszerei (совм. с Ю.К.Беляевым и А.Д.Соловьевым. Müszaki könyvkiadó, Budapest, 1-463).
626. О некоторых нерешенных задачах теории массового обслуживания («VI International Teletreffic Congress, Munchen, 9 - 15 IX 1970», 227/1 - 227/17).
627. Статистические методы в теории надежности (ВНИИПТМАШ, «Труды. Надежность подъемно-транспортных машин», вып. 1(96), 6 - 12).
628. О математических методах в теории надежности (Венгрия, «Труды совещания по теории надежности в Тихани, 14 - 19 сентября 1970»).
629. Проблемы надежности («Техника и вооружение», № 4, 38 - 39).
630. Развитие прикладных методов теории вероятностей (Киев. «История отечественной математики», т.4, книга 2, 7 - 13, 52 - 62).

631. Итоги дискуссии по поводу статьи П.С. Суханова «Об одном противоречии системы предпочтительных чисел» (совм. с С.В. Крейтером. «Стандарты и качество», № 8, 25 - 27).
632. Проблемы математизации современного естествознания (Изд-во «Наука». Сборник «Диалектика и современное естествознание», 82 - 102).
633. Научно-технический прогресс и математика (Изд-во «Знание», серия «Математика и кибернетика», № 10, 3 - 17).
634. О будущем прикладной математики (Изд-во «Знание». Сборник "Будущее науки", вып. 3, 82 - 102), («Наука и жизнь», № 1, 42 –47, 71).
635. Nauczanie a efektiwnośċ badań naukowych (Warszawa. Polska Akademia Nauk. «Zagadnienia naukoznawstwa», tom VI, zeszyt 3 (23), 70 - 78).
636. Предисловие к брошюре А.Реньи "Письма о вероятности" (Изд-во «Мир», 5 - 15).
637. В.И.Ленин и развитие математики в Советском Союзе («Математика в школе», № 1, 4 - 12).
638. В.И.Ленин и методологические проблемы математики (Изд-во «Знание», серия «Математика и кибернетика», № 1, 1 - 32).
639. В.И.Ленин и методологические вопросы математики («Успехи математических наук», т. 25, вып. 2, 5 - 14).
640. Ленинская теория познания и математическое образование («Вестник высшей школы», № 4, 77 - 81).
641. Lenin a matematyka w Zwianzku Radzieckim (Warszawa. Polska Akademia Nauk. «Zagadnienia naukoznawstwa», tom VI, zeszyt 2 (22), 3 - 31).
642. Фридрих Енгелс за философските проблеми на математиката (Българска Академия на науките, «Физико математическо списание», том 13(46), кн. 4, 296 – 306).
643. Рецензия на книгу Beard R.T., Pentikainen T., Pesonen E. «Теория риска» («Новые книги за рубежом», серия А, № 4, 31 – 33).
644. Рецензия на книгу Szaby A. «Начала греческой математики» (совм. с И.Б. Погребысским, «Новые книги за рубежом», серия А, №5 ,5 –6).
645. Рецензия на книгу Onicescu O. «Теория вероятностей и ее применения» («Новые книги за рубежом», серия А, № 6, 17 – 19).
646. Рецензия на книгу Weinberg F. «Основы теории вероятностей и статистики и их применение к исследованию операций» («Новые книги за рубежом», серия А, № 6, 19 – 21).
647. Рецензия на книгу Hajek J. «Курс по непараметрической статистике» («Новые книги за рубежом», серия А, № 7, 16 – 17).
648. Практичность абстракции (газета «Неделя», 18 - 24 мая, № 21).
649. Счет время бережет (совм. с В. Падня. «Известия», 13 июля № 164).

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650. Einfuhrung in die Bedienungstheorie (zus. I.N.Kovalenko. Berlin, Academie-Verlag, 1 - 450).
651. Wstep do teorii obslugi masowej (совм. с И.Н. Коваленко. Warszawa, Panstwowe Wydawnictwo Naukowe, 1-431).
652. Математические методы в теории надежности (совм. с Ю.К.Беляевым и А.Д.Соловьёвым. Япония, т. I, 1-267).
653. Курс теории вероятностей (Япония, т. I, 1-271).
654. Лекции по теории суммирования случайного числа независимых величин (Варшава. Изд-во Варшавского университета, 1 - 42).
655. Введение к сборнику «Большие системы: теория, методология, моделирование» (АН СССР, общество «Знание», Научный совет по комплексной проблеме «кибернетика». М. Изд-во «Наука». 7-9).

656. Применение теории массового обслуживания к задачам больших систем (совм. с И.Н.Коваленко. М. Изд-во «Наука». АН СССР, общество «Знание», Научный совет по комплексной проблеме «кибернетика». Сборник «Большие системы, теория, методология, моделирование», 105 - 122).
657. Theorie und Praxis der Productionssicherheit (Deutsche Verlags-Anstalt, Stuttgart. «Ideen des exakten Wissens», № 6, 411 – 414).
658. Свойства решений задачи с потерями в случае периодичности интенсивностей (совм. с И.П. Макаровым. Минск. Изд-во «Наука и техника». «Дифференциальные уравнения», т. VII, № 9, 1696 – 1698).
659. Беседы за математическата статистика (София. Изд-во «Техника». «Малка математическа библиотека», 5-61).
660. За бъдещето на приложната математика (София, «Математика», година X, книжка 5, 5 – 9).
661. Сообщение на заседании НТС (Москва, ЦНИИ информации и технико-экономических исследований рыбного хозяйства, «Тезисы докладов, конференций и совещаний», вып. 3, 28 – 30).
662. Об источниках нового в математике (Белград, «Dijalektika», broj. 3, godina VI, 19-35).
663. О роли математики в ускорении темпов научно-технического прогресса («Математика в школе», № 5, 4 – 11).
664. Zum sechsten Hilbertschen Problem («Die Hilbertschen Probleme». Unter die Redaktion von P.S.Alexandrov. Leipzig: Akademische Verlaggesellschaft. Geest&Portig K.-G. (Ostwalds Klassiker. Bd. 252). 145-150). (В последующие годы было еще несколько изданий этой книги).
665. Mathematik und Leben (Berlin, «Wissenschaft und fortschritt», № 6, 256 – 259).
666. Фридрих Энгельс о философских проблемах математики («Вестник МГУ. Философия», № 2, 20 - 27; «Математика в школе», № 1, 4 – 11).
667. Гордость отечественной науки. К 150-летию со дня рождения П.Л.Чебышева. («Вестник высшей школы», № 5, 76 - 80).
668. Иосиф Бенедиктович Погребысский (некролог) («Математика в школе», № 6, 91 – 92).
669. Естественные факультеты МГУ («Математика в школе», № 1, 50 - 55).
670. Введение к сборнику «Бюльшие системы. Теория, методология, моделирование» (М. «Наука», 7 – 9).
671. Предисловие к книге Т.Саати "Элементы теории массового обслуживания и ее приложения" (М. Изд-во "Советское Радио", 2-е изд., 5 - 13).
672. Предисловие и послесловие составителя сборника «Проблемы современной математики» (Москва, Знание, серия «Математика и кибернетика», № 10, 3, 45 – 48).
673. Рецензия на книгу «История математики» (т. I, т. II) («Вестник АН СССР», № 10, 123 – 124).
674. Рецензия на «Французско–русский математический словарь» («Успехи математических наук», т. XXVI, вып. 3, 249 – 251).
675. Рецензия на книгу Broad C.D. «Индукция, вероятность и причинность» (совм. с И.Б. Погребысским, «Новые книги за рубежом», серия А, №3 ,5 –6).
676. Рецензия на книгу Sturmer H. «Полумарковские процессы с конечным множеством состояний» («Новые книги за рубежом», серия А, № 4, 24 – 25).
677. Пафнутий Львович Чебышев (журнал «Знание–сила», № 10, 22 – 23).
678. Практичность абстракции (еженедельник «Неделя», 18-24 мая,, № 21).
679. Счет время бережет (совм. с В. Падня. «Известия», 13 июля, № 164).
680. Математика и технический прогресс («Приокская правда», 29 сентября, № 231).

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681. Математические методы в теории надежности (совм. с Ю.К. Беляевым и А.Д. Соловьёвым. Япония, т.II, 1-317).
682. Курс теории вероятностей (Япония, т.II, 273-471).
683. Methodes Mathematiques en Theorie de la Fiabilite (совм. с Ю.К. Беляевым и А.Д. Соловьёвым. На французском языке. Москва. Изд-во «Мир», 1-535).
684. О задачах теории массового обслуживания (М. Изд-во Московского университета. «Сборник трудов II Всесоюзного совещания-школы по теории массового обслуживания. Дилижан 1970», 41 - 51).
685. Limit theorems for sums of a random number of positive independent random variables (Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, v. II, 537 - 549).
686. Асимптотические методы в вопросах исследования операций (Изд-во «Наука». Сборник "Исследование операций. Методологические аспекты", 29 - 42).
687. О статистических методах в социальных науках (Москва. АН СССР. Научный Совет по философским вопросам естествознания. «Математизация научного знания», вып.V, 50 - 60).
688. О некоторых вопросах надежности как предмета исследования и преподавания («Надежность и долговечность машин и оборудования», 62 - 71).
689. Беседи върху теория на вероятностите и комбинаторика (София, «Математика», вып. 4, 2 - 8; вып. 5, 1 - 6).
690. Математика многонациональной советской страны и научно-технический прогресс (Изд-во "Знание", сборник "Математика и научно-технический прогресс", серия "Математика, кибернетика", № 11, 29 - 58).
691. Математика - наука древняя и молодая (Изд-во «Знание». «Архитектура математики», серия «Математика, кибернетика», № 1, 19 - 32).
692. Иосиф Бенедиктович Погребысский (некролог) («Успехи математических наук», т. XXVII, вып.1, 227 - 235).
693. Георгий Федорович Рыбкин (некролог) (совм. с П.С.Александровым, А.Н.Колмогоровым, А.И.Маркушевичем и др. «Успехи математических наук», т.27, вып.5, 223-225).
694. XIII Международный конгресс по истории науки (совм. с С.С. Демидовым. «Математика в школе», № 1, 94 - 96).
695. О математике в СССР за 50 лет его существования («Математика в школе», № 6, 5 – 12).
696. Математика и научно-технический прогресс (Изд-во "Просвещение", сборник «Школьникам о XXIV съезде КПСС», 110 - 119).
697. Технический прогресс и математическое образование (М. Изд-во "Высшая школа". «Математика» (сборник научно-методических статей по математике), вып. 2, 22 - 27).
698. Математизация науки и математическое образование («Вестник высшей школы», № 1, 40 – 45).
699. Статистическое мышление и школьный курс математики (Изд-во "Знание", сборник «Новое в школьной математике», 165 - 180).
700. Обзор статей, посвященных факультативному курсу теории вероятностей («Математика в школе», № 2, 47 – 48).
701. Статистическо образование в училищата и университетите (Българска Академия на науките, «Физико математическо списание», т. 15, кн. 4, 321 - 327).
702. Предисловие (М. Изд-во Московского университета. «Сборник трудов II Всесоюзного совещания-школы по теории массового обслуживания. Дилижан 1970», 8-9).
703. Предисловие (Изд-во «Знание». «Архитектура математики», серия «Математика, кибернетика», № 1, 3).

704. Наше всеобщее достояние (к 25-летию Всесоюзного общества "Знание") (журнал "Знание-сила", № 6, 1-2).
705. Відповідь без чернеток (газета «Радянська Україна», 16 января, № 13).
706. В единстве к свету (газета «Московский университет», 7 марта, № 10).
707. Сучасна школа. Здібності і підготовка до самостійной праці (газета «Радянська Україна», 16 января, № 13).
708. Выполнению обязательств – все силы (газета «Московский университет», 29 сентября, № 31).

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709. The theory of probability (Moscow. Mir Publishers, second printing).
710. Беседы о теории массового обслуживания (Изд-во «Знание», серия "Математика, кибернетика", № 9, 3 - 63).
711. Приоритетные системы обслуживания (совм. с Э.А. Даниеляном, Б.Н. Димитровым, Г.П. Климовым, В.Ф. Матвеевым. Изд-во Московского университета, 3 - 447).
712. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (und A.Chintschin. Berlin, Deutscher Verlag der Wissenschaften. Neunte, uberarbeitete und erweiterte Auflage. 1 - 174).
713. О работах по приоритетным системам обслуживания (совм. с Э.А.Даниеляном. Дополнение к книге Н.Джейсуола "Очереди с приоритетами", изд-во "Мир", 255 - 271).
714. Asymptotic Problems in Queueing Theory («Proceedings of the Prague symposium on asymptotic statistics 3-6 september 1973», 107 - 125).
715. Über einige neue Probleme der Bedienungstheorie (Leipzig, Urania Verlag, heft 4, 72 – 75; heft 5, 72 - 75).
716. Об условиях существования финальных вероятностей у Марковского процесса (совм. с А.Д.Соловьевым. «Math. Operationsforsch. und Statist.», 4, heft 5, 379 - 390).
717. Statistical Problems in Teletraffic Theory (with M.A. Schneps-Schneppe. Stockholm, Seventh International Teletraffic Congress, VI, p. 141/1 – 141/6).
718. Математико-статистические методы на службу стандартизации и контроля качества (Минск, Тезисы докладов конференции «Проблемы подготовки и повышения квалификации специалистов в области стандартизации», 32 - 35).
719. Математика и современное естествознание (М. Изд-во «Наука». АН СССР, Научный совет по философским вопросам современного естествознания, Институт философии. Сборник «Синтез современного научного знания», 143 - 158).
720. Полвека советской математической науки (Изд-во «Знание». «Слово лектора», № 1, 38 – 44; № 2, 32 - 38).
721. Математиката в СССР за 50 години от неговото съществуване (София. «Физико математическо списание», т. 16, кн. 1, 3 - 14).
722. Mathematik und Ausbildung von Ingenieuren (Wissenschaftliche Zeitschrift der Technischen Universitat Dresden, 22, heft 5, 777-785).
723. Некоторые проблемы преподавания теории телетрафика и статистического моделирования (совм. с Г.П.Башариным. "Электросвязь", № 9, 73 - 78).
724. Беседи въерху теория на вероятностите и комбинаторика (София. «Математика», кн. 1, 6 - 14).
725. Принцип отражения и математика (Академия Наук СССР, Институт философии. Болгарская Академия Наук, Институт философии. Изд-во «Наука и изкуство», София. «Ленинская теория отражения и современная наука. Принцип отражения и естествознание», т. 2, 78-86.).
726. Методологические предпосылки применения количественных методов в педагогических исследованиях (Москва. Научно-исследовательский семинар общей педагогики АПН СССР. Тезисы докладов к семинару по методологии педагогики и методике

педагогических исследований, VI сессия, 13-16 марта 1973 года. «Объективные характеристики, критерии, оценки и измерения педагогических явлений и процессов», 3 - 4).

727. Колмогоров Андрей Николаевич (БСЭ-3, т. 12, стр. 437).
728. Андрей Николаевич Колмогоров. К 70-летию со дня рождения («Успехи математических наук», т.XXVIII, вып. 5, 5 -15).
729. Ученый и педагог. К 70-летию А.Н. Колмогорова ("Математика в школе", № 2, 88 - 89).
730. Андрей Николаевич Колмогоров (София, «Физико математическо списание», т. 16, кн. 3, 226 - 228).
731. Математик (о творческом пути А.Н.Колмогорова) ("Московский комсомолец", 6.05.1973).
732. Математика и современность (газета «Московский университет», 27 апреля, № 16).
733. Науки о случайном (газета «Московский университет», 13 февраля).

1974 год

734. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. На арабском языке. Москва. Изд-во «Мир»).
735. О работах по статистическим методам теории надежности и теории массового обслуживания в АН СССР (совм. с Ю.К.Беляевым. «Известия АН СССР. Техническая кибернетика», № 3, 19 -23).
736. Об исследованиях по теории информации в системе АН СССР («Известия АН СССР. Техническая кибернетика», № 3, 24-26).
737. О математической теории надежности (Изд-во «Знание». Сборник "Математика в нашей жизни", серия «Математика, кибернетика», № 10, 43 - 62).
738. Новите задачи на теорията на масовото обслужване (София. Изд-во «Наука и изкуство». «Проблеми на съвременната математика», т. 2, 179 - 185).
739. Беседи върху теория на вероятностите и комбинаторика (София. Издание на ЦК на ДКМС и МНП за средношколци, «Математика», година XIII, № 1, 5 – 13; № 2, 4 – 11; № 3, 10 – 19; № 4, 6 - 11).
740. О дефиницији математике (Beograd. «Nastava Matematike», № 1, 81 - 84).
741. Об исследованиях по истории математики, проводящихся в Советском Союзе («Proceedings of the XVII International Congress of Mathematicians, Vancouver, B.C., 21-29 VIII 1974», vol. 2, p. 549 - 560).
742. Role of practice in development of the theory of probability (XIV-th International Congress of the History of Science, Tokyo&Kyoto, Japan, 19 - 27 august 1974, abstracts of Papers, 14, Science Council of Japan).
743. Вплив П.Л.Чебишова на розвиток теорії ймовірностей (Київ, «Нариси з історії природознавства і техніки», вип. XVIII, 13 - 23)
744. Академия наук и прогресс математики («Квант», № 4, 3 – 11; № 5, 18 - 25).
745. Академия наук и развитие математики («Математика в школе», № 1, 4 - 11).
746. Академия наук и развитие математического просвещения в СССР («Математика в школе», № 2, 7 - 14).
747. Прикладные аспекты преподавания математики (Изд-во «Знание». Сборник "Математическое образование сегодня", № 6, 30 - 52).
748. Роль преподавателя вуза в научно-техническом прогрессе (М. Изд-во «Высшая школа». «Математика» (сборник научно-методических статей по математике), вып. 4, стр. 13 – 17).
749. Заведующий кафедрой («Вестник высшей школы», № 3, 51 - 59).
750. Нужны специализированные группы («Вестник высшей школы», № 8, 57 -58).
751. Политехнические аспекты преподавания математики в средней школе («Математика в школе», № 6, 18 - 24).
752. Приобщение к мышлению (Изд-во «Знание». Сборник "Этюды о лекторах", 204 - 211).

753. Научно-технический прогресс и математика (Стенограмма кинозаписи лекции, изд - во «Знание», 3 - 18).
754. 20-й Международный конгресс математиков (Вестник высшей школы, № 12, 45 - 48).
755. Лев Аркадьевич Калужнин (к шестидесятилетию со дня рождения) («Успехи математических наук», т. XXIX, вып.4, 193 - 197).
756. Александр Яковлевич Маргулис (к шестидесятилетию со дня рождения) («Математика в школе», № 1, 84).
757. Предисловие (М. Изд-во «Знание». Сборник "Математическое образование сегодня", серия «Математика, кибернетика», № 6, 3 - 4).
758. Предисловие (М. Изд-во «Знание». Сборник "Математика в нашей жизни", серия «Математика, кибернетика», № 10, 3 - 4).
759. Послесловие к статье Пичурина Л.Ф. "Школьная математика и вузовское преподавание" («Вестник высшей школы», № 7, 25-27).
760. Мой учитель (газета «Вечерняя Москва», 5 ноября).

1975 год

761. О надежности дублированной системы с восстановлением и профилактическим обслуживанием (совм. с М.Динич, Ю.Насром. «Известия АН СССР. Техническая кибернетика», № 1, 66 – 71).
762. Приближенная модель одной физической задачи (Саранск, сборник "Управление, надежность, навигация", вып. 3, 125 - 127).
763. О некоторых вопросах управления научными исследованиями («Тезисы докладов к семинару "Вопросы целевого управления", 9-10 декабря», 5 - 7).
764. Об источниках нового в математике (Изд - во «Знание». «Современная культура и математика», серия «Математика, кибернетика", № 8, 35 - 51), (София, Поредица "Математика, физика, химия", № 1, 32 - 46).
765. Проблемы современной математики (Изд-во «Знание». «Материалы в помощь лектору, выступающему по проблемам физики и математики», 5 - 10).
766. Научно-технический прогресс и математика (Изд-во «Знание». «Материалы в помощь лектору, выступающему по проблемам физики и математики», 11 - 16).
767. О математизации научного знания ("Коммунист", № 5, 73 - 80).
768. Die Wahrscheinlichkeitsrechnung und der wissenschaftlich-technische Fortschritt (Berlin, «ALPHA», Mathematische Schulerzeitschrift, № 1, S. 1 – 2, 24).
769. Научно-технический прогресс и математика («Слово лектора», № 7, 57 - 64).
770. Высшее математическое образование в СССР за 50 лет (М. Изд-во «Высшая школа». «Математика» (сборник научно-методических статей по математике), вып. 5, стр. 3 – 10).
771. Чтобы лучше готовить математиков в университетах («Вестник высшей школы», № 9, 54 - 57).
772. Полезная форма повышения квалификации математиков («Вестник высшей школы», N7, 84 - 87).
773. Об исследованиях по истории математики в Советском Союзе («Математика в школе», № 6, 8 - 16).
774. Теория отражения и математика («Математика в школе», № 4, 4 - 12).
775. Алексей Дмитриевич Семушин (к шестидесятилетию со дня рождения) (совм. с А.Я.Маргулисом, Г.Г.Масловой. «Математика в школе», № 1, 89).
776. Предисловие к книге "Статистические задачи обработки систем и таблицы для числовых расчетов показателей надежности" (Москва, "Высшая школа").
777. Предисловие к книге Б.А. Козлова и И.А. Ушакова «Справочник по расчету надежности» (Москва, «Советское радио»).

778. Gruβschreiben von Prof. Dr. B.Gnedenko (Leipzig. «Tagung der Konferenz der Mathematikmethodiker, 25 - 26 September», 5 - 6).
779. Математика в наступлении (газета «Красное знамя», Харьков, 14 августа).
780. Стахановцам нужны знания… («Московский университет», 25 ноября)

1976 год

781. The Theory of Probability (Moscow, Mir Publishers, third printing).
782. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. "Наука", 8-ое изд., 5 - 167).
783. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (zum A.J. Chintschin. Berlin, Deutscher Verlag der Wissenschaften, 10 Auflage, 3 - 174).
784. О длительности безотказной работы дублированной системы с восстановлением и профилактикой (совм. с И.М.Махмудом. «Известия АН СССР, Техническая кибернетика», № 3, 86 - 91).
785. Приближенная модель одной физической задачи (Саранск. Межвузовский сборник научных работ «Управление, надежность и навигация», вып. 3, 125-127).
786. Предсловие к книге Перроте А.И. "Вопросы надежности радиоэлектронной аппаратуры (Москва. Изд-во "Советское Радио").
787. Ташмухамед Алиевич Сарымсаков (к шестидесятилетию со дня рождения) (совм. с П.С.Александровым, А.Н.Кологоровым, Ю.В.Прохоровым. «Успехи математических наук», т. XXXI, вып. 2, 241-246).
788. Всесоюзное совещание-семинар заведующих математическими кафедрами университетов (совм. с Б.Р.Вайнбергом и др. «Успехи математических наук», т. XXXI, вып 2, 247 – 253).
789. О некоторых вопросах управления научными исследованиями (Московский городской Совет научно-технических обществ, Институт экономики АН СССР, МИНХ им. Г.В.Плеханова, Центральный научно-исследовательский институт информации и технико-экономических исследований в электротехнике. Московский городской научно-практический семинар «Вопросы целевого управления» 9-10 декабря 1976 года, тезисы докладов, 5-7).
790. О математическом образовании в итальянской школе (совм. с М.Клерико. «Математика в школе», № 5, 90 - 93).
791. О некоторых вопросах преподавания математики в средних специальных учебных заведениях (Министерство высшего и среднего специального образования СССР. Изд-во "Высшая школа". «Методические рекомендации по математике», выпуск № 1, стр. 5 – 12).
792. Мястото на приложния и на абстрактния аспект на обучението по математика в средното училище (София. Изд-во «Народна просвета». Сборник "Осъвременяване на обучението по математика", ч.1, 151 - 162).
793. За източниците на новото в математиката (София. Изд-во «Наука и изкуство». Библиотека на дружество «Георги Кирков», поредица «Математика, физика, химия», выпуск 1, сборник статей «Современната култура и математиката», 32-46).
794. О развитии мышления и речи на уроках математики («Математика в школе», № 3, 8 - 13).
795. Важные аспекты проблемы качества обучения («Математика в школе», № 1, 6 - 10).
796. Сообщество наук («Московский университет», 24 февраля).
797. О специальности математика («Московский университет», 2 апреля).
798. Что делать с «неспособными»? (газета «Советская культура», 14 сентября).
799. День рождения – рождение года (интервью провел Н. Марунов. «Московский университет», 1 января).

1977 год

800. О развитии теории массового обслуживания и теории надежности в СССР (совм. с Ю.К. Беляевым, И.А. Ушаковым. «Известия АН СССР, Техническая кибернетика», № 5, 69 - 87).
801. Беседы о теории массового обслуживания (Япония, Гэндай - сугаку, № 10, 11 – 16; № 11, 72 – 77; № 12, 55 - 58, на японском языке).
802. Научно-технический прогресс и математизация знаний (М. Изд - во «Знание», 3 - 61).
803. Математика - народному хозяйству (М. Изд - во «Знание», 3 - 63. Переведена на датский язык и опубликована в Дании в 1978 году (см. № 824)).
804. За советом в природу (Заметки о надежности в технике и живом мире) (совм. с Я.М. Сориным, М.Б. Славиным. Москва. Изд - во «Знание», 3 - 128).
805. Главное направление научно-технического прогресса («Слово лектора», № 7, 31 - 39).
806. Математика: мода или необходимость? (Москва. Изд-во «Знание».«Просто о сложном» (материалы Всесоюзной научно-методической конференции), 80 - 83).
807. Высшее математическое образование в СССР на современном этапе (Киев. Изд-во «Вища школа», сборник "Проблемы высшей школы", вып. 28, 8 - 9).
808. О развитии математики в нашей стране за 60 лет Советской власти («Математика в школе», № 5, 12 - 19), (сокращенный вариант, «Квант», № 11, 18 – 26).
809. Высшее математическое образование за 60 лет Советской власти («Математика в школе», № 3, 8 - 16).
810. О математике Страны Советов ("Квант", № 11, 19-26).
811. Естественные факультеты Московского университета («Математика в школе», № 1, 47 - 51).
812. Current Studies in the history of mathematics in the Soviet Union (Amer. Math. Soc. Transl. (2) v. 109, 119 - 129).
813. Исследования по истории математики в Советском Союзе ("Нариси історіі природознавства і техніки", вып. 23, 3-13).
814. Пьер Симон Лаплас (Българска Академия на науките, «Физико математическо списание», т. 20, кн. 3, 252 - 259).
815. Abbildtheorie und Mathematik (Berlin, «Mathematik in der Schule», № 9, 449 - 456).
816. О воспитании научного мировоззрения на уроках математики («Математика в школе», № 4, 13 - 19).
817. За развитието на мислинето и речта при уроците по математика (София, "Обучинието по математика", № 5, 6 - 12).
818. Рецензия на книгу "Хрестоматия по истории математики" под ред. А.П. Юшкевича (совм. с С.С. Петровой. «Успехи математических наук», т. XXXII, вып. 1, 249 - 251).
819. Нужен эксперимент (газета «Московский университет», 18 марта).

1978 год

820. The Theory of Probability (Moscow. Mir Publishers, fourth printing).
821. Lehrbuch der Wahrscheinlichkeitsrechnung (Academy Verlag, Berlin, 7 Auflage, 3 - 399).
822. Teoria della probabilita (Roma, traduzione dal ruso, Editori Riuniti, Edizioni Mir, 5 - 391).
823. Математика и контроль качества продукции (Изд-во «Знание», серия "Математика, кибернетика", № 11, 3 - 64).
824. Математика - народному хозяйству (№ 803 опубликована в Дании, с добавлением статьи из газеты «Правда» от 4 января 1978, № 4 (см. № 852)).
825. Matematikkens forhold til samfundsokonomien (Tekster fra IMFUFA, № 7, 1 - 77).
826. О методах теории массового обслуживаия (Изд-во «Наука». «Кибернетика и диалектика», 116 - 140).

827. О математических методах кибернетики. Теория массового обслуживания (Москва. Изд-во «Энергия». Сборник "Кибернетику - на службу коммунизму", т. 9, 11 - 27).
828. On some problems in queueing theory (Hungary. «Colloguia mathematica societatis Janos Bolyai», 85 - 92).
829. К вопросу о профилактике технических систем (Саранск, сборник "Управление, надежность, навигация", вып. 4, 97 - 100).
830. Беседы о теории массового обслуживания (Япония, Гэндай - сугаку, № 2, 74 - 76, на японском языке).
831. La mathematisation de la sciece (Alap-Paris, Novosti Moscou, «La Science au 20-e siecle», t. 5, 99 - 127).
832. Математика і науково-технічний прогрес (совм. с B.C. Сологубом. Київ. Изд-во «Знання». 3 - 48).
833. Научно-технический прогресс и математика (Минск. Изд-во «Вышейшая школа». «Хрестоматия по лекторскому мастерству», 122 - 131).
834. Теория вероятностей (совм. с О.Б. Шейниным. Москва. Изд-во "Наука". "Математика XIX века", 184 - 240).
835. О Всесоюзном совещании-семинаре заведующих математическими кафедрами механико-математических и физико-математических факультетов университетов (Москва. Изд-во «Высшая школа». «Математика» (сборник научно-методических статей по математике), выпуск 7, 120-123).
836. Математизация знаний и особенности ее пропаганды (Общество «Знание». «Слово лектора», № 11, 41 - 46).
837. Математика и оборона страны («Математика в школе», № 2, 56 - 61).
838. О математическом образовании в вузах в период научно-технического прогресса (М. Изд-во «Высшая школа». «Математика» (сборник научно-методических статей по математике), выпуск 7, 3-9).
839. Научно-технический прогресс и математическое образование во втузах (М. Изд-во «Высшая школа». «Математика» (сборник научно-методических статей по математике), выпуск 8, 6 - 11).
840. Wybrane problemy nauczania matematyki w szkolach wyzszych (Warszawa. «Zycie szkoly wyzszei», 27 - 42).
841. Совершенствовать мастерство преподавателя («Вестник высшей школы», № 3, 57 -61).
842. Статистическое мышление и школьное математическое образование (Москва. Изд-во «Просвещение». Сборник «На путях обновления школьного курса математики», 56 - 68).
843. Политехнические аспекты преподавания математики в средней школе (Москва. Изд-во «Просвещение». Сборник «На путях обновления школьного курса математики», 121 - 132).
844. Мнение кафедры теории вероятностей МГУ им. М.В. Ломоносова об учебниках для средней школы по математике («Математика в школе», № 5, 26 - 27).
845. Предисловие к четвертому изданию книги А.Я. Хинчина "Цепные дроби" (Москва. Изд-во «Наука». 3 - 4).
846. Über einige grundsatzliche Fragen zur Entwicklung der Mathematik im Zusammenhang mit der Erziehung zu einer wissenschaftlichen Weltanschauung (Berlin, «Mathematik in der Schule», No 9, 451 - 455).
847. Ученый, педагог, реформатор (к 75-летию со дня рождения А.Н. Колмогорова) («Математика в школе», № 2, 93-94).
848. Алексей Иванович Маркушевич (к 70–летию со дня рождения) (совм. с Б.В. Шабатом и др.) («Математика в школе», № 2, 95-96).
849. Комсомол и развитие советской математики («Математика в школе», № 5, 22 - 24).

850. Памяти Бориса Осиповича Солоноуца (совм с Е.С. Вентцель и др.) (Москва. Изд-во «Высшая школа». «Математика» (сборник научно-методических статей по математике), вып. 7, 137-139).
851. Памяти Рафаила Самойловича Гутера (совм. с И.В. Чувило и др.) (Москва. Изд-во «Высшая школа». «Математика» (сборник научно-методических статей по математике), вып. 8, 112 – 113).
852. Математика на каждый день (газета «Правда», 4 января, № 4. Статья переведена на датский язык и опубликована в Дании в 1978 году (см. № 824)).
853. Слова, идущие в атаку, как бойцы… (интевью провел И.Н. Егоров. Газета Московского округа противовоздушной обороны «На боевом посту», 27 января).

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854. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (zum A.J. Chintschin. Berlin, Veb Deutscher Verlag der Wissenschaften, 11 Auflage, 3 - 174).
855. Теория на вероятностите (совм. с А.А. Гешевым. Пловдив. Изд-во Пловдивского университета. 3 - 219).
856. Вероятностей теория (Украинская Советская Энциклопедия, т. 2, 191).
857. Zum sechsten Hilbertschen Problem (Leipzig, Ostwalds Klassiker der exakten Wissenschaften, b. 252, 145 - 150).
858. Mathematics in Sceintific Research and Education (в книге «Computers in the life sceinces», printed in Great Britain, by Biddles Ltd.Guildford, Surrey, Croom Helm London, 23 – 25).
859. Popularisation of Mathematics, Mathematical Ideas and Results in the USSR (Denmark, Tekster fra IMFUFA, nr. 18, 60 - 62).
860. О математическом образовании математика («Вестник высшей школы», № 10, 21 - 24).
861. The Mathematical Education of Engineers (совм. с Z. Khalil. «Educational Studies in Mathematics», 10 (1979), 71 - 83, D. Reidel Publishing Co., Dordrecht, Holland, and Boston, USA).
862. Педагог, коллектив и воспитание творческих начал («Вестник высшей школы», № 4, 38 - 41).
863. Как подготовить творческого специалиста? («Вестник высшей школы», № 3, 11).
864. О математическом творчестве («Математика в школе», № 6, 16 - 22).
865. Школьный курс математики и воспитание мировоззрения («Математика в школе», № 3, 3 - 6).
866. Предисловие к книге Х.Крамера "Полвека с теорией вероятностей: наброски воспоминаний" (Изд-во «Знание», серия "Математика, кибернетика", № 2, 3 - 4).
867. Предисловие к книге И.Г. Башмаковой "Становление алгебры" (Изд-во «Знание», серия «Математика, кибернетика», № 9, 3 - 7).
868. Предисловие к книге А.Я. Маргулиса "Серия "Математика, кибернетика" за 12 лет" (Изд - во «Знание», серия "Математика, кибернетика", № 10, 3 - 8).
869. Алексей Иванович Маркушевич (некролог) (совм. с А.Н. Колмогоровым и др. «Математика в школе», № 5, 77 – 78).
870. Петр Сергеевич Моденов (некролог) (совм. с А.Г. Свешниковым. «Математика в школе», № 1, 79 - 80).
871. Рецензия на книгу "Хрестоматия по истории математики" под ред. А.П. Юшкевича (совм. с С.С. Петровой. «Успехи математических наук», т. 34, вып. 1, 262 - 264).
872. Радость творчества («Учительская газета», 10 марта).
873. Нет доброты по выбору (интервью провела Л. Артомонова, газета «Социалистическая индустрия», 16 июня).
874. ЭВМ: перспективы и опыт применения (газета «Вышка», 17 июля).

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875. Математические методы управления качеством продукции (Изд-во «Знание», 4 - 32).
876. Математика в современном мире (Москва, Изд-во «Просвещение», 3 - 128).
877. Теоретическая и прикладная математика (изд-во «Знание», «Что такое прикладная математика», серия "Математика, кибернетика", № 10, 50 - 62).
878. Математика в Московском Университете за первые 100 лет (Изд-во «Знание». "Математическая наука в МГУ", серия "Математика, кибернетика", № 4, 5 - 20).
879. О московской школе теории вероятностей (изд-во «Знание». «Математическая наука в МГУ», серия "Математика, кибернетика", № 4, 30 - 44).
880. Математика в Московском Государственном университете («Квант», № 2, 2 - 9).
881. Московский университет и математическое просвещение («Математика в школе», № 2, 14 - 19).
882. Развитие математики и математического образования в СССР («Математика в школе», № 6, 3 - 8).
883. НТП и математическое образование ("Вестник высшей школы", № 9, 52 - 54).
884. Кафедра и подготовка творческой смены ("Вестник высшей школы", № 3, 43 - 47).
885. Възпитаване на научен мироглед в уроците по математика (София. Изд-во «Народна просвета», сборник статей «За някои въпроси на обучинието по математика», 5-18).
886. Върху развитието на мислинето и речта в уроците по математика (София. Изд-во «Народна просвета», сборник статей «За някои въпроси на обучинието по математика», 18 - 28).
887. Елементи от историята на науката в уроците по математика (София. Изд-во «Народна просвета», сборник статей «За някои въпроси на обучинието по математика», 28 - 41).
888. Ленин и математика («Математика в школе», № 1, 3 - 8).
889. Предисловие к сборнику «Математика как профессия» (изд-во «Знание», Серия "Математика, кибернетика", № 6, 3 - 23).
890. Предисловие к книге А. Реньи "Трилогия о математике" (Москва. Изд-во «Мир», 5 - 16).
891. Предисловие к книге Н.А. Плохинского, "Алгоритмы биометрии" (Изд-во Московского университета, 3 - 4).
892. Предисловие к книге Н.Б. Вассоевича и др. "Коэффициент ранговой корреляции Спирмена" (Изд-во Московского университета, 3).
893. Наум Яковлевич Виленкин. К 60-летию со дня рождения (совм. с С.И. Шварцбурдом, А.Г. Мордковичем. «Математика в школе», № 6, 63 - 64).
894. О серии брошюр "Математика, кибернетика" («Математика в школе», № 5, 76 - 77).
895. О книге "Биографический словарь деятелей в области математики" («Математика в школе», № 4, 64 - 65).
896. Математик (К 1000-летию со дня рождения Абу Али Ибн Сины) («Комсомольская правда», 21 августа).
897. Дух поиска научной истины (интервью провел В. Прошкин, газета «Московский университет», 28 марта, № 19).
898. Желаю вам расцвета (интервью провела С. Козлова, «Тувинская правда», 24 мая, № 121).

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899. Из истории науки о случайном (изд-во «Знание», серия "Математика, кибернетика", № 6, 3 - 64).
900. Математическое образование в вузах (Москва, изд. «Высшая школа», 3 - 173).
901. Предельные теоремы для сумм случайного числа случайных слагаемых (совм. с Д.Б. Гнеденко. Ивановский ГУ, межвузовский сборник "Алгебраические системы", 78 - 88).

902. О формулах Эрланга для систем с потерями (совм. с О. Аннаоразовым. «Известия АН Туркменской ССР», серия физико-технических, химических и геологических наук, № 6, 99-100).
903. Математика в Московском университете («Москва, изд-во «Высшая школа». «Сборник научно-методических статей по математике», вып. 9, 124 - 136,).
904. О месте лекции в математическом образовании (Москва, изд-во «Высшая школа». «Сборник научно-методических статей по математике», вып. 9, 25 - 37,).
905. О призвании учителя («Математика в школе», № 5, 5 - 11).
906. Роль математики в формировании у учащихся научного мировозрения («Сурган Хумуужуулэгч», № 1, 35 - 45).
907. О воспитании научного мировоззрения на занятиях по математике (Ивановский ГУ, межвузовский сборник "Алгебраические системы", 10 - 18).
908. Слово, зажигающее сердца (Изд-во «Знание». Сборник "Живое слово науки", 184 - 189).
909. Константин Петрович Сикорский. К 85-летию со дня рождения (совм. с Р.С. Черкасовым, Н.А. Курдюмовым. «Математика в школе», № 5, 66).
910. Симеон Дени Пуассон («Математика в школе», № 3, 64 - 67; Болгария, "Физико-математическое списание", 23(56), № 3, 242-246).
911. Предисловие к книге А.Н. Колмогорова и др. "Физико-математическая школа при МГУ" (Изд-во «Знание», серия "Математика, кибернетика", № 5, 3 - 7).
912. Предисловие к книге Л.Н. Дашевского, Е.А. Шкабара "Как это начиналось" (Изд-во «Знание», серия "Математика, кибернетика", № 1, 3 - 6).
913. Введение к сборнику "Труды IV Всесоюзной школы-семинара. Теория массового обслуживания. Баку. 1978" (Москва, ВНИИСИ, 3).
914. Историко-математические исследования (к выходу XXV тома) («Успехи математических наук», т. 36, вып. 4, 242-243).
915. Изабелла Григорьевна Башмакова. К 60-летию со дня рождения (совм. с П.С. Александровым, А.Н. Колмогоровым и др.. «Успехи математических наук», т. 36, № 5(221), 211-214).
916. Изабелла Григорьевна Башмакова (совм. с П.С.Александровым, А.Н.Колмогоровым и др.. "Математика в школе", № 1, 73-74).
917. Алексей Иванович Маркушевич (совм. с П.С.Александровым, А.Н.Колмогоровым и др.. Болгария, "Физико-математическое списание", 23(56), № 2, 150-152).
918. Сагды Хасанович Сираждинов (к 60-летию со дня рождения) (совм. с А.Н.Колмогоровым и др.. «Успехи математических наук», т. XXXVI, № 1, 73-74).
919. Мордухай Моисеевич Вайнберг (некролог) («Математика в щколе», № 1, 80).
920. Комментарий (газета «Ленинская смена», 21 апреля).
921. Нам не нужна война («Московский университет», 16 декабря).
922. Шабыт берер бул кадам («КАЗАКСТАН МУГАЛIМI», 24 апреля).
923. В интересах науки, в интересах практики (интервью провел Е. Марченко. «Московский университет», 27 ноября).

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924. The Theory of Probability (Moscow. Mir Publishers, fifth printing, 1-392).
925. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. Изд-во «Наука», 9-ое изд., 3 - 156).
926. Математика и теория надежности (совм. с А.Д. Соловьевым. Изд-во «Знание», серия "Математика, кибернетика", № 10, 3 - 63).
927. Формирование мировоззрения учащихся в процессе обучения математике (Москва. Изд-во «Просвещение», 3 - 144).

928. Математические модели старения полимерных изоляционных материалов (совм. с Р.П. Брагинским, С.А. Молчановым и др. «Доклады АН СССР», т. 268, № 2, 281 – 284).
929. Об одном свойстве предельных распределений для максимального и минимального членов вариационного ряда (совм. с Л. Сенуси-Берекси. «Доклады АН СССР», т. 267, № 5, 1039 - 1040).
930. Об одном свойстве логистического распределения (совм. с Л. Сенуси-Берекси. «Доклады АН СССР», т. 267, № 6, 1293 - 1295).
931. О распределениях Лапласа и логистическом как предельных в теории вероятностей (совм. с Д.Б. Гнеденко. «Сердика. Българско математическо списание», т. 8, 229 - 234).
932. Теория надежности (совм. с Ю.К. Беляевым. «Математическая энциклопедия», т. III, 854 - 858).
933. Статистически методи за контрол на качеството на масовата промишлена продукция (София. "Математика", № 7, 2 - 9).
934. Математическое образование и математика в СССР за 60 лет («Математика в школе», № 6, 6 - 10).
935. Статья В.И. Ленина "О значении воинствующего материализма" и математическое образование («Математика в школе», № 4, 5 - 8).
936. Московский государственный университет (О вступительных экзаменах в вузы в 1981 году. «Математика в школе», № 2, 57 - 59).
937. Какъв трябва да бъде учебникът по математика за ученици (София. "Обучението по математика", № 1, 10 - 18).
938. Математика в современном мире («Вечерняя средняя школа», № 1, 30 - 33).
939. О математических способностях и их развитии («Математика в школе», № 1, 31 - 34).
940. Математика в СССР («Квант», № 11, 2 - 4).
941. Михаил Васильевич Остроградский («Квант», № 10, 5 - 10).
942. Александр Яковлевич Маргулис (некролог) («Математика в школе», № 1, 80).
943. Предисловие к книге М.А.Ястребенецкого «Надёжность технических средств в АСУ технологическими процессами» (Москва. «Энергоиздат»).
944. Gruβadresse aus der UdSSR (An den Mathematiker-Kongreβ der DDR 1981. «ALPHA», Mathematische Schulerzeitschrift, № 1, S. 8).

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945. Предисловие (совм. с Д. Кёнигом) и глава I в "Handbuch der Bedienungstheorie I" (Academie - Verlag, Berlin, 7-9, 19-38).
946. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (und A.J. Chintschin. VEB Deutcher Verlag der Wissenschften, Berlin, 3 - 174).
947. On limit theorems for a random number of random variables (Proceedings of the Fourth USSR-Japan Symposium "Probability Theory and Mathematical Statistics", august 23-29, 1982. Springer-Verlag, Berlin, 167 - 176).
948. On some stability theorems (Proceedings of the 6th International Seminar "Stability Problems for Stochastic Models", april 1982. Springer-Verlag, Berlin, 24 - 31).
949. О свойстве продолжимости предельных распределений для максимального члена последовательности (совм. с Л. Сенуси-Берекси. «Вестник Московского университета», серия 1, "Математика. Механика", № 3, 11 - 20).
950. Предельные теоремы для крайних членов вариационного ряда (совм. с А. Шерифом. «Доклады АН СССР», т. 270, № 3, 523 - 525).
951. A characteristic property of one class of limit distributions (and S. Janjic. Math. Nachr., 113, 145 - 149).
952. Теоремы устойчивости для предельных распределений членов вариационного ряда (Изд-во «Наука», «Теория вероятностей и ее применения», т. 28, вып. 4, 809-810).

953. Математические модели старения полимерных изоляционных материалов (совм. с Р.П. Брагинским и др. «Доклады АН СССР», т. 268, № 2, 281 - 284).
954. О математических задачах теории массового обслуживания и надежности (совм. с Ю.К. Беляевым, И.А. Ушаковым. «Известия АН СССР, Техническая кибернетика», № 6, 3 - 12).
955. Учет периодичности при оценке коэффициента загрузки диспетчера (совм. с Л.Г. Афанасьевой и Н.А. Дроздовым. Москва. АН СССР. «II Всесоюзная конференция по управлению воздушным движением. Тезисы докладов». 51 – 53)
956. Теория вероятностей и математическая статистика (в сборнике "Очерки развития математики в СССР". Киев, изд-во "Наукова думка", 500-513).
957. Математика и научное познание (изд-во «Знание», серия "Математика, кибернетика", № 7, 3-64).
958. О преподавании предметов теоретико-вероятностного цикла во втузах (Москва. Изд-во «Высшая школа». «Сборник научно-методических статей по математике», вып. 10, 189-191).
959. О математических способностях (Москва. Изд-во «Высшая школа». «Сборник научно-методических статей по математике», вып. 10, 154-163).
960. Об учебниках по математике для высших учебных заведений (Москва. Изд-во «Высшая школа». «Сборник научно-методических статей по математике», вып. 11, 40-51).
961. О математическом творчестве (Москва. Изд-во «Высшая школа». «Сборник научно-методических статей по математике», вып. 11, 141-156).
962. Колос и машина ("Изобретатель и рационализатор", № 6, стр. 6 - 7).
963. О Продовольственной программе и математике («Математика в школе», № 2, 4 - 9).
964. Математика и производство («Квант», № 1, 3 - 6, 11).
965. Карл Маркс и математиката (София, «Физико-математическо списание», т. 25, вып. 4, 267 - 276).
966. Андрей Николаевич Колмогоров (к 80-летию со дня рождения) (совм. с Н.Н. Боголюбовым, С.Л. Соболевым. «Успехи математических наук», т. 38, вып. 4, 11 - 23). (Эта статья также помещена в третий том избранных работ А.Н. Колмогорова «Теория информации и теория алгоритмов», М., «Наука», 1987, 7-23)..
967. Андрей Николаевич Колмогоров (к 80-летию со дня рождения) («Математика в школе», № 2, 76 – 78).
968. Павел Сергеевич Александров (некролог) (совм. с А.Н. Колмогоровым. «Математика в школе», № 1, 47 - 48).
969. Иван Федорович Тесленко (к 75-летию со дня рождения) (совм. с М.И. Бурда, Р.С. Черкасовым. «Математика в школе», № 2, 78 – 79).
970. Предисловие к книге Е.Ю. Барзиловича и др. "Вопросы математической теории надежности" (Москва, изд-во «Радио и связь», 3 - 8).
971. Предисловие к книге «Геометрия гильбертова пространства и три принципа функционального анализа» (Изд-во «Знание», серия "Математика, кибернетика", № 6, 3).
972. О книге А.Я. Халамайзера "Математика гарантирует выигрыш" («Математика в школе», № 6, 69).
973. «Ценю увлеченность…» (газета «Московский университет», 4 января).
974. Путь в большую науку (газета «Брянский комсомолец», 2 марта, № 25).
975. Дорогу осилит идущий (газета «Московский университет», 14 апреля).
976. Пътят до истината (интервью провел Дончо Христов. «АБВ» информационен седмичник за книгата, 8 ноември).

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977. Теория на вероятностите и математическа статистика (совм. с А.А. Гешевым. София, изд-во «Наука и изкуство», 3 - 229).
978. Matematika siuolaikiniame pasaulyje (Математика в современном мире) (Kaunas, Sviesa, 4 - 102).
979. Service systems with the time-dependent input and service intensities (Fundamentals of teletraffic theory. «Proceedings of the Third International Seminar on Teletraffic Theory, Moscow, june 20-26, 1984», 142 - 146).
980. Принципы аналитического моделирования диспетчера в секторе управления воздушным движением (совм. с Л.Г. Афанасьевой. Пермь, «Тезисы докладов Всесоюзной научно-технической конференции "Применение статистических методов в производстве и управлении", 31 мая - 2 июня», 101 - 102).
981. Особенности анализа эффективности криогенных систем и установок с резервными режимами работы (совм. с М.В. Козловым и др.. Ленинград. «Межвузовский сборник научных трудов "Криогенная техника и кондиционирование"», 3 – 8).
982. О распределении медианы (совм. с С. Стоматовичем, А. Шукри. «Вестник Московского университета», серия "Математика. Механика", № 2, 59 - 63).
983. За управлението на качеството на промишлената продукция (София, «Социално управление», № 1, 3 - 10).
984. К истории понятия вероятности случайного события (совм. с М.Т. Перес. «АН СССР, Вопросы истории естествознания и техники», № 1, 71 - 75).
985. Международный математический конгресс в Варшаве (16-24 августа 1983 г.) («Математика в школе», № 4, 67 - 69).
986. Математическое творчество и общественный прогресс («Квант», № 2, 2 - 5).
987. Воспитание моральных принципов и математика («Математика в школе», № 5, 6 - 10).
988. Математические рукописи К.Маркса и вопросы математического образования («Математика в школе», № 2, 7 - 12).
989. Слово, зажигающее сердца (Изд-во «Знание». «Владимир Васильевич Голубев», серия "Математика, кибернетика", № 10, 52 - 57).
990. Михаил Васильевич Остроградский (Изд-во «Знание», серия "Математика, кибернетика", № 5, 3 - 63).
991. Евгений Евгеньевич Слуцкий (Киев. "У світі математики", вып. 15, стр. 40).
992. А.Я. Халамайзер (к 60-летию со дня рождения) (совм. с З.А. Скопец и П.В. Стратилатовым. «Математика в школе», № 3, 77-78).
993. Предисловие к книге «Handbuch der Bedienungstheorie II» (und D. König. Academie-Verlag, Berlin, 7 - 8).
994. Какими быть X-XI классам? (коллективное письмо девяти академиков (Б. Гнеденко, Н. Дубинин, И. Кикоин, А. Колмогоров, М. Нечкина, С. Никольский, Н. Семенов, С. Соболев, Д. Эпштейн) с предложением к проекту реформ школы) («Известия», 26 января).
995. О ценности знаний («Правда», 27 февраля).
996. Яш зехинлер тапланяр («Мугаллымлар газети», 30 мая)
997. Что может статистика (Пермь, газета «Звезда», 23 июня, № 145).
998. Математика и жизнь («Учительская газета», 5 июля).
999. Вторая грамотность XX века («Известия», 18 декабря).
1000. О цирке (интервью, еженедельник «Советский цирк», 6 июля, № 1).
1001. Верю в победу чемпиона (интервью провел Гагик Карапетян, газета «Вечерняя Казань», 25 октября).

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1002. Математика и контроль качества продукции (на монгольском языке, Улан-Батор, Улсын Хэвлэпийн Газар, Улаанбаатор, 3-69).
1003. Математика и математическое образование в современном мире (Изд-во «Просвещение», 3-191).
1004. О продолжимости предельных совместных распределений для членов вариационного ряда (совм. с Х. Баракат и С. Хемида. «Доклады АН СССР», т. 284, 789 – 790)
1005. Вероятностно-статистическое моделирование управления воздушным движением (совм. с Л.Г. Афанасьевой. Тарту, «Тезисы докладов III Всесоюзной научно-технической конференции "Применение многомерного статистического анализа в экономике и оценке качества продукции", 17-18 сентября 1985 года», 134 - 144).
1006. О некоторых актуальных проблемах надежности (совм. с Е.Ю. Барзиловичем. Изд-во «Машиностроение», сборник "Проблемы надежности летательных аппаратов", 4 – 9).
1007. И не только в биологии (Вестник высшей школы)», № 10, 31 - 32).
1008. К читателям (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 5-7).
1009. Вероятность (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 36-37).
1010. Вероятностей теория (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 37-39).
1011. Андрей Андреевич Марков (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 39-40).
1012. Математика (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 172-178).
1013. Математическая статистика (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 183-184).
1014. Программа педагогических вузов по истории математики (составлена совм. с. А.П.Юшкевичем, И.Г.Башмаковой, Б.А.Розенфельдом, С.С.Демидовым) («Математика в школе», № 3, 57-60).
1015. О двух совещаниях в Болгарии по вопросам школьного образования ("Информатика и вычислительная техника", 68).
1016. Математика и математики в Великой Отечественной войне («Квант», № 5, 9-15).
1017. О нашем товарище (газета «Московсий университет», 1 апреля).
1018. Не стареет наука, а значит – и кафедра (газета «Московский университет», 11.XII).
1019. Любовь к математике (интервью провела О. Огнева. «Студенческий меридиан», № 3, 10-11).

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1020. Формиране на мироглед у учениците при обучинието по математика (София. Изд-во «Народна просвета», 2 - 156).
1021. Предельные теоремы для членов вариационного ряда (Москва. Изд-во "Наука", «Первый Всемирный конгресс Общества математической статистики и теории вероятностей им. Бернулли. Тезисы», т. 1, 194).
1022. Математические основы исследования (Москва. Изд-во «Машиностроение», справочник «Надежность и эффективность в технике», т.1, 54 - 58).
1023. Место математической статистики в научно-техническом прогрессе ("Заводская лаборатория", т. 52, № 12, 1 - 2).
1024. Вторая грамотность века (изд-во «Знание», «Слово лектора», № 3, 14 - 18).

1025. К истории основных понятий теории вероятностей («История и методология естественных наук», вып. XXXII, 81 - 88).
1026. Из истории начального периода истории теории вероятностей (Москва. Изд-во "Наука", «Первый Всемирный конгресс Общества математической статистики и теории вероятностей им. Бернулли. Тезисы», т. 2, 939).
1027. Я. Бернулли и теория вероятностей (совм. с С.Х. Сираждиновым. Производственно-издательский комбинат ВИНИТИ, сборник «Бернулли ученые и общество Бернулли», 24 – 37, сборник «Bernoulli scholars. Bernoulli society», 21-31).
1028. Математической подготовке - прикладную направленность («Вестник высшей школы», № 9, 49 - 52).
1029. Математизация знания и вопросы математического образования (М. АН СССР, Центральный совет философских (методологических) семинаров при Президиуме АН СССР, Московский государственный университет. «Сборник трудов симпозиума «Математизация современной науки: предпосылки, проблемы, перспективы», проведенного в январе-феврале 1983 года в г. Пущино», 23 - 32).
1030. Математиката и изучаването на заобикалящата ни действителност (София, "Проблеми на ученическото техническо творчество", № 3, 25 - 29).
1031. О двух совещаниях в Болгарии по вопросам школьного образования («Математика в школе», № 1, 68 - 69).
1032. Об упражнениях по математике (совм. с М.В. Потоцким. Москва. Изд-во "Высшая школа", «Сборник научно-методических статей по математике», вып. 13, 6-15).
1033. Об исследовании по истории школьного математического образования в нашей стране, проводимом в Японии (совм. с Р.С. Черкасовым. «Математика в школе», № 4, 75 - 76).
1034. Из воспоминаний о В.В. Голубеве (Москва. Изд-во ВВИА им. Н.Е. Жуковского, «Голубев Владимир Васильевич. К 100-летию со дня рождения», 60-61).
1035. Великий русский ученый и просветитель Михаил Васильевич Ломоносов (совм. с Н.П. Жидковым. «Математика в школе», № 5, 49 - 54).
1036. Адольф Павлович Юшкевич (совм. с Башмаковой И.Г. и др.. "Математика в школе", № 4, 72-74).
1037. Вечният стремеж към открития (Интервью. София, "Наука и техника за младежта", № 3, 14 - 18).
1038. Математика в инженерном деле (НТР. Проблемы и решения. Бюллетень ордена Ленина Всесоюзного общества «Знание». № 10 (25), 20 мая – 2 июня).

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1039. Введение в теорию массового обслуживания (совм. с И.Н. Коваленко. «Наука», 2-е изд., перераб. и дополн., 1-336).
1040. Lehrbuch der Wahrscheinlichkeitsrechnung (Berlin. Academie-Verlag, eighth edition).
1041. Современные задачи теории и практики надежности («Надежность и контроль качества», № 11, 3 - 10).
1042. Работы академика А.А.Маркова по теории вероятностей (Москва. Изд-во "Наука". С.Я.Гродзенский "Андрей Андреевич Марков", стр. 223-237).
1043. В единстве теории и практики (совм. с Д.Б. Гнеденко. «Вестник высшей школы», № 4, 48 – 50).
1044. Университеты и научно-технический прогресс (Изд-во «Высшая школа». «Сборник научно-методических статей по математике», вып. 14, 3 - 11).
1045. Посев научный - жатве народной (Изд-во «Советская Россия», сборник статей "Октябрь, наука, прогресс", 108 – 117).
1046. Мечта и НТР («Слово лектора», № 9, 63 - 64).
1047. О математике Страны Советов («Квант», № 11, 3 - 8).

1048. Развитие школьного математического образования в Советском Союзе за 70 лет (совм. с Г.Г. Масловой, Р.С. Черкасовым. «Математика в школе», № 6, 6 - 14).
1049. Математика и математическое образование в Стране Советов («Математика в школе», № 4, 6 - 12; № 5, 3 - 7).
1050. К вопросу о содержании факультатива по теории вероятностей («Математика в школе», № 3, 24 -25).
1051. О педагогической деятельности кафедры теории вероятностей Московского университета («Вестник Московского университета», сер. 1, Математика. Механика; № 2, 91 - 94).
1052. По поводу первого Всемирного конгресса Общества им. Я. Бернулли (совм. с М.А. Мирзахмедовым, Х.П. Ариповым. «Математика в школе», № 1, 75 - 76).
1053. Итоги работы приложения «Надежность и контроль качества» за 1987 год («Надежность и контроль качества», № 12, 3 - 6).
1054. Предисловие ко второму тому справочника «Надежность и эффективность в технике» (Москва. Изд-во «Машиностроение», т. 2 «Математические методы в теории надежности и эффективности», 8-10).
1055. Предисловие редактора к книге С.Я.Гродзенского "Андрей Андреевич Марков" (Москва, изд-во "Наука", стр. 5-10).
1056. Рецензия на книгу «Справочник по надежности технических систем» («Надежность и контроль качества», № 3, 57 – 58).
1057. Рецензия на книгу «Надежность систем энергетики» («Надежность и контроль качества», № 9, 60).
1058. Журнал "Вопросы истории естествознания и техники" («Математика в школе», № 3, 73 - 74).
1059. О двух сборниках трудов по философским вопросам математики (совм. с В.Н. Пономаревым, А.А. Григоряном. «Математика в школе», № 5, 74 - 75).
1060. Строки из биографии (интервью провел З. Табатадзе. «Московский университет», 22 января).
1061. Алгебра здоровья (интервью провел Ю. Грот. «Советский спорт», 31 января).
1062. Профессия – математик («Московский университет», 5 апреля).
1063. Решать нестандартные задачи (интервью провел А. Баршай. «Советская Киргизия», 26 мая).
1064. Для чего нужны олимпиады (интервью провела А. Лазутина. «Учителькая газета», 11 июня).
1065. Воспитание творчеством («Учительская газета», 8 августа).
1066. Математика и экономика (совм. с Е.В. Морозовым. Газета «Ленинская правда», Петрозаводск, 22 сентяря).
1067. Жизнь во имя науки (совм. с И.М. Гельфандом, С.М. Никольским, С.Л. Соболевым и др., «Учительская газета», 26 ноября).

1988 год

1068. Курс теории вероятностей (Изд-во «Наука», изд. 6-е, переработанное и дополненное, 3-447).
1069. The theory of probability (Moscow. Mir Publishers, sixth printing, 1-529).
1070. Курс теории вероятностей (Египет, на арабском языке, совм. с изд-ом «Мир»).
1071. Особенности перколяционной модели старения полимеров (совм. с Р.П. Брагинским, В.В. Малуновым и др. «Доклады АН СССР», т. 303, № 3, 535 – 537).
1072. Теоретическое и статистическое исследование дефектного множества в эмаль-лаковых электроизоляционных покрытиях (совм. с Р.П. Брагинским и др.. «Доклады АН СССР», т. 303, № 2, 270-274).

1073. О некоторых современных проблемах теории и практики надежности (совм. с И.А.Ушаковым. «Вестник машиностроения», № 12, 3-9).
1074. Нормирование надежности и "перестройка" взглядов (совм. с И.А. Ушаковым. «Стандарты и качество», № 7, 35 - 38).
1075. Совершенствование математического образования в университете (Саранск. Министерство высшего и среднего специального образования РСФСР, Мордовский государственный университет. Сборник "Совершенствование содержания математического образования в школе и ВУЗе", 13 - 19).
1076. О специальных курсах и семинарах естественнонаучного и прикладного характера (Изд-во «Высшая школа», «Сборник научно-методических статей по математике», вып. 15, 4 - 9).
1077. О некоторых вопросах перестройки математического образования в университетах (совм. с Д.Б. Гнеденко. «Современная высшая школа», № 3, 81 - 90).
1078. Роль математических методов исследования в кардинальном ускорении научно-технического прогресса (совм. с А.И. Орловым. «Заводская лаборатория», т. 54, № 1, 1-4).
1079. О математике Страны Советов (Япония. Изд-во Гэндай-Сугакуся, журнал «Бэйсик Сугаку», № 3, 47-52), (№ 798 и № 1031 – разные статьи. Данная статья -- перепечатка с № 798).
1080. О курсе математики в школах Японии (совм. с Р.С. Черкасовым. «Математика в школе», № 5, 72 – 76).
1081. Отстаивая исследовательский поиск (Изд-во «Наука», сборник «Путь в большую науку: академик Аксель Берг», 147 - 150).
1082. Относиться с уважением («Московский университет», 11 октября).

1989 год

1083. The theory of probability (Chelsea Publishing Company, New York. 1-529).
1084. Introduction to Queueing Theory (and I.N. Kovalenko. Birkhäuser, Boston, 2nd edition, revised and supplemented, 1-315).
1085. Об оценке неизвестных параметров распределения при случайном числе независимых наблюдений («Академия наук ГССР, Труды Тбилисского математического института», т. 92, 146-150).
1086. О работах кафедры теории вероятностей по математической теории надежности (совм. с Ю.К. Беляевым, А.Д. Соловьевым. «Теория вероятностей и ее применения», т. XXXIV, вып. 1, 191 – 196).
1087. Современная теория надежности: состояние, проблемы, перспективы (совм. с И.А.Ушаковым. «Надежность и контроль качества», № 1, 6-22).
1088. Кафедра теории вероятностей Московского университета (Изд-во «Наука». «Теория вероятностей и ее применения», т. XXXIV, вып. 1, 119 - 127).
1089. К читателям (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 2-ое изд., 5-7).
1090. Вероятность (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 2-ое изд., 36-37).
1091. Вероятностей теория (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 2-ое изд., 37-39).
1092. Андрей Андреевич Марков (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 2-ое изд., 39-40).
1093. Математика (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 2-ое изд., 172-178).

1094. Математическая статистика (Москва. Изд-во «Педагогика», «Энциклопедический словарь юного математика», 2-ое изд., 183-184).
1095. Об образовании преподавателя математики средней школы («Математика в школе», № 3, 19 - 22).
1096. Математика как орудие педагогического исследования (Свердловск. Министерство народного образования РСФСР, Свердловский государственный педагогический институт. Сборник "Применение математических методов и ЭВМ в педагогических исследованиях", 6 - 22).
1097. О роли математики в формировании у учащихся научного мировоззрения и нравственных принципов («Математика в школе», № 5, 19 - 26).

1990 год

1098. Теория вероятностей (совм. с И.Н. Коваленко. Киев. Изд-во «Вища школа», 3-328,).
1099. Предисловие к книге В.М.Круглова и В.Ю.Королёва "Предельные теоремы для случайных сумм" (Москва. Изд-во МГУ. 5-8).
1100. Вначале было слово («Вестник высшей школы», № 1, 23 - 27).
1101. Ученый, учитель, гражданин («Математика в школе», № 5, 56 - 59).
1102. Воспоминания о Вячеславе Васильевиче Степанове (К 100-летию со дня рождения) («Успехи математических наук», т.45, вып. 6, 165 - 169).
1103. Советская школа и В.И. Ленин («Математика в школе», № 3, 2- 8).
1104. О применении современных статистических методов в управлении качеством продукции (совм. с А.И.Орловым. «Надежность и контроль качества», № 3, 92).

1991 год

1105. Einfuhrung in die Wahrscheinlichkeitstheorie (Academy Verlag, Berlin, 9 Auflage, 1-469).
1106. Введение в специальность математика (Москва, Изд-во «Наука», 3 - 237).
1107. Павел Сергеевич Александров (совм. с А.Н.Колмогоровым. Изд-во "Наука", сборник "Математика в её историческом развитии", 125-130).
1108. Математика в современном мире и математическое образование («Математика в школе», № 1, 2 - 4).
1109. Развитие мышления и речи при изучении математики («Математика в школе», № 4, 3 - 9).
1110. Памяти Ганса Фройденталя (совм. с А.Я. Халамайзером. «Математика в школе», № 2, 79-80).

1992 год

1111. Probability Theory (совм. с О.Б. Шейниным. «Mathematics in the 19th Century», Birkhäuser, Boston).
1112. Введение (совм. с В.И. Анискиным. НИИ механизации сельского хозяйства. Сборник "Применение вероятностных методов решения задач технического обеспечения агропромышленного производства", 4 - 10).
1113. Математика и проблемы надежности и безопасности современной техники («Математика в школе», № 1, 3 - 7).
1114. Математика в Московском университете (1755 - 1933) (совм. с О.Б. Лупановым, К.А. Рыбниковым. Изд-во Московского университета, сборник "Математика в Московском университете", 3 - 19).
1115. Кафедра теории вероятностей (Изд-во Московского университета, сборник "Математика в Московском университете", 217 - 237).

1116. Ростислав Семенович Черкасов. К 80-летию со дня рождения. (совм. с Л.С. Атанасяном, И.Г. Башмаковой и др. «Математика в школе», № 4 - 5, 42 – 43).
1117. Интервью (интервью провели Нозер Сингпурвалла и Ричард Смит в июне 1991. «Statistical Science», v. 7, № 2, 273-283).
1118. Интервью (интервью провел А.Н. Ширяев в январе 1992. Изд-во «Наука», «Теория вероятностей и ее применения», т. 37, № 4, 724-746).

1993 год

1119. Елементарни увод у теорију вероватноhе (совм. с А.Я. Хинчиным. Перевод на сербскохорватский язык. Београд. 7-168).
1120. О прошлом и будущем (Київ. «Теорія імовірностей та математична статистика», 49, 3-26,).
1121. Учитель и друг (Издательская фирма «Физико-математическая литература» ВО «Наука». Сборник "Колмогоров в воспоминаниях", 173 - 208).
1122. Педагогические взгляды Н.И. Лобачевского. К 200-летию со дня рождения («Математика в школе», № 1, 2 - 5).
1123. Знание истории науки - преподавателю школы («Математика в школе», № 3, 30 - 32).

1994 год

1124. Введение в специальность математика (Китай. Шанхайское издательство популярной научной литературы. Перевод профессора математического факультета педагогического университета Восточного Китая Лю Хун-Куня). (Предисловие к этому изданию написано и отослано в издательство в 1992 году).
1125. Стандарт образования – взгляд в будущее (совм. с Д.Б. Гнеденко. «Математика в школе», № 3, 2 - 3).
1126. Абак, десятичная позиционная система счисления и десятичные дроби («Математика в школе», № 1, 75 – 77).
1127. Одна русская народная задача («Математика в школе», № 2, 65).
1128. Александр Яковлевич Хинчин («Математика в школе», № 4, 70 - 73).
1129. Александр Яковлевич Хинчин («Квант», № 6, 2-6).
1130. Лобачевский как педагог и просветитель («Вестник Московского университета», сер. 1, Математика. Механика, № 2, 15 - 23).
1131. Preface to American edition («Handbook of Reliability Engineering», John Wiley, New York, XIX - XXI).
1132. Послесловие к публикации «Евгений Неглинкин» (журнал «Новое литературное обозрение», № 6, 182).
1133. Адольф-Андрей Павлович Юшкевич (некролог) (совм. с И.Г.Башмаковой, С.С.Демидовым и др. «Успехи математических наук», т. 49, вып. 4(298), 75-76).

1995 год

1134. Probabilistic Reliability Engineering (and I. Ushakov. John Wiley, New York, 1-518).
1135. О случайных величинах, обусловленных суммами независимых случайных величин (совм. с Э.М. Кудлаевым. «Вестник Московского университета», № 5, 23 – 31).
1136. Предисловие к книге А.Я. Хинчина «Избранные труды по теории вероятностей» (Москва. Научное изд-во ТВП. VII - IX).
1137. Александр Яковлевич Хинчин (19.7.1894 – 18.11.1959) (Москва. Научное изд-во ТВП. В книге А.Я. Хинчина «Избранные труды по теории вероятностей», XI -- XIV).

1138. Александр Яковлевич Хинчин (перевод Ю.С.Хохлова, оригинал смотри № 327 этой библиографии) (Москва. Научное изд-во ТВП. В книге А.Я. Хинчина «Избранные труды по теории вероятностей», XXI – XXXVIII).
1139. Рассказ–воспоминание в книге Б.Н. Рудакова «Много лет пронеслось…» (Изд-во Московского университета, 132 - 136).

1996 год

1140. Random summation: limit theorems and applications (and V.Y. Korolev. New York, CRC Press Boca Raton, 1 – 267).
1141. Theoria de las Probabilidades (Madrid, Rubinos – 1860, Moscu, Euro – Omega, 1-392).
1142. Павел Сергеевич Александров (совм. с А.Н. Колмогоровым. «Математика в школе», № 2, 2 - 4).
1143. О преподавании математики в предстоящем тысячелетии (совм. с Р.С. Черкасовым. «Математика в школе», № 1, 52 - 54).

1997 год

1144. Lehrbuch der Wahrscheinlichkeitstheorie (Verlag Harri Deutsch, 10 korrigierte Auflage, 1-469).
1145. Развитие теории вероятностей (Изд-во Московского университета. «Очерки по истории математики», 247 - 338).
1146. Мої університетські роки (Киев. «У світі математики», т. 3, вип. 2, 73 - 82).
1147. Викладання і творчість (Киев. «У світі математики», т. 3, вип. 2, 95 - 100).

1998 год

1148. Theory of Probability (Gordon and Breach Science Publishers, New York, sixth ed., 1-497).
1149. Эйлер и Украина (Киев. "У світі математики", т.4, в. 4, 32).

1999 год

1150. Statistical Reliability Engineering (совм. с И.В. Павловым и И.А. Ушаковым. John Wiley, New York, 3-499).
1151. Статистическое мышление и школьное математическое образование («Математика в школе», № 6, 2 – 6).
1152. Учитель в математике, учитель в жизни (Москва. Изд-ва «Фазис» и «Мирос». «Явление чрезвычайное. Книга о Колмогорове», 40 – 48).

2000 год

1153. Математика и жизнь (Москва. Изд-во «Едиториал УРСС». В книге «О математике», 8 - 85).
1154. Об обучении математике в университетах и педвузах на рубеже двух тысячелетий (совм. с Д.Б. Гнеденко. Москва. Изд-во «Едиториал УРСС». В книге «О математике», 88 – 207).

2001 год

1155. Курс теории вероятностей (Москва. Изд-во «Едиториал УРСС», 7-е изд., исправленное, 1-318).
1156. Очерк по истории теории вероятностей (Москва. Изд-во «Едиториал УРСС». 1-86).

2002 год

1157. Математика и жизнь (Москва. Изд-во «Едиториал УРСС». В книге «О математике», изд. 2-е, 8-85).
1158. Об обучении математике в университетах и педвузах на рубеже двух тысячелетий (совм. с Д.Б. Гнеденко. Москва. Изд-во «Едиториал УРСС». В книге «О математике», изд. 2-е, 88 – 207).

2003 год

1159. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. Москва. Изд-во «Едиториал УРСС». 10-е изд., исправленное, 1-206).
1160. Беседы о математике, математиках и Механико-математическом факультете (Москва. Изд-во Центра прикладных исследований при механико-математическом факультете МГУ. 1-149).

2004 год

1161. Курс теории вероятностей (Москва. Изд-во "Едиториал УРСС". Серия "Классический университетский учебник", 8-е изд., исправленное, 1-446).
1162. О месте лекции в математическом образовании («Математика в высшем образовании», № 2, 107-120).
1163. Декан механико-математического факультета Андрей Николаевич Колмогоров (в книге "Математики и механики -- ректоры Московского университета и деканы механико-математического факультета МГУ". Изд-во Центра прикладных исследований при механико-математическом факультете МГУ. 101-102).
1164. О некоторых постановках задач и результатах теории массового обслуживания (Москва, изд-во "Едиториал УРСС". В книге А.Я. Хинчина "Работы по математической теории массового обслуживания", 2-е изд., 221-235).
1165. Предисловие редактора (Москва, изд-во "Едиториал УРСС". В книге А.Я. Хинчина "Работы по математической теории массового обслуживания", 2-е изд., 4-6).
1166. Предисловие к книге А. Реньи "Диалоги о математике" (Москва. Изд-во "Едиториал УРСС". 2-е изд., 5-19).
1167. Предисловие к книге А.Я. Хинчина "Цепные дроби" (Москва. Изд-во "Едиториал УРСС". 4-е изд., 3).

2005 год

1168. Введение в теорию массового обслуживания (совм. с И.Н. Коваленко. Москва. Изд-во «КомКнига». 3-е изд., исправленное и дополненное, 1-397).
1169. Очерки по истории математики в России (предисловие ко второму изданию и комментарии С.С. Демидова) (Москва. Изд-во «КомКнига». 2-е изд., исправленное и дополненное, 1-292).
1170. Пять интервью (интервью провел В.Д.Дувакин 19 июня и 12 октября 1976 года, 5 и 9 декабря 1983 года, 16 января 1984 года. Москва. Изд-во «Минувшее». «Математики рассказывают», 37-112).

2006 год

1171. Математика и жизнь (Москва, изд-во «КомКнига». 3-е изд., 1-125).

1172. Об обучении математике в университетах и педвузах на рубеже двух тысячелетий (совм. с Д.Б. Гнеденко. Москва. Изд-во «КомКнига». 3-е изд., 1-158).
1173. Учитель и друг (М. МЦНМО. «Колмогоров в воспоминаниях учеников», 128-151).
1174. Александр Яковлевич Хинчин (Москва. Изд-во МЦНМО. В книге А.Я. Хинчина "Избранные труды по теории чисел", VII-XX).
1175. Александр Яковлевич Хинчин (Москва. Изд-во «КомКнига». В книге А.Я. Хинчина "Педагогические статьи", 2-е изд., 180-196).

2007 год

1176. Курс теории вероятностей (Москва. Изд-во ЛКИ. 9-е изд., исправленное, 1-446).
1177. Введение в теорию массового обслуживания (совм. с И.Н. Коваленко. Москва. Изд-во ЛКИ. 4-е изд., исправленное, 1-397).
1178. Очерки по истории математики в России (комментарии С.С. Демидова) (Москва. Изд-во ЛКИ. 3-е изд., исправленное, 1-292).
1179. Математика и контроль качества продукции (Москва. Изд-во ЛКИ. 2-е изд., исправленное, 1-61).

2008 год

1180. Очерк по истории теории вероятностей (Москва. Изд-во «Едиториал УРСС». 2-е изд., 1-86).

2009 год

1181. Беседы о математической статистике (Москва. Книжный дом «ЛИБРОКОМ». 2-е изд., исправленное, 1-86).
1182. Беседы о теории массового обслуживания (Москва. Книжный дом «ЛИБРОКОМ». 2-е изд., исправленное, 1-68).
1183. Очерки по истории математики в России (комментарии С.С. Демидова) (Москва. Книжный дом «ЛИБРОКОМ». 4-е изд., 1-292).
1184. Беседы о математике, математиках и Механико-математическом факультете МГУ (Москва. Книжный дом «ЛИБРОКОМ». 2-ое изд., 1-162).
1185. Каким быть X-XI классам? (коллективное письмо девяти академиков (Б. Гнеденко, Н. Дубинин, И. Кикоин, А. Колмогоров, М. Нечкина, С. Никольский, Н. Семенов, С. Соболев, Д. Эпштейн) с предложением к проекту реформ школы) (см. № 994).
1186. «Кикоин, Колмогоров», М. ФМШ МГУ, 2-е изд., пересмотренное и дополненное, 225-227).

2010 год

1187. Введение в теорию массового обслуживания (совм. с И.Н. Коваленко. Москва. Изд-во ЛКИ. 5-е изд., исправленное, 1-397).
1188. Курс теории вероятностей (Москва. Книжный дом «Либроком». 10-е изд., юбилейное, исправленное, дополненное, 1-485).
1189. Предисловие к книге Т.Л.Саати «Элементы теории массового обслуживания и её приложения» (Москва. Книжный дом «ЛИБРОКОМ». 2-е изд., 5-13).
1190. Курс теорії ймовірностей (Киев. Изд-во Киевского университета. 1-463).

2011 год

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2012 год

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2015 год

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2016 год

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2017 год

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