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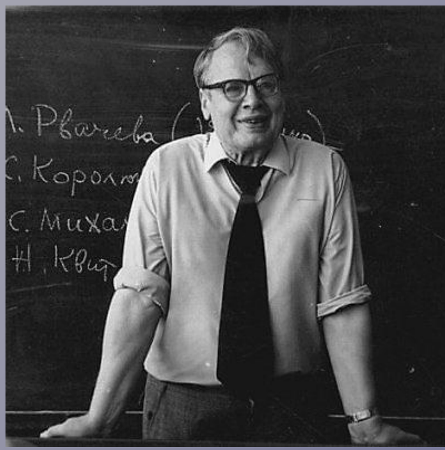

# RELIABILITY:
## THEORY&APPLICATIONS


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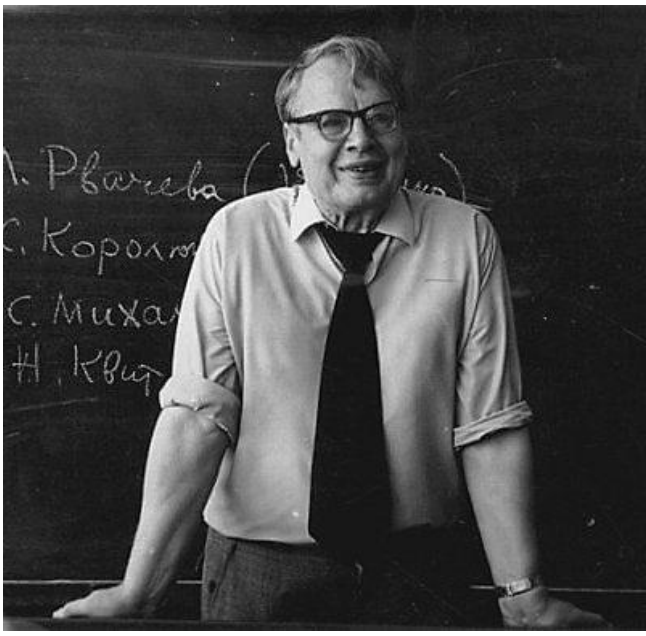

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## THEORY & APPLICATIONS

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# Statistical Analysis for Type-I Progressive Hybrid Censored Data from Burr Type XII Distribution under Step-Stress Partially Accelerated Life Test Model

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## Abstract

Progressive hybrid censoring scheme is now quite common in the experiment of life testing and reliability analysis. In this article the data, failure life times of units, is obtained by using type-I progressive hybrid censoring scheme. It is assumed that data follows Burr Type XII distribution. The point and interval estimation of the Burr Type XII distribution parameters and acceleration factor are performed using maximum likelihood estimator under stress partially accelerated life test model. Monte Carlo simulation study is used to obtain the biases and mean square errors of the estimators.

**Keywords:** Reliability, type I progressive hybrid censoring scheme, Burr Type XII distribution, maximum likelihood function, Monte Carlo simulation, Step-Stress partially accelerated life test model.

## 1. Introduction

Nowadays, due to quick and rapid advances in technology and increasing global competition, pressure on manufacturers to produce high quality products has increased. Life testing and reliability experiments are often used to gain knowledge about product failure time distribution. But the information of such high reliable products cannot be obtained at usual level of stress (or normal stress). So to collect the quick information of the products accelerated life test (ALT) is used. In ALT, we put the items or products at higher than normal stress. Here the relationship between the stress and lifetime is known or can be assumed or acceleration factor is known. But sometimes we face the situation when neither these relationships are known nor it can be assumed. At this point of time partially accelerated life test (PALT) is used to test the items and to gather information on lifetimes of products. In PALT, first the products or testing items have been put at use condition or normal stress, after a specified time, we increase the stress. Therefore, in PALT the items run at normal as well as accelerated condition, see DeGroot and Goel(1979).

Nelson (1990) described the ways by which stress(es) can be applied into the experiment of life test. The common stresses are constant stress, step stress and progressive or linearly increasing stress. In constant stress test, each unit runs at a prespecified level of stress which does not vary with time that is every unit is put at only one stress level until unit is failed or the experiment is terminated at some specific point of time. In step stress test, the items are being tested at some specified stress level after a certain time the stress level is increased and the test continues until all the items get failed or the experiment is terminated at some pre specified time.

There are many situations in reliability and life testing experiments in which units (or subjects) are lost or removed from the experimentation before failure. Complete information on failure times may not be obtained by the experimenter for all experimental units. The data obtained from the experiment are called censored data and the method is censoring method or censoring scheme. Conventional type-I and type-II censorings are the most common censoring schemes. In type-I censoring scheme, the test will be terminated at a pre specified time $T$ and in type-II censoring scheme, the test will run upto rth failure (where r is pre-specified). The mixture of type-I and type-II censoring schemes is known as the hybrid censoring scheme. The hybrid censoring scheme was first introduced by Epstein(1954). But recently it becomes quite popular in the reliability and life testing experiments, e.g. Chen and Bhattacharya(1988), Childs et al.(2003), Draper and Guttman (1987), Fairbanks, Madason and Dykstra(1982), Gupta and Kundu (1998), Jeong, Park and Yan(1996), Lin, Ng and Chan(2009), Ling, Xu and Li (2009) etc.

The major drawback of these censoring schemes is that they do not allow the removal of the units from the experiment other than the terminal point. To deal with this problem, a more general censoring scheme called progressive type-II right censoring is introduced. It can be described as follows: consider an experiment in which n(>m) units are placed on a life test. At the time of first failure $Y_{1:m:n}, R_1$ units are randomly removed from the remaining (n-1) surviving units. Next at the time of the second failure $Y_{2:m:n}, R_2$ of the remaining $(n-2-R_1)$ units are removed randomly. The test continues until the m-th failure. At the time of m-th failure, all the remaining $R_m = n - m - R_1 - \cdots - R_{m-1}$ units are removed from the test. The m integers $(R_1, R_2, \cdots, R_m)$ are fixed prior to the study. They are determined by the experimenter to control the total test time or to observe the more failures which in return have good efficiency in statistical inference. Conventional type-II right censoring is a special case when $R_1 = R_2 = \cdots = R_{m-1} = 0$ and $R_m = n - m$. For further details the reader may refer to Balakrishnan and Aggrawala (2000), Balakrishnan (2007), Balakrishnan and Cramer(2014).

Kundu and Joarder(2006) and Childs et al.(2008) suggested a progressive hybrid censoring (PHC) scheme, named as type-I progressive hybrid censoring scheme which is described as follows: The life test experiment with progressive censoring scheme $(R_1, R_2, \cdots, R_m)$ is stopped at a random time $\min\{y_{m:m:n}, \eta\}$ where $\eta \in (0, \infty)$ *and* $1 \le m \le n$ are fixed prior to the study. The ordered failure times collected from the experiment is $Y_{1:m:n} < Y_{2:m:n} < \cdots Y_{m:m:n}$. If the mth progressively failure occurs before time $\eta\,(i.e. Y_{m:m:n} < \eta)$ the experiments terminates at time $Y_{m:m:n}$ and if m-th failure does not occur before time $\eta$, the experiment will be terminated at time point with J failures such as $Y_{j:m:n} < \eta < Y_{j+1:m:n}$ and all the remaining $(n - J - \sum_{i=1}^{J} R_i)$ surviving items are censored at time $\eta$. Therefore, J, the number of failures upto time $\eta$ is the random variable. Lin, Ng and Chan (2009) indicated the purpose of it to control the total test time of the experiment.

Literature available on the PALT has been studied using censoring schemes, for example, see Goel(1971), DeGroot and Goel(1979), Bai, Chung and Chen(1993), Bhattcharya and Soejoeti(1989), Bai and Chung(1992), Abdel-Ghaly et al.(2011), Abdel-Ghani(2004), Ismail(2010), Aly and Ismail (2008), Ismail and Sarhan (2009), Ismail and Aly (2010), Ismail(2012), Lone, Rahman and Islam (2016), Rahman, Lone and Islam(2016), Zarrin et al.(2012), Kamal et al.(2013), also, SSPALT has been studied under hybrid censoring, see Ismail(2012). In addition, Ismail (2012) has considered SSPALT, using the progressive Type-II censoring scheme.

Ismail(2014) has first studied progressive type-I hybrid censoring scheme under SSPALT. After that Ismail (2014) has considered progressive hybrid censored data from Weibull distribution under SSPALT. This scheme under SSPALT will be described in the next section.

This article is arranged as follows. In Section 2, the model and test method are described. Based on the data obtained from section 2, the parameters of the distribution are estimated under SSPALT using maximum likelihood (ML) estimation technique in Section 3. Also, the asymptotic confidence bounds for the model parameters are constructed based on the asymptotic distribution of ML estimators. The simulation study has been performed in Section 4 to check and evaluate the performance of the estimators based on the PHC scheme. Conclusion and suggestion for future work on the PHC is described in Section 5.

## 2. Description of the Model

It is assumed that the random variable $Y$ representing the lifetimes of the product has Burr Type XII distribution with parameter (c, k). The pdf of the distribution is given as follows:

$$f(y,c,k) = kcy^{c-1}(1+y^c)^{-(k+1)} \qquad y>0, c>0 \text{ and } k>0 \qquad (2.1)$$

Where c and k are the shape parameters of the distribution.
The cumulative distribution function is

$$F(y,c,k) = 1-(1+y^c)^{-k} \quad y>0, c>0 \text{ and } k>0 \qquad (2.2)$$

The reliability function of the Burr Type XII distribution

$$R(y,c,k) = (1+y^c)^{-k} \qquad (2.3)$$

The hazard function of the Burr Type XII distribution

$$h(y,c,k) = kcy^{c-1}(1+y^c)^{-1} \qquad (2.4)$$

The Burr (c, k) distribution was first introduced as a lifetime model by Dubey (1972,1973). Evans and Simons (1975) studied further the distribution as a failure model and they also derived maximum likelihood estimators as well as moments of the Burr (c, k) probability density function. Lewis (1981) noted that the Weibull and exponential distributions are special limiting cases of the parameter values of the Burr (c, k) distribution. She proposed the use of the Burr(c, k) distribution as a model in accelerated life test data.

**Assumptions**

(a) The lifetimes of the items follow Burr type-XII distribution with parameters (c, k).
(b) The total lifetime Y of an item is defined as

$$Y = \begin{cases} T, & 0 < T \le \tau \\ \tau + \beta^{-1}(T-\tau) & T > \tau \end{cases}$$

Where T is the lifetime of the items at normal stress, τ is the time at what stress is to be increased and β is the acceleration factor which is the ratio of the lifetime at normal stress to that at accelerated condition.

(c) The lifetimes of test items are independent and identically distributed random variable.
(d) Under type I progressive hybrid censoring, the test is terminated at min $\{Y_{m:m:n}, \eta\}$.

**Test procedure**

(a) All n identical and independent items are placed on life test and run at used condition.
(b) Change the level of experiment at time $\tau$ to accelerated condition and observe the lifetimes of the items before the test is terminated at min $\{Y_{m:m:n}, \eta\}$.
(c) Once experiment is started, the items begin to fail. At the time of the ith failure we remove the $R_i$ units from the remaining units. Finally at the time of min $\{Y_{m:m:n}, \eta\}$, all the remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ or $R_J = n - J - \sum_{i=1}^{J-1} R_i$ units are removed accordingly from the test and test is terminated.

The description of progressively Type-I hybrid censoring scheme is as follows. Suppose that n identical and independent units are placed on a life test. All of them are run first under the normal stress (use condition). The normal stress level is changed to an accelerated condition at time τ, put all the remaining units at accelerated condition and the test is continued. At the time of first failure $Y_1, R_1$ of the units are removed randomly from the remaining (n-1) units, when second failure occurs $Y_2$, $R_2$ units from the remaining $(n-2-R_1)$ units are removed randomly. If the m-th failure (m < n) occurs at a time $Y_{m:m:n}$ before a prefixed time η > τ, the experiment terminates at the time point $Y_{m:m:n}$. But if $Y_{m:m:n} > \eta$, then all the remaining units are removed and the experiment terminates at the time η. This censoring scheme is called the progressively Type-I hybrid censoring scheme. It is noted that compared to the conventional Type-I censoring scheme, the termination time of the progressively Type-I hybrid censoring scheme is at most η. Let $n_u$ be the number of units that fail before time τ, $n_a$ be the number of units that fail before time η at accelerated condition and $n_t$ be the total number of units that fail on the experiment. So we have

$$n_t = \begin{cases} n_u + n_a = m, & if\ \tau < y_{m:m:n} \le \eta \\ n_u + n_a = J, & if\ y_{m:m:n} > \eta \end{cases}$$

We observe the following samples under type I progressively hybrid censoring scheme

Set 1: $y_{1:m:n} < y_{2:m:n} < \ldots < y_{n_u:m:n} \le \tau < y_{n_u+1:m:n} < \ldots\ldots < y_{J:m:n} < \eta, \quad if\ y_{m:m:n} > \eta$

Set 2: $y_{1:m:n} < y_{2:m:n} < \ldots < y_{n_u:m:n} \le \tau < y_{n_u+1:m:n} < \ldots\ldots < y_{m:m:n} \le \eta, \quad if\ \tau < y_{m:m:n} \le \eta$

The pdf of Y under step stress partially accelerated life test is given by

$$f(y) = \begin{cases} 0 & y \le 0 \\ f_1(y) = f_Y(y; c, k) & 0 < y \le \tau \\ f_2(y) & y > \tau \end{cases} \tag{2.5}$$

Where

$$f_2(y) = kc\beta[\tau + \beta(y-\tau)]^{c-1}\left[1 + \{\tau + \beta(y-\tau)\}^c\right]^{-(k+1)} \tag{2.6}$$

## 3. **Estimation Process**

In this section, the process of obtaining the estimates of the parameters and acceleration factor based on the data observed by progressively type I hybrid censoring scheme under SSPALT model have been discussed. Also, consider both point and interval estimates of the parameters. Maximum likelihood estimation technique is used to estimate the parameters.

### 3.1. Point Estimation

In this section the likelihood function for the data observed based on the progressively type I hybrid censoring scheme are constructed under SSPALT.

The likelihood function is given by

$$L(x : c, k, \beta) \propto \prod_{i=1}^{n_u} f_1(x_i)[S_1(\tau)]^{R_i} \prod_{i=n_u+1}^{J} f_2(x_i)[S_2(\eta)]^{R_J^*} \tag{3.1}$$

$$L(x : c, k, \beta) \propto \prod_{i=1}^{n_u} \{k\, c\, x_i^{c-1}(1 + x_i^c)^{-(k+1)}[(1+\tau^c)^{-k}]^{R_i}\} \times$$

$$\prod_{i=n_u+1}^{J} k\, c\, \beta\{\tau + \beta(x_i - \tau)\}^{c-1}[1 + \{\tau + \beta(x_i - \tau)\}^c]^{-(k+1)}\{[1 + \{\tau + \beta(\eta - \tau)\}^c]^{-k}\}^{R_J^*} \tag{3.2}$$

where, $R_J^* = n - J - \sum_{i=1}^{J} R_i$ .

The log likelihood function is maximized. The natural logarithm of the likelihood function is as follows.

$$\ln L = J\ln k + J\ln c + n_a \ln\beta + (c-1)\{\sum_{i=1}^{n_u}\ln x_i + \sum_{i=n_u+1}^{J}\ln\phi_i\} - (k+1)\left[\sum_{i=1}^{n_u}\ln(1+x_i^{\,c}) + \sum_{i=n_u+1}^{J}\ln(1+\phi_i)\right]$$
$$-\sum_{i=1}^{n_u} R_i\, k\ln(1+\tau^c) - kn_a R_J^* \ln(1+\phi_\eta^{\,c}) \qquad (3.3)$$

The first order partially derivatives of Eq.(3.3) with respect to $c$, $k$ and $\beta$ are obtained and are equated to zero.

$$\frac{\partial \ln L}{\partial c} = 0 = \frac{J}{c} + \sum_{i=1}^{n_u}\ln x_i + \sum_{i=n_u+1}^{J}\ln\phi_i - k\frac{\tau^c\ln\tau}{1+\tau^c}\sum_{i=1}^{n_u}R_i - (k+1)\left(\sum_{i=1}^{n_u}\frac{x_i^{\,c}\ln x_i}{1+x_i^{\,c}} + \sum_{i=n_u+1}^{J}\frac{\phi_i^{\,c}\ln\phi_i}{1+\phi_i}\right)$$
$$-kn_a R_J^*\frac{\phi_\eta^{\,c}\ln\phi_\eta}{1+\phi_\eta} \qquad (3.4)$$

$$\frac{\partial \ln L}{\partial k} = 0 = \frac{J}{k} - \left(\sum_{i=1}^{n_u}\ln(1+x_i^{\,c}) + \sum_{i=n_u+1}^{J}\ln(1+\phi_i^{\,c})\right) - \ln(1+\tau^c)\sum_{i=1}^{n_u}R_i - n_a R_J^*\ln(1+\phi_\eta^{\,c}) \qquad (3.5)$$

$$\frac{\partial \ln L}{\partial \beta} = 0 = \frac{n_a}{\beta} + (c-1)\sum_{i=n_u+1}^{J}\frac{(x_i-\tau)}{\phi_i} - c(k+1)\sum_{i=n_u+1}^{J}\frac{\phi_i^{\,c-1}(x_i-\tau)}{(1+\phi_i^{\,c})} - \frac{kcn_a R_J^*(\eta-\tau)\phi_\eta^{\,c-1}}{(1+\phi_\eta^{\,c})} \qquad (3.6)$$

Obviously, it is very difficult to obtain the closed form solution for three nonlinear equations (3.4)-(3.6). Newton-Raphson iterative process is used to get the MLE solutions $(\hat{c},\hat{k},\hat{\beta})$.

## 3.2.Interval Estimation

The most common method to construct the approximate confidence interval of parameters is based on the asymptotic distribution of the ML estimators of the unknown parameters $\Omega = (c,k,\beta)$. The asymptotic distribution of the ML estimators of $\Omega$ is given as

$$\left((\hat{c}-c),(\hat{k}-k),(\hat{\beta}-\beta)\right) \to N(0, I^{-1}(c,k,\beta))$$

where $I^{-1}(c,k,\beta)$ is the variance covariance matrix of the unknown parameters $\Omega = (c,k,\beta)$. The matrix is of 3×3 dimension and its elements $I_{ij}(c,k,\beta),\ i,j = 1,2,3$ can be approximated by $I_{ij}(\hat{c},\hat{k},\hat{\beta})$.

where, $$I_{ij}(\Omega) = -\left.\frac{\partial^2 \ln L(\Omega)}{\partial\Omega_i\partial\Omega_j}\right|_{\Omega=\hat{\Omega}}$$

So the elements of the matrix are given as follows

$$\frac{\partial^2 \ln L}{\partial c^2} = -\frac{J}{c^2} - (k+1)\sum_{i=1}^{n_u} x_i^{\,c}\left(\frac{\ln x_i}{1+x_i^{\,c}}\right)^2 - (k+1)\sum_{i=n_u+1}^{J}\phi_i^{\,c}\left(\frac{\ln\phi_i}{1+\phi_i^{\,c}}\right)^2 - k\tau^c\left(\frac{\ln\tau}{1+\tau^c}\right)^2\sum_{i=1}^{n_u}R_i$$
$$-kn_a\phi_\eta^{\,c}\left(\frac{\ln\phi_\eta}{1+\phi_\eta^{\,c}}\right)^2 R_J^*$$

$$\frac{\partial^2 \ln L}{\partial k^2} = -\frac{J}{k^2}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{n_a}{\beta^2} - (c-1)\sum_{i=n_u+1}^{J}\left(\frac{x_i-\tau}{\phi_i}\right)^2 - c(k+1)\sum_{i=n_u+1}^{J}\phi_i^{\,c-2}(c-1-\phi_i^{\,c})\left(\frac{x_i-\tau}{1+\phi_i^{\,c}}\right)^2$$
$$-kcn_aR_J^*\phi_\eta^{\,c-2}(c-1-\phi_\eta^{\,c})\left(\frac{\eta-\tau}{1+\phi_\eta^{\,c}}\right)^2$$

$$\frac{\partial^2 \ln L}{\partial k \partial c} = -\sum_{i=1}^{n_u}\frac{x_i^{\,c}\ln x_i}{1+x_i^{\,c}} - \sum_{i=n_u+1}^{J}\frac{\phi_i^{\,c}\ln\phi_i}{1+\phi_i^{\,c}} - \frac{\tau^c\ln\tau}{1+\tau^c}\sum_{i=1}^{n_u}R_i - n_aR_J^*\frac{\phi_\eta^{\,c}\ln\phi_\eta}{1+\phi_\eta^{\,c}}$$

$$\frac{\partial^2 \ln L}{\partial k \partial \beta} = -c\sum_{i=n_u+1}^{J}\frac{\phi_i^{\,c-1}(x_i-\tau)}{1+\phi_i^{\,c}} - \frac{n_acR_J^*(\eta-\tau)\phi_\eta^{\,c-1}}{1+\phi_\eta^{\,c}}$$

$$\frac{\partial^2 \ln L}{\partial c \partial \beta} = \sum_{i=n_u+1}^{J}\frac{(x_i-\tau)}{\phi_i} - (k+1)\sum_{i=n_u+1}^{J}\frac{(x_i-\tau)\phi_i^{\,c-1}(1+c\ln\phi_i+\phi_i^{\,c})}{(1+\phi_i^{\,c})^2} - kn_a\,R_J^*(\eta-\tau)\frac{\phi_\eta^{\,c-1}(1+c\ln\phi_\eta+\phi_\eta^{\,c})}{(1+\phi_\eta^{\,c})^2}$$

Thus, the approximate *100(1-ε)%* two sided confidence intervals for *c*, *k* and *β* are respectively given by

$$\hat{c} \pm z_{\varepsilon/2}\sqrt{I_{11}^{-1}(\hat{c},\hat{k},\hat{\beta})}\ ,\ \hat{k} \pm z_{\varepsilon/2}\sqrt{I_{22}^{-1}(\hat{c},\hat{k},\hat{\beta})}\ ,\ \ \hat{\beta} \pm z_{\varepsilon/2}\sqrt{I_{11}^{-1}(\hat{c},\hat{k},\hat{\beta})}$$

Where $z_{\varepsilon/2}$ is the upper $(\varepsilon/2)^{th}$ percentile of a standard normal distribution.

## 4. Simulation Studies

In this section simulation study is performed to evaluate the performance of the MLEs in terms of their mean squared errors(MSEs) for various choices of n, m, τ and η values. Also, 95% asymptotic confidence bounds are made based on the asymptotic distribution of the ML estimators. It is performed using the R software.

The considered schemes are as follows:

$$Scheme1: R_1 = \cdots = R_{m-1} = 0\ and\ R_m = n-m$$
$$Scheme2: R_1 = n-m\ \ and\ \ R_2 = \cdots = R_m = 0$$
$$Scheme3: R_1 = \cdots = R_{m-1} = 1\ and\ R_m = n-2m+1$$

The algorithm of the simulation study is given as

(1) Specify the values of n, m, τ and η.

(2) Choose values of c, k and β.

(3) To generate the data from the Burr type XII distribution,a random sample of size n from uniform random variable [0,1]. Then we use iverse cdf in eq(2.2) to generate data from the distribution $y = \left[\exp\left(-\ln(1-u)/k\right)-1\right]^{1/c}$.

(4) The data set can be considered to generate progressively type I hybrid censored data for the given values of n, m, τ, η(η>τ), c, k and β.

(5) Parameters are estimated using the above data. Newton-Raphson iterative method is used for solving the system of nonlinear equations.

(6) Replicate step 3-5, 10,000 times to avoid randomness.

(7) Compute the average values of biases and MSEs associated with the ML estimators of the parameters.

**Table 1:** The average of MLEs and its Biases and MSEs at the values of parameters (c=1.3, k=1.4, β=1.1) for different sample sizes under different schemes of type-I progressive hybrid censoring

| | | Estimate of c | | | Estimate of k | | | Estimate of β | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (n,m) | Schemes | MLE | Bias | MSE | MLE | Bias | MSE | MLE | Bias | MSE |
| (30,15) | 1 | 1.251 | 0.453 | 0.438 | 1.393 | 0.431 | 0.404 | 1.121 | 0.378 | 0.343 |
| | 2 | 1.261 | 0.496 | 0.451 | 1.524 | 0.463 | 0.440 | 1.072 | 0.417 | 0.386 |
| | 3 | 1.352 | 0.467 | 0.449 | 1.436 | 0.451 | 0.429 | 1.182 | 0.385 | 0.370 |
| (30,20) | 1 | 1.285 | 0.389 | 0.354 | 1.387 | 0.410 | 0.395 | 1.142 | 0.336 | 0.304 |
| | 2 | 1.271 | 0.425 | 0.382 | 1.426 | 0.443 | 0.428 | 1.213 | 0.369 | 0.337 |
| | 3 | 1.286 | 0.403 | 0.368 | 1.408 | 0.436 | 0.416 | 1.128 | 0.362 | 0.317 |
| (50,30) | 1 | 1.318 | 0.381 | 0.352 | 1.387 | 0.313 | 0.286 | 0.978 | 0.295 | 0.254 |
| | 2 | 1.273 | 0.412 | 0.379 | 1.478 | 0.407 | 0.348 | 1.218 | 0.331 | 0.286 |
| | 3 | 1.306 | 0.396 | 0.360 | 1.386 | 0.353 | 0.303 | 1.017 | 0.318 | 0.275 |
| (50,40) | 1 | 1.279 | 0.347 | 0.304 | 1.385 | 0.296 | 0.228 | 1.005 | 0.273 | 0.228 |
| | 2 | 1.347 | 0.423 | 0.374 | 1.437 | 0.341 | 0.285 | 1.193 | 0.309 | 0.263 |
| | 3 | 1.286 | 0.369 | 0.325 | 1.404 | 0.318 | 0.259 | 1.247 | 0.297 | 0.237 |
| (70,50) | 1 | 1.317 | 0.276 | 0.239 | 1.392 | 0.243 | 0.198 | 1.204 | 0.247 | 0.190 |
| | 2 | 1.330 | 0.328 | 0.283 | 1.378 | 0.328 | 0.263 | 1.185 | 0.283 | 0.234 |
| | 3 | 1.282 | 0.292 | 0.244 | 1.413 | 0.277 | 0.221 | 0.996 | 0.268 | 0.210 |
| (70,60) | 1 | 1.313 | 0.202 | 0.176 | 1.408 | 0.185 | 0.129 | 1.135 | 0.193 | 0.155 |
| | 2 | 1.338 | 0.283 | 0.217 | 1.389 | 0.238 | 0.183 | 1.217 | 0.249 | 0.192 |
| | 3 | 1.316 | 0.227 | 0.183 | 1.418 | 0.208 | 0.167 | 0.952 | 0.241 | 0.178 |

**Table 2**: The average of MLEs and its Biases and MSEs at the values of parameters (c=1.3, k=1.4, β=1.25) for different sample sizes under different schemes of type-I progressive hybrid censoring

| | | Estimate of c | | | Estimate of k | | | Estimate of β | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (n,m) | Schemes | MLE | Bias | MSE | MLE | Bias | MSE | MLE | Bias | MSE |
| (30,15) | 1 | 1.263 | 0.427 | 0.387 | 1.372 | 0.408 | 0.382 | 1.118 | 0.354 | 0.326 |
| | 2 | 1.258 | 0.463 | 0.418 | 1.496 | 0.445 | 0.398 | 1.289 | 0.423 | 0.352 |
| | 3 | 1.326 | 0.448 | 0.392 | 1.419 | 0.437 | 0.387 | 1.127 | 0.361 | 0.333 |
| (30,20) | 1 | 1.274 | 0.394 | 0.327 | 1.361 | 0.382 | 0.349 | 1.187 | 0.318 | 0.273 |
| | 2 | 1.267 | 0.418 | 0.353 | 1.418 | 0.436 | 0.384 | 1.221 | 0.349 | 0.318 |
| | 3 | 1.283 | 0.386 | 0.342 | 1.426 | 0.408 | 0.327 | 1.162 | 0.327 | 0.280 |
| (50,30) | 1 | 1.313 | 0.344 | 0.302 | 1.438 | 0.305 | 0.276 | 1.402 | 0.279 | 0.247 |
| | 2 | 1.259 | 0.382 | 0.328 | 1.464 | 0.374 | 0.317 | 1.481 | 0.303 | 0.268 |
| | 3 | 1.338 | 0.365 | 0.317 | 1.373 | 0.329 | 0.281 | 1.183 | 0.284 | 0.252 |
| (50,40) | 1 | 1.262 | 0.317 | 0.273 | 1.357 | 0.298 | 0.242 | 1.386 | 0.267 | 0.217 |
| | 2 | 1.320 | 0.384 | 0.328 | 1.446 | 0.352 | 0.263 | 1.282 | 0.283 | 0.243 |
| | 3 | 1.265 | 0.349 | 0.294 | 1.429 | 0.321 | 0.259 | 1.350 | 0.295 | 0.225 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (70,50) | 1 | 1.276 | 0.255 | 0.218 | 1.387 | 0.240 | 0.186 | 1.228 | 0.225 | 0.173 |
| | 2 | 1.321 | 0.293 | 0.237 | 1.364 | 0.331 | 0.238 | 1.452 | 0.267 | 0.217 |
| | 3 | 1.277 | 0.270 | 0.225 | 1.436 | 0.288 | 0.207 | 1.197 | 0.239 | 0.184 |
| (70,60) | 1 | 1.325 | 0.216 | 0.167 | 1.427 | 0.174 | 0.115 | 1.183 | 0.178 | 0.138 |
| | 2 | 1.326 | 0.273 | 0.224 | 1.356 | 0.204 | 0.153 | 1.237 | 0.203 | 0.162 |
| | 3 | 1.309 | 0.230 | 0.176 | 1.433 | 0.194 | 0.129 | 1.376 | 0.187 | 0.147 |

Table 3: Confidence intervals of the parameters (c=1.3, k=1.4, β=1.1) at confidence level 0.95.

| (n,m) | Schemes | Estimate of c | | Estimate of k | | Estimate of β | |
|---|---|---|---|---|---|---|---|
| | | LCL | UCL | LCL | UCL | LCL | UCL |
| (30,15) | 1 | 0.758 | 1.820 | 0.736 | 2.002 | 0.585 | 1.987 |
| | 2 | 0.791 | 1.836 | 0.717 | 2.253 | 0.562 | 2.014 |
| | 3 | 0.773 | 1.847 | 0.749 | 1.995 | 0.571 | 1.973 |
| (30,20) | 1 | 0.784 | 1.816 | 0.758 | 1.974 | 0.606 | 1.954 |
| | 2 | 0.759 | 1.838 | 0.733 | 2.117 | 0.579 | 1.997 |
| | 3 | 0.803 | 1.782 | 0.746 | 1.981 | 0.593 | 1.941 |
| (50,30) | 1 | 0.836 | 1.748 | 0.772 | 1.939 | 0.628 | 1.885 |
| | 2 | 0.825 | 1.785 | 0.748 | 1.972 | 0.582 | 1.916 |
| | 3 | 0.841 | 1.753 | 0.753 | 1.946 | 0.643 | 1.894 |
| (50,40) | 1 | 0.842 | 1.742 | 0.815 | 1.895 | 0.662 | 1.833 |
| | 2 | 0.812 | 1.763 | 0.786 | 1.938 | 0.604 | 1.876 |
| | 3 | 0.828 | 1.737 | 0.799 | 1.887 | 0.677 | 1.828 |
| (70,50) | 1 | 0.849 | 1.724 | 0.842 | 1.858 | 0.690 | 1.784 |
| | 2 | 0.827 | 1.756 | 0.801 | 1.896 | 0.651 | 1.839 |
| | 3 | 0.861 | 1.747 | 0.857 | 1.840 | 0.683 | 1.768 |
| (70,60) | 1 | 0.863 | 1.718 | 0.895 | 1.818 | 0.727 | 1.730 |
| | 2 | 0.842 | 1.750 | 0.864 | 1.854 | 0.686 | 1.782 |
| | 3 | 0.875 | 1.746 | 0.887 | 1.809 | 0.744 | 1.741 |

Table 4: Confidence intervals of the parameters (c=1.3, k=1.4, β=1.25) at confidence level 0.95.

| (n,m) | Schemes | Estimate of c | | Estimate of k | | Estimate of β | |
|---|---|---|---|---|---|---|---|
| | | LCL | UCL | LCL | UCL | LCL | UCL |
| (30,15) | 1 | 0.778 | 1.809 | 0.747 | 2.093 | 0.524 | 1.782 |
| | 2 | 0.815 | 1.817 | 0.738 | 2.187 | 0.503 | 1.814 |
| | 3 | 0.794 | 1.883 | 0.763 | 2.066 | 0.548 | 1.763 |
| (30,20) | 1 | 0.795 | 1.792 | 0.778 | 1.986 | 0.580 | 1.747 |
| | 2 | 0.830 | 1.781 | 0.754 | 1.947 | 0.552 | 1.764 |
| | 3 | 0.821 | 1.799 | 0.768 | 1.974 | 0.594 | 1.726 |
| (50,30) | 1 | 0.854 | 1.756 | 0.838 | 1.968 | 0.603 | 1.718 |
| | 2 | 0.862 | 1.743 | 0.823 | 1.946 | 0.578 | 1.746 |
| | 3 | 0.857 | 1.767 | 0.882 | 1.903 | 0.639 | 1.689 |
| (50,40) | 1 | 0.835 | 1.828 | 0.901 | 1.881 | 0.626 | 1.693 |
| | 2 | 0.804 | 1.782 | 0.889 | 1.853 | 0.617 | 1.720 |
| | 3 | 0.817 | 1.753 | 0.917 | 1.819 | 0.654 | 1.671 |
| (70,50) | 1 | 0.867 | 1.773 | 0.952 | 1.807 | 0.651 | 1.660 |
| | 2 | 0.836 | 1.791 | 0.937 | 1.792 | 0.637 | 1.682 |
| | 3 | 0.859 | 1.762 | 0.961 | 1.799 | 0.668 | 1.627 |
| (70,60) | 1 | 0.883 | 1.745 | 0.995 | 1.716 | 0.705 | 1.638 |
| | 2 | 0.862 | 1.767 | 0.968 | 1.704 | 0.678 | 1.664 |
| | 3 | 0.894 | 1.738 | 0.983 | 1.738 | 0.714 | 1.609 |

## Findings:

Simulation study has been performed and the results are summarized in table 1-4. To get the smooth results and to avoid the randomness, the procedures are replicated 10000. From table 1 & 2, it is observed that the biases and MSEs are decreasing as the values of sample size are increased for all cases. When number of failures increases, the RABs and MSEs also are decreased. Only Scheme 2 has a slightly larger biases and MSEs than scheme 1 and scheme 3, since the experiment is censored heavily in the beginning of it. It can also be observed that Table 2 has RABs and MSEs less than Table 1 because when the acceleration factor increases the errors and biases decreases. Additionally, the confidence intervals also get narrower as the sample size and acceleration factor increases.

## 4.5. Conclusion

In this study, it is considered that the lifetimes of the units follow Burr Type XII distribution. To estimate the acceleration factor and parameters of the distribution maximum likelihood estimation technique is used under step-stress partially accelerated life test method using type-I progressive hybrid censored data. Newton-Raphson method is used to obtain the point estimates of the parameters and tampering coefficient. Their performances are analysed and discussed in terms of biases and MSEs. It has been observed that all the statistical assumptions are fulfilled. This shows that the assumptions of experiment, model considered and data used are correct. Bayesian inferences under the SSPALT assuming the same censoring proposed in this article can be considered as future work plan.

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# Reliability Assessment of Deteriorating System

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## Abstract

Industrial equipment and technical systems are exposed various degree of deterioration ranging from minor deterioration, medium to major deterioration and subsequently failed thereafter and are replaced at failure. Such deteriorations can slightly reduce system performance and will ultimately lead to random failure. This paper presents modelling and evaluation of reliability characteristics such as availability, profit and mean time to failure (MTTF) of a system subjected to three consecutive stages of deterioration (minor, medium and major) before failure. Markov models of the system are derived through the system state transition probabilities and differential equations which are further used to evaluate the system availability, busy period, profit and mean time to failure (MTTF). Based on assumed numerical values given to system parameters, graphical illustrations are given to highlight important results.

**Keywords**: , Deterioration, Reliability, Availability, profit.
Mathematical Subject Classification: 90B25

## Introduction

The process industry comprises of large complex engineering systems, subsystems arranged in standby, series, parallel or a combination of them. For efficient and economical operation of a process plant, each system or the subsystem should work failure free under the existing operative plant conditions. Most of these systems are subjected to random deterioration which can result in unexpected failures and disastrous effect on the system availability and the prospect of the economy. Therefore, it is important to find a way to slow down the deterioration rate, and to prolong the equipment's life span. Maintenance policies are vital in the analysis of deterioration and deteriorating systems as they help in improving reliability and availability of the systems. Maintenance models can assume minor maintenance, major maintenance before system failure, perfect repair (as good as new), minimal repair (as bad as old), imperfect repair and replacement at system failure.

Several models on deteriorating systems under different conditions have been studied by several researchers. Liu *et al.* [1] presented the reliability analysis of a deteriorating system with delayed vacation of repairman. Pandey *et al.*[2] discussed the influence of temporal uncertainty of deterioration on life-cycle management of structures. Rani *et al.* [3] discussed the replacement time for a deteriorating system. Tuan *et al.* [4] dealt with reliability-based predictive maintenance modelling for k-out-of-n Deteriorating Systems. Vinayak and Dharmaraja [5] presented semi-Markov modeling approach for deteriorating systems with preventive maintenance. Xiao *et al.* [6] studied the Bayesian reliability estimation for deteriorating systems with limited samples. Yuan *et* *al.* [7] analyzed modeling of a deteriorating system with repair satisfying general distribution. Yuan and Xu [8] studied deteriorating system with its repairman having multiple vacations. Yusuf *et al* [9] presented modeling the reliability and availability characteristics of a system with three stages of deterioration.

In this paper, a single system with four consecutive modes minor, medium and major deterioration failure modes is considered and derived its corresponding mathematical models. Furthermore, we study availability of the system using linear first order differential equations. The focus of our analysis is primarily to capture the effect of minor and medium deterioration rate, minor and major maintenance rates on steady-state availability and profit.

The organization of the paper is as follows. Section 2 contains a description of the system under study. Section 3 presents formulations of the models. The results of our numerical simulations are presented in section 4. Finally, we make some concluding remarks in Section 5.

## Description and States of the System

In this paper, a single system with three consecutive modes of deterioration: a minor, medium and major deterioration and failure mode is considered. At early state of the system life, the unit is exposed to minor deterioration with rate $\lambda_1$ **and this deterioration is rectified through minor maintenance** $\alpha_1$ **which revert the unit to its earliest position before deterioration**. If not maintained, the unit is allowed to continue operating under the condition of minor deterioration which later results to medium deterioration with rate $\lambda_2$ . At this stage, the strength of the unit still strong that it can rectified to early state with major maintenance with rate $\alpha_2$ . However, the system can move to major deterioration stage with rate $\lambda_3$ where the and subsequently failed with parameter $\lambda_4$ and replaced by with a new one with rate $\alpha_3$ .

Table 1: Transition rate table

| | $s_0$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
|---|---|---|---|---|---|
| $s_0$ | | $\lambda_1$ | | | |
| $s_1$ | $\alpha_1$ | | $\lambda_2$ | | |
| $s_2$ | $\alpha_2$ | | | $\lambda_3$ | |
| $s_3$ | | | | | $\lambda_4$ |
| $s_4$ | $\alpha_3$ | | | | |

Table 2: States of the System

| State | Description |
|---|---|
| $S_0$ | Initial state, the unit is working. The system is working. |
| $S_1$ | The working unit is in minor deterioration mode and is under online minor maintenance. The system is working. |
| $S_2$ | The working unit is in medium deterioration mode and is under online major maintenance. The system is working. |
| $S_3$ | The working unit is in major deterioration mode. The system is working. |
| $S_4$ | The working unit has failed. The system is inoperative. . |

## Formulation of the Models

In order to analyze the system availability of the system, define $P_i(t)$ to be the probability that the system at $t \geq 0$ is in state $S_i$. Also let $P(t)$ be the row vector of these probabilities at time $t$. The initial condition for this problem is:

$$P(0) = [p_0(0), p_1(0), p_2(0), p_3(0), p_4(0)] \\ = [1, 0, 0, 0, 0]$$

the following differential difference equations are obtained from Figure 1:

$$\left.\begin{aligned} \frac{d}{dt} p_0(t) &= -\lambda_1 p_0(t) + \alpha_1 p_1(t) + \alpha_2 p_2(t) + \alpha_3 p_4(t) \\ \frac{d}{dt} p_1(t) &= -(\alpha_1 + \lambda_2) p_1(t) + \lambda_1 p_0(t) \\ \frac{d}{dt} p_2(t) &= -(\alpha_2 + \lambda_3) p_2(t) + \lambda_2 p_1(t) \\ \frac{d}{dt} p_3(t) &= -\lambda_4 p_4(t) + \lambda_3 p_2(t) \\ \frac{d}{dt} p_4(t) &= -\alpha_3 p_4(t) + \lambda_4 p_3(t) \end{aligned}\right\} \quad \textbf{(1)}$$

This can be written in the matrix form as

$$\dot{P} = MP, \quad \textbf{(2)}$$

where

$$M = \begin{pmatrix} -\lambda_1 & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 & -(\alpha_1 + \lambda_2) & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\alpha_2 + \lambda_3) & 0 & 0 \\ 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\ 0 & 0 & 0 & \lambda_4 & -\alpha_3 \end{pmatrix}$$

Equation (2) is expressed explicitly in the form

$$\begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{pmatrix} = \begin{pmatrix} -\lambda_1 & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 & -(\alpha_1 + \lambda_2) & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\alpha_2 + \lambda_3) & 0 & 0 \\ 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\ 0 & 0 & 0 & \lambda_4 & -\alpha_3 \end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{pmatrix}$$

The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) and busy periods (the sum of the probabilities of states involving minor and major maintenance and replacement) are given by

$$A_V(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) \quad (3)$$

$$B_{P1}(\infty) = p_1(\infty) \quad (4)$$

$$B_{P2}(\infty) = p_2(\infty) \quad (5)$$

$$B_{P3}(\infty) = p_8(\infty) \quad (6)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (2) become

$$MP = 0 \quad (7)$$

this is in matrix form

$$\begin{pmatrix} -\lambda_1 & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\ 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\ 0 & 0 & 0 & \lambda_4 & -\alpha_3 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Subject to following normalizing conditions:

$$p_0(\infty)+p_1(\infty)+p_2(\infty)+p_3(\infty)+p_4(\infty)=1 \quad (8)$$

Substitute (8) in the last row of (7) to compute the steady-state probabilities $p_0(\infty), p_1(\infty), p_2(\infty), p_3(\infty), p_4(\infty)$

$$\begin{pmatrix} -\lambda_1 & \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\ 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The expressions for the steady-state availability and busy periods involving minor and major maintenance and replacement given in equations (3), (4), (5) and (6) above are given by

$$A_T(\infty)=\frac{\alpha_3\lambda_4(\lambda_2\lambda_3+\alpha_2\lambda_2+\alpha_1\lambda_3+\alpha_1\alpha_2)+\alpha_3\lambda_1\lambda_4(\alpha_2+\lambda_3)+\alpha_3\lambda_1\lambda_2\lambda_4+\alpha_3\lambda_1\lambda_2\lambda_3}{\lambda_2\lambda_3(\alpha_3\lambda_4+\lambda_1\lambda_4+\alpha_3\lambda_1)+\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_1)+\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_1)+\alpha_2\alpha_3\lambda_4(\alpha_1+\lambda_1)}$$

$$B_{P1}(\infty)=p_1(\infty)=\frac{\alpha_3\lambda_1\lambda_4(\alpha_2+\lambda_3)}{\lambda_2\lambda_3(\alpha_3\lambda_4+\lambda_1\lambda_4+\alpha_3\lambda_1)+\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_1)+\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_1)+\alpha_2\alpha_3\lambda_4(\alpha_1+\lambda_1)}$$

$$B_{P2}(\infty)=\frac{\alpha_3\lambda_1\lambda_2\lambda_4}{\lambda_2\lambda_3(\alpha_3\lambda_4+\lambda_1\lambda_4+\alpha_3\lambda_1)+\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_1)+\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_1)+\alpha_2\alpha_3\lambda_4(\alpha_1+\lambda_1)}$$

$$B_{P3}(\infty)=\frac{\lambda_1\lambda_2\lambda_3\lambda_4}{\lambda_2\lambda_3(\alpha_3\lambda_4+\lambda_1\lambda_4+\alpha_3\lambda_1)+\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_1)+\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_1)+\alpha_2\alpha_3\lambda_4(\alpha_1+\lambda_1)}$$

From Figure 1, the system is under minor and major maintenance due to minor and medium deterioration and replacement due to failure as can be observed in the states 1, 2,5, 6 and 8 respectively. Let $C_0, C_1, C_2$ and $C_3$ be the revenue generated when the system is in a working state , equivalently loss of income when in an inoperative/failed state and the cost of each maintenance and replacement respectively. The expected total profit per unit time generated by the system in the steady-state is

Profit=total revenue generated – (total maintenance and replacement cost).

$$PF=C_0A_V(\infty)-C_1B_{P1}(\infty)-C_2B_{P2}(\infty)-C_3B_{P3}(\infty) \quad \mathbf{(9)}$$

It is difficult to evaluate the transient solutions, hence we follow Wang and Kuo (2000) and Wang et al. (2006) and delete the rows and columns of absorbing state of matrix $M$ and take the transpose to produce a new matrix, say $Q$. The expected time to reach an absorbing state is obtained from

$$MTTF=P(0)(-Q^{-1})[1,1,1,1]^T=\frac{\lambda_4(\alpha_1+\lambda_2)(\alpha_2+\lambda_3)+\lambda_1\lambda_4(\alpha_2+\lambda_3)+\lambda_1\lambda_2(\lambda_3+\lambda_4)}{\lambda_1\lambda_2\lambda_3\lambda_4} \quad (10)$$

Where

$$Q = \begin{pmatrix} -\lambda_1 & \lambda_1 & 0 & 0 \\ \alpha_1 & -(\alpha_1+\lambda_2) & \lambda_2 & 0 \\ \alpha_2 & 0 & -(\alpha_2+\lambda_3) & \lambda_3 \\ 0 & 0 & 0 & -\lambda_4 \end{pmatrix}$$

## Numerical Examples and Discussion

Numerical examples are presented to demonstrate the impact of deterioration and maintenance rates on steady-state availability and net profit of the system based on given values of the parameters. For the purpose of numerical example, the following sets of parameter values are used: $\lambda_1 = 0.1$, $\lambda_2 = 0.5$, $\lambda_3 = 0.9$, $\lambda_4 = 0.1$, $\alpha_1 = 0.1$, $\alpha_2 = 0.3$, $\alpha_3 = 0.3$, $C_0 = 500{,}000$, $C_1 = 10{,}000$, $C_2 = 12{,}000$, $C_3 = 15{,}000$.

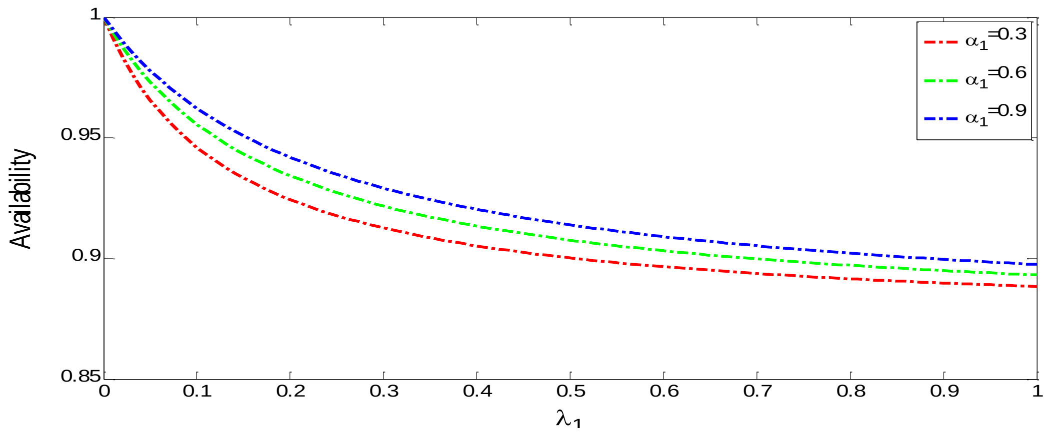


Figure 1: Availability against minor deterioration rate $\lambda_1$ **for** different values of $\alpha_1(0.3, 0.6, 0.9)$

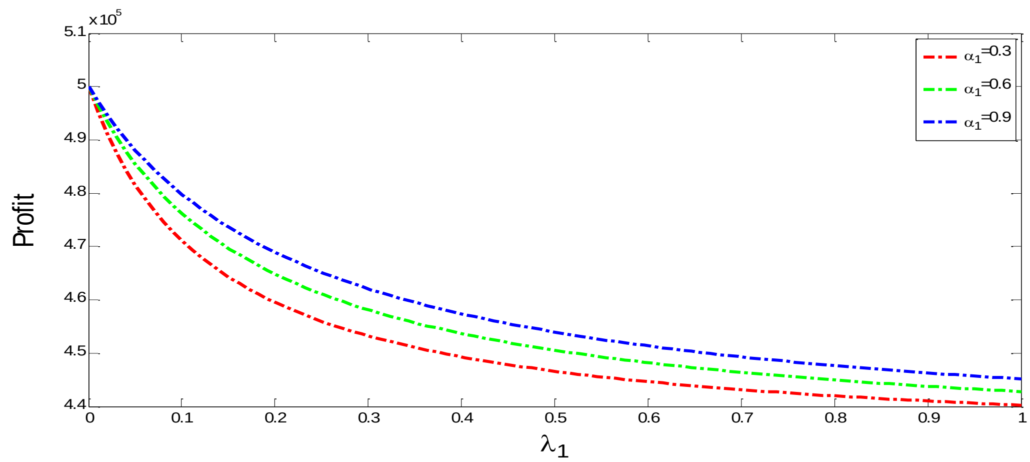


Figure 2: Profit against minor deterioration rate $\lambda_1$ **for** different values of $\alpha_1(0.3, 0.6, 0.9)$

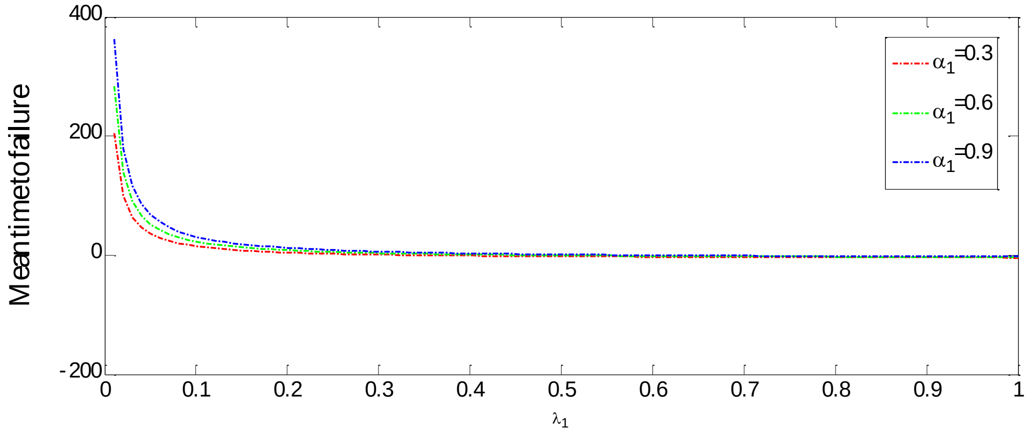


Figure 3: MTTF against minor deterioration rate $\lambda_1$ **for** different values of $\alpha_1(0.3, 0.6, 0.9)$

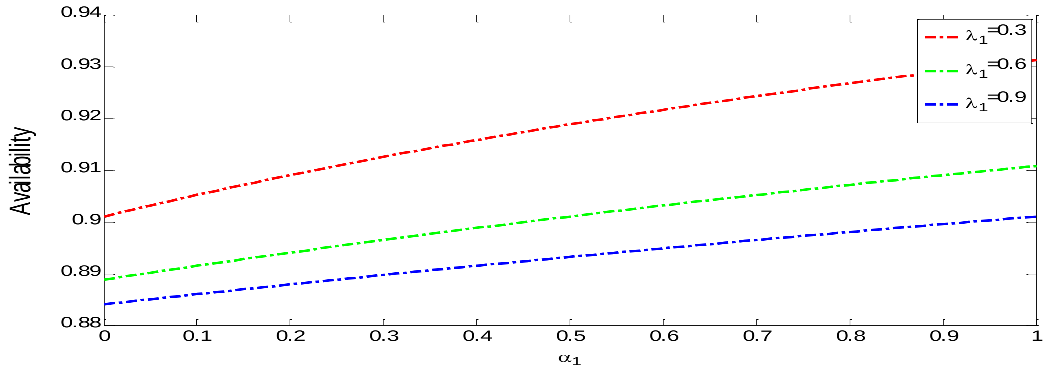


Figure 4: Availability against minor maintenance rate $\alpha_1$ **for** different values of $\lambda_1(0.3, 0.6, 0.9)$

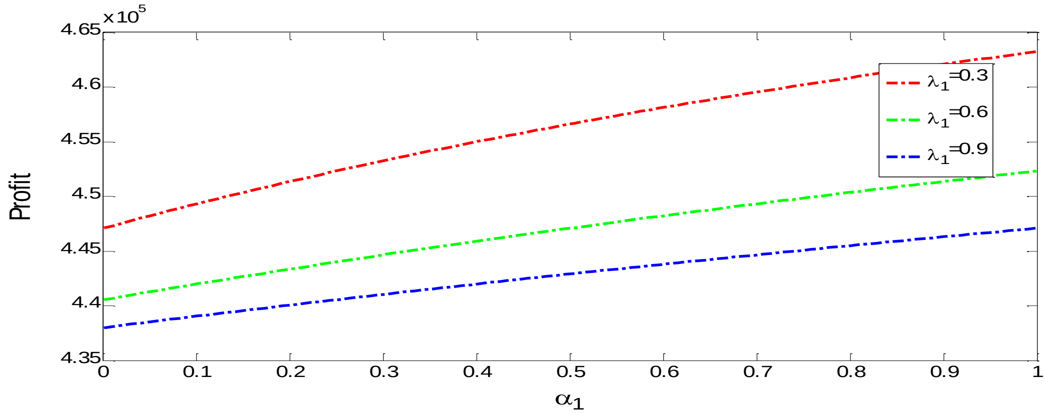


Figure 5: Profit against minor maintenance rate $\alpha_1$ **for** different values of $\lambda_1(0.3, 0.6, 0.9)$

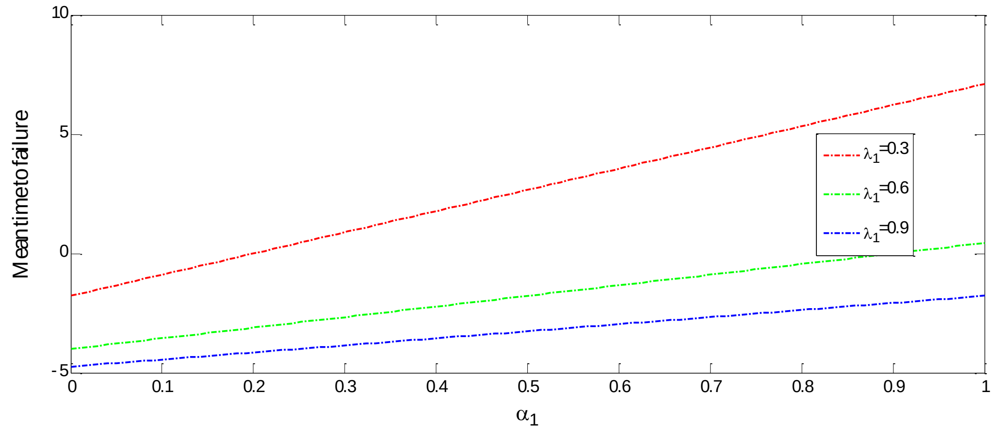


Figure 6: MTTF against minor maintenance rate $\alpha_1$ **for** different values of $\lambda_1(0.3, 0.6, 0.9)$

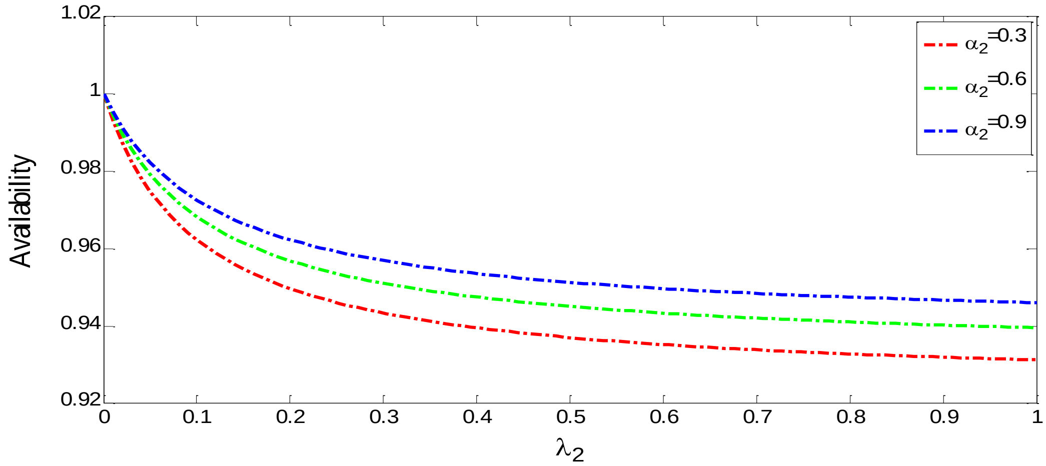


Figure 7: Availability against medium deterioration rate $\lambda_2$ **for** different values of $\alpha_2(0.3, 0.6, 0.9)$

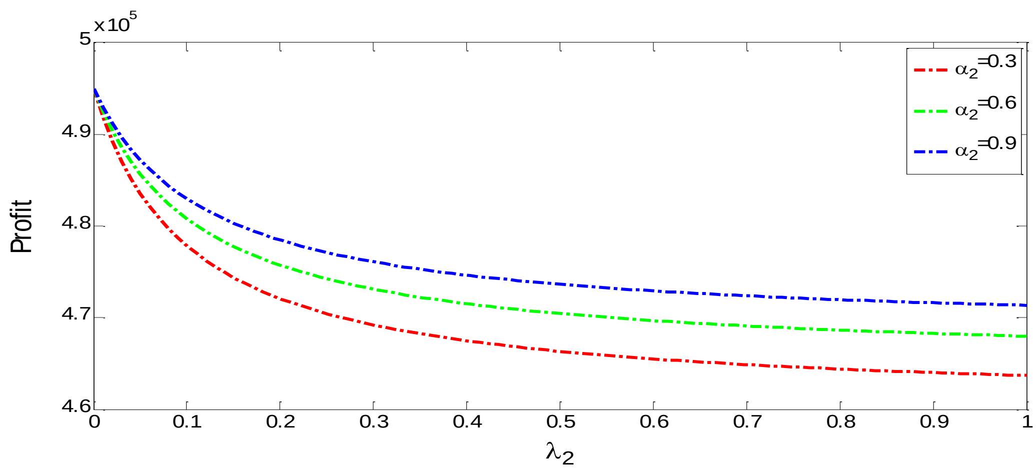


Figure 8: Profit against medium deterioration rate $\lambda_2$ **for** different values of $\alpha_2(0.3, 0.6, 0.9)$

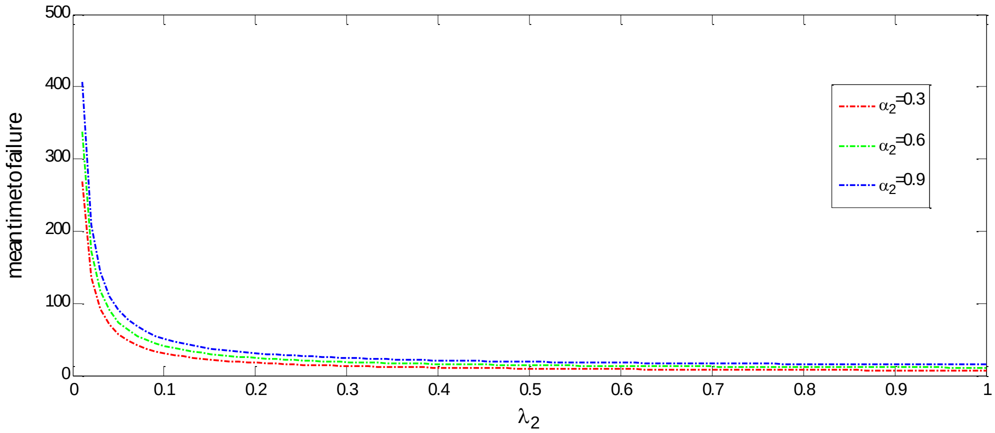


Figure 9: MTTF against medium deterioration rate $\lambda_2$ **for** different values of $\alpha_2(0.3, 0.6, 0.9)$

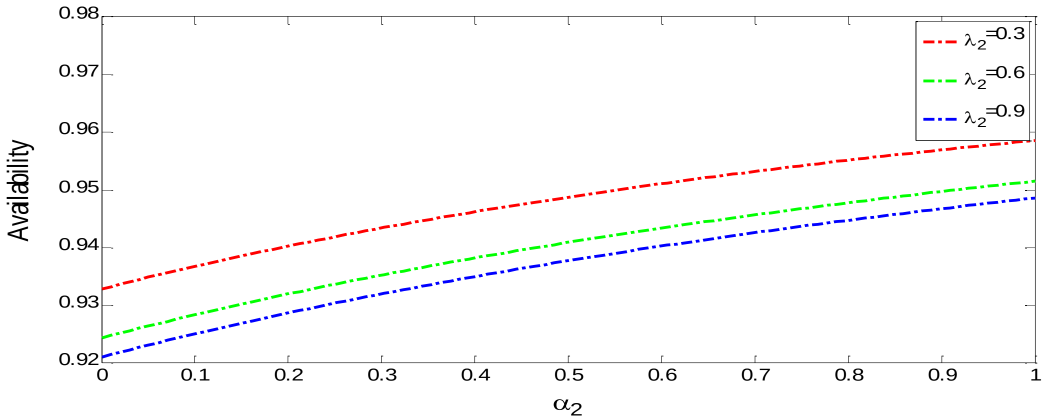


Figure 10: Availability against major maintenance rate $\alpha_2$ **for** different values of $\lambda_2(0.3, 0.6, 0.9)$

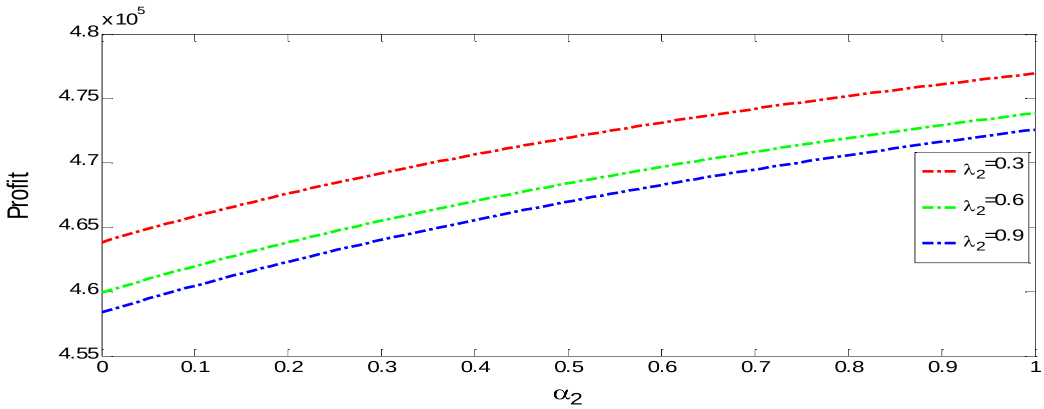


Figure 11: Profit against major maintenance rate $\alpha_2$ **for** different values of $\lambda_2(0.3, 0.6, 0.9)$

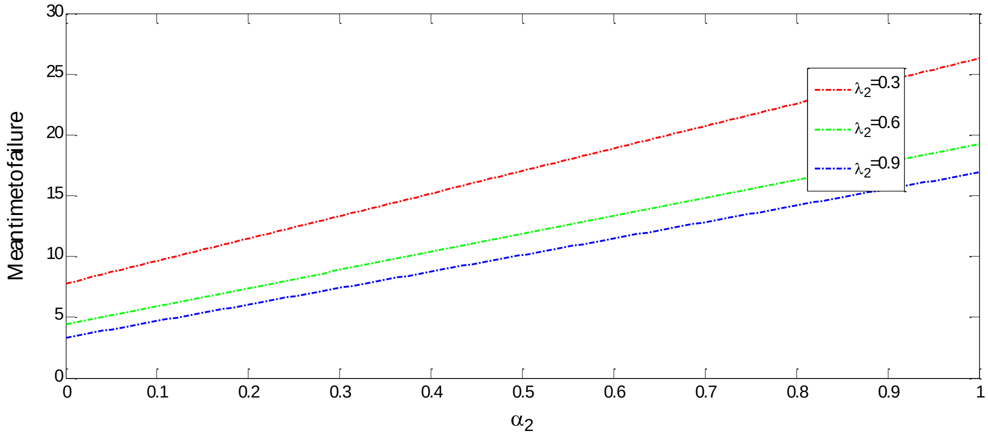


Figure 12: MTTF against major maintenance rate $\alpha_2$ **for** different values of $\lambda_2(0.3, 0.6, 0.9)$

The results which compare the steady state availability and profit with respect to $\lambda_1$ for different values of $\alpha_1$, are depicted in Figures 1-3. From these Figures, it is evident that system availability, profit and MTTF decrease as $\lambda_1$ increases. It is clear from these that availability and profit tend to be higher when $\alpha_1 = 0.9$. This shows that minor maintenance could make a significant difference to the system availability and profit. Numerical results of availability, profit and MTTF with respect to minor maintenance $\alpha_1$ for different values of $\lambda_1$ are depicted in Figures 4-6. It is clear from these Figures that availability and profit increases as $\alpha_1$ increases. It is evident from these Figures that minor maintenance significantly slow down minor deterioration from 0.9 to 0.3 as depicted in the Figures. Thus, availability, profit and MTTF are higher when $\lambda_1 = 0.3$. Similar observations can be seen in Figures 7-9 with respect to medium deterioration $\lambda_2$ and major maintenance $\alpha_2$. Availability, profit and MTTF displayed decreasing pattern as $\lambda_2$ increases for different values of $\alpha_2$ in Figures 7 and 8. It is clear from these that availability and profit tend to be higher when $\alpha_2 = 0.9$. This shows that higher major maintenance could make a great impact to the system availability and profit. On the other hand, Results of availability, profit and MTTF with respect to $\alpha_2$ for different values of $\lambda_2$ are depicted in Figures 10-12. It is clear from these Figures that availability, profit and MTTF increases as $\alpha_2$ increases. It is evident from these Figures that major maintenance significantly slow down minor deterioration from 0.9 to 0.3 as depicted in the Figures. Thus, availability and profit are higher when $\lambda_2 = 0.3$.

## Conclusion

In this paper, a single unit system with four modes: minor deterioration, medium deterioration, and major deterioration and failure modes is studied. The paper presents modelling and evaluation of reliability characteristics such as availability, profit and MTTF of the system. Explicit expressions for the steady-state availability, busy period for minor and major maintenance, and replacement, profit function and MTTF have been developed.

On the basis of the numerical examples presented in Figures 2-13, it is suggested that the availability, profit and MTTF of a system can be enhance by

(i) By taking emphasis to maintenance (preventive maintenance) before or at early stage of deterioration.
(ii) By increasing maintenance rate.
(iii) Adding more spares/ cold standby units

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# How Entropy Infiltrates the Reliability Domain: Two Theoretical Inquiries

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**Abstract**

The reliability theory includes an enormous amount of works but does not yet become a mature discipline. Many factors account for this fail; as first the reliability domain is lacking the comprehensive and unifying frame.
The entropy concept demonstrates to be capable of offering unified views and providing integrated mathematical tools in various scientific sectors. The present paper presents a concise review of two recent theoretical studies that introduce the entropy in the reliability domain in order to treat broad issues. The first of these applies thermodynamic entropy to degradation mechanisms and provides a model for a wide range of degradation processes. The second employs the Boltzmann-like entropy to describe the spontaneous decay of systems during the entire lifespan.

**Keywords:** The present state of the reliability theory, thermodynamic degradation, entropy generation theorem, Boltzmann-like entropy, bathtub curve, reparability function.

## I. Introduction

### I.1 The present state of the reliability theory

Reliability theory was boosted after the Second World War when experts of the US and the USSR elaborated effective answers to practical issues arising from companies and institutions. Reliability studies continued to progress in various directions in the following decades. However, those mathematicians who were mostly attracted by applied problems paid little attention to broad issues and general principles. Nowadays, reliability theory looks like a 'heap' of works addressing an assortment of topics that are sometimes not clearly connected in terms of logic due to the missing unifying conceptual frame.
There are several small theories and specialist approaches under the umbrella of the reliability domain, as examples we mention the following self-explanatory parts of modern reliability studies: physics-of-failure methods, the cumulative and extreme shock theory, availability calculations, human factors and reliability-centered maintenance. They provide a rather effective aid to practitioners, though the large number of data-driven inquiries about system dependability can be compared to a collection of things placed randomly one on top of the other because of the missing comprehensive frame.

### I.2 The unifying concept of entropy

By the end of the seventeenth century, steam engines triggered the Industrial Revolution, although those engines were somewhat inefficient. Rudolf Clausius tackled this kind of issues and stated the second law of thermodynamics by means of the entropy $H_C$ that is a function of the exchanged amount of heat $Q$ and temperature $T$

$$dH_C = \frac{\delta Q}{T} \leq 0. \qquad (1)$$

The quantity $dH_C$ is capable of explaining the irreversibility of real systems that are steam engines and also living beings, nuclear plants, stars etc. [1]. Ludwig Boltzmann fixed on the entropy $H_B$ as a measure of the statistical disorder caused by the number Ω of complexions typical of the equilibrium state of $S$ [2]. When the temperature reaches absolute zero, a perfect lattice of molecules has only one complexion, and $H_B$ illustrates the third law of thermodynamics, which is a universal rule

$$H_B = k_B \ln \Omega = 0. \qquad (2)$$

Claude Shannon defined this entropy for a source of discrete signals

$$H_S = -\sum_{i}^{n} P_i \log_2 P_i \ . \qquad (3)$$

Where $P_i$ is the probability of the generic signal emitted by the source. He established that the average code-word length $L$ for any distortionless source coding is bounded as

$$L \geq H_S. \qquad (4)$$

The entropy $H_S$ proves the source coding theorem that is a fundamental statement in information theory.
Scholars devised other entropy equations, yet the present summary should be sufficient to recall how the entropy is able to express general laws that belong to a variety of sectors. The entropy unifies several empirical expressions in steam engineering and also in areas that are rather distant from thermodynamics, such as telecommunications, quantum mechanics, computing, economics, and biology. The entropy notion enhances experts' knowledge and at the same time improves the effectiveness of practitioners. Once a comprehensive equation has been formulated with the aid of entropy, then convenient specializations to particular structural elements proceed on a more informed basis.

### I.3 An assortment of specialist issues

A group of theorists employ the concept of entropy to solve specific issues in the reliability sector. For example, some put information redundancy near to operation redundancy and use the Shannon entropy to optimize the parallels between systems [4], [5]; others aim at controlling the complexity of intelligent systems [6]. The present paper makes a short review of two inquiries whose authors address broad issues. They seek a unified view and integrated mathematical instruments namely they tackle the issue presented in Section I.1. The first study looks at degradation processes, and the second answers some problems that lay at the basis of the reliability theory.

## II. Entropy and Energy Dissipation

Degradation processes affect several systems and have a notable impact on modern economies. Speaking in general, degradation processes involve different mechanisms with distinctive features, types, and rates. Experts use collective terms such as corrosion, erosion, wear, fatigue, thermal degeneration, and plasticity to group together the varieties of failure mechanisms. For example, by the term 'corrosion' experts mean galvanic corrosion, pitting, dealloying, crevice corrosion, microbiologically-influenced corrosion, stress corrosion cracking, intergranular corrosion, fretting, and hydrogen damage [7].
Normally a degradation process evolves over a certain period of time, and the path to failure depends on an assortment of factors that requires systematic inspections of the system components and the collection of data from different sources such as photographic documentation, microscopic scanning, chemical analysis etc. The dynamics of each degeneration are based on particular physical principles, and often a formal model is limited primarily to a single type of failure. The complex and specific traits of each degradation process require engineers to use a non-negligible set of variables. The science of materials employs a large apparatus of concepts, formal definitions and mathematical tools, while instead the agile theory of degenerative phenomena should be derived from fruitful generalization and uniform principles. The present complicated intellectual situation stimulated the search for a unifying frame capable of facilitating the work of practitioners

### II.1 Entropy generation theorem

A group of researchers started by observing how different degeneration processes share a common and intriguing feature: *the loss of energy*. Progressive decay consists of dissipative transformations, and the entropy $H_C$ offers the appropriate lens to investigate these transformations with unifying mathematical support, as an alternative to the partial and heterogeneous models currently in use. The change in irreversible entropy can be calculated via the thermodynamic equations when the heat or energy dissipated is known.
The early attempts in this new direction of study may be ascribed to Klamecki, who developed the thermodynamic analysis of friction and wear for bodies in sliding contact [8], [9]. Another pioneer, Zmitrowicz, conducted a complex enquiry to predict the friction and wear of bodies in contact [10]. Progressively, research went in various directions: for instance, Dai and others analyzed the production of entropy associated with fretting wear [11].
Cemal Basaran set up a ponderous inquiry assuming the corrosion process as an irreversible entropy generating chemical process. According to second law of thermodynamics, irreversible entropy generation gradually increases as system evolves toward the final failure. Entropy production and its rate are used as the sole measures of system evolution in the place of a large number of specialist measures [13], [14]. Basaran and other scientists experimentally verified his unified scheme for several degradation factors such as mass transport, electromigration, thermomigration, creep, thermo-mechanical degradation, phase change, fatigue tribology and loading agents [12].
Sosnovskiy and Sherbakov following the ideas of Basaran examine a mechano-thermodynamic system which they conceive as a continuum including scattered solids which interact with each other and with the continuum. Sosnovskiy and Sherbakov prepare a generalized theory of system evolution which is based on the concept of tribo-fatigue entropy, in this way they make attempts to place under a single umbrella classical mechanics and thermodynamics [15]. Essence of the proposed approach is that tribo-fatigue entropy is determined by thermodynamic and mechanical effects causing to the change of states of the system.
Finally, we mention Bryant and others who develop a thermodynamic characterization of degradation dynamics using $H_C$ as the fundamental measure. The paper [16] formulate the *entropy generation theorem* (EGT) which relate the entropy production and decay, via generalized thermodynamic forces and degradation forces. The authors assume that $n$ dissipative processes $p_i = p_i \ \xi_i^j$ ($i$ = 1,2,...$n$) are characterized by a set of time-dependent variables $\xi_i^j$ ($j$ = 1, 2,...$m_i$), while

the 'degradation measure' $w = w\{p_1, p_2,..., p_i,.., p_k\}$ determines the characters of the collective, including $k$ processes. The results of the EGT can be summarized in the following three points:

(i) The generalized 'degradation forces' $Y_i^j$ are linear functions of the generalized 'thermodynamic forces' $X_i^j$.

(ii) The degradation component $\dot{w}_i = \sum_j Y_i^j J_i^j$ proceeds at the rate $J_i^j$ that is determined by the entropy production.

(iii) The degradation rate $\dot{w} = \sum_i \dot{w}_i$ is a linear combination of the components that produce the overall entropy.

The entropy generation theorem shows how linear expressions, which are consistent with the laws of thermodynamics, govern the decadence of systems. EGT suggests a simplified approach for degradation analysis and speeds up a methodology for the accelerated testing of degradation.

**II.2 Multiple degradation**

Some researchers are aiming to explain the damage mechanism for single materials as well as the synergistic effect amongst different mechanisms. Amiri and Modarres [17], [18] exploit EGT and write the total entropy of $S$ as the sum of the entropies of reversible and irreversible changes

$$dH_C = dH_C^r + dH_C^d. \quad (5)$$

The reversible entropy $dH_C^r$ is caused by the transfer of mass and heat

$$\frac{dH_C^r}{dt} = -\int_{\Omega} J_s d\Omega. \quad (6)$$

Where $J_s$ is a vector of the total entropy flow per unit area. The entropy of irreversible processes $dH_C^d$ is produced by the physical components of the system when $\sigma$ is the entropy dissipated per unit volume and unit time

$$\frac{dH_C^d}{dt} = \int_{V} \sigma dV. \quad (7)$$

Using the conservation of energy, mass, and momentum, Amiri and Modarres calculate various dissipative phenomena pertaining to (6) and (7) in this way

$$J_s = -\nabla \left( \frac{J_q - \sum_k c_m \zeta + \mu_k \ J_k}{T} \right). \quad (8)$$

$$\sigma = \frac{1}{T^2} J_q \nabla T - \sum_k J_k \left( \nabla \frac{\mu_k}{T} \right) + \frac{1}{T} \ \tau : \varepsilon_p \ + \frac{1}{T} \left( \sum_i v_i D_i \right) + \frac{1}{T} \left( \sum_m c_m J_m \ - \nabla \zeta \ \right). \quad (9)$$

This entropic frame illustrates the multifold degeneration dynamics and their interrelations; more precisely, eqn. (8) is the result of the exchange of heat and mass with the surroundings. Eqn. (9) includes five terms that, in order, calculate internal heat conduction, diffusion dynamics, plastic deformation, chemical reactions, and external forces. More precisely, $T$ is the temperature, $J_q$ the heat flux, $J_k$ the diffusion flow, $c_m$ the coupling constant, $\zeta$ the potential of the external field (magnetic or electric), $\mu_k$ is the chemical potential, $J_m$ the magnetic or electrical flux, $v_i$ the chemical reaction rate, $D_i$ the chemical reaction potential difference, $\tau$ the stress tensor, and $\varepsilon_p$ the plastic strain rate tensor. In conclusion, this approach, which could be labeled as 'thermodynamic degradation' or 'thermodynamic reliability', provides a few equations for multiple degradation processes. The authors calculate the competing mechanisms that contribute to damage and can also account for the synergistic effects.

## III. In Search of Ideal Models

Boris V. Gnedenko – often recognized as the 'father' of the reliability theory – conceives the reliability domain as a new science and lays the first theoretical stone for building up this scientific edifice in [19]. He defines the *hazard rate* $\lambda(t)$ (also called the *failure* or *mortality rate*) as the instantaneous incidence rate

$$\lambda\ (t) = -\ P'\ (t)/P(t). \tag{10}$$

And deduces the following exponential function through mathematical proof

$$P(t) = e^{-\int_0^t \lambda(t)dt}. \tag{11}$$

Where $P(t)$ – normally termed the *reliability* of $S$ – is the probability of functioning with no failure in the interval from time 0 to time $t$. Several writers in the reliability sector assume that $\lambda(t)$ complies with the *bathtub curve*, however numerous evidence disproves this curve [20], [21], [22], which was set up through generic intuition and not after mathematical demonstration. A rigorous study that relates $\lambda(t)$ in all particulars to the system lifespan is lacking and the construction inaugurated by Gnedenko is no longer progressing because of the discrepancy between theory and practice.

### III.1 The Boltzmann-like entropy

The preparation of a unified construct is a challenging duty, but a new form of entropy – the *Boltzmann-like entropy* – offers aid. The *irreversibility* and the *reversibility* (I/R) of the generic state $A_i$ ($i$=1,2,..$n$) of the stochastic system $S$ are coupled properties, and the Boltzmann-like entropy qualifies the I/R of the state $A_i$ in accordance to the criteria adopted by Boltzmann [23]

$$H(A_i) = H(P_i) = \log\ (P_i). \tag{12}$$

Where $P_i$ is the probability of the state $A_i$. Let us confine our attention to the *functioning state* $A_f$ and the *recovery state* $A_r$ during which $S$ works steadily and is repaired/maintained respectively. The following intuitive remarks can help the reader to grasp the physical significance of the *functional entropy* $H_f = H(A_f) = H(P_f)$ and the *recovery entropy* $H_r = H(A_r) = H(P_r)$. In consequence of the coupled properties I/R we obtain the following paired remarks a and b:

1.a) When $H_f$ is 'high', the functioning state is irreversible and the system works steadily. In particular, the higher $H_f$ is, the more irreversible $A_f$ is, and $S$ is capable of working.
1.b) On the other hand, when $H_f$ is low, $S$ often abandons $A_f$ and switches to $A_r$, and we say that $S$ is incapable of working.

2.a) When $H_r$ is 'high', the recovery state is irreversible, and the workers operate on *S* with effort. In particular, the higher $H_r$ is, the more stable $A_r$ is, and in practice *S* is hard to repair and/or maintain in the world.

2.b) On the other hand, when $H_r$ is low, *S* leaves $A_r$ and we say that *S* can be easily restored or maintained.

In summary, the Boltzmann-like entropy qualifies the behavior of the systems this way: *the reliability entropy expresses the aptitude of S to work without failures; the recovery entropy illustrates the disposition of S toward reparation.*

**III.2 Ideal tripartite function**

The reliability entropy furnishes an aid for detailing (11). If one assumes that the capability of good functioning decreases regularly with time, [24] demonstrates

$$\lambda(t) = c, \quad c > 0. \tag{13}$$

When the system's components have a certain degree of deterioration, a damaged part spoils a neighboring part and this in turn can affect another part and so on. If this cascade effect is linear, $H_f$ leads to

$$\lambda(t) = at^n, \quad a,n > 1. \tag{14}$$

When the cascade effect evolves in various directions, $H_f$ leads to

$$\lambda(t) = d\exp(t), \quad d > 1. \tag{15}$$

The precise assumptions that lead to (13), (14) and (15) enable an expert to deduce the position of these results within the system lifespan. The cascade effect occurs if the system components have a certain level of deterioration and this hypothesis is appropriate for aging. More precisely, (14) is suitable for machines and (15) for living systems that have intricate structures. The cascade effects do not work during system maturity when the constant hazard rate (13) is occurring. Generally, manufacturers' burn-in and infant mortality cause a decreasing distribution of $\lambda(t)$, and hence one can depict the hazard rate in linear terms during the system infancy

$$\lambda(t) \propto -mt, \quad m > 0. \tag{16}$$

Let $t_1$ and $t_2$ be the extreme times of the useful lifespan, then eqns. from (13) to (16) can be joined into the following tripartite function

$$\lambda(t) \begin{cases} \propto -mt, & 0 < t < t_1. \\ = c, & t_1 < t < t_2. \\ = at^n, \ \text{ OR } d\exp(t), & t > t_2. \end{cases} \tag{17}$$

This result can be classified as the *ideal model* of systems' hazard rate since it is based on exact hypotheses. An ideal model is different from a *statistical model*: the first is developed through *deductive logic* and the second through *inductive logic*. The ideal model is true provided that the hypotheses are true, and this means that (17) is not a general rule but is true as long as the hypotheses are plausible. When the conditions for (13), (14), (15) and (16) do not occur in the real world, then the hazard rate function $\lambda(t)$ does not conform to (17) and in practice presents peaks, troughs and humps. The special features of the ideal model (17) conform with the wide usage of the bathtub curve in the literature and at the same time justify the exceptions that derive from the assumptions of (17) that do not occur in the physical reality.

**III.3 Reparability function**

When one maps the reliability and the recovery entropies, one obtains *the reparability function*

$$H_f = f\left(H_r\right) = \ln\left(1 - \exp\left(H_r\right)\right) \tag{18}$$

An assortment of empirical criteria and heuristic methods regulate the repair and maintenance of systems, while some achievements even seem contradictory. The mathematical development of (18) qualifies the results that have until now been obtained by trial and error. For example, eqn. (18) confirms that generally the state of a system after repair will be *as-good-as-new* (AGAN) or *as-bad-as-old* (ABAO), while an intermediate quality outcome is not readily achievable. The reparability function explains also that the amelioration of a repaired system depends on the initial state of *S*. An identical intervention in two systems – e.g. the replacement of an old component with a new one – can bring either great benefit or trivial progress. Ultimately, function (18) proves how the working capacity of a reparable system follows a *saw-shaped curve*, which is the *ideal model for describing repair cycles.*

## IV. Discussion

This paper reminds the readers that the concept of entropy is used in various fields and everywhere it supports unifying theories and comprehensive frames in the place of heterogeneous empirical statements. This paper makes a concise survey of two entropy functions introduced in the reliability domain: the well-known thermodynamic entropy and the new Boltzmann-like entropy. Both functions seek to explain self-generated phenomena typical of degraded systems in an exhaustive manner. This pair of theoretical frames derive all their results from rigorous assumptions, and they furnish intriguing answers in the areas of interest. The first applies thermodynamic entropy to degradation mechanisms and provides a model for various degradation processes that can even interfere one another. The second deduces two distinct ideal models for functioning systems and repaired systems.

The theoretical achievements supported by the entropy functions cover a broad range of situations and thus can help the reliability theory to evolve and reach the status of 'science' on a par with disciplines such as mechanics and chemistry.

The inquiries – briefly reviewed in this paper – are relatively recent and need experimental validation. The exhaustive intent of the authors implies that testing will not be trivial, and the corroboration phase will have the last word in the present argument.

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# A 'One Parameter' Bathtub Shaped Failure Rate Distribution

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## Abstract

Most real life system exhibit bathtub shapes for their failure rate functions. Generalized Lindley, Generalized Gamma, Exponentiated Weibull and x-Exponential distributions are proposed for modeling lifetime data having bathtub shaped failure rate model. This paper considered a simple model but exhibiting bathtub shaped failure rate and discuss the failure rate behavior. The proposed distribution has only one parameter. A Little works are available in literature with one parameter. Computation of moments requires software. Applications in reliability study is discussed.

**Keywords:** Bathtub failure rate, Reliability

## I. Introduction

There are many distributions for modeling lifetime data. Among the known parametric models, the most popular are the Lindley, Gamma, log-Normal, Exponentiated Exponential and the Weibull distributions. These five distributions are suffer from a number of drawbacks. None of them exhibit bathtub shape for their failure rate functions. Most real life system exhibit bathtub shapes for their failure rate functions. Generalized Lindley (GL), Generalized Gamma (GG), Exponentiated Weibull (EW) and x-Exponential distributions are proposed for modeling lifetime data having bathtub shaped failue rate model. In this paper we consider a simple model but exhibiting bathtub shaped failure rate and discuss the failure rate behavior of the distribution. The inference procedure also become simple than GL, GG and EW distributions.

Section II, discussed new distribution and their properties, Maximum likelihood estimator is obtained in section III. Conclusions are given at the final section.

### II. New Bathtub shaped failure rate model

In this section we consider failure rate function

$$h(x) = \frac{1 + ax + x^2}{1 + x + x^2}, x > 0, a > 0.$$

a is considered to be arbitrary

$$\int \frac{1+ax+x^2}{1+ax+x^2}dx = \int \left(\frac{(a-1)x}{1+x+x^2}+1\right)dx$$
$$= (a-1)\int \frac{x}{1+x+x^2}dx + \int 1dx$$
$$= (a-1)\int \frac{2x+1}{2(1+x+x^2)}dx - \int \frac{1}{2(1+x+x^2)}dx + \int 1dx$$
$$= \frac{(a-1)}{2}\int \frac{1}{w}dw - \frac{2}{2\sqrt{3}}\int \frac{1}{(1+u^2)}du + \int 1dx,$$

by substituting $w = \frac{1}{1+x+x^2}$, $u = \frac{1+2x}{\sqrt{3}}$.

$$= \frac{(a-1)}{2}ln(1+x+x^2) - (a-1)\frac{1}{\sqrt{3}}arctan\frac{(1+2x)}{\sqrt{3}} + x$$

Here, we consider a simplified form of distribution function,

$$F(x) = 1 - e^{-(x+(a-1)(\frac{\log(1+x+x2)}{2}-\frac{\arctan((1+2x)/\sqrt{3})}{\sqrt{3}})}, x > 0, -\infty < a < \infty \quad (1)$$

It is an alternative model GL, GG, EW distributions. Clearly F(0)=0, $F(\infty) = 1$, $F$ is non-decreasing and right continuous. More over F is absolutely continuous. The probability density function (pdf) is given by

$$f(x) = \frac{1+ax+x^2}{1+x+x^2}\, e^{-(x+(a-1)(\frac{\log(1+x+x2)}{2}-\frac{\arctan((1+2x)/\sqrt{3})}{\sqrt{3}})}, x > 0,\ -\infty < a < \infty.$$

It is positively skewed distribution.

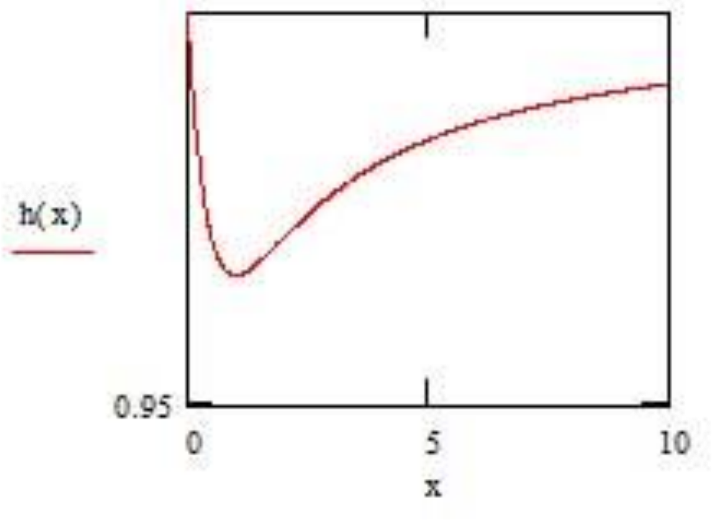


Figure 1. Failure rate function for a=0.9

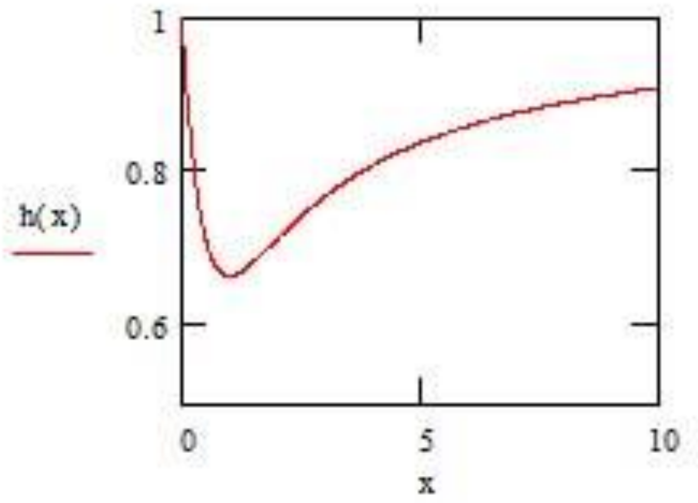


Figure 2. Failure rate function for a=0.001

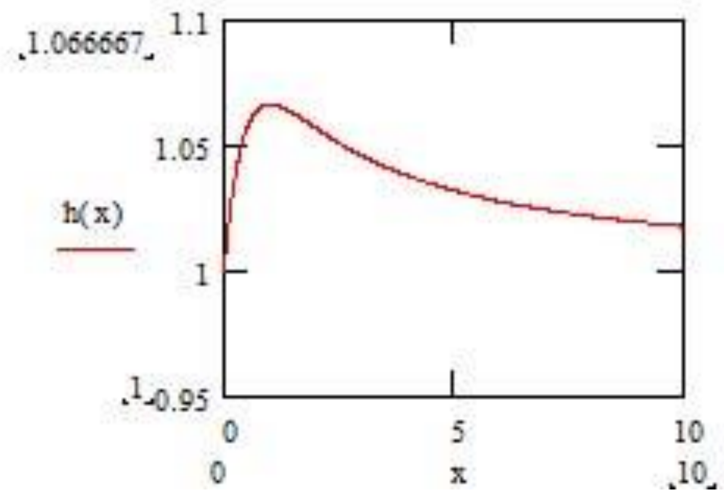


Figure 3. Failure rate function for a=0.001

From Figure 1 and 2, the shape of the hazard rate function appears monotonically decreasing or to initially decrease and then increase, a bathtub shape. The proposed distribution allows only bathtub shapes for its hazard rate function. Fig. 3 shows Upside down bathtub shape in its failure rate function, for a=1.2. When a=1, it becomes constant failure rate model.

**Proposition 1:** The proposed distribution is a generalization of Exponential distribution.
When a=1, it becomes exponential distribution $f(x) = e^{-x}, x > 0.$

**A generalization to Two parameter distribution**

Here, we consider a simplified form of distribution function,

$$F(x) = 1 - e^{-\lambda(x+(a-1)(\frac{\log(1+x+x2)}{2} - \frac{\arctan((1+2x)/\sqrt{3})}{\sqrt{3}})}, x > 0, \lambda > 0 - \infty < a < \infty \qquad (1)$$

It is an alternative model GL, GG, EW distributions. Clearly F(0)=0, F($\infty$) = 1, $F$ is non-decreasing and right continuous. More over F is absolutely continuous. The probability density function (pdf) with scale parameter $\lambda$ is given by

$$f(x) = \frac{\lambda(1+ax+x^2)}{1+x+x^2}\; e^{-\lambda(x+(a-1)(\frac{\log(1+x+x2)}{2}-\frac{\arctan((1+2x)/\sqrt{3})}{\sqrt{3}})}, x > 0,\ \lambda > 0, -\infty < a < \infty.$$

It is positively skewed distribution.

The failure rate function is

$$h(x) = \frac{\lambda(1 + ax + x^2)}{1 + x + x^2}\ , x > 0\ , \lambda > 0.$$

**Moments**

All the raw and central moments, moment generating functions etc exist, since the function f(x) is having countable number of discontinuities, and integrable but the resulting function require more mathematical treatment. It can be done by softwares like Matlab. It left to reader.

**Estimation**

Here, we consider estimation by the method maximum likelihood.

$$L(a, x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f(x_i)$$

$$L(a, x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} \frac{1 + ax_i + x_i^2}{1 + x_i + x_i^2}\ e^{-(x_i+(a-1)(\frac{\log(1+x_i+x_i^2)}{2}-\frac{\arctan((1+2x_i)/\sqrt{3})}{\sqrt{3}})}$$

$$logL = \sum_{i=1}^{n}[\ \log(1 + ax_i + x_i^2) - \ \log(1 + x_i + x_i^2)] - \sum_{i=1}^{n}[x_i + (a-1)(\frac{\log(1 + x_i + x_i^2)}{2} - \frac{arctan\ \ (\frac{1+2x_i}{\sqrt{3}})}{\sqrt{3}})]$$

$$= \sum_{i=1}^{n} \log(1 + ax_i + x_i^2) - \sum_{i=1}^{n} \log(1 + x_i + x_i^2) - -\sum_{i=1}^{n} x_i + (a-1)\sum_{i=1}^{n}(\frac{\log(1 + x_i + x_i^2)}{2} - \frac{arctan\ (\frac{1+2x_i}{\sqrt{3}})}{\sqrt{3}})]$$

$$\frac{\partial}{\partial a}\ log\ L\ = 0\ \Rightarrow \sum_{i=1}^{n} \frac{x_i}{(1 + ax_i + x_i^2)} - \sum_{i=1}^{n}\left[\frac{\log(1 + x_i + x_i^2)}{2} - \frac{arctan\ \ \left(\frac{1+2x_i}{\sqrt{3}}\right)}{\sqrt{3}}\right] = 0$$

$$\Rightarrow \frac{\sum_{i=1}^{n} x_i}{(n + a\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} x_i^2)} = \sum_{i=1}^{n}\left[\frac{\log(1 + x_i + x_i^2)}{2} - \frac{arctan\ \ \left(\frac{1+2x_i}{\sqrt{3}}\right)}{\sqrt{3}}\right]$$

$$\Longrightarrow \frac{(n + a\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} x_i^2)}{\sum_{i=1}^{n} x_i} = \frac{1}{\sum_{i=1}^{n}\left[\frac{\log(1 + x_i + x_i^2)}{2} - \frac{tan^{-1}\left(\frac{1+2x_i}{\sqrt{3}}\right)}{\sqrt{3}}\right]}$$

$$\Rightarrow \left(n + a\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} x_i^2\right) = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n}\left[\frac{\log(1 + x_i + x_i^2)}{2} - \frac{tan^{-1}\left(\frac{1+2x_i}{\sqrt{3}}\right)}{\sqrt{3}}\right]}$$

$$\Rightarrow a\sum_{i=1}^{n} x_i = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n}\left[\frac{\log(1 + x_i + x_i^2)}{2} - \frac{tan^{-1}\left(\frac{1+2x_i}{\sqrt{3}}\right)}{\sqrt{3}}\right]} - n - \sum_{i=1}^{n} x_i^2$$

$$\Rightarrow \hat{a} = \frac{1}{\sum_{i=1}^{n}\left[\frac{\log(1 + x_i + x_i^2)}{2} - \frac{tan^{-1}\left(\frac{1+2x_i}{\sqrt{3}}\right)}{\sqrt{3}}\right]} - \frac{(n - \sum_{i=1}^{n} x_i^2)}{\sum_{i=1}^{n} x_i}$$

Thus we obtained Maximum likelihood estimator for the parameter.

## III. Applications and Conclusions

Identifying the failure rate model is crucial to the maintenance and replacement policies. The optimal burn in time can be computed for the Bathtub shaped failure rate models. The model suggested here provide Bathtub shaped failure rate distributions which is more flexible and simple than many existing distributions, in the sense of estimation. We considered Arset data [5] parameter is a=0.813225

Table 1. Aarset Data

| |
|---|
| 0.1 0.2 1 1 1 1 1 2 3 6 7 11 12 18 18 18 18 18 21 32 36 40 45 46 47 50 55 60 63 63 67 67 67 67 72 75 79 82 82 83 84 84 84 85 85 85 85 85 86 86 |

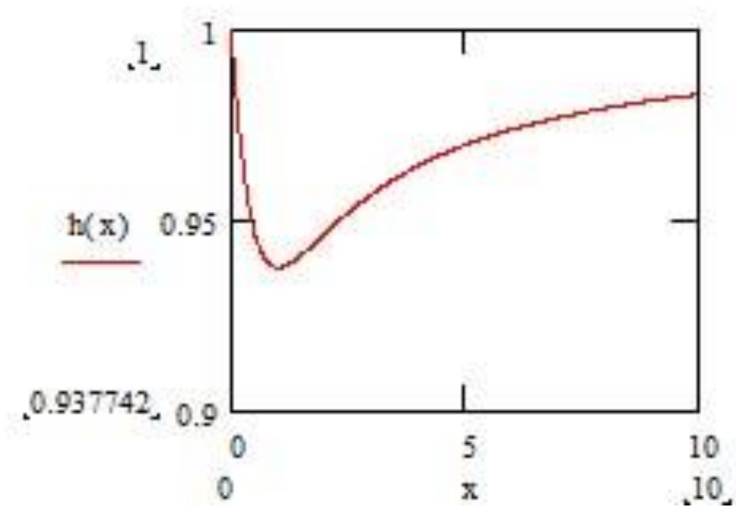


Figure 4. Failure rate function a=0.813225

We obtained bathtub shaped curve for the Aarset data as in figure 4.

## IV. Discussion

There are many distributions in reliability which exhibit Bathtub shaped failure rate model, but most of them are complicated in finding estimators. The complication in using GL,GG,GE distributions is reduced in the proposed model. Any way the problem of computing Moments, characteristic functions etc still remains.

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# Preface

Dear Friends!

On the cover of this issue there is a "special issue". What is its difference from the usual numbers issued earlier? The fact is that with this number, we wanted to pay tribute to the memory of our friend and colleague Igor Ushakov. It is he who owes his origin and existence to the Gnedenko Forum and our journal, it is with his name that all our initiatives of the last 11 years are connected. We decided to post three articles by Igor, printed in our magazine in different years. They very clearly demonstrate its versatility.

The first article is a philosophical reflection on the ways of developing the theory of reliability and its recent "golden age". These arguments are all the more valuable because their author is one of those who stood at the roots of what is now called the theory of reliability.

The second article is an example of the scientific work of Igor Ushakov, in which he discusses the interesting practical application of generating functions.

The third is his recollections about the teacher - about Boris Vladimirovich Gnedenko, with whom he was associated by kind friendship.

I have no hesitation to see Igor as my teacher, through him I learned a lot in life and in science. And so, without hesitation, I can say that it was true for me and all understand each. It has been almost a year since he passed away, and this year has been very difficult. It does not take the loneliness, not enough of our letters and conversations. They stayed site Gnedenko, stayed the electronic journal, which I hope I will be able to save. Were his books.

And stayed my debt to him: I promised help Igor to make him personal website. Now this site will be a memorial - the site of human memory, memory of scientist and friend.

**Alexander Bochkov**

# Is Reliability Theory Still Alive?

Igor Ushakov

At the banquet held during closing of the MMR-2004 Conference (Santa Fe, USA), one of the most prominent specialists on Reliability Theory, Professor of The George Washington University Nozer Singpurwalla was a host of the discussion during the dinner.The topic he chose was a bit provocative: "IS RELIABILITY THEORY STILL ALIVE ?" Even the question itself led to a furious reaction of the conference participant: "Yes! It is alive! It is flourishing!"

What is going now if even such a question was suggested to the audience by such a serious mathematician who dedicated all his talent to developing Reliability Theory?

It seems to me that Professor Singpurwalla is right asking such a question. Though an answer to this question is not so simple. Being in a position a "mammoth" (if not a dinosaur ☺) in Reliability Theory, I take a brevity to discuss this difficult question.

## Factors That Determined in the Past and Determine Now Reliability Theory

1. A theory always germinates in the depth of practical problems.

Let us recollect when the first boom of Reliability Theory happened. It was the Korean War time (1950-53). Military equipment of the both opposing sides developed in the years of the "Cold War" very intensively: Soviet and American hawks competed at armament race. Equipment became more and more sophisticated, more and more complex and – as a result – more and more unreliable. Both sides lose huge money due to unreliability, and of course Americans were the first who began to develop Reliability Theory: they always could count money better.

First, the US engineers paid more attention to quality control, reliability engineering and maintenance. Institute of Radio Engineers (IRE) and later Institute of Electrical and Electronics Engineers (IEEE) called annual Symposiums on Reliability and Quality Control (R&QC) and published Proceedings. At the beginning of 60s, a real tsunami of publication on reliability hit the engineering communities...

A little later (as usual!) activity in this area began in the former Soviet Union. Academician Axel Berg coined a phrase: "Reliability is the problem number №1 !'

Thus, there appeared the problem that had to be solved fast and efficiently.

2. Decreasing interest to Reliability Theory.

First reason is objective: equipmen noe is much more reliable than earlier. If vacuum lamps in electronic equipment in 50-60s had MTTF about at most hundreds hours, today's microchips that can perform much more complex operations have failure rate $10^{-8}$ 1/h and less.

It is clear that reliability problems moved to the system level rather than component level.

3. Oversaturation of the "scientific market".

A theory should always go ahead of needs of practice. Otherwise it will take a hand on tha pulse of a dead man ☺... However, one can say that modern reliability theory ran too far from practical engineering needs or even went to dead ends of "exotic" and practically useless mathematical exercises. Actually, practical reliability engineering has enough first class solution for today's problems. New "local" problems can be solved on the local levels.

Probably, for engineering companies, it is more effective way to solve current reliability problems id to invite specialists on a contract basis.

4. Beginning "theory for theory".

If you take a look at the first works on reliability of the end of 50s and of the beginning of 60s, you could see pure pragmatic nature of those works. Even "pure mathematicians" wrote for users rather than for themselves: their results were transparent and their applicability was evident. However, in the middle of 70s there appeared papers considering unrealistic models, math results began to be non-understandable with no common sense interpretation.

That situation led to definite discredit of Reliability Theory as a whole. This situation was expressed by one of leading specialist in reliability engineering: "The reliability Theory is for those who understand nothing in reliability. Those who understand reliability, they design and produce reliable equipment!"

(Unfortunately, such position led to a catastrophe with Soviet "Soyuz-1" when due to a failure at the cabin sealing three Soviet astronauts died during landing: Sputnik's designers forgot that relay schemes have two types of failures: false opening and false closing.)

Nevertheless, indeed, pragmatism of theoretical reliability works went down dramatically...

5. Aspects of "modern fashion" in technology.

Once I asked my old friend Robert Machol, who is known for his book "System Engineering", why did a new direction "Management Science" appear? Initially, it was Cybernetics, then Operations Research has been coined, and now we have Management Science… "You already answered on your own question: this is a problem of fashion changing! Who will pay for an old dress? It is assumed that new is better than old!" – answered Machol.

Of course, it was a joke though, as it said, any joke contents a *bit of joke*.

6. Moving a "center of gravity" of the problem.

At its first steps, Reliability Theory paid its main attention to problems od field data gathering methodology and data inference. In the modern theory the system analysis became the main topic. At the same time, giant technological systems like telecommunication, transportation, computer networks or oil and gas distributing systems need specific methods rather than general ones. Very often a solution for one particular type of the system is absolutely inapplicable for another. However, any specific solution is based on the fundamental results of common reliability theory.

Thus, as Marc Twain said, the hearsay about the death of reliability are premature, though the age of its flourish doubtlessly is behind...

## Reliability Works in the Former Soviet Union

In the end of 50s there appeared first publications on reliability, and in 1958 the First All-Union Conference on Reliability took place in Moscow.

Informal scientific groups began to form in Moscoe, Leningrad, Kiev and Riga...

**Moscow school of Reliability.**

First group was formed in Zhukovsky (B. Vasilyev, G. Druzhinin. M. Sinitsa)and one of the Military R&D Institute of Defense Ministry (V. Kuznetsov, I. Morozov, K. Tsvetaev).

At the same time at the Popov Society, a brilliant manager Jacov Sorin organized Reliability Chapter where the main role played R. Levin. Then in 1959 J. Sorin established the very first Reliability Department at one of the industrial institutes of the Military-IndustrialComplex of the former USSR.

From the very first days of the department existence, Academician Boris Gnedenko and Professors of the Moscow State University Alexander Solovyev and Yuri Belyaev collaborate with this department. A well known statistician – Jacov Shor from one of Military R&D Institutes joined them. Those scientists with J. Sorin and the first employee of the department Igor Ushakov became official consultants on reliability at the State Bureau on Standartization (Gosstandard) and later form the Scientific Counsil on Reliability.

In 1962 B. Gnedenko I J. Sorin established at the Moscow State University weekly Seminar on Reliability for engineers. It was a very popular event attended by tens of practical engineers. That Seminar was led by B. Gnedenko with help of A. Solovyev, Yu.Belyaev and I. Kovalenko.

Tandem "Sorin-Gnedenko" has been successfully existing about 25 years and has performed a huge organizational and educational work.

Approximately in a year, J. Sorin established Moscow Reliability Consulting Center, and as the Director of the Center appointed B. Gnedenko as a Scientific Lead of the organization and I. Ushakov as its Scientific Coordinator.

A number of Doctors of Sciences and Professors collaborated with the Center, among them A.Aristov, I. Aronov, Yu. Belyaev, B. Berdichevsky, E. Dzirkal, F. Fishbein, J. Shor, A. Solovyev, R. Ulinich, I. Ushakov, and others. They performed everyday's consulting for industrial engineers and twice a month there were tree 2-hour lectures. More than 50% of attendees were not from Moscow. They came from various arts of the former Soviet Union: Far East and Baltic Republics, Ukraine and Caucasus Republics.

In 1969 J.Sorin established the journal titled "Reliabiity and Quality Control" and became its first Editor, taking B. Gnedenko, J. Shor and I. Ushakov as his deputies.

Approximately at the same time, the Publishing House "Soviet Radio" (later "Radio and Telecommunication") established Editorial Council headed by B. Gnedenko. It began to publish series named "Library of ReliabilityEngineers". Books of the series played significant role in educating reliability engineers all over the former Soviet Union.

In the middle of 70s, a respectful academic journal "Technical Cybernetics" (translated and published in the USA as "Soviet Journal of Computer and System Sciences") established a special Section "Reliability Theory".

It is difficult to name all those who belong toe the Moscow reliability school, nevertheless I should mention A. Aristov, I. Aronov, V. Gadasin, Yu. Konyonkov, G. Kartashov, I. Pavlov, A. Rajkin, R. Sudakov, O. Tyoskin, V. Shper.

Talking about Moscow Reliability School, it is reasonable to mention two books that reflected many results in Reliability Theory.

First of all, it was an excellent book "Mathematical Methods in Reliability" by B. Gnedenko, Yu. Belyaev and A. Solovyev [ 1 ]. The book was translated into English [ 2 ]. Even now, 40 years after the publication, this book and the book by R. Barlow and F. Proschan book [ 3, 4 ] that was translated into Russian [ 5, 6 ], remain the best best monographs on the subject.

Secondly. It was "Handbook on Reliability" by B. Kozlov and I. Ushakov [ 7 ] that had several editions [ 8 – 9 ] and translations [ 10 – 14 ]. This handbook remainded many years a table book for reliability engineers.

**Leningrad Reliability School.**

In 1959 at one of Leningrad R&D Institutes of Shipbuilding Ministry has been established the first Reliability Department headed by I. Malikov. In the same year I. Malikov, A. Polovko, N. Romanov and P. Chukreev, who led the Leningrad Reliability School, published first Russian book "Fundamentals of Reliability Calculation" [ 15 ]. The book contained only 139 pages, but it was the first book where one could find systematic description of an elementary knowledge in reliability theory.

Soon in Leningrad A. Polovko founded Leningrad Reliability Center.

In 1964 A. Polovko published the very first monograph on Perliability Theory [16] that was the first Russian book on the subject translated into English [ 17 ].

Leningrad Reliability School gave several significant names: G. Cherkesov, L. Gorsky, I. Ryabinin, N. Sedyakin, I. Shubinsky and others.

**Kiev Reliability School.**

In Kiev Military Radio Engineering Academy flourished a group headed by N. Shishonok: L. Barvinsky, B. Kredentser, M. Lastovchenko, A. Perrote, V. Repkin, S. Senetsky. Under Shishonok's editorial leadership it was published "Fundamentals of Reliability Theory for Electronic Equipment" [ 18 ].

In parallel, at Kiev State University and later in Cybernetics Institute appears a very strong group consisted mostly of pupils of B. Gnedenko. This group dealt with general stochastic processes theory applied to queuing and reliability problems. In this group there were such outstanding scientists like Academicians I. Kovalenko and V. Korolyuk, and such specialists like V. Anisimov, V. Volkovich, T.Maryanovich, A Turvin, V. Zaslavsky and others.

**Riga Reliability School.**

Founder od Riga Reliability School was Kh. Kordonsky who was a Chair of Department at Riga Instute of Civil Aviation. His pupils – A. Andronov, I. Gertsbakh and Yu. Paramonov.

Probably this group was specifically practice oriented. In 1963 Kh. Kordonsky published his book [ 19 ], in which some reliability models were discussed, then in 1969 I. Gertsbakh published his book [ 21 ], that is, probably, the best book on maintenance problem.

Kh. Kordonsky, following his Moscow and Leningrad colleagues open a regular seminar on reliability theory for engineers.

Independently at the same time in the same area V. Leontiev and V. Levin have been working.

**Irkutsk Reliability School**.

Reliability problems in Siberia were related mostly to energy systems. Director of Siberian Energy Instutute Academician Yu. Rudenko led those researches gathering a group of young scientists (N. Voropai, G. Kolosok, L. Krivorutsky, V Zorkaltsev and other). For the work related to survivability analysis of All-Union Energy system, Yu. Rudenko and I. Ushakov were honored by prestigious Academy of Sciences' Krzhizhanovsky Prize. They published together the first book on energy systems reliability [ 22, 23 ].

Famous Rudenko's Seminars in Baikal Lake area attracted not only by exotic place… Among participants there were such specialists like E. Chervony, Yu. Guk, N. Manov, E. Stavrovsky, M. Sukharev, E. Farkhad-Zadeh, M. Cheltsov, M. Yastrebenetsky and other.

Of course, the list could be continued: Tashkent, Gorky, Kharkov, Minsk, Tbilisi, Erevan and Vladivostok should be mentioned here.

## Brief History of Development Reliability Theory in the Former Soviet Union

As already was mentioned, the first steps in Reliability Theory developing were done in the USA. However, Soviet statisticians and engineers bagan to work in that direction with a small delay.

This brief review does not target to be complete, though I believe that some analysis of theoretical ideas developed in the Soviet Reliability School should be done.

Interesting method of analysis of confidence estimates of system reliability based on non-failure tests of its components was suggested byR. Mirny and A. Solovyev [ 24 ]. Then some general results based on Monte Carlo simulation were obtained by Yu. Belyaev [ 25, 26 ]. Many new analytical results afterwards were obtained by I. Pavlov [ 27 – 29], R. Sudakov [ 30 ] and O. Tyoskin [ 31 ].

Many works were related to analysis of complex systems with degradation of the operational level (partial failures). Indeed, hardly a complex system might be characterized by simple binary criteria of type "yes-no" [ 32-34 ].

The profs of too limit theorem for stochastic point processes played significant role in further development of methods of analysis of repairable system.

First. Hungarian A. Renyi [ 35 ] proved theorem concerning asymptotical "sifting" of stochastic point process, and approximately at the same time G. Ososkov [36] proved theorem concerning asymptotical superposition of the processes of the same type. Afterwards Yu. Belyaev, B. Grigelionis and I. Pogozhev generalized those results. Their results permitted to develop convenient approximate practical methods for reliability analysis of vomplex repairable (renewable) systems [ 37 ].

B. Gnedenko [ 38, 39 ] was the first investigator of asymptotic methods of reliability analysis of repairable (renewable) systems I the beginning of 60-s. He considered a duplicated renewable system and proved that asymptotic distribution (under condition of "fast repair") of the system time to failure is exponential and does not depend on the distribution of the repair time. This work opened a new direction in Reliability Theory that was successfully developed, first of all, by I. [ 40 - 42 ] Kovalenko and A. Solovyev [ 43 - 46 ].

Interesting ideas of semi-Markov processes aggregation related to reliability problems were suggested by V. Korolyuk and A. Turbin [ 47 – 48 ], and afterwards these ideas were developed in a series of works [ 49 – 50 ]. Interesting applications to Reliability Theory contains in the works by V. Anisimov [51] and D. Silvestrov [ 52 ].

Methods of optimal redundancy were developed in [ 53 - 57 ]. Some results from these works were used for preparation of Military Standards.

Such important direction of Reliability Theory as accelerated testing appeared in the very beginning of activity of Soviet specialists on reliability. Here works by N. Sedyakin [58], I. Gertsbakh and Kh. Kordonsky, [59], G. Kartashov, A. Perrote and K. Tsvetaev [ 60 ] have to be mentioned first of all. Models od accelerated tests with time-dependent loading were considered by V. Bagdanavichus and M. Nikulin [ 61 ].

Concluding this brief review, it is necessary to mention an excellent book edited by B. Gnedenko [ 62 ], in which many results of Soviet School on Reliability Theory have been summed up.

* * *

Evidently, these brief notes could not mention everybody who made an input into Reliability Theory and its practical implementation. Moreover, such brief review almost always suffer from author's subjective viewpoint. Actually, writing such review is a very dangerous thing: the author can offend his friends and colleagues who appears out of the review…

The flow of publications in Reliability Theory is very intensive. A new generation of specialists in reliability can loose their orientation in these trouble waters of books and papers on the subject.

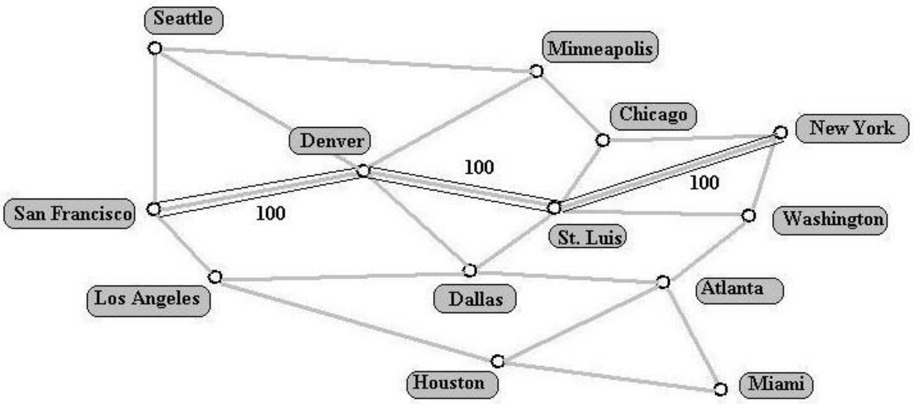
We have our Gnedenko Forum. Maybe it is reasonable to arrange rating of books on reliability?

Below I am presenting examples of some practical problems that I solved last years, working for several American companies.

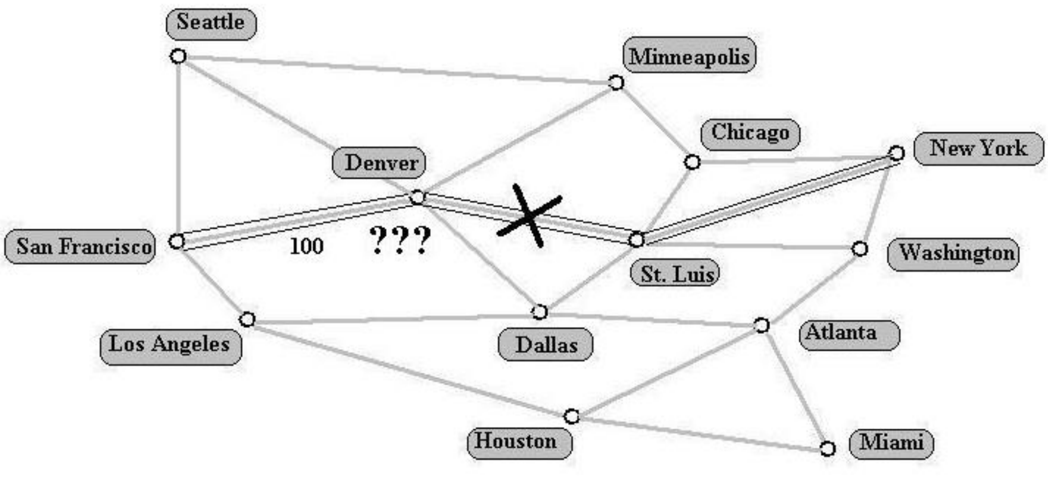
## Examples of Solution of Practical Problems

**Computer model of survivability analysis of the telecommunication network (for US company MCI)**

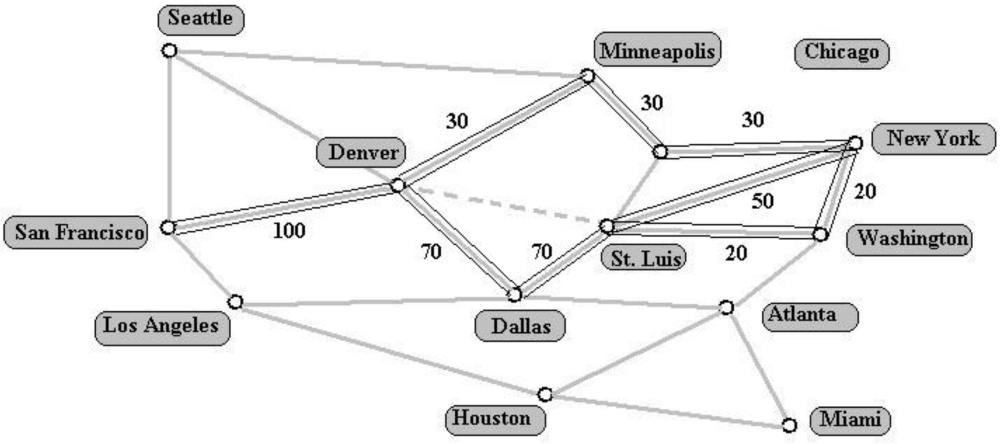
The problem of optimal allocation of traffic after catastrophic failure is considered. Matrix of traffic between various pairs of nods and capacity of trunks are taken into account. Let us assume that the traffic between San Francisco and New York iz such as presented in the figure below.




The model is working in interactive regime: a user would like to look at the network reaction on failure (or emergency turn off) of the trunk between Denver and St. Luis.



The model calculates new input data (loss of the trunk) and finds a new optimal traffic allocation between San Francisco and New York, taking into account minimum "harm" for other system users.



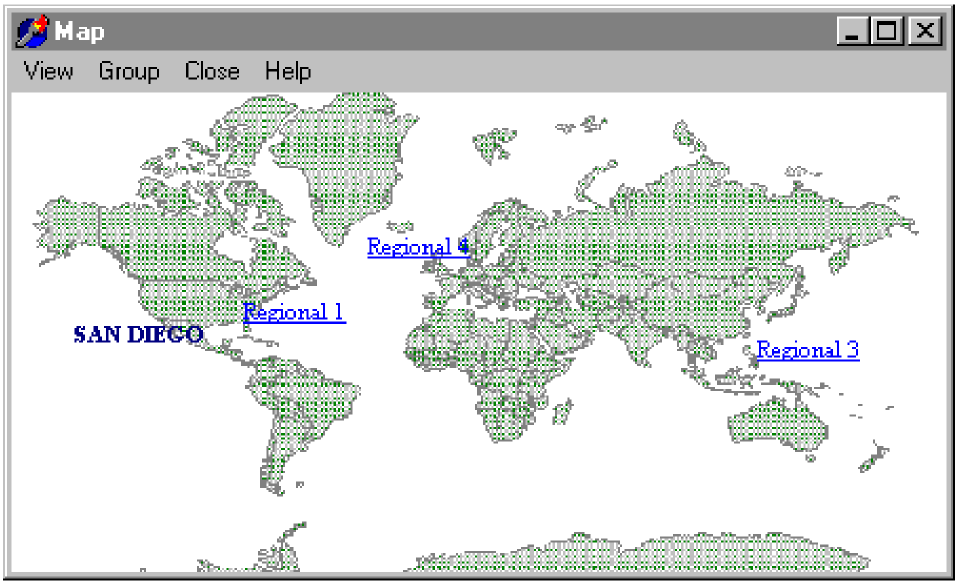

This computer model has been used for control of real telecommunication network.

**<u>Computer model for optimal allocation of spare parts for base stations of satellite telecommunication system GlobalStar</u>**


GlobalStar system uses low-orbit satellites that move around the Earth by spiral trajectories, covering practically al regions. It was planned to have about hundred ground base stations. Each such station might have its own configuration depending on the population density in the station zone, access to other communication systems, etc.

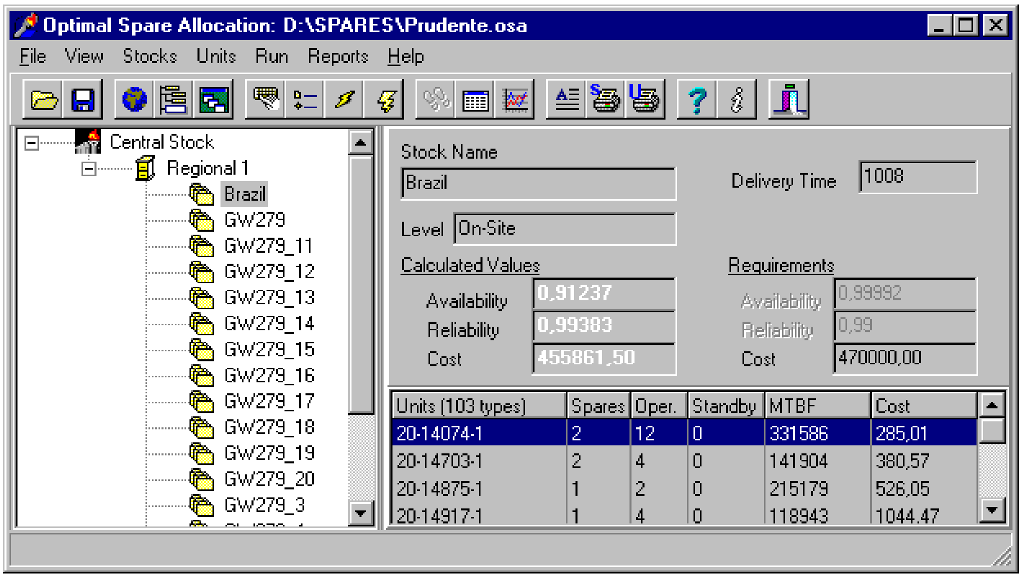
In a situation when each station might have an individual optimal allocation plan, the only possibility to solve the problem was designing of a computer model. Educated managers almost immediately understood that Neanderthal methods of type "5% of operating units, though not less than one" did not work.

It was also clear that spare supply from a single center is absolutely unreasonable. So, there were Central storage in San Diego (California) and three regional storages.

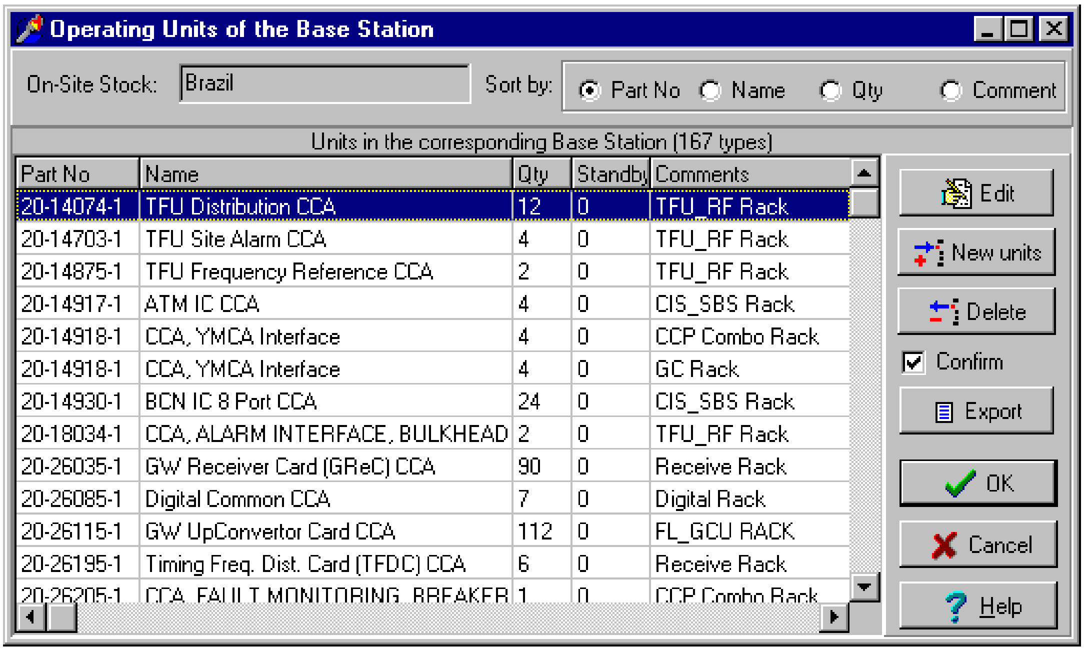



A computer model of optimal spare allocation allowed to get lists of spares for each individual base station taking into account capacity of the base station, the type of spares replenishment (periodical or by request), time of delivery and so on. Input data (failure rates of various units and its costs) were kept in a special database.

The user's window with the list of basestations within one of the regions is presented below.

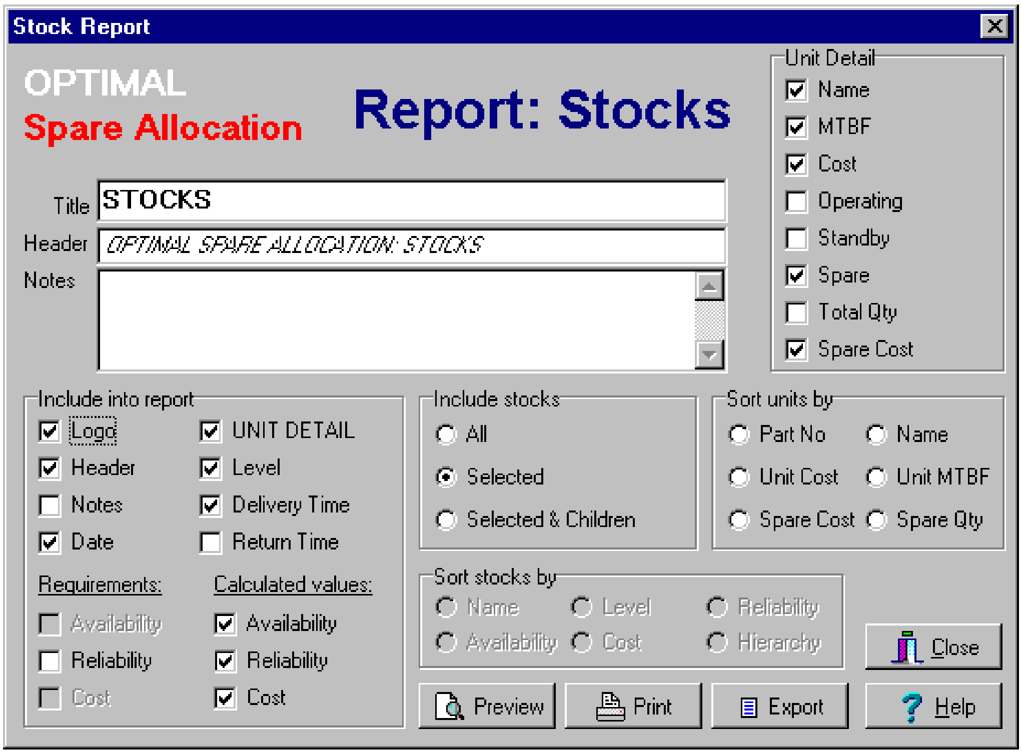



For each ground base station, the model kept all necessary input data for calculating optimal spare allocation.




Two problems can be solved: (1) Find optimal number of spare units of each type to warranty maximum base station availability under limited total expenses; and (2) Find optimal number of spare units of each type that delivered total expenses under condition that availability was not less than specified level.

After the computation, the report printing was available in the form defined by the user.



An example of the report is given below.

QUALCOMM

**OPTIMAL SPARE ALLOCATION**

***STOCKS***

*Stock:* Brazil *Level:* On-Site *Availability:* 0,912372575375
*Spare unit delivery time:* 1008 *Reliability:* 0,993832202067
*Cost:* 455861,50

*Unit data:*

| Part No | Name | MTBF | Cost | Spare | Spare Cost |
|---|---|---|---|---|---|
| 20-14074-1 | TFU Distribution CCA | 331586 | 285,01 | 2 | 570,02 |
| 20-14703-1 | TFU Site AlarmCCA | 141904 | 380,57 | 2 | 761,14 |
| 20-14875-1 | TFU Frequency Reference CCA | 215179 | 526,05 | 1 | 526,05 |
| 20-14917-1 | ATM IC CCA | 118943 | 1044,47 | 1 | 1044,47 |
| 20-14918-1 | CCA, YMCA Interface | 66667 | 92,42 | 3 | 277,26 |
| 20-14930-1 | BCN IC 8 Port CCA | 102364 | 609,74 | 3 | 1829,22 |
| 20-18034-1 | CCA, ALARM INTERFACE, BULKHEAD | 166667 | 178,44 | 1 | 178,44 |
| 20-26035-1 | GW Receiver Card (GReC) CCA | 78468 | 1301,53 | 6 | 7809,18 |
| 20-26085-1 | Digital Common CCA | 133333 | 788,14 | 2 | 1576,28 |

**<u>Finding size of maintenance zones, number of servicemen and location of the maintenance center within the zone for serving users of satellite telecommunication system</u>**

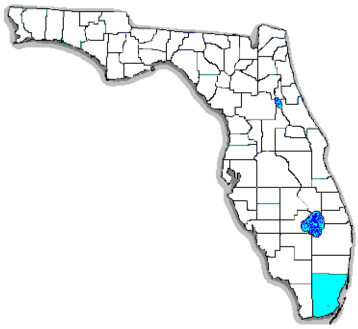
There were data of request rate obtained from a previous history of the maintenance system operation in different counties of Florida State (there are several tens of such counties)

| County | Number of requests | Area | Rate (number of requests per day) |
|---|---|---|---|
| Alachua | 8 | 902 | 0.148148 |
| Baker | 0 | 585 | 0 |
| Bay | 9 | 758 | 0.166667 |
| Bradford | 3 | 293 | 0.055556 |
| Brevard | 16 | 995 | 0.296296 |
| Broward | 70 | 1211 | 1.296296 |
| . . . | . . . | . . . | . . . |
| Wakulla | 3 | 601 | 0.055556 |
| Walton | 8 | 1066 | 0.148148 |

The designed computer model gave a possibility of interactive solution. Such method has been chosen because the problem had a lot of non-formalized factors. For instance, a maintenance center of the zone should be chosen at some town rather than from pure geometrical considerations.

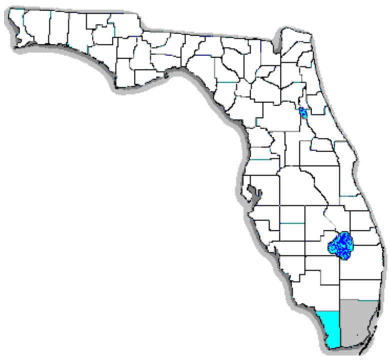
The designed algorithm based on directed enumeration with local step-by-step optimization. It was also taken into account an intuitive hypothesis that solution for, say, South Florida counties did not influence on the solution for Northern Florida counties.

The first county was chosen arbitrarily, though the maximum population density has been taken into account. Such county occurred to beDade.

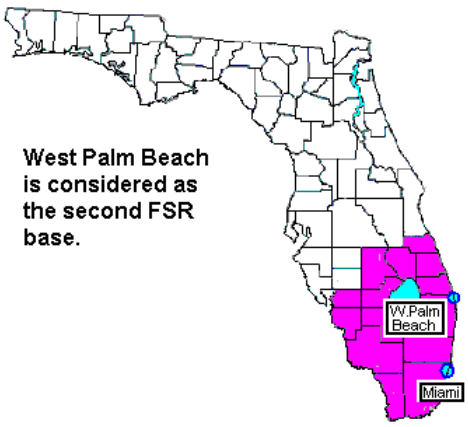


.After computing obtained maintenance parameters, it was clear that it is possible to add some neighbor county. Again informal hint for choosing the next county was that new two county should form a "compact area", i.e. this solution based on expert opinion. In this particular case the added county was

Monroe.



After multiple application of the described procedure, the first maintenance zone has been constructed.




Then in this zone one tried to split a single maintenance center into two (keeping the same total number of servicemen). It gave a possibility (again in interactive regime) to widen the maintenance zone.

After this first "macro step", the first maintenance zone became "frozen" and the same procedure is applied to find a next zone.

As the result of constructing new maintenance zones, only in Florida State alone estimated save was about $400,000 a year due to best zoning, best location of maintenance centers and decreasing the sraff.

## Conclusion

Reliability Theory is alive! However, it should be applied in a right direction. Probably, needs in pure theoretical researches is decreasing, nevertheless, there are many practical problems, which are waiting solutions.

Thus, since life is continuing, the need of solving practical problems in reliability and maintainability will exist always!

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# Object Oriented Commonalities in Universal Generating Function for Reliability and in C++.

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Sumantra Chakravarty[1]

## Abstract

The main idea of Universal Generating Function is exposed in reliability applications. Some commonalities in this approach and the C++ language are discussed.

**Keywords**: Universal Generating function (UGF), C++, reliability.

## Introduction

Usually, binary systems are considered in the reliability theory. However, this approach does not describe systems with several levels of performance sufficiently. Analysis of multi-state systems forms now a special branch of the reliability theory.

For analysis of such systems consisting of multi-state subsystems/elements, one can use the method of Universal Generating Functions (UGF), which is described below.

## 1. Generating function

One frequently uses an effective tool in probabilistic combinatorial analysis: the method of generating functions. For a distribution function of a discrete random variable $\xi$ such that $\Pr\{\xi = k\} = p_k$ for any natural *k*, the generating function has the form

$$\varphi(x) = \sum_k p_k x^k$$

Advantages of using a generating function are well established in this field, and we list a few of those:

(1) For many discrete distributions (e.g., binomial, geometrical, Poisson), there are compact forms of generating functions, which allows one to get analytical solutions quickly and easily.

(2) Moments of statistical distributions can be written in convenient forms. For example, the mathematical expectation of random variable $\xi$ can be found as

$$E\{\xi\} = \frac{\partial}{\partial x}\varphi(x)\bigg|_{x=1}.$$

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(3) If there are $n$ independent random variables $\xi_1$, $\xi_2$, ..., $\xi_n$ with the respective generating functions $\varphi_1(x)$, $\varphi_2(x)$,..., $\varphi_n(x)$ , then the following generation function can be written for the convolution of these distributions:

$$\varphi(x) = \prod_{j=1}^{n} \varphi_j(x).$$

where $\varphi_j(x) = \sum_k p_{jk} x^k$ , and $p_{jk}$ is the probability that $j$-th random variable takes value $k$.

## 2. Computer alorithm for calculation product of GF's

Let us present a generating function as a set of objects. Each object corresponds to a term in the generating function polynomial. It means that object is a pair of two values: the first is the coefficient, i.e. probability, p, and the second is the power of the argument, a, i.e. the corresponding random variable.

Consider a computational algorithm for calculation of the convolution of two distributions. One makes the following formal operations.

♦ Take two sets of objects: set { $(p_{11}, a_{11})$, $(p_{12}, a_{12})$,..., $(p_{1k}, a_{1k})$ } for generating function $\varphi_1(x)$, and set { $(p_{21}, a_{21})$, $(p_{22}, a_{22})$,..., $(p_{2m}, a_{2m})$ } for generating function $\varphi_2(x)$.

♦ Find all cross "interactions" of objects of the first set with all objects of the second set, using the following rule:

[Interacting objects: $(p_{1k}, a_{1k})$ and $(p_{2m}, a_{2m})$ ] →
[Resulting object: $(p_{1k} p_{2m}; a_{1k} + a_{2m})$ ].

♦ For all resulting objects with different $a_{1k_1}$ for object-1 and $a_{2m_2}$ for object-2, but such that $a_{1k_1} + a_{2m_2} = a$, one forms a new final resulting object: $(\sum p_{1k_1} p_{2m_2};\ a)$. The total set of such final resulting objects gives us the needed solution: from here we can get probabilities for any $a$.

## 3. Universal generating function

We have described a formalized procedure on sets of objects interaction coresponding to product of polynomials. But in practice, we meet a number of situations when this operation is not enough. Consider the following simple examples.

Example 1. Assume that there is a series connection of two (statistically independent) capacitors (Fig. 1).

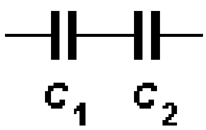


**Fig. 1.** Series connection of two capacitors.

Assume that c1 and c2 are random with discrete distributions: p1k=Pr{c1=k} and p2j=Pr{c2=j}. One is interested in distribution of total capacity. It is impossible to find the solution with the help of a common generating function. However, there is a possibility to use formal algorithm, described above with the use of corresponding operations over the elements of the objects.

The following procedure can be suggested:

♦ Take two sets of objects, $S_1$ and $S_2$:

$$S_1=\{(p_{11},c_{11}),(p_{12},c_{12}),\ldots,(p_{1k},c_{1k})\}$$

and

$$S_2=\{(p_{21},c_{11}),(p_{22},c_{22}),\ldots,(p_{2m},c_{2m})\},$$

where $k$ is the number of discrete values of the first capacitor, and $m$ is the same for the second one. Here the first element of the object is the probability and the second element is the respective capacity.

♦ Find all cross "interactions", Ω, of objects of set $S_1$ with all objects of set $S_2$, using the following rule:

$$\Omega\{(p_{1i},c_{1i}),(p_{2j},c_{2j})\}=(p_{ij}^*;c_{ij}^*).$$

Here $p_{ij}^*$ is the resulting probability calculated in accordance with the multiplication rule (under assumption of independence) as

$$p_{ij}^*=\Omega_{(p)}\{p_{1i},p_{2j}\}=p_{1i}p_{2j},$$

where $\Omega_{(p)}$ is the rule of interaction of parameters $p$, which in this particular case is multiplication.

Value of $c_{ij}^*$ is the resulting capacity calculated in accordance with the harmonic sum rule for capacities:

$$c_{ij}^*=\Omega_{(c)}\{c_{1i},c_{2j}\}=\left(c_{1i}^{-1}+c_{2j}^{-1}\right)^{-1},$$

where $\Omega_{(c)}$ is the rule of interaction of parameters $c$.

♦ Assume that in result we obtain all $R=km$ possible resulting objects of kind $(p^*;c^*)$. Let us order all these resulting pairs in increase of value of $c^*$: $(p_1^*;c_1^*)$, ..., $(p_R^*;c_R^*)$. For some resulting pairs with numbers, say, $i$, $i+1$,… , $i+j$ values of $c^*$ can be the same and equal some $C$. We converge such objects into a single aggregated object with parameters: $(\sum_{i\le s\le i+j} p_s^*;C)$. The total set of such final resulting objects gives us the needed solution.

The procedure can be easily expanded on a series connection of several independent capacitors.

$$\Omega_{(p)}^{SER}\{p_{1i},\ p_{2j},\ \ldots,\ p_{nr}\}=p_{1i}\cdot p_{2j}\cdot\ldots\cdot p_{nr},$$

and

$$\Omega_{(c)}^{SER}\{c_{1i},\ c_{2j},\ \ldots,c_{nr}\}=\left[c_{1i}^{-1}+c_{2j}^{-1}+\ldots+c_{nr}^{-1}\right]^{-1}.$$

Example 2. Pipeline consists of *n* series sections (pipes). Section *j* is characterized by random capacity, for which each value $v$ is realized with some probability $p$. In this case,

$$\Omega_{(p)}^{PAR}\{p_{1i},\ p_{2j},\ ...,\ p_{nr}\} = p_{1i}\cdot p_{2j}\cdot ...\cdot p_{nr},$$

and

$$\Omega_{(c)}^{SER}\{v_{1i}, v_{2j}, ..., v_{nr}\} = \min\{v_{1i},\ v_{2j},\ ...,\ v_{nr}\},$$

Example 3. One measures a sum of values, each summand of which is random. With probability $p_{js}$ value *j* is measured with standard deviation (STD) equal to $\sigma_{js}$. In this case, using notation similar to above, one has:

$$\Omega_{(p)}^{PAR}\{p_{1i},\ p_{2j},\ ...,\ p_{nr}\} = p_{1i}\cdot p_{2j}\cdot ...\cdot p_{nr},$$

and

$$\Omega_{(c)}\{\sigma_{1i},\ \sigma_{2j},\ ..., \sigma_{nk_n}\} = \sqrt{\sigma_{1i}^2 + \sigma_{2j}^2 + ... + \sigma_{nr}^2}\ .$$

Examples can be continued and not necessarily with probabilistic parameters.

## 4. Formal description of the Method of Universal Generating Functions

After these simple examples, let us begin with formal description of the Method of Universal Generating Function (UGF[2]). For a more vivid presentation, let us use special terminology to distinguish the UGF from the common generation function. This will relieve us from using traditional terms in a new sense, which may lead to some confusion. Moreover, we hope that this new terminology can help us, in a mnemonic sense, to remember and perhaps even to explain some operations.

In the ancient Roman army, a cohort (C) was the main combat unit. Each cohort consisted of maniples (M), which were independent and sometimes specialized combat units with several soldiers of different profiles. Several cohorts composed a legion (L). The use of this essentially military terminology appears to be convenient in this essentially peaceful mathematical application. A legion is close by its sense to a generating function, a cohort is close to a term of the generating function written in the form of expanded polynomial, and a maniple is close to a parameter of each term.

Starting with polynomial multiplication, in our approach, we will consider less restrictive operations (not only multiplication of terms) and more general parameters. For instance, multiplication of polynomials assumes getting products of coefficients and summation of powers. In our case, we will expand on such restrictive limits on operations.

Let's denote legion j by Lj. This legion includes vj different cohorts, Cjk:

$$L_j = \left(C_{j1}, C_{j2}, ..., C_{jv_j}\right).$$

The number of cohorts within different legions might be different. However, in our approach, maniples, which consist of a cohort, must be similar by its structure.

Each cohort $C_{jk}$ is composed of some maniples, $M$, each of which represents different parameters, special characteristics, and auxiliary attributes. Each cohort consists of <u>the same set</u> of maniples:

$$C_{jk} = \left(M_{jk}^{(1)}, M_{jk}^{(2)}, ..., M_{jk}^{(s)}\right).$$

[2] UGF might be also read as Ushakov's Generating Function ☺.

To make description of the method more transparent, let us start with the examples of two legions, $L_1$ and $L_2$: each of which consists of the following cohorts, $L_1=(C_{12},C_{12},C_{13})$ and $L_2=(C_{21},C_{22})$, and each cohort $C_{jk}$ includes two maniples $M_{jk}^{(1)}$ and $M_{jk}^{(2)}$,i.e. $C_{jk}=(M_{jk}^{(1)}, M_{jk}^{(2)})$. Denote the operation of legion interaction by $\Omega_L$. This operator is used to obtain the resulting legion $L_{RES}$. In this simple case, one can write:

$$L_{RES} = \Omega_L \{L_1, L_2\}. \tag{1}$$

This interaction of legions produces six pairs of interactions between different cohorts, which generate the following resulting cohorts:

$$C_{RES-1} = \Omega_C \{C_{11}, C_{21}\}, C_{RES-2} = \Omega_C \{C_{11}, C_{22}\},$$

$$C_{RES-3} = \Omega_C \{C_{12}, C_{21}\}, C_{RES-4} = \Omega_C \{C_{12}, C_{22}\},$$

$$C_{RES-5} = \Omega_C \{C_{13}, C_{21}\}, C_{RES-6} = \Omega_C \{C_{13}, C_{22}\}.$$

Here $\Omega_C \{\bullet\}$ denotes the interaction of cohorts.

Interaction of cohorts consists of interaction between its costituent maniples. All cohorts contain maniples of the same types though with individual values of parameters. Let us take, for instance, resulting cohort $C_{RES-5}$, which is obtained as interaction of cohorts $C_{13}$ and $C_{21}$. In turn, interaction of these particular cohorts consists in interaction of their corresponding maniples:

$$M_{RES-5}^{(1)} = \Omega_M^{(1)} \{M_{13}^{(1)}, M_{21}^{(1)}\}$$

$$M_{RES-5}^{(2)} = \Omega_M^{(2)} \{M_{13}^{(2)}, M_{21}^{(2)}\}$$

The rules of interaction between maniples of different types, i.e. $\Omega_M^{(1)} \{M_{1i}^{(1)}, M_{2j}^{(1)}\}$ and $\Omega_M^{(2)} \{M_{1i}^{(2)}, M_{2j}^{(2)}\}$ are (or might be) different.

Interaction of $n$ legions can be written as:

$$L = \Omega_L (L_1, L_2, ..., L_n).$$

Operator $\Omega_L$ denotes a kind of "$n$-dimensional Cartesian product" of legions and special final "reformatting" of the resulting cohorts (like converging polynomial terms with the equal power for a common generating function). Since each legion $j$ consists of $v_j$ cohort, the total number of resulting cohorts in the final legion (after all legion interaction) is equal to

$$v = \prod_{1 \le j \le n} v_j .$$

Number $v$ corresponds to the total number of cohorts' interactions.

## 5. Implementing UGF philosophy in computer language C++

We would like use the UGF (Universal Generating Function) philosophy in an analysis tool and perform reliability calculations for real-world systems. Because we are talking about an (reliability) engineering discipline, all philosophies present the need to be converted into numerical results and predictions. Thus, the UGF philosophy begs an implementation! The implementation task is to identify objects (maniple, cohort, legion) and program all interactions between them. Unfortunately, we run into a combinatoric explosion of possible interactions for a sysem consisting of a large number of (atomic) units. Even moderm computers are not able to enumerate astronomically large (21000) number of interaction states in system consisting of 1000 binary atomic units. Fortunately, for a class of frequently occuring practical systems, the situation is not as hopeless as it may first appear. For a system to be useful in engineering, it may only fail very infrequently. In a highly reliable system, the failure probability of all atomic units much smaller that the system failure probability. This fact makes most of the interactions exceedingly rare and they can be systematically ignored in an approximation scheme that retains only the dominant contributions.

Let us proceed to find an approximate implementation of the UGF philosophy for highly relaible systems in a system simulator. It should be reasonably easy to identify an atomic unit in reliability theory as a maniple. Independence of the maniples corresponds to statistical independence of the atomic units. A cohort is defined to be a collection of maniples. The same definition holds in the context of reliability theory, where the collection is defined by a failure criterion. In a series system, each atomic unit is assumed to provide distinct and critical functionality. This maps on to the notion of specialized combat units. In a parallel system, all atomic units are statistically identical. This improves survival probability during operation, either in the military or in system reliability! Thus, we may identify a subsystem in reliability engineering as a cohort in UGF formalism.

Interactions between the objects are identified in the simulator by their natural reliability names. k-out-of-n combinations are of primary interest. But this class includes the two most frequently appearing reliability structures: series (n-out-of-n) and parallel (1-out-of-n). In fact, probability of failure of a parallel system is negligible (higher order in numerical smallness) with an additional assumption of high availability of the atomic units. Obviously a series system can be made up of distinct units providing separate functionality to the system.

As an illustration let us consider a system S of two subsystems A and B in series. Let A be atomic and B be composed of two atomic units X and Y in parallel. One possible C++ coding for this (simple) system is

*B*=*Parallel*(X,*Y*); S = *Series*(A,*B*);

Properties (MTBF, MTTR etc.) of all atomic units are specified at the start of analysis. Operations like Series and Parallel are C++ member functions for the instances of class “unit”. We will not specify unit composition rules in this work. Most of these rules can be found in standard textbooks on reliability engineering. Interested readers may find the remaining ones (involving switching time and PEI) in Chakravarty and Ushakov (2000, 2002).

It remains to identify the “legion”. The preceding paragraphs almost suggest that a legion be identified with the entire system in reliability theory, where the system is further assumed to be represented by its generating function. We would like to note that that this analogy cannot be taken literally sometimes. It is common for a real world reliability system to have deeper hierarchies (e.g., system, equipment shelves, equipment racks, electornic cards) like modern day militaries. In such an elaborate system, we still identify the atomic units as maniples. At the other end, we identify the entire system as a “legion”! All intermediate stages in the hierarchy are considered generalized "cohorts".

In Chakravarty and Ushakov (2000) implementation, any subsystem can be composed from other subsystems at the next lower level of hierarchy (or atomic units which are always at the lowest level). A newly formed subsystem provides an effective reliability description of all units that compose this subsystem. This composition can be continued indefinitely to obtain an effectiveness measure for the entire system. They have shown that this can be recast as an approximation from a system generating function when all atomic units satisfy binary failure criteria (on/off) they are statistically independent, the system itself is highly reliable and reliability design of the system consists of hierarchical blocks.

## 6. Reliability analysis of GlobalstarTM Gateways

Globalstar is a low-earth-orbit (LEO) based telephony system with global coverage. The gateways make its ground segment that connect to the orbiting satellites. The gateways are cpmlex systems with more than a thousand components (e.g., electronic cards). Ushakov (1998), Chakravarty and Ushakov (2002) used the UGF approach for the reliability (performance) analysis of GlobalstarTM gateways (fixed ground segment of a low earth orbit satellite communications system). Given the prominence of object oriented abstractions and operations in Globalstar design, it should not be surprising that the reliability analysis naturally fits into the UGF philosophy. Further, these ideas can be naturally implemented in the computer using an object oriented language.

Because of the object oriented nature of system reliability design in Globalstar (interaction between objects like system, racks, shelves, cards are triggered by failure, switching of failed units and changing user demand), Ushakov (1998) proposed that a system reliability simulator should be coded in an object oriented computer language like C++. Later, Chakravarty and Ushakov (2002) implemented a simulator for the GlobalstarTM Gateway in C++.

In Chakravarty and Ushakov implementation for Globalstar, C++ objects are in one-to-one correspondence with reliability objects. An object is specified by mean time between failures (MTBF), mean time to repair/replace (MTTR) and an effectiveness weight (partial effectiveness index: PEI). By definition, PEI=1 for binary atomic units. All failure distributions are implicitly assumed to be Exponential. If failed units were to be automatically swapped, a switching time was also assigned by Chakravarty and Ushakov (2000). Even small switching time is important because it changes a parallel system "on paper" to a series system with small MTTR. This may have dramatic effect overall on system reliability.

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# Thurst of Life: Two Gnedenko's Visits to the United States

Igor Ushakov

It won't be an exaggeration to say that I never met a man with a stronger thirst for life, creating good around him, and being a courageous man who also faced life's test and terrible illness…

I was really lucky: I had been working with Boris Vladimirovich for many years shoulder-to-shoulder, traveled with him many business trips, spent many evenings with his hospitable family, he was my guest as well many times…

It was my great privilege: Gnedenko visited me twice in the United States when I was working at The George Washington University: in spring of 1991 and in summer of 1993. I will try to present a "photo report" of these events using only few words for comments.

## *USA-91*

Just before B.V.'s visit I was appointed to an open-heart surgery. I begged my surgeon to postpone the surgery for two days because I had to meet my teacher at airport who flew from Moscow. My surgeon agreed with me that I will survive extra couple days without an artificial valve...

Below: we met at the Washington's Dallas Airport. As you can see B.V. – as usual, ~~was~~ strong and smiling: nothing showed that he was already very sick… On his right – his son, Dimitri.

In an hour, we were back at our place. Back then we lived in Arlington VA, which was close enough to The George Washington University.

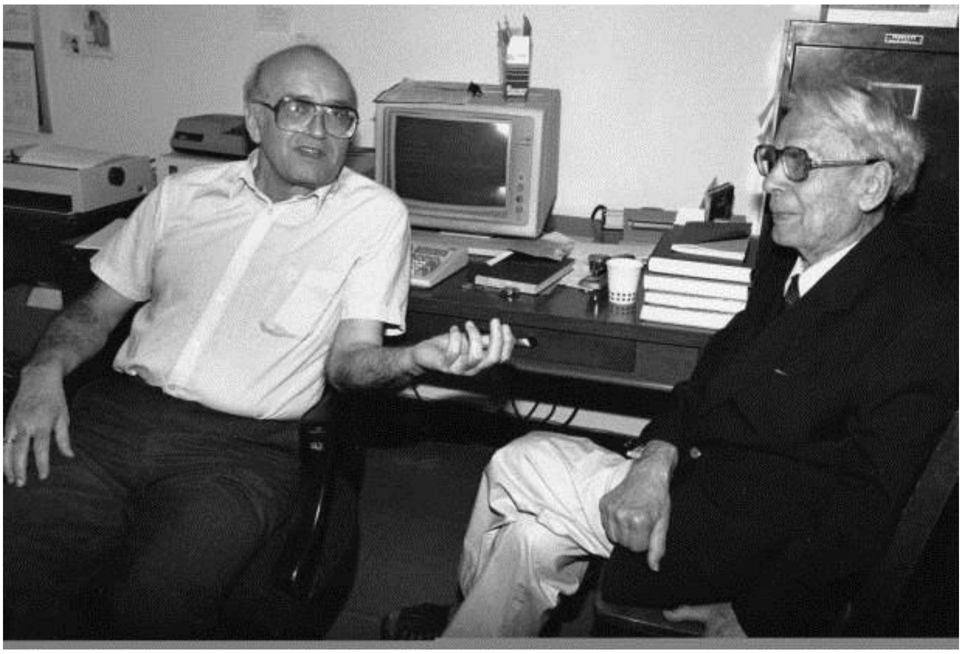
It seems the long flight from Moscow did not make B.V. tired.

Our first dinner: from left to right – Tatyana Ushakov, Dimitri Gnedenko and B.V

Another dinner: the table is full of everything (including, of course, a bottle of "Stoli").

B.V. with his unavoidable glass of water and his constant kind smile. He seemed not tired though the day was tough enough: lecturing, visits…

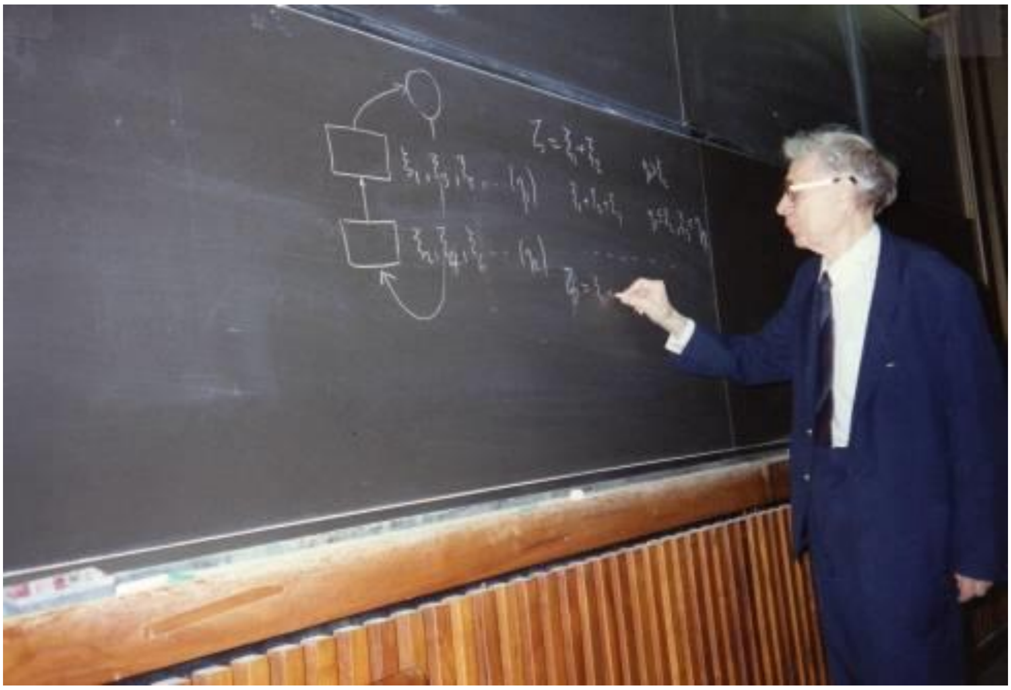
When local university "paparazzi" had known about B.V.'s visit to the Operations Research Department, they came immediately. This photo made for the University weekly newspaper at my office.

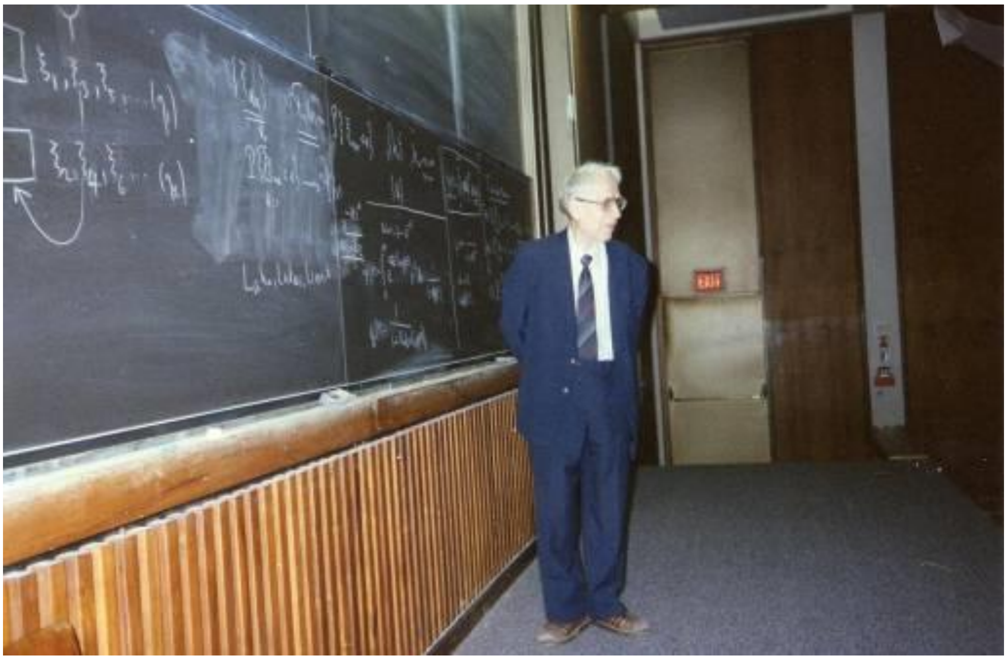


Here B.V. and I visited Professor James Falk who later was the editor of the book written by B.V. and myself *Probabilistic Reliability Engineering* (*John Wiley and Sons*, *New York*, 1995).

Three days later after B.V. arrival I was at the University Hospital for the surgery. During those days, B.V. visited Professor Richard Smith at the University of North Carolina. By the time he came back, it has been five days pass, I was back on my feet: American hospitals are fast! The next day, still with pain in my broken chest, I was at B.V.'s lecture as an interpreter...

As usual, B.V. was lecturing tremendously. Of course, he did not need me as an interpreter, though I stayed at the podium hiding behind a lectern: I could not step down without someone's help. So I sat there very still, it was extremely painful to move…



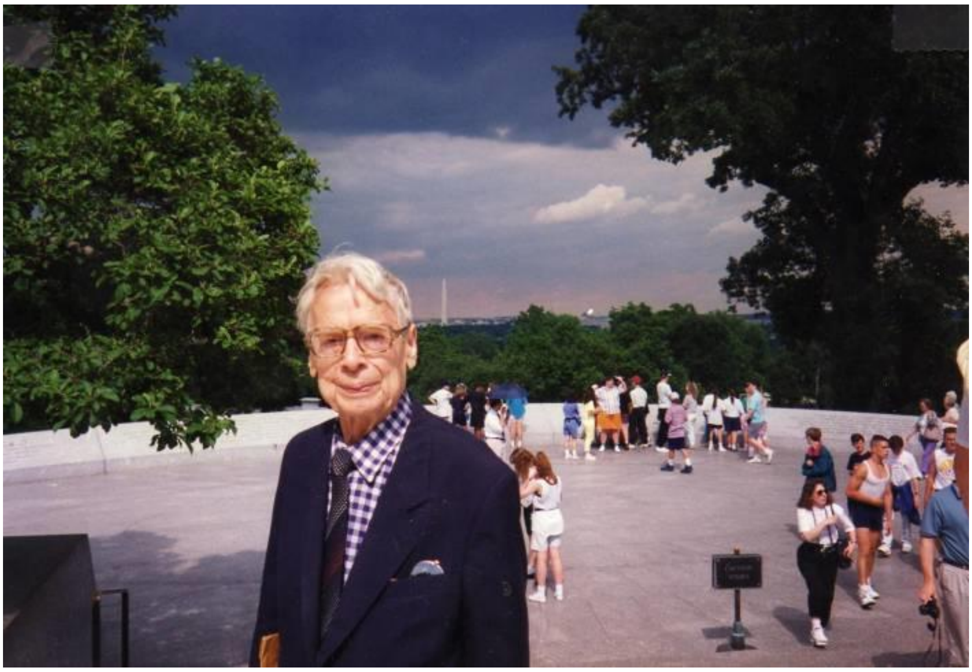


When B.V. finished the lecture and applause became silent, B.V. helped me rise to my feet. He took my elbow and we slowly went down the steps off the podium. At that moment I joked: "B.V., can you imagine what everybody is thinking, it is I who should be supporting you, not the other way around…" B.V. stopped in place and, shaking from laughing, said: "Igor, don't joke like this! I am afraid that we both might loose our balance and fall down…"

Once there was B.V.'s interview with Professors Nozer Singpurwalla and Richard Smith at which Dimitri and I were attending. (One could find that interview at No. 1 of our Journal.)

At one of our dinners, Professor Falk with his wife Jean were our guests. It seems me that then we asked Jim to be the editor of our book.

B.V. and myself spent a lot of time walking in Washington, D.C. He was very attentive to my conditions after the surgery, though my believe is that those frequent promenades made me physically stronger in a very short time. When we walked through Arlington cemetery, B.V. sadly joked: "Here we are, at the meeting with our future…"



We spoke about various things, though almost never on professional themes. B.V. was connoisseur in poetry, music, fine art… Once I remarked that I did not like anything created be Felix Mendelssohn but his Violin Concerto… B.V. did not point out my mistakes, but simply told me: "Igor, try to listen to Mendelssohn's music more. I'm sure that you will love him…" And that is exactly what happened! Now Mendelssohn's CD's are next to Rachmaninov, Beethoven and Mozart.

Next, I confessed that I did not like Pushkin[3]: "I understand that he is a great poet but emotionally I do not connect with his writing…" B.V. responded: "Understanding of Pushkin came with age…" Well… I guess I am still too young for it!

Our evenings were social. Washington mathematicians invited B.V. for dinners where he always a center of gravitation. In those international communities his knowledge of English, German and French was very useful...

[3] Alexander Pushkin is a great Russian poet.

## USA-91

In two years B.V. came to visit again. This time I was able to arrange his visit to one of the leading telecommunication company MCI. Though B.V.'s illness was progressing, nobody except us could tell anything was wrong. Being so much around him we began to notice that he got tired earlier.

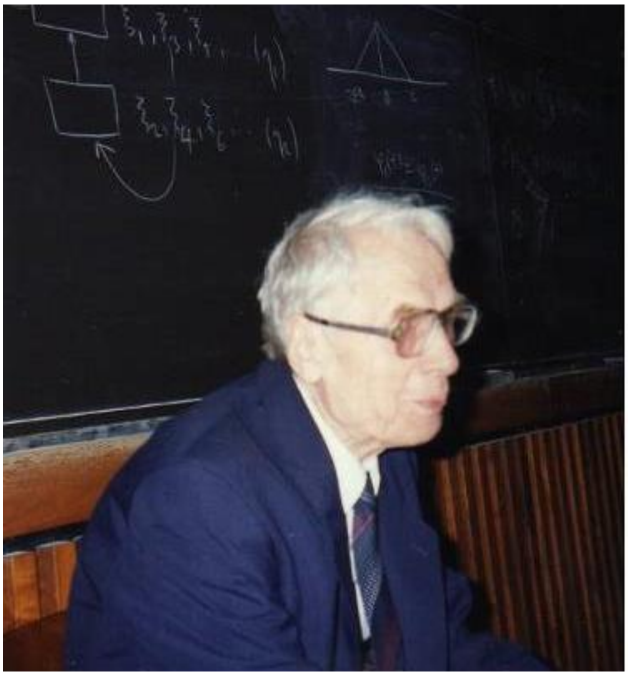
-

Nevertheless, he was always in the epicenter of any discussion, his eyes were always glistering with sincere interest to various problems. He compiled a dense plan of visits around the country.

His first visit was to MCI Headquarter near Dallas (TX). He was introduced to the audience by Chief Scientist Chris Hardy who first of all told how he convinced MCI top managers to invite Gnedenko: "I told ~~to~~ the President of the company that visit of Professor Gnedenko to us is equivalent to visit Norbert Wiener to Los Alamos Labs. It is a great honor for us!"

The photo below shows how Chris introduced B.V. to the MCI scientific community.

After the introduction, B.V. began ~~with~~ his lecture touching on some problems similar to the company interests. That time he lectured sitting down on the chair: it was right after a long flight from Washington.



The next day B.V. was accompanied by Dimitri and myself, took a plane to Boston where we were met by my former PhD student Eugene Litvak from the Harvard University. Photo below: E. Litvak, D. Gnedenko, B. Gnedenko and the author.


Since the audience was not "too mathematical", B.V. chose an intriguing topic: "Probability Theory from Medieval to Modern Times". It is time to point out that B.V. had always felt the audience and possessed an astonishing ability of adaptation and changing the style and the level of his presentation.

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# COLLOQUIUM SERIES

## Professor Boris V. Gnedenko

Chairman, Department of Probability Theory
Moscow University

will speak on

## "Probability Theory and Mathematical Statistics from Medieval to Modern Times"

Monday, December 13, 1993
4:00 - 5:00 p.m.
Kresge, Room G-3

Coffee, tea and cookies
3:30 - 4:00 p.m.
Outside Kresge, Room G-3

**ABOUT THE SPEAKER:** Professor Gnedenko is one of the greatest living probabilists. His contributions to probability theory and statistics have ensured his place in the history of Mathematics. His results in limit distributions of ordered statistics and summation of random variables are classical. He was one of the creators of Queuing Theory and Reliability Theory. Professor Gnedenko was a pupil, colleague, co-author and a personal friend of Professor Andrey Kolmogorov, one of the creators of Modern Probability Theory. Among their joint publications is the well known book *Limit Distributions for Sums of Random Variables*.

Professor Gnedenko wrote a comprehensive and intriguing historical review which was published in the last edition of his famous *The Theory of Probability*. This textbook has been standard for many generations of mathematicians.

Professor Gnedenko is an Honorary Member of the American Statistical Society and the Royal Statistical Society as well as an Honorary Professor at the University of Berlin and the University of Athens.

**This talk is co-sponsored by SOTAS Inc., Rockville, Maryland.**

B.V. was at his best. I knew that history of mathematics was "his love" but never imagined that it was possible to tell about such "dry subject" so vividly!


Immediately after the lecture Dimitri measured B.V.'s blood pressure. He was an excellent "family doctor" who knows when and what medicine should be given to his father...

It seems to me that it was the last serious B.V.'s trip… Time was inexorable… The illness became out of control. Nevertheless, he continued to work, wrote several books simultaneously.

When I visited B.V. the last time in Moscow in the summer of 1995, he practically did not leave his chair in the dining room. I brought with me our book *Probabilistic Reliability Engineering* that has been published recently by John Wiley. Our second book *Statistical Reliability Engineering* in co-authorship with my pupil Igor Pavlov was published in 1999... Sadly Boris Vladimirovich already has passed away...

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