The FCFS-RQ system by Laslo Lacatos and its modifications

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Abstract

The A. is proud of his being a disciple and co-worker of the world-wide known scholar Boris Vladimirovich Gnedenko and of being a participant of his scientific school, especially in the scope of queueing and reliability. The attempt is made to outline the contribution of prominent Gnedenko's colleagues Professors M.A. Fedotkin, L.G. Afanasyeva and G.I. Falin to the theory and practice of transportation processes. In 1994, a talented Hungarian probabilist Laslo Lakatos invented a new class of queueing systems, FCFS RQ systems motivated by an aviation problem. Such models were generalized by the Author's disciple E.V. Koba. The A. makes a further step in the study of this problem considering a Lakatos type system with hyper-Erlangian inter-arrival and service times.

Keywords: FCFS-RQ system, Laslo Lacators, transportation processes

1. Introduction

My dear master, a world-wide known mathematician Boris Vladimirovich Gnedenko is a one of prominent founders of queueing theory. He was also interested to its applications to wide-ranged practical problems. Concentrating attention to transportation problems, I can confirm that he fruitfully contacted to experts in many branches of the transport as M.N. Zubkov (naval ports), V.A. Padnya (rail transport), E.Yu. Barzilovich (aviation), M.A. Fedotkin (highway traffic) etc etc.

In 1960 I have defended a Ph.D. Thesis conducted by B.V. Gnedenko in Kyiv. Very soon I moved to Moscow where I worked on reliability problems. From June, 1971 I returned to Kyiv to be engaged to the Institute of Cybernetics.

My brightest memories from the Moscow decade are in that I was a participant and a co-conductor (with A.D. Soloviev and Yu.K. Belyaev) in the Seminar "Queueing & Reliability Theory" organized by B.V. Gnedenko. It was a real scientific school: just 20 its participants became full Drs. of science. In 1979, a collective of realibility experts headed by B.V. Gnedenko was awarded a State Prise of the USSR. This collective included mathematicians as well as leading engineering experts, among them two admirals: N.A. Severtsev and I.A.Ryabinin.

I should like to give a very short account on results of the three talented participants of the Gnedenko's Seminar who highly contributed the theory as well as applied areas. They are: Mikhail A. Fedotkin, Larisa G. Afanasyeva, Gennady I. Falin.

2. Investigation of conflicting transportation flows

A problem of control of transportation flows is very difficult, especially in modern metropolises. A considerable contribution to the solution of this problem was done by the Nizhny Novgorod mathematical school led by a talented mathematician M.A. Fedotkin [1-8]. A number of models describing the interaction of vehicles queueing by a crossing, were elaborated by this school. An iterative-majority method was invented basing on complex Markov chains. In papers [9-10] by M.A. Fedotkin and N.Yu. Kuznetsov an algorithm of the control of flows was presented, considerably (up to 2 to 3 times) shortening mean queueing time of vehicles.

3. Investigation of retrial queueing systems

The theory of retrial queueing systems (RQ systems) became an important branch of queueing theory. I would confine myself with outlying the contribution of two distinguished scientists, professors of M.V. Lomonosov Moscow State University Larisa G. Afanasyeva (L.A.), and Gennady I. Falin (G.F.). Both of them belong to Gnedenko's probability school.

L.A. has studied several classes of queueing systems with different queueing disciplines and probabilistic models of the input and service time [11-16]. A special attention was paid to RQ systems. L.A. fruitfully associates with prominent engineering experts in the aviation service. She studies mathematical RQ models of the airport service using analytical as well as Monte Carlo models. In particular she studied the stability conditions of some RQ systems.

G.F. is a much fruitful author. Among his publications [17-24] and many others there are monographs, surveys, studies of many classes of queueing systems. The most of his papers are connected to the analysis of RQ systems. It must be noted that G.F. is highly skilled in analytical methods. So he applies a diffusion approximation of the queueing process, see [18].

4. FCFS retrial queueing systems

An FCFS RQ system was invented by a Hungarian mathematician Laslo Lakatos [25] following the order of a Croatian aviation engineer Vaclav Ceric. The queueing system by Lacatos can be described in the following way.

The flow of aircrafts is Poissonian, with a parameter λ. The service time of an aircraft is distributed exponentially, with a parameter μ. Let t_n be the enter time of n th customer, W_n be its waiting time whereas Y_n be its service time. Then $W_n = K_n T$, where a cardinal number $K_n = \min\left\{m : mT \geq W_{n-1} + Y_{n-1} - \left(t_n - t_{n-1}\right)\right\}$. So the service discipline is FCFS and a constant T is a so-called orbit time.

The $\left(K_n\right)$ is an homogeneous Markov chain with states $0,1,2,\ldots$ As Lakatos proved, this chain is ergodic iff

$$\frac{\lambda}{\mu} < \frac{e^{-\lambda T}\left(1-e^{-\mu T}\right)}{1-e^{-\lambda T}}.$$

See also [26-37]. For other papers on this problem see [38-44].

My disciple Mrs. Elena V. Koba (E.K.) generalized this result to FCFS system with a recurrent flow and a generally distributed service time Y_n. Besides orbit times T_n can be considered as i.i.d.r.v.[45]. See also [46-48].

In a special case, where $T_n = T = const$, the ergodicity condition can be presented in the following way.

Let $F(x)$ be the d.f. of $Y - X$ where Y is a service time and X is an inter-arrival time. Then the ergodicity condition can be expressed as $\left\{\mathbf{E}N^{+} - \mathbf{E}N^{-} < 0\right\}$ where $\mathbf{E}N^{+} = \sum_{k=0}^{\infty}\left(1 - F\left(kT\right)\right)$,

$$\mathbf{E}N^{-} = \sum_{k=1}^{\infty} F\left(-kT\right).$$

5. Further generalization

Consider an FCFS system with a constant orbit time T which differs from the original Lakatos [25] in the following points. The inter-arrival time X possesses the p.d.f.

$$a\left(x\right) = \frac{1}{\left(l-1\right)!}\lambda^{l} x^{l-1} e^{-\lambda x}, \quad x > 0,$$

and service time Y possesses the p.d.f.

$$b\left(x\right) = \frac{1}{\left(m-1\right)!}\mu^{m} x^{m-1} e^{-\mu x}, \quad x > 0.$$

Thus the corresponding complimentary d.f. are given by Eqs

$$\bar{A}(x) = \sum_{i=0}^{l-1} \frac{1}{i!}\lambda^{i} x^{i-1} e^{-\lambda x}, \ x \ge 0;$$

$$\bar{B}(x) = \sum_{i=0}^{m-1} \frac{1}{i!}\mu^{i} x^{i-1} e^{-\mu x}, \ x \ge 0.$$

The d.f. of $Y - X$ can be obtained from Eqs

$$\mathbf{P}\left\{Y - X > x\right\} = \int_{0}^{\infty} a\left(t\right)\bar{B}\left(t + x\right)dt, \quad x \ge 0;$$

$$\mathbf{P}\left\{X - Y > x\right\} = \int_{0}^{\infty} b\left(t\right)\bar{A}\left(t + x\right)dt.$$

After cumbersome algebra one obtains the expressions

$$\mathbf{P}\left\{Y - X > x\right\} = e^{-\mu x}\sum_{j=0}^{m-1}\left(\mu x\right)^{j} g_{j}, \quad x \ge 0;$$

$$\mathbf{P}\left\{X - Y > x\right\} = e^{-\lambda x}\sum_{j=0}^{l-1}\left(\lambda x\right)^{j} g'_{j}, \quad x \ge 0,$$

where

$$g_{j} = \frac{1}{j!}\left(\frac{\lambda}{\lambda+\mu}\right)^{l}\sum_{i=0}^{m-j-1}\left(\frac{\mu}{\lambda+\mu}\right)^{i} C_{l+i-1}^{i};$$

$$g'_{j} = \frac{1}{j!}\left(\frac{\mu}{\lambda+\mu}\right)^{m}\sum_{i=0}^{l-j-1}\left(\frac{\lambda}{\lambda+\mu}\right)^{i} C_{m+i-1}^{i}.$$

Hence

$$\mathbf{E}N^{+} = \sum_{k=0}^{\infty}\mathbf{P}\left\{Y - X > kT\right\} = \sum_{j=0}^{l-1}\mu^{j} g_{j}\sum_{k=0}^{\infty}\left(kT\right)^{j} e^{-\mu kT},$$

$$\mathbf{E}N^{-} = \sum_{k=1}^{\infty}\mathbf{P}\left\{X - Y > kT\right\} = \sum_{j=0}^{m-1}\lambda^{j} g'_{j}\sum_{k=1}^{\infty}\left(kT\right)^{j} e^{-\lambda kT}.$$

The Markov chain $\left(W_{n}\right)$ of waiting times is ergodic iff

$$\mathbf{E}N^{+} < \mathbf{E}N^{-}.$$

It is possible to avoid infinite series while computing $\mathbf{E}N^{\pm}$. So, denoting $\lambda T = z$ or $\mu T = z$, we start with an identity

$$\sum_{k=0}^{\infty} e^{-kz} = \frac{1}{1 - e^{-z}}.$$

Applying the derivation one obtains the identity

$$\sum_{k=0}^{\infty} ke^{-kz} = \sum_{k=0}^{\infty}\left(-\frac{d}{dz}e^{-kz}\right) = -\frac{d}{dz}\frac{1}{1-e^{-z}}$$

and generally

$$\sum_{k=0}^{\infty} k^j e^{-kz} = \left(-1\right)^j \frac{d^j}{\left(dz\right)^j}\frac{1}{1-e^{-z}}$$

for any natural number j.

A further generalization consists of the following. We shall consider a Lakatos type FCFS RQ system with inter-arrival p.d.f. $a\left(x\right)$ and service time p.d.f. $b\left(x\right)$ the both being mixtures of Erlangian p.d.f., so that

$$a\left(x\right) = \sum_{p=1}^{P} c_p e\left(l_p, \lambda_p; x\right),$$

$$b\left(x\right) = \sum_{q=1}^{Q} d_q e\left(m_q, \mu_q; x\right)$$

where the e s are Erlangian p.d.f. with form parameters l_p, m_q and scale parameters λ_p, μ_q. We also assume that $\left(X_n\right)$ and $\left(Y_n\right)$ are i.i.d.r.v. As for c_p and d_q, they may posses any sign, provided $a\left(x\right)$, $b\left(x\right)$ are nonnegative.

One can observe that the bilinearity property holds:

$$\mathbf{E}N^{\pm}\left(a,b\right) = \sum_{p,q} c_p d_q \mathbf{E}N^{\pm}\left(e\left(l_p,\lambda_p\right), e\left(m_q,\mu_q\right)\right)$$

where the $N^{\pm}$ in the RHS of this equation relate to the $p-$ and $q-$components of the mixtures of $a\left(x\right)$ and $b(x)$.

Evidently the bilinearity implies the property that $\mathbf{E}N^{\pm}\left(a,b\right)$ and hence the associated ergodicity condition, can be expressed by elementary functions, avoiding infinite series.

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