The problem of large deviations. Comparison of the classical and alternative representations, p.2

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Abstract

This work is a continuation of the previous article (see [1]). Therefore, I note only that here are considered representations of large deviations for 5 other distributions of the summands of the mentioned normalized sums. In addition, if [1] explains what views are compared and how to get them, here the introduction clarifies the main results of analysis, i.e. whether it is difficult to get them and how they are practically useful.

Keywords: classical and alternative versions of representations of large deviations, the deviation function and its "analogue".

1 Introduction

As we have noted in the first article [1] on this topic, the increased interest in the problem of large deviations in the last 20 years actually has stimulated our desire to attempt to compare the representations of the large deviations from the title. Moreover, Zhulenev S. V. is the author of an alternative approach. In the same article, variants of the large deviations representations from [2] and [3], which were offered for comparison, were described in sufficient detail. In addition, there it was decided to compare the representations of two types for 10 different distributions of the terms of the normalized sums of random variables. More precisely, in the [1], we have compared the representations of two types in which the terms had the following 5 distributions: geometric, Poison, exponential, chi-squared and triangle. In this paper, we compare the representations for the other 5 distributions: normal, binomial, uniform, Bernoulli (0,1) and gamma.

The results of the comparison are given in the conclusion, but they are the simplest, because the process of obtaining them turned out to be very laborious. The only serious comment is the following. For 2 of 10 considered distribution we could not obtain a classical representation, but all 10 alternative representations were obtained. The cause of this is the need to always solve the equations for λ to find the function of deviations, which for uniform $(1.)$ and triangle $(2.)$ distributions look like:

$$1.\, g\left\{\frac{e^{\lambda g}+e^{-\lambda g}}{e^{\lambda g}-e^{-\lambda g}}\right\}-\frac{1}{\lambda}=\alpha,\ 2.\, \frac{e^{2\lambda\gamma}-e^{-2\lambda\gamma}}{4\lambda\gamma}-\left(\frac{e^{\lambda\gamma}-e^{-\lambda\gamma}}{2\lambda\gamma}\right)^2=\frac{\alpha\lambda}{2}.$$

So instead of an analytical solution we will have to look for an approximate.

Firstly we present the results of calculations of the basic elements of the sought representations. Then we observe the shape of the obtained representations.In conclusion, we note our attitude to the main results

2 Results

1. Arbitrary normal distribution In this case random variable ξ has density $p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-m)^2/2\sigma^2}, \quad -\infty < x < \infty$, And,

$$E\xi = m, \quad d^2 = D\xi = \sigma^2, \quad f(t) = Ee^{t\xi} = e^{mt+(t\sigma)^2/2}.$$

1.1 From this we obtain ($X_1 = \xi - E\xi$)

$$\varphi(\lambda) = Ee^{\lambda X_1} = e^{m\lambda+(\sigma\lambda)^2/2-m\lambda} = e^{(\sigma\lambda)^2/2} < \infty, \quad 0 < \lambda < \infty = \Delta,$$

$$\varphi'(\lambda) = \sigma^2\lambda e^{(\sigma\lambda)^2/2}, \quad m(\lambda) = E\zeta = \frac{\varphi'(\lambda)}{\varphi(\lambda)} = \sigma^2\lambda, \quad \sigma^2(\lambda) = D\zeta = m'(\lambda) = \sigma^2.$$

Thus,

$$\sigma(z) = \sigma, \quad \varphi(z) = e^{(\sigma z)^2/2}, \quad m(z) = \sigma^2 z.$$

$\alpha(z)$ is clearly defined in (6) from [1].

Note, that in this case in (1) from 1 part of this article $x = \lambda$, because

$$x = \frac{n}{s_n} m(z), \quad m(z) = \sigma^2 z, \quad s_n^2 = n\sigma^2, d^2 = \sigma^2.$$

1.2 In this case ($X_1 = (\xi - E\xi)/\sqrt{D\xi}$)

$$\psi(\lambda) = Ee^{\lambda X_1} = e^{m\lambda/\sigma+\lambda^2/2-m\lambda/\sigma} < \infty, \quad 0 < \lambda < \infty = \lambda_+,$$

$$\psi'(\lambda) = \lambda e^{\lambda^2/2}, \quad m(\lambda) = \lambda, \quad \alpha_+ = m(\lambda_+) = \infty.$$

Then we get $\lambda(\alpha)$:

$$\frac{\psi'(\lambda(\alpha))}{\psi(\lambda(\alpha))} = \alpha \Leftrightarrow \lambda(\alpha) = \alpha.$$

Further $\Lambda(\alpha)$:

$$\psi(\lambda(\alpha)) = e^{\alpha^2/2}, \quad \ln\psi(\lambda(\alpha)) = \frac{\alpha^2}{2}, \quad \Lambda(\alpha) = \alpha^2 - \alpha^2/2 = \alpha^2/2.$$

It remains to find σ_α^2:

$$\sigma_\alpha^2 = m'(\lambda(\alpha)) \equiv 1.$$

2. Binomial distribution

In this case the distribution of a random variable is given by probabilities

$$P(\xi = l) = C_k^l p^l q^{k-l}, \quad q = 1 - p, \quad l = 0, \cdots, k, \quad 0 < p < 1.$$

The distribution characteristics are

$$E\xi = kp, \quad d^2 = D\xi = kpq, \quad f(t) = Ee^{t\xi} = (q + pe^t)^k.$$

2.1 It follows that

$$\varphi(\lambda) = Ee^{\lambda X_1} = Ee^{\lambda(\xi-kp)} = e^{-\lambda kp}(q + pe^\lambda)^k < \infty, \quad 0 < \lambda < \infty = \Delta,$$

$$\varphi'(\lambda) = -kpe^{-\lambda kp}(q + pe^\lambda)^k + k(q + pe^\lambda)^{k-1}pe^\lambda e^{-\lambda kp},$$

$$m(\lambda) = E\zeta = \frac{\varphi'(\lambda)}{\varphi(\lambda)} = \frac{kpe^{-\lambda kp}(q+pe^\lambda)^{k-1}(e^\lambda-q-pe^\lambda)}{e^{-\lambda kp}(q+pe^\lambda)^k} = \frac{kpq(e^\lambda-1)}{q+pe^\lambda}, \quad \zeta = \zeta(\lambda, X_1),$$

$$\sigma^2(\lambda) = D\zeta = m'(\lambda) = \frac{kpq^2e^\lambda+kp^2qe^{2\lambda}-kp^2qe^{2\lambda}+kp^2qe^\lambda}{(q+pe^\lambda)^2} = \frac{kqpe^\lambda}{(q+pe^\lambda)^2}.$$

Thus,

$$\sigma(z) = \frac{\sqrt{kqpe^z}}{q+pe^z}, \quad \varphi(z) = e^{-zkp}(q + pe^z)^k, \quad m(z) = \frac{kqp(e^z-1)}{q+pe^z}.$$

$\alpha(z)$ is clearly defined in (6) from [1].

2.2 In this case

$$\psi(\lambda) = Ee^{\lambda X_1} = Ee^{\lambda(\xi-kp)/\sqrt{kpq}} = e^{-\lambda\sqrt{kp/q}}(q + pe^{\lambda/\sqrt{kpq}})^k < \infty, \quad 0 < \lambda < \infty = \lambda_+,$$

$$\psi'(\lambda) = -\sqrt{\frac{kp}{q}} e^{-\lambda\sqrt{kp/q}}(q + pe^{\lambda/\sqrt{kpq}})^k + kp\frac{1}{\sqrt{kpq}} e^{\lambda/\sqrt{kpq}} e^{-\lambda\sqrt{kp/q}}(q + pe^{\lambda/\sqrt{kpq}})^{k-1},$$

$$m(\lambda)=\frac{\psi'(\lambda)}{\psi(\lambda)}=\frac{\sqrt{kp/q}e^{-\lambda\sqrt{kp/q}}(q+pe^{\lambda/\sqrt{kpq}})^{k-1}(e^{\lambda/\sqrt{kpq}}-q-pe^{\lambda/\sqrt{kpq}})}{e^{-\lambda\sqrt{kp/q}}(q+pe^{\lambda/\sqrt{kpq}})^{k}}=\frac{\sqrt{kpq}(e^{\lambda/\sqrt{kpq}}-1)}{q+pe^{\lambda/\sqrt{kpq}}},$$

$$\alpha_{+}=m(\lambda_{+})=d/p.$$

Further we get $\lambda(\alpha)$:

$$\frac{\psi'(\lambda(\alpha))}{\psi(\lambda(\alpha))}=\alpha\Leftrightarrow\frac{\sqrt{kpq}(e^{\lambda(\alpha)/\sqrt{kpq}}-1)}{q+pe^{\lambda(\alpha)/\sqrt{kpq}}}=\alpha\Rightarrow\lambda(\alpha)=\sqrt{kpq}\ln\left(\frac{\alpha q+\sqrt{kpq}}{\sqrt{kpq}-\alpha p}\right).$$

Then $\Lambda(\alpha)$:

$$\psi(\lambda(\alpha))=\frac{(\sqrt{kqp})^{k}(\sqrt{kqp}-\alpha p)^{pk}}{(\sqrt{kqp}-\alpha p)^{k}(\alpha q+\sqrt{kqp})^{kp}}$$

,

$$\ln\psi(\lambda(\alpha))=k\ln\sqrt{kpq}-kq\ln(\sqrt{kpq}-\alpha p)-kp\ln(\alpha q+\sqrt{kpq}),$$

$$\Lambda(\alpha)=\alpha\sqrt{kpq}\ln\left(\frac{\alpha q+\sqrt{kpq}}{\sqrt{kpq}-\alpha p}\right)-k\ln\sqrt{kpq}+kq\ln(\sqrt{kpq}-\alpha p)+kp\ln(\alpha q+\sqrt{kpq}).$$

And σ_α^2

$$m'(\lambda)=\frac{e^{\lambda/\sqrt{kpq}}(q+pe^{\lambda/\sqrt{kpq}})-pe^{\lambda/\sqrt{kpq}}(e^{\lambda/\sqrt{kpq}}-1)}{(q+pe^{\lambda/\sqrt{kpq}})^2}\Rightarrow\sigma_\alpha^2=m'(\lambda(\alpha))=\frac{(\alpha q+\sqrt{kpq})(\sqrt{kpq}-\alpha p)}{kpq}.$$

3. Uniform distribution on the segment [a,b]

In this case the distribution of a random variable is given by density function $p(x)=\frac{1}{b-a},\quad a\le x\le b$. And,

$$E\xi=\frac{a+b}{2}=c,\quad d^2=D\xi=\frac{(b-a)^2}{12},\quad f(t)=Ee^{t\xi}=\frac{e^{tb}-e^{ta}}{t(b-a)}.$$

3.1 It follows that ($h=\frac{b-a}{2}$)

$$\varphi(\lambda)=Ee^{\lambda X_1}=Ee^{\lambda(\xi-c)}=\frac{e^{\lambda(b-c)}-e^{\lambda(a-c)}}{\lambda(b-a)}=\frac{e^{\lambda h}-e^{-\lambda h}}{2\lambda h}<\infty,\quad 0<\lambda<\infty=\Delta,$$

$$\varphi'(\lambda)=\frac{h(e^{\lambda h}+e^{-\lambda h})2\lambda h-2h(e^{\lambda h}-e^{-\lambda h})}{4\lambda^2h^2},$$

$$m(\lambda)=E\zeta=\frac{\varphi'(\lambda)}{\varphi(\lambda)}=\frac{h(e^{\lambda h}+e^{-\lambda h})}{e^{\lambda h}-e^{-\lambda h}}-\frac{1}{\lambda}=h\left\{\frac{e^{\lambda h}+e^{-\lambda h}}{e^{\lambda h}-e^{-\lambda h}}-\frac{1}{\lambda h}\right\},$$

$$\sigma^2(\lambda)=D\zeta=m'(\lambda)=h\left\{\frac{h(e^{\lambda h}-e^{-\lambda h})^2-h(e^{\lambda h}+e^{-\lambda h})^2}{(e^{\lambda h}-e^{-\lambda h})^2}+\frac{1}{\lambda^2h}\right\}\quad=$$

$$=\quad h^2\left\{1-\frac{(e^{\lambda h}+e^{-\lambda h})^2}{(e^{\lambda h}-e^{-\lambda h})^2}\right\}+\frac{1}{\lambda^2}=\frac{1}{\lambda^2}-\frac{4h^2}{(e^{\lambda h}-e^{-\lambda h})^2}.$$

Thus,

$$\sigma(z)=\sqrt{\frac{1}{z^2}-\frac{4h^2}{(e^{zh}-e^{-zh})^2}},\quad\varphi(z)=\frac{e^{zh}-e^{-zh}}{2zh},\quad m(z)=h\left\{\frac{e^{zh}+e^{-zh}}{e^{zh}-e^{-zh}}-\frac{1}{zh}\right\}.$$

$\alpha(z)$ is clearly defined in (6) from [1].

3.2 In this case ($g=h/d=\sqrt{3}$)

$$\psi(\lambda)=Ee^{\lambda X_1}=Ee^{\lambda(\xi-c)/d}=\frac{e^{\lambda g}-e^{-\lambda g}}{2\lambda g}<\infty,\quad 0<\lambda<\infty=\lambda_{+},$$

$$m(\lambda)=g\left\{\frac{e^{\lambda g}+e^{-\lambda g}}{e^{\lambda g}-e^{-\lambda g}}\right\}-\frac{1}{\lambda},\quad\alpha_{+}=m(\lambda_{+})=g=\sqrt{3}.$$

$\lambda(\alpha)$ is determined from equation:

$$\frac{\psi'(\lambda(\alpha))}{\psi(\lambda(\alpha))}=\alpha\Leftrightarrow g\left\{\frac{e^{\lambda g}+e^{-\lambda g}}{e^{\lambda g}-e^{-\lambda g}}\right\}-\frac{1}{\lambda}=\alpha,\qquad(1)$$

which so far can not be solved. The remaining elements are expressed in terms of $\lambda(\alpha)$:

$$\Lambda(\alpha)=\alpha\lambda(\alpha)-\ln\psi(\lambda(\alpha)),\quad\sigma_\alpha^2=m'(\lambda(\alpha))=\frac{\psi''(\lambda(\alpha))}{\psi(\lambda(\alpha))}-\alpha^2.$$

4. Bernoulli distribution $(\mathbf{0},\mathbf{1})$

In this case $P(\xi=1)=p,\quad P(\xi=0)=q,\quad p+q=1$. And

$$E\xi=p,\quad d^2=D\xi=pq,\quad f(t)=Ee^{t\xi}=q+pe^t.$$

4.1 It follows that

$$\varphi(\lambda)=Ee^{\lambda X_1}=Ee^{\lambda(\xi-p)}=pe^{\lambda q}+qe^{-\lambda p}<\infty,\quad 0<\lambda<\infty=\Delta,$$

$$\varphi'(\lambda)=pq(e^{\lambda q}-e^{-\lambda p}),$$

$$m(\lambda)=E\zeta=\frac{\varphi'(\lambda)}{\varphi(\lambda)}=\frac{pq(e^{\lambda}-1)}{pe^{\lambda}+q},\quad \zeta=\zeta(\lambda,X_1),$$

$$\sigma^2(\lambda)=D\zeta=m'(\lambda)=pq\frac{e^{\lambda}(pe^{\lambda}+q)-pe^{\lambda}(e^{\lambda}-1)}{(pe^{\lambda}+q)^2}=\frac{pqe^{\lambda}}{(pe^{\lambda}+q)^2}.$$

Thus,

$$\sigma(z)=\frac{\sqrt{qpe^z}}{q+pe^z},\quad \varphi(z)=e^{-zp}(q+pe^z),\quad m(z)=\frac{qp(e^z-1)}{q+pe^z}.$$

$\alpha(z)$ is clearly defined in (6) from [1].

4.2 In this case

$$\psi(\lambda)=Ee^{\lambda X_1}=Ee^{\lambda(\xi-p)/\sqrt{pq}}=pe^{\lambda\sqrt{q/p}}+qe^{-\lambda\sqrt{p/q}}<\infty,\quad 0<\lambda<\infty=\lambda_+,$$

$$\psi'(\lambda)=\sqrt{pq}(e^{\lambda\sqrt{q/p}}-e^{-\lambda\sqrt{p/q}}),$$

$$m(\lambda)=\frac{\psi'(\lambda)}{\psi(\lambda)}=\sqrt{pq}\frac{e^{\lambda/\sqrt{pq}}-1}{pe^{\lambda/\sqrt{pq}}+q},\quad \alpha_+=m(\lambda_+)=\sqrt{q/p}$$

Further $\lambda(\alpha)$ $(c=\sqrt{pq})$:

$$\frac{\psi'(\lambda(\alpha))}{\psi(\lambda(\alpha))}=\alpha\Leftrightarrow e^{\lambda/c}-1=\frac{\alpha}{c}(pe^{\lambda/c}+q)\Leftrightarrow e^{\lambda/c}(1-\alpha p/c)=1+\alpha q/c,$$

$$\lambda(\alpha)=\sqrt{pq}\ln\left(\frac{1+\alpha\sqrt{q/p}}{1-\alpha\sqrt{p/q}}\right)$$

Then we get $\Lambda(\alpha)$:

$$\psi(\lambda(\alpha))=\frac{p(1+\alpha\sqrt{q/p})+q(1-\alpha\sqrt{p/q})}{(1-\alpha\sqrt{p/q})^q(1+\alpha\sqrt{q/p})^p}=\frac{1}{(1-\alpha\sqrt{p/q})^q(1+\alpha\sqrt{q/p})^p},$$

$$-\ln\psi(\lambda(\alpha))=q\ln(1-\alpha\sqrt{p/q})+p\ln(1+\alpha\sqrt{q/p}),$$

$$\Lambda(\alpha)=(q-\alpha\sqrt{pq})\ln(1-\alpha\sqrt{p/q})+(p+\alpha\sqrt{pq})\ln(1+\alpha\sqrt{q/p}).$$

It remains to calculate σ_α^2

$$m'(\lambda)=\frac{e^{\lambda/c}}{(pe^{\lambda/c}+q)^2}\Rightarrow\sigma_\alpha^2=m'(\lambda(\alpha))=1-\alpha^2+\frac{\alpha(q-p)}{\sqrt{pq}}=\frac{(d+\alpha q)(d-\alpha p)}{d^2}.$$

5. Gamma distribution

In this case random variable has a density function

$$p(x)=\frac{e^{-\mu x}x^{\beta-1}\mu^{\beta}}{\Gamma(\beta)},\quad x>0\quad(\beta>0,\mu>0).$$

The distribution characteristics are

$$E\xi=\frac{\beta}{\mu},\quad d^2=D\xi=\frac{\beta}{\mu^2},\quad f(t)=Ee^{t\xi}=\frac{1}{(1-t/\mu)^{\beta}}.$$

5.1 In this case $(\nu=\lambda/\mu)$

$$\varphi(\lambda)=Ee^{\lambda X_1}=Ee^{\lambda(\xi-k)}=\left(\frac{e^{-\nu}}{1-\nu}\right)^{\beta}<\infty,\quad 0<\lambda<\mu=\Delta,$$

$$\varphi'(\lambda) = \beta \left(\frac{e^{-\nu}}{1-\nu}\right)^{\beta-1} \frac{-\frac{1}{\mu}e^{-\nu}(1-\nu)+\frac{1}{\mu}e^{-\nu}}{(1-\nu)^2} = \beta \left(\frac{e^{-\nu}}{1-\nu}\right)^{\beta} \frac{\lambda}{\mu(\mu-\lambda)},$$

$$m(\lambda) = E\zeta = \frac{\varphi'(\lambda)}{\varphi(\lambda)} = \frac{\beta\lambda}{\mu(\mu-\lambda)}, \quad \zeta = \zeta(\lambda, X_1),$$

$$\sigma^2(\lambda) = D\zeta = m'(\lambda) = \frac{\beta}{(\mu-\lambda)^2}.$$

$\alpha(z)$ is clearly defined in (6) from [1].

5.2 This time ($d = \sqrt{\beta}/\mu$)

$$\psi(\lambda) = Ee^{\lambda X_1} = Ee^{\lambda(\xi-\beta/\mu)/d} = \frac{e^{-\lambda\sqrt{\beta}}}{(1-\lambda/\sqrt{\beta})^{\beta}} < \infty, \quad 0 < \lambda < d\mu = \sqrt{\beta} = \lambda_+,$$

$$\psi'(\lambda) = \frac{\lambda e^{-\lambda\sqrt{\beta}}}{(1-\lambda/\sqrt{\beta})^{\beta+1}}, \quad m(\lambda) = \frac{\psi'(\lambda)}{\psi(\lambda)} = \frac{\lambda}{1-\lambda/\sqrt{\beta}}, \quad \alpha_+ = m(\lambda_+) = \infty.$$

Further $\lambda(\alpha)$:

$$\frac{\psi'(\lambda(\alpha))}{\psi(\lambda(\alpha))} = \alpha \Leftrightarrow \frac{\lambda(\alpha)}{1-\lambda(\alpha)/\sqrt{\beta}} = \alpha \Rightarrow \lambda(\alpha) = \frac{\alpha\sqrt{\beta}}{\alpha+\sqrt{\beta}}.$$

Then using (12), we get $\Lambda(\alpha)$:

$$\psi(\lambda(\alpha)) = \exp\left(-\frac{\alpha\beta}{\alpha+\sqrt{\beta}}\right)(1+\alpha/\sqrt{\beta})^{\beta},$$

$$\Lambda(\alpha) = \frac{\alpha^2\sqrt{\beta}+\alpha\beta}{\alpha+\sqrt{\beta}} - \beta\ln(1+\alpha/\sqrt{\beta}) = \alpha\sqrt{\beta} - \beta\ln(1+\alpha/\sqrt{\beta}).$$

σ_α^2 is equal to

$$m'(\lambda) = \frac{1-\lambda/\sqrt{\beta}+\lambda/\sqrt{\beta}}{(1-\lambda/\sqrt{\beta})^2} = \frac{1}{(1-\lambda/\sqrt{\beta})^2} \Rightarrow \sigma_\alpha^2 = m'(\lambda(\alpha)) = (1+\alpha/\sqrt{\beta})^2.$$

Two types of representations that arise

1. Representation

$$P(S_n > xs_n) = \frac{\varphi^n(z)e^{-\lambda x}}{\sqrt{2\pi}c\sigma(z)}\left(1+\delta_n(\lambda)\right), \tag{2}$$

in which main parameter is related with λ by the equality

$$x = \frac{n}{s_n}m(z), \quad z = \frac{\lambda}{s_n},$$

takes place under the conditions (3), (4) from [1] for any $0 < \lambda < s_n\Delta$.

2. Asymptotic equivalence

$$P(S_n > y) \sim \frac{\exp(-n\Lambda(\alpha))}{\sqrt{2\pi n}\sigma_\alpha\lambda(\alpha)}, \quad \alpha = \frac{y}{n}, \tag{3}$$

occurs if the conditions $(8)-(10)$ from [1] are met, and also ch. function $\varphi^m(t)$ (for example $\varphi(t) = Ee^{it\xi}$) is integrable for some integer $m \geq 1$.

Now we give these representations in each of 5 cases. We only note, that according theorem 1 from [3] the first term $\delta_n(\lambda) = O(1/\sqrt{n})$, $n \to \infty$, and the smallness of the second determines fast convergence $J(x) \uparrow 1, x \to \infty$. So it turns out that we will compare two equivalences.

1. Arbitrary normal distribution

$$1.1 \quad P(S_n > xs_n) \sim \frac{e^{n\sigma^2z^2/2}e^{-x^2}}{\sqrt{2\pi}\lambda}, \quad 0 < \lambda < \infty, \quad x = \lambda.$$

$$1.2 \quad P(S_n > y) \sim \frac{e^{-n\alpha^2/2}}{\sqrt{2\pi n}\alpha}, \quad (\lambda_+ = \infty, \alpha_+ = \infty).$$

2. Binomial distribution

$$2.1 \quad P(S_n > xs_n) \sim \frac{(pe^{zq}+qe^{-zp})^{kn} e^{-\lambda x}(q+pe^z)}{\sqrt{2\pi}\lambda e^{z/2}}, \quad x = \frac{d\sqrt{n}(e^z-1)}{q+pe^z}.$$

$$2.2 \quad P(S_n > y) \sim \frac{(d+\alpha q)^{-n(kp+d\alpha)}(d-\alpha p)^{-n(kq-\alpha d)} d^{kn}}{\sqrt{2\pi n(d+\alpha q)(d-\alpha p)}(\ln(d+\alpha q)-\ln(d-\alpha p))},$$

$$\lambda_+ = \infty, \alpha_+ = d/p.$$

3. Uniform distribution

$$3.1 \quad P(S_n > xs_n) \sim \frac{d(e^{zh}-e^{-zh})^n e^{-\lambda x}}{\sqrt{2\pi\left(\frac{1}{z^2}-\frac{4h^2}{(e^{zh}-e^{-zh})^2}\right)\lambda(2zh)^n}}, \quad 0 < \lambda < \infty,$$

$$x = \sqrt{3n}\left\{\frac{e^{zh}+e^{-zh}}{e^{zh}-e^{-zh}} - \frac{1}{zh}\right\}.$$

3.2 The value $\lambda(\alpha)$ could not be found. So the representation of the second type could not be obtained.

4. Bernoulli distribution $(0, 1)$

$$4.1 \quad P(S_n > xs_n) \sim \frac{e^{-zpn}(q+pe^z)^{n+1} e^{-\lambda x}}{\sqrt{2\pi}\lambda e^{z/2}}, \quad 0 < \lambda < \infty,$$

$$x = \frac{d\sqrt{n}(e^z-1)}{q+pe^z}.$$

$$4.2 \quad P(S_n > y) \sim \frac{(d+\alpha q)^{-n(p+\alpha d)}(d-\alpha p)^{-n(q-\alpha d)} d^n}{\sqrt{2\pi n(d+\alpha q)(d-\alpha p)}(\ln(d+\alpha q)-\ln(d-\alpha p))},$$

$$\lambda_+ = \infty, \quad \alpha_+ = \sqrt{q/p}.$$

5. Gamma distribution

$$5.1 \quad P(S_n > xs_n) \sim \frac{e^{-\lambda x} e^{-\lambda\sqrt{n\beta}}}{\sqrt{2\pi}x(1-\lambda/\sqrt{n\beta})^{n\beta}}, \quad 0 < \lambda < d\mu\sqrt{n},$$

$$x = \frac{\lambda}{1-\lambda/\sqrt{\beta n}}.$$

$$5.2 \quad P(S_n > y) \sim \frac{e^{-n\alpha\sqrt{\beta}}(1+\alpha/\sqrt{\beta})^{n\beta}}{\alpha\sqrt{2\pi n}}, \quad \lambda_+ = \sqrt{\beta}, \quad \alpha_+ = \infty.$$

3 Conclusion

The aim of this paper was to attempt to compare the difficulty of obtaining elements of two representations and their final form. Other questions were not considered, because the comparison proved to be a very laborious task. The problems that was not discussed here, but whose appearance is natural, is following:

- Try to simplify equation (1) and find an approximate solution
- Simplify the final form of representation (2) by substitution z for λ and

representation(3) by substitution α for y

- Compile a table for comparisons of representations (2) and (3). Such as this way

	1	2	3	4	5
Δ	∞	∞	∞	∞	μ
λ_+	∞	∞	∞	∞	$\sqrt{\beta}$
α_+	∞	d/p	$\sqrt{3}$	$\sqrt{q/p}$	∞

It is also worth noting that the representations in both approaches have a very complex form, therefore it is desirable to simplify it. However, we confine ourselves to simple conclusions that require no transformations. In cases 2,4,5 representation (2) is less cumbersome than representation (3). In case 3 the classical representation could not be obtained, in contrast to the alternative approach. In case 1 representations are almost identical. The complexity of calculating of elements of representation (2) is much lower than the complexity of calculating of elements of representation (3). (In the case of an arbitrary normal distribution, the complexity of obtaining the elements of representations is almost identical).

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