Reliability Analysis of a Maintenance Scheduling Model Under Failure Free Warranty Policy

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Abstract

This paper considers a maintenance scheduling model by using the concepts of failure free warranty policy. In this model, all the repairs during warranty are cost-free to the users, provided failures are not due to the negligence of the users. However, the users will have to repair the failed unit at their own expenses beyond warranty. During their formulation, the failure rate of the system is considered to be negative exponential distribution while the preventive maintenance (PM), repair and replacement time distributions are taken to be arbitrary with different probability density functions. Under these assumptions, using the supplementary variable technique, the various expressions which depict the behavior of the system such as reliability of the system, Mean Time to System Failure (MTSF), availability and profit function have been derived. Further, steady-state behavior of the system has also been derived. To substantiate the proposed approach, the effect of the parameters of the system has been analyzed through the system reliability, and expected profit through an illustrative example.

Keywords: Warranty, Reliability, Maintenance, Inspection, degradation, Mean Time to System Failure

1. Introduction

In the present era of industrial growth, the optimal efficiency and minimum hazards are more challengeable to maintain. To overcome these issues, reliability technology can play an important role. Reliability is measured as the ability of a system to perform its intended function, successfully, for a specified period, under predetermined conditions. This attribute has far-reaching consequences on the durability, availability, and life cycle cost of a product or system and is of great importance to the end user/engineer [8]. Typically, high-reliability targets or specifications are set for the system, and ways to achieve them are then examined, taking into account resource constraints. Apart from the limitations of resources, the targets set may be in dispute. For instance, high reliability generally means a high cost, weight, and volume. Also at the same time the unfortunate penalty of low availability and high maintenance cost need to be improved for their survival. To achieve this end, availability and reliability of equipment in the process must be maintained at the higher order. Thus, reliability and maintainability concepts are mainly applicable at the design stage of a machinery or plant layout, while the availability concept is mostly applicable after commissioning the plant or after a steady state of production is reached

[6, 7]. Modern technology has developed a tendency to design and manufacture equipment and systems of greater capital cost, sophistication, complexity, and capacity. In the literature, a variety of methods exists for failure analysis which includes reliability block diagrams, Markov Modeling, failure mode and effect analysis, Petri nets, fault tree analysis, and so forth [1, 4, 6, 7, 10, 11].

In a system where a certain amount of failures is allowed, the efficient repair and/or replacement of these failures is critical to the continued usefulness of the system. This repair and replacement of failures are called maintenance. Maintenance has a definite influence on operating costs, either through its own (maintenance) labor or through its effect of system downtime and efficiency. In reliability, maintainability can also be used to increase the probability that a system will continue to operate efficiently, given that it is allowed a certain amount of downtime of repairs. The purpose of maintainability is to return a failed or deteriorating system to a satisfactory operating state. To do this, there are two extreme maintenance policies that can be applied. The first is to unplanned (corrective) maintenance while the second one is planned (preventive) maintenance. In unplanned, corrective strategy, no maintenance action is carried out until the component or structure breaks down or when its cost of operation becomes creeping or wear-out failures. This is called corrective maintenance (CM) or emergency repair. A study on the effect of CM in maintenance policies was done by Samrout et al. [35]. However, to avoid failures at occasions that have high cost consequences preventive maintenance (PM) is normally chosen. The main function of planned maintenance is to restore equipment to the "as good as new" condition; periodical inspections must control equipment condition and both actions will ensure equipment availability. PM of the systems is necessary after a pre-specific period of time not only to maintain the operational power but may also reduce the failure rate. A study on the effect of PM on a single-unit system was done by Kapur et al. [19]. PM can increase the performance of two or more unit system model if it starts for an operative unit only when the other unit is in standby as discussed by Gupta [12].

Since the importance of PM during the reliability analysis, various authors have put forth the different approaches to enhance the reliability of the system. For instance, Mokaddis et al. [25] developed a three-unit standby redundant system with repair and PM. Yanagi and Sasaki [38] evaluated the availability of a parallel system with PM. Rander et al. [33] examined cost analysis of a two dissimilar cold standby system with preventive maintenance and replacement of standby. Hadidi and Rahim [13] analyzed the reliability for multiple units adopting sequential imperfect maintenance policies. Jin et al. [15] presented an option model for joint production preventive maintenance system. Garg et al. [9] presented different PM models to analyze the reliability of the system. Liao [22] evaluated optimum policy for a production system with major repair and preventive maintenance. Malik [24] studied reliability modeling of a computer system with preventive maintenance and priority subject to maximum operation and repair times while Kadyan [18] analyzed reliability and profit of a single-unit system with PM subject to maximum operation time. Apart from these, in the literature, numerous attempts have been made by the researchers to analyze the reliability of the system using different approaches [20, 18, 38, 32, 17, 17, 27].

From the above study, it is revealed that most of the above mentioned work considered that the unit works as new after PM and repair. Since, the working capability and efficiency of a unit after repair depend more or less on the quality of the unit and repair mechanism adopted, so in general, the assumption of considering the unit as good as new after repair is not always true. Moreover, continuous operation and ageing of the systems gradually reduce the performance of a system and repair action cannot bring the system to the good stage, but can make it operational [30]. Further, in such a situation, unit after its repair works with reduced capacity. To minimize the error in the study, Kumar et al. [20] and Malik [23] analyzed a single-unit system with degradation and maintenance by using degraded unit. Eryilmaz [5] studied a three

state reliability model in which degradation rates are random and statistically dependent. Apart from all these work, in our day-today life, there is always occur a situation where the repair of the failed degraded systems is neither possible nor economical to both manufacturer and the user due to wearout and other unforeseen conditions. In such cases, inspection can play an important role to see the feasibility of repair. On system reliability models, the concept of inspection with different maintenance policies has been discussed by various researchers such as Tuteja and Malik [36], Hariga [14], Leung [21], Zequeira and Berenguer [39], Nailwal and Singh [26], Pietruczuk and Wojciechowska [31], and Wang et al [37].
Since all the above system models are analyzed without considering the concept of failure free warranty policy. Under failure free warranty policy, the items are replaced/repaired free of cost to the users during warranty. Customers need assurance that the product they are buying will perform satisfactorily and warranty provides such assurance. Providing warranty to the system for a certain period of operation is one of the effective ways to ensure reliability of a sold product (or system) [29]. Also, it is an essential part of sale for commercial and industrial products. With these objectives, Kadyan and Niwas [16] and Niwas et al. [29] discussed reliability models of a single-unit system with warranty and different repair policies by using Supplementary Variable Technique [3]. Further, Niwas et al. [28] analyzed that replacement of failed degraded unit by new one is not economically beneficial. However, the unit or product can be restored to operate the required functions by repairing it rather than replacing the entire product.

Keeping these views in mind here we proposed a single-unit repairable system model with the concept of failure free Warranty Policy with PM and inspection for feasibility of repair of degraded unit by using Supplementary Variable Technique. In the proposed approach, to avoid unnecessary expenses on replacement of the entire product, inspection can be done to see the feasibility of repair of the degraded unit. On the other hand, if repair is feasible then the failed degraded unit will be repaired by the repairman otherwise it is replaced by a new one. Due to failure free warranty policy, users are secure about early failures of the products/system because all the repairs are cost free during warranty. Further, after expiration of failure free warranty policy, PM is conducted to improve the condition of the deteriorated product/system. The concepts of PM, degradation and inspection are conducted beyond warranty. So, these factors do not have any impact on failure free warranty policy but they have an impact on the overall performance of the system/product. And, for users prospective by using these concepts after expiration of warranty, can improve the overall performance of the product/system.

The remainder of this paper is organized as follows. Section 2 gives the description of the System containing the assumptions of the model, state-specifications and Notations related to the proposed system model, Section 3 presents the model analysis in which different system performance measures are computed such as steady-state behavior of the system, reliability and MTSF of the system, Section 4 shows the results and discussion with special case containing availability of the system and profit analysis of the user and provides a numerical result for these special cases. Finally, Section 5 presents concluding remarks.

2. Description of the System

2.1.Assumptions

(i) The system has a single unit.
(ii) There is a single repairman, who is always available with the system to do repair or replacement, PM and inspection of the failed unit.
(iii) The cost of repair during warranty is borne by the manufacturer provided failures are not due to the negligence of users such as cracked screen, accident, misuse, physical damage, damage due to liquid and unauthorized modifications etc.

(iv) Beyond warranty, the unit goes under PM and works as new after PM but works with some reduced capacity after its repair and so is called a degraded unit.
(v) Repairman inspects the failed degraded unit to see the feasibility of repair.
(vi) The distribution of failure time is taken as negative exponential while the PM, repair and replacement times are considered as arbitrary.

2.2. *State-Specification*

s_0 / s_1 : The unit is operative under warranty/ beyond warranty.

s_2 / s_4 : The unit is in failed state under warranty/ beyond warranty.

s_3 : The unit is under PM.

s_5 : The degraded unit is operative.

s_6 : The failed degraded unit is under inspection.

2.3. *Notations*

λ / λ_1	Constant failure rate of the new unit within/beyond warranty.
λ_2	Constant failure rate of the degraded unit beyond warranty.
λ_m	Transition rate with which a unit goes under PM for improvement.
α	Transition rate with which warranty of the system is completed.
p / q	Probability that repair is feasible/not feasible.
$\mu(x), S(x) / \mu_1(x), S_1(x)$	Repair rate of the unit and probability density function, for the elapsed repair time x within/ beyond warranty.
$\mu_2(y), S_2(y)$	PM rate of the unit and probability density function, for the elapsed PM time y .
$h(z), S_3(z)$	Inspection rate of the failed unit and probability density function, for the elapsed inspection time z .
$p_0(t) / p_1(t)$	Probability density that at time t, the system is within/ beyond warranty and in good state.
$p_i(x,t)$	Probability density that at time t, the system is in state $S_i, i = 2,4$ and the system is under repair with elapsed repair time x .
$p_3(y,t)$	Probability density that at time t, the system is in state s_3 and the unit is under PM with elapsed PM time y .
$p_5(t)$	Probability density that at time t, the system is operable and in degraded state.
$p_6(z,t)$	Probability density that at time t, the system is in state s_6 and the failed degraded unit is under inspection with elapsed inspection time z.
$p(s)$	Laplace transform of function $p(t)$

$$S(x) = \mu(x)\, e^{\left[-\int_0^x \mu(x)dx\right]}$$

$$S_1(x) = \mu_1(x)\, e^{\left[-\int_0^x \mu_1(x)dx\right]}$$

$$S_2(y) = \mu_2(y)\, e^{\left[-\int_0^y \mu_2(y)dy\right]}$$

$$S_3(z) \quad = \quad h(z)\, e^{\left[-\int_0^z h(z)dz\right]}$$

3. Model Analysis

The system model consists of a single-unit in which there is a single repairman who always remains with the system and monitoring its performance. Initially, the unit is in operating within warranty and when it fails within warranty then it goes to repair with free of cost. On the other hand, if warranty is completed due to negligence of the users then system remain in working condition. In this case, the unit goes under PM and works as new after PM but with some reduced capacity after its repair and so is called a degraded unit. Degraded unit is inspected for feasibility of repair after its failure. It has been assumed the failure times of the system follow a negative exponential distribution while during the PM, replacement and/or repair time, its distribution is taken as arbitrary. The transition diagram of this system by considering all the states, namely up (i.e., good or working), failed and degrade states is shown in Fig. 1

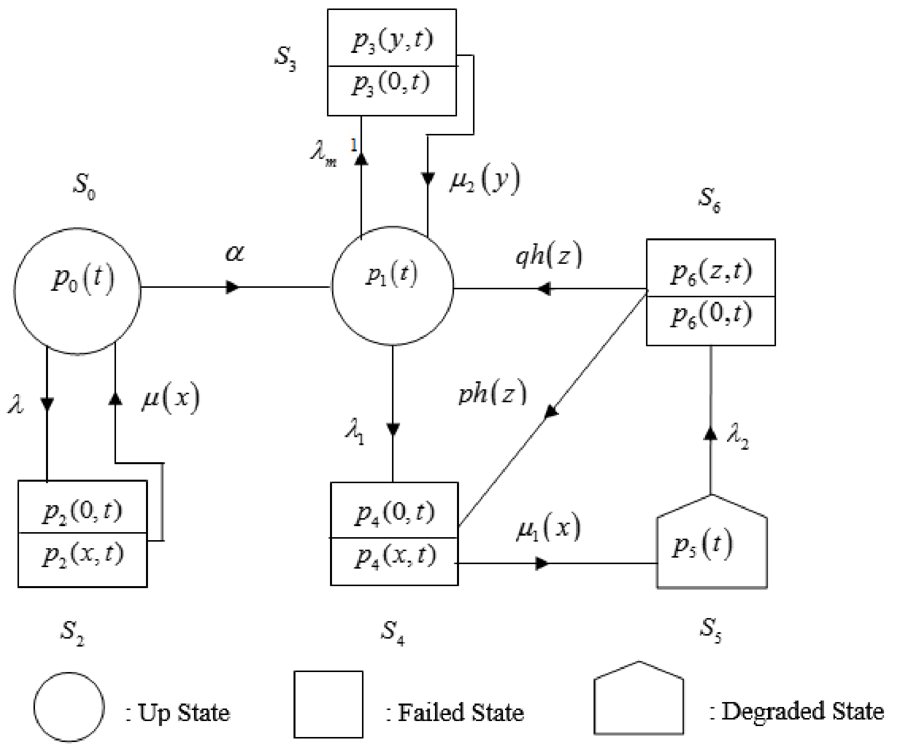

Figure 1: Transition diagram of the model

3.1. *Formulation of mathematical model*

Based on this diagram, we can formulate the difference-differential equations by using the probabilistic arguments of each state of the system and are summarized as follows [3], [29]:

$$\left[\frac{d}{dt}+\lambda+\alpha\right]p_0(t)=\int_0^\infty \mu(x)\,p_2(x,t)dx \tag{1}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_m\right]p_1(t)=\alpha p_0(t)+q\int_0^\infty h(z)p_6(z,t)dz+\int_0^\infty \mu_2(y)p_3(y,t)dy \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu(x)\right]p_2(x,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_2(y)\right]p_3(y,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_1(x)\right]p_4(x,t)=0 \tag{5}$$

$$\left[\frac{d}{dt}+\lambda_2\right]p_5(t)=\int_0^\infty \mu_1(x)p_4(x,t)dx \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+h(z)\right]p_6(z,t)=0 \tag{7}$$

Whereas, the boundary conditions [**Ошибка! Источник ссылки не найден.**] for the system are

$$p_2(0,t)=\lambda p_0(t), \tag{8}$$

$$p_3(0,t)=\lambda_m p_1(t) \tag{9}$$

$$p_4(0,t)=\lambda_1 p_1(t)+p\int_0^\infty h(z)p_6(z,t)dz, \tag{10}$$

$$p_6(0,t)=\lambda_2 p_5(t) \tag{11}$$

and the initial conditions are $p_i(0)=\begin{cases}1 & ; \quad i=0\\ 0 & ; \quad i\neq 0\end{cases}$ (12)

3.2.Solution of the equations

In order to solve the above formulated Eqs. (1) – (11), we use the Laplace transforms corresponding to initial condition given in Eq. (12) and get

$$\left[s+\lambda+\alpha\right]p_0(s)=1+\int_0^\infty \mu(x)p_2(x,s)dx \tag{13}$$

$$\left[s+\lambda_1+\lambda_m\right]p_1(s)=\alpha p_0(s)+q\int_0^\infty h(z)p_6(z,s)dz+\int_0^\infty \mu_2(y)p_3(y,s)dy \tag{14}$$

$$\left[\frac{\partial}{\partial x}+s+\mu(x)\right]p_2(x,s)=0 \tag{15}$$

$$\left[\frac{\partial}{\partial y}+s+\mu_2(y)\right]p_3(y,s)=0 \tag{16}$$

$$\left[\frac{\partial}{\partial x}+s+\mu_1(x)\right]p_4(x,s)=0 \tag{17}$$

$$\left[s+\lambda_2\right]p_5(t)=\int_0^\infty \mu_1(x)p_4(x,s)dx \tag{18}$$

$$\left[\frac{\partial}{\partial z}+s+h(z)\right]p_6(z,s)=0 \tag{19}$$

$$p_2(0,s)=\lambda p_0(s) \tag{20}$$

$$p_3(0,s)=\lambda_m p_1(s) \tag{21}$$

$$p_4(0,s) = \lambda_1 p_1(s) + p\int_0^\infty h(z)p_6(z,s)dz \tag{22}$$

$$p_6(0,s) = \lambda_2 p_5(s) \tag{23}$$

Thus, by integrating Eq. (15) and further using Eq. (20) we get

$$p_2(x,s) = p_2(0,s)e^{\left[-sx-\int_0^x \mu(x)dx\right]} \tag{24}$$

Similarly, by integrating Eqs. (16), (17) and (19) and using their corresponding Eqs. (21), (22) and (23) respectively, then we get

$$p_3(y,s) = p_3(0,s)e^{\left[-sy-\int_0^y \mu_2(y)dy\right]} \tag{25}$$

$$p_4(x,s) = p_4(0,s)e^{\left[-sx-\int_0^x \mu_1(x)dx\right]} \tag{26}$$

$$p_6(z,s) = p_6(0,s)e^{\left[-sz-\int_0^z h(z)dz\right]} \tag{27}$$

Further, by using Eq. (24), Eq. (13) yields

$$\left[s+\lambda+\alpha\right]p_0(s) = 1 + p_2(0,s)\int_0^\infty \mu(x)e^{\left[-sx-\int_0^x \mu(x)dx\right]}dx$$

$$= 1 + \lambda p_0(s)S(s) \tag{28}$$

$$\Rightarrow \quad p_0(s) = \frac{1}{T(s)} \tag{29}$$

$$\text{Where } T(s) = s + \alpha + \lambda\left(1 - S(s)\right) \tag{30}$$

Now, by using Eq. (27), the Eq. (22) yields

$$p_4(0,s) = \lambda_1 p_1(s) + \int_0^\infty ph(z)p_6(0,s)e^{\left[-sz-\int_0^z h(z)dz\right]}$$

$$p_4(0,s) = \lambda_1 p_1(s) + p\lambda_2 p_5(s)S_3(s) \tag{31}$$

Using Eq. (31), Eq. (26) yields

$$p_4(x,s) = \left(\lambda_1 p_1(s) + p\lambda_2 p_5(s)S_3(s)\right)e^{\left[-sx-\int_0^x \mu_1(x)dx\right]} \tag{32}$$

On the other hand, by using Eq. (32), Eq. (18) yields

$$\left[s+\lambda_2\right]p_5(s) = \left(\lambda_1 p_1(s) + p\lambda_2 p_5(s)S_3(s)\right)\int_0^\infty \mu_1(x)e^{\left[-sx-\int_0^x \mu_1(x)dx\right]}dx$$

$$p_5(s) = A(s)p_1(s) \tag{33}$$

$$\text{Where, } A(s) = \frac{\lambda_1 S_1(s)}{\left(s+\lambda_2 - p\lambda_2 S_1(s)S_3(s)\right)} \tag{34}$$

Now, the Eq. (14) can be simplified by using Eqs. (25), (27) and (33) and get

$$\left[s+\lambda_1+\lambda_m\right]p_1(s) = \alpha p_0(s) + qp_6(0,s)\int_0^\infty h(z)e^{\left[-sz-\int_0^z h(z)dz\right]} + p_3(0,s)\int_0^\infty \mu_2(y)e^{\left[-sy-\int_0^y \mu_2(y)dy\right]}$$

$$p_1(s) = \frac{B(s)}{T(s)} \tag{35}$$

Where, $B(s) = \dfrac{\alpha}{\left(s+\lambda_1+\lambda_m-\lambda_m S_2(s)-q\lambda_2 A(s)S_3(s)\right)}$ (36)

Using Eq. (35) in Eq. (33), we get

$$p_5(s) = \frac{A(s)B(s)}{T(s)} \tag{37}$$

Now, the Laplace transform of the probability that the system is in the failed state is given by

$$p_2(s) = \int_0^\infty p_2(s,x)dx = \lambda p_0(s)\frac{\left(1-S(s)\right)}{s}$$

$$p_2(s) = \frac{\lambda C(s)}{T(s)} \tag{38}$$

Where $C(s) = \dfrac{\left(1-S(s)\right)}{s}$ (39)

Similarly $p_3(s) = \displaystyle\int_0^\infty p_3(s,y)dy = \lambda_m p_1(s)\frac{\left(1-S_2(s)\right)}{s}$

$$p_3(s) = \frac{\lambda_m B(s)D(s)}{T(s)} \tag{40}$$

Where $D(s) = \dfrac{\left(1-S_2(s)\right)}{s}$ (41)

Similarly $p_4(s) = \displaystyle\int_0^\infty p_4(s,x)dx = \left(\lambda_1 p_1(s) + p\lambda_2 p_5(s)S_3(s)\right)\frac{\left(1-S_1(s)\right)}{s}$

$$p_4(s) = \frac{\left(\lambda_1 B(s) + p\lambda_2 B(s)A(s)S_3(s)\right)E(s)}{T(s)} \tag{42}$$

Where $E(s) = \dfrac{\left(1-S_1(s)\right)}{s}$ (43)

Now, $p_6(s) = \displaystyle\int_0^\infty p_6(s,z)dz = \lambda_2 p_5(s)\frac{\left(1-S_3(s)\right)}{s}$

$$p_6(s) = \frac{\left(\lambda_2 A(s)B(s)F(s)\right)}{T(s)} \tag{44}$$

Where, $F(s) = \dfrac{\left(1-S_3(s)\right)}{s}$ (45)

It is worth noticing that

$$p_0(s)+p_1(s)+p_2(s)+p_3(s)+p_4(s)+p_5(s)+p_6(s) = \frac{1}{s} \tag{46}$$

3.3. Evaluation of Laplace transforms of up and down state probabilities

The Laplace transforms of the probabilities that the system is in up (i.e. good State) and down (i.e. failed State) at time *t* are as follows

$$Av(s) \text{ or } P_{up}(s) = p_0(s)+p_1(s)+p_5(s)$$

$$Av(s) = \frac{\left(1+A(s)+B(s)A(s)\right)}{T(s)} \tag{47}$$

$$P_{down}(s) = p_2(s)+p_3(s)+p_4(s)+p_6(s)$$

$$P_{down}(s)=\frac{\left(\lambda C(s)+\lambda_m B(s)D(s)+\left(\lambda_1+p\lambda_2 S_3(s)A(s)\right)B(s)E(s)+\lambda_2 B(s)A(s)F(s)\right)}{T(s)} \tag{48}$$

3.4.Steady-State Behavior of the System

Using Abel's Lemma [26] i.e., $\lim_{s\to 0} s\left[F(s)\right]=\lim_{t\to\infty}\left[F(t)\right]=F$

in Eqs. (47) and (48), Provided the limit on the right hand side exists, the following time independent probabilities have been obtained.

$$Av=\frac{\lambda_1+q\lambda_2}{\left(\lambda_1+q\lambda_2-q\lambda_2\lambda_m S_2^{'}(0)-\lambda_1\lambda_2 S_1^{'}(0)-\lambda_1\lambda_2 S_3^{'}(0)\right)} \tag{49}$$

$$p_{down}=\frac{-q\lambda_m\lambda_2 S_2^{'}(0)-\lambda_1\lambda_2 S_1^{'}(0)-\lambda_1\lambda_2 S_3^{'}(0)}{\left(\lambda_1+q\lambda_2-q\lambda_m\lambda_2 S_2^{'}(0)-\lambda_1\lambda_2 S_1^{'}(0)-\lambda_1\lambda_2 S_3^{'}(0)\right)} \tag{50}$$

3.5. Reliability of the system

Reliability, $R(t)$ is the probability that the system functions well in a specified period of time. Using the method similar to that in section 3.1, the differential–difference equations for reliability are [4]:

$$\left[\frac{d}{dt}+\lambda+\alpha\right]p_0(t)=0 \tag{51}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_m\right]p_1(t)=\alpha\, p_0(t) \tag{52}$$

Taking Laplace transforms of Eqs. (51) and (52) and using Eq. (12) we get

$$\left[s+\lambda+\alpha\right]p_0(s)=1 \tag{53}$$

$$\left[s+\lambda_1+\lambda_m\right]p_1(s)=\alpha\, p_0(s) \tag{54}$$

Using the initial conditions, the solution can be written as

$$p_0(s)=\frac{1}{\left(s+\alpha+\lambda\right)} \tag{55}$$

$$p_1(s)=\frac{\alpha}{\left(s+\alpha+\lambda\right)\left(s+\lambda_1+\lambda_m\right)} \tag{56}$$

$$R(s)=p_0(s)+p_1(s)=\frac{1}{\left(s+\alpha+\lambda\right)}+\frac{\alpha}{\left(s+\alpha+\lambda\right)\left(s+\lambda_1+\lambda_m\right)}$$

Taking inverse Laplace transform, we get

$$R(t)=e^{-(\lambda+\alpha)t}\left[\frac{\left(\lambda-\lambda_m-\lambda_1\right)}{\left(\lambda-\lambda_m-\lambda_1+\alpha\right)}\right]+\left[\frac{\alpha}{\left(\lambda-\lambda_m-\lambda_1+\alpha\right)}\right]e^{-(\lambda_1+\lambda_m)t} \tag{57}$$

Now, based on Eq. (57), the Mean Time to System Failure (MTSF) is defined as:

$$MTSF=\int_0^\infty R(t)dt$$

$$=\int_0^\infty\left\{e^{-(\lambda+\alpha)t}\left(\frac{\left(\lambda-\lambda_m-\lambda_1\right)}{\left(\lambda-\lambda_m-\lambda_1+\alpha\right)}\right)+\left(\frac{\alpha}{\left(\lambda-\lambda_m-\lambda_1+\alpha\right)}\right)e^{-(\lambda_1+\lambda_m)t}\right\}dt$$

$$=\left[\frac{(\lambda-\lambda_m-\lambda_1)}{(\lambda-\lambda_m-\lambda_1+\alpha)(\lambda+\alpha)}\right]+\left[\frac{\alpha}{(\lambda-\lambda_m-\lambda_1+\alpha)(\lambda_1+\lambda_m)}\right] \tag{58}$$

4. Results and discussions

In this section, firstly, we have deduced the expression of the availability and the cost-benefit analysis by taking a particular case of the distribution function of the component of the system.

4.1. Availability of the system $A_v(t)$

Assume that the repairs, PM and Inspection time follow negative exponential distribution i.e., $S(s)=\frac{\mu}{(s+\mu)}, S_1(s)=\frac{\mu_1}{(s+\mu_1)}$, $S_2(s)=\frac{\mu_2}{(s+\mu_2)}$ and $S_3(s)=\frac{h}{(s+h)}$ where μ, and μ_1 are constant repair rates, μ_2 is constant PM rate and h is constant inspection rate. Putting these values in Eqs. (28)-(37) we get

$$p_0(s)=\frac{1}{I(s)} \tag{59}$$

$$\text{Where } I(s)=\frac{\left(s^2+s(\lambda+\alpha+\mu)+\alpha\mu\right)}{(s+\mu)} \tag{60}$$

$$p_1(s)=\frac{J(s)}{I(s)} \tag{61}$$

Where

$$J(s)=\left[\frac{\alpha(s+h)(s+\mu_2)}{(s+\lambda_1+\lambda_m)(s+\mu_2)(s+h)-\lambda_m\mu_2(s+h)-qh\lambda_2K(s)(s+\mu_2)}\right] \tag{62}$$

$$p_5(s)=\frac{J(s)K(s)}{I(s)} \tag{63}$$

Where

$$K(s)=\left[\frac{\mu_1\lambda_1(s+h)}{(s+\mu_1)(s+\lambda_2)(s+h)-ph\lambda_2\mu_1}\right] \tag{64}$$

$$A_v(s) \text{ or } P_{up}(s)=p_0(s)+p_1(s)+p_5(s)$$

$$=\left[\frac{\left(s^5+b_4s^4+b_3s^3+b_2s^2+b_1s+b_0\right)(s+\mu)}{s\left(s^2+s(\lambda+\alpha+\mu)+\alpha\mu\right)\left(s^4+s^3a_3+s^2a_2+sa_1+a_0\right)}\right] \tag{65}$$

Where

$$b_4=(\lambda_1+\mu_1+\mu_2+\alpha+h+\lambda_2+\lambda_m),$$

$$b_3=\begin{pmatrix}\lambda_1\mu_1+\lambda_1\mu_2+h\mu_1+\mu_2\mu_1+h\mu_2+\lambda_2\mu_2+\lambda_1\lambda_2+\lambda_2\mu_1+\lambda_2h\\+\lambda_1h+\lambda_m\mu_1+\lambda_mh+\lambda_m\lambda_2+\lambda_2\alpha+\mu_1\alpha+\mu_2\alpha+h\alpha\end{pmatrix},$$

$$b_2=\begin{pmatrix}\lambda_1\mu_1h+\lambda_2\mu_1h+\lambda_2\mu_1\mu_2+\lambda_2\mu_2h+\mu_1\mu_2h+\lambda_1\lambda_2\mu_1+\lambda_1\lambda_2h+\lambda_1\mu_1\mu_2+\lambda_1\mu_2h+\lambda_1\lambda_2\mu_2\\+\lambda_m\mu_1h+\lambda_2\lambda_m\mu_1+\lambda_2\lambda_mh+\lambda_2\mu_1\alpha+\mu_1h\alpha+\lambda_2h\alpha+\mu_1\mu_2\alpha+\mu_2h\alpha+\lambda_2\mu_2\alpha+\alpha\lambda_1\mu_1\end{pmatrix},$$

$$b_1=\begin{pmatrix}\lambda_1\lambda_2\mu_1\mu_2+\lambda_1\lambda_2h\mu_2+\lambda_m\lambda_2\mu_1h-ph\lambda_2\lambda_m\mu_1-p\lambda_2\mu_1\mu_2h+\alpha\lambda_2\mu_1h\\+\alpha\lambda_2\mu_1\mu_2+\alpha\lambda_2\mu_2h+\alpha\lambda_1\mu_1h+\alpha\lambda_1\mu_1\mu_2+\alpha\mu_1\mu_2h-p\alpha\lambda_2\mu_1h\end{pmatrix}$$

and $b_0 = \alpha qh\lambda_2\mu_1\mu_2 + \alpha h\lambda_1\mu_1\mu_2$

and $a_3 = (\mu_1 + \mu_2 + \lambda_1 + h + \lambda_2 + \lambda_m)$,

$$a_2 = \begin{pmatrix} \lambda_1\mu_1 + \lambda_1 h + \mu_1 h + \lambda_1\lambda_2 + \mu_1\lambda_2 + \mu_1\mu_2 + \mu_2 h \\ +\mu_2\lambda_2 + \lambda_1\mu_2 + \lambda_m\mu_1 + \lambda_m h + h\lambda_2 + \lambda_m\lambda_2 \end{pmatrix},$$

$$a_1 = \begin{pmatrix} \lambda_1\mu_1 h + \lambda_2\mu_1 h + \lambda_2\mu_1\mu_2 + \lambda_2\mu_2 h + \mu_1\mu_2 h + \lambda_1\lambda_2\mu_1 + \lambda_1\lambda_2 h \\ +\lambda_1\mu_1\mu_2 + \lambda_1\mu_2 h + \lambda_m\mu_1 h + \lambda_2\lambda_m\mu_1 + \lambda_2\lambda_m h + \lambda_2\lambda_1\mu_2 \end{pmatrix}$$

and

$$a_0 = (\lambda_1\lambda_2\mu_1\mu_2 + \lambda_2 h\mu_1\mu_2 + \lambda_1\lambda_2 h\mu_2 + \lambda_m\lambda_2\mu_1 h - ph\lambda_2\lambda_m\mu_1 - p\lambda_2\mu_1\mu_2 h)$$

Taking inverse Laplace transforms of Eq. (65) we get

$$\begin{aligned} A_v(t) = {} & \frac{(\alpha q\mu_2\lambda_2\mu_1 h + \alpha\mu_2\lambda_1\mu_1 h)\mu}{z_1z_2z_3z_4z_5z_6} + \left\{\frac{(z_1^5 + b_4z_1^4 + b_3z_1^3 + b_2z_1^2 + b_1z_1 + b_0)(z_1+\mu)}{z_1(z_1-z_2)(z_1-z_3)(z_1-z_4)(z_1-z_5)(z_1-z_6)}\right\}e^{z_1t} \\ & + \left\{\frac{(z_2^5 + b_4z_2^4 + b_3z_2^3 + b_2z_2^2 + b_1z_2 + b_0)(z_2+\mu)}{z_2(z_2-z_1)(z_2-z_3)(z_2-z_4)(z_2-z_5)(z_2-z_6)}\right\}e^{z_2t} + \left\{\frac{(z_3^5 + b_4z_3^4 + b_3z_3^3 + b_2z_3^2 + b_1z_3 + b_0)(z_3+\mu)}{z_3(z_3-z_1)(z_3-z_2)(z_3-z_4)(z_3-z_5)(z_3-z_6)}\right\}e^{z_3t} \\ & + \left\{\frac{(z_4^5 + b_4z_4^4 + b_3z_4^3 + b_2z_4^2 + b_1z_4 + b_0)(z_4+\mu)}{z_4(z_4-z_1)(z_4-z_2)(z_4-z_3)(z_4-z_5)(z_4-z_6)}\right\}e^{z_4t} + \left\{\frac{(z_5^5 + b_4z_5^4 + b_3z_5^3 + b_2z_5^2 + b_1z_5 + b_0)(z_5+\mu)}{z_5(z_5-z_1)(z_5-z_2)(z_5-z_3)(z_5-z_4)(z_5-z_6)}\right\}e^{z_5t} \\ & + \left\{\frac{(z_6^5 + b_4z_6^4 + b_3z_6^3 + b_2z_6^2 + b_1z_6 + b_0)(z_6+\mu)}{z_6(z_6-z_1)(z_6-z_2)(z_6-z_3)(z_6-z_5)(z_6-z_4)}\right\}e^{z_6t} \end{aligned} \tag{66}$$

z_1 and z_2 are roots of the equation $s^2 + s(\lambda+\alpha+\mu) + \alpha\mu = 0$ and z_3, z_4, z_5 and z_6 are roots of the equation $s^4 + s^3a_3 + s^2a_2 + sa_1 + a_0 = 0$

4.2. *Profit analysis of the user*

Suppose that the warranty period of the system is $(0, w]$ includes the second state. Since the repairman is always available with the system, therefore beyond warranty period, it remains busy for time $(t-w)$ during the interval $(w,t]$. Let K_1 be the revenue per unit time and K_2 be the repair cost per unit time respectively, then the expected profit $H(t)$ during the interval $(0,t]$ is given by [29] $H(t) = K_1\int_0^t A_v(t)dt - K_2(t-w)$.

By using Eq. (66) and after solving, we get

$$H(t) = K_1\left\{\begin{array}{l} \dfrac{(\alpha q\mu_2\lambda_2\mu_1 h + \alpha\mu_2\lambda_1\mu_1 h)\mu t}{z_1z_2z_3z_4z_5z_6} \\ + \left\{\dfrac{(z_1^5 + b_4z_1^4 + b_3z_1^3 + b_2z_1^2 + b_1z_1 + b_0)(z_1+\mu)}{z_1^2(z_1-z_2)(z_1-z_3)(z_1-z_4)(z_1-z_5)(z_1-z_6)}\right\}(e^{z_1t}-1) \\ + \left\{\dfrac{(z_2^5 + b_4z_2^4 + b_3z_2^3 + b_2z_2^2 + b_1z_2 + b_0)(z_2+\mu)}{z_2^2(z_2-z_1)(z_2-z_3)(z_2-z_4)(z_2-z_5)(z_2-z_6)}\right\}(e^{z_2t}-1) \\ + \left\{\dfrac{(z_3^5 + b_4z_3^4 + b_3z_3^3 + b_2z_3^2 + b_1z_3 + b_0)(z_3+\mu)}{z_3^2(z_3-z_1)(z_3-z_2)(z_3-z_4)(z_3-z_5)(z_3-z_6)}\right\}(e^{z_3t}-1) \\ + \left\{\dfrac{(z_4^5 + b_4z_4^4 + b_3z_4^3 + b_2z_4^2 + b_1z_4 + b_0)(z_4+\mu)}{z_4^2(z_4-z_1)(z_4-z_2)(z_4-z_3)(z_4-z_5)(z_4-z_6)}\right\}(e^{z_4t}-1) \\ + \left\{\dfrac{(z_5^5 + b_4z_5^4 + b_3z_5^3 + b_2z_5^2 + b_1z_5 + b_0)(z_5+\mu)}{z_5^2(z_5-z_1)(z_5-z_2)(z_5-z_3)(z_5-z_4)(z_5-z_6)}\right\}(e^{z_5t}-1) \\ + \left\{\dfrac{(z_6^5 + b_4z_6^4 + b_3z_6^3 + b_2z_6^2 + b_1z_6 + b_0)(z_6+\mu)}{z_6^2(z_6-z_1)(z_6-z_2)(z_6-z_3)(z_6-z_5)(z_6-z_4)}\right\}(e^{z_6t}-1) \end{array}\right\} - K_2(t-w) \tag{67}$$

To analyze the behavior of the system, we conducted an analysis where we vary the values of the parameter such as failure rates (λ and λ_1), transition rate (λ_m) and transition rate of completion of warranty (α). Based on it, the ranges of the system reliability and profit are computed and depicted in Tables 1 and 2 respectively. Further, to investigate the effect of individual component onto the system reliability, we vary the parameter λ from 0.01 to 0.03 and then further to 0.05 by fixing other parameters. By doing this, we compute that at time say 15 units, reliability of system decreased by 25.58% and further to 25.51% respectively. However, the complete variation of reliability with λ is summarized in Figure 2(a). On the other hand, if we increase the parameter λ_1 from 0.02 to 0.04 and further to 0.06 then reliability of the system decreases with the passage of time from 0.849405 to 0.84619 and then further to 0.843514 for the time 15units. The complete variation of this parameter is shown in Figure 2(b). However, Figures 2(c) and 2(d) respectively depicts the variation of the reliability with respect to the parameters α and λ_m .From these graphs, we conclude that the effect of α is more on to the system reliability than λ_m.

Table-1: Effect of failure rates (λ and λ_1), transition rate (λ_m) and transition rate of completion of warranty (α) on Reliability of the system $\left(R(t)\right)$

Time (t)	λ_1=0.02, α=0.003, λ_m=0.04	λ_1=0.02, α=0.003, λ_m=0.04	λ=0.01, α=0.003, λ_m=0.04	λ=0.01, λ_1=0.02, λ_m=0.04	λ=0.01, λ_1=0.02, α=0.003
	$R(t)$ (for λ =0.01)	$R(t)$ (for λ =0.03)	$R(t)$ (for λ_1 =0.04)	$R(t)$ (for α =0.005)	$R(t)$ (for λ_m =0.06)
10	0.899114	0.7378251	0.897294	0.895363	0.897294
11	0.889088	0.7154459	0.886992	0.884676	0.886992
12	0.8791	0.6937016	0.876723	0.873994	0.876723
13	0.869154	0.672577	0.866496	0.863327	0.866496
14	0.859254	0.652057	0.856317	0.852681	0.856317
15	0.849405	0.6321266	0.846192	0.842066	0.846192
16	0.83961	0.6127712	0.836125	0.831487	0.836125
17	0.829873	0.5939763	0.826122	0.820952	0.826122

Table 2: Effect of repair cost (K_2), PM rate (μ_2), transition rate of completion of warranty (α), inspection rate (h) and failure rate of degraded unit (λ_2) on expected profit ($H(t)$)

Time (t)	λ=0.01, μ=0.2, λ_2=0.04, λ_m=0.04, α=0.003, λ_1=0.02, p=0.6, μ_1=0.1 q=0.4 μ_2=0.4, h=0.5, w=3, K_1=500	λ=0.01, μ=0.2, λ_1=0.02, λ_2=0.04, λ_m=0.04, α=0.003, p=0.6, μ_1=0.1 q=0.4 μ_2=0.4, h=0.5, w=3, K_1=500	λ=0.01, λ_1=0.02, λ_2=0.04, λ_m=0.04, K_2=150, α=0.003, p=0.6, μ_1=0.1 q=0.4, μ=0.2, h=0.5, w=3, K_1=500	λ=0.01, λ_1=0.02, λ_2=0.04, λ_m=0.04, K_2=150, p=0.6, μ_1=0.1, μ=0.2 q=0.4, μ_2=0.4 h=0.5, w=3, K_1=500	λ=0.01, λ_1=0.02, λ_2=0.04, λ_m=0.04, μ_1=0.1, μ=0.2, K_2=150, α=0.003, q=0.4, μ_2=0.4 p=0.6, w=3, K_1=500	λ=0.01, λ_1=0.02, μ=0.2, λ_m=0.04, μ_1=0.1, h=0.5, K_2=150, α=0.003, q=0.4, μ_2=0.4 p=0.6, w=3, K_1=500
	$H(t)$	$H(t)$	$H(t)$	$H(t)$ (For α =0.006)	$H(t)$	$H(t)$

	(For K_2=150)	(For K_2=100)	(For μ_2=0.45)		(For h=0.7)	(For λ_2=0.02)
10	3984.972	4334.972	4211.116	3803.535	3801.078	3846.46
11	4293.353	4693.353	4563.899	4119.926	4101.091	4157.451
12	4593.596	5043.596	4908.402	4430.105	4393.336	4462.041
13	4885.589	5385.589	5244.42	4733.752	4677.603	4760.034
14	5169.253	5719.253	5571.809	5030.638	4953.735	5051.281
15	5444.534	6044.534	5890.476	5320.608	5221.625	5335.673
16	5711.41	6361.41	6200.37	5603.571	5481.207	5613.137
17	5969.883	6669.883	6501.481	5879.487	5732.451	5883.631

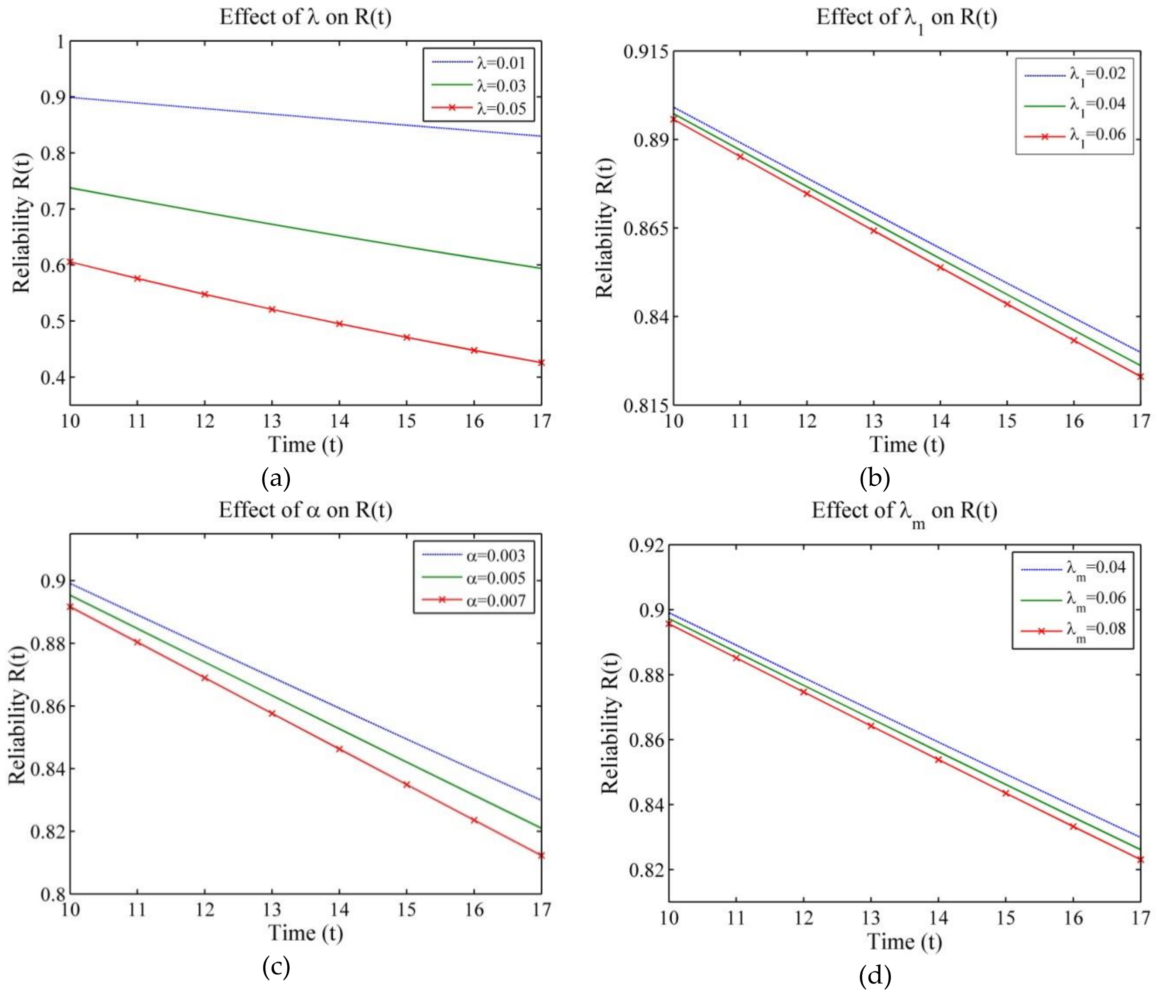

Figure 2: **Effect of the parameters $\lambda, \lambda_1, \alpha, \lambda_m$ on to the system reliability $R(t)$**

On the other hand, if we analyze the effect of the various parameters on to the expected profit $H(t)$ during the interval $(0,t]$ as given in Eq. (67). Firstly, we fix the different parameters as λ =0.01, λ_1=0.02, λ_2=0.03, λ_m=0.04, α=0.003, μ=0.2, p=0.7, μ_1=0.1, q=0.3 μ_2=0.3, h=0.4, w=3, K_1=500. Now, the effect of the parameters K_2, μ_2, α and h is analyzed on $H(t)$ are analyzed with the passage of the time. For it, firstly if we decrease the repair cost K_2 from 150 to 100 and then to 50, then the expected profit at a time 15 units is increased from 5444.534 to 6044.534 and to 6644.534. The complete variation of the profit is summarized in Figure 3(a). On the other hand, the expected profit increases by 8.19% and further to 8.03% when value of μ_2 changes from 0.4 to 0.45 then further to 0.5. The variation corresponding to this is given in Figure 3(b). Finally, in Figures 3(c) and 3(d) respectively give the variation of the profit values with α and h. From these graphs, it is interpreted that the expected profit increases if i decrease the values of α and h and conclude that the effect of h is more on to expected profit $H(t)$ than α. Also, Table 2 depicts that whenever the failure rate of degraded unit (λ_2) changes from 0.04 to 0.02 then expected profit $H(t)$ decreases from 5444.534 to 5335.763 corresponding to time 15 units. Similar observations have been found for different time periods in it.

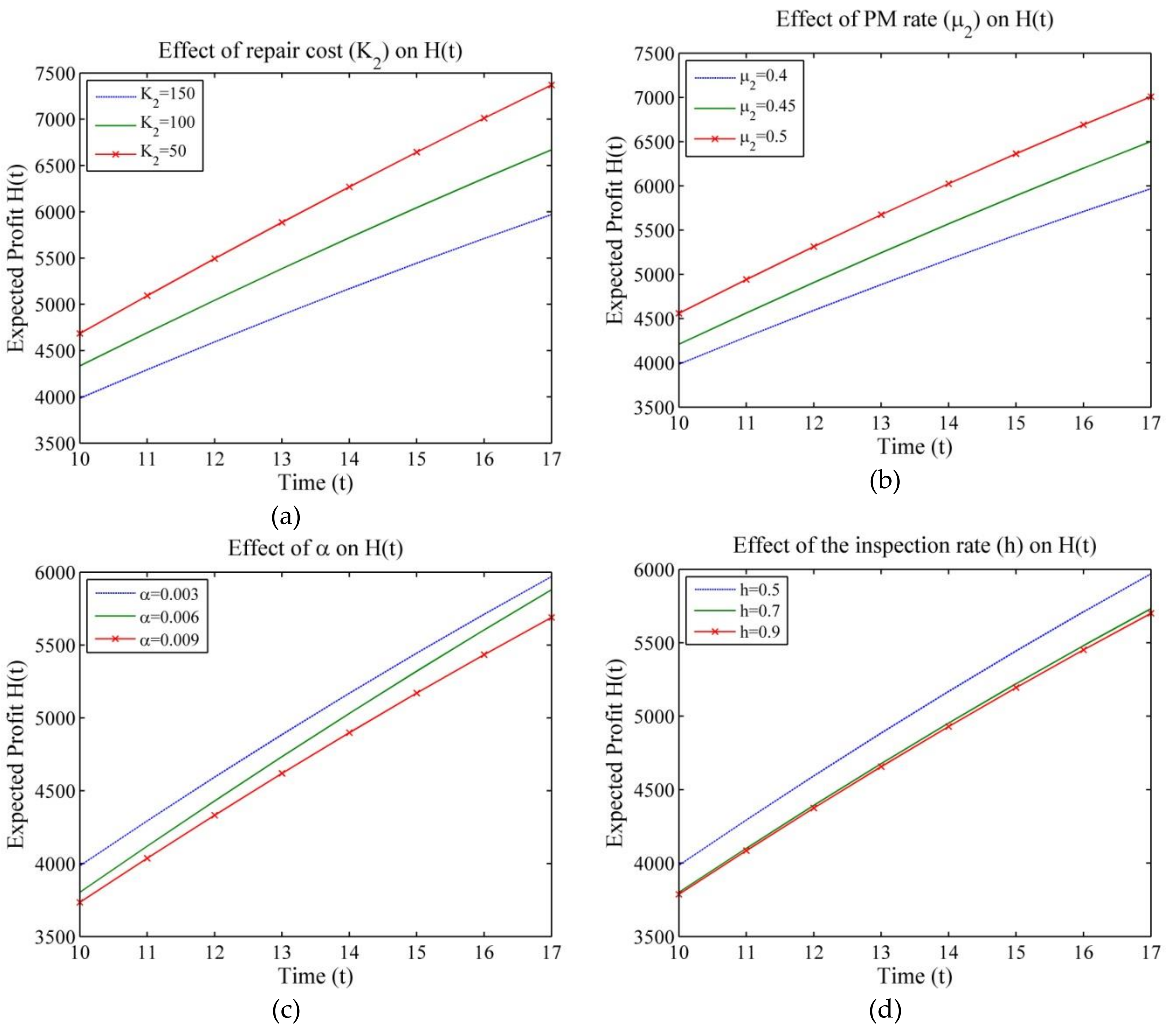

Figure 3. Effect of the parameters K_2, μ_2, α, and h on expected profit $H(t)$

As compared to the existing model proposed by Kadyan and Ramniwas [16], when we set parameters $\lambda_m, \mu_2, \lambda_2, p, q, h$ are all zero i.e., when beyond warranty, the system does not go under PM, there is no inspection of failure unit and the unit works as like a new unit after its repair, then the proposed model reduced to Kadyan and Niwas [16]. Additionally, it is observed that when parameters $\lambda_m = \mu_2 = \lambda_2 = 0$ i.e., the unit neither maintained nor works with reduced capacity, then the current model reduces to Niwas et al. [29] model. Thus, it is clearly seen that the proposed model is an extension of these existing model. Therefore, the study reveals that after getting PM beyond warranty, a system in which unit works with reduced capacity after its repair will be economically beneficial; if failed degraded unit is inspected for feasibility of repair. So, our studying model is more reasonable and advance than the existing models.

5. Conclusion

In the present paper, we have proposed an approach for analyzing a maintenance scheduling model using failure free warranty policy. In it, all the repairs during warranty are cost-free to the users, provided failures are not due to the negligence of users. For improving the performance of system PM is conducting beyond warranty and the unit works as new after PM but becomes degraded after its repair. Degraded unit is inspected by the repairman for feasibility of repair after its failure. Further, the effect of the various parameters on to system reliability and expected profit have been analyzed and found that by varying μ_2, h and α , expected profit is increased. Based on it, the system analyst may focus on μ_2 , h and α parameters so as to increase the performance and productivity of the system. In future work, we shall extend our work to different approaches such as reliability-cost optimization model, fuzzy reliability, and geometric process for two or more unit system models using Weibull- Gnedenko distribution [2, 34].

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