Imperfect Production Model for Price Sensitive Demand with Shortage

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Abstract

In this paper, we have presented an economic production inventory model considering non-linear demand depanding on selling price. Here, all imperfect quality items are reworked after the regular production process and the reworked items are considered as similar as good quality items. Rework is important in those businesses where last product is expensive and raw materials are insuficient. Now, our objective is to find out the optimal ordering lot size, optimal selling price and shortage, for which the profit of the system is maximum. A numerical example is presented to illustrate the validity of the model. Manageral implications has been presented based on sensitivity analysis.

Keywords: Dynamic pricing, Non linear demand, Optimal price settings, Imperfect item, Rework, Partial backlogging

AMS Subject Classification: 90B05, 90B30, 90B50

1 Introduction

Inventory control is an important part of business because it ensure quality control in business. Inventory management secure the business and help to smooth runing of business affair. Today, pricing and production strategies are two fundamental components of the daily operations for manufacturers, particularly in the presence of imperfect production system. Production system is one of the most important ascepts of company's business strategy. To avoid the off overeges and shortages of products, firm should carefully design the production process to enrich the business.

As a consequence of this paper, the topic of pricing with production system has recently been the focus of acadmic research increase diverse as economics, marketing and operation managemant. There has been several studies analysing condition under which different pricing strategys optimize the compnies profitability Bose *et al.* (1995) desinged an economoc order quantity inventory model for deteriorating products with linear demand and positive trend under allowable shortage and backlogging. Chakrabarti and chauduri (1997) presented an inventory model for perishable items. In this model the demend was taken as linear function and shortage in each cycle. Wee (1999) developed an inventory model for deteriorating items. In which, shortage was partially backlogged at constant rate and demand was taken to be linear function of selling price.

The production is an essential part of inventory system and not produced hundred persent perfect items. Many researchers designed a production inventory model under backlogging situation such as Chern *et al.* (2008); Dye *et al.* (2007); Lodree (2007); Leung (2008) Goyal and Imran (2008); Thannyam and Uthayakumar (2008); Cardenas and Berson (2009); Taleizadeh (2011) Roy *et al.* (2011). Das *et al.* (2011) presented an economic order quantity model for imperfect quality items with partialy backlogging. In this model, they also considered the cost of lost sale. Taleizadeh *et al.* (2012) proposed an EOQ model in which they considered a special sale price along with partial backlogging, and customer may take the advantage of discount in price. Lee and Dye (2012) formulated an economic order quantity model with shortage, in that model demand was taken stock dependent. They also considered the optimal ordering and preservation policies to maximize the total profit.

Several inventory model considered demand dependence on other factors such as product selling price and quality. Datta (2013) investigated an inventory model assuming that the demand depends on both the selling price and quality. Kumar *et al.* (2013) proposed EOQ model under the consideration of price-dependent demand, where the carrying cost is a function of the trade credit for deteriorating products. Sana (2010) designed an economic order quantity (EOQ) model in that model, the demand was considered as function of selling price and they also assumed the deterioration rate of defective item is time proportional. Sana (2011) suggested an inventory model in which, they taken the demand function as quadratic function and the selling price increases in each cycle, but demand decreases quadraticly with selling price.

By use of item preservation concept for deteriorating items, Khedlekar *et al.* (2016) conceptualized an EOQ in that model the demand was considered as function of selling price and linearly decreases. They considered as the profit is the concave function of the optimal selling price, also calculated the optimal selling price, the length of the replenishment cycle and the optimal preservation concept investment simultaneously. Mishra (2016) proposed a single-manufacturer single-retailer inventory model by incorporating preservation technology cost for defective items and determined optimal retail price, replenishment cycle and the cost of preservation lechnology.

Taleizadeh and Noori-daryan (2016) studid a production inventory model with a three-level decentralised supply chain with price sensitive demand. Haider *et al.* (2016) proposed an economic production quantity (EPQ) model from this they reveal if we make the discount in defective item and apply rework process then we get maximum profit. Teksan and Geunes (2016) reported an economic order quantity model for finished goods. In this model they they assumed that the demand rate was more price sensitive for supplier and customer both. Taleizadeh *et al.* (2017) outlined an imperfect production inventory model without shortages. Pal and Adhikari (2017) conceptualized an imperfect production inventory model with exponential partially backlogging with rework, in that model they assumed that all imperfect quality products are reworked after the regular production process and demand rate was price sensitive and it was monotonic decreasing function selling price. Among other researcher in the exposure, the notworthy contribution of Sarkar, Sana and Chaudhuri (2011); Yu, and Chen (2007); Wee, and Kuo (2013); Pal, Sana, and Chaudhuri (2014); Sarkar (2012, 2013); Haider, Salameh, and Nasr (2016); Tyyab and Sarkar (2016) should be mentioned.

We have considered an imperfect production model which depend on the selling price. We assumed, all the defective products are reworked just after the regular production process and no any scarp product is produced during production as well as reworking run time. Shortage occurs at the beginning of the cycle and production starts after backorder time and backlogging rate is variable. The price of goods is definitely shown to the customer at the beginning of time cycle in many situations. So it is very difficult to take the different price within same inventory cycle. In this paper we deal with the three issues: first, what will be selling price for the items, second one how much inventory should be produced and third one what time period shortage would be allowed in order to optimum profit.

2 Assumptions & Notations

2.1 Assumptions

The model is designed for infinite time horizon, This model is developed for single item, Production rate if perfect item p is constant and production rate of defective items is $p_d = xp$, where x is continuous randam variable, In this model the shortages occur at the beginning of the cycle and during the shortage time interval a fraction of the demand varying with waiting time is backlogged for the clients, who have patience to wait, assume that customers impatient function by

$B(\tau) = e^{-\alpha\tau}$, $\alpha > 0$, After the continue production process all imperfect items are reworked, The holding cost for both type (perfect and imperfect) items is the same, Every constant costs as inspection cost and purchasing cost are included within the production cost of the items, The demand function of the product is $D(s) = \varphi s^{-\eta}$; $\eta > 0$.

2.2 Notations

$[D(s)]$ – Demand function for good products,

$[I(t)]$ – On-hand inventory of product at time t in j^{th} cycle,

$[p]$ – Production rate for perfect item per units per unit time, [p_d] – Production rate for imperfect quantity items unit per unit time, $[x]$ – Percentage of produced imperfect quality items which is randam variable, $[f(x)]$ – Probability density function of x, $[r]$ – Rework rate of imperfect quality item per unit per unit time, $[\omega]$ – Backorder level, $[B(\tau)]$ – Customers impatient function, where τ is the waiting time for customer, $[c_h]$ – Holdig cost per item per unit time, $[c_{h_1}]$ – Holdig cost of reworked item per item per unit time, $[c_p]$– Production cost per unit of item, $[c_b]$– Backorder cost per item, $[c_k]$– Per production set-up cost, $[c_l]$– Lost sale cost per item, $[s]$– Selling price per item, $[\varphi]$– Stock dependent parameter, [Π]– The total profit,

1. – Average total profit,
2. – Excepted average total profit.

3 The Mathematical Model

Suppose a business start with shortage of products which are partially backlogged. The backlogging rate is a function of customer waiting time as $B(\tau) = e^{-\alpha\tau}$, $a > 0$, where τ is waiting time $\tau = t_1 - t$. Suppose the production start at time t_1 and it continue up to time t_3. Due to production run, all the products which are backlogged, during time period $[0, t_2]$ are provide at the time t_2. The production rate is considered constant. The qx amount of defective item is produced by the total production. The rework rate of defective products is r, and these are reworked after the regular production process. $\frac{qx}{r}$ is the amount of time required for reworking of defective items, where qx is total items produced and r is rework rate. There is the same price of good products and reworked product and demand rate is depend on selling price and defined as,

$$D(s) = \varphi s^{-\eta} \tag{1}$$

We take $T_i = t_i - t_{i-1}$.

For the Time period $0 \le t \le t_1$, the differential equation governing the inventory level is

$$\frac{dI}{dt} = -D(s)B(\tau) \tag{2}$$

with the boundary condition$I(0) = 0$ and $I(t_1) = -\omega$ where $\tau = t_1 - t$.

The solution of above differential equation by using the boundary condition is

$$I(t) = \frac{D(s)e^{-}at - e^{a(t_1 - t)}}{a} \tag{3}$$

and using the boundary condition $I(t_1) = -\omega$, we get

$$\omega = \frac{D(s)(1-e^{-at_1})}{a} \tag{4}$$

The backorder cost during $0 \le t \le t_1$ is

$$c_b \int_0^{t_1} (I(t))dt = \frac{c_b D(s)\{1-at_1e^{-at_1}-e^{-at_1}\}}{a^2} \tag{5}$$

The demand rate is D(s), out of this only $D(s)e^{-a(t_1-t)}$ is fullfilld during $[0, t_1]$ and $D(s) - D(s)e^{-a(t_1-t)}$ wich not fullfilld. Then the cost of lost sale is given by

$$c_l \int_0^{t_1} D(s)\{1-e^{-a(t_1-t)}\}dt = \frac{c_l D(s)(at_1-1+e^{-at_1})}{a} \tag{6}$$

For the time interval $t_1 \le t \le t_2$, the governing differential equation of inventory level is

$$\frac{dI}{dt} = p - p_d - D(s) \tag{7}$$

with boundary condition $I(t_1) = -\omega$, $I(t_2) = 0$

Then the solution of above differential equation is

$$I(t) = \{(1-x)p - D(s)\}(t - t_2) \tag{8}$$

using the condition $I(t) = -\omega$, we have

$$\omega = \{(1-x)p - D(s)\}T_2, \tag{9}$$

where $T_2 = t_2 - t_1$

The cost of backorder in time interval $t_1 \le t \le t_2$ is

$$c_b \int_0^{t_1} (I(t))dt = \frac{c_b \omega T_2}{2} \tag{10}$$

Eq. (9) & Eq. (10) leads the back order cost during $t_1 \le t \le t_2$

$$= \frac{c_b \omega^2}{2\{(1-x)p-D(s)\}} \tag{11}$$

For the time interval $t_2 \le t \le t_3$, the governing differential equation of inventory level is

$$\frac{dI}{dt} = p - p_d - D(s) \tag{12}$$

with boundary condition $I(t_2) = 0$, $I(t_3) = z_3$ where z_3, is inventory level of good product.

Then the solution of above differential equation is

$$I(t) = \{(1-x)p - D(s)\}(t - t_2) \tag{13}$$

using $I(t_3) = z_3$, we get

$$z_3 = \{(1-x)p - D(s)\}T_3 \tag{14}$$

The holding cost for good items in time period $t_2 \le t \le t_3$ is

$$c_h \int_{t_2}^{t_3} (I(t))dt = \frac{c_h z_3 T_3}{2} \tag{15}$$

Now $T_2 + T_3 = \frac{q}{p}$, using the Eq. (9) & Eq. (14) the holding cost is

$$= \frac{c_h}{2}\{(1-x)p - D(s)\}\frac{q^2}{p^2} - \frac{c_h q\omega}{p} + \frac{c_h \omega^2}{2\{(1-x)p-D(s)\}} \tag{16}$$

The differential equation for time period $t_3 \le t \le t_4$, is

$$\frac{dI}{dt} = r - D(s) \tag{17}$$

with boundary condition $I(t_3) = z_3$, $I(t_4) = z_4$, where z_4 is the highest inventory level of good items

$$I(t) = z_3 + \{r - D(s)\}(t - t_3) \tag{18}$$

by using the condition $I(t_4) = z_4$

$$z_4 - z_3 = \{r - D(s)\}T_4 \tag{19}$$

After some simplification and putting $T_4 = \frac{qx}{r}$, we get

$$z_4 = q\{1 - \frac{D(s)(r+x)}{pr}\} - \omega \tag{20}$$

Holding cost for good poducts for the time interval $t_3 \le t \le t_4$ is given by

$$c_h \int_{t_2}^{t_3} (I(t))dt = \frac{c_h}{2}(z_3 + z_4)T_4 \tag{21}$$

Putting the value from Eq. (19) then holding cost

$$\begin{aligned}
&= \frac{c_h T_4}{2}\{z_3 + z_3 + \{r - D(s)\}T_4\} \\
&= c_h T_4 z_3 + \frac{c_h\{r-D(s)\}T_4{}^2}{2} \\
&= c_h\{(1-x)p - D(s)\}T_3 T_4 \quad .2cm by Eq. (3.14) \\
&= c_h\{(1-x)p - D(s)\}\left(\frac{q}{p} - \frac{\omega}{\{(1-x)p - D(s)\}}\right)\frac{qx}{r} + \{r - D(s)\}\frac{q^2x^2}{r^2} \\
&= c_h\{(1-x)p - D(s)\}\frac{q^2x}{pr} - \frac{c_h\omega qx}{r} + \frac{c_h}{2}\{r - D(s)\}\frac{q^2x^2}{r^2}
\end{aligned} \tag{22}$$

Now it can be seen that the difective products produced during the time interval $t_1 \le t \le t_3$ at rate p_d. The defective products are reworked perfectly during the time interval $[t_3, t_4]$ by the rework rate r. In this system there is no defective items after time $t = t_4$.

The differential equation for time period $t_4 \le t \le t_5$, that show inventory level is

$$\frac{dI}{dt} = -D(s) \tag{23}$$

with boundary conditions $I(t_4) = z_4$ and $I(t_5) = 0$

Then the solution of this differential equation

$$I(t) = D(s)(t_5 - t) \tag{24}$$

$$By using \quad .2cm I(t) = z_4, \quad .5cm z_4 = D(s)T_5 \tag{25}$$

Holding cost for the time interval $t_4 \le t \le t_5$ is given by

$$\begin{aligned}
c_h \int_{t_4}^{t_4} (I(t))dt &= \frac{c_h}{2} z_4 T_5 \\
&= \frac{c_h z_4{}^2}{2D(s)} \\
&= \frac{c_h}{2D(s)}\left[q\{1 - \frac{\beta(r+x)}{pr}\} - \omega\right]^2
\end{aligned} \tag{26}$$

The inventory of defective products is given figure(2) then the differential equation for time period $t_1 \le t \le t_3$

$$\frac{dI_d}{dt} = p_d, \quad .2cm with boudary condition \quad .2cm I_d(t_1) = 0, \quad .2cm I_d(t_3) = qx \tag{27}$$

Then the solution is

$$I_d(t) = p_d(t - t_1) \tag{28}$$

Holding cost for the defective products is

$$c_h \int_{t_1}^{t_3} (I_d(t))dt = \frac{c_h q^2 x}{2p} \tag{29}$$

For time interval $t_3 \le t \le t_4$ the governing differential equation inventory level of the defective item, is given by

$$\frac{dI_d}{dt} = -r, \quad .2cm with boudary condition \quad .2cm I_d(t_3) = qx, \quad .2cm I_d(t_4) = 0 \tag{30}$$

Then the solution is

$$I_d(t) = r(t_4 - t) \tag{31}$$

The holding cost of reworked items

$$c_{h_r} \int_{t_3}^{t_4} (I_d(t))dt = \frac{c_{h_1} q^2 x^2}{2r} \tag{32}$$

The total profit = Revenue - total cost

= Revenue - (backorder cost + cost of lost sale + holding cost for good and defective products + holding cost for reworked items + purchase cost + repairing cost for defective items + set-up cost)

$$\begin{aligned}\Pi(q,t_1,s) = \quad & sq - \frac{c_b D(s)\{1-at_1e^{-at_1}-e^{-at_1}\}}{a^2} - \frac{c_l D(s)(at_1-1+e^{-at_1})}{a} \\ & -\frac{c_b\omega^2}{2\{(1-x)p-D(s)\}} - \frac{c_h}{2}(1-x)\frac{q^2}{p} + \frac{c_h}{2}\frac{D(s)q^2}{p^2} + \frac{c_h q\omega}{p} \\ & -\frac{c_h\omega^2}{2\{(1-x)p-D(s)\}} - c_h\{(1-x)p-D(s)\}\frac{q^2x}{pr} + \frac{c_h\omega qx}{r} \\ & -\frac{c_h}{2}\{r-D(s)\}\frac{q^2x^2}{r^2} - \frac{c_h}{2D(s)}\left[q\{1-\frac{\beta(r+x)}{pr}\}-\omega\right]^2 \\ & -\frac{c_h q^2 x}{2p} - \frac{c_{h_1}q^2x^2}{2r} - c_p q - c_r qx - k\end{aligned} \tag{33}$$

The total average profit of the model

$$\begin{aligned}\Pi_{\tilde{a}\tilde{t}\tilde{p}} = \quad & \frac{D(s)}{q}\Pi(q,t_1,s) \\ = \quad & \frac{D(s)}{q}[sq - \frac{c_b D(s)\{1-at_1e^{-at_1}-e^{-at_1}\}}{a^2} - \frac{c_l D(s)(at_1-1+e^{-at_1})}{a} \\ & -\frac{c_b\omega^2}{2\{(1-x)p-D(s)\}} - \frac{c_h}{2}(1-x)\frac{q^2}{p} + \frac{c_h}{2}\frac{D(s)q^2}{p^2} + \frac{c_h q\omega}{p} \\ & -\frac{c_h\omega^2}{2\{(1-x)p-D(s)\}} - c_h\{(1-x)p-D(s)\}\frac{q^2x}{pr} + \frac{c_h\omega qx}{r} \\ & -\frac{c_h}{2}\{r-D(s)\}\frac{q^2x^2}{r^2} - \frac{c_h}{2D(s)}\left[q\{1-\frac{\beta(r+x)}{pr}\}-\omega\right]^2 \\ & -\frac{c_h q^2 x}{2p} - \frac{c_{h_1}q^2x^2}{2r} - c_p q - c_r qx - k]\end{aligned} \tag{34}$$

The total expected average profit of the model

$$\begin{aligned}\Pi_{\tilde{a}\tilde{t}\tilde{p}} = \quad & \frac{D(s)}{q}[sq - \frac{c_b D(s)\{1-at_1e^{-at_1}-e^{-at_1}\}}{a^2} - \frac{c_l D(s)(at_1-1+e^{-at_1})}{a} \\ & -\frac{c_b\omega^2}{2\{(1-m)p-D(s)\}} - \frac{c_h}{2}(1-m)\frac{q^2}{p} + \frac{c_h}{2}\frac{D(s)q^2}{p^2} + \frac{c_h q\omega}{p} \\ & -\frac{c_h\omega^2}{2\{(1-m)p-D(s)\}} - c_h\{(1-m)p-D(s)\}\frac{q^2m}{pr} + \frac{c_h\omega qm}{r} \\ & -\frac{c_h}{2}\{r-D(s)\}\frac{q^2(m^2+\sigma^2)}{r^2} - \frac{c_h}{2D(s)}\left[q\{1-\frac{\beta(r+m)}{pr}\}-\omega\right]^2 \\ & -\frac{c_h q^2 m}{2p} - \frac{c_{h_1}q^2(m^2+\sigma^2)}{2r} - c_p q - c_r qm - k]\end{aligned} \tag{35}$$

Eq. (4) & Eq. (35) leads to

$$\Pi_{\tilde{e}\tilde{a}\tilde{t}\tilde{p}} = f_1(q,s,t_1) = u_0(s) + u_1(s,t_1) + \frac{u_2(s,t_1)}{\Psi(s)q} \tag{36}$$

where

$$\begin{aligned}
u_0(s) = \quad & x_{00} + x_{01}D(s) + x_{02}D(s)^2 \\
u_1(s,t_1) = \quad & w_1(s) + w_2(s)e^{-at_1} \\
u_2(s,t_1) = \quad & v_1(s)e^{-2at_1} + \{v_2(s) + t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s) \\
\Psi(s) = \quad & 2a^2\{(1-m)p - D(s)\} \\
v_1(s) = \quad & \lambda_{11}D(s)^2 + \lambda_{12}D(s)^3 \\
v_2(s) = \quad & \lambda_{21}D(s) + \lambda_{22}D(s)^2 + \lambda_{22}D(s)^3 \\
v_3(s) = \quad & \lambda_{31}D(s)^2 + \lambda_{32}D(s)^3 \\
v_4(s) = \quad & \lambda_{41}D(s)^2 + \lambda_{42}D(s)^3 \\
v_5(s) = \quad & \lambda_{51}D(s) + \lambda_{52}D(s)^2 + \lambda_{53}D(s)^3 \\
w_1(s) = \quad & x_{11}D(s) + x_{12}D(s)^2 \\
w_2(s) = \quad & x_{21}D(s) + x_{22}D(s)^2 \\
\lambda_{11} = \quad & -c_hp(1-m); \lambda_{12} = -c_b; \lambda_{21} = 0 \\
\lambda_{22} = \quad & 2c_bp(1-m) + 2\{c_hp(1-m) - c_lp(1-m)a\}; \lambda_{23} = 2c_la \\
\lambda_{31} = \quad & 2c_bp(1-m)a; \lambda_{32} = -2c_ba; \lambda_{41} = -2c_lp(1-m)a^2 \\
\lambda_{51} = \quad & 2kp(1-m)a^2; \lambda_{52} = 2c_bp(1-m) + \{-c_hp(1-m) + c_lp(1-m)a\} \\
\lambda_{53} = \quad & c_b - 2c_la; x_{00} = \frac{-c_h}{2}; \\
x_{01} = \quad & -\frac{c_h(1-m)}{2p} - \frac{c_hm}{2p} - \frac{c_h(1-m)m}{r} - \frac{c_h(m^2+\sigma^2)}{2r} - \frac{c_{h_1}(m^2+\sigma^2)}{2r} + \frac{c_h(m+r)}{pr} \\
x_{02} = \quad & \frac{c_h}{2p^2} + \frac{c_hm}{pr} + \frac{c_h(m^2+\sigma^2)}{2r^2} - \frac{c_h(m+r)^2}{2p^2r^2}; x_{11} = -c_p + s - c_rm + \frac{c_h}{a} \\
x_{12} = \quad & \frac{c_hm}{ra} - \frac{c_hm}{pra}; x_{21} = -\frac{c_h}{a}; x_{22} = -\frac{c_hm}{ra} + \frac{c_hm}{pra}
\end{aligned}$$

Proposition. The profit function $f_1(q,s,t_1)$ is concave if the corresponding Hessian matrix H of expected profit function is negative definite. where

$$H=\begin{pmatrix} \frac{\partial^2 f_1}{\partial q^2} & \frac{\partial^2 f_1}{\partial s\,\partial q} & \frac{\partial^2 f_1}{\partial q\,\partial t_1} \\ \frac{\partial^2 f_1}{\partial s\,\partial q} & \frac{\partial^2 f_1}{\partial s^2} & \frac{\partial^2 f_1}{\partial t_1\,\partial s} \\ \frac{\partial^2 f_1}{\partial q\,\partial t_1} & \frac{\partial^2 f_1}{\partial t_1\,\partial s} & \frac{\partial^2 f_1}{\partial {t_1}^2} \end{pmatrix}$$

Proof: We have

$$\Pi_{\tilde{e}\tilde{a}\tilde{t}\tilde{p}} = f_1(q,s,t_1) = u_0(s) + u_1(s,t_1) + \frac{u_2(s,t_1)}{\Psi(s)q}$$

$$\frac{\partial f_1}{\partial q} = x_{00} + x_{01}D(s) + x_{02}D(s)^2 - \frac{v_1(s)e^{-2at_1} + \{v_2(s)+t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)}{q^2\Psi(s)}$$

$$\frac{\partial f_1}{\partial s} = w_1'(s) + w_2'(s)e^{-at_1} + q\{x_{01}D'(s) + 2x_{02}D'(s)D(s)\} - \frac{\{v_1(s)e^{-2at_1} + \{v_2(s)+t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)\}\Psi'(s)}{q\Psi(s)^2} + \frac{v_1'(s)e^{-2at_1} + \{v_2'(s)+t_1v_3'(s)\}e^{-at_1} + v_4'(s)t_1 + v_5'(s)}{q\Psi(s)}$$

$$\frac{\partial f_1}{\partial t_1} = -aw_2(s)e^{-at_1} + \frac{-2av_1(s)e^{-2at_1} + e^{-at_1}v_3(s) - \{v_2(s)+t_1v_3(s)\}ae^{-at_1} + v_4(s)}{q\Psi(s)}$$

Solve above equations by puting

$$\frac{\partial f_1}{\partial q} = 0, \frac{\partial f_1}{\partial s} = 0, \frac{\partial f_1}{\partial t_1} = 0$$

and get the values of variable q, s, t_1

$$x_{00} + x_{01}D(s) + x_{02}D(s)^2 - \frac{v_1(s)e^{-2at_1} + \{v_2(s)+t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)}{q^2\Psi(s)} = 0$$

Then

$$q = \sqrt{\frac{v_1(s)e^{-2at_1} + \{v_2(s)+t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)}{\{x_{00} + x_{01}D(s) + x_{02}D(s)^2\}\Psi(s)}} \tag{37}$$

Substituting the value of q in the Eq. $\frac{\partial f_1}{\partial s} = 0$ & $\frac{\partial f_1}{\partial t_1} = 0$ and solving them, we get the solution of decision variable q, s, t_1 of the model.

If the second order condition of of optimization method will be satisfied then above solution will be optimal.

Now the second order derivatives

$$\frac{\partial^2 f_1}{\partial q^2} = \frac{2[v_1(s)e^{-2at_1} + \{v_2(s)+t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)]}{q^2\Psi(s)} \tag{38}$$

$$\frac{\partial^2 f_1}{\partial q\,\partial t_1} = -\frac{-2v_1(s)e^{-2at_1} + v_3(s)e^{-at_1} + v_4(s) - \{v_2(s)+t_1v_3(s)\}ae^{-at_1}}{q^2\Psi(s)} \tag{39}$$

$$\begin{aligned} \frac{\partial^2 f_1}{\partial s^2} = & -\frac{2\{v_1'(s)e^{-2at_1} + \{v_2'(s)+t_1v_3'(s)\}e^{-at_1} + v_4'(s)t_1 + v_5'(s)\}\Psi'(s)}{q\Psi(s)^2} \\ & + \frac{2\{v_1(s)e^{-2at_1} + \{v_2(s)+t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)\}\{\Psi'(s)\}^2}{q\Psi(s)^3} \\ & + \frac{v_1''(s)e^{-2at_1} + \{v_2''(s)+t_1v_3''(s)\}e^{-at_1} + v_4''(s)t_1 + v_5''(s)}{q\Psi(s)} \\ & + w_1''(s) + w_2''(s)e^{-2at_1} + q[x_{01}D''(s) + 2x_{02}D''(s)D(s) + 2x_{02}\{D'(s)\}^2] \end{aligned} \tag{40}$$

$$\begin{aligned} \frac{\partial^2 f_1}{\partial s\,\partial q} = & -\frac{\{v_1'(s)e^{-2at_1} + \{v_2'(s)+t_1v_3'(s)\}e^{-at_1} + v_4'(s)t_1 + v_5'(s)\}}{q^2\Psi(s)} \\ & + \frac{\{v_1(s)e^{-2at_1} + \{v_2(s)+t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)\}\{\Psi'(s)\}}{q\Psi(s)^2} \\ & + 2x_{01}x_{02}D'(s) + D'(s)D(s) \end{aligned} \tag{41}$$

$$\frac{\partial^2 f_1}{\partial {t_1}^2} = a^2 w_2(s)e^{-at_1} + \frac{4a^2 v_1(s)e^{-2at_1} - 2e^{-at_1}v_3(s) + \{v_2(s)+t_1 v_3(s)\}a^2 e^{-at_1}}{q\Psi(s)} \tag{42}$$

$$\frac{\partial^2 f_1}{\partial s\,\partial t_1} = \frac{-2av_1'(s)e^{-2at_1} + v_3'(s)e^{-at_1} + v_4'(s) - \{v_2'(s)+t_1 v_3'(s)\}ae^{-at_1}}{q\Psi(s)} - aw_2'(s)e^{-at_1} - \frac{[-2av_1(s)e^{-2at_1} + e^{-at_1}v_3(s) - \{v_2(s)+t_1 v_3(s)\}ae^{-at_1} + v_4(s)]\Psi'(s)}{q\Psi(s)^2} \tag{43}$$

putting all values of second derivatives in Hessian matrix

$$H = \begin{pmatrix} \frac{\partial^2 f_1}{\partial q^2} & \frac{\partial^2 f_1}{\partial s\,\partial q} & \frac{\partial^2 f_1}{\partial q\,\partial t_1} \\ \frac{\partial^2 f_1}{\partial s\,\partial q} & \frac{\partial^2 f_1}{\partial s^2} & \frac{\partial^2 f_1}{\partial t_1\,\partial s} \\ \frac{\partial^2 f_1}{\partial q\,\partial t_1} & \frac{\partial^2 f_1}{\partial t_1\,\partial s} & \frac{\partial^2 f_1}{\partial {t_1}^2} \end{pmatrix}$$

If all eigen values are nagetive i.e Hessian matrix H of expected profit function is negative definite, then the profit function is concave.

4 Numerical Example & Sensitivity Analysis

Consider a numerical example taking the demand function as given in Eq. (1)

4.1 Example

We consider the demand function $D(s) = \varphi s^{-\eta}$ and the value of the parameter in appropriate units are as follows $\eta = 1.2$, $c_l = 2$ per unit per unit time, $c_b = 1.5$ per unit per unit time, $k = 500$, $c_h = 1$ per unit per unit time, $c_{h_1} = 1$ per unit per unit time, $c_r = 1.5$ per unit, $c_p = 4$ per unit, $\varphi = 3000$, $r = 1200$ units per unit time, $\alpha = 1.6$, $m = 0.05$, $\sigma^2 = \frac{1}{1200}$, $p = 800$ units per unit time, and randam variable follows uniform distribution in the interval (0,0.1). Then the optimal values for the model are $f_1^* = 1107.4$, $s^* = 39.15$, $q^* = 206$, $t_1^* = 0.69$. These values are optimal as the eigen value of the Hessian matrix $\begin{pmatrix} \frac{\partial^2 f_1}{\partial q^2} & \frac{\partial^2 f_1}{\partial s\,\partial q} & \frac{\partial^2 f_1}{\partial q\,\partial t_1} \\ \frac{\partial^2 f_1}{\partial s\,\partial q} & \frac{\partial^2 f_1}{\partial s^2} & \frac{\partial^2 f_1}{\partial t_1\,\partial s} \\ \frac{\partial^2 f_1}{\partial q\,\partial t_1} & \frac{\partial^2 f_1}{\partial t_1\,\partial s} & \frac{\partial^2 f_1}{\partial {t_1}^2} \end{pmatrix}$ are negative. i.e -31.88, -0.39, -0.002. So the profit function is concave.

4.2 Sensitive Analysis

We observed the sensitiveness of the key parameters which help the decision makers to take appropriate decision on their marketing strategy.

From Table 1, we observed that, with the increasing values of holding cost of products there is a minor change in the optimal lot size and selling price, but the expected average profit decreases shortlly and there is negligible changes in the period of shortage. It is clear that higher holding cost reduce the lot size. So smaller commodity causes the increas in shortage period. In this situation the expected average total profit in decreasing order.

From Table 2, we noticed that, the optomal lot size, shortage period and selling price are increasing with increasing production cost and we also fund that expected profit decreases with increasing the production cost.

From Table 3, we observed that, with the increasing values of backorder cost there is a minor changes in the optimal lot size and selling price, and there is negligible changes in the expected profit and shortage period.

We observed that, with the increasing values of parameter η there is a major change in the optimal lot size and selling price, the expected average profit decreases and there is negligible changes in the period of shortage (table 5). With the changes of parameter a, ther are minor change in optimal lot size, selling price and expected average profit with the increasing values of parameter a shortage period decreases. (table 4). If the demand function parameter φ increases, the expected average profit, and lot size increases highly while the selling price and shortage period decreases (from table 6).

Now we have followed graphical analysis method three-dimensional (3D) plots for the profit function $\Pi_{\tilde{e}\tilde{a}\tilde{t}\tilde{p}}$, Figure 1 and 2 present the piecewise 3D plots for the profit function, $\Pi_{\tilde{e}\tilde{a}\tilde{t}\tilde{p}}$, versus the two corresponding variables subsequently out of the three variables, s, q and t_1 . In each Figure 1 and 2, 3D plot of function, t_1 using the other two variables, s and q at a fixed shortage time period t_1 and 3D plot of function, $\Pi_{\tilde{e}\tilde{a}\tilde{t}\tilde{p}}$, using the other two variables, s and t_1 at a fixed lot-size q.

Table 1: Changges in c_h

c_h	s	t_1	q	f_1
1	39.15	0.69	206	1107.4
1.1	39.49	0.72	195	1099.26
.2	37.30	0.72	197	1090.8
.3	25.69	0.70	275	1052.24

Table 2: Changges in c_p

c_p	s	t_1	q	f_1
3	31.30	0.64	240	1144.37
4	39.15	0.69	106	1107.4
	46.83	0.74	183	1074.41
	54.39	0.78	166	1047.31

Table 3: Changges in c_b

c_b	s	t_1	q	f_1
.5	19.88	0.73	446	1027.81
1	39.06	0.72	207	1107.82
.5	39.15	0.64	206	1107.4
	39.24	0.66	206	1107.01

Table 4: Changges in a

a	s	t_1	q	f_1
1.3	38.90	0.82	209	1109.22
1.4	31.78	0.73	245	1103.37
.5	32.62	0.70	240	1103.95
.6	39.15	0.69	206	1107.4

Table 5: Changges in η

η	s	t_1	q	f_1
1.1	77.56.15	0.78	167	1687.86
1.2	39.15	0.69	206	1107.4
.3	22.29	0.68	289	744.67
.4	22.71	0.69	210	519.93

Table 6: Changges in φ

φ	s	t_1	q	f_1
3000	39.15	0.69	206	1107.4
3500	37.77	0.65	235	1308.24
	36.69	0.62	254	1510.38
	35.83	0.59	277	1713.58

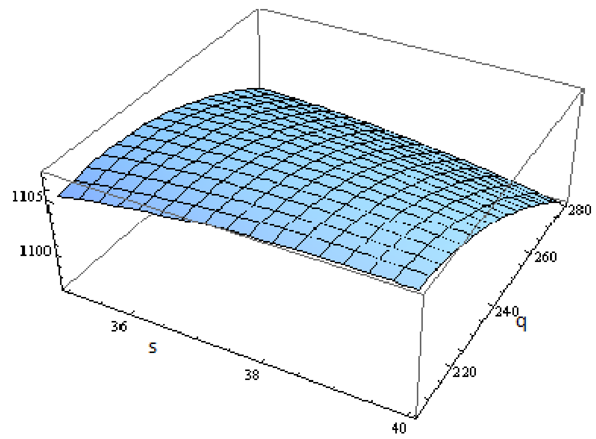

Fig.1. Expected average total profit versus quantity and price

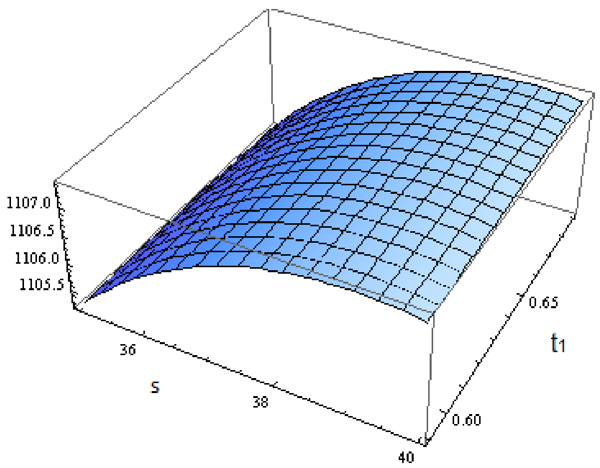

Fig.2. Expected average total profit versus shortage time and price

5 Conclusion

Several manufacturers have to call back their items after use and rework on them to make protect. satisfy the demands with new ones in recent years. This type of remanufacturing system may prevent disposal cost and reduce environment dilemmas. To overcome this problem, an economic production quantity model has been portrayed for imperfect items with rework and production.

We have presented an imperfect production inventory model by considering demand as nagetive power function of selling price. The shortage occurs in begning bears the more cost for inventory manager, but it helps to project the product and optimize the selling price also. We have also illustrated the model numerically for demand depending on selling price. In the sensitivity of parameters of the model, we observed that the optimal expected average profit decreases with higher holding cost of items and optimal expected average profit increases with higher value of parameter φ.

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