A Discrete Parametric Markov-Chain Model Of A Two Unit Cold Standby System With Appearance And Disappearance Of Repairman

Rakesh Gupta, Parul Bhardwaj

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Department of Statistics,
Ch. Charan Singh University, Meerut
E-mails: smprgccsu@gmail.com,
parul.bhardwaaj@gmail.com

Abstract

The paper deals with the stochastic analysis of a two-identical unit cold standby system model assuming two modes- normal and total failure. A single repair facility appears in and disappears from the system randomly. The random variables denoting the failure time, repair time, time to appearance and time to disappearance of repairman are independent of discrete nature having geometric distributions with different parameters. The various measures of system effectiveness are obtained by using regenerative point technique.

Keywords: Transition probability, mean sojourn time, regenerative point, reliability, MTSF, availability of system, busy period of repairman.

1. Introduction

In reliability modeling repair maintenance is concerned with increasing system reliability, availability and net expected profit earned by the system with the implementation of major changes in the failed components of a unit. In order to achieve this goal, the failed components of a unit may be either repaired or replaced by new ones.

Numerous authors [1, 5, 6, 7, 8] have analyzed various system models considering different repair policies. Goyal and Murari [1] analyzed a two-identical unit standby system model considering two types of repairmen- regular and expert. The regular repairman is always available with the system whereas expert repairman can be made available from the outside instantaneously. Mokaddis et. al. [6] obtained the busy period analysis of a man-machine system model assuming different physical conditions of repairman. Pandey and Gupta [7] analyzed a two-unit standby redundant system model assuming that a delay occurs due to some administrative action in locating and getting repairman available to the system. Sharma et. al. [8] considered a two-unit parallel system assuming dependent failure rates and correlated working and rest time of repairman.

Gupta et. al. [5] investigated a two unit standby system with correlated failure and repair and random appearance and disappearance of repairman. They have also assumed that the failure time, repair time, time to appearance and time to disappearance of a repairman are continuous random variables. Few works by the authors [3, 4, 5] is carried out in the literature of reliability analyzing the system models taking geometric failure and repair time distributions.

The purpose of the present paper is to analyze a two-unit cold standby system model with appearance and disappearance of repairman under discrete parametric Markov-Chain i.e. **failure and repair times and appearance and disappearance times of repairman follow geometric distributions with different parameters.** The phenomena of discrete failure and repair time distributions may be observed in the following situation.

Let the continuous time period $(0,\infty)$ is divided as 0, 1, 2,…, n,… of equal distance on real line and the probability of failure of a unit during time (i, i+1); i = 0, 1, 2,….. is p, then the probability that the unit will fail during (t, t+1) i.e. after passing successfully t intervals of time is given by $p(1-p)^t$; t = 0, 1, 2,…. This is the p.m.f of geometric distribution. Similarly, if r denotes the probability that a failed unit is repaired during (i, i+1); i = 0, 1, 2,... then the probability that the unit will be repaired during (t, t+1) is given by $r(1-r)^t$; t = 0, 1, 2,…. On the same way, the random variables denoting appearance and disappearance of repairman may follow geometric distributions.

The following economic related measures of system effectiveness are obtained by using regenerative point technique-

i) Transition probabilities and mean sojourn times in various states.
ii) Reliability and mean time to system failure.
iii) Point-wise and steady-state availabilities of the system as well as expected up time of the system during interval (0, t).
iv) Expected busy period of the repairman during time interval (0, t).
v) Net expected profit earned by the system during a finite interval and in steady-state.

1. Model Description and Assumptions

i) The system comprises of two-identical units. Initially, one unit is operative and other is kept into cold standby.
ii) Each unit of the system has two modes- Normal (N) and Total failure (F).
iii) There is a single repair facility which appears in and disappears from the system randomly. Once the repairman starts the repair of a failed unit, he does not leave the system till all the units are repaired that failed during his stay in the system.
iv) All random variables denoting failure time, repair time, time to appearance and disappearance of repairman are independent of discrete nature and follow geometric distributions with different parameters.
v) The system failure occurs when both the units are in total failure mode.
vi) The repaired unit works as good as new.

2. Notations and States of the System

a) Notations:

pq^t	:	p.m.f. of failure time of an operating unit $(p+q=1)$.
rs^t	:	p.m.f. of repair time of a failed unit $(r+s=1)$.

ab^t	:	p.m.f. of disappearance of repairman from the system $(a+b=1)$.
cd^t	:	p.m.f. of appearance of repairman in the system $(c+d=1)$.
$q_{ij}(\cdot), Q_{ij}(\cdot)$	:	p.m.f. and c.d.f. of one step or direct transition time from state S_i to S_j.
p_{ij}	:	Steady state transition probability from state S_i to S_j. $p_{ij} = Q_{ij}(\infty)$
$Z_i(t)$	:	Probability that the system sojourns in state S_i at epochs 0, 1, 2,……,up to (t-1).
ψ_i	:	Mean sojourn time in state S_i.
$*, h$	:	Symbol and dummy variable used in geometric transform e. g. $GT\left[q_{ij}(t)\right] = q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$
©	:	Symbol for ordinary convolution i.e. $A(t)©B(t) = \sum_{u=0}^{t} A(u)B(t-u)$

b) Symbols for the states of the system:

N_O / N_S : Unit in normal (N) mode and operative/standby.

F_r / F_w : Unit in total failure (F) mode and under repair/waiting for repair

A/ NA : Repairman is available/not available with the system.

TRANSITION DIAGRAM

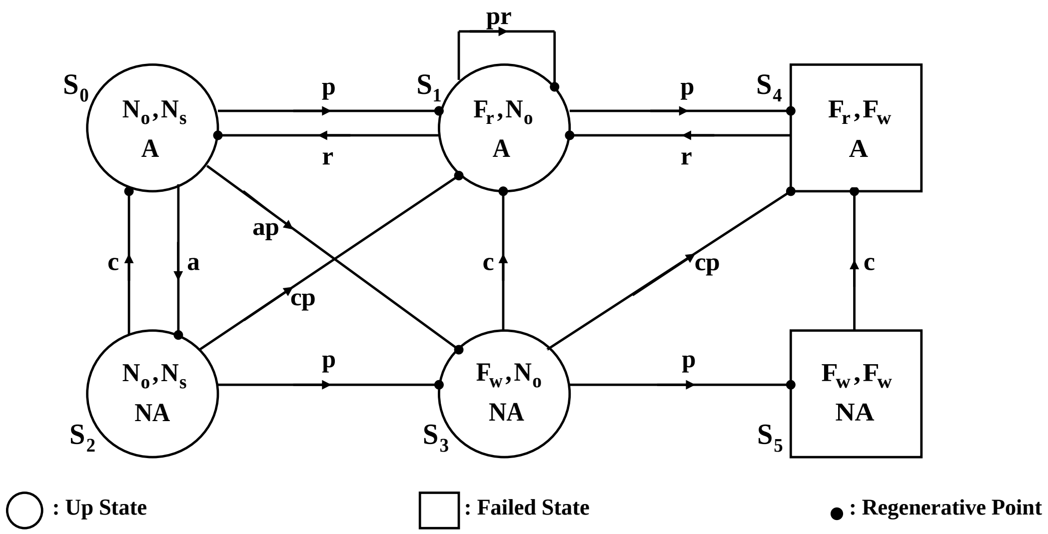

Fig.1

With the help of above symbols the possible states of the system along with failure and repair rates are shown in the transition diagram (Fig.1)

3. Transition Probabilities

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then $Q_{ij}(t)=P\left[T_{ij}\le t\right]$. In particular, $Q_{01}(t)$ is the probability that the operative unit fails at an epoch u between (0,t) and repairman does not disappear up to the epoch u.

$Q_{01}(t)=\sum_{u=0}^{t}$ P (Operative unit fails at epoch u) X P (Repairman does not disappear up to the epoch u) $=\sum_{u=0}^{t} pq^u b^{u+1}=\frac{bp}{(1-bq)}\left\{1-(bq)^{t+1}\right\}$

Similarly,

$$Q_{02}(t)=\frac{aq}{(1-bq)}\left\{1-(bq)^{t+1}\right\}, \qquad Q_{03}(t)=\frac{ap}{(1-bq)}\left\{1-(bq)^{t+1}\right\}$$

$$Q_{10}(t)=\frac{qr}{(1-qs)}\left\{1-(qs)^{t+1}\right\}, \qquad Q_{11}(t)=\frac{pr}{(1-qs)}\left\{1-(qs)^{t+1}\right\}$$

$$Q_{14}(t)=\frac{ps}{(1-qs)}\left\{1-(qs)^{t+1}\right\}, \qquad Q_{20}(t)=\frac{cq}{(1-dq)}\left\{1-(dq)^{t+1}\right\}$$

$$Q_{21}(t)=\frac{cp}{(1-dq)}\left\{1-(dq)^{t+1}\right\}, \qquad Q_{23}(t)=\frac{dp}{(1-dq)}\left\{1-(dq)^{t+1}\right\}$$

$$Q_{31}(t)=\frac{cq}{(1-dq)}\left\{1-(dq)^{t+1}\right\}, \qquad Q_{34}(t)=\frac{cp}{(1-dq)}\left\{1-(dq)^{t+1}\right\}$$

$$Q_{35}(t)=\frac{dp}{(1-dq)}\left\{1-(dq)^{t+1}\right\}, \qquad Q_{41}(t)=\left(1-s^{t+1}\right)$$

$$Q_{54}(t)=\left(1-d^{t+1}\right) \qquad (1-14)$$

The steady state transition probabilities from state S_i to S_j can be obtained from $(1-14)$ by taking $t\to\infty$, as follows:

$$p_{01}=\frac{bp}{(1-bq)}, \quad p_{02}=\frac{aq}{(1-bq)}, \quad p_{03}=\frac{ap}{(1-bq)}, \quad p_{10}=\frac{qr}{(1-qs)}$$

$$p_{11}=\frac{pr}{(1-qs)}, \quad p_{14}=\frac{ps}{(1-qs)}, \quad p_{20}=\frac{cq}{(1-dq)}, \quad p_{21}=\frac{cp}{(1-dq)}$$

$$p_{23}=\frac{dp}{(1-dq)}, \quad p_{31}=\frac{cq}{(1-dq)}, \quad p_{34}=\frac{cp}{(1-dq)}, \quad p_{35}=\frac{dp}{(1-dq)}$$

$$p_{41}=1, \quad p_{54}=1$$

We observe that the following relations hold-

$$p_{01}+p_{02}+p_{03}=1, \quad p_{10}+p_{11}+p_{14}=1, \quad p_{20}+p_{21}+p_{23}=1$$

$$p_{31}+p_{34}+p_{35}=1, \quad p_{41}=p_{54}=1 \qquad (15-19)$$

4. Mean Sojourn Times

Let ψ_i be the sojourn time in state S_i (i = 0, 1, 2, 3, 4, 5) then mean sojourn time in state S_i is given by

$$\psi_i = \sum_{t=1}^{\infty} P\left[T \geq t\right]$$

In particular,

$$\psi_0 = \frac{bq}{(1-bq)}, \qquad \psi_1 = \frac{qs}{(1-qs)}, \qquad \psi_2 = \psi_3 = \frac{dq}{(1-dq)} = \psi \text{, say}$$

$$\psi_4 = \frac{s}{r}, \qquad \psi_5 = \frac{d}{c} \tag{20-24}$$

The evaluation of steady-state transition probabilities and mean sojourn time play the vital role as the various measures of system effectiveness are obtained in these terms.

5. Methodology for Developing Equations

In order to obtain various interesting measures of system effectiveness we develop the recurrence relations for reliability, availability and busy period of repairman as follows-

a) Reliability of the system

Here we define $R_i(t)$ as the probability that the system does not fail up to t epochs 0, 1, 2,….,(t-1) when it is initially started from up state S_i. To determine it, we regard the failed states S_4 and S_5 as absorbing states. Now, the expressions for $R_i(t)$; i = 0, 1, 2, 3; we have the following set of convolution equations.

$$R_0(t) = b^t q^t + \sum_{u=0}^{t-1} q_{01}(u) R_1(t-1-u) + \sum_{u=0}^{t-1} q_{02}(u) R_2(t-1-u) + \sum_{u=0}^{t-1} q_{03}(u) R_3(t-1-u)$$

$$= Z_0(t) + q_{01}(t-1) © R_1(t-1) + q_{02}(t-1) © R_2(t-1) + q_{03}(t-1) © R_3(t-1)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{10}(t-1) © R_0(t-1) + q_{11}(t-1) © R_1(t-1)$$

$$R_2(t) = Z_2(t) + q_{20}(t-1) © R_0(t-1) + q_{21}(t-1) © R_1(t-1) + q_{23}(t-1) © R_3(t-1)$$

$$R_3(t) = Z_3(t) + q_{31}(t-1) © R_1(t-1)$$

$(25-28)$

Where,

$$Z_1(t) = b^t q^t, \qquad Z_2(t) = Z_3(t) = d^t q^t = Z(t) \text{, say}$$

b) Availability of the system

Let $A_i(t)$ be the probability that the system is up at epoch (t-1), when it initially starts from state S_i. By using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $A_i(t)$; i = 0 to 5.

$$A_0(t)=Z_0(t)+q_{01}(t-1)©A_1(t-1)+q_{02}(t-1)©A_2(t-1)+q_{03}(t-1)©A_3(t-1)$$
$$A_1(t)=Z_1(t)+q_{10}(t-1)©A_0(t-1)+q_{11}(t-1)©A_1(t-1)+q_{14}(t-1)©A_4(t-1)$$
$$A_2(t)=Z(t)+q_{20}(t-1)©A_0(t-1)+q_{21}(t-1)©A_1(t-1)+q_{23}(t-1)©A_3(t-1)$$
$$A_3(t)=Z(t)+q_{31}(t-1)©A_1(t-1)+q_{34}(t-1)©A_4(t-1)+q_{35}(t-1)©A_5(t-1)$$
$$A_4(t)=q_{41}(t-1)©A_1(t-1)$$
$$A_5(t)=q_{54}(t-1)©A_4(t-1)$$

$$(29-34)$$

Where,

The values of $Z_i(t)$; i = 0, 1 and $Z(t)$ are same as given in section 6(a).

c) Busy period of repairman

Let $B_i(t)$ be the respective probability that the repairman is busy at epoch (t-1) in the repair of each unit, when system initially starts from S_i. Using simple probabilistic arguments as in case of reliability, the recurrence relations for $B_i(t)$; i = 0 to 5 can be easily developed as below-

$$B_0(t)=q_{01}(t-1)©B_1(t-1)+q_{02}(t-1)©B_2(t-1)+q_{03}(t-1)©B_3(t-1)$$
$$B_1(t)=Z_1(t)+q_{10}(t-1)©B_0(t-1)+q_{11}(t-1)©B_1(t-1)+q_{14}(t-1)©B_4(t-1)$$
$$B_2(t)=q_{20}(t-1)©B_0(t-1)+q_{21}(t-1)©B_1(t-1)+q_{23}(t-1)©B_3(t-1)$$
$$B_3(t)=q_{31}(t-1)©B_1(t-1)+q_{34}(t-1)©B_4(t-1)+q_{35}(t-1)©B_5(t-1)$$
$$B_4(t)=Z_4(t)+q_{41}(t-1)©B_1(t-1)$$
$$B_5(t)=q_{54}(t-1)©B_4(t-1)$$

$$(35-40)$$

Where,

$Z_1(t)$ has the same values as in section 6(a) and $Z_4(t)=s^t$.

6. Analysis of Characteristics

a) Reliability and MTSF

Taking geometric transforms of $(6.1-6.4)$ and simplifying the resulting set of algebraic equations for $R_0^*(h)$, we get

$$R_0^*(h)=\frac{N_1(h)}{D_1(h)} \qquad (41)$$

Where,

$$N_1(h)=\left(1-hq_{11}^*\right)\left[Z_0^*+hq_{02}^*Z^*+\left(hq_{03}^*+h^2q_{02}^*q_{23}^*\right)Z^*\right]$$

$$+\left[hq_{01}^{*}+hq_{02}^{*}\left(hq_{21}^{*}+h^{2}q_{23}^{*}q_{31}^{*}\right)+h^{2}q_{03}^{*}q_{31}^{*}\right]Z_{1}^{*}$$

$$D_{1}(h)=\left(1-hq_{11}^{*}\right)\left(1-h^{2}q_{02}^{*}q_{20}^{*}\right)-hq_{10}^{*}\left[hq_{01}^{*}+hq_{02}^{*}\left(hq_{21}^{*}+h^{2}q_{23}^{*}q_{31}^{*}\right)+h^{2}q_{03}^{*}q_{31}^{*}\right]$$

Collecting the coefficient of h^{t} from expression (7.1), we can get the reliability of the system $R_{0}(t)$. The MTSF is given by-

$$E(T)=\lim_{h\to 1}\sum_{t=1}^{\infty}h^{t}R(t)=\frac{N_{1}(1)}{D_{1}(1)}-1 \tag{42}$$

Where, on noting that $q_{ij}^{*}(1)=p_{ij}$, $Z_{i}^{*}(0)=\psi_{i}$, we have

$$N_{1}(1)=(1-p_{11})\left[\psi_{0}+\left\{p_{03}+p_{02}(1+p_{23})\right\}\psi\right]+\left[p_{01}+p_{02}(p_{21}+p_{23}p_{31})+p_{03}p_{31}\right]\psi_{1}$$

$$D_{1}(1)=(1-p_{11})(1-p_{02}p_{20})-p_{10}\left[p_{01}+p_{02}(p_{21}+p_{23}p_{31})+p_{03}p_{31}\right]$$

b) Availability Analysis. On taking geometric transform of $(6.5-6.10)$ and simplifying the resulting equations, we get

$$A_{0}^{*}(h)=\frac{N_{2}(h)}{D_{2}(h)} \tag{43}$$

Where,

$$N_{2}(h)=\left(1-hq_{11}^{*}-h^{2}q_{14}^{*}q_{41}^{*}\right)\left[Z_{0}^{*}+hq_{12}^{*}Z^{*}+\left(hq_{03}^{*}+h^{2}q_{02}^{*}q_{23}^{*}\right)Z^{*}\right]$$
$$+\left[\left(hq_{01}^{*}+h^{2}q_{02}^{*}q_{21}^{*}\right)+\left\{hq_{31}^{*}+hq_{41}^{*}\left(hq_{03}^{*}+h^{2}q_{02}^{*}q_{23}^{*}\right)\right\}\left(hq_{03}^{*}+h^{2}q_{02}^{*}q_{23}^{*}\right)\right]Z_{1}^{*}$$

$$D_{2}(h)=\left(1-hq_{11}^{*}-h^{2}q_{14}^{*}q_{41}^{*}\right)\left(1-h^{2}q_{02}^{*}q_{20}^{*}\right)-hq_{10}^{*}\left[\left(hq_{01}^{*}+h^{2}q_{02}^{*}q_{21}^{*}\right)\right.$$
$$\left.+\left\{hq_{31}^{*}+hq_{41}^{*}\left(hq_{34}^{*}+h^{2}q_{35}^{*}q_{54}^{*}\right)\right\}\left(hq_{03}^{*}+h^{2}q_{02}^{*}q_{23}^{*}\right)\right]$$

The steady state availability of the system is given by-

$$A_{0}=\lim_{t\to\infty}A_{0}(t)=\lim_{h\to 1}(1-h)\frac{N_{2}(h)}{D_{2}(h)}$$

As $D_{2}(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$A_{0}=-\frac{N_{2}(1)}{D_{2}'(1)} \tag{44}$$

Where,

$$N_{2}(1)=p_{10}\left[\psi_{0}+p_{02}\psi+(p_{03}+p_{02}p_{23})\psi\right]+(1-p_{02}p_{20})\psi_{1}$$

and

$$D_{2}'(1)=p_{10}\left[\psi_{0}+p_{02}\psi+(p_{03}+p_{02}p_{23})\psi+p_{35}\psi_{5}+(1-p_{31})\psi_{4}\right]$$
$$+(1-p_{02}p_{20})(\psi_{1}+p_{14}\psi_{4})$$

Now the expected up time of the system up to epoch (t-1) is given by

$$\mu_{up}(t)=\sum_{x=0}^{t-1}A_{0}(x)$$

so that

$$\mu_{up}^{*}(h)=\frac{A_0^{*}(h)}{(1-h)} \tag{45}$$

c) Busy Period Analysis. On taking geometric transforms of $(6.11-6.16)$ and simplifying the resulting equations, we get

$$B_0^{*}(h)=\frac{N_3(h)}{D_2(h)} \tag{46}$$

Where,

$$\begin{aligned}N_3(h)=&\Big[\left(hq_{01}^{*}+h^2q_{02}^{*}q_{21}^{*}\right)+\left\{hq_{31}^{*}+hq_{41}^{*}\left(hq_{03}^{*}+h^2q_{02}^{*}q_{23}^{*}\right)\right\}\left(hq_{03}^{*}+h^2q_{02}^{*}q_{23}^{*}\right)\Big]Z_1^{*}\\&+\Big[hq_{14}^{*}\left(hq_{01}^{*}+h^2q_{03}^{*}q_{31}^{*}\right)+\left(1-q_{11}^{*}\right)\left(hq_{34}^{*}+h^2q_{35}^{*}q_{54}^{*}\right)\left(hq_{03}^{*}+h^2q_{02}^{*}q_{23}^{*}\right)\\&+h^2q_{02}^{*}q_{14}^{*}\left(hq_{21}^{*}+h^2q_{23}^{*}q_{31}^{*}\right)\Big]Z_4^{*}\end{aligned}$$

and $D_2(h)$ is same as in availability analysis.

In the long run, the respective probability that the repairman is busy in the repair of each unit is given by-

$$B_0=\lim_{t\to\infty}B_0(t)=\lim_{h\to 1}(1-h)\frac{N_3(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0=-\frac{N_3(1)}{D_2'(1)} \tag{47}$$

Where,

$$N_3(1)=(1-p_{02}p_{20})(\psi_1+p_{14}\psi_4)+p_{10}(1-p_{31})(p_{03}+p_{02}p_{23})$$

and $D_2'(1)$ is same as in availability analysis.

The expected busy periods of the repairman in the repair of both units up to epoch $(t-1)$ is given by-

$$\mu_b(t)=\sum_{x=0}^{t-1}B_0(x)$$

So that,

$$\mu_b^{*}(h)=\frac{B_0^{*}(h)}{(1-h)} \tag{48}$$

7. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier sections.

Let us consider,

K_0 = revenue per-unit time by the system when it is operative.

K_1 = cost per-unit time when repairman is busy in the repair of the failed units.

Then, the net expected profit incurred up to epoch (t-1) is given by

$$P(t)=K_0\mu_{up}(t)-K_1\mu_b(t) \tag{49}$$

The expected profit per unit time in steady state is as follows-

$$P=\lim_{t\to\infty}\frac{P(t)}{t}$$

$$=K_0\lim_{h\to 1}(1-h)^2\frac{A_0^*(h)}{(1-h)}-K_1\lim_{h\to 1}(1-h)^2\frac{B_0^*(h)}{(1-h)}$$

$$=K_0A_0-K_1B_0 \tag{50}$$

8. Graphical Representation

The curves for MTSF and profit function have been drawn for different values of parameters p, a, c Fig. 2 depicts the variations in MTSF with respect to the rate of appearance of repairman (c) in the system for three different values of failure rate (p = 0.08, 0.10, 0.12) of an operative unit and two different values of rate of disappearance of repairman from the system (a = 0.5, 0.6). From these curves we observe that MTSF increases uniformly as the value of c increases. It also reveals that the MTSF decreases with the increase in p and decreases with the increase in a.

Similarly, Fig. 3 reveals the variations in profit (P) with respect to c for varying values of p and a, when the values of other parameters are kept fixed as r = 0.5, K_0=15 and K_1=80. From the curves we observe that profit decreases uniformly as the value of c increases. It also reveals that the profit decreases with the increase in p and decreases with the increase in a. From this figure it is clear from the dotted curves that the system is profitable only if the rate of appearance of repairman (c) in the system is less than 0.21, 0.41 and 0.70 respectively for p = 0.08, 0.10, and 0.12 for fixed value of a = 0.5. From smooth curves, we conclude that the system is profitable only if c is less than 0.19, 0.35 and 0.56 respectively for p = 0.08, 0.10, and 0.12 for fixed value of a = 0.

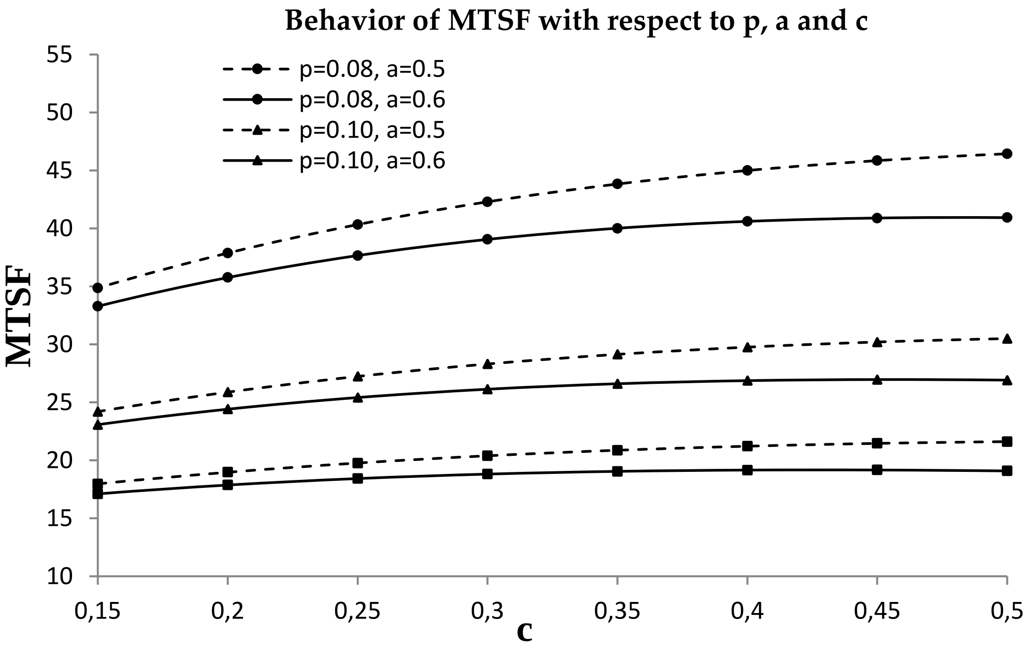

Fig. 2

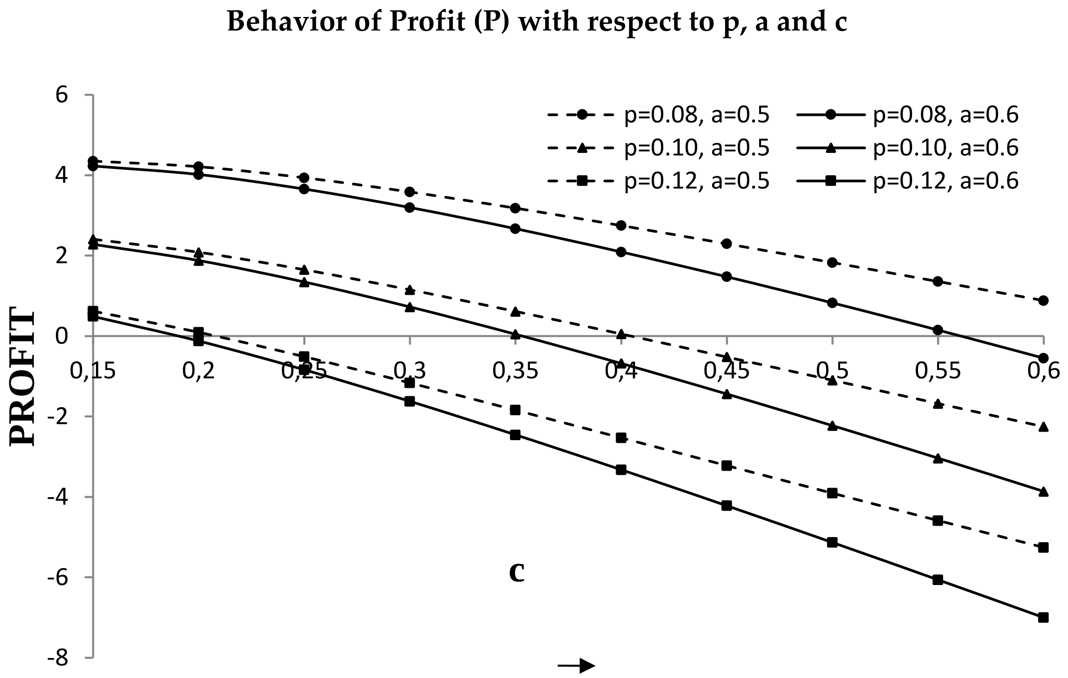

Fig. 3

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