A Discrete Parametric Markov-Chain Model Of A Two-Unit Cold Standby System With Repair Efficiency Depending On Environment

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Abstract

The paper deals with stochastic analysis of a two identical unit cold standby system model with two modes of the units-normal (N) and total failure (F). A single repairman is always available with the system to repair a failed unit. Two physical conditions-good and poor of repairman depending upon the perfect and imperfect environment are considered. The system transits from perfect to imperfect environment and vice-versa after random periods of time. The failure and repair times of a unit are taken as independent random variables of discrete nature having geometric distributions with different parameters.

Keywords: Transition probability, Regenerative point, reliability, MTSF, availability of system, busy period of repairman.

1. Introdution

In order to fight with the competitive situations in modern business and industrial systems, the redundancy plays a vital role in the improvement of the various measures of system effectiveness. Numerous authors including [4,5,6,7] have studied two unit redundant systems with different sets of assumptions such as slow and imperfect switches, two types of repairmen, special warranty schemes, preparation time for repair etc. Various authors including [1,2] have analyzed the reparable system models by assuming that the repair rate of a failed unit is affected by the physical conditions of repairman. Goel et al [3] analyzed a two identical unit cold standby system model assuming the physical conditions of repairman and the repair time distributions are affected by good and poor physical conditions of repairman. They have not mentioned the cause of the change of physical conditions of repairman. All the above authors have assumed continuous distributions of various random variables such as failure time, repair time etc. Sometimes we may come across the situations when the physical conditions of repairman depend upon the changing in environmental conditions. For example- The repair efficiency of repairman is good in air-condition atmosphere as compared to non-air-condition atmosphere.

Keeping the above fact in view, the present paper deals with the analyses of a two identical unit standby system model assuming that the physical conditions of repairman depends upon the environmental conditions i.e. the perfect and imperfect environment. Here the parametric space of Markov-chain involved is taken of discrete nature and the random variables denoting failure times, repair time and time to change environments are assumed to follow geometric distributions with different parameters. Initially the repairman starts the repair of a failed unit in good physical condition. The following economic related measures of system effectiveness are obtained by using regenerative point technique-

vi) Transition probabilities and mean sojourn times in various states.
vii) Reliability and mean time to system failure.
viii) Point-wise and steady-state availabilities of the system as well as expected up time of the system during a finite interval of time.
ix) Expected busy period of the repairman in the perfect and imperfect environments during a finite interval of time.
x) Net expected profit incurred by the system during a finite interval of time and steady-state are obtained.

2. Model Description And Assumptions

i) The system comprises of two identical units with two modes of a unit- normal (N) total failure (F).
ii) Initially one unit is operative and other is kept into cold standby.
iii) A single repairman is always available with the system to repair a failed unit.
iv) Two physical conditions- good and poor of repairman depending upon the perfect and imperfect environments are considered. Initially the repairman starts the repair of a failed unit in good physical condition.
v) The system transits from perfect to imperfect environment and vice-versa after a random period of time.
vi) The repair rate of a failed unit in perfect environment is higher than the imperfect environment.
vii) A repaired unit works as good as new.
viii) Failure and repair times of the units follow independent geometric distributions with different parameters.

3. Notations And States Of The System

a) Notations :

pq^t : p.m.f. of failure time of a unit $(p+q=1)$.

rs^t : p.m.f. of repair time of a unit in perfect environment $(r+s=1)$.

$r's'^t$: p.m.f. of repair time of a unit in imperfect environment $(r'+s'=1)$.

ab^t : p.m.f. of time to change the environment from perfect to imperfect $(a+b=1)$.

cd^t : p.m.f. of time to change the environment from imperfect to perfect $(c+d=1)$

$q_{ij}(\cdot), Q_{ij}(\cdot)$: p.m.f. and c.d.f. of one step or direct transition time from state S_i to S_j.

p_{ij} : Steady-state transition probability from state S_i to S_j.

$$p_{ij} = Q_{ij}(\infty)$$

$Z_i(t)$: Probability that the system sojourn in state S_i at epochs 0, 1, 2 … (t-1).

ψ_i : Mean sojourn time in state S_i.

$*, h$: Symbol and dummy variable used in geometric transform e. g.

$$GT\left[q_{ij}(t)\right] = q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$$

b) Symbols for the states of the systems:

N_0 / N_S : Unit in normal (N) mode and operative/standby.

F_r / F_w : Unit in total failure (F) mode in perfect environment and under repair/waiting for repair

$F_{r'} / F_{w'}$: Unit in total failure (F) mode in imperfect environment and under repair/ waiting for repair

G / P : System in perfect/ imperfect Environment.

With the help of above symbols the possible states S_0 to S_4 of the system are shown in **Fig.1,** where, S_0, S_1 and S_2 are the up states whereas S_3 and S_4 are the failed states

TRANSITION DIAGRAM

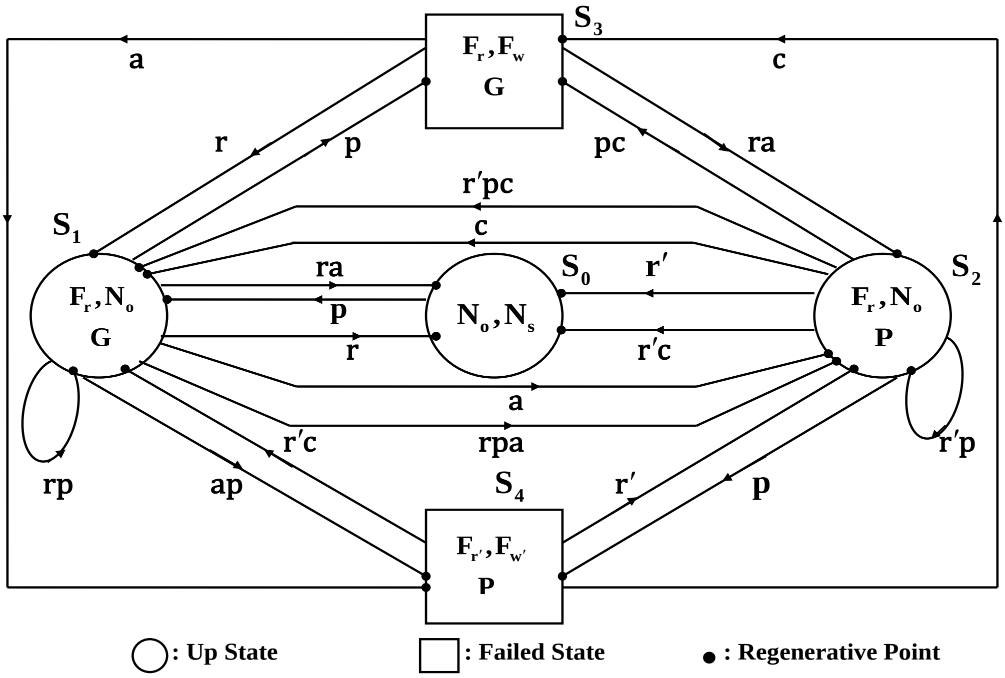

Fig.1

4. Transition Probabilities

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \le t\right]$$

By using simple probabilistic arguments we have

$$Q_{01}(t)=1-q_1^{t+1},\quad Q_{10}(t)=rqA_1,\quad Q_{11}(t)=rbpA_1,$$
$$Q_{12}(t)=a(sq+rp)A_1$$
$$Q_{13}(t)=bpsA_1,\quad Q_{14}(t)=apsA_1,\quad Q_{20}(t)=r'qA_2,$$
$$Q_{21}(t)=c(qs'+pr')A_2$$
$$Q_{22}(t)=r'pdA_2,\quad Q_{23}(t)=s'cpA_2,\quad Q_{24}(t)=s'dpA_2,\quad Q_{31}(t)=rbA_3$$
$$Q_{32}(t)=raA_3,\quad Q_{34}(t)=saA_3,\quad Q_{41}(t)=r'cA_4,\quad Q_{42}(t)=r'dA_4$$
$$Q_{43}(t)=s'cA_4$$

(1-17)

Where, $A_1=\left[1-(sbq)^{t+1}\right]\big/(1-sbq)$, $A_2=\left[1-(s'qd)^{t+1}\right]\big/(1-s'qd)$,

$A_3=\left[1-(sb)^{t+1}\right]\big/(1-sb)$, $A_4=\left[1-(s'd)^{t+1}\right]\big/(1-s'd)$

The steady state transition probabilities from state S_i to S_j can be obtained from (1-17) by taking $t\to\infty$, as follows:

$$p_{01}=1,\; p_{10}=rqC,\; p_{11}=rbpC,\; p_{12}=a(rq+sq)C$$
$$p_{13}=sbpC,\; p_{14}=sapC$$

Where, $C=\dfrac{1}{1-sbq}$

Similarly, the values of other transition probabilities p_{20}, p_{21}, p_{22}, p_{23}, p_{24}, p_{31}, p_{32}, p_{34}, p_{41}, p_{42} and p_{43} can be evaluated.

We observe that the following relations hold-

$$p_{01}=1,\quad p_{10}+p_{11}+p_{12}+p_{13}+p_{14}=1,$$
$$p_{20}+p_{21}+p_{22}+p_{23}+p_{24}=1$$
$$p_{31}+p_{32}+p_{34}=1,\quad p_{41}+p_{42}+p_{43}=1 \qquad (18\text{-}22)$$

5. Mean Sojourn Times

Let T_i be the sojourn time in state S_i (i=0, 1, 2, 3, 4) then mean sojourn time in state S_i is given by

$$\psi_i=\sum_{t=1}^{\infty}P[T\ge t]$$

In particular,

$$\psi_0=\sum_{t=1}^{\infty}P[\text{ The operating unit in state } S_0 \text{ doesn't fail up to epoch t-1}]$$
$$=\sum_{t=1}^{\infty}q^t=\frac{q}{p}$$

Similarly,

$$\psi_1=sbqC,\quad \psi_2=\frac{s'dq}{1-s'dq},\quad \psi_3=\frac{sb}{1-sb},\quad \psi_4=\frac{s'd}{1-s'd} \qquad (23\text{-}27)$$

6. Methodology For Developing Equations

In order to obtain various interesting measures of system effectiveness we developed the recurrence relations for reliability, availability and busy period of repairman as follows

d) Reliability of the system- Here we define $R_i(t)$ as the probability that the system does not fail up to epochs 0, 1, 2,.., (t-1) when it is initially started from up state S_i. To determine it, we regard the failed state S_3 and S_4 as absorbing state. Now, the expression for $R_i(t)$; i=0, 1, 2 we have the following set of convolution equations.

$$R_0(t) = q^t + \sum_{u=0}^{t-1} q_{01}(u) R_1(t-1-u)$$
$$= Z_0(t) + q_{01}(t-1) © R_1(t-1)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{10}(t-1) © R_0(t-1) + q_{11}(t-1) © R_1(t-1) + q_{12}(t-1) © R_2(t-1)$$
$$R_2(t) = Z_2(t) + q_{20}(t-1) © R_0(t-1) + q_{21}(t-1) © R_1(t-1) + q_{22}(t-1) © R_2(t-1)$$

(28-30)

Where,

$$Z_0(t) = q^t, \qquad Z_1(t) = b^t s^t q^t, \qquad Z_2(t) = d^t s'^t q^t$$

e) Availability of the system- Let $A_i(t)$ be the probability that the system is up at epoch (t-1), when it initially started from state S_i. By using simple probabilistic arguments as illustrated in case of reliability, the following recurrence relations can be easily developed for $A_i(t)$; i=0 to 4.

$$A_0(t) = Z_0(t) + q_{01}(t-1) © A_1(t-1)$$
$$A_1(t) = Z_1(t) + q_{10}(t-1) © A_0(t-1) + q_{11}(t-1) © A_1(t-1) + q_{12}(t-1) © A_2(t-1) + q_{13}(t-1) © A_3(t-1) + q_{14}(t-1) © A_4(t-1)$$
$$A_2(t) = Z_2(t) + q_{20}(t-1) © A_0(t-1) + q_{21}(t-1) © A_1(t-1) + q_{22}(t-1) © A_2(t-1) + q_{23}(t-1) © A_3(t-1) + q_{24}(t-1) © A_4(t-1)$$
$$A_3(t) = q_{31}(t-1) © A_1(t-1) + q_{32}(t-1) © A_2(t-1) + q_{34}(t-1) © A_4(t-1)$$
$$A_4(t) = q_{41}(t-1) © A_1(t-1) + q_{42}(t-1) © A_2(t-1) + q_{43}(t-1) © A_3(t-1)$$

(31-35)

Where,

The values of $Z_i(t)$; i=0 to 2 are same as given in section 8(a).

c) Busy period of repairman- Let $B_i^G(t)$ and $B_i^P(t)$ be the respective probabilities that the repairman is busy at epoch (t-1) in the repair of failed unit in good and poor conditions depending upon the perfect and imperfect environment are conditions when system initially starts from state S_i. Using simple probabilistic arguments as illustrated in case of availability analysis, the relations for $B_i^k(t)$; i=0 to 4 and k=G, P can be easily developed on replacing the following in expressions (31-35)-

i) A_i by B_i^k, $Z_0(t)$ by 0, $Z_1(t)$ by $(1-\delta)Z_1(t)$, $Z_2(t)$ by $\delta Z_2(t)$ and

ii) Considering one more contingency respectively $(1-\delta)Z_3(t)$ and $\delta Z_4(t)$ in expressions (34)

and (35). The resulting expressions may be denoted by (36-40).

Where, $\delta = 0$ and 1 respectively for k = G and P. Also, $Z_1(t)$ and $Z_2(t)$ are same as given in section 7(a). and $Z_3(t)$, $Z_4(t)$ are as follows-

$Z_3(t) = s^t b^t$, $Z_4 = s^t d^t$

7. Analysis Of Reliability And Mtsf

Taking geometric transforms of relations (28-30) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h) = \frac{N_1(h)}{D_1(h)} \tag{41}$$

Where,

$$N_1(h) = \left[(1-hq_{11}^*)(1-hq_{22}^*) - h^2 q_{12}^* q_{21}^*\right] Z_0^* + \left[hq_{01}^*(1-hq_{22}^*)\right] Z_1^* + h^2 q_{01}^* q_{12}^* Z_2^*$$

$$D_1(h) = (1-hq_{11}^*)(1-hq_{22}^*) - h^2 q_{12}^* q_{21}^* - h^2 q_{01}^* q_{10}^* (1-hq_{22}^*) - h^3 q_{01}^* q_{12}^* q_{20}^*$$

Collecting the coefficient of h^t from expression (41), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T) = \lim_{h\to 1} \sum_{t=1}^{\infty} h^t R(t) = \frac{N_1(1)}{D_1(1)} - 1 \tag{42}$$

Where,

$$N_1(1) = \psi_0\left[(1-p_{11})(1-p_{22}) - p_{12}p_{21}\right] + \psi_1\left[p_{01}(1-p_{22})\right] + p_{01}p_{12}\psi_2$$

$$D_1(1) = (1-p_{11})(1-p_{22}) - p_{12}p_{21} - p_{01}(1-p_{22}) - p_{01}p_{12}p_{20}$$

8. Availability Analysis

On taking geometric transforms of relations (31-35) and simplifying the resulting equations we get-

$$A_0^*(h) = \frac{U_0^* Z_0^* + U_1^* Z_1^* + U_2^* Z_2^*}{U_0^* - hq_{10}^* U_1^* - hq_{20}^* U_2^*} = \frac{N_2(h)}{D_2(h)} \tag{43}$$

Where,

$$U_0^* = (1-hq_{11}^*)\left\{(1-hq_{22}^*)(1-h^2 q_{34}^* q_{43}^*) - hq_{23}^*(hq_{32}^* + h^2 q_{34}^* q_{42}^*) - hq_{24}^*(hq_{42}^* + h^2 q_{43}^* q_{32}^*)\right\}$$
$$-hq_{12}^*\left\{hq_{21}^*(1-h^2 q_{34}^* q_{43}^*) + hq_{23}^*(hq_{31}^* + h^2 q_{34}^* q_{41}^*) + hq_{24}^*(h^2 q_{43}^* q_{31}^* + hq_{41}^*)\right\}$$
$$-hq_{13}^*\left\{hq_{21}^*(hq_{32}^* + h^2 q_{34}^* q_{42}^*) + (1-hq_{22}^*)(hq_{32}^* + h^2 q_{34}^* q_{42}^*) + hq_{24}^*(h^2 q_{41}^* q_{32}^*\right.$$

$$-h^2q^*_{31}q^*_{42}\Big)\Big\}-hq^*_{14}\Big\{hq^*_{21}\left(h^2q^*_{43}q^*_{32}+hq^*_{42}\right)+\left(1-hq^*_{22}\right)\left(hq^*_{41}+h^2q^*_{43}q^*_{31}\right)$$

$$+hq^*_{22}\left(h^2q^*_{31}q^*_{42}-h^2q^*_{32}q^*_{41}\right)\Big\}$$

$$U^*_1=\left[\left(1-hq^*_{22}\right)\left(1-h^2q^*_{34}q^*_{43}\right)-hq^*_{23}\left(hq^*_{32}+h^2q^*_{34}q^*_{42}\right)-hq^*_{24}\left(hq^*_{42}+h^2q^*_{43}q^*_{32}\right)\right]hq^*_{01}$$

$$U^*_2=\left[hq^*_{12}\left(1-h^2q^*_{34}q^*_{43}\right)+hq^*_{13}\left(hq^*_{32}+h^2q^*_{34}q^*_{42}\right)+hq^*_{14}\left(hq^*_{42}+h^2q^*_{43}q^*_{32}\right)\right]hq^*_{01}$$

The steady state availability of the system is given by-

$$A_0=\lim_{t\to\infty}A_0(t)=\lim_{h\to 1}(1-h)\frac{N_2(h)}{D_2(h)}$$

Now as $D_2(h)$ at h=1 is zero, therefore by applying L. hospital rule we get-

$$A_0=-\frac{U_0\psi_0+U_1\psi_1+U_2\psi_2}{U_0\psi_0+U_1\psi_1+U_2\psi_2+U_3\psi_3+U_4\psi_4}=-\frac{N_2(1)}{D'_2(1)} \tag{44}$$

Where,

$$U_0=p_{10}\left\{p_{21}\left(1-p_{34}p_{43}\right)+p_{23}\left(p_{31}+p_{34}p_{41}\right)+p_{24}\left(p_{41}+p_{43}p_{31}\right)\right\}+p_{20}\left\{p_{12}\left(1-p_{34}p_{43}\right)\right.$$
$$\left.+p_{13}\left(p_{32}+p_{34}p_{42}\right)+p_{14}\left(p_{42}+p_{43}p_{32}\right)\right.$$

$$U_1=\left(1-p_{22}\right)\left(1-p_{34}p_{43}\right)-p_{23}\left(p_{32}+p_{34}p_{42}\right)-p_{24}\left(p_{42}+p_{43}p_{32}\right)$$

$$U_2=p_{12}\left(1-p_{34}p_{43}\right)+p_{13}\left(p_{32}+p_{34}p_{42}\right)+p_{14}\left(p_{42}+p_{43}p_{32}\right)$$

$$U_3=p_{12}\left(p_{23}+p_{24}p_{43}\right)+p_{13}\left(1-p_{22}+p_{24}p_{42}\right)+p_{14}\left(p_{43}\left(1-p_{22}\right)-p_{42}p_{23}\right)$$

$$U_4=p_{12}\left(p_{24}+p_{23}p_{34}\right)+p_{13}\left(\left(1-p_{22}\right)p_{34}+p_{32}p_{24}\right)+p_{14}\left(\left(1-p_{22}\right)-p_{23}p_{32}\right)$$

Now, the expected up time of the system up to epoch t-1 (total t epochs) is given by-

$$\mu_{up}(t)=\sum_{x=0}^{t-1}A_0(x)$$

So that, $\mu^*_{up}(h)=A^*_0(h)/(1-h)$ (45)

9. Busy Period Analysis

On taking geometric transform of (36-40) and simplifying the resulting equations for k=G and P we get

$$B_0^{G*}(h)=\frac{U_1^*Z_1^*+U_3^*Z_3^*}{D_2(h)} \quad \text{and} \quad B_0^{P*}(h)=\frac{U_2^*Z_2^*+U_4^*Z_4^*}{D_2(h)}$$

(46-47)

Where,

$$U^*_3=\left[hq^*_{12}\left(hq^*_{23}+h^2q^*_{24}q^*_{43}\right)+hq^*_{13}\left(1-hq^*_{22}+h^2q^*_{24}q^*_{42}\right)+hq^*_{14}\left(hq^*_{43}\left(1-hq^*_{22}\right)-h^2q^*_{42}q^*_{23}\right)\right]hq^*_{01}$$

$$U^*_4=\left[hq^*_{12}\left(hq^*_{24}+h^2q^*_{23}q^*_{34}\right)+hq^*_{13}\left(hq^*_{34}\left(1-hq^*_{22}\right)+h^2q^*_{32}q^*_{24}\right)+hq^*_{14}\left(1-hq^*_{22}+h^2q^*_{24}q^*_{32}\right)\right]hq^*_{01}$$

and $D_2(h)$ is same as given in section 10.

In the long run the respective probabilities that the repairman is busy in the repair of failed unit in perfect and imperfect environment conditions are given by-

$$B_0^G = \lim_{t\to\infty} B_o^G(t) = \lim_{h\to 1}(1-h)\frac{N_3(h)}{D_2(h)}$$

$$B_0^P = \lim_{t\to\infty} B_o^P(t) = \lim_{h\to 1}(1-h)\frac{N_4(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0^G = -\frac{U_1\psi_1 + U_3\psi_3}{D_2'(1)} \qquad \text{and} \qquad B_0^P = -\frac{U_2\psi_2 + U_3\psi_3}{D_2'(1)}$$

(48-49)

and $D_2'(1)$ is same as given in section 10.

Now the expected busy period of the repairman is busy in the repair of failed unit in perfect and imperfect environment conditions up to epoch (t-1) are respectively given by-

$$\mu_b^G(t) = \sum_{x=0}^{t-1} B_0^G(x), \qquad \mu_b^P(t) = \sum_{x=0}^{t-1} B_0^P(x)$$

So that,

$$\mu_b^{G*}(h) = \frac{B_0^{G*}(h)}{(1-h)}, \qquad \mu_b^{P*}(h) = \frac{B_0^{P*}(h)}{(1-h)} \tag{50-51}$$

10. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier section.

Let us consider,

K_0 =revenue per-unit time by the system when it is operative.

K_1 =cost per-unit time when repairman is busy in the repairing failed unit in perfect environment condition.

K_2 =cost per-unit time when repairman is busy in the repairing failed unit in imperfect environment condition.

Then, the net expected profit incurred up to epoch (t-1) given by

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b^G(t) - K_2\mu_b^P(t) \tag{52}$$

The expected profit per unit time in steady state is given by-

$$P = \lim_{t\to\infty}\frac{P(t)}{t} = \lim_{h\to 1}(1-h)^2 P^*(h)$$

$$= K_0 \lim_{h\to 1}(1-h)^2 \frac{A_0^*(h)}{(1-h)} - K_1 \lim_{h\to 1}(1-h)^2 \frac{B_0^{G*}(h)}{(1-h)} - K_2 \lim_{h\to 1}(1-h)^2 \frac{B_0^{P*}(h)}{(1-h)}$$

$$= K_0 A_0 - K_1 B_0^G - K_2 B_0^P \tag{53}$$

11. Graphical Representation

The curves for MTSF and profit function have been drawn for different values of parameters. Fig.2 depicts the variations in MTSF with respect to failure rate (p) of operative unit for different values of repair rate (r) of failed unit in perfect environmental condition and rate of the change of environment from perfect to imperfect (a) when repair rate of failed unit in imperfect environmental condition and rate of change of environment from imperfect to perfect are kept fixed as $r' = 0.07$ and $c = 0.1$.

The smooth curves shows the trends for three different values 0.2, 0.3 and 0.4 of r when 'a' is taken as 0.01 whereas dotted curves shows the trends for same three values of 'r' as above when 'a' is taken as 0.10. From these curves we observed that MTSF decreases uniformly as the values of 'p' and 'a' increase and increases with the increase in 'r'. From the curve of MTSF we also conclude that to achieve at least a specified value of expected life of the system say 3000 units, the failure rate p of a unit should not exceed 0.0124 and 0.0142 respectively for a = 0.10 and 0.01 when r is fixed as 0.4. Similarly when r = 0.3 and 0.2 one can find the upper bonds for a = 0.10 and 0.01.

Similarly, Fig. 3 reveals the variations in profit (P) with respect to p for varying values of r and a as in case of MTSF, when the values of other parameters are kept fixed as $r' = 0.07$, $c = 0.1$ $K_0 = 11$, $K_1 = 280$ and $K_2 = 400$. From this figure same trends in respect of p, r, a have been observed as in MTSF. Further it is also revealed by smooth curves that system is profitable only if p is less than 0.0153, 0.026, 0.041 respectively for r = 0.2, 0.3 and 0.4 for fixed $a = 0.01$. From dotted curves it is obvious that system is profitable only if p is less than 0.0105, 0.017 and 0.025 respectively for r = 0.2, 0.3 and 0.4 for fixed $a = 0.10$.

Thus the above graphical study reveals that the bonds of any parameter can be evaluated for fixed values of other parameters to get non-negative profit. Moreso, one can also obtain the upper bond of any parameter (in case the curve is of decreasing nature w.r.t. this paramter) to achieve at least any specific value of MTSF and the lower bond of any parameter (in case the curve is of increasing nature w.r.t this parameter) to achieve at least any particular value of MTSF.

This study will help the industrial manager to take decision to reduce the failure rate of a unit by incorporating its redundancy or to increase the repair rate of a failed unit by adopting various repair policies to get a specified value of expected life time and non-negative expected profit by the system.

Behavior of MTSF with respect to p, r and a

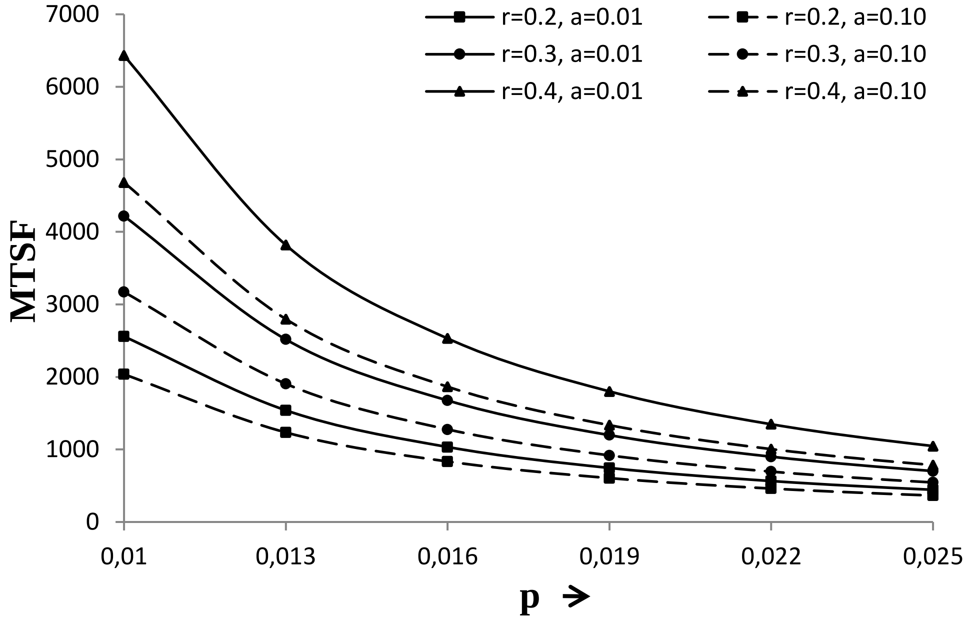

Fig. 2

Behavior of Profit (P) with respect to p, r and a

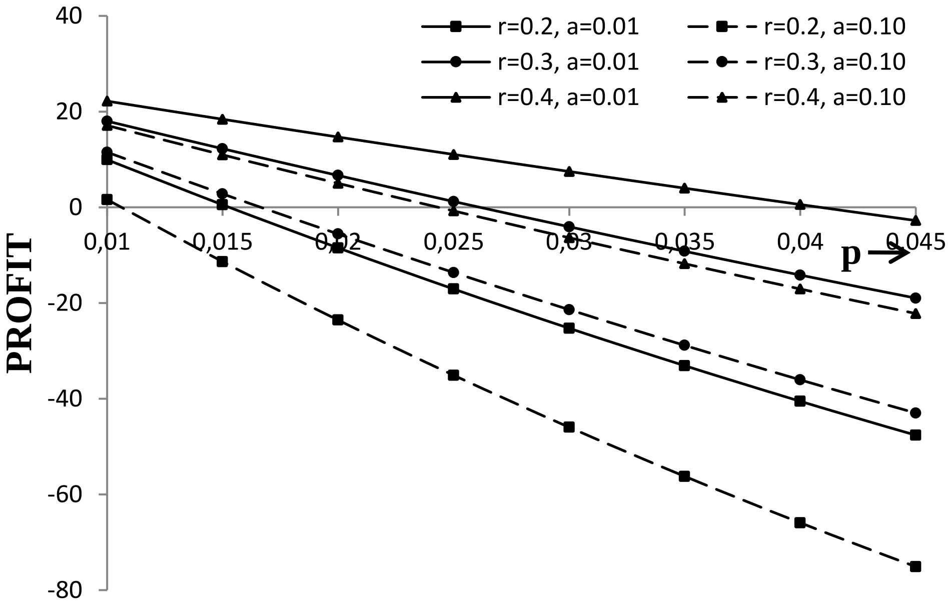

Fig. 3

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