A Two Identical Unit Cold Standby System Subject To Two Types Of Failures

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Abstract

The paper deals with a system model composed of a two identical unit standby system in which initially one is operative and other is kept as cold standby. Each unit of the system has two possible modes – Normal (N) and Total Failure (F). An operating unit may fail either due to normal or due to chance causes. A single repairman is always available with the system to repair a unit failed due to any of the above causes. The system failure occurs when both the units are in total failure mode. The failure time distributions of a unit failed due to both the causes are taken as exponentials with different parameters whereas the repair time distributions of a failed unit in both types of failure are taken as general with different CDFs. Using regenerative point technique, the various important measures of system effectiveness have been obtained:

Keywords: Reliability, Mean time to system failure, availability, expected busy period of repairman, net expected profit.

1. Introduction

The two unit cold standby systems have been widely studied in the literature of reliability as they are frequently used in modern business and industries. It is obvious that the standby unit is switched to operate when the operating unit fails and the switching device which is used to put the standby unit into operation may be perfect or imperfect at the time of need. In past years various authors including [1,2,4,5,6,9,10,11] analyzed the two identical and non-identical units standby redundant system models with different sets of assumptions such as imperfect switching device, slow switching device, waiting time distribution of repairman, repair machine failure etc. They have analyzed the two identical and non-identical unit system models by taking the single failure mode of an operating unit i.e. due to normal (ageing effect).

In many realistic situations, the systems are subject to two types of failure .One occurs by a normal cause and the other due to chance cause such as (i) abnormal environmental condition i.e. temperature, pressure, vibration etc. (ii) defective design (iii) misunderstanding the process variables (iv) operator's negligence and mishandling of the system etc. Keeping this fact in view few authors [3,7,8] analyzed the system models assuming two failure modes of each unit.

The purpose of the present paper is to deal with a stochastic model of a two identical unit cold standby redundant system subject to two types of failure in each of the operating unit. By using regenerative point technique, the following important measures of system effectiveness are obtained.

i. Transient-state and steady-state transition probabilities.
ii. Mean sojourn time in various regenerative states.
iii. Reliability and mean time to system failure (MTSF).
iv. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
v. The expected busy period of repairman in time interval (0, t).
vi. Net expected profit earned by the system in time interval (0, t) and in steady-state.

2. System Description and Assumptions

1. The system consists of two identical units. Initially, one unit is operative and other is kept as cold standby.
2. Each unit of the system has two possible modes: Normal (N) and Total Failure (F).
3. The switching device used to put the standby unit into operation is always perfect and instantaneous.
4. An operative unit may fail either due to normal cause i.e. due to ageing effect or due to chance cause.
5. The system failure occurs when both the units are in total failure mode.
6. A single repairman is always available at the system to repair a unit failed due to normal cause or due to chance cause.
7. The failure time distributions of the units to reach into the failure mode either due to normal or due to chance cause are taken as exponential whereas the repair time distributions of a unit failed due to both causes are taken as general with different CDF's.
8. A repaired unit always works as good as new.

3. Notations and States of the System

We define the following symbols for generating the various states of the system-

N_0, N_s	:	Unit is in N-mode and operative/standby
F_{1r}, F_{2r}	:	Unit is in failure mode due to normal cause/due to chance cause and under repair.
F_{1w}, F_{2w}	:	Unit is in failure mode due to normal cause/due to chance cause and waiting for repair.

Considering the above symbols in view of assumptions stated in section-2, the possible states of the system are shown in the transition diagram represented by **Fig. 1**. It is to be noted that the epochs of transitions into the state S_4 from S_1, S_3 from S_1, S_5 from S_2, S_6 from S_2 are non-regenerative, whereas all the other entrance epochs into the states of the system are regenerative. The states S_0, S_1 and S_2 are the up-states of the system and the states S_3, S_4, S_5 and S_6 are the failed states of the system.

TRANSITION DIAGRAM

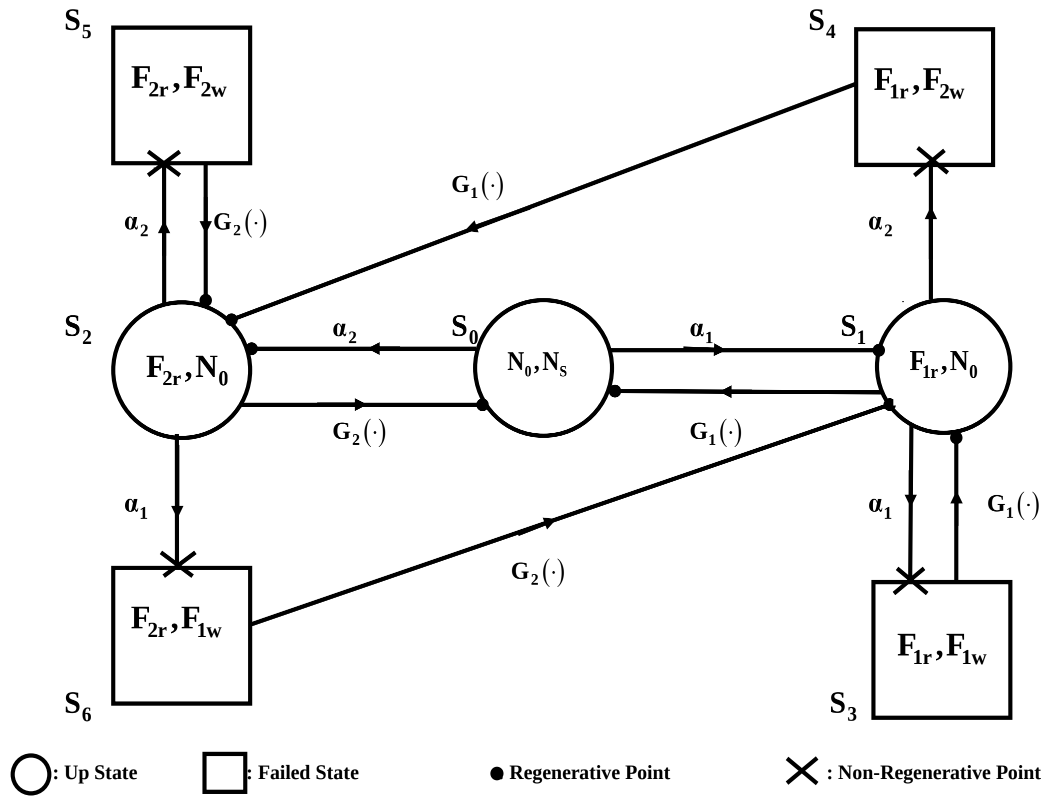

Fig. 1

The other notations used are defined as follows:

E : Set of regenerative states $\equiv \{S_0, S_1, S_2\}$

$\overline{E}$: Set of non-regenerative states $\equiv \{S_3, S_4, S_5, S_6\}$

α_1, α_2 : Constant failure rate of an operative unit due to normal cause/chance cause

$G_1(\cdot), G_2(\cdot)$: CDF of repair time of failed unit due to normal cause/chance cause.

$q_{ij}(\cdot)$: p.d.f of transition time from regenerative state S_i to S_j.

$q_{ij}^{(k)}(\cdot)$: p.d.f of transition time from regenerative state S_i to S_j via non-regenerative state S_k.

$p_{ij}(\cdot)$: One[1] step steady-state transition probability from regenerative state S_i to $S_j = \int q_{ij}(u)du$.

$p_{ij}^{(k)}(\cdot)$: Two step steady-state transition probability from regenerative state S_i to S_j via non-regenerative state $S_k = \int q_{ij}^{(k)}(u)du$.

n_1, n_2 : Mean repair times of operative unit and standby unit

[1] The limits of integration are taken to be 0 to ∞ whenever they are not mentioned

$$=\int \bar{G}_1(t)dt \text{ and } \int \bar{G}_2(t)dt$$

⊔ : Symbol for Laplace Stieltjes Transform. i.e. $\tilde{Q}_{ij}(s)=\int e^{-st}dQ_{ij}(t)$

© : Symbol for ordinary convolution i.e. $A(t)©B(t)=\int_0^t A(u)©B(t\text{-}u)du$

∗ : Symbol for Laplace Transform. i.e. $q_{ij}^{*}(s)=\int e^{-st}q_{ij}(u)du$

4. Transition Probabilities and Sojourn Times

(a) The direct or one step steady-state transition probabilities are as follows

$$p_{01}=\int e^{\alpha_2 t}\alpha_1 e^{\alpha_1 t}dt=\frac{\alpha_1}{\alpha_1+\alpha_2}$$

$$p_{02}=\int e^{-\alpha_1 t}\alpha_2 e^{-\alpha_2 t}dt=\frac{\alpha_2}{\alpha_1+\alpha_2}$$

$$p_{10}=\int e^{-(\alpha_1+\alpha_2)t}dG_1(t)=\tilde{G}_1(\alpha_1+\alpha_2)$$

$$p_{20}=\int e^{-(\alpha_1+\alpha_2)t}dG_2(t)=\tilde{G}_2(\alpha_1+\alpha_2)$$

(b) The two step steady-state transition probabilities are given by

$$p_{11}^{(3)}=\int \alpha_1 e^{-\alpha_1 u}du\, e^{\alpha_2 u}\bar{G}_1(u)\int_u^\infty \frac{d\bar{G}_1(t)}{\bar{G}_1(u)}$$

$$=\alpha_1\int dG_1(t)\int_0^t e^{-(\alpha_1+\alpha_2)u}du$$

$$=\frac{\alpha_1}{\alpha_1+\alpha_2}\int\left[1-e^{-(\alpha_1+\alpha_2)t}\right]dG_1(t)$$

$$=\frac{\alpha_1}{\alpha_1+\alpha_2}\left[1-\tilde{G}_1(\alpha_1+\alpha_2)\right]$$

Similarly,

$$p_{12}^{(4)}=\frac{\alpha_2}{\alpha_1+\alpha_2}\left[1-\tilde{G}_1(\alpha_1+\alpha_2)\right]$$

$$p_{22}^{(5)}=\frac{\alpha_2}{\alpha_1+\alpha_2}\left[1-\tilde{G}_2(\alpha_1+\alpha_2)\right]$$

$$p_{21}^{(6)}=\frac{\alpha_1}{\alpha_1+\alpha_2}\left[1-\tilde{G}_2(\alpha_1+\alpha_2)\right]$$

We observe the following relationship

$$p_{01}+p_{02}=1, \qquad p_{10}+p_{11}^{(3)}+p_{12}^{(4)}=1, \qquad p_{20}+p_{22}^{(5)}+p_{21}^{(6)}=1$$

(1-3)

(a) The mean sojourn times in various states are as follows:

$$\psi_0 = \int e^{-(\alpha_1+\alpha_2)t} dt = \frac{1}{\alpha_1+\alpha_2}$$

Similarly,

$$\psi_1 = \int e^{-(\alpha_1+\alpha_2)t} \bar{G}_1(t) dt$$

$$\psi_2 = \int e^{-(\alpha_1+\alpha_2)t} \bar{G}_2(t) dt$$

5. Analysis of Characteristics

(a) RELIABILITY AND MTSF

Let $R_i(t)$ be the probability that the system is operative during $(0,t)$ given that at t=0 it starts from state $S_i \in E$. By simple probabilistic arguments, we have the following recurrence relations in $R_i(t)$; i = 0, 1, 2

$$R_0(t) = Z_0(t) + q_{01}(t) © R_1(t) + q_{02}(t) © R_2(t)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{10}(t) © R_0(t)$$

$$R_2(t) = Z_2(t) + q_{20}(t) © R_0(t) \qquad (4\text{-}6)$$

Where,

$$Z_0(t) = e^{-(\alpha_1+\alpha_2)t}, \quad Z_1(t) = e^{-(\alpha_1+\alpha_2)t}\bar{G}_1(t), \qquad Z_2(t) = e^{-(\alpha_1+\alpha_2)t}\bar{G}_2(t)$$

Taking Laplace Transforms of the relation (4-6) and solving the resulting set of algebraic equations for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*}{1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*} \qquad (7)$$

We have omitted the argument's' from $q_{ij}^*(s)$ and $Z_i^*(s)$.

The expression of mean time to system failure is given by

$$E(T_0) = \lim_{s \to 0} R_0^*(s)$$

Observing that $q_{ij}^*(0) = p_{ij}$ and $Z_i^*(0) = \psi_i$, we get

$$E(T_0) = \frac{\psi_0 + p_{01}\psi_1 + p_{02}\psi_2}{1 - p_{01}p_{10} - p_{02}p_{20}} \qquad (8)$$

b) AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is up at epoch t, when initially it starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $A_0(t)$ in terms of its Laplace transforms i.e. $A_0^*(s)$ given as follows-

$$A_0^*(s)=\frac{N_1(s)}{D_1(s)} \tag{9}$$

Where,

$$N_1(s)=Z_0^*\left[\left(1-q_{11}^{(3)*}\right)\left(1-q_{22}^{(5)*}\right)-q_{12}^{(4)*}q_{21}^{(6)*}\right]+Z_1^*\left[q_{02}^*q_{21}^*+q_{01}^*\left(1-q_{22}^{(5)*}\right)\right]$$
$$+Z_2^*\left[q_{01}^*q_{12}^{(4)*}+\left(1-q_{11}^{(3)*}\right)q_{02}^*\right]$$
$$D_1(s)=\left[\left(1-q_{11}^{(3)*}\right)\left(1-q_{22}^{(5)*}\right)-q_{12}^{(4)*}q_{21}^{(6)*}\right]-q_{01}^*\left[q_{20}^*q_{12}^{(4)*}+q_{10}^*\left(1-q_{22}^{(5)*}\right)\right]$$
$$-q_{02}^*\left[q_{10}^*q_{21}^{(6)*}+\left(1-q_{11}^{(3)*}\right)q_{20}^*\right]$$

The steady-state availability of the system is given by

$$A_0=\lim_{s\to 0} sA_0^*(s)=\lim_{s\to 0} s\frac{N_1(s)}{D_1(s)} \tag{10}$$

We observe that

$$D_1(0)=0$$

Therefore, by using L.Hospital's rule the steady state availability is given by

$$A_0=\frac{N_1(0)}{D_1'(0)} \tag{11}$$

Where,

$$N_1(0)=\psi_0\left[p_{10}\left(1-p_{22}^{(5)}\right)-p_{20}p_{12}^{(4)}\right]+\psi_1\left[p_{01}\left(1-p_{22}^{(5)}\right)+p_{02}p_{20}\right]$$
$$+\psi_2\left[p_{01}p_{12}^{(4)}+p_{02}\left(1-p_{11}^{(3)}\right)\right]$$

$$D_1'(0)=\left[p_{10}\left(1-p_{22}^{(5)}\right)+p_{12}^{(4)}p_{20}\right]\psi_0+\left(p_{01}\left(1-p_{22}^{(5)}\right)+p_{02}p_{21}^{(6)}\right)n_1$$
$$+\left(p_{01}p_{12}^{(4)}+p_{02}\left(1-p_{11}^{(3)}\right)\right)n_2 \tag{12}$$

The expected up time of the system in interval (0, t) is given by

$$\mu_{up}(t)=\int_0^t A_0(u)\,du$$

so that,

$$\mu^*_{up}(s)=\frac{A_0^*(s)}{s} \tag{13}$$

(c) BUSY PERIOD ANALYSIS

Let $B_i^1(t)$ and $B_i^2(t)$ be the probability that the repairman is busy in the repair of a failed unit due to normal cause and due to chance shock at time t when system initially starts from state $S_i \in E$. Using the simple probabilistic arguments in regenerative point technique and the tools of Laplace transforms, one can obtain the value of $B_i^1(t)$ and $B_i^2(t)$ in terms of their Laplace transforms as follows-

$$B_0^{1*}(s)=\frac{N_2(s)}{D_1(s)} \quad \text{and} \quad B_0^{2*}(s)=\frac{N_3(s)}{D_1(s)} \tag{14-15}$$

where,

$$N_2(s)=\left[q_{01}^*\left(1-q_{22}^{(5)*}\right)+q_{02}^*q_{21}^{(6)*}\right]\bar{G}_1^*$$

$$N_3(s)=\left[q_{02}^*\left(1-q_{11}^{(3)*}\right)+q_{01}^*q_{12}^{(4)*}\right]\bar{G}_2^*$$

and $D_1(s)$ is already defined in section 5(b). $\bar{G}_1^*$ and $\bar{G}_2^*$ are the L.T. of $\bar{G}_1(t)$ and $\bar{G}_2(t)$.

In the long run, the probabilities that the repairman will be busy in repair of normal cause and chance causes are as follows-

$$B_0^1=\frac{N_2(0)}{D_1'(0)} \quad \text{and} \quad B_0^2=\frac{N_3(0)}{D_2'(0)} \tag{16-17}$$

where,

$$N_2(0)=\left[p_{01}\left(1-p_{22}^{(5)}\right)+p_{02}p_{21}^{(6)}\right]n_1$$

$$N_3(0)=\left[p_{02}\left(1-p_{11}^{(3)}\right)+p_{01}p_{12}^{(4)}\right]n_2$$

The value of $D_1'(0)$ is same as given in expression (12).

The expected busy period of the repairman in repair in repairing during (0, t) are given by

$$\mu_b^1(t)=\int_0^t B_0^1(u)du \quad \text{and} \quad \mu_b^2(t)=\int_0^t B_0^2(u)du$$

so that,

$$\mu_b^{1*}(s)=\frac{B_0^{1*}(s)}{s} \quad \text{and} \quad \mu_b^{2*}(s)=\frac{B_0^{2*}(s)}{s} \tag{18-19}$$

(d) PROFIT FUNCTION ANALYSIS

The net expected total profit incurred by the system in time interval (0, t) is given by

$P(t)=$ Expected total revenue in (0, t) - Expected cost of repair in (0, t)

$$=K_0\mu_{up}(t)-K_1\mu_b^1(t)-K_2\mu_b^2(t) \tag{20}$$

Where, K_0 is the revenue per- unit up time by the system during its operation. K_1 and K_2 are the amounts paid to the repairman per-unit of time when he is busy in repair of a unit failed due to normal cause and due to chance cause respectively.

The expected total profit incurred per unit time in steady-state is given by

$$P=K_0A_0-K_1B_0^1-K_2B_0^2 \tag{21}$$

6. Particular Cases

Case 1: When the repair time of both the units also follow exponential distribution with p.d.fs as follows-

$$g_1(t) = \eta_1 e^{-\eta_1 t}, \qquad g_2(t) = \eta_2 e^{-\eta_2 t}$$

The Laplace Transform of above density functions are as given below.

$$g_1^*(s) = \tilde{G}_1(s) = \frac{\eta_1}{s+\eta_1}, \qquad g_2^*(s) = \tilde{G}_2(s) = \frac{\eta_2}{s+\eta_2}$$

Here $\tilde{G}_i(s)$ are the Laplace-Stieltjes Transforms of the c.d.fs $G_i(t)$ corresponding to the p.d.fs $g_i(t)$.

In view of above, the changed values of transition probabilities and mean sojourn times are given below-

$$p_{01} = \frac{\alpha_1}{\alpha_1+\alpha_2}, \qquad p_{02} = \frac{\alpha_2}{\alpha_1+\alpha_2}, \qquad p_{10} = \frac{\eta_1}{\alpha_1+\alpha_2+\eta_1}$$

$$p_{11}^{(3)} = \frac{\alpha_1}{\alpha_1+\alpha_2+\eta_1}, \qquad p_{12}^{(4)} = \frac{\alpha_2}{\alpha_1+\alpha_2+\eta_1}, \qquad p_{20} = \frac{\eta_2}{\alpha_1+\alpha_2+\eta_2}$$

$$p_{22}^{(5)} = \frac{\alpha_2}{\alpha_1+\alpha_2+\eta_2}, \qquad p_{21}^{(6)} = \frac{\alpha_1}{\alpha_1+\alpha_2+\eta_2}$$

$$\psi_0 = \frac{1}{\alpha_1+\alpha_2}, \qquad \psi_1 = \frac{1}{\alpha_1+\alpha_2+\eta_1}, \qquad \psi_1 = \frac{1}{\alpha_1+\alpha_2+\eta_2}$$

7. Graphical Study Of Behaviour

The curves for **MTSF** and **profit function** are drawn for the two particular cases: case 1 and case 2 in respect of different parameters. **In Case 1, when the repair time of unit-1 also follow exponential distribution**. We plot curves for **MTSF** and **profit function** in **Fig. 2** and **Fig. 3** w.r.t. $\boldsymbol{\alpha_1}$ for three different values of $\boldsymbol{\eta_1}$ and two different values of $\boldsymbol{\eta_2}$ while the other parameters are kept fixed as $\boldsymbol{\alpha_2 = 0.029}$. From the curves of **Fig. 2** we observe that MTSF increases uniformly as the value of η_1 and η_2 increase and it decreases with the increase in $\boldsymbol{\alpha_1}$. Further, we also observed from **Fig. 2** that the value of $\boldsymbol{\alpha_1}$ must be less than **0.012, 0.114** and **0.017** corresponding to $\boldsymbol{\eta_1 = 0.1}$, **0.2** and **0.3** to achieve at least **300** units of **MTSF** when $\boldsymbol{\eta_2 = 0.9}$ is fixed as. Similarly, we can find the upper bounds of $\boldsymbol{\alpha_1}$ corresponding to the values of $\boldsymbol{\eta_1}$ to achieve **300** units of **MTSF** when $\boldsymbol{\eta_2}$ is kept fixed as **0.4**.

Similarly, **Fig. 3** reveals the variations in profit w.r.t. $\boldsymbol{\alpha_1}$ for varying values of $\boldsymbol{\eta_1}$ and $\boldsymbol{\eta_2}$, when the values of other parameters are kept fixed as $\boldsymbol{\alpha_2 = 0.00009}$, $\mathbf{K_0 = 70}$, $\mathbf{K_1 = 40}$ and $\mathbf{K_2 = 500}$. Here also the same trend in respect of $\boldsymbol{\alpha_1}$, $\boldsymbol{\eta_1}$ and $\boldsymbol{\eta_2}$ are observed as in case of **MTSF.** From the figure it is clearly observed from the smooth curves, that the system is profitable if the value of parameter $\boldsymbol{\alpha_1}$ is less than **0.41, 0.56** and **0.86** respectively for $\boldsymbol{\eta_1 = 0.2}$, **0.3** and **0.5** for fixed value of $\boldsymbol{\eta_2 = 0.9}$. From dotted curves, we conclude that system is profitable if the value of parameter α_1 is less than **0.44, 0.60** and **0.90** respectively for $\eta_1 = 0.2$, 0.3 and 0.5 for fixed value of $\eta_2 = 0.05$.

Behaviour of MTSF w.r.t. α_1 for different values of λ_1 and λ_2

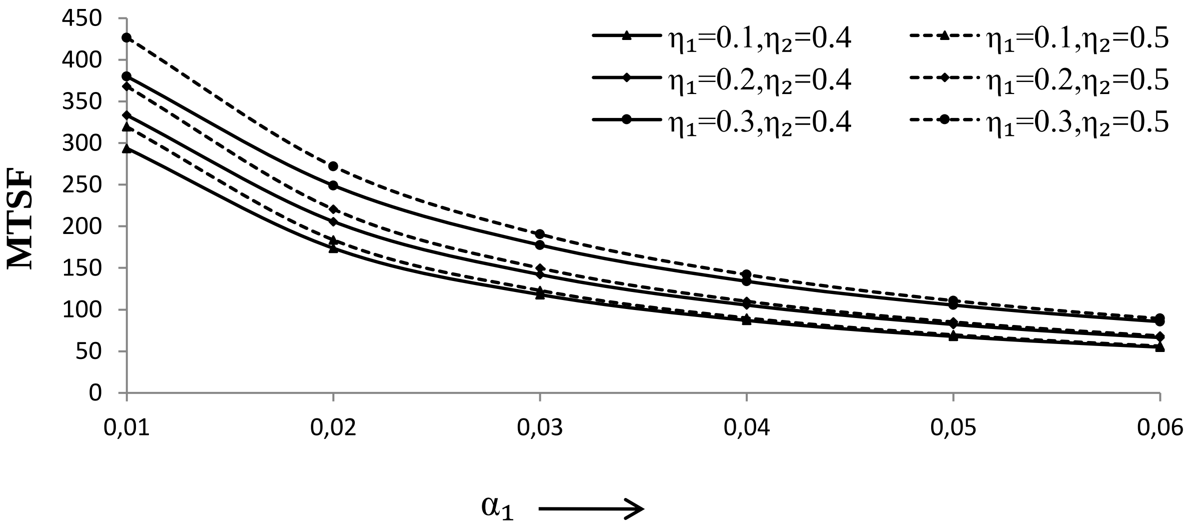

Fig. 2

Behaviour of PROFIT (P) w.r.t. α_1 for different values of λ_1 and λ_2

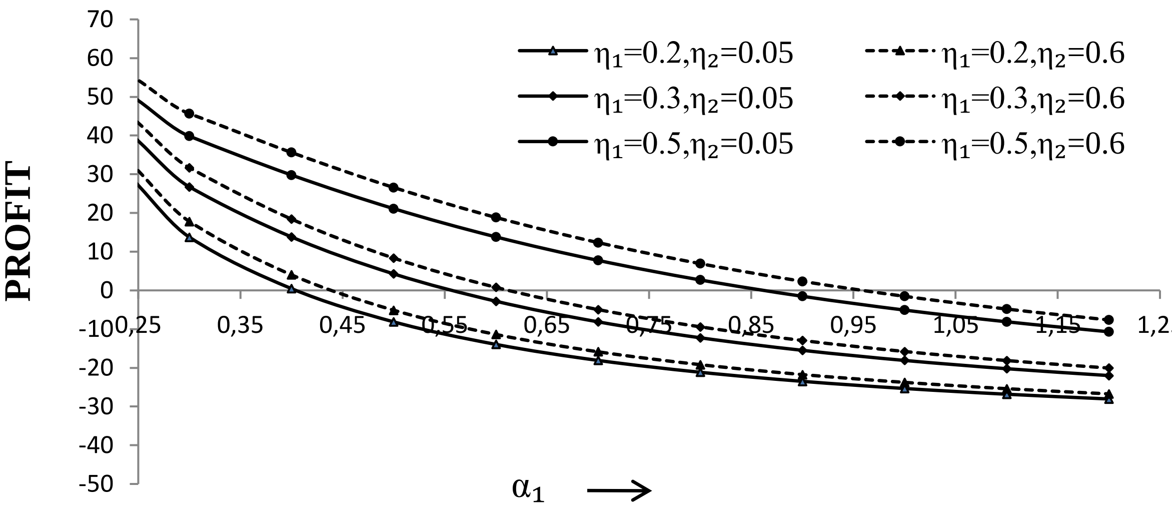

Fig. 3

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