Analysis of Reliability Measures of Two Identical Unit System with One Switching Device and Imperfect Coverage

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Abstract

The present paper deals with the reliability analysis of a two identical unit system model with safe and unsafe failures, switching device and rebooting. Initially one of the units is in operative state and other is kept in standby mode. A single repairman is always available with the system for repairing and rebooting the failed units. In case of unsafe failure, repair cannot be started immediately but first rebooting is done which transforms the unsafe failure to safe failure and thereafter repair is carried out as usual. Switching device is used to put the repaired and standby units to operation. The failure time distributions of both the units and switch are also assumed to be exponential while the repair time distributions are taken general in nature. Reboot delay time is assumed to be exponentially distributed. Using regenerative point techniques, various measures of system effectiveness such as transition probabilities, availability, busy period, expected numbers of repairs etc. have been obtained, to make the study more informative some of them have been studied graphically.

Key Words: reliability, availability, regenerative point technique, rebooting, coverage probability

1. Introduction

In the context of global competition and paced development, it has become the foremost concern to make the apt decisions in order to increase the reliability and profit margin of every institution. With the advent of complexity of machines and more advancements in industrial sectors, the focus on increasing reliability and profit margins of any firm is increasing day by day as it is the sole aim on which most of the firms/industries are flourishing. It has become an important point which has to be kept in mind that the designs and layout of complex equipments should be in such a way that it enhances the reliability of the system and try to minimize the loopholes which are responsible for its degradation. Hence, designing the reliable systems and determining their availability have become the relevant steps in almost every sector.

In many situations from daily life, we find that the breakdown of the units' results into machine failure which results into huge losses and one of the ways to increase reliability is to introduce standby units which increase its reliability. There also arises certain situations when reason for the failure of unit is not detected immediately which leads to the situation of imperfect coverage, which is further tackled by reboot. Depending upon the complexity the timings of reboot delay vary from system to system.. In the last few decades, elaborated and comprehensive research work regarding the reliability, availability, standby systems, imperfect coverage, reboot etc has been carried out. The concept of reboot is discussed by Trivedi [8] in his book 'Probability and Statistics with Reliability, Queueing and Computer Science Applications'. Several empirical studies are proposed by P.A. Keiller and D.R. Miller [3] to increase the reliability of system. The imperfect coverage models with various status and trends were given by Amari, et al. [1]. Hsu, et al. [4] have studied the machine repair problem with standby system, repair and reboot delay. The reliability measure of repairable system with standby switching failures and reboot delay is studied by Jyh-Bin, et al. [5]. The other important contributions are made by Amari, et al. [2], Wang and Chen [9], Ke and Liu [7]. Ke, et al. [6] has also done the analysis by considering detection, imperfect coverage and reboot as major factor.

The present paper here deals with the reliability analysis of a system model with two identical units and one switching device. The switching device is used to turn the unit from standby or repaired state to operative state and is assumed to be in good condition when the system initially starts. The failure in any of the identical unit or switch may result into safe/ unsafe failures. Unsafe failure is the situation when reason for any of the breakdown is not known which is cleared by reboot first. Reboot delay time, failure time of both the units and switch are assumed to be exponentially distributed while the repair time distributions are general in nature. Other measures of system effectiveness such as mean time to system failure, reliability, availability, expected number of repair have been evaluated using regenerative point techniques.

2. System Description and Assumptions

1. The system comprises of two identical units N_0 and N_s, one switch S is attached to it.
2. Initially one of the units is in operative state and other is kept in standby mode. Switching device is used to put the repaired and standby units to operation. In the initial phase, switching device is assumed to be in good condition.
3. The failures of units and switch in system might be safe and unsafe. Whenever any of the unit or switch of system results in safe failure, it may be immediately detected and located with coverage probability c and it will be repaired immediately if the repairman is available.
4. In case of unsafe failure, repair cannot be started immediately but first rebooting is done which transforms the unsafe failure to safe failure and thereafter repair is carried out as usual. Reboot delay times for units and switch are considered to be exponentially distributed random variables with different parameters.
5. A single repairman is always available with the system for repair and rebooting the failed units. Switch is always given preference over the failed units in the system for its repair.

6. The failure time distributions of both the units and switch are assumed to be exponential while the repair time distributions are taken general in nature.
7. Once a component is repaired it is as good as new.

3. Notations And Symbols

α : Failure rate of identical units
β : Failure rate of switching device
$H_1(.)$: Repair rate of failed unit
$H_2(.)$: Repair rate of switching device
c : Coverage probability
γ : Rebooting delay rate for unsafe failure for units
δ : Rebooting delay rate for unsafe failure for switching device

SYMBOLS FOR THE STATES OF THE SYSTEM

$N_0/N_s/N_g$: Unit is in operative / standby / good condition.
N_{usf}/S_{usf} : Unit/ switching device has undergone unsafe failure
N_r/N_{wr} : Unit is under repair / waiting for repair
S_r : Switch is under repair

With the help of the symbols defined above, the possible states of the system are:

$S_0 = [N_0, N_s, S_g]$ $S_1 = [N_r, N_0, S_g]$ $S_2 = [N_{usf}, N_g, S_g]$ $S_3 = [N_0, N_s, S_r]$
$S_4 = [N_g, N_s, S_{usf}]$ $S_5 = [N_g, N_g, S_r]$ $S_6 = [N_{usf}, N_g, S_r]$ $S_7 = [N_{wr}, N_g, S_r]$
$S_8 = [N_r, N_{usf}, S_g]$ $S_9 = [N_r, N_{wr}, S_g]$ $S_{10} = [N_r, N_g, S_{usf}]$

The transition diagram along with all transitions is shown in fig.1

TRANSITION DIAGRAM

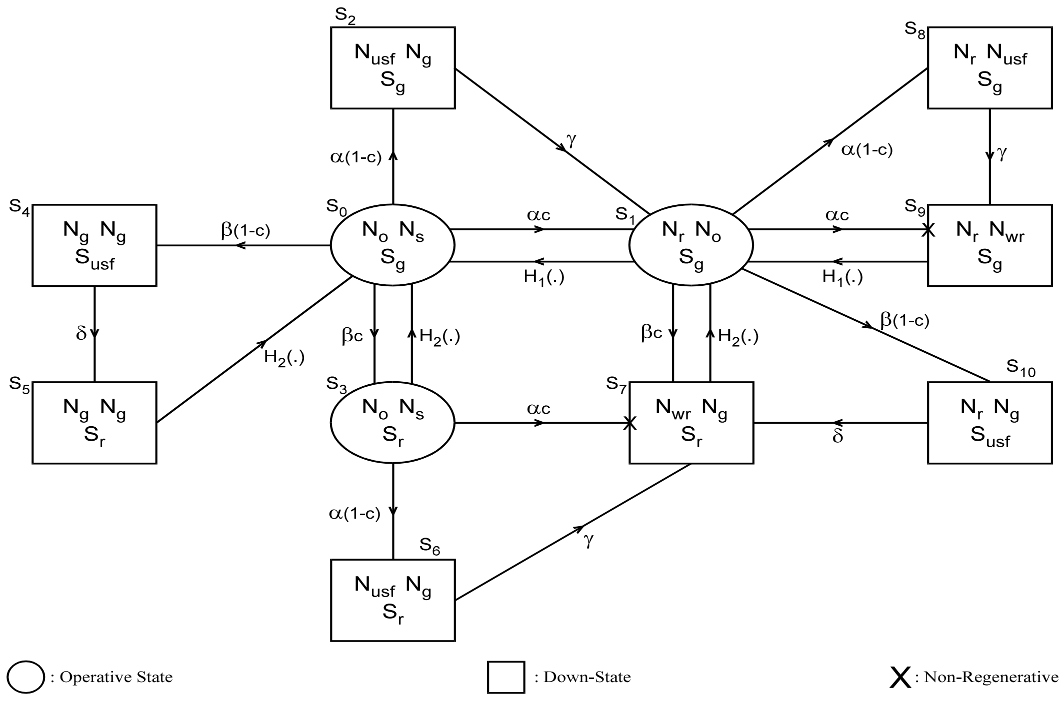

Fig.1

4. Transition Probabilities And Sojourn Times

Let X_n denotes the state visited at epoch T_{n^+} just after the transition at T_n, where T_1, T_2 .... represents the regenerative epochs. Then, Markov-Renewal process is constituted by $\{X_n, T_n\}$ with state space E representing set of regenerative states and

$$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$$

is the semi Markov kernel over E.

Then the transition probability matrix of the embedded Markov chain is

$$P = p_{ij} = Q_{ij}(\infty) = Q(\infty)$$

First we obtain the following direct steady-state transition probabilities:

$$p_{01} = \alpha c \int e^{-(\alpha+\beta)u}\, du = \frac{\alpha c}{(\alpha+\beta)}$$

Similarly,

$$p_{02} = \frac{\alpha(1-c)}{(\alpha+\beta)} \qquad p_{03} = \frac{\beta c}{(\alpha+\beta)}$$

$$p_{04} = \frac{\beta(1-c)}{(\alpha+\beta)} \qquad p_{10} = \widetilde{H}_1(\alpha+\beta)$$

$$p_{17} = \frac{\beta c}{(\alpha+\beta)}\left[1 - \widetilde{H}_1(\alpha+\beta)\right] \qquad p_{18} = \frac{\alpha(1-c)}{(\alpha+\beta)}\left[1 - \widetilde{H}_1(\alpha+\beta)\right]$$

$$p_{1,10} = \frac{\beta(1-c)}{(\alpha+\beta)}\left[1 - \widetilde{H}_1(\alpha+\beta)\right] \qquad p_{30} = \widetilde{H}_2(\alpha)$$

$$p_{36} = (1-c)\left[1 - \widetilde{H}_2(\alpha)\right]$$

$$p_{21} = p_{45} = p_{50} = p_{67} = p_{71} = p_{89} = p_{91} = p_{10,7} = 1$$

The indirect transition probability may be obtained as follows:

$$Q_{11}^{(9)}(t) = \alpha c \int_0^t e^{-(\alpha+\beta)u}\, \overline{H}_1(u) du \int_t^v \frac{dH_1(v)}{\overline{H}_1(u)}$$

$$= \alpha c \int_0^t dH_1(v) \int_0^v e^{-(\alpha+\beta)u}\, du$$

$$= \frac{\alpha c}{\alpha+\beta} \int_0^t (1 - e^{-(\alpha+\beta)v})\, dH_1(v)$$

By taking $t \to \infty$, we obtain the following indirect steady-state transition probability:

$$p_{11}^{(9)} = \frac{\alpha c}{\alpha+\beta} \int (1 - e^{-(\alpha+\beta)v}) dH_1(v) = \frac{\alpha c}{\alpha+\beta}\left[1 - \widetilde{H}_1(\alpha+\beta)\right]$$

Similarly,

$$p_{31}^{(7)} = c\left[1 - \widetilde{H}_2(\alpha)\right] \tag{1}$$

From these steady state probabilities obtained above, it can be easily verified that the following results holds good:

$$p_{01} + p_{02} + p_{03} + p_{04} = 1, \qquad p_{10} + p_{11}^{(9)} + p_{17} + p_{18} + p_{1,10} = 1$$

$$p_{30} + p_{36} + p_{31}^{(7)} = 1, \qquad p_{21} = p_{45} = p_{50} = p_{67} = p_{71} = p_{89} = p_{91} = p_{10,7} = 1 \tag{2}$$

Mean sojourn times

Mean sojourn time is defined as the expected time taken by the system in a state before making transition to any other state. Let Ψ_i be the mean sojourn time for state S_i , then to obtain mean sojourn time Ψ_i in state S_i ,we observe that there is no transition from S_i to any other state as long as the system is in state S_i.If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is:

$$\Psi_i = E[T_i] = \int P(T_i > t) dt$$

Hence, using it following expressions for mean sojourn time is obtained:

$$\Psi_0 = \frac{1}{(\alpha+\beta)} \qquad \Psi_1 = \frac{1}{(\alpha+\beta)}(1 - \widetilde{H}_1) \qquad \Psi_2 = \Psi_6 = \Psi_8 = \frac{1}{\gamma}$$

$$\Psi_3 = \frac{1}{\alpha}(1 - \widetilde{H}_2) \qquad \Psi_4 = \Psi_{10} = \frac{1}{\delta} \qquad \Psi_5 = \Psi_7 = \int \widetilde{H}_2(t) dt$$

$$\Psi_9 = \int \widetilde{H}_1(t)\, dt \tag{3}$$

5. Analysis Of Reliability And MTSF

Let T_i be the random variable denoting time to system failure when system starts up from state $S_i \in E_i$, then the reliability of the system is given by
$R_i(t) = P[T_i > t]$
To obtain $R_i(t)$, we consider failed states as absorbing states.
By referring to the state transition diagram, the recursive relations among $R_i(t)$ can be formulated on the basis of probabilistic arguments. Taking their Laplace Transform and solving the resultant set of equations for $R_0^*(s)$, we get

$$R_0^*(s) = N_1(s)/D_1(s) \tag{4}$$

where,
$N_1(s) = Z_0^* + q_{01}^* Z_1^* + q_{03}^* Z_3^*$
and
$D_1(s) = 1 - q_{01}^* q_{10}^* - q_{03}^* q_{30}^*$
Taking inverse Laplace Transform of **(4),** we get reliability of the system.
To get MTSF, we use the well known formula
$E(T_0) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) = N_1(0)/D_1(0)$
where,
$N_1(0) = \Psi_0 + p_{01}\Psi_1 + p_{03}\Psi_3$
and
$D_1(0) = 1 - p_{01}p_{10} - p_{03}p_{30}$
Since, we have $q_{ij}^*(0) = p_{ij}$ *and* $\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \Psi_i$

6. Availability Analysis

Define $A_i(t)$ as the probability that the system is available at time 't' given that initially started from state $S_i \in E_i$. Point wise availability is another measure of system effectiveness and is defined as the probability that the system will be able to work satisfactorily within tolerances at any instant of time. By using simple stochastic arguments, the recurrence relations among different point wise availabilities are obtained and taking the Laplace transforms and solving the resultant set of equations for $A_0^*(s)$, we have

$$A_0^*(s) = N_2(s)/D_2(s) \tag{5}$$

where,

$$N_2(s) = (1 - q_{11}^{(9)*} - q_{18}^* q_{89}^* q_{91}^* - q_{1,10}^* q_{10,7}^* q_{71}^* - q_{17}^* q_{71}^*)(Z_0^* + q_{03}^* Z_3^*) + Z_1^*(q_{01}^* + q_{03}^* q_{31}^{(7)*} + q_{03}^* q_{36}^* q_{67}^* q_{71}^* + q_{02}^* q_{21}^*) \tag{6}$$

and,

$$D_2(s) = (1 - q_{11}^{(9)*} - q_{18}^* q_{89}^* q_{91}^* - q_{1,10}^* q_{10,7}^* q_{71}^* - q_{17}^* q_{71}^*)(1 - q_{03}^* q_{30}^* - q_{04}^* q_{45}^* q_{50}^*) - q_{01}^* q_{10}^* - q_{10}^* q_{03}^* q_{31}^{(7)*} - q_{10}^* q_{03}^* q_{36}^* q_{67}^* q_{71}^* - q_{10}^* q_{02}^* q_{21}^* \tag{7}$$

The steady state availability is given by
$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\, A_0^*(s) = N_2(0)/D_2(0)$
as we know that, $q_{ij}(t)$ is the pdf of the time of transition from state S_i to S_j and $q_{ij}(t)dt$ is the probability of transition from state S_i to S_j during the interval $(t, t + dt)$, thus
$q_{ij}^*(s)/_{s=0} = q_{ij}^*(0) = p_{ij}$
Also we know that
$\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \Psi_i$

Therefore,

$$N_2(0) = (1 - p_{11}^{9} - p_{18}p_{89}p_{91} - p_{17}p_{71} - p_{1,10}p_{10,7}p_{71})(\Psi_0 + p_{03}\Psi_3) + \Psi_1(p_{01} + p_{03}p_{31}^{7} + p_{02}p_{21} + p_{03}p_{36}p_{67}p_{71}) \quad (8)$$

$$D_2(0) = (1 - p_{11}^{9} - p_{18}p_{89}p_{91} - p_{17}p_{71} - p_{1,10}p_{10,7}p_{71})(1 - p_{03}p_{30} - p_{04}p_{45}p_{50}) - p_{10}p_{01} - p_{10}p_{03}p_{31}^{7} - p_{10}p_{03}p_{36}p_{67}p_{71} - p_{10}p_{02}p_{21} \quad (9)$$

The steady state probability that the system will be up in the long run is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\, A_0^*(s)$$

$$\lim_{s\to 0}\frac{sN_2(s)}{D_2(s)} = \lim_{s\to 0} N_2(s) \lim_{s\to 0}\frac{s}{D_2(s)}$$

Since as $s \to 0, D_2(s)$ becomes zero.

Hence, on using L'Hospital's rule, A_0 becomes

$$A_0 = N_2(0)/D_2'(0) \quad (10)$$

where,

$$D_2'(0) = p_{10}(\Psi_0 + \Psi_2 p_{02} + \Psi_3 p_{03}) + p_{10}p_{04}(\Psi_4 + \Psi_5) + p_{10}p_{03}p_{36}(\Psi_6 + \Psi_7) + (1 - p_{03}p_{30} - p_{04})[\Psi_1 + p_{110}(\Psi_7 + \Psi_{10}) + \Psi_7 p_{17}] + (1 - p_{03}p_{30} - p_{04})p_{18}(\Psi_8 + \Psi_9) \quad (11)$$

Using **(8)** and **(11)** in **(10)**, we get the expression for A_0.

The expected up time of the system during (0, t] is given by

$$\mu_{up}(t) = \int_0^t A_0(u)\, du$$

So that,

$$\mu_{up}^*(s) = A_0^*(s)/s.$$

7. Busy Period Analysis

$B_i\,(t)$ is defined as the probability that the system having started from regenerative state $S_i \in E$ at that *t*=0, is under repair at time *t* due to failure of the unit. Now to determine these probabilities, we use simple probabilistic arguments and further taking the Laplace transform and solving the resultant set of equations for $B_0^*(s)$, we have

$$B_0^*(s) = N_3(s)/D_2(s) \quad (12)$$

where,

$$N_3(s) = (1 - q_{11}^{(9)*} - q_{18}^*q_{89}^*q_{91}^* - q_{1,10}^*q_{10,7}^*q_{71}^* - q_{17}^*q_{71}^*)[q_{02}^*Z_2^* + q_{03}^*Z_3^* + q_{04}^*(Z_4^* + q_{45}^*Z_5^*) + q_{03}^*q_{36}^*Z_6^*] + \left[q_{01}^* + q_{02}^*q_{21}^* + q_{03}^*(q_{31}^{(7)*} + q_{36}^*q_{67}^*q_{71}^*)\right]\left[Z_1^* + q_{18}^*(Z_8^* + q_{89}^*Z_9^*) + q_{1,10}^*Z_{10}^*\right] + Z_7^*\left\{q_{03}^*q_{36}^*q_{67}^*(1 - q_{11}^{(9)*} - q_{17}^*q_{71}^* - q_{18}^*q_{89}^*q_{91}^* - q_{1,10}^*q_{10,7}^*q_{71}^*) + (q_{1,10}^*q_{10,7}^* + q_{17}^*)\left[q_{01}^* + q_{02}^*q_{21}^* + q_{03}^*(q_{31}^{(7)*} + q_{36}^*q_{67}^*q_{71}^*)\right]\right\} \quad (13)$$

and, $D_2(s)$ is same as given by (7).

In the steady state, the probability that the repairman will be busy is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} s\, B_0^*(s) = N_3(0)/D_2'(0) \quad (14)$$

where,

$$N_3(0) = (1 - p_{11}^{(9)} - p_{17}p_{71} - p_{18}p_{89}p_{91} - p_{1,10}p_{10,7}p_{71})[\,p_{02}p_{04} + p_{03}\Psi_3 + p_{04}(p_{04} + p_{45}\Psi_5) + p_{03}p_{36}\Psi_6] + \left[p_{01} + p_{02}p_{21} + p_{03}(p_{31}^{(7)} + p_{36}p_{67}p_{71})\right]\left[\Psi_1 + p_{18}(\Psi_8 + p_{89}\Psi_9) + p_{1,10}\Psi_{10}\right] + \Psi_7\left\{p_{03}p_{36}p_{67}(1 - p_{11}^{(9)} - p_{17}p_{71} - p_{18}p_{89}p_{91} - p_{1,10}p_{10,7}p_{71}) + (p_{1,10}p_{10,7} + p_{17})\left[p_{01} + p_{02}p_{21} + p_{03}(p_{31}^{(7)} + p_{36}p_{67}p_{71})\right]\right\} \quad (15)$$

and $D_2'(0)$ is same as obtained in **(11)**.

The expected busy period of the repairman during (*0, t*] is given by

$\mu_b(t) = \int_0^t B_0(u)\,du$

So that, $\mu_b^*(s) = B_0^*(s)/s$

8. Expected number of Repairs

$V_i(t)$ is defined as the expected number of repairs of the failed units during the time interval $(0, t]$ when the system initially starts from regenerative state S_i. Further, using the definition of $V_i(t)$ and by probabilistic reasoning the recurrence relations are easily obtained and taking their Laplace- Stieltjes transforms and solving the resultant set of equations for $\tilde{V}_0(s)$, we get

$$\tilde{V}_0(s) = N_4(s)/D_3(s) \tag{16}$$

where,

$$N_4(s) = \left(1 - \tilde{Q}_{11}^9 - \tilde{Q}_{17}\tilde{Q}_{71} - \tilde{Q}_{18}\tilde{Q}_{89}\tilde{Q}_{91} - \tilde{Q}_{1,10}\tilde{Q}_{10,7}\tilde{Q}_{71}\right)\left(\tilde{Q}_{03}\tilde{Q}_{30} + \tilde{Q}_{04}\tilde{Q}_{45}\tilde{Q}_{50}\right) + \tilde{Q}_{10}(\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21} + \tilde{Q}_{03}\tilde{Q}_{31}^{(7)} + \tilde{Q}_{03}\tilde{Q}_{36}\tilde{Q}_{67}\tilde{Q}_{71}) \tag{17}$$

and $D_3(s)$ can be written on replacing q_{ij}^* and $q_{ij}^{(k)*}$ by $\tilde{Q}_{ij}$ and $\tilde{Q}_{ij}^{(k)}$ respectively in the equation (7) .

In the steady state, the expected number of repairs per unit time is given by

$$V_0 = \lim_{t\to\infty}[V_0(t)/t] = \lim_{s\to 0} s\,\tilde{V}_0(s) = N_4(0)/D_2'(0) \tag{18}$$

where,

$$N_4(0) = (1 - p_{11}^9 - p_{17}p_{71} - p_{18}p_{89}p_{91} - p_{1,10}p_{10,7}p_{71})(p_{03}p_{30} + p_{04}p_{45}p_{50}) + p_{10}(p_{01} + p_{02}p_{21} + p_{03}p_{31}^{(7)} + p_{03}p_{36}p_{67}p_{71}) \tag{19}$$

9. Profit Function Analysis

With the help of reliability characteristics obtained, the profit function $P(t)$ for the system can be obtained easily. Profit is excess of revenue over the cost, therefore, the expected total profits generated during (*0, t*] is:

$P(t) =$ Expected total revenue in (*0,t*]− Expected total expenditure in (*0,t*]

$$= K_0\mu_{up}(t) - K_1\mu_b(t) - K_2V_0 \tag{20}$$

where,

K_0 : Revenue per unit up time of the system.

K_1 : Cost per unit time for which repair man is busy in repairing the failed unit.

K_2 : Cost of repair per unit.

In steady state, the expected total profits per unit time, is

$P = \lim_{t\to\infty}[P(t)/t] = \lim_{s\to 0} s^2P^*(s)$

Therefore, we have

$$P = K_0A_0 - K_1B_0 - K_2V_0 \tag{21}$$

10. Graphical Study Of The System Model

Graphical study of the system model gives a more perceived picture of system behaviour. So, for more concrete study, we plot MTSF and Profit function with respect to α, failure rate of identical unit for different values of h_1, repair rate of identical unit.

Fig. 2 represents the variations in MTSF with respect to α for different values of $h_1$as 0.05, 0.40, 0.80 by keeping all the other parameters fixed at $\beta = 0.03,\ h = 0.20,\ \gamma = 0.35,\ \delta = 0.45$. The coverage probability c for the system is set at 0.70. It can be clearly seen from the graph of MTSF that it decreases continuously with increase in failure rate α and by increasing repair rate h_1, the value for MTSF also increases, thereby concluding that repair facility increases the lifetime of the system.

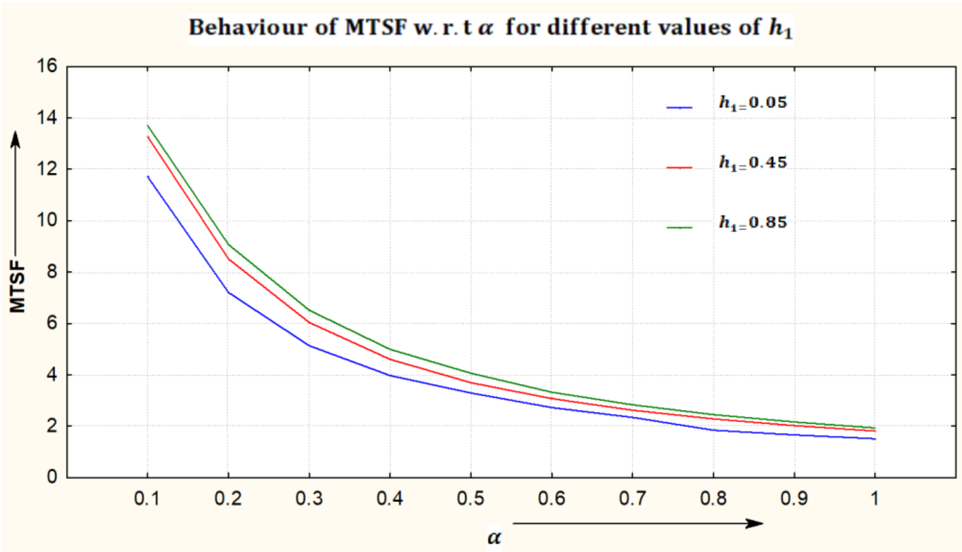

Fig. 2

Fig. 3 represents the change in Profit function P with respect to α for different values of h_1 as 0.05, 0.40, 0.80 by keeping all the other parameters fixed at $\beta = 0.03,\ h = 0.20,\ \gamma = 0.35,\ \delta = 0.45$. The coverage probability c for the system is set at 0.70. Clearly, it is observed that profit function decreases with increase in failure rate α but increases with increase in repair rate h_1. Hence, repairing the system from time to time will result in better performance of the system.

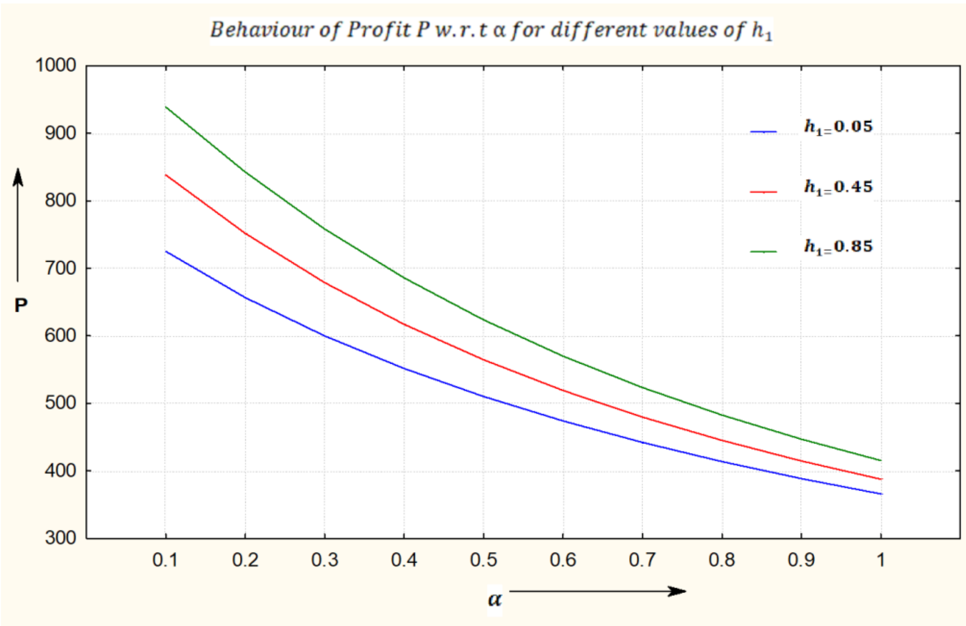

Fig. 3

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