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Performance Measures Of A Two Non-Identical Unit System Model With Repair And Replacement Policies

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Abstract

The present paper deals with two non-identical units A and B, both are in operative mode. If the unit A fails then it is taken up for preparation of repair before entering into repair mode and the unit-B gives a signal for repair before going into failure mode. If the unit gets repaired then it becomes operative otherwise it is replaced by the new one. A single repairman is always available with the system to repair the failed units and the priority in repair is always given to the unit-A. The failure time distributions of both the units are taken as exponential and the repair time distributions are taken as general. Using regenerative point technique the various characteristics of the system effectiveness have been obtained such as Transition Probabilities and Mean Sojourn times, Mean time to system failure (MTSF), Availability of the system, Busy Period of repairman, Expected number of Replacement, Expected profit incurred.

Keyword: Preparation, Signal, Repair and Replacement.

1.1 INTRODUCTION

Several researchers have considered and studied numerous reliability system models having identical units. In view of their growing use in modern technology the study of reliability characteristics of different stochastic models have attracted the attention of the researchers in the field of reliability theory and system engineering. To help system designers and operational managers, various researchers including [1,2,3] in the field of reliability theory have analysed two unit system models with two types of repairs, replacement policy, signal concept etc. They obtained various economic measures of system effectiveness by using regenerative point technique. The common assumption which is taken in most of these models is that a single repairman is always available with the system to repair the failed unit and after the repair the unit becomes as good as new. But in many practical situations, it is not possible that a single repairman perform the whole process of repair particularly in case of complicated unit/machine. Goel [1] analyzed that the multi standby, multi failure mode system model with repair and replacement policy and there are various authors who have carried out study on repair and replacement policies.

In the present paper, we study a two non-identical units system model. The units are named as A and B and are taken to be in operative mode. If the unit A fails then goes for preparation for repair before entering into repair. Unit-B while in operation gives a signal for its repair before going in to failure mode and if it gets repaired it starts its functioning in usual manner otherwise it is replaced by the new one. A single repairman is always available with the system to repair the failed units and the priority in repair is always given to the unit-A The failure time distributions of both the units are taken as exponential and the repair time distribution is taken as general. All random variable are statistically independent.

Using semi- Markov process and regenerative point technique the expressions for the following important performance measures of the system have been derived in steady state -

1. Transition Probabilities and mean Sojourn times.
2. Mean time to system failure (MTSF).
3. Availability of the system.
4. Busy period of repairman.
5. Expected number of replacement of the unit.
6. Net expected profit earned by the system during the interval (0,t) and in steady state.

1.2 MODEL DESCRIPTION AND ASSUMPTIONS

1. The system comprises of two non-identical units A and B initially both are in operative mode.

2. Upon the failure of unit A, it will go for preparation for repair before taken up for repair.

3. Unit-B while in operation gives a signal for its repair before going in to failure mode and if it is not repaired in a stipulated time it is replaced by the new one.

4. A single repairman is always available with the system to repair and replace the failed units and the priority in repair is always given to the unit A over unit B

5. The failures of the units are independent and the failure time distributions of the units are taken as Exponential.

6. The repair time distributions of the units are taken as general.

1.3 NOTATIONS AND STATES OF THE SYSTEM

We define the following symbols for generating the various states of the system.

A_{10}, B_{20} : Unit A and unit B are in operative mode.
A_{1r}/ A_{1P} : Unit A under repair/ preparation for repair.
B_{2sr}/ B_{2srw} : Unit B in operative mode and gives signal for repair/waiting of signal for repair.
B_{2r}/ B_{2rw} : Unit B under repair/waiting for repair.
B_{2R}/B_{2Rw} : Unit B under replacement/waiting for replacement

b) NOTATIONS:

E : Set of regenerative states $= \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_8\}$
$\overline{E}$: Set of non − regenerative states $= \{S_7, S_9, S_{10}, S_{11}\}$
α_1 : Failure rate of unit − A
α_2 : Repair rate of unit − B
β_1 : Parameter for signal
β_2 : Repair rate of unit − A
β_3 : Repalcement rate of unit − B
H_1 : cdf of repair time of unit − B
H_2 : cdf of repair time of unit − A
G_1 : cdf of replacement time of unit − B

TRANSITION DIAGRAM

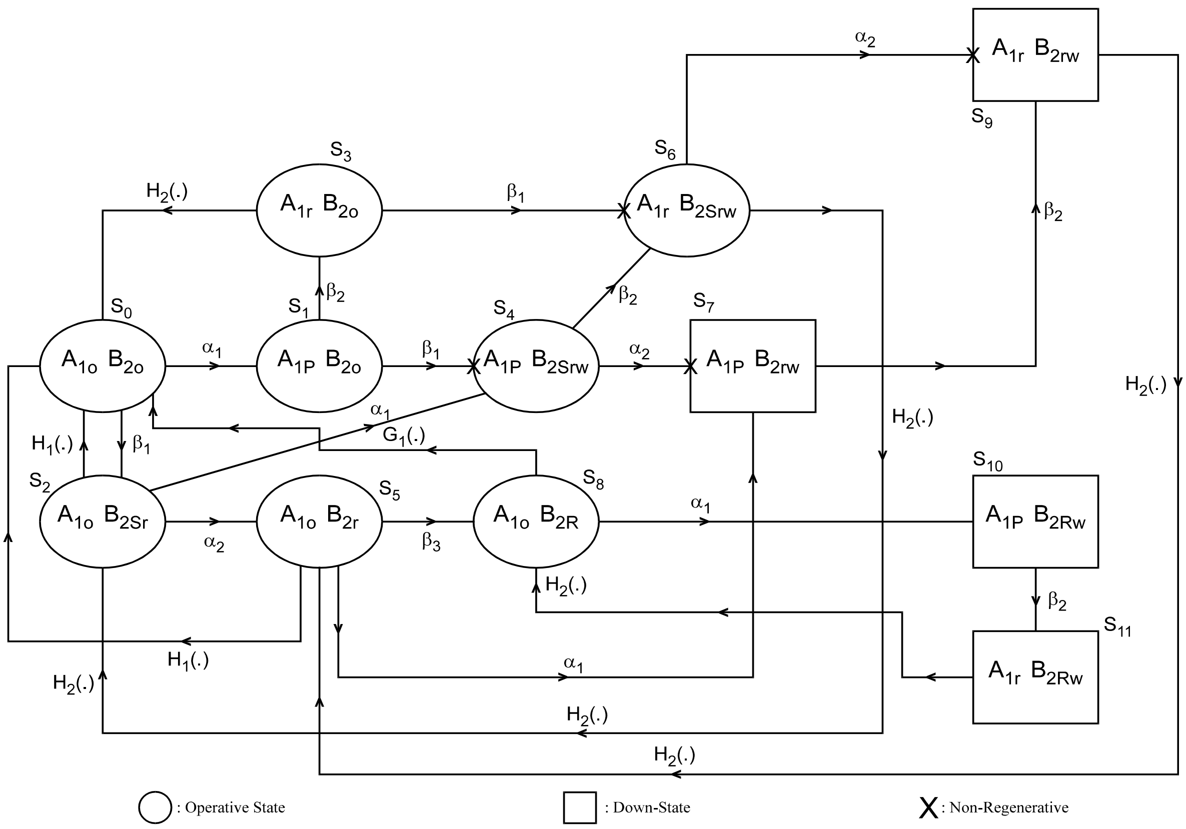

Fig.1.1

1.4 TRANSITION PROBABLITIES

Let X_n denotes the state visited at epoch T_{n^+} just after the transition at T_n, where T_1, T_2 .... represents the regenerative epochs, then$\{X_n, T_n\}$ constitute a Markov-Renewal process with state space E and

$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$

is the semi Markov kernel over E.

Then the transition probability matrix of the embedded Markov chain is

$P = p_{ij} = Q_{ij}(\infty) = Q(\infty)$

We obtain the following direct steady-state transition probabilities:

$$p_{01} = \alpha_1 \int e^{-(\alpha_1+\beta_1)u} du = \frac{\alpha_1}{(\alpha_1+\beta_1)}$$

Similarly,

$$p_{02} = \frac{\beta_1}{(\alpha_1+\beta_1)}, \quad p_{13} = \frac{\beta_2}{(\beta_2+\beta_1)}, \quad p_{20} = H_1^*(\alpha_2+\alpha_1)$$

$$p_{24} = \frac{\alpha_1}{(\alpha_2+\alpha_1)}[1 - H_1^*(\alpha_2+\alpha_1)], \quad p_{25} = \frac{\alpha_2}{(\alpha_2+\alpha_1)}[1 - H_1^*(\alpha_2+\alpha_1)]$$

$$p_{30} = H_2^*(\beta_1), \quad p_{46} = \frac{\beta_2}{\alpha_2+\beta_2}, \quad p_{50} = H_1^*(\alpha_1+\beta_3)$$

$$p_{57} = \frac{\alpha_1}{\alpha_1+\beta_3}[1 - H_1^*(\alpha_1+\beta_3)], \quad p_{58} = \frac{\beta_3}{\beta_3+\alpha_1}[1 - H_1^*(\alpha_1+\beta_3)]$$

$$p_{62} = H_2^*(\alpha_2), \quad p_{80} = G_1^*(\alpha_1), \quad p_{8,10} = 1 - G_1^*(\alpha_1)$$

$$p_{79} = p_{95} = p_{10,11} = p_{11,8} = 1$$

The indirect transition probability may be obtained as follows:

$$p_{16}^{(4)} = \frac{\beta_1\beta_2}{(\beta_1-\alpha_2)} \int e^{-(\beta_2+\alpha_2)v} - e^{-(\beta_2+\beta_1)v} du$$

$$= \frac{\beta_1\beta_2}{(\beta_2+\alpha_2)(\beta_2+\beta_1)}$$

Similarly,

$$p_{19}^{(4,7)} = 1 + \frac{\beta_2\alpha_2}{(\beta_1-\alpha_2)(\beta_2+\beta_1)} - \frac{\beta_1\beta_2}{(\beta_2-\alpha_2)(\beta_2+\alpha_2)}$$

$$p_{35}^{(6,9)} = 1 - \frac{\beta_1 H_2^*(\alpha_2)}{(\beta_1-\alpha_2)} + \frac{\alpha_2 H_2^*(\beta_1)}{(\beta_1-\alpha_2)}, \quad p_{32}^{(6)} = \frac{\beta_1}{(\beta_1-\alpha_2)}[H_2^*(\alpha_2) - H_2^*(\beta_1)]$$

$$p_{49}^{(7)} = \frac{\alpha_2}{\alpha_2+\beta_2} \quad p_{65}^{(9)} = 1 - H_2^*(\alpha_2)$$

It can be easily verified that

$$p_{01} + p_{02} = 1, \quad p_{13} + p_{19}^{(4,7)} + p_{16}^{(4)} = 1, \quad p_{20} + p_{24} + p_{25} = 1$$

$$p_{30} + p_{35}^{(6,9)} + p_{32}^{(6)} = 1, \quad p_{46} + p_{49}^{(7)} = 1, \quad p_{50} + p_{57} + p_{58} = 1$$

$$p_{62} + p_{65}^{(9)} = 1, \quad p_{80} + p_{8,10} = 1, \quad p_{79} = p_{95} = p_{10,11} = p_{11,8} = 1$$

A) MEAN SOJOURN TIMES

The mean sojourn time in state S_i denoted by μ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time μ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time μ_i in state S_i is:

$$\mu_i = E[T_i] = \int P(T_i > t) dt$$

Therefore,

$$\mu_0 = \frac{1}{\alpha_1+\beta_1}, \quad \mu_1 = \frac{1}{\beta_1+\beta_2}, \quad \mu_2 = \frac{1}{\alpha_1} - H_1^*(\alpha_1)$$

$$\mu_3 = \frac{1}{\beta_1} - H_2^*(\beta_1), \quad \mu_4 = \frac{1}{\alpha_2+\beta_2}, \quad \mu_5 = \frac{1}{\beta_3+\alpha_1} - H_1^*(\beta_3+\alpha_1)$$

$$\mu_6 = \frac{1}{\alpha_2} - H_2^*(\alpha_2), \quad \mu_8 = \frac{1}{\alpha_1} - G_1^*(\alpha_1) \quad \mu_7 = \mu_{10} = \frac{1}{\beta_2}$$

$$\mu_9 = \mu_{11} = \int \overline{H}_2(t) dt$$

1.5 ANALYSIS OF RELIABILITY

Let T_i be the random variable denoting time to system failure when system starts up from state $S_i \in E_i$, then the reliability of the system is given by

$R_i(t) = P[T_i > t]$

To obtain $R_i(t)$, we consider failed states as absorbing states.

The recursive relations among $R_i(t)$ can be developed on the basis of probabilistic arguments. Taking their Laplace Transform and solving the resultant set of equations for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (1.5.1)$$

Where

$$N_1(s) = [(1 - q_{24}^* q_{46}^* q_{62}^*)(Z_0^* + q_{01}^* Z_1^* + q_{01}^* q_{13}^* Z_3^*)] + \left[q_{01}^* \left(q_{13}^* q_{32}^{(6)*} + q_{16}^{(4)*} q_{62}^*\right) + q_{02}^*\right](Z_2^* + q_{24}^* Z_4^* + q_{25}^* Z_5^* + q_{25}^* q_{58}^* Z_8^*) + \left[q_{01}^* q_{16}^{(4)*} + q_{24}^* q_{46}^* \left(q_{01}^* q_{13}^* q_{32}^{(6)*} + q_{02}^*\right)\right] Z_6^*$$

$$D_1(s) = [(1 - q_{24}^* q_{46}^* q_{62}^*)(1 - q_{01}^* q_{13}^* q_{30}^*)] - \left[\left(q_{13}^* q_{32}^{(6)*} + q_{16}^{(4)*} q_{62}^*\right) q_{01}^* - q_{02}^*\right](q_{20}^* + q_{50}^* + q_{25}^* q_{58}^* q_{80}^*)$$

Taking the Inverse Laplace Transform of **(1.5.1)**, one gets the reliability of the system.

To get MTSF, we use the well known formula

$$E(T_0) = \int R_0(t)dt = \lim_{s \to 0} R_0^*(s) = N_1(0)/D_1(0)$$

where,

$$N_1(0) = [(1 - p_{24} p_{46} p_{62})(\mu_0 + \mu_1 p_{01} + \mu_3 p_{01} p_{13})] + \left[p_{01} \left(p_{13} p_{32}^{(6)} + p_{16}^{(4)} p_{62}\right) + p_{02}\right](\mu_2 + p_{24}\mu_4 + p_{25}\mu_5 + p_{25} p_{58} \mu_8) + \left[p_{01} p_{16}^{(4)} + p_{24} p_{46} \left(p_{01} p_{13} p_{32}^{(6)} + p_{02}\right)\right] \mu_6$$

$$D_1(0) = [(1 - p_{24} p_{46} p_{62})(1 - p_{01} p_{13} p_{30})] - \left[\left(p_{13} p_{32}^{(6)} + p_{16}^{(4)} p_{62}\right) p_{01} - p_{02}\right](p_{20} + p_{50} + p_{25} p_{58} p_{80})$$

Since, we have $q_{ij}^*(0) = p_{ij}$ and $\lim_{s \to 0} Z_i^*(s) = \int Z_i(t)dt = \mu_i$

1.6 AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is available at epoch t, when it initially starts from $S_i \in E$. Using the regenerative point technique and the tools of L.T., one can obtain the value of above probabilities in terms of their L.T. i.e$A_i^{n*}(s)$.Solving the resultant set of equations and simplifying for $A_0^*(s)$, we have

$$A_0^{n*}(s) = N_2(s)/D_2(s) \quad (1.6.1)$$

$$N_2(s) = q_{80}(1 - q_{57})(1 - q_{24} q_{46} q_{62})[Z_0 + q_{01} Z_1] + q_{01}[q_{13} q_{80}(1 - q_{57})(1 - q_{24} q_{46} q_{62}) Z_3] + q_{80}(1 - q_{57})\big(Z_2 + q_{24}(Z_4 + q_{46} Z_6)\big)\left[q_{01} q_{13} q_{32}^{(6)} + q_{02}\right] + \left\{(q_{80} Z_5 + q_{58} Z_8)\left[\left(q_{65}^{(9)} q_{46} + q_{4,9}^{(7)}\right) q_{24} + q_{25}\right]\right\}\left(q_{01} q_{32}^{(6)} + q_{02}\right) + q_{01}(q_{80} Z_5 + q_{58} Z_8)(1 - q_{24} q_{46} q_{62})\left[q_{35}^{(6,9)} + q_{19}^{(4,7)}\right] + q_{01} q_{16}^{(4)}\left[q_{80}(1 - q_{57})\left[(Z_6 + q_{62} Z_2 + q_{24} q_{62} Z_4) + (q_{80} Z_5 + q_{58} Z_8)\left\{q_{6,5}^{(9)} + q_{62} q_{25} + q_{25} q_{4,9}^{(7)}\right\}\right]\right] \quad (1.6.2)$$

and

$$D_2(s) = q_{80}(1-q_{57})(1-q_{24}q_{46}q_{62}) - \left\{q_{01}\left[q_{13}q_{80}(1-q_{57})(1-q_{24}q_{46}q_{62})\left(1-q_{32}^{(6)}\right)\right]\right\} - q_{01}q_{13}q_{80}q_{32}^{(6)}q_{80}(1-q_{57})(1-q_{24}q_{46}q_{62}) - q_{01}q_{80}q_{19}^{(4,7)}(1-q_{57})(1-q_{24}q_{46}q_{62}) - q_{01}q_{80}q_{16}^{(4)}(1-q_{57})(1-q_{24}q_{46}q_{62}) - q_{02}q_{80}(1-q_{57})(1-q_{24}q_{46}q_{62}) \quad (1.6.3)$$

The steady state availability is given by

$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2(0)}$

As we know that, $q_{ij}(t)$ is the pdf of the time of transition from state S_i to S_j and $q_{ij}(t)dt$ is the probability of transition from state S_i to S_j during the interval $(t, t+dt)$, thus $\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \mu_i$ and $q_{ij}^*(s) = q_{ij}^*(0) = p_{ij}$, we get

Therefore,

$$N_2(0) = p_{80}(1-p_{57})(1-p_{24}p_{46}p_{62})[\mu_0 + p_{01}\mu_1] + p_{01}[p_{13}p_{80}(1-p_{57})(1-p_{24}p_{46}p_{62})\mu_3] + p_{80}(1-p_{57})(\mu_2 + p_{24}(\mu_4 + p_{46}\mu_6))\left[p_{01}p_{13}p_{32}^{(6)} + p_{02}\right] + \left\{(p_{80}\mu_5 + p_{58}\mu_8)\left[\left(p_{65}^{(9)}p_{46} + p_{4,9}^{(7)}\right)p_{24} + p_{25}\right]\right\}\left(p_{01}p_{32}^{(6)} + p_{02}\right) + p_{01}(p_{80}\mu_5 + p_{58}\mu_8)(1-p_{24}p_{46}p_{62})\left[p_{35}^{(6,9)} + p_{19}^{(4,7)}\right] + p_{01}p_{16}^{(4)}\left[p_{80}(1-p_{57})\left[(\mu_6 + p_{62}\mu_2 + p_{24}p_{62}\mu_4) + (p_{80}\mu_5 + p_{58}\mu_8)\left\{p_{6,5}^{(9)} + p_{62}p_{25} + p_{25}p_{4,9}^{(7)}\right\}\right]\right] \quad (1.6.4)$$

$$D_2(0) = p_{80}(1-p_{57})(1-p_{24}p_{46}p_{62}) - \left\{p_{01}\left[p_{13}p_{80}(1-p_{57})(1-p_{24}p_{46}p_{62})\left(1-p_{32}^{(6)}\right)\right]\right\} - p_{01}p_{13}p_{80}p_{32}^{(6)}p_{80}(1-p_{57}) - p_{01}p_{80}p_{19}^{(4,7)}(1-p_{57})(1-p_{24}p_{46}p_{62}) - p_{01}p_{80}p_{16}^{(4)}(1-p_{57})(1-p_{24}p_{46}p_{62}) - p_{02}p_{80}(1-p_{57})(1-p_{24}p_{46}p_{62})$$

The steady state probability that the system will be up in the long run is given by

$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) \lim_{s\to 0}\frac{sN_2(s)}{D_2(s)} = \lim_{s\to 0} N_2(s) \lim_{s\to 0}\frac{s}{D_2(s)}$

as $s \to 0$, $D_2(s)$ becomes zero.

Therefore, by L' Hospital's rule, A_0 becomes

$$A_0 = N_2(0)/D_2'(0) \quad (1.6.5)$$

where,

$$D_2'(0) = \mu_0\{p_{80}(1-p_{57})(1-p_{24}p_{46}p_{62})\} + \mu_1\{p_{01}p_{80}(1-p_{57})(1-p_{24}p_{46}p_{62})\} + \mu_2\left\{p_{80}(1-p_{57})p_{01}\left[p_{13}p_{32}^{(6)} + p_{16}^{(4)}p_{62}\right] + p_{80}(1-p_{57})p_{02}\right\} + \mu_3\{p_{01}p_{80}p_{13}(1-p_{57})(1-p_{24}p_{46}p_{62})\} + \mu_4\left\{p_{80}(1-p_{57})p_{24}\left[p_{01}p_{16}^{(4)}p_{62} + p_{01}p_{13}p_{32}^{(6)} + p_{02}\right]\right\} + (\mu_5 + \mu_7 + \mu_8 + \mu_9 + \mu_{10} + \mu_{11})(1) + \mu_6\left\{p_{02}p_{80}(1-p_{57})p_{24}p_{46} + p_{01}p_{80}(1-p_{57})p_{24}p_{46}p_{32}^{(6)}\right\} \quad (1.6.6)$$

Using the results **(1.6.4)** and **(1.6.6)** in **(1.6.5),** we get the expressions for A_0.

The expected up (operative) time of the system during (0, t] is given by

$$\mu_{up}(t) = \int_0^t A_0(u)du$$

So that,

$$\mu_{up}^*(s) = \frac{A_0^*(s)}{s}$$

1.7 BUSY PERIOD OF REPAIRMAN

Let $B_i(t)$ be the probability that the repairman is busy in the repair of failed unit at epoch t, when the system initially starts operation from state $S_i \in E$. Developing the recursive relations among $B_i(t)$'s and solving the resultant set of equations and simplifying for $B_0^*(s)$, we have

$$B_0^*(s) = N_3(s)/D_2(s) \tag{1.7.1}$$

where

$$N_3(s) = \Big[q_{01}^*(1 - q_{24}^* q_{46}^* q_{62}^*)\left[q_{13}^* q_{35}^{(6,9)*} + q_{65}^{(9)*} + q_{19}^{(4,7)*}\right] + q_{01}^*(1 - q_{20}^* - q_{24}^* q_{46}^* q_{62}^*)\left(q_{32}^{(6)*} + q_{16}^{(4)*} q_{62}^*\right) + q_{02}^*(1 - q_{20}^* - q_{24}^* q_{46}^* q_{62}^*)\Big]\{q_{80}^* M_5^* + q_{8,10}^* q_{58}^* M_{11}^* + q_{80}^* q_{57}^* q_{79}^* M_9^*\} + \left\{q_{01}^* q_{80}^*(1 - q_{57}^*)(1 - q_{24}^* q_{46}^* q_{62}^*)\left[q_{13}^* M_3^* + q_{16}^{(4)*} M_6^*\right] + q_{01}^* q_{16}^{(4)*} M_9^*\right\} + q_{80}^*(1 - q_{57}^*)\left[M_2^* + q_{24}^*\left(q_{46}^* M_6^* + q_{49}^{(7)*} M_9^*\right)\right]\left\{q_{01}^*\left(q_{13}^* q_{32}^{(6)*} + q_{16}^{(4)*} q_{62}^*\right) + q_{02}^*\right\}$$

In the long run, the expected fraction of time for which the expert server is busy in the repair of failed unit is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} B_0^*(s) = \frac{N_3(0)}{D_2'(0)} = \frac{N_3}{D_2} \tag{1.7.2}$$

$$N_3(0) = \Big[p_{01}(1 - p_{24}p_{46}p_{62})\left[p_{13}p_{35}^{(6,9)} + p_{65}^{(9)} + p_{19}^{(4,7)}\right] + p_{01}(1 - p_{20} - p_{24}p_{46}p_{62})\left(p_{32}^{(6)} + p_{16}^{(4)}p_{62}\right) + p_{02}(1 - p_{20} - p_{24}p_{46}p_{62})\Big]\{p_{80}\mu_5 + p_{8,10}p_{58}\mu_{11} + p_{80}p_{57}p_{79}\mu_9\} + \left\{p_{01}p_{80}(1 - p_{57})(1 - p_{24}p_{46}p_{62})\left[p_{13}\mu_3 + p_{16}^{(4)}\mu_6\right] + p_{01}p_{19}^{(4,7)}\mu_9\right\} + p_{80}(1 - p_{57})\left[\mu_2 + p_{24}\left(p_{46}\mu_6 + p_{49}^{(7)}\mu_9\right)\right]\left\{p_{01}\left(p_{13}p_{32}^{(6)} + p_{16}^{(4)}p_{62}\right) + p_{02}\right\} \tag{1.7.3}$$

and $D_2(s)$ is same as given by **(1.6.6)**.

Thus using **(1.7.3)** and **(1.6.6)** in **(1.7.2)**, we get the expression for B_0.

The expected busy period of repairman during the time interval (0,t] is given by

$$\mu_b(t) = \int_0^t B_0(u)du$$

So that

$$\mu_b^*(s) = \frac{B_0^*(s)}{s}$$

1.8 EXPECTED NUMBER OF REPLACEMENTS

Let $V_i^{rp}(t)$ be the expected number of replacements by the server in (0,t] given that the system entered the regenerative state S_i at t=0. Framing the relations among $V_i^{rp}(t)$, taking L.S.T and solving for $\widetilde{V}_0^{r}(s)$, we get

$$\widetilde{V}_0^{rp}(s) = \frac{N_4^{rp}(s)}{D_2(s)} \tag{1.8.1}$$

where,

$$N_4^{rp}(s) = \widetilde{Q}_{01}\left[\widetilde{Q}_{12}\widetilde{Q}_{21} + \widetilde{Q}_{25}\widetilde{Q}_{21}(\widetilde{Q}_{78}\widetilde{Q}_{89} + \widetilde{Q}_{79})\widetilde{Q}_{57}\widetilde{Q}_{90} + \widetilde{Q}_{13}\widetilde{Q}_{34}\left(\widetilde{Q}_{46}\widetilde{Q}_{69}^{(8)} + \widetilde{Q}_{48}^{(7)}\widetilde{Q}_{89} + \widetilde{Q}_{49}^{(7)}\right) + \widetilde{Q}_{13}\widetilde{Q}_{37}^{(5)}(\widetilde{Q}_{78}\widetilde{Q}_{89} + \widetilde{Q}_{79})\widetilde{Q}_{90}\right]$$

and $D_2(s)$ can be obtained on replacing q_{ij}'s by Q_{ij}'s in **1.6.6**

In steady-state per-unit of time expected number of replacement by server is given

$$V_0^{rp} = \lim_{t\to\infty}\frac{V_0^{rp}(t)}{t} = \lim_{s\to 0}\widetilde{V}_0^{rp}(s) = \frac{N_4^{rp}(0)}{D_2'(0)} = \frac{N_4^{rp}}{D_2} \tag{1.8.2}$$

Where

$$N_4^{rp} = p_{01}\left[p_{12}p_{21} + p_{12}p_{25}(p_{78}p_{89} + p_{79})p_{57}p_{90} + p_{13}p_{34}\left(p_{46}p_{69}^{(8)} + p_{89}p_{48}^{(7)} + p_{49}^{(7)}\right) +\right.$$

$p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79})p_{90}\Big]$ (1.8.3)

Thus using **(1.8.3)** and **(1.6.6)** in **(1.8.2),** we get the expression for V_0^{rp}.

1.9 PROFIT FUNCTION ANALYSIS

The net profit incurred during (0,t) is given by

$P(t) =$ Expected total revenue in (0,t]− Expected total expenditure in (0,t]

$= K_0\mu_{up}(t) - K_1\mu_b^r(t) - K_2\mu_n^{rp}(t)$

Where K_0 is the revenue per unit up time by the system, and K_1 repair cost per unit of time in repairing the failed unit by repairman and K_2 is per unit replacement cost of the failed unit.

Also,

$$\mu_{up}(t) = \int_0^t A_0(u)\,du$$

So that, $\mu_{up}^*(t) = \frac{A_0^*(s)}{s}$

In the similar way $\mu_b^r(t), \mu_n^{rp}(t)$can be defined.

Now the expected profit per unit of time in steady state is given by

$$P = \lim_{t\to\infty}\frac{P(t)}{t} = \lim_{s\to 0} s^2P^*(s)$$

$$= K_0A_0 - K_1B_0 - K_2V_1$$

1.10 CONCLUSION

To study the behavior of MTSF and profit function through graphs w.r.t *v*arious parameters, curves are plotted for these characteristics w.r.t failure parameter α_1 in Fig.2.1 and Fig.2.2 respectively for three different values of repair rate $\beta_2 = (0.20, 0.50, 0.60)$ whereas other parameters are kept fixed as $\alpha_2 = 0.03, \beta_1 = 0.25, \beta_3 = 0.20, h_1 = 0.30, h_2 = 0.02, g_1 = 0.03$.

Fig.2.1 represents variation in MTSF for varying values of failure parameter α_1 for three different values of repair rate β_2 . The graph shows decrease in MTSF with the increase in failure rate and an increase with the increase in repair rate. The curves also indicates that for the same value of failure rate, MTSF is higher for higher values of repair rate .So we conclude that the repair facility has a better understanding of failure phenomenon resulting in longer lifetime of the system.

Fig.2.2 represents the variation pattern in profit function w.r.t. varying values of failure parameter α_1 for three different values of repair rate β_2, it is observed from graph that profit decreases with the increase in failure rate α_1 and increases with increase in repair rate β_2 irrespective of other parameters. The curve also indicates that for the same value of repair rate, profit is lower for higher values of failure rate and decrease in both MTSF and profit function is almost exponential.

Hence, it can be concluded that the expected life of the system can be increased by decreasing failure rate and increasing repair rate of the unit which in turn will improve the reliability and hence the effectiveness of the system.

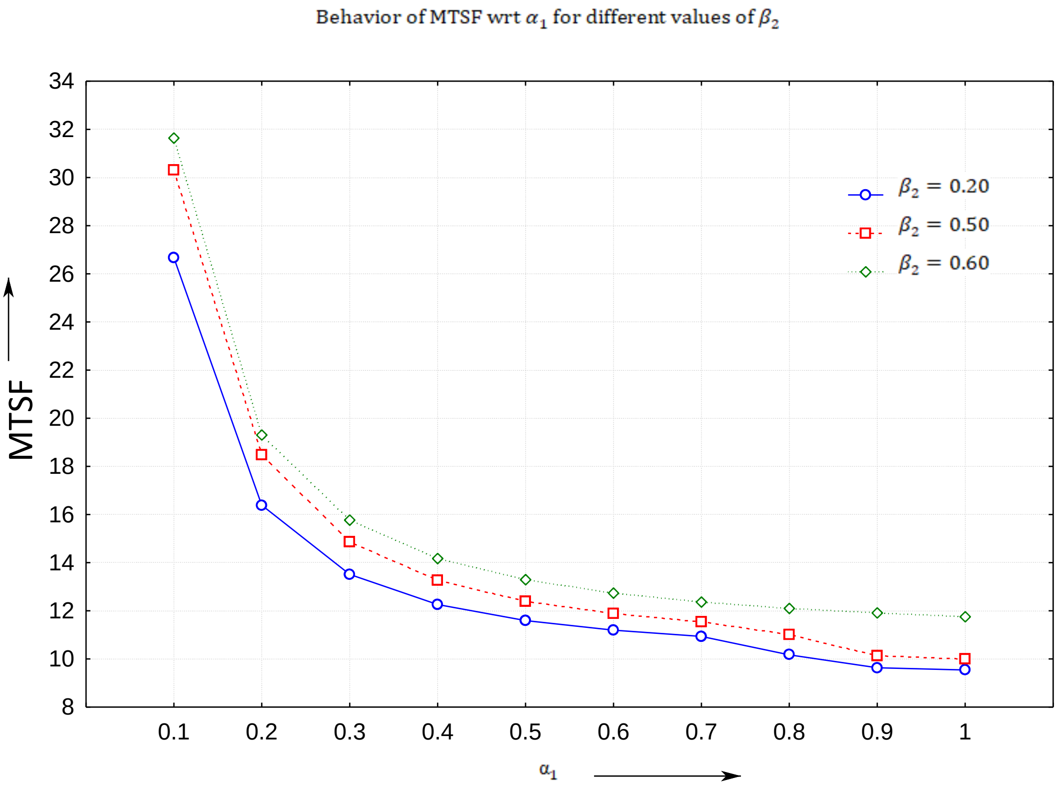
Behavior of MTSF wrt α_1 for different values of β_2
MTSF
α_1
$\beta_2 = 0.20$
$\beta_2 = 0.50$
$\beta_2 = 0.60$
34
32
30
28
26
24
22
20
18
16
14
12
10
8
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIG 2.1

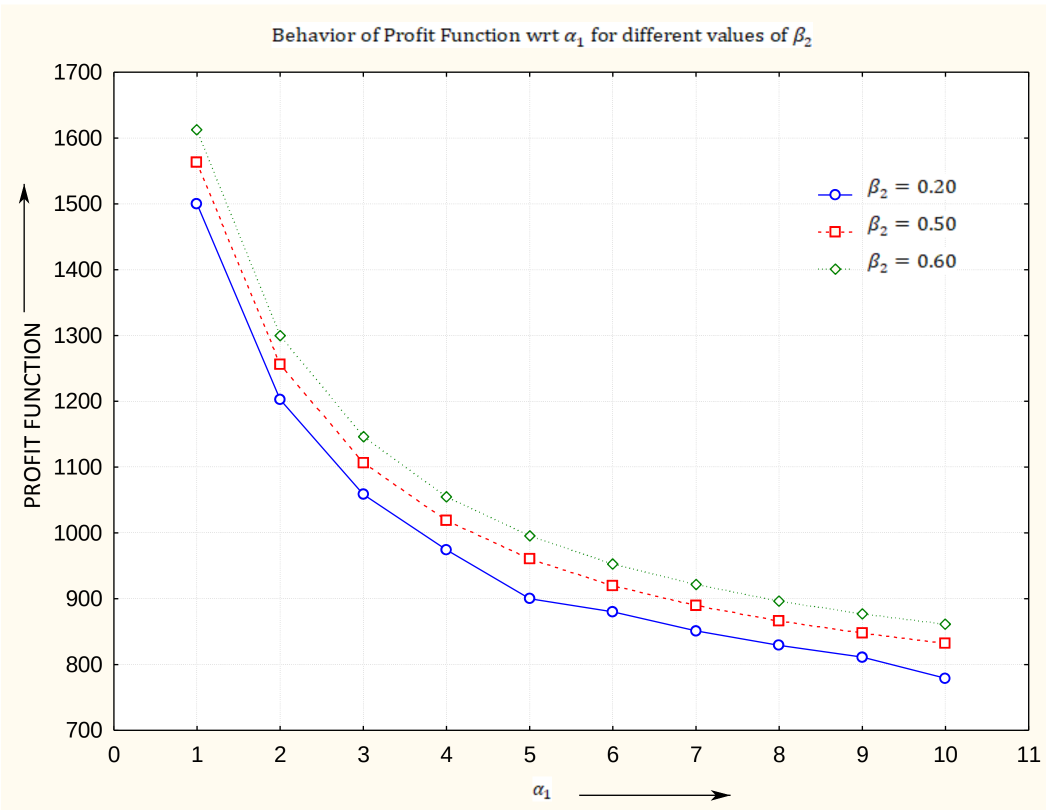
Behavior of Profit Function wrt α_1 for different values of β_2
PROFIT FUNCTION
α_1
$\beta_2 = 0.20$
$\beta_2 = 0.50$
$\beta_2 = 0.60$
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
0
1
2
3
4
5
6
7
8
9
10
11

FIG 2.2

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