Cost-profit Anslysis of Stochastic Heterogeneous Queue with Reverse Balking, Feedback and Retention of Impatient Customers

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Abstract

Consider a system operating as an M/M/2/N queue. Increasing system size influences newly arriving customers to join the system (reverse balking). As the system size increases, the customers' waiting in the queue become impatient. After a threshold value of time, the waiting customer abandons the queue (reneging). These reneging customers can be retained with some probability (retention). Few customers depart dissatisfied with the service and rejoin the system as feedback customers. In this paper a feedback queuing system with heterogeneous service, reverse balking, reneging and retention is developed. The model is solved in steady-state recursively. Necessary measures of performance are drawn. Numerical interpretation of the model is presented. Cost-profit analysis of the system is performed by developing a cost model. Sensitivity analysis of the model is also presented arbitrarily.

Keyword – reverse balking, heterogeneous service, queuing theory, customer impatience

1. INTRODUCTION

Balking and reneging (impatience) are fundamental concepts in queuing literature introduced by Anker & Gafarian (1963a, 1963b). Further Haight (1957, 1959) and Bareer (1957) studied notion of customer reneging and balking in various ways. They state that an arriving customer shows least interest in joining a system which is already crowded. This behavior is termed as *Balking*. Since then researchers applied balking at various places and a number of research papers appeared on balking. Singh (1970) studied a two-server Markovian queues with Balking. He compared two heterogeneous servers with homogeneous servers. He also obtained the efficiency of heterogeneous system functioning under balking, Hassin, (1986) applied balking in customer information in markets with random product quality; they consider a revenue server suppressing the information using balking function. Falin (1995) approximated multi-server queues with balking/retrial discipline. They studied congestion in communication with balking discipline. Kumar (2006) further studied multi-server feedback retrial queues with balking and control retrial rate. They analyzed system as quasi-birth-and-death process and discuss stability conditions. They also obtained optimization of retrial rate. Wang (2011) studied balking with delayed repairs. They investigated equilibrium threshold balking strategies for fully and partially observable single-server queues with server breakdowns and delayed repairs. Kumar (2018) studied transient and steady-state behavior of two-heterogeneous servers' queuing systems with balking retention of reneging customers. They obtained time-dependent and steady state solution of the model.

On contrary to balking Jain et. al., (2014) stated that when it comes to businesses like healthcare, restaurant, investment, service station etc a large customer base becomes an attracting factor for newly arriving customer i.e. a customer is more willing to join a firm that already has a large customer base. This behavior of customers is termed as *Reverse Balking*. Kumar (2015a, 2015b) studied queuing systems with reverse balking, reverse reneging and feedback. Further Som et. al. (2016) studied a heterogeneous queuing system with reverse balking reverse reneging. The notion of reverse balking is studied further by Kumar (2017) and Som (2018a, 2018b). A limited number of publications appeared on reverse balking as it is an evolving concept.

Reverse balking results in increasing queue length and longer waiting times. A customer waiting in queue to get served may get impatience after certain period of time and decides to abandon the queue without completion of service. This behavior of customers is termed as *reneging* Anker & Gafarian (1963a, 1963b). Reneging has gained popularity due to its practical viability. Researchers studied applications of reneging in detail. Rao (1971) studies reneging and balking in M/G/1 system. He investigated the busy period using supplementary variable technique and transforms. Abou-El-Ata et. al. (1992) studied a truncated general queue with reneging and general balk function. They derived steady-state solution of the model. Wang et. al. (2002) performed cost analysis of finite M/M/R queuing system with balking reneging and server breakdowns. They developed a cost-model of the system under study as well. Singh et. al. (2016) studied single-server finite queuing system with varying speed of server in random environment. Bakuli et. al., (2017) investigated $M/M^{(a,b)}/1$ queuing model with impatient customers. They derived solution of the model and found measures of performance. Further Kumar et. al. (2017) studied transient analysis of a multi-server queuing model with discouraged arrivals and impatient customers. Reneging has gained wide popularity due to its practically viable implication.

As reneging causes loss of customers hence it leaves a negative impact on goodwill and revenue of the firm. Kumar et. al. (2012) introduced the idea of *retention of impatient customers* in queuing literature. They mentioned that if a retention strategy is employed in form of offers and discount; a reneging customer may be retained with some probability. Kumar et. al. (2013, 2014) further performed economic analysis of M/M/c/N queue with retention of impatient customers. They obtained steady-state solution of the model and obtained various measures of effectiveness. They also optimize a queuing system with reneging and retention of impatient customers. Since then a lot of paper appeared on retention of reneged customers such as Som et. al. (2017, 2018c) discussed a various queuing system with encouraged arrivals and retention of impatient customers.

Further a serviced customer may depart from the system dissatisfied. These customers may rejoin the system for completion of incomplete or dissatisfied service. These customers are termed as feedback customers in queuing literature. Takas (1963) introduced the feedback mechanism in queues. He used instant Bernoulli's feedback in a M/G/1 queue. Nakamura (1971) studied a delayed feedback system using Bernoulli's decision process.

Further DAvignon et. al. (1976) studied state dependent M/G/1 feedback under the assumption of general state. The obtained the stationary queue length along with busy period and queue length. They applied single-server feedback queue with respect to computer time sharing system. Santhakumaran et. al. (2000) studied a single-server queue with impatient and feedback customers. They studied stationary process of the arrival distribution. Choudhary et. al. (2005) have discusses an M/G/1 queue with two phases of heterogeneous service. Further Som (2018a, 2018b) has studied a feedback queue with various queuing systems. This is also evident that servers vary in their capacity of service and provide service at heterogeneous rate.

Though the queuing models with reverse balking, reneging, retention and feedback are developed and studied but none of these models studies a facility undergoing reverse balking, feedback, and retention of impatient customers with heterogeneous service all together. Practically, all of these contemporary phenomenons occur simultaneously. Therefore it is worthy to study and measure such a system. Hence in this paper we study a feedback queuing system with reverse balking, reneging, retention and heterogeneous service. The necessary measures of performance are obtained in steady-state. The model is tested with arbitrary values. Later the cost model is developed and economic analysis of the model is performed.

2. THE MODEL

The model proposed in the paper can be presented through following state diagram;

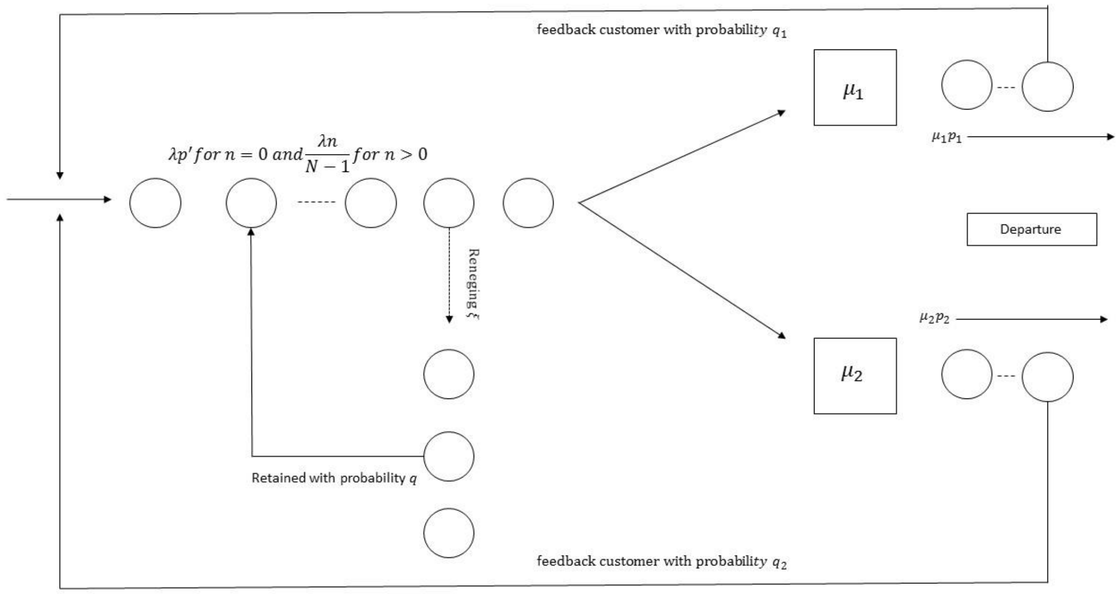

Figure -1

Consider the arrivals occur one by one in accordance with Poisson process. Inter-arrival times are exponentially distributed with parameter $1/\lambda$. Customers are serviced through two servers with heterogeneous service times distributed exponentially with parameters μ_1 and μ_2. An arriving customer joins the system in front of an empty server i.e. with probability π_1 in front of server 1 and with π_2(=1- π_1) in front of server two. Capacity of system is finite say, N. When system is empty, an arriving customers reverse balks with probability q' and joins the system with probability $q' = (1 - p')$. When number of customers in the system are ≥0, an arriving customer reverse balks with probability $\left(1 - \frac{n}{N-1}\right)$and does not reverse balk with probability $\left(\frac{n}{N-1}\right)$. A customer waiting for service in queue may get impatient after time T and decides to abandon the queue with an exponentially distributed parameter ξ. Arrivals are served in order of their arrival i.e. the queue discipline is first come first serve. A reneging customer may be retained with probability $q = (1 - p)$. A serviced customer may not get satisfied with the service of first server μ_1 and rejoin the system as a feedback customer with probability $q_1 = (1 - p_1)$. While a serviced customer may not be satisfied with the service of second server $\mu_2$and rejoin to the system as a feedback customer with probability $q_2 = (1 - p_2)$.

2.1 BALANCE EQUATIONS AND STEADY STATE SOLUTION

Let $P_n(t)$ = probability of n customers in the system at time t. $P_{ij}(t)$= probability that there are i customers in front of first server one and j customers in front of second server at time t. In steady-state as $t \to \infty$, $P_n(t) = P_n, P_{ij}(t) = P_{ij}$ and $P_n'(t) = P_{ij}'(t) = 0$. The system of steady-state equations governing the model is given by;

$$\lambda p' P_{00} = \mu_1 p_1 P_{10} + \mu_2 p_2 P_{01}; n = 0 \ (1)$$
$$\mu_2 p_2 P_{11} = \left(\frac{\lambda}{N-1} + \mu_1 p_1\right) P_{10} - \lambda \pi_1 p' P_{00}; n = 1 (2)$$
$$\mu_1 p_1 P_{11} = \left(\frac{\lambda}{N-1} + \mu_2 p_2\right) P_{01} - \lambda \pi_2 p' P_{00}; n = 1 (3)$$
$$(\mu_1 p_1 + \mu_2 p_2 + \xi p) P_3 = \left(\frac{2\lambda}{N-1} + \mu_1 p_1 + \mu_2 p_2\right) P_2 - \frac{\lambda}{N-1} P_1; n = 2 \ (4)$$
$$\{\mu_1 p_1 + \mu_2 p_2 + n\, \xi p\} P_{n+1} = \left\{\frac{n\lambda}{N-1} + \mu_1 p_1 + \mu_2 p_2 + (n-2)\xi p\right\} P_n - \frac{\lambda(n-1)}{N-1} P_{n-1}; n \leq N-1 \ (5)$$
$$\{\mu_1 p_1 + \mu_2 p_2 + (N-2)\xi p\} P_N = \lambda P_{N-1}; n = N \ (6)$$

Steady-state solution

On solving (1) – (6), we get

$$P_{10} = \left\{\frac{\lambda + (\mu_1 p_1 + \mu_2 p_2)\pi_1 (N-1)}{2\lambda + (\mu_1 p_1 + \mu_2 p_2)(N-1)}\right\} \left(\frac{\lambda}{\mu_1 p_1}\right) p' P_{00} \ (7)$$
$$P_{01} = \left\{\frac{\lambda + (\mu_1 p_1 + \mu_2 p_2)\pi_2 (N-1)}{2\lambda + (\mu_1 p_1 + \mu_2 p_2)(N-1)}\right\} \left(\frac{\lambda}{\mu_2 p_2}\right) p' P_{00} \ (8)$$

Adding (7) and (8)

$$P_1 = \left\{\frac{\lambda + (\pi_1 \mu_2 p_2 + \pi_2 \mu_1 p_1)(N-1)}{2\lambda + (\mu_1 p_1 + \mu_2 p_2)(N-1)}\right\} \frac{\lambda(\mu_1 p_1 + \mu_2 p_2)}{\mu_1 p_1 u_2 p_2} p' P_{00} \ (9)$$

Adding equation (2) and (3) and using (4)

$$P_2 = \frac{1}{N-1} \left\{\frac{\lambda + (\pi_1 \mu_2 p_2 + \pi_2 \mu_1 p_1)(N-1)}{2\lambda + (\mu_1 p_1 + \mu_2 p_2)(N-1)}\right\} \frac{\lambda}{\mu_1 p_1} \frac{\lambda}{\mu_2 p_2} p' P_{00} \ (10)$$

Using equation (5)

$$P_n = \frac{(n-1)!}{(N-1)^{n-1}}\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda}{\mu_1p_1}\frac{\lambda}{\mu_2p_2}\prod_{k=3}^{n}\frac{\lambda}{\mu_1p_1+\mu_2p_2+(k-2)\xi p}p'P_{00} \quad (11)$$

Using (6) and (11)

$$P_N = \frac{(N-2)!}{(N-1)^{N-2}}\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda}{\mu_1p_1}\frac{\lambda}{\mu_2p_2}\prod_{k=3}^{N}\frac{\lambda}{\mu_1p_1+\mu_2p_2+(k-2)\xi p}p'P_{00} \quad (12)$$

Using condition of normality $\sum_{n=0}^{N}P_n = 1$ we get,

$$P_0 = \left[\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda(\mu_1p_1+\mu_2p_2)}{\mu_1p_1u_2p_2}p' + \frac{1}{N-1}\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda}{\mu_1p_1}\frac{\lambda}{\mu_2p_2}p'\right.$$
$$+\sum_{n=3}^{N-1}\frac{(n-1)!}{(N-1)^{n-1}}\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda}{\mu_1p_1}\frac{\lambda}{\mu_2p_2}\prod_{k=3}^{n}\frac{\lambda}{\mu_1p_1+\mu_2p_2+(k-2)\xi p}p'$$
$$\left.+\frac{(N-2)!}{(N-1)^{N-2}}\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda}{\mu_1p_1}\frac{\lambda}{\mu_2p_2}\prod_{k=3}^{N}\frac{\lambda}{\mu_1p_1+\mu_2p_2+(k-2)\xi p}p'\right]^{-1}$$

3 MEASURES OF PERFORMANCE

In this section necessary measures of performance are derived. Apart from these other measures of performance such as average waiting time in queue, average queue length, and average waiting time in the system can be drawn by using classical queuing theory relations.

3.1 EXPECTED SYSTEM SIZE

$$L_s = \sum_{n=1}^{N} nP_n$$
$$L_s = \sum_{n=1}^{N} nP_n = P_1 + 2P_2 + \sum_{n=3}^{N-1} nP_n + NP_N$$

$$L_s = \left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda(\mu_1p_1+\mu_2p_2)}{\mu_1p_1u_2p_2}p'P_{00}$$
$$+2\frac{1}{N-1}\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda}{\mu_1p_1}\frac{\lambda}{\mu_2p_2}p'P_{00}$$
$$+\sum_{n=3}^{N-1} n\frac{(n-1)!}{(N-1)^{n-1}}\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda}{\mu_1p_1}\frac{\lambda}{\mu_2p_2}\prod_{k=3}^{n}\frac{\lambda}{\mu_1p_1+\mu_2p_2+(k-2)\xi p}p'P_{00}$$
$$+N\frac{(N-2)!}{(N-1)^{N-2}}\left\{\frac{\lambda+(\pi_1\mu_2p_2+\pi_2\mu_1p_1)(N-1)}{2\lambda+(\mu_1p_1+\mu_2p_2)(N-1)}\right\}\frac{\lambda}{\mu_1p_1}\frac{\lambda}{\mu_2p_2}\prod_{k=3}^{N}\frac{\lambda}{\mu_1p_1+\mu_2p_2+(k-2)\xi p}p'P_{00}$$

3.2 AVERAGE RATE OF REVERSE BALKING

$$R_b' = q'\lambda P_0 + \sum_{n=1}^{N-1}\left(1-\frac{n}{N-1}\right)\lambda P_n$$

$$R_b' = q'\lambda P_0 + \frac{N-2}{N-1}\lambda P_1 + \frac{N-3}{N-1}\lambda P_2 + \sum_{n=3}^{N-1}\left(1-\frac{n}{N-1}\right)\lambda P_n$$

$$R_b' = q'\lambda P_0 + \left[\frac{N-2}{N-1}\lambda\left\{\frac{\lambda+(\pi_1\mu_2 p_2+\pi_2\mu_1 p_1)(N-1)}{2\lambda+(\mu_1 p_1+\mu_2 p_2)(N-1)}\right\}\frac{\lambda(\mu_1 p_1+\mu_2 p_2)}{\mu_1 p_1 u_2 p_2}p' \right.$$
$$+\frac{N-3}{N-1}\lambda\frac{1}{N-1}\left\{\frac{\lambda+(\pi_1\mu_2 p_2+\pi_2\mu_1 p_1)(N-1)}{2\lambda+(\mu_1 p_1+\mu_2 p_2)(N-1)}\right\}\frac{\lambda}{\mu_1 p_1}\frac{\lambda}{\mu_2 p_2}p'$$
$$\left.+\left(1-\frac{n}{N-1}\right)\sum_{n=3}^{N-1}\frac{(n-1)!}{(N-1)^{n-1}}\left\{\frac{\lambda+(\pi_1\mu_2 p_2+\pi_2\mu_1 p_1)(N-1)}{2\lambda+(\mu_1 p_1+\mu_2 p_2)(N-1)}\right\}\frac{\lambda}{\mu_1 p_1}\frac{\lambda}{\mu_2 p_2}\prod_{k=3}^{n}\frac{\lambda}{\mu_1 p_1+\mu_2 p_2+(k-2)\xi p}p'\right]$$

3.3 AVERAGE RAE OF RENEGING

$$R_r = \sum_{n=1}^{N}(n-2)\xi p P_n$$

$$R_r = \sum_{n=3}^{N-1}(n-2)\xi p P_n + (N-2)\xi p P_N$$

$$R_r = \left[(n-2)\xi p\sum_{n=3}^{N-1}\frac{(n-1)!}{(N-1)^{n-1}}\left\{\frac{\lambda+(\pi_1\mu_2 p_2+\pi_2\mu_1 p_1)(N-1)}{2\lambda+(\mu_1 p_1+\mu_2 p_2)(N-1)}\right\}\frac{\lambda}{\mu_1 p_1}\frac{\lambda}{\mu_2 p_2}\prod_{k=3}^{n}\frac{\lambda}{\mu_1 p_1+\mu_2 p_2+(k-2)\xi p}p' \right.$$
$$\left.+(N-2)\xi p\frac{(N-2)!}{(N-1)^{N-2}}\left\{\frac{\lambda+(\pi_1\mu_2 p_2+\pi_2\mu_1 p_1)(N-1)}{2\lambda+(\mu_1 p_1+\mu_2 p_2)(N-1)}\right\}\frac{\lambda}{\mu_1 p_1}\frac{\lambda}{\mu_2 p_2}\prod_{k=3}^{N}\frac{\lambda}{\mu_1 p_1+\mu_2 p_2+(k-2)\xi p}p'\right]P_0$$

3.4 AVERAGE RATE OF RETENTION

$$R_r = \sum_{n=1}^{N}(n-2)\xi q P_n$$

$$R_r = \sum_{n=3}^{N-1}(n-2)\xi p P_n + (N-2)\xi p P_N$$

$$R_R = \left[(n-2)\xi q\sum_{n=3}^{N-1}\frac{(n-1)!}{(N-1)^{n-1}}\left\{\frac{\lambda+(\pi_1\mu_2 p_2+\pi_2\mu_1 p_1)(N-1)}{2\lambda+(\mu_1 p_1+\mu_2 p_2)(N-1)}\right\}\frac{\lambda}{\mu_1 p_1}\frac{\lambda}{\mu_2 p_2}\prod_{k=3}^{n}\frac{\lambda}{\mu_1 p_1+\mu_2 p_2+(k-2)\xi p}p' \right.$$
$$\left.+(N-2)\xi q\frac{(N-2)!}{(N-1)^{N-2}}\left\{\frac{\lambda+(\pi_1\mu_2 p_2+\pi_2\mu_1 p_1)(N-1)}{2\lambda+(\mu_1 p_1+\mu_2 p_2)(N-1)}\right\}\frac{\lambda}{\mu_1 p_1}\frac{\lambda}{\mu_2 p_2}\prod_{k=3}^{N}\frac{\lambda}{\mu_1 p_1+\mu_2 p_2+(k-2)\xi p}p'\right]P_0$$

4 IMPACT ANALYSIS AND NUMERICAL ILLUSTRATION

In this section we, measure the impact of feedback, impatience and retention on various measures of performance by varying one parameter at a time. Figure -1 and 2 studies the impact of reneging on the system. Figure -3 studies the impact of retention, while figure -4 studies the impact of feedback on relevant measures of performance.

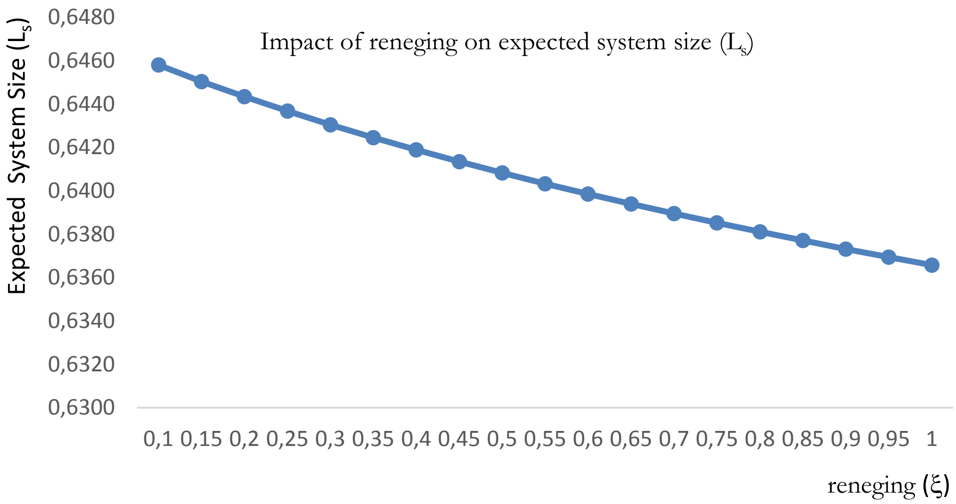

Figure -1 ($\lambda = 4, \mu_1 = 2, \mu_2 = 3, q' = 0.6, q_1 = 0.2, q_2 = 0.2, q = 0.8, N = 15$)

It can be observed from figure-1 that increasing rate of reneging leaves a negative impact on expected system size. The expected system size gradually reduces as, increasing rate of reneging leads to more and more customers leaving the system without completion of service. Reducing system size is not good for any system.

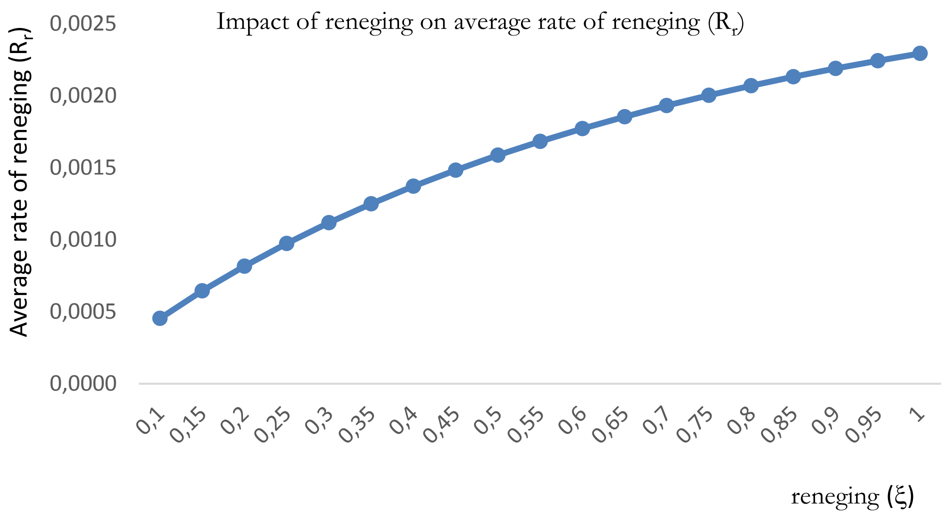

Figure -2 ($\lambda = 4, \mu_1 = 2, \mu_2 = 3, q' = 0.6, q_1 = 0.2, q_2 = 0.2, q = 0.8, N = 15$)

From figure-2 it is clear that with increase in rate of reneging average rate of reneging (R_r)

increases.

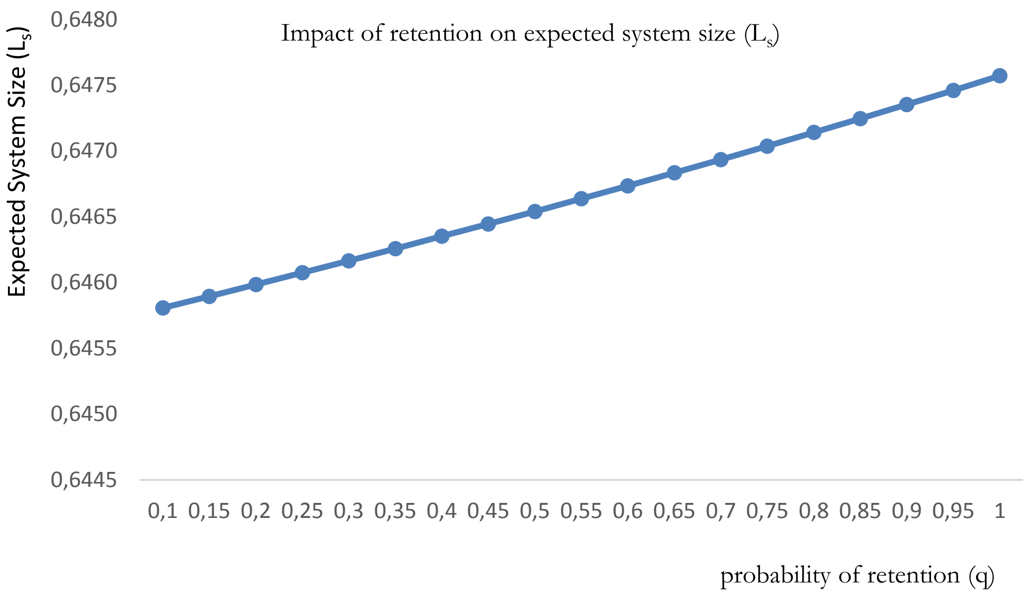

Figure -3 ($\lambda = 4, \mu_1 = 2, \mu_2 = 3, q' = 0.6, q_1 = 0.2, q_2 = 0.2, \xi = 0.1, N = 15$)

From figure -3, it is clear that with increasing rate of retention more and more customers get retained and system size increases gradually. Increasing system size is good for health of any organization as they can earn larger revenue.

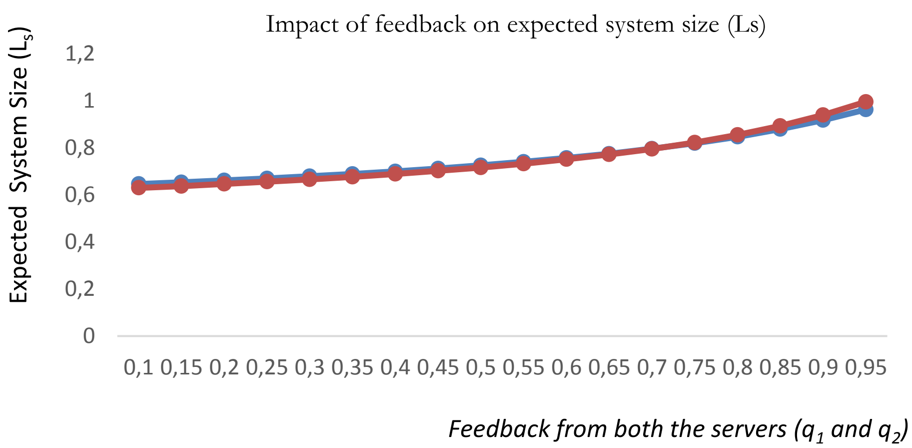

Figure -4 ($\lambda = 4, \mu_1 = 2, \mu_2 = 3, q' = 0.6, \xi = 0.1, q_1 = 0.2\ (varying\ q_2), q_2 = 0.2\ (varying\ q_1), q = 0.8, N = 15$)

From figure -4 it can be observed that more and more customers retiring in to the system. This results in increasing system size.

It can be observed here that both retention of reneging customers and feedback of customers result in increasing system size. Increasing in system size due to retention is good because a customer is retained which otherwise was lost, on other hand increasing system size due to feedback indicates dissatisfaction in service.

Now we will present numerical illustration of the model. Let us consider facility in which arrivals occur in accordance to Poisson process with an average rate of arrival 5 customers per unit time, there are two servers providing service in accordance to exponential distribution with average service rate of 2 and 3 units per unit time. Reneging times are exponentially distributed with a rate of 0.1 per unit time. Firms employ different strategies to retain reneging customers and a reneging customer may be retained with a 60% chance. While due to unsatisfactory service 20% customers rejoin the system from each servers per unit time. Initially, an arriving customer shows least interest in the facility due to n =0 and it may not join (reverse balk) the system with a probability of 0.8. An arriving customer may join on server one with probability 0.4 and server two with a probability 0.6.

The cost of service is Rs 4 per server per customer, holding cost id Rs 2, reverse balking cost is Rs 7, cost of retaining a reneging customer is Rs 2, and cost of a reneging customer is Rs 3 while feedback cost of a customer is Rs 2. The facility earns a revenue of Rs 50 on each customer on an average.

Calculate;

(i) Probability of zero customers in the system
(ii) Expected System Size
(iii) Expected waiting time in the system
(iv) Average rate of reverse balking
(v) Average rate of reneging
(vi) Average rate of retention
(vii) Total Expected Cost
(viii) Total Expected Revenue
(ix) Total Expected Profit

Solution

Measure of Performance	**Numerical Output**
Probability of zero customers in the system (P_0)	0.65885
Expected System Size (L_s)	0.533313
Expected waiting time in the system (W_s)	0.0067
Average rate of reverse balking (R_b')	1.140397
Average rate of reneging (R_r)	0.0002
Average rate of retention (R_R)	0.00013
Total Expected Cost (TEC)	Rs 71.93
Total Expected Revenue (TER)	Rs 128.59
Total Expected Profit (TEP)	Rs 56.67

In next session we develop cost model for the system discusses above and perform cost-profit analysis.

5 COST-PROFIT ANALYSIS

In this section economic analysis of the model is presented. The cost-model is developed with the functions of total expected cost, total expected revenue and total expected profit. The hypothetical values are taken to test the model. Here, TEC = Total Expected Cost, TER = Total Expected Revenue , TEP = Total Expected Profit C_s = Cost of service per unit, C_h = Holding cost per unit , C_f = Feedback cost per unit , C_r = Reneging cost per unit, C_R = Retention cost per unit

The functions of total expected cost, revenue and profit are described as under;

Total expected cost of the model is given by;

$$TEC = C_s(\mu_1 + \mu_2) + C_h L_s + C_b R_b' + C_r R_r + C_R R_R + C_f(\mu_1 p_1 + \mu_2 p_2)$$

Total expected revenue if given by;

$$TER = R \times \mu \times (1 - P_0)$$

Total expected profit is given by;

$$TEP = TER - TEC$$

Following tables present sensitivity analysis of the model with respect to arbitrary inputs of variables.

Table -1
(System performance with change in rate of reneging ξ)
$\lambda = 5, \mu_1 = 2, \mu_2 = 3, q' = 0.6, q_1 = 0.2, q_2 = 0.2, q = 0.8, N = 15$
$C_s = 4, C_h = 2, C_R = 2, C_r = 3, C_b = 7, C_f = 4, R = 50$

Rate of Reneging (ξ)	Expected System Size (L_s)	Average Rate of Reneging (R_r)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
0.1	0.548	0.0003	71.928	128.564	56.637
0.2	0.547	0.0006	71.927	128.512	56.585
0.3	0.545	0.0008	71.926	128.465	56.538
0.4	0.544	0.0010	71.926	128.421	56.496
0.5	0.544	0.0012	71.925	128.380	56.456
0.6	0.543	0.0014	71.924	128.342	56.418
0.7	0.542	0.0015	71.923	128.305	56.382
0.8	0.541	0.0017	71.922	128.270	56.348
0.9	0.541	0.0018	71.922	128.237	56.315
1.0	0.540	0.0019	71.921	128.205	56.284

Table -1, shows that reneging leaves a negative impact on the system, as more and more customers leave the system without completion of service. Expected system size with TER, TEC and TEP reduces. And average rate of reneging increases gradually.

Table -2

(System performance with change in probability of reverse balking when n=0)
$\lambda = 5, \mu_1 = 2, \mu_2 = 3, \xi = 0.2, q_1 = 0.2, q_2 = 0.2, q = 0.8, N = 15$
$C_s = 4, C_h = 2, C_R = 2, C_r = 3, C_b = 7, C_f = 4, R = 50$

Probability of Reverse Balking at n=0 (q')	Expected System Size (L_s)	Average Rate of Reneging ($R_{b'}$)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
0.1	0.7489	3.401	69.31	176.04	106.73
0.2	0.7222	3.458	69.66	169.76	100.10
0.3	0.6905	3.526	70.07	162.32	92.25
0.4	0.6524	3.607	70.56	153.35	82.79
0.5	0.6056	3.707	71.17	142.35	71.18
0.6	0.5467	3.833	71.93	128.51	56.58
0.7	0.4705	3.996	72.91	110.60	37.68
0.8	0.3679	4.215	74.24	86.48	12.24
0.9	0.2224	4.525	76.12	52.29	-23.84
1.0	0.0000	5.000	79.00	0.00	-79.00

Table -2 shows that, increasing probability of reverse balking when system is empty leaves a bad effect on revenue. We can see that system goes under loss when probability of reverse balking raises a certain limit. The system size obviously reduces to zero as no customer joins the system.

Table -3
(System performance with change in probability of reverse balking when n=0)
$\lambda = 5, \mu_1 = 2, \mu_2 = 3, \xi = 0.2, q' = 0.6, q_1 = 0.2, q_2 = 0.2, N = 15,$
$C_s = 4, C_h = 2, C_R = 2, C_r = 3, C_b = 7, C_f = 4, R = 50$

Probability of Retention (q)	Expected System Size (L_s)	Average Rate of Retention (R_R)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
0.1	0.5440	0.0001	71.92	128.40	56.48
0.2	0.5445	0.0003	71.92	128.42	56.50
0.3	0.5449	0.0004	71.92	128.44	56.52
0.4	0.5455	0.0006	71.92	128.46	56.54
0.5	0.5461	0.0007	71.93	128.49	56.56
0.6	0.5467	0.0009	71.93	128.51	56.58
0.7	0.5474	0.0011	71.93	128.54	56.61
0.8	0.5483	0.0014	71.93	128.56	56.64
0.9	0.5494	0.0017	71.93	128.59	56.66
1.0	0.5507	0.0021	71.93	128.63	56.69

Table -3, discusses the effect of retention on system. As retention pulls the customers back to system the system size gets higher and higher and firms make more profit and revenue.

Table -4
(System performance with change in probability of feedback from first server)
$\lambda = 5, \mu_1 = 2, \mu_2 = 3, \xi = 0.2, q_1 = 0.2, q = 0.8, q' = 0.6, N = 15$
$C_s = 4, C_h = 2, C_R = 2, C_r = 3, C_b = 7, C_f = 4, R = 50$

Probability of feedback on server one (q_1)	Expected System Size (L_s)	Total Expected Cost (TEC)
0.1	0.5310	70.93
0.2	0.5483	71.92
0.3	0.5691	72.96
0.4	0.5948	74.06
0.5	0.6274	75.23
0.6	0.6704	76.50
0.7	0.7299	77.94
0.8	0.8167	79.64
0.9	0.9487	70.93

Feedback is a negative process. Increasing feedback depicts poor quality of service table-4 shows increasing probability of feedback from server 1 and hence the system size increases. The facility is crowded with people on which either no or very less revenue is earned. We can observe the rising cost with increase in probability of feedback. Figure-1, shows change in total expected cost with respect to increasing probability of feedback at second server.

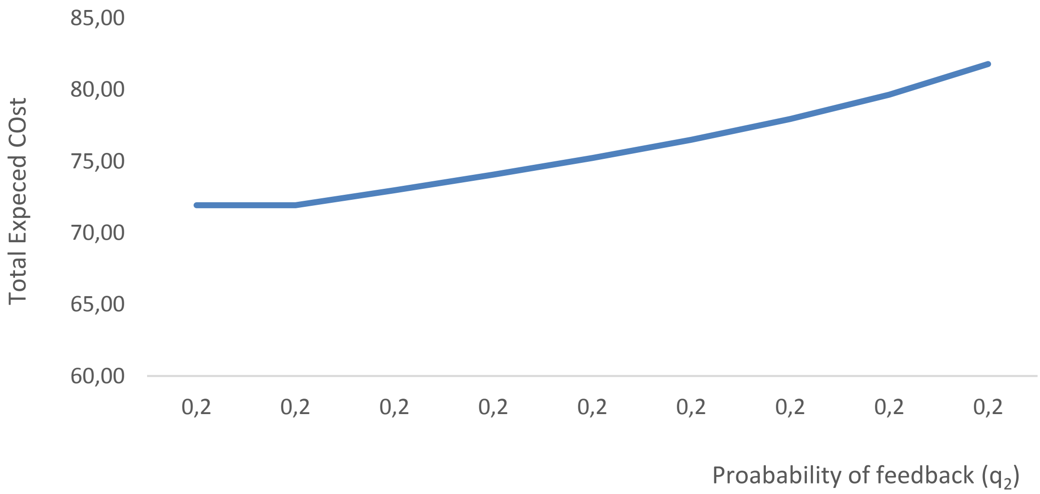

Figure -1
Total Expected Cost w.r.t probability of feedback on server 2 (q_2)
$\lambda = 5, \mu_1 = 2, \mu_2 = 3, \xi = 0.2, q_2 = 0.2, q = 0.8, q' = 0.6, N = 15$
$C_s = 4, C_h = 2, C_R = 2, C_r = 3, C_b = 7, C_f = 4, R = 50$

8 CONCLUSION AND FUTURE WORK

In this paper a feedback queuing model with heterogeneous service, reverse balking, and retention of impatient customers is formulated. The model is solved in steady-state. Necessary measures of performance, numerical illustration and cost-profit analysis of the model is performed. The model is useful for firms that are going through mentioned contemporary challenges. The model can be used for designing effective administrative strategies. The future scope of the work is to test the model in real time environment. The optimization of the model with respect to various parameters can also be obtained thereafter.

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