Transient Analysis of a Single-Server Queuing System with Correlated Inputs and Reneging

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Abstact

In this paper, we study a continuous time single-server queuing system, wherein the arrivals at two consecutive transition marks are correlated. The service times and the reneging times are exponential distributed. The time-dependent behavior of the model is studied using Runge-Kutta method.

Keywords: Correlated input, Exponential, Queuing model, Reneging, Transient analysis

1 Introduction

Queuing modelling has been playing a very vital role since its inception. It has a great role in modelling and designing communication systems. A lot of work has been done in queuing theory with reference to its applications in inventory management, manufacturing, supply chain management, population studies, genetic studies and in transportation management. Mohan and Murari [9] obtained the transient solution for a correlated queuing system with variable capacity. Murari [10] studied the steady-state behavior of single server queuing system in which both the arrivals and phase-type service were correlated. Andrade Parra [2] studied the correlated nature of cell traffic in broadband communications. Kamoun and Ali [7] considered a two-node tandem network with correlated arrivals and discussed its application in ATM networks. Takine, Suda and Hasegawa [11]studied the ATM switching nodes with the correlated cell arrivals. They also proved that the cell loss and output process characteristics are affected by correlation and variation of cell arrivals. Drezner [4] obatined the performance measures of for $M^c/G/1$ queuing system with dependent arrivals. Jain and Kumar [5] considered the correlated queuing problem with variable capacity and catastrophes and obtained the transient solution by probability generating technique. Jain and Kumar [6] incorporated the concept of restoration in a queuing system with correlated arrivals, variable capacity and catastrophes. Kumar [8] studied the correlated queuing system with catastrophe, restoration and customer impatience. Banerjee [3] studied a workload dependent service queuing system with Markovian Arrival Process. Vishnevskii and Dudin [12] did the review of the queuing systems with correlated inputs with their applications to modeling telecommunication networks.

In this paper, we obtain transient solution of a single-server queuing system with correlated inputs and reneging where the service times are exponentially distributed. Rest of the paper is as follows: In section 2, we described the queuing model. In section 3, the differential-difference equation of the model is presented. Section 4 deals with transient analysis of the model. In section 5 we concluded our paper.

2 Queuing Model Description

The queuing model considered is based on the following assumptions: The customers arrive at a service facility and form a queue. The arrivals can occur only at the transition marks $t_0, t_1, t_2, \ldots$ where $\theta_r = t_r - t_{r-1}, r = 1,2,3\ldots$, are negative exponentially distributed random variables with parameter λ. The arrivals of customers at the two consecutive transition marks t_{r-1} and $t_r, r = 1,2,3\ldots$, are governed by the following transition probability matrix:

$$\begin{array}{cc} & \begin{array}{cc} & t_r \\ 0 & 1 \end{array} \\ t_{r-1} \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \end{array}$$

where $p_{00} + p_{01} = 1$ and $p_{10} + p_{11} = 1$, where 0 refers to no arrival and 1 refers to the occurrence of arrival. Hence, the arrivals are correlated The system has finite capacity, say N. There is a single server and the customers are served one by one on FCFS basis. The service time distribution is negative exponential with parameter μ. Every customer that enters the system will wait for a certain period of time after which he becomes impatient and leaves the queue. This behaviour of a customer is known as reneging. The reneging times of the customers are assumed to be distributed according to negative exponential distribution with parameter ξ.

3 Mathematical Model

Defining the following probabilities

$Q_{0,0}(t)$ = Probability that at time t the queue length is empty, the server is idle and no arrival has occurred at the previous transition mark.

$Q_{0,1}(t)$ = Probability that at time t the queue length is empty, the server is idle and an arrival has occurred at the previous transition mark.

$P_{0,0}(t)$ = Probability that at time t the queue length is empty, the server is not idle and no arrival has occurred at the previous transition mark.

$P_{0,1}(t)$ = Probability that at time t the queue length is empty, the server is not idle and an arrival has occurred at the previous transition mark.

$P_{n,0}(t)$ = Probability that at time t the queue length is equal to n $(1 \leq n < N)$, the server is not idle and no arrival has occurred at the previous transition mark.

$P_{n,1}(t)$ = Probability that at time t the queue length is equal to n $(1 \leq n < N)$, the server is not idle and an arrival has occurred at the previous transition mark.

$P_{N,0}(t)$ = Probability that at time t the queue length is equal to N, the server is not idle and no arrival has occurred at the previous transition mark.

$P_{N,1}(t)$ = Probability that at time t the queue length is equal to N, the server is not idle and an arrival has occurred at the previous transition mark.

The differential-difference equations of the model are:

$$\frac{d}{dt}Q_{0,0}(t) = -\lambda Q_{0,0}(t) + \mu P_{0,0}(t) + \lambda[p_{00}Q_{0,0} + p_{10}Q_{0,1}]$$

$$\frac{d}{dt}Q_{0,1}(t) = -\lambda Q_{0,1}(t) + \mu P_{0,1}(t)$$

$$\frac{d}{dt}P_{0,0}(t) = -(\lambda+\mu)P_{0,0}(t) + (\mu+\xi)P_{1,0}(t) + \lambda[p_{00}P_{0,0} + p_{10}P_{0,1}]$$

$$\frac{d}{dt}P_{0,1}(t) = -(\lambda+\mu)P_{0,1}(t) + (\mu+\xi)P_{1,1}(t) + \lambda[p_{01}Q_{0,0} + p_{11}Q_{0,1}]$$

$$\frac{d}{dt}P_{n,0}(t) = -(\lambda+\mu+n\xi)P_{n,0}(t) + [\mu+(n+1)\xi]P_{n+1,0}(t) + \lambda[p_{00}P_{n,0}(t) + p_{10}P_{n,1}(t)]$$

$$\frac{d}{dt}P_{n,1}(t) = -(\lambda+\mu+n\xi)P_{n,1}(t) + [\mu+(n+1)\xi]P_{n+1,1}(t) + \lambda[p_{01}P_{n-1,0}(t) + p_{11}P_{n-1,1}(t)]$$

$$\frac{d}{dt}P_{N,0}(t) = -(\mu+N\xi)P_{N,0}(t) + \lambda[p_{00}P_{N,0}(t) + p_{10}P_{N,1}(t)]$$

$$\frac{d}{dt}P_{N,1}(t) = -(\mu+N\xi)P_{N,1}(t) + \lambda[p_{01}P_{N-1,0}(t) + p_{11}P_{N-1,1}(t)]$$

4 Transient Analysis of the Model

In this section, the transient analysis of the model is carried out. Runge -Kutta method of fourth order is used o obtain the solution. The $''ode45''$ function of MATLAB software is used to find the transient numerical results corresponding to the differential-difference equation of the model.

Here we take $N = 6, \lambda = 1.8, \mu = 2.5, \xi = 0.15, p_{00} = 0.2, p_{01} = 0.8, p_{10} = 0.3$ and $p_{11} = 0.7$. In Fig. 1, we plot the system size probabilities with time. We observed that initially $P_{0,0}$ is higher and with the passage of time it decreases becomes steady. The probabilities of the system have lower values initially but they increase gradually and after sometime these become steady.

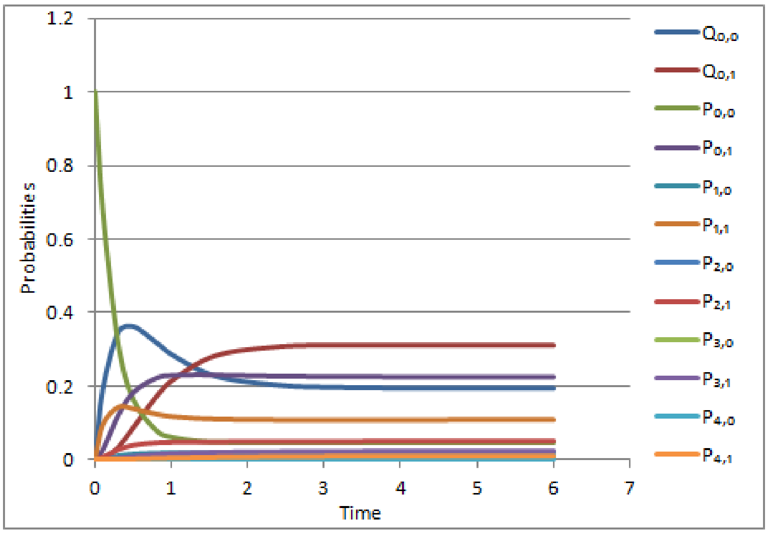

Figure 1: Time dependent behavior of probabilities.

In Fig. 2, we show a graph between expected system size and time. Further we consider two queuing models: one with correlated arrivals and reneging and the other with

Poisson arrivals and reneging. It can be seen from the graph that the expected system size is relatively lower in case of correlated queuing model than the simple model.

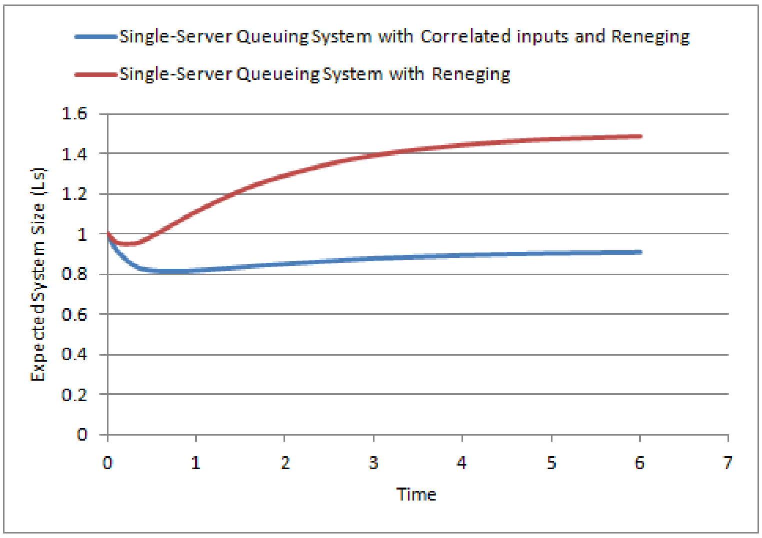

Figure 2: Expected system size vs time

In Fig. 3, the variation in expected waiting time with time is shown. We can see that the expected waiting time of customers is lower in case of correlated queuing system then the system with simple poisson arrivals. This sort of comparison indicates that the correlated input queuing system performs better than the one without correlated arrivals.

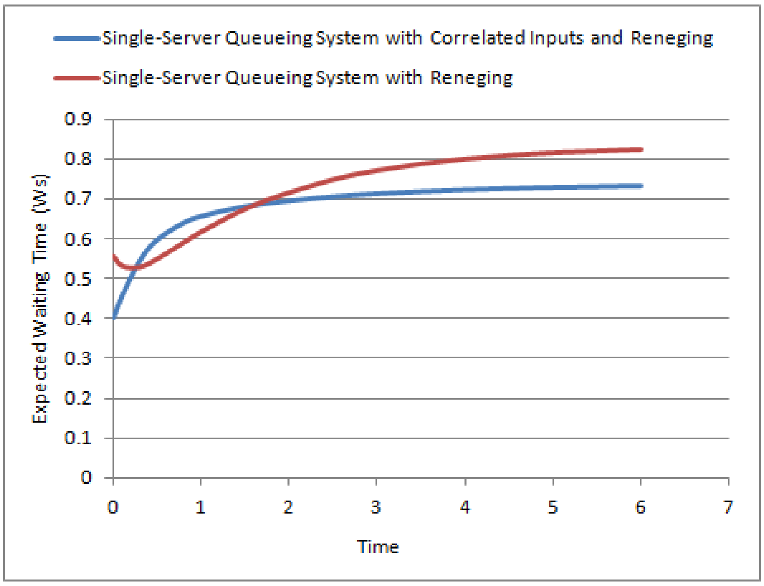

Figure 3: Expected waiting time vs time

5 Conclusion

In this paper we have performed the transient numerical analysis of a single server queuing model with correlated inputs and reneging. We have compared our model with a single server queuing model with reneging and have observed that our model performs better than the other.

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