Defuzzification of Healthcare Related Problem by Using (λ, ρ) Intervalued Trapezoidal Fuzzy Numbers and Their Functional

Dr. Rajesh Dangwal

•

Professor, B.G.R. Campus H.N.B. Garhwal
University, Srinagar, Uttarakhand,
E-Mail: dangwalrajeshhnbgu@gmail.com

Kapil Naithani

•

Research Scholar, B.G.R. Campus
H.N.B. Garhwal University, Srinagar, Uttarakhand,
E-Mail: kapilpintu88@gmail.com

Abstract

In healthcare related problems there exist some medical error due to involvement of human errors and technologies error. In general the available data is not sufficient to assess the clinical process. This study uses level (λ, ρ) intervalued trapezoidal fuzzy numbers and their functional to evaluate the estimate reliability of system in the fuzzy sense, In this paper we uses fault tree diagram of the mixed system (series and parallel system) and we change their crisp probability to trapezoidal fuzzy number and a new approach of functional of fuzzy number has developed and Computed results have been compared with results obtained from other existing techniques.

Key Words: Healthcare, Fuzzy sets, inter-valued trapezoidal fuzzy numbers, Functional of fuzzy numbers, Fault tree analysis, Defuzzification.

I. Introduction

In the traditional set theory, an element either belong to the set or not, that is the answer become yes or no rather than more or less. Fuzzy set theory provide means of uncertainty. Probability theory also a primary tool for representing uncertainty but they are random. And all the uncertainty are not random. Fuzzy set theory is marvellous tool for modelling such kind of uncertainty associated with vagueness which are not random.

Healthcare is a series of process for a patient to receive medication..A Joint Health Commission report indicates that medical errors result in the death of between 44,000 and 98,000 patients every year and concludes that healthcare is a high risk [5] There are several examples where reliability analysis methods such as root cause analysis (RCA), failure mode and effect analysis (FMEA), fault tree analysis (FTA) and event tree analysis (ETA) have been applied for patient safety risk modelling in healthcare [6,7–10]. Fault tree analysis has been extensively used as a powerful technique in health related risk analysis from both qualitative and quantitative perspectives [8–10]. . Some of the suggested healthcare areas where FTA can be used are equipment failures and malfunctions, material faults, human errors, environment-related risks, management deficiencies, communication and measurement errors, etc

II. Fuzzy Sets

In real world there exist fuzzyness i.e vagueness , uncertainty. Fuzzy sets were introduced independently by Lotfi A. Jadeh and Dieter Klaua in 1965 as an extension of classical notion of a set .In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent conditions that is an element either belong to set or not, fuzzy set theory permits the gradual assessment of the membership of elements in a set this is described with the aid of a membership function valued belong to the unit interval [0,1]. Fuzzy sets generalize classical sets, since the indicator functions of classical sets are special case of membership functions of fuzzy sets, if the latter only take values 0 or 1. Classical bivalent sets are usually called crisp sets it is used in a wide range of domains in which information is incomplete or imprecise

A fuzzy set is defined by a membership function from the universal set to the interval [0,1], as given below;

$$\mu_A(x) : X \to [0,1] \tag{1}$$

where $\mu_A(x)$ gives the degree of belongingness of x in the set A. A fuzzy set A can be expressed as follows:

$$\tilde{A} = \{(x, \mu_A(x)) : x \in X\} \tag{2}$$

Fuzziness can be found in many areas of daily life such as in engineering, medicine, manufacturing and others. In all areas in which human judgements, evaluation and decision are important. These are the areas of decision making reasoning, learning and so on.

III. Fuzzification and Inter-Valued Trapezoidal Fuzzy Number

In various situations, the exact values of any parameters of a system are not known due to unavailability of data and complete knowledge about the system, and thus the uncertainty arise. In order to quantify uncertainty, Fuzzification of parameter's value or collected data are done by system experts. In the process of fuzzification, crisp value is transformed into fuzzy value with the help of fuzzy membership functions. For analysing safety and healthcare related problems, inter-valued trapezoidal fuzzy membership functions or more simply inter-valued trapezoidal fuzzy number (IVTPFNs) are often utilized to provide more precise descriptions and to obtain more accurate solutions [27]. In this paper, IVTPFNs are used for quantifying data uncertainty associated with basic events. Mathematically, An interval-valued fuzzy set $\tilde{A}(i-v\ fuzzzyset)$ on R is derived by $\tilde{A} \equiv \{(x, [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^u}(x)]) / x \in R\}$ $0 \le \mu_{\tilde{A}^L}(x) \le \mu_{\tilde{A}^u}(x) \le 1 \, \forall \, x \in R$, It is denoted by $\mu_{\tilde{A}}(x) = \lfloor \mu_{\tilde{A}^L}(x)\, \mu_{\tilde{A}^U}(x), \rfloor\ x \in R$ or $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]$

The i-v fuzzy set $\tilde{A}$ indicates that, when the membership grade of x belongs to the interval $\lfloor \mu_{\tilde{A}^L}(x)\, \mu_{\tilde{A}^U}(x), \rfloor$ the largest grade is $\mu_{\tilde{A}^U}(x)$ and the smallest grade is $\mu_{\tilde{A}^L}(x)$

$$\mu_{\tilde{A}^L}(x) = \begin{cases} \dfrac{\lambda(x-a)}{b-a} & a \le x \le b \\ \lambda & b \le x \le c \\ \dfrac{\lambda(d-x)}{d-c} & c \le x \le d \\ 0 & otherwise \end{cases} \tag{3}$$

Therefore, $\tilde{A}^L = (a, b, c, d, \lambda)\ a < b < c < d$

Let

$$\mu_{\tilde{A}^U}(x) = \begin{cases} \dfrac{\rho(x-e)}{b-e} & e \le x \le b \\ \rho & b \le x \le c \\ \dfrac{\rho(f-x)}{(f-c)} & c \le x \le f \\ 0 & otherwise \end{cases} \quad (4)$$

Therefore $\tilde{A}^U = (e, b, c, f, \rho), e < b < c < f$, Consider the case in which $0 < \lambda \le \rho \le 1$ and $e < a < b < c < d < f$, from (3) and (4) we obtain $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]$ $[(a, b, c, d; \lambda), (e, b, c, f; \rho)]$, Which is called the level (λ, ρ) $i - v$ fuzzy number.

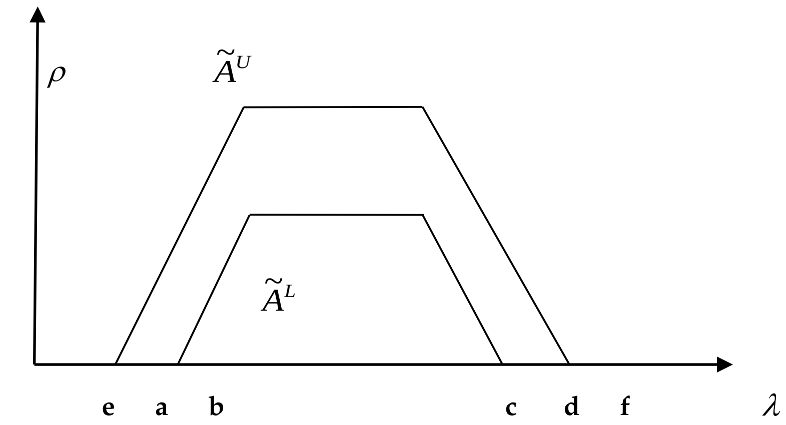

Fig1: i-v trapezoidal fuzzy number $\tilde{A}$

IV. Fault Tree Analysis and Fuzzy probability

Fault-tree analysis (FTA) is a top-down. Deductive failure analysis in which an undesired state of a system is analyzed using Boolean logic to combine a series of lower level basic events it is to evaluate probability of an accident resulting from sequences and combinations of faults and failure events. A fault tree describes an accident-model and interprets the relations between components.

Thus, the fault tree is useful for understanding logically the mode of occurrence of an accident. Furthermore, given the failure probabilities of system components, the probability of the top event can be calculated. In fault tree diagram there are two gates are used one is “AND” and another is “OR” AND (conjunction) means the failure probability is depend on all those event which are associated with AND gate and in OR gate the event associated with OR gate they are work independently to failure next event.

The fuzzy failure probability can be calculated by following arithmetic operation on fuzzy numbers, here when we take probability in fuzzy sense the FTA is called FFTA(fuzzy fault tree analysis)

Table1:

Row. No.	Approach	Gate	Operation	Equation
(1)	**Traditional**	**OR**	**Conjunction**	**P**OR $=1-[(1-q_1)\times(1-q_2)\times\ldots\times(1-q_n)]$
	FTA	**AND**	**Intersection**	**P**AND $= q_1 \times q_2 \times \ldots. \times q_n$
(2)	**Traditional**	**OR**	**Conjunction**	**P**OR $= \tilde{1}\Theta[(\tilde{1}\Theta\tilde{q}_1)\otimes(\tilde{1}\Theta\tilde{q}_2)\otimes\ldots.\otimes(\tilde{1}\Theta\tilde{q}_n)]$
	FFTA	**AND**	**Intersection**	**P**AND $= \tilde{q}_1 \otimes \tilde{q}_2 \otimes \ldots\ldots \otimes \tilde{q}_n$

V. Defuzzification

Defuzzification is the process of producing a quantifiable result in crisp logic. Given fuzzy sets and corresponding memberships degree it is the process to transformed fuzzy numbers to crisp value there are many rule to transform a number of variable into a fuzzy set and then defuzzified. Defuzzification is interpreting the membership degrees of the fuzzy sets into a specific decision or real value the simplest method to defuzzification is choose to set of highest membership function another a common and useful defuzzification technique is centre of gravity. For the interval valued trapezoidal fuzzy number we will take the mean of the COG of upper and lower trapezoidal fuzzy numbers. Also we use the signed distance method and bisector of area method to evaluate the defuzzified the fuzzy numbers.

VI. Steps of the methodology

Step1. Construction of fault-tree

Construct fault-tree for some healthcare related problems (e.g. patient transfer without infection control, etc.) by using fault-tree logical symbols.

Step2. Obtain fundamental events failure probabilities in the form of level $(\boldsymbol{\lambda}, \boldsymbol{\rho})$ interval-valued trapezoidal fuzzy numbers

Possible failure probability of each fundamental event is obtained by aggregating experts knowledge and experience, and represented in terms of level (λ, ρ) interval-valued trapezoidal fuzzy numbers.

Step3. Computation of system top event fuzzy failure probability $(\tilde{\mathbf{q}}_{\mathbf{T}})$

Using fault-tree diagram and possible failure of fundamental events represented in terms (λ, ρ) interval-valued trapezoidal fuzzy numbers, the system top event fuzzy failure probability $(\tilde{q}_T)$ can be computed by using operations given in table 4. Also, the defuzzified value of system top event can be easily computed using various defuzzification methods as COG, bisector of area, middle of maxima.

Step4.Compute system top event fuzzy reliability

Compute system top event fuzzy reliability which is equal to one minus the fuzzy failure probability of the top event.

Step5. Find the most and least influential fundamental events of the problem.

Tanaka et. al. V −index will be extended for level (λ, ρ) interval-valued trapezoidal fuzzy numbers, the most and least influential fundamental events of the considered problems will be evaluated by finding $\max\{V(\tilde{q}_T, \tilde{q}_{T_i}) \forall\, i\}$ and $\min\{V(\tilde{q}_T, \tilde{q}_{T_i}) \forall\, i\}$ values respectively for the system, where $\tilde{q}_{T_i}$ is the system top event fuzzy failure probability after eliminated i^{th} basic event.

Step6. Analyze the results and give suggestions based on it for improving the efficiency of considered healthcare related problems.

Index V, measure the difference between $\tilde{q}_T$ and $\tilde{q}_{T_i}$, and defined as

$$V(\tilde{q}_T, \tilde{q}_{T_i}) = \begin{matrix} (\tilde{q}^L{}_{T_1} - \tilde{q}^L{}_{Ti_1}) + (\tilde{q}^L{}_{T_2} - \tilde{q}^L{}_{Ti_2}) + (\tilde{q}^L{}_{T_3} - \tilde{q}^L{}_{Ti_3}) + (\tilde{q}^L{}_{T_4} - \tilde{q}^L{}_{Ti_4}) + \\ (\tilde{q}^U{}_{T_5} - \tilde{q}^U{}_{Ti_5}) + (\tilde{q}^U{}_{T_2} - \tilde{q}^U{}_{Ti_2}) + (\tilde{q}^U{}_{T_3} - \tilde{q}^U{}_{Ti_3}) + (\tilde{q}^U{}_{T_6} - \tilde{q}^U{}_{Ti_6}) \end{matrix} > 0 \qquad (5)$$

$V(\tilde{q}_T, \tilde{q}_{T_i})$ indicates the extent of improvement in eliminating the failure of the ith component.

If $V(\tilde{q}_T, \tilde{q}_{T_i}) \geq V(\tilde{q}_T, \tilde{q}_{T_j})$ then preventing failure of i-th component is more effective than j-th component

VII. Example

Fault tree analysis of patient transfer without infection control precaution here is an example of study extent and execution of redundant process during in patient transfer to radiology, and their impact on errors during the transfer process; in which there are given some basic events which are fundamental event to reliable whole system. There were four ways to communicate the required infection control precautions to a porter: (1) verbal handover at Radiology; (2) written handover at Radiology using a transfer form; (3) verbal handover at the ward during patient collection; and (4) verification of the transfer form by the ward nurse fig 2 depicts a fault tree for the events leading to inadequate infections control precautions during transfers. The basic events are given as follow.

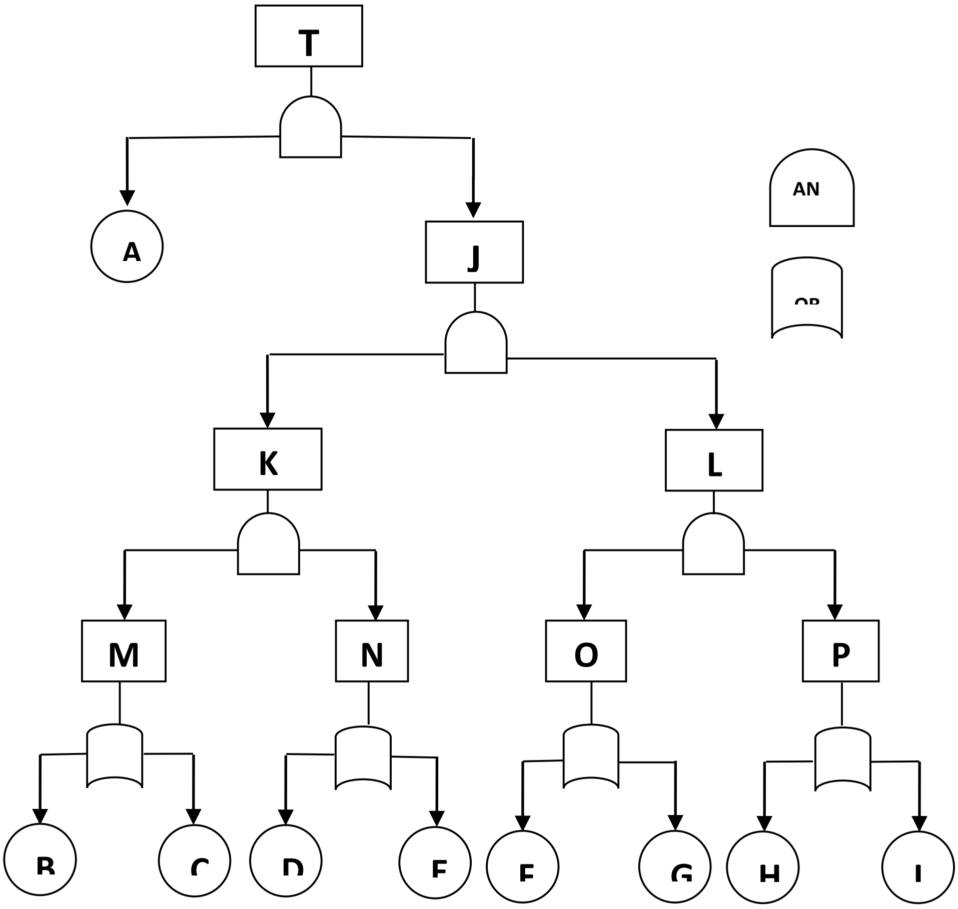

Fig 2: Fault tree of patient transfer without infection control precaution

T=Inadequate infections control precautions

A= Patient was infectious

B= patient infectious status was not communicated through verbal handover at radiology

C= Patient infectious status was communicated through verbal handover at radiology, but error still occurred

D= Patient infectious status was not communicated through written handover at radiology

F= Patient infectious status was not communicated through verbal handover at ward

G= Patient infectious status was not communicated through verbal handover at ward, but error still occurred

H= Patient infectious status was not communicated through written handover at ward.

I= Patient infectious status was not communicated through written handover at ward, but error still occurred

The fault tree diagram of "**Patient transfer without infection control precaution**" is follow, here the Shape show "AND" and for "OR" gate

We express and gate by Intersection $\cap$ and OR gate by conjunction (union) $\cup$. The fault tree can be expressed as

$$
\begin{aligned}
T &= A \cap J \\
&= A \cap (k \cap L) \\
&= A \cap [(M \cap N) \cap (O \cap P)] \\
&= A \cap [(B \cup C) \cap (D \cup E) \cap (F \cup G) \cap (H \cup I)]
\end{aligned} \tag{6}
$$

The mathematical formula of failure probability o top event T is given by

$$
\begin{aligned}
q_{\tilde{T}} &= q_{\tilde{A}} \times q_{\tilde{J}} \\
&= q_{\tilde{A}} \times q_{\tilde{K}} \times q_{\tilde{L}} \\
&= q_{\tilde{A}} \times [(q_{\tilde{M}} \times q_{\tilde{N}}) \times (q_{\tilde{O}} \times q_{\tilde{P}})] \\
&= q_{\tilde{A}} \times [\{1-(1-q_{\tilde{B}})(1-q_{\tilde{C}})\} \times \{1-(1-q_{\tilde{D}})(1-q_{\tilde{E}})\} \times \{1-(1-q_{\tilde{F}})(1-q_{\tilde{G}})\} \times \{1-(1-q_{\tilde{H}})(1-q_{\tilde{I}})\}]
\end{aligned} \tag{7}
$$

Using this formula the failure probability $q_{\tilde{T}}$ by various method can be calculated and the reliability of system calculated by $1-q_{\tilde{T}}$.

TABLE 2: Crisp values and their corresponding trapezoidal fuzzy numbers

S.No.	Failure Probability	Crisp value	IVTPFN
1	$q_{\tilde{A}}$	0.27	$\begin{pmatrix} 0.23, 0.25, 0.29, 0.31 : 0.8 \\ 0.21, 0.25, 0.29, 0.33 : 1.0 \end{pmatrix}$
2	$q_{\tilde{B}}$	0.93	$\begin{pmatrix} 0.89, 0.91, 0.95, 0.97 : 0.8 \\ 0.87, 0.91, 0.95. 0.99 : 1.0 \end{pmatrix}$
3	$q_{\tilde{C}}$	0	$\begin{pmatrix} 0.00, 0.00, 0.00, 0.00 : 0.8 \\ 0.00, 0.00, 0.00, 0.00 : 1.0 \end{pmatrix}$
4	$q_{\tilde{D}}$	0.52	$\begin{pmatrix} 0.48, 0.50, 0.54, 0.56 : 0.8 \\ 0.46, 0.50, 0.54, 0.58 : 1.0 \end{pmatrix}$
5	$q_{\tilde{E}}$	0.22	$\begin{pmatrix} 0.18, 0.20, 0.24. 0.26 : 0.8 \\ 0.16, 0.20, 0.24, 0.28 : 1.0 \end{pmatrix}$
6	$q_{\tilde{F}}$	0.74	$\begin{pmatrix} 0.70, 0.72, 0.76, 0.78 : 0.8 \\ 0.68, 0.72, 0.76, 0.80 : 1.0 \end{pmatrix}$

S.No.	Failure Probability	Crisp value	IVTPFN
7	$q_{\tilde{G}}$	0	$\begin{pmatrix} 0.00, 0.00, 0.00, 0.00 : 0.8 \\ 0.00, 0.00, 0.00, 0.00 : 1.0 \end{pmatrix}$
8	$q_{\tilde{H}}$	0.56	$\begin{pmatrix} 0.52, 0.54, 0.58, 0.60 : 0.8 \\ 0.50, 0.54, 0.58, 0.62 : 1.0 \end{pmatrix}$
9	$q_{\tilde{I}}$	0.30	$\begin{pmatrix} 0.26, 0.28, 0.32, 0.34 : 0.8 \\ 0.24, 0.28, 0.32, 0.36 : 1.0 \end{pmatrix}$

TABLE 3: Fuzzy operation of two intwervalued fuzzy numbers

OPERATION	TRAPEZOIDAL FUZZY INTERVALUED NUMBERS
MULTIPPLICATION	$\begin{pmatrix} a_1, b_1, c_1, d_1 : \lambda \\ e_1, b\ , c\ , f_1 : \rho \end{pmatrix} \times \begin{pmatrix} a_2, b_2, c_2, d_2 : \lambda \\ e_2, b_2, c_2, f_2 : \rho \end{pmatrix} = \begin{pmatrix} a_{1.} a_2, b_1 b_2, c_1 c_2, d_1 d_2 : \lambda \\ e_1 e_2, b_1 b_2, c_1 c_2, f_1 f_2 : \rho \end{pmatrix}$
COMPLEMENT	$1 - \begin{pmatrix} a, b, c, d : \lambda \\ e, b, c, f : \rho \end{pmatrix} = \begin{pmatrix} 1-d, 1-c, 1-b, 1-a : \lambda \\ 1-f, 1-c, 1-b, 1-a : \rho \end{pmatrix}$

VIII. Failure probability by various method

VIII(I). Traditional Method: The fuzzy failure probability of top event by traditional method taking crisp value of failure probability is given by **0.080441705** and reliability is **0.919558295**

VIII(II). Max Min Method:-It is for crisp value of failure probability, it is introduced by huang.et.al.,in this method we take maximum for union and minimum for intersection. In given example the fuzzy failure probability can be calculated as follow from equation (5)

$$q_M = \max(q_B, q_C) = \max(0.93, 0.00) = 0.93$$
$$q_N = \max(q_D, q_E) = \max(0.52, 0.22) = 0.52$$
$$q_O = \max(q_F, q_G) = \max(0.74, 0.00) = 0.74$$
$$q_P = \max(q_H, q_I) = \max(0.56, 0.30) = 0.56$$
$$q_K = \min(q_M, q_N) = \min(0.93, 0.52) = 0.52$$
$$q_L = \min(q_O, q_P) = \min(0.74, 0.56) = 0.56$$
$$q_J = \min(q_K, q_L) = \min(0.52, 0.56) = 0.52$$
$$q_T = \min(q_A, q_J) = \min(0.27, 0.52) = 0.27$$

That is fuzzy failure probability of top event is 0.27 and fuzzy reliability of top event is 0.73

VIII(III). TANAKA ET AL METHOD: The fuzzy failure probability of top event of fault tree by tanaka. et. al method from table 2 and is given by $\begin{pmatrix} 0.0527192568, 0.0668251584, 0.0986636751, 0.1179379848 : 0.8 \\ 0.0418668562, 0.0668251584, 0.0986636751, 0.1396448571 : 1.0 \end{pmatrix}$ and the fuzzy reliability of top event is $\begin{pmatrix} 0.8820620152, 0.9013363249, 0.9331748416, 0.9472807432 : 0.8 \\ 0.8603551429, 0.9013363249, 0.9331748416, 0.9581331438 : 1.0 \end{pmatrix}$

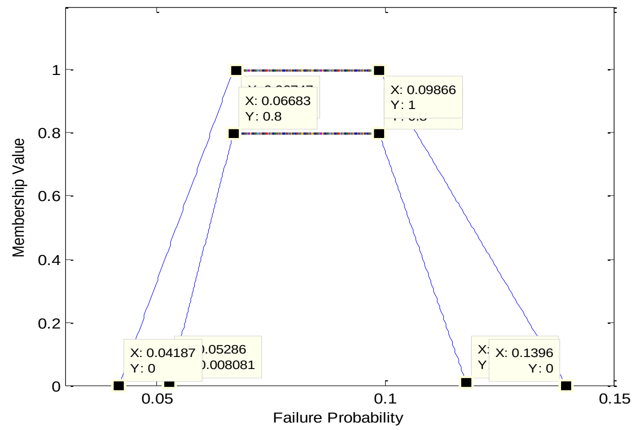

Fig3: Failure Probability Of Top Event In Trapezoidal Fuzzy Number

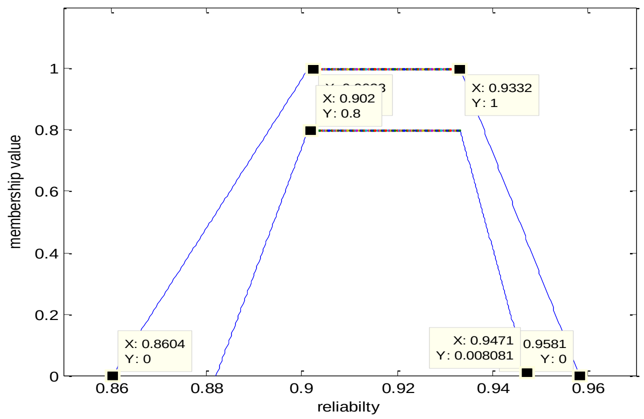

Fig4: Fuzzy Reliability Of Top Event In Trapezoidal Fuzzy Numbers

VIV. DEFUZZIFICATION METHODS

Defuzzificztion is a process when we transformed fuzzy numbers to discrete value which is most probable value among all the probabilities there are some important defuzzification methods, which are following.

VIV(I) . MEAN OF COG METHOD

COG method is one of the most defuzzification method. In this method we take mean value of lower and upper trapezoidal fuzzy numbers, the upper left and right and similarly lower left and right membership function are

$$\mu_{A_L^U}(x) = \rho\left(\frac{x-e}{b-e}\right) \qquad \mu_{A_R^U}(x) = \rho\left(\frac{x-f}{c-f}\right)$$

$$\mu_{A_L^L}(x) = \lambda\left(\frac{x-a}{b-a}\right) \qquad \mu_{A_R^L}(x) = \lambda\left(\frac{x-d}{c-d}\right)$$

Now centre of gravity is given by $\bar{x} = \dfrac{\int x.\mu_A(x)dx}{\int \mu_A(x)dx}$

$$\bar{x}^U = \frac{\int x^U.\mu_{A^U}(x)dx}{\int \mu_{A^U}(x)dx} \qquad \bar{x}^L = \frac{\int x^L.\mu_{A^L}(x)dx}{\int \mu_{A^L}(x)dx}$$

MEAN OF COG = $\dfrac{1}{2}\left(\bar{x}^U + \bar{x}^L\right)$

$$x = \frac{1}{2}\left[\left\{\frac{\int_e^b x.\rho\left(\frac{x-e}{b-e}\right)dx + \int_b^c x\rho dx + \int_c^f x.\rho\left(\frac{x-f}{c-f}\right)dx}{\int_e^b \rho\left(\frac{x-e}{b-e}\right)dx + \int_b^c \rho dx + \int_c^f \rho\left(\frac{x-f}{c-f}\right)dx}\right\} + \left\{\frac{\int_a^b x.\rho\left(\frac{x-a}{b-a}\right)dx + \int_b^c x\lambda dx + \int_c^d x.\lambda\left(\frac{x-d}{c-d}\right)dx}{\int_e^b \rho\left(\frac{x-a}{b-a}\right)dx + \int_b^c \lambda dx + \int_c^d \lambda\left(\frac{x-d}{c-d}\right)dx}\right\}\right]$$

(8)

$$x = \frac{1}{6}\left[\left\{\frac{c^2 + f^2 - b^2 - e^2 - fc - eb}{c + f - e - b}\right\} + \left\{\frac{c^2 + d^2 - b^2 - a^2 + cd - ab}{c + d - a - b}\right\}\right] \tag{9}$$

In this method the fuzzy failure probability obtained by tanaka. et. al. of top event can be calculated and is equal to 0.0504603867 and fuzzy reliability of top event is **0.949539613**

VIV(II). BISECTOR OF AREA METHOD:

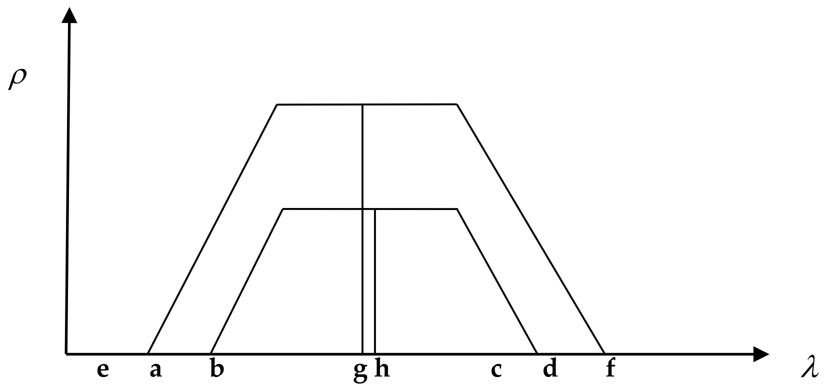

FIG 5: Bisector Of Area

in this method we evaluate the point where the area are bisected let 'g" and "h" be the point where the upper and lower trapezoidal region is bisected for upper trapezoidal area

$$\frac{1}{2}[(g-b)+(g-e)]\times\rho = \frac{1}{2}[(f-g)+(c-g)]\times\rho$$
$$2g-(b+e)=(f+c)-2g$$
$$4g=(f+c)+(b+e)$$
$$g=\frac{1}{4}(f+c+b+e) \tag{10}$$

Similarly for lower trapezoidal area $h=\dfrac{1}{4}(d+c+b+a)$ (11)

Therefore mean of the bisector of upper and lower trapezoidal fuzzy number will be given by

$$\frac{1}{2}(g+h)=\frac{1}{8}(a+e+d+f+2b+2c) \tag{12}$$

In this method the fuzzy failure top event is = 0.0853933277 and the fuzzy reliability of top event is **0.914606672**

VIV(III). MIDDLE OF MAXIMA: –

In this method we get the value of x which is middle of maximum membership function which is given by $x_{MOM} = \frac{1}{2}(b+c)$ (13)

By this method the fuzzy failure probability of top event is =0.0827444168 and the fuzzy reliability is =**0.917255583**

VIV(IV) FUNCTIONAL OF FUZZY NUMBERS:

It is defined as a function of function of x that is if we give the membership value to membership function .it is defined as follow

$\mu_A(x): X \to [0,1] \quad \mu(\mu_A(x)): \mu_A(x) \to [0,1]$

$\tilde{A} = \{((x, \mu_A(x)), \mu(\mu_A(x))) : x \in X\}$

Let $\mu(\mu_A(x)) = \frac{1}{x+1}$ and $0 \le \frac{1}{x+1} \le 1$ for any non negative value of x then the fuctional form of fuzzy number as following

TABLE 4

S.No.	Failure Probabiliy	Crisp value	Functional value of IVTPFN $((x, \mu_A(x)), \mu(\mu_A(x)))$
1	$q_{\tilde{A}}$	0.27	$\begin{pmatrix} 0.23:0.8:0.813,0.25:0.8:0.8,0.29:0.8:0.775,0.31:0.8:0.763 \\ 0.21:1.0:0.826,0.25:1.0:0.8,0.29:1.0:0.775,0.33:1.0:0.751 \end{pmatrix}$
2	$q_{\tilde{B}}$	0.93	$\begin{pmatrix} 0.89:0.8:0.529,0.91:0.8:0.523,0.95:0.8:0.512,0.97:0.8.0.507 \\ 0.87:1.0:0.534,0.91:1.0:0.523,0.95:1.0:0.512,0.99:1.0:0.502 \end{pmatrix}$
3	$q_{\tilde{C}}$	0	$\begin{pmatrix} 0.00:0.8:1,0.00:0.8:1,0.00:0.8:1,0.00:0.8:1 \\ 0.00:1.0:1,0.00:1.0:1,0.00:1.0:1,0.00:1.0:1 \end{pmatrix}$
4	$q_{\tilde{D}}$	0.52	$\begin{pmatrix} 0.48:0.8:0.675,0.50:0.8:0.666,0.54:0.8:0.649,0.56:0.8:0.641 \\ 0.46:1.0:0.684,0.50:1.0:0.666,0.54:1.0:0.649,0.58:1.0:0.632 \end{pmatrix}$
5	$q_{\tilde{E}}$	0.22	$\begin{pmatrix} 0.18:0.8:0.847,0.20:0.8:0.833,0.24:0.8:0.806.0.26:0.8:0.793 \\ 0.16:1.0:0.862,0.20:1.0:0.833,0.24:1.0:0.806,0.28:1.0:0.781 \end{pmatrix}$
6	$q_{\tilde{F}}$	0.74	$\begin{pmatrix} 0.70:0.8:0.588,0.72:0.8:0.581,0.76:0.8:0.568,0.78:0.8:0.561 \\ 0.68:1.0:0.595,0.72:1.0:0.581,0.76:1.0:0.568,0.80:1.0:0.555 \end{pmatrix}$
7	$q_{\tilde{G}}$	0	$\begin{pmatrix} 0.00:0.8:1,0.00:0.8:1,0.00:0.8:1,0.00:0.8:1 \\ 0.00:1.0:1,0.00:1.0:1,0.00:1.0:1,0.00:1.0:1 \end{pmatrix}$
8	$q_{\tilde{H}}$	0.56	$\begin{pmatrix} 0.52:0.8:0.657,0.54:0.8:0.649,0.58:0.8:0.632,0.60:0.8:0.625 \\ 0.50:1.0:0.666,0.54:1.0:0.649,0.58:1.0:0.632,0.62:1.0:0.612 \end{pmatrix}$
9	$q_{\tilde{I}}$	0.30	$\begin{pmatrix} 0.26:0.8:0.793,0.28:0.8:0.781,0.32:0.8:0.757,0.34:0.8:0.746 \\ 0.24:1.0:0.806,0.28:1.0:0.781,0.32:1.0:0.757,0.36:1.0:0.735 \end{pmatrix}$

The fuzzy functional failure probability of top event is given by from tanaka.et.al. method from table 2 and is given by

$$\begin{pmatrix} 0.0527192568:0.8:0.949,0.0668251584:0.8:0.937,0.0986636751:0.8:0.910,0.1179379848:0.8:0.988 \\ 0.0418668562:1.0:0.959,0.0668251584:1.0:0.937,0.0986636751:1.0:0.910,0.1396448571:1.0:0.877 \end{pmatrix}$$

and reliability of top event given by

$$\begin{pmatrix} 0.8820620152\ 0.8:0.531, 0.9013363249\ 0.8:0.525, 0.9331748416\ 0.8:0.517, 0.9472807432\ 0.8:0.513 \\ 0.8603551429\ 1.0:0.537, 0.9013363249\ 1.0:0.525, 0.9331748416\ 1.0:0.517, 0.9581331438\ 1.0:0.510 \end{pmatrix}$$

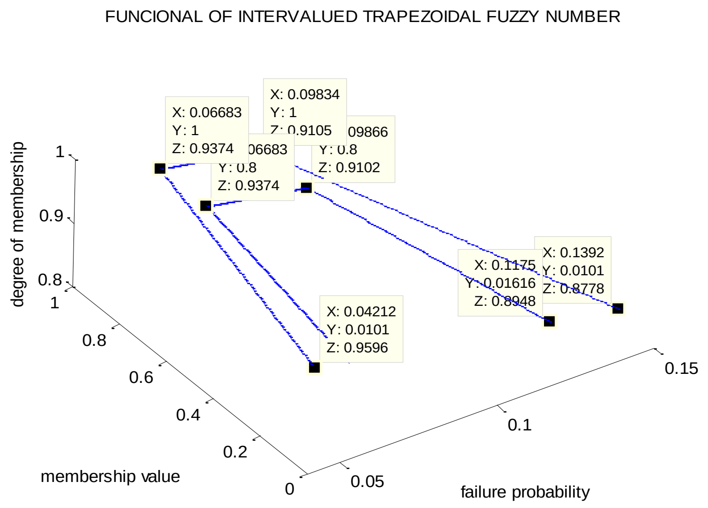

Fig6: Graph Of Functional Value Of Failure Probability F Top Event

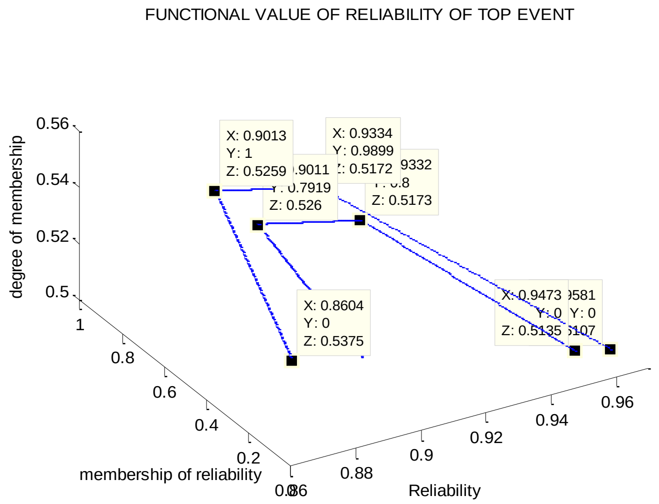

Fig7: Graph Of Functional Value Of Reliabilty Of Top Event

For defuzzification of functional of fuzzy numbers we are using the centre of gravity method(centre of volume)

Now to determine the centre of gravity we can use the formula

$$x^* = \frac{\int x.\mu_A(x).\mu(\mu_A(x))dx}{\int \mu_A(x).\mu(\mu_A(x))dx} \tag{14}$$

We evaluate the defuzzified value of each lower and upper trapezoidal fuzzy numbers and then we take the mean value of both defuzzification value.

$$x^{U^*}=\frac{\int_e^b x.\rho\left(\frac{x-e}{b-e}\right).\frac{1}{x+1}dx+\int_b^c x.\rho.\frac{1}{x+1}dx+\int_c^f x.\rho\left(\frac{f-x}{f-c}\right).\frac{1}{x+1}dx}{\int_e^b \rho\left(\frac{x-e}{b-e}\right).\frac{1}{x+1}dx+\int_b^c \rho.\frac{1}{x+1}dx+\int_c^f \rho\left(\frac{f-x}{f-c}\right).\frac{1}{x+1}dx}$$

$$x^{L^*}=\frac{\int_a^b x.\lambda\left(\frac{x-a}{b-a}\right).\frac{1}{x+1}dx+\int_b^c x.\lambda.\frac{1}{x+1}dx+\int_c^d x.\lambda\left(\frac{d-x}{d-c}\right).\frac{1}{x+1}dx}{\int_a^b \lambda\left(\frac{x-a}{b-a}\right).\frac{1}{x+1}dx+\int_b^c \lambda.\frac{1}{x+1}dx+\int_c^d \lambda\left(\frac{d-x}{d-c}\right).\frac{1}{x+1}dx}$$

$$x^*=\frac{1}{2}\left[x^{U^*}+x^{L^*}\right]$$

$$x^{U^*}=\frac{\left[\frac{f+c-b-e}{2}+\frac{e+1}{b-e}\ln\left(\frac{b+1}{e+1}\right)-\ln\left(\frac{c+1}{b+1}\right)-\frac{f+1}{f-c}\ln\left(\frac{f+1}{c+1}\right)\right]}{\left[\frac{f+1}{f-c}\ln\left(\frac{f+1}{c+1}\right)-\frac{e+1}{b-e}\ln\left(\frac{b+1}{e+1}\right)+\ln\left(\frac{c+1}{b+1}\right)\right]} \tag{15}$$

$$x^{L^*}=\frac{\left[\frac{d+c-b-a}{2}+\frac{a+1}{b-a}\ln\left(\frac{b+1}{a+1}\right)-\ln\left(\frac{c+1}{b+1}\right)-\frac{d+1}{d-c}\ln\left(\frac{d+1}{c+1}\right)\right]}{\left[\frac{d+1}{d-c}\ln\left(\frac{d+1}{c+1}\right)-\frac{a+1}{b-a}\ln\left(\frac{b+1}{a+1}\right)+\ln\left(\frac{c+1}{b+1}\right)\right]} \tag{16}$$

Defuzzified value of failure probability of top event of upper trapezoidal fuzzy number is 0.083528098 and defuzzified value of failure probability of lower trapezoidal fuzzy number is 0.083528098.Similarly defuzzified value of reliabilty of lower trapezoidal fuzzy number is 0.916471902 and defuzzified value of reliability of lower trapezoidal fuzzy number is is 0.912979297 .and the mean value of both values is **0.9147255995**

In traditional method the fuzzy failure probability can be calculated as follow

$$=0.27\times\left[\{1-(1-0.93)(1-0.00)\}\times\{1-(1-0.52)(1-0.22)\}\times\{1-(1-0.74)(1-0.00)\}\times\{1-(1-0.56)(1-0.30)\}\right]$$

=**0.080441705**

And the fuzzy reliability of top event is **0.919558295**

The difference of reliability value from traditional method and the tanaka et al method is 0.004832695 to the left of trapezoidal fuzzy numbers.

Table 5.Ranking of basic event of Example using failure difference

Eliminated event	$\tilde{q}_{Ti_1}$	$V\left(\tilde{q}_{T_1},\tilde{q}_{T_{1i}}\right)$	Rank
$A(i=1)$	$\begin{pmatrix}0.00,0.00,0.00,0.00\\0.00,0.00,0.00,0.00\end{pmatrix}$	**0.6831466219**	**1**
$B(i=2)$	$\begin{pmatrix}0.00,0.00,0.00,0.00\\0.00,0.00,0.00,0.00\end{pmatrix}$	**0.6831466219**	**1**
$C(i=3)$	$\begin{pmatrix}0.0527192568,0.0668251584,0.0986636751,0.1179379848\\0.0418668562,0.0668251584,0.0986636751,0.1396448571\end{pmatrix}$	**0.00**	**6**
$D(i=4)$	$\begin{pmatrix}0.0162490860,0.0219098880,0.0358994573,0.0448827226\\0.0120220692,0.0219098880,0.0358994573,0.0553832294\end{pmatrix}$	**0.4389908241**	**2**

Eliminated event	$\tilde{q}_{Ti_1}$	$V(\tilde{q}_{T_1}, \tilde{q}_{T_{1i}})$	Rank
$E(i=5)$	$\begin{pmatrix} 0.043330896, 0.05477472, 0.0807737789, 0.0966704794 \\ 0.0345634491, 0.05477472, 0.0807737789, 0.1147221028 \end{pmatrix}$	**0.1227626968**	**5**
$F(i=6)$	$\begin{pmatrix} 0.00, 0.00, 0.00, 0.00 \\ 0.00, 0.00, 0.00, 0.00 \end{pmatrix}$	**0.6831466219**	**1**
$G(i=7)$	$\begin{pmatrix} 0.0527192568, 0.0668251584, 0.0986636751, 0.1179379848 \\ 0.0418668562, 0.0668251584, 0.0986636751, 0.1396448571 \end{pmatrix}$	**0.00**	**6**
$H(i=8)$	$\begin{pmatrix} 0.0217571536, 0.02797704, 0.0441942554, 0.054482212 \\ 0.0166138318, 0.02797704, 0.0441942554, 0.0664272576 \end{pmatrix}$	**0.3615252583**	**3**
$I(i=9)$	$\begin{pmatrix} 0.0435143026, 0.05395572, 0.0801020878, 0.0961450963 \\ 0.0346121496, 0.05395572, 0.0801020878, 0.1144024992 \end{pmatrix}$	**0.1263569586**	**4**

Conclusion

It is seen that when the membership function of fuzzy failure probability is also in fuzzy sense than reliability of top event can also go to another value**.** Here the error outcome from trapezoidal fuzzy failure probability to left side, and using equation 5 and table 5, the value of index is analyzed that the most critical basic events are A B and F whereas least critical basic events is C and G. The order of all critical basic events are given below in decreasing manner (A, B, F) > D > H > I > E > C > G.

References

[1] Reason J. Achieving a safe culture: theory and practice. Work Stress 1998; 12:293e306.
[2] Hollnagel E. Barrier and accident prevention. Aldershot, UK: Ashgate, 2004:68e80.
[3] Dzik WH. Emily Cooley lecture 2002: transfusion safety in the hospital. Transfusion 2003; 43:1190e8.
[4] McClelland DBL, Phillips P. Errors in blood transfusion in Britain: Survey of hospital haematology departments. BMJ 1994; 308: 1205e6
[5] The Joint Commission, Improving Patient and Worker Safety: Opportunities for Synergy, Collaboration and Innovation, The Joint Commission, Oakbrook Terrace, IL, November 2012
[6] G. Yucel, S. Cebi, B. Hoege, A.F. Ozok, A fuzzy risk assessment model for hospital information system implementation, Expert Syst. Appl. 39 (2011) 1211–1218.
[7] A.C. Cagliano, S. Grimaldi, C. Rafele, A systemic methodology for risk management in healthcare sector, Saf. Sci. 49 (2011) 695–708.
[8] W.A. Hyman, E. Johnson, Fault tree analysis of clinical alarms, J. Clin. Eng. (April/June) (2008) 85–94.
[9] A. Park, S.J. Lee, Fault tree analysis on handwashing for hygiene management, Food Control 20 (2009) 223–229.
[10] Z.A. Abecassis, L.M. McElroy, R.M. Patel, R. Khorzad, C. Carroll IV, S. Mehrotra, Applying fault tree analysis to the prevention of wrong-site surgery, J. Surg. Res. (2014) 1–7.
[11] D. Raheja, M.C. Escano, Reducing patient healthcare safety risks through fault tree analysis, J. Syst. Saf. (September–October) (2009).
[12] R. Ferdous, F. Khan, B. Veitch, P.R. Amyotte, Methodology for computer aided fuzzy fault tree analysis, Process Saf. Environ. Prot. 87 (2009) 217–226.